In-medium heavy quarkonium from a Bayesian point of view

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References:

Y. Burnier (EPFL), A.R.: **PRL 111 (2013) 18, 182003**

S. Kim (Sejong-U.), P. Petreczky (BNL), A.R.: **arXiv:1409.3630**

Y.Burnier, O.Kaczmarek (Bielefeld-U.), AR.: in preparation

Deutsche Forschungsgemeinschaft

Outline

- **Physics Motivation:** Relativistic heavy-ion collisions and heavy quarkonium
- **Technical progress:** Bayesian spectral function reconstruction in lattice QCD
- **Project I:** The static in-medium heavy quark potential
- **Project II:** Bottomonium spectral functions from lattice NRQCD
- **Conclusion**

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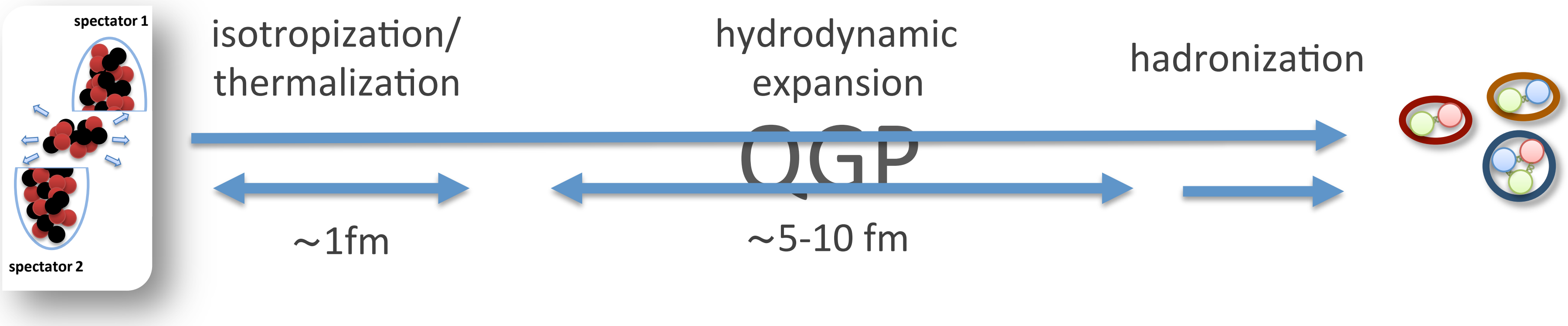
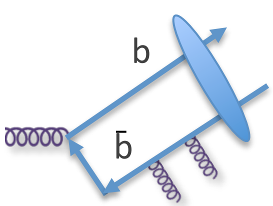
Relativistic Heavy-Ion collisions



- Probes that are susceptible to medium but distinguishable from it: $Q_{probe} >> T_{med}$

Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $m_Q >> T_{med}$



- $b\bar{b}$ produced in the early stages of the collision (M_b=4.65GeV)
- rapid bound state formation expected
- long lifetime due to OZI rule (Γ^{Υ}=54keV)

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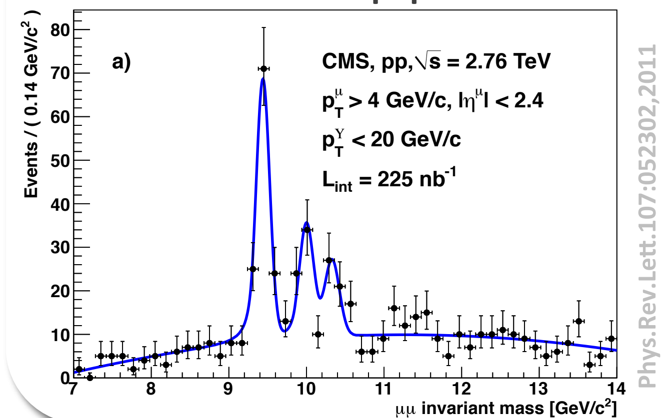
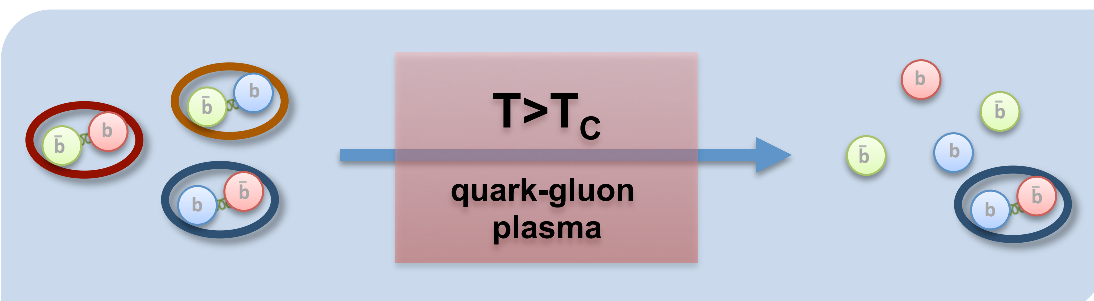
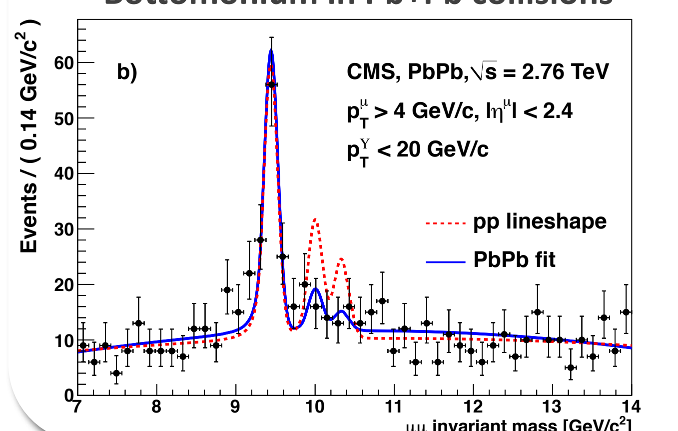
Bottomonium as QGP probe

Bottomonium in p+p collisions





Bottomonium in Pb+Pb collisions



- QGP is strongly interacting at T∼220-305MeV
  see e.g. ALICE NPA 904-905 (2013) and PHENIX PRL 104 (2010) 132301
- **Goal: Non-perturbative understanding of in-medium Bottomonium via lattice QCD**

Two distinct paths, One common challenge

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The T>0 static interquark potential

- Simplification: Infinitely heavy quarks
- Allows a real-time description of the approach towards equilibrium
- Also describes in-medium spectra of thermalized QQbar

in collaboration with Y. Burnier and O. Kaczmarek

In-medium Bottomonium spectra

- Realistic heavy quark masses
- Determination of melting/survival
- Full kinetic equilibration
- Based on the effective field theory NRQCD, also used in T=0 lattice QCD

in collaboration with S. Kim and P. Petreczky

- Lattice simulations in Euclidean time, no direct access to dynamical information
  - Analytic continuation from a finite and noisy dataset necessary: ill-defined problem

    M. Jarrell, J. Gubernatis, , Physics Reports 269 (3) (1996)

➡ Approach via lattice spectral functions: Improve on the Maximum Entropy Method

M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 46, 459 (2001)

Technical Progress

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Bayesian spectral function reconstruction in lattice QCD

Novel Bayesian Spectral Reconstruction

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- Inversion of Laplace transform required to obtain spectra from correlators

$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

- Give meaning to problem by incorporating prior knowledge: Bayesian approach

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

  - Bayes theorem: Regularize the naïve χ^2 functional P[D|ρ] through a prior P[ρ|I]

$$P[\rho|D,I] \propto P[D|\rho]\; P[\rho|I]$$

  - New prior enforces: ρ positive definite, smoothness of ρ, result independent of units

$$P[\rho|I] \propto e^S \qquad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

Y.Burnier, A.R. PRL 111 (2013) 18, 182003

  - **Different from Maximum Entropy Method**: S not entropy, no more flat directions

$$\frac{\delta}{\delta\rho} P[\rho|D,I]\Big|_{\rho=\rho^{BR}} = 0$$

  - No apriori restriction on the search space
  - Convergence to unique global extremum

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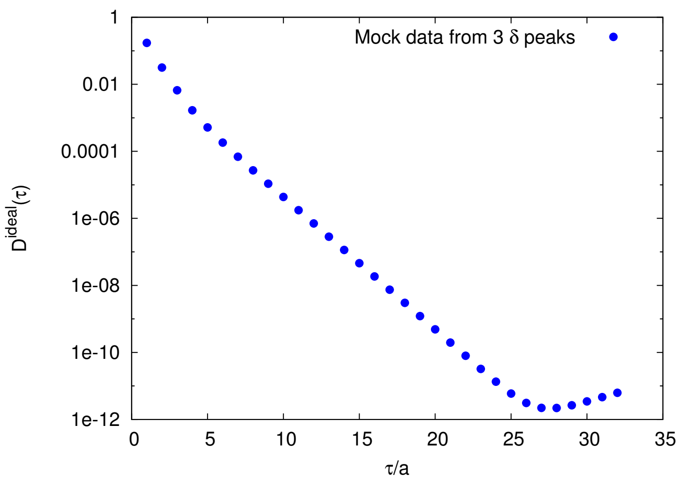
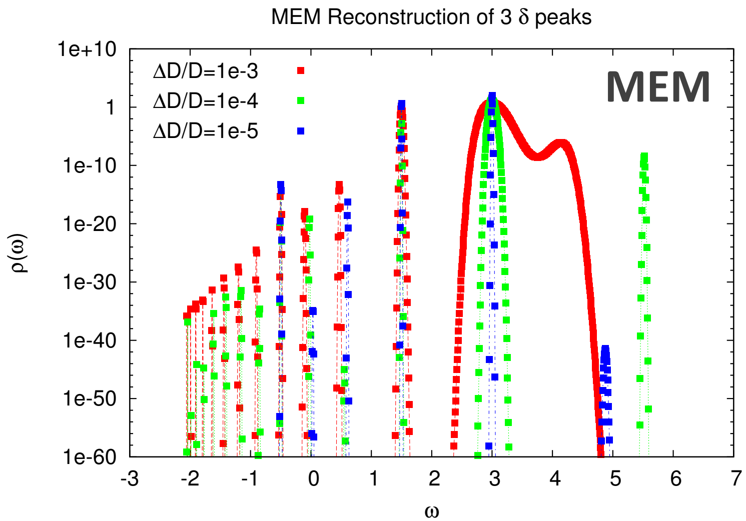
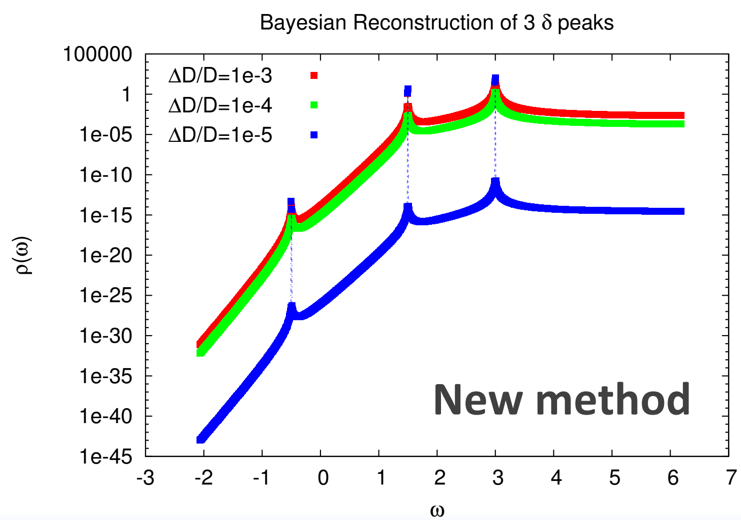
A first mock data test

Mock analysis:
three delta peaks in the spectrum







Project I

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The static in-medium interquark potential

The static inter-quark potential at T>0

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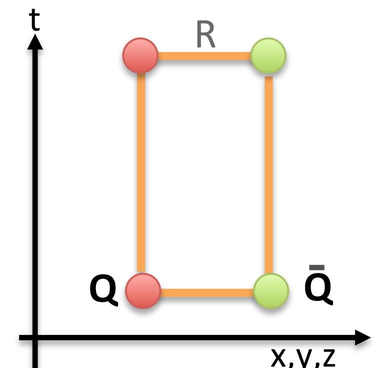
- A lot of intuition has been accumulated over the years:
  - Lattice QCD at T=0: Confining linear rise + string breaking
  - Analogy with an Abelian plasma: Debye screening Matsui & Satz 1986
  - Modeling: Color singlet free energies or internal energies from lattice QCD see e.g. Nadkarni 1986

- For static quarks a clean definition of the potential from QCD is available
  - Use heavy meson operators: $M(x,y,t)= Q(x,t)\, \Gamma\, U(x,y)\, \bar{Q}(y,t)$

$$D^{>}(R,t) = \langle M(x,y,t)\; M^{\dagger}(x,y,0)\rangle_{\rm med}$$

  - In the static limit: $D^{>}$ becomes the **real-time Wilson loop**

$$D^{>}(R,t) \overset{m\to\infty}{=} W_{\square}(R,t) = \langle {\rm Tr}\Big(\exp\Big[-ig \int_{\square} dz^{\mu} A_{\mu}(z) \Big]\Big)\rangle$$



see e.g.: Barchielli et. al. Nucl.Phys. B296 (1988) 625

  - Potential emerges at late times: QQbar timescale much slower than gluons

$$i\partial_t W_{\square}(R,t) \overset{t\to\infty}{=} V^{QCD}(R) W_{\square}(R,t) \quad \Rightarrow \quad V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_{\square}(R,t)}{W_{\square}(R,t)}$$

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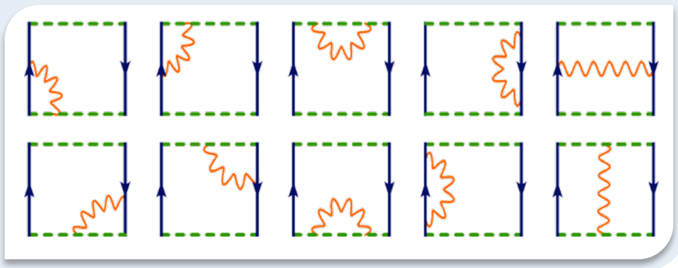
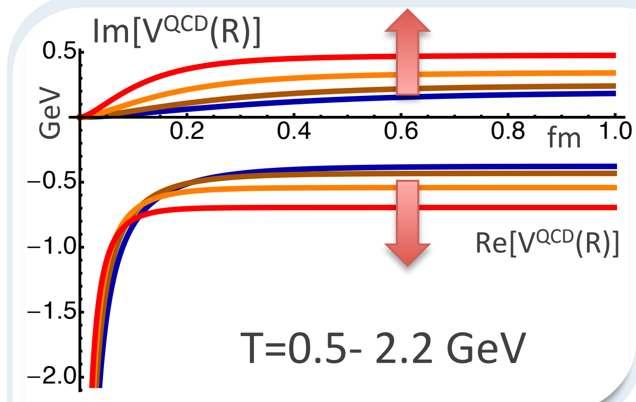
The high temperature potential

- T>>T_C: Asymptotic freedom of QCD allows weak coupling evaluation

Laine et al. JHEP03 (2007) 054

$W_\square(R,t) =$





$$V^{QCD}_{HTL}(R) = -\frac{gC_F}{4\pi}\left[m_D + \frac{e^{-m_D R}}{R}\right] - i\frac{g^2 T C_F}{4\pi}\phi(m_D R)$$

$$\phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Debye screening **Landau damping**

- Re[V] from Debye screening: presence of deconfined color charges
- Im[V] from gluon scattering (Landau damping) and absoprtion (singlet octet transition)

Beraudo et. al. NPA 806:312,2008 Brambilla et. al. PRD 78 (2008) 014017

- Presence of Im[V] is a QCD result not a model assumption

Extracting V^{QQ} from lattice QCD

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- Real-time not directly accessible!
- How to connect to the Euclidean domain: **spectral functions**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

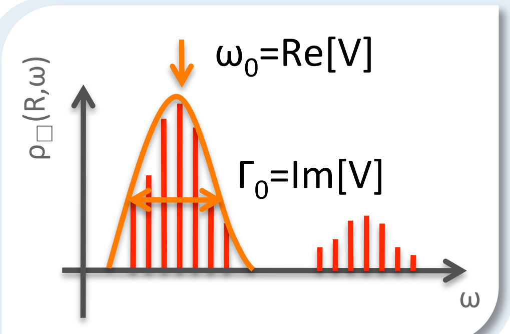
A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)}$$

Bayesian spectral analysis

Y.Burnier, A.R. PRL 111 (2013) 182003

- How are the spectrum and the potential related?



$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \dots$$

$$\lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)} = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

The extraction strategy

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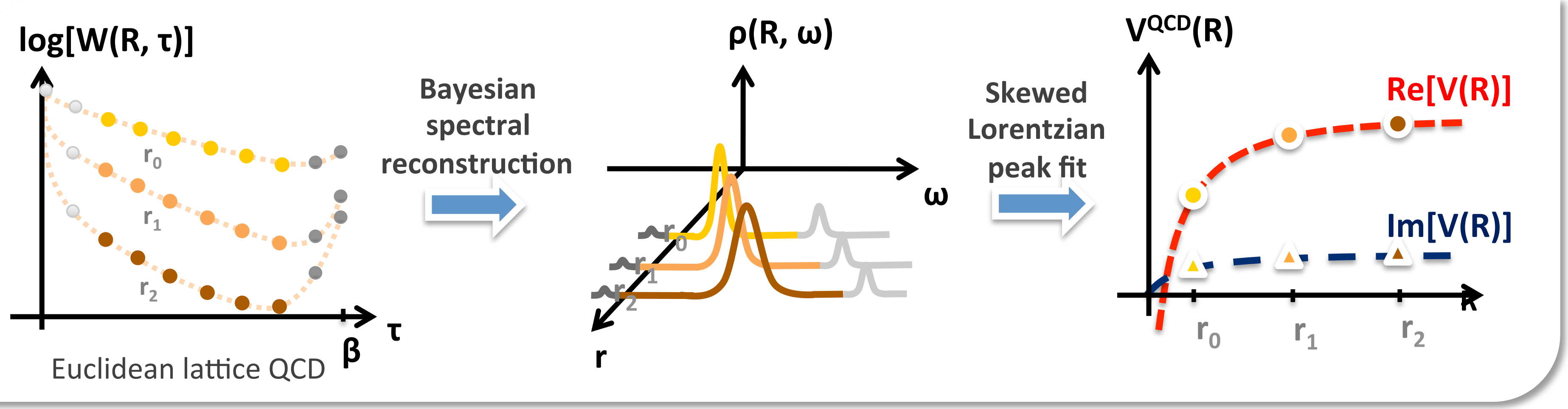
- From lattice QCD Euclidean Wilson loops to the complex heavy quark potential



  - technical detail: avoid cusp divergences using Wilson line correlators in CG

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 x N_\tau$

N_τ	24	32	40	48	56	64	72	80	96
T/T_C	3.11	2.33	1.86	1.55	1.33	1.17	1.04	0.93	0.78
N_{meas}	2750	1570	1680	1110	760	1110	700	940	690

Towards $V^{QQ}(r)$ on quenched lattices

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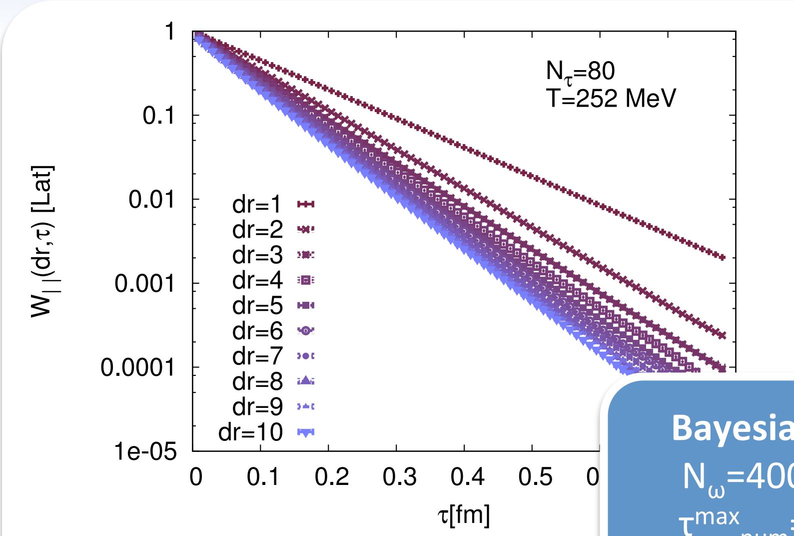
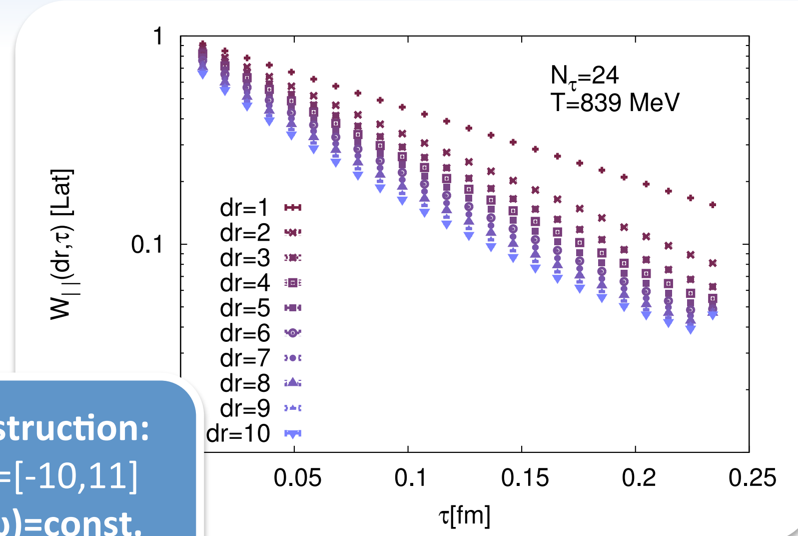
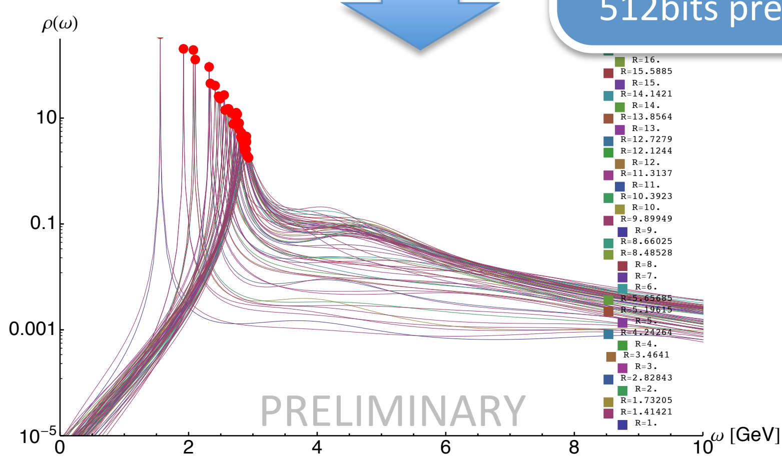
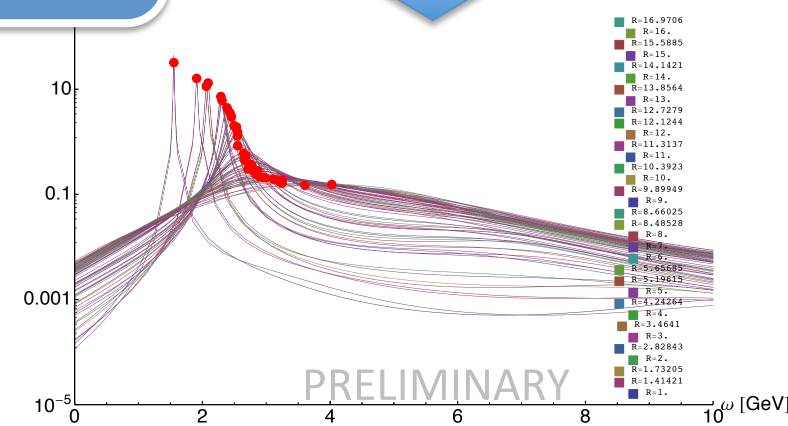




Bayesian reconstruction:
N_ω=4000, I_ω^{num}=[-10,11]
τ^{max}_{num}=20, **m(ω)=const.**
512bits precision, $\Delta^{min}=10^{-60}$





- Presence of Im[V] at high T already visible from curvature in the correlator data

The potential in quenched lattice QCD

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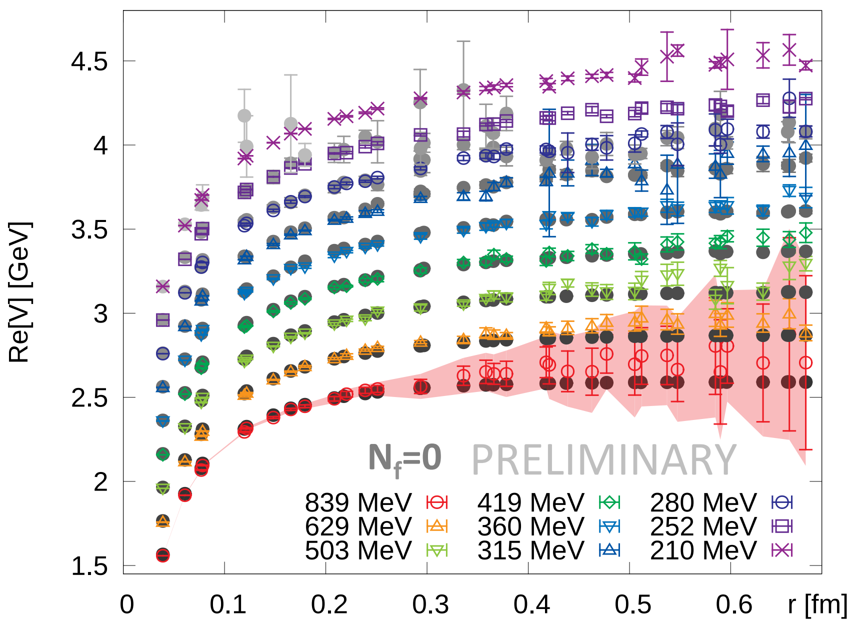

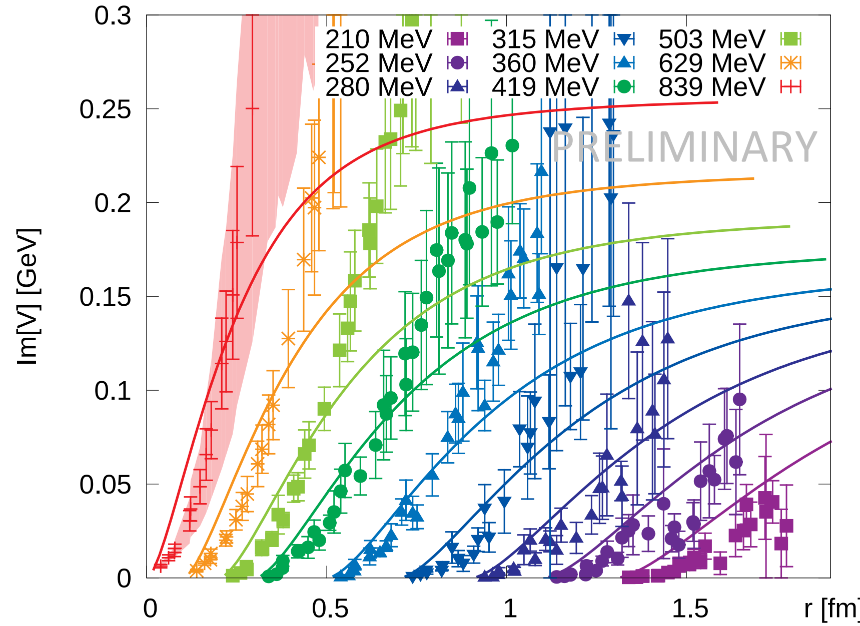

- Transition from a confining to a Debye screened behavior
- Re[V] lies close to the color singlet free energies $F^1(r)$

$$F^{(1)}(r) = -\frac{1}{\beta}\log\left[W_{||}(r,\tau=\beta)\right]_{CG}$$

- For small r: good agreement between Im[V] and HTL prediction down to $1.17T_C$

Project II

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In-medium Bottomonium
spectral functions
from lattice QCD

A Lattice QCD Challenge

- PRACTICAL: High cost if light and heavy d.o.f share the same spacetime grid

$$a \ll \frac{1}{2m_b} \approx 0.02\text{fm} \qquad \frac{1}{T} = N_\tau a \sim 1\text{fm}$$

→ Turn the separation of scales into an advantage: effective field theory NRQCD

Thacker, Lepage Phys.Rev. D43 (1991) 196-208

Effective Field Theory: Lattice NRQCD

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$$L_{NRQCD} = \psi^\dagger\left(iD_t + \frac{D_i^2}{2M_Q} + \ldots\right)\psi + \xi^\dagger(\ldots)\xi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{q}(\ldots)q$$

Lepage et.al, Phys.Rev. D46 (1992) 4052-4067
Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Hevay quark ψ and antiquark ξ as separate non- relativistic Pauli spinors

Light medium d.o.f. from a fully relativistic lattice simulation

- Separation of scales $T/M_Q << 1$, $\Lambda_{QCD}/M_Q<<1$, $p/M_Q<< 1$: systematic expansion in $1/M_Qa$
- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau)=<\psi(\tau)\psi^\dagger(0)>$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau)\left(1 - \frac{\mathbf{p}_{lat}^2}{4M_Q a} + \ldots\right)G(\mathbf{x}, \tau)$$

well behaved if $M_Qa > 1.5$

Davies, Thacker Phys.Rev. D45 (1992)

- **$^3S_1$ (ϒ)** and **$^3P_1$ (χ_{b1})** channel correlators D(τ) from products of heavy quark propagators G(τ)

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) G_{\mathbf{x}\tau} O^\dagger(\mathbf{x}_0, \tau_0) G_{\mathbf{x}\tau}^\dagger \rangle_{med} \qquad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

Thacker, Lepage Phys.Rev. D43 (1991)

A Medium With Nf=2+1 Light HISQ Flavors

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- Light d.o.f. (gluons, u d s quarks) represented by HotQCD configurations

  A. Bazavov et. al., Phys. Rev. D 85 (2012) 054503

  - 48^3x12 with relatively light pions $M_\pi \sim$ 161MeV and a T_C=159±3MeV

HotQCD HISQ/tree action $48^3 \times N_\tau$ $m_{u,d}/m_s = 0.05$

β	6.664	6.700	6.740	6.770	6.800	6.840	6.880
a[fm]	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau = 12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
a[fm]	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
$M_b a$	2.187	2.107	2.030	1.956	1.835	1.753	1.559
$T/T_C(N_\tau = 12)$	1.151	1.194	1.240	1.286	1.371	1.436	1.614

- Important property for the use with lattice NRQCD: $2.759 > M_b a > 1.559 > 1.5$
- Temperature changed by variation of the lattice spacing 140MeV < T < 249MeV

  For a study based on the fixed scale approach see: FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097, JHEP 1111 (2011) 103
- Low temperature configurations available at b=6.664, 6.8, 6.95, 7.28

Spectral Functions In NRQCD

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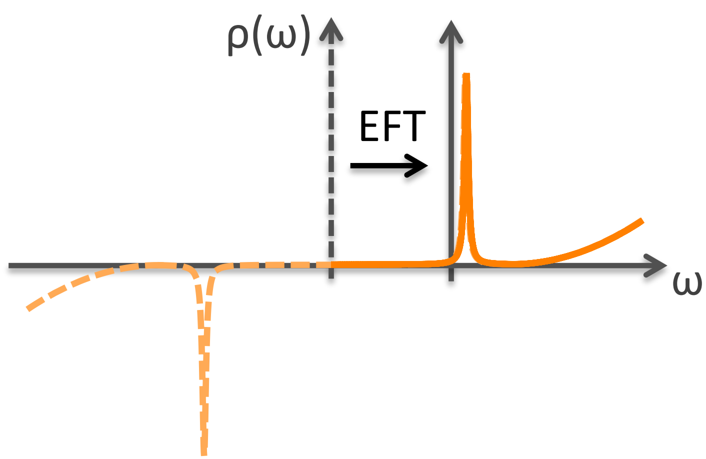
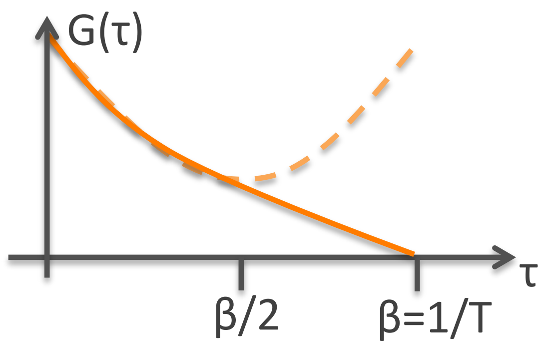
- "Integrating out M_b" in setting up NRQCD introduces a scale dependent frequency shift

Drawback: setting absolute frequency scale at T>0 requires additional T=0 calibration

Advantage: Correlator not periodic in 1/T and linked to spectra via simple T=0 Kernel,

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$





Bottomonium Correlators Close To T=0

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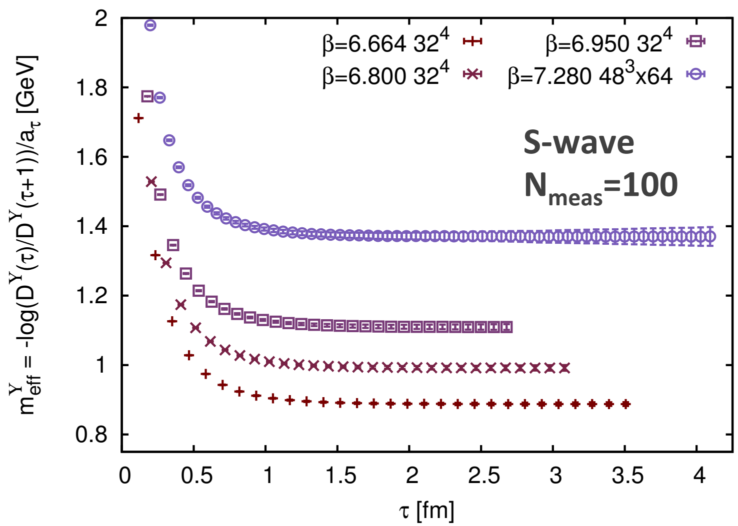
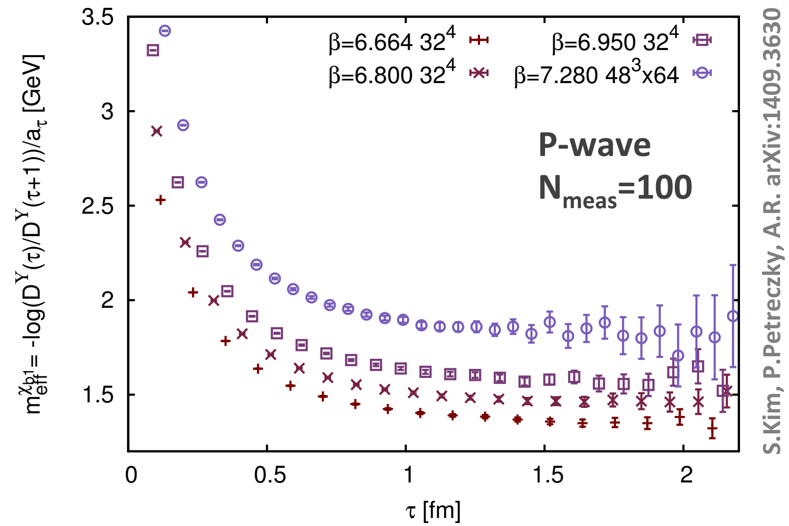


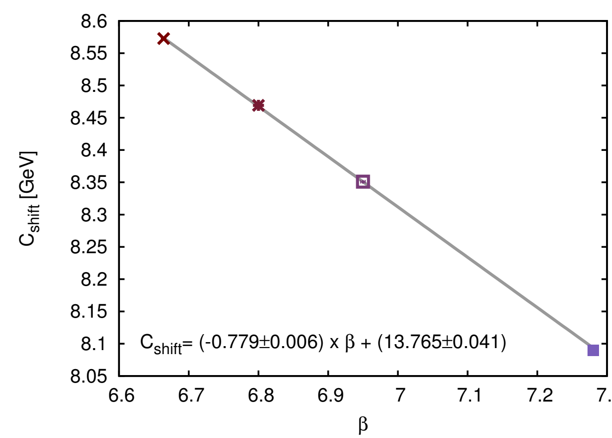


- Set absolute scale by comparison to experiment

$$M^{\rm exp}_{\Upsilon(1S)} = M^{\rm NRQCD}_{\Upsilon(1S)} + 2(Z_{M_b} M_b - E_0)$$

$$C_{\rm shift}(\beta)$$

$$M^{\rm exp}_{\Upsilon(1S)} = 9.46030(26)\,{\rm GeV}$$



- Linear dependence: interpolated values to calibrate mass shift at intermediate β

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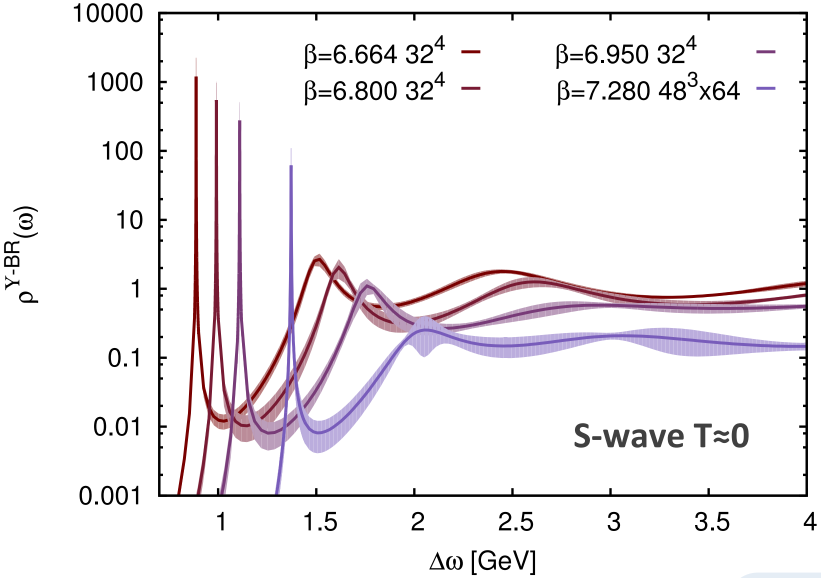
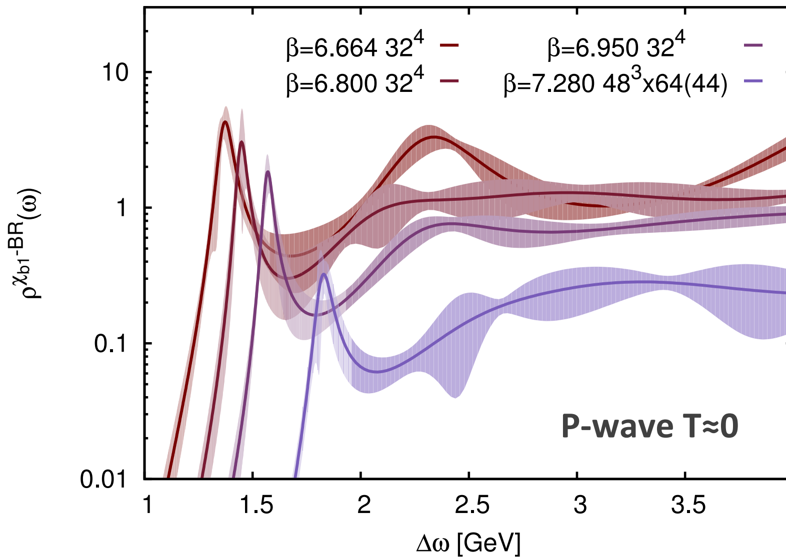
Spectral Functions Close To T=0





S.Kim, P.Petreczky, A.R. arXiv:1409.3630

- Bayesian reconstruction:

N_ω=1200 I_ω=[-0.5,30] β^{num}=20 N_{jack}=10

m_l=const, 512 bit precision, Δtol= 10^{-60}

- S-wave ground state peak very well resolved, next peak mostly from Υ(2S)
- P-wave ground state broader: worse s/n ratio and smaller physical peak size

$$M_{\chi_{b1}(1P)} = M^{\rm NRQCD}_{\chi_{b1}} + C(\beta) = 9.917(3)\,{\rm GeV} \;\; > \;\; M^{\rm exp}_{\chi_{b1}(1P)} = 9.89278(26)(31)\,{\rm GeV}$$

Reconstruction Accuracy: S-wave

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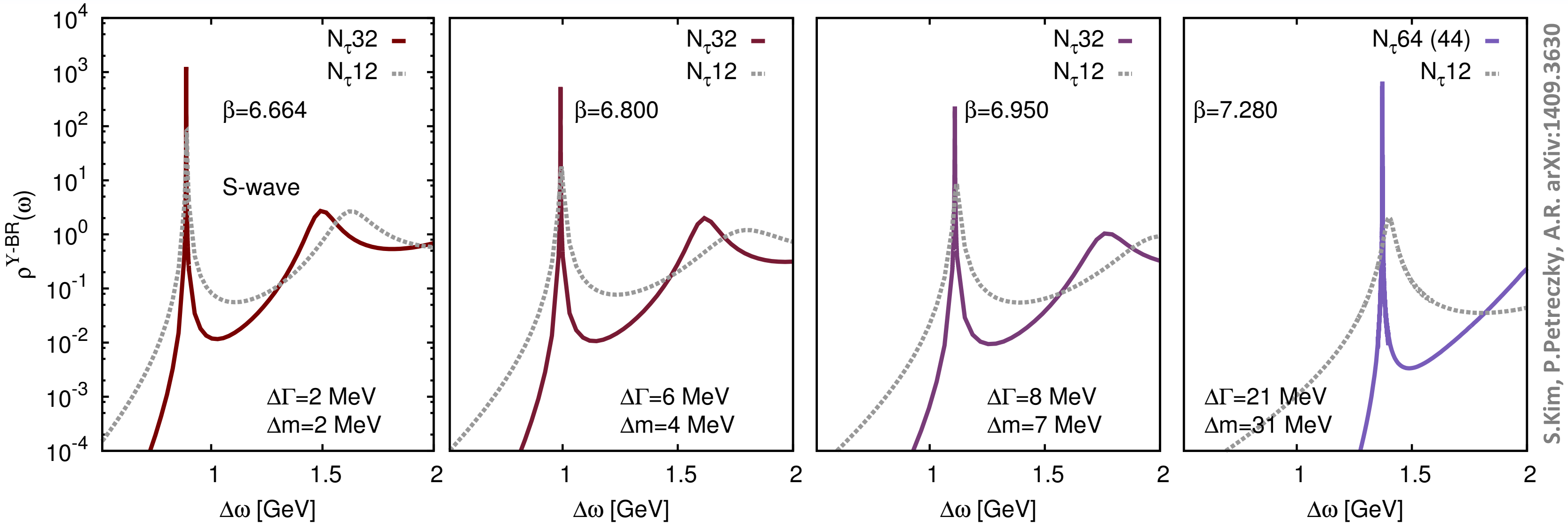

- High precision of the improved Bayesian reconstruction (narrow width resolved)
- How does accuracy suffer from limited available information at T>0 (Nτ=12) ?
- One of the tests we ran: truncate T=0 dataset (Nτ=32/64) to Nτ=12

Overall Limits:

$$\beta = 6.664: \quad \Delta m_T < 2\text{MeV}, \quad \Delta\Gamma_T < 5\text{MeV}$$
$$\beta = 7.280: \quad \Delta m_T < 40\text{MeV}, \quad \Delta\Gamma_T < 21\text{MeV}$$

Reconstruction Accuracy: P-wave

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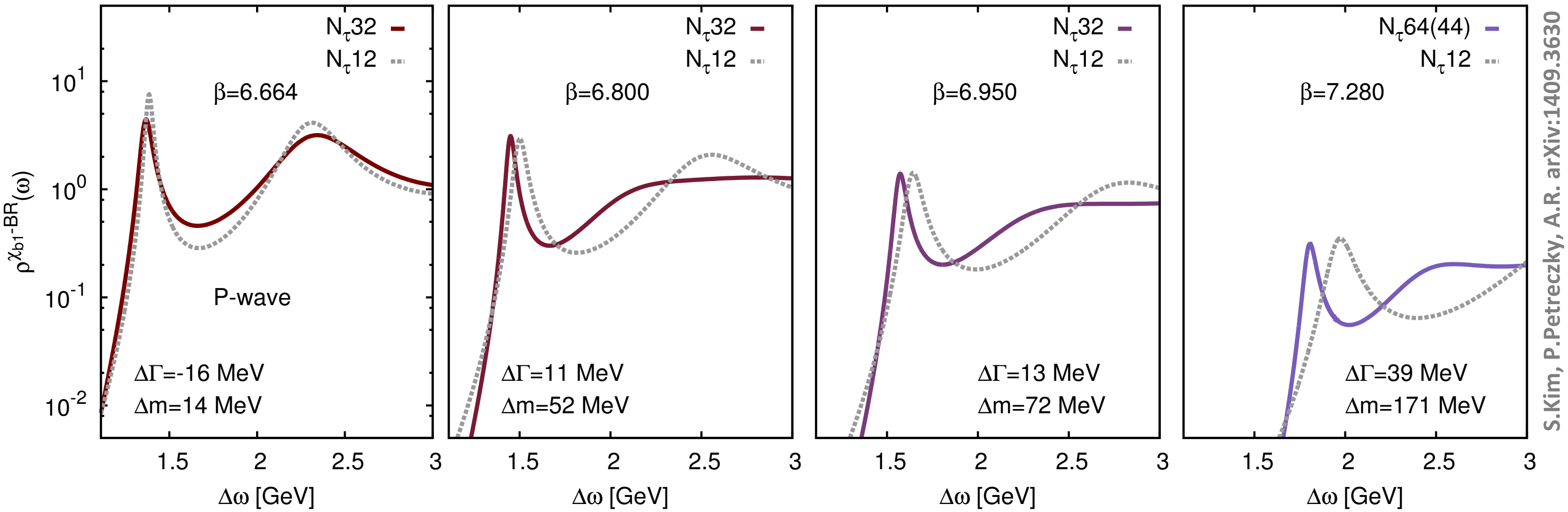

- Estimate systematics: truncate T=0 dataset (Nτ=32/64) to Nτ=12
- Due to a worse signal-to noise ratio, effect in P-wave is larger than for S-wave

Overall Limits:

$$\beta = 6.664: \quad \Delta m_T < 60\text{MeV}, \quad \Delta\Gamma_T < 20\text{MeV}$$
$$\beta = 7.280: \quad \Delta m_T < 200\text{MeV}, \quad \Delta\Gamma_T < 40\text{MeV}$$

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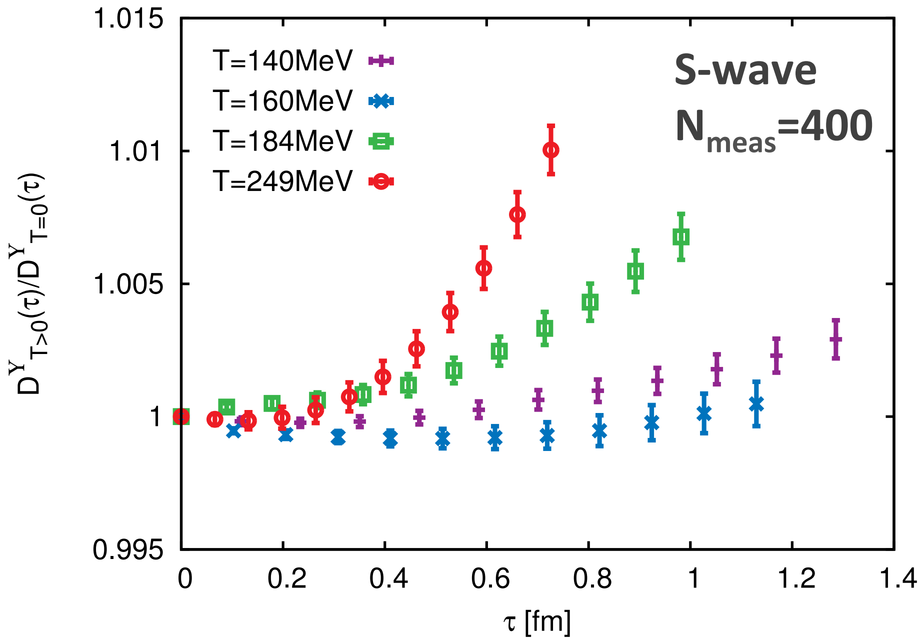
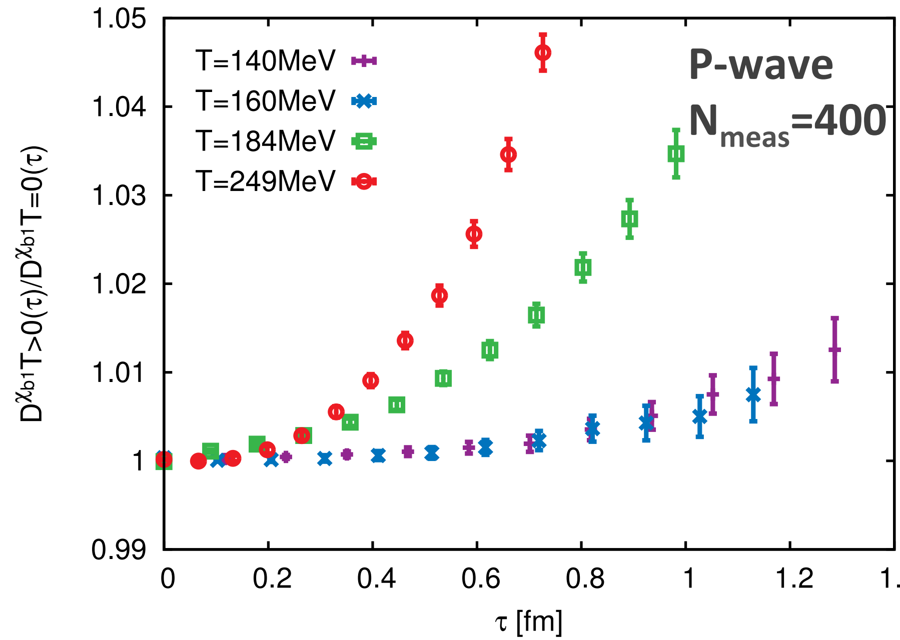
Bottomonium Correlators At Finite T



S-wave at most 1% change



P-wave at most 5% change

S.Kim, P.Petreczky, A.R. arXiv:1409.3630

- Statistically significant in-medium modification above T=160MeV
- Side remark: similar qualitative and quantitative behavior for η_b and h_b (scalar)

S-wave Spectral Functions At T>0

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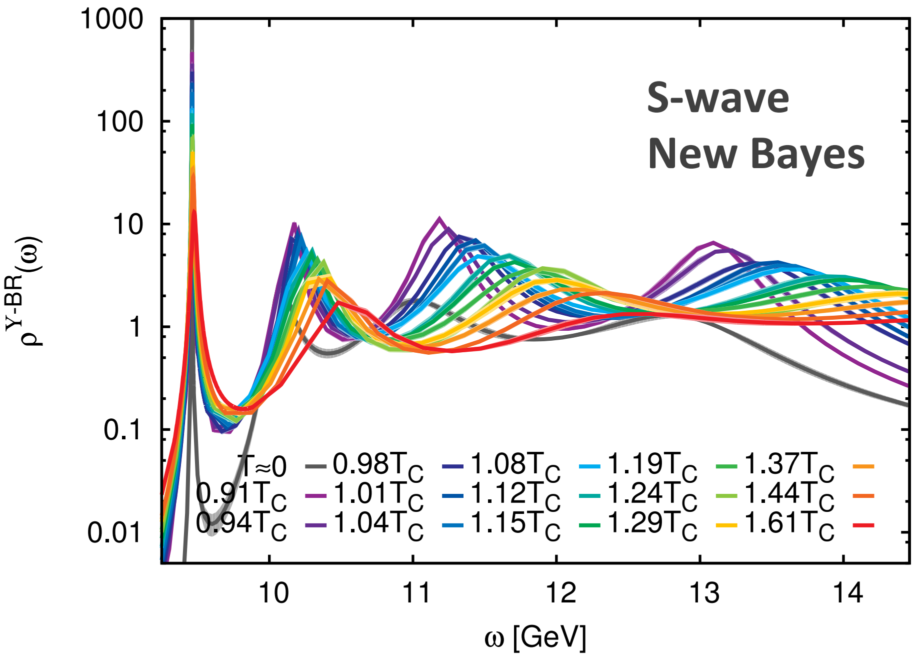

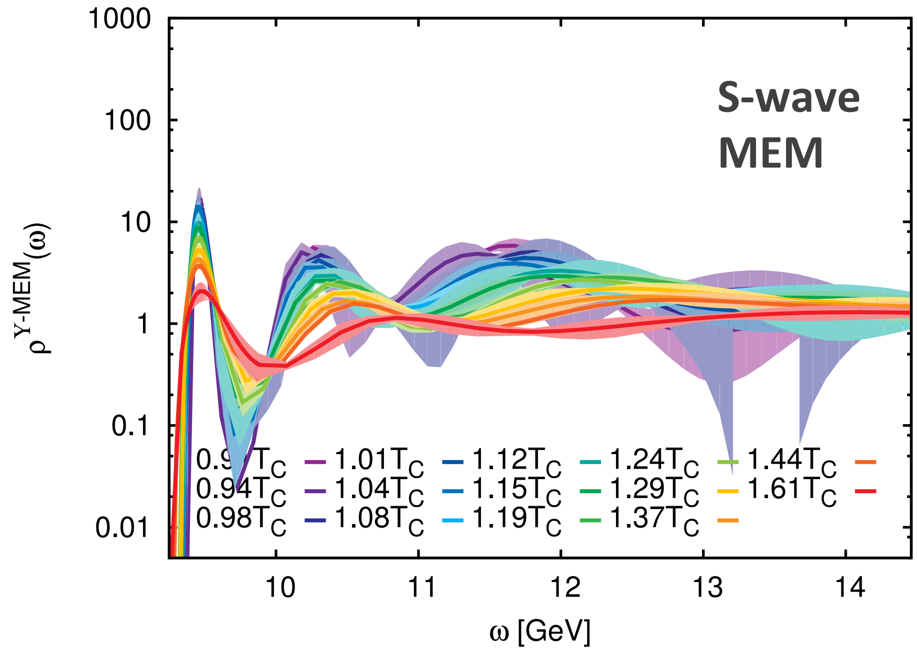

S.Kim, P.Petreczky, A.R. arXiv:1409.3630

- Bayesian reconstruction:

  N_ω=1200 I_ω=[-1,25] β^{num}=20 N_{jack}=10

  m_l=const 512 bit precision, $\Delta tol=10^{-60}$

- New Bayesian method resolves peaks much better than MEM
  - observed broadening and peak shifts at finite T smaller than accuracy limits
- Well defined **ground state peak present up to 1.61T_C**

P-wave Spectral Functions At T>0

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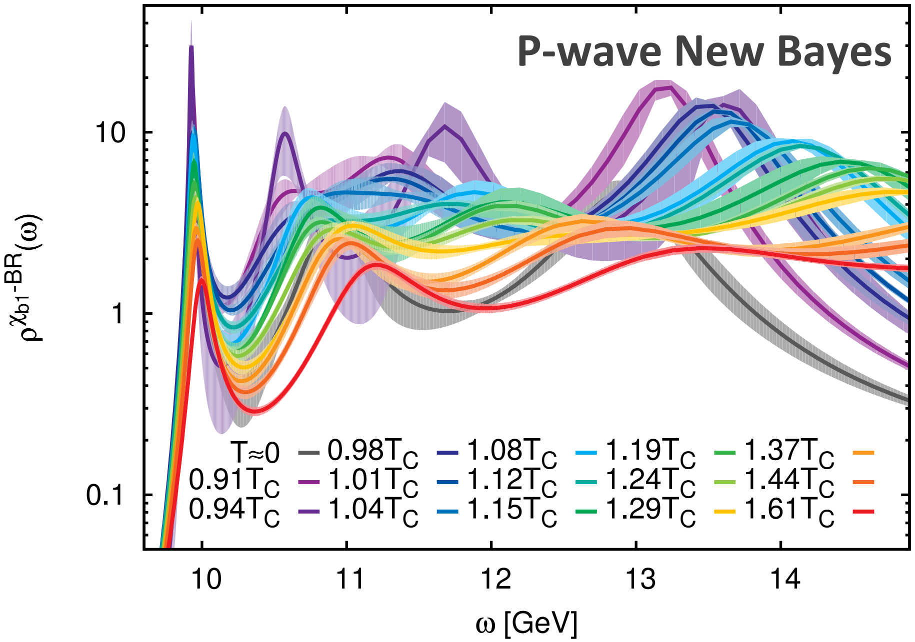
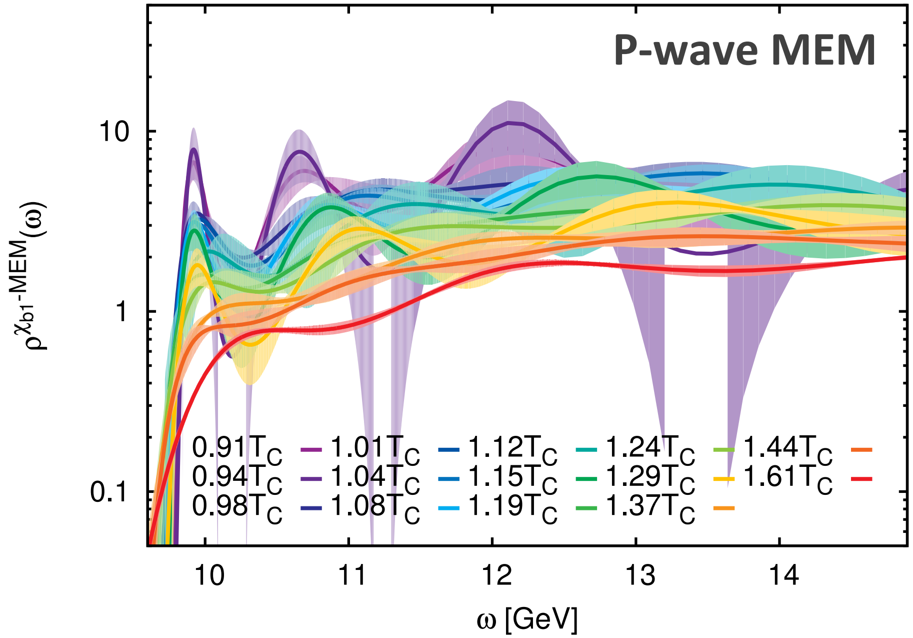


Ground state peak well defined up to $T=1.61T_C$



Ground state peak disappears for $T>1.29T_C$

S.Kim, P.Petreczky, A.R. arXiv:1409.3630

- Worse signal to noise ratio leads to larger Jackknife errors than for S-wave
  - observed broadening and peak shifts also smaller than accuracy limits
- New approach finds well defined peak up to highest T investigated 249 MeV

MEM result similar to FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097

How To Verify Survival Of A Bound State?

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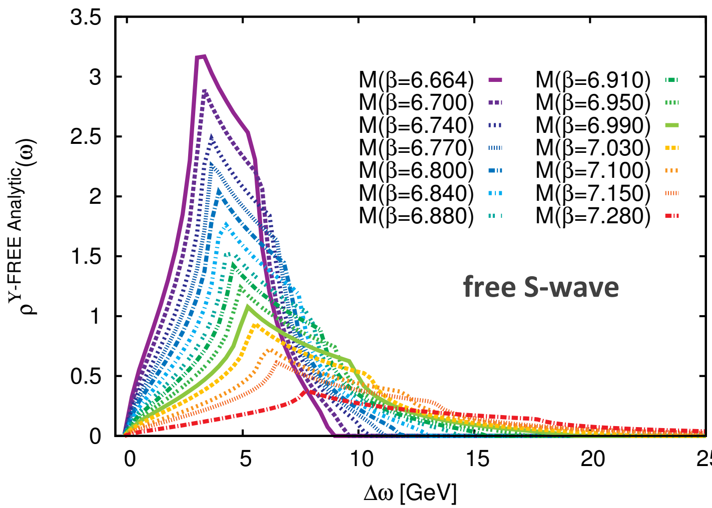
- Inspection by eye insufficient: systematic comparison to non-interacting spectra
  - **Analytically**: From free NRQCD dispersion relation:

$$a_\tau E_{\mathbf{p}} = -\log\Big(1 - \frac{\mathbf{p}^2_{\rm lat}}{8 M_b a_s}\Big)$$

$$\rho_S(\omega) = \frac{4\pi N_c}{N_s^2}\sum_{\mathbf{p}} \delta(\omega - 2E_{\mathbf{p}}) \quad \rho_P(\omega) = \frac{4\pi N_c}{N_s^2}\sum_{\mathbf{p}} \mathbf{p}^2\delta(\omega - 2E_{\mathbf{p}})$$

G.Aarts et. al., JHEP 1111 (2011) 103



S.Kim, P.Petreczky, A.R. arXiv:1409.3630

  - **Numerically**: Reconstruct from free NRQCD correlator (U_μ=1)
  - Expectation: Presence of peaked features due to numerical **Gibbs ringing**

S-wave And P-wave Survival At T=249MeV

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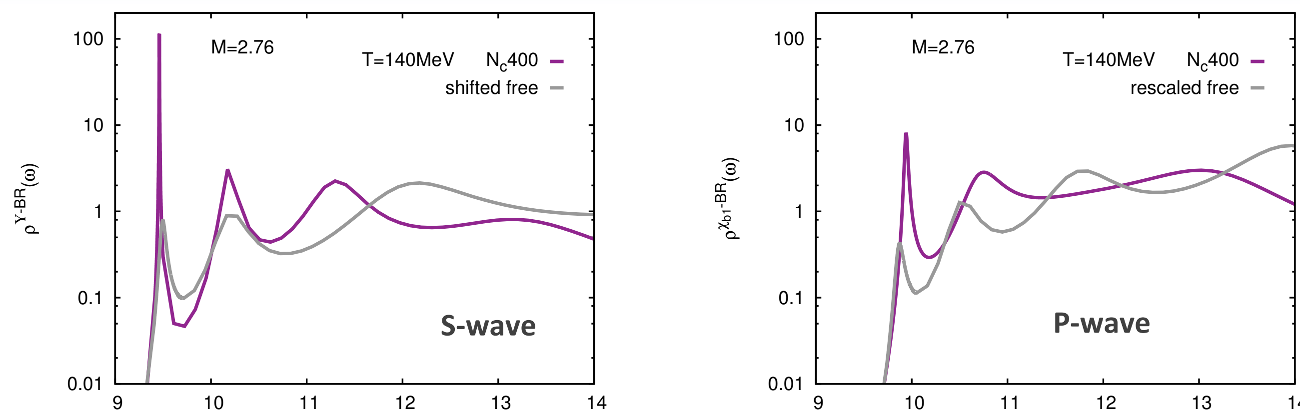
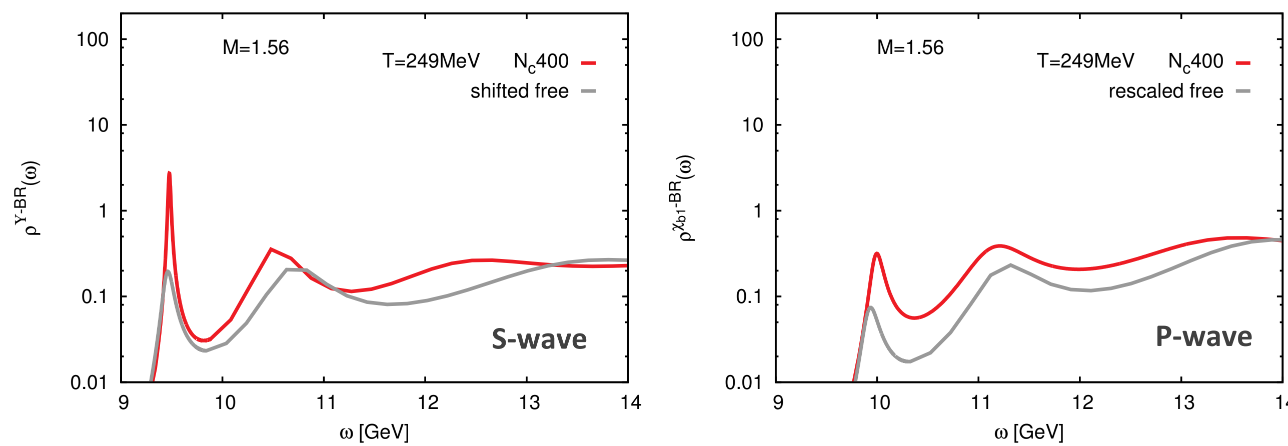


S.Kim, P.Petreczky, A.R. arXiv:1409.3630

- At T=140MeV clear difference between ground state peak and numerical ringing



- At T=249 MeV: Ground state peak still stronger than numerical ringing

Conclusion

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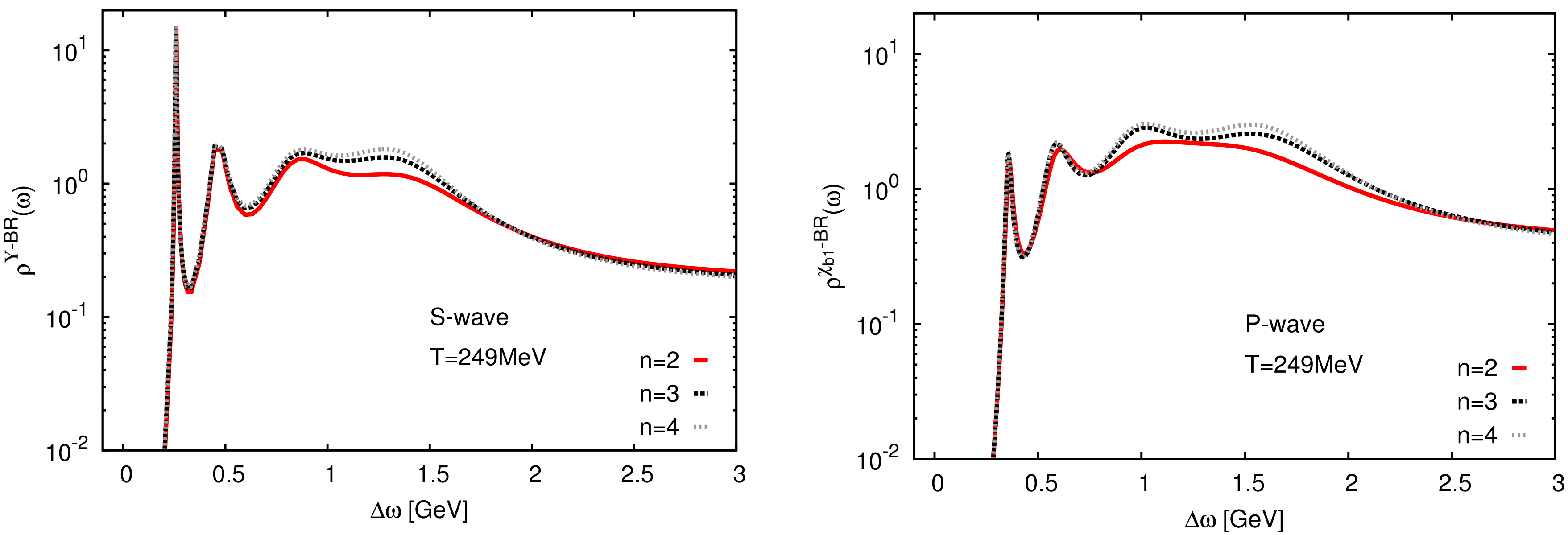
- Improved Bayesian approach to spectral function reconstruction is promising
  - Outperforms MEM consistently: higher resolution on same datasets
  - No restricted search space: accuracy suffers from loss of information alone
- The in-medium potential between static quarks can be accessed in lattice QCD
  - Re[V] lies close to color singlet free energies in Coulomb gauge at all T
  - Im[V] in quenched QCD: same order of magnitude as HTL perturbation theory at $T>T_C$
- Bottomonium spectra on HotQCD lattices with $N_f=2+1$ light HISQ flavors
  - In-medium modification of correlators above T=160MeV [up to 1% (ϒ) and 5% (χ_{b1})]
  - $N_\tau=12$ datapoints allow us to set upper bounds on in-medium modification
  - A systematic comparison between free and interacting spectra show:

S-wave and P-wave ground state survive up to at least T=249MeV

Thank you for your attention

Dependence On The NRQCD Discretization

- Reduce the effective temporal step size for NRQCD propagator E.O.M.



- As expected: high momentum behavior changes but IR unaffected

Default Model Dependence

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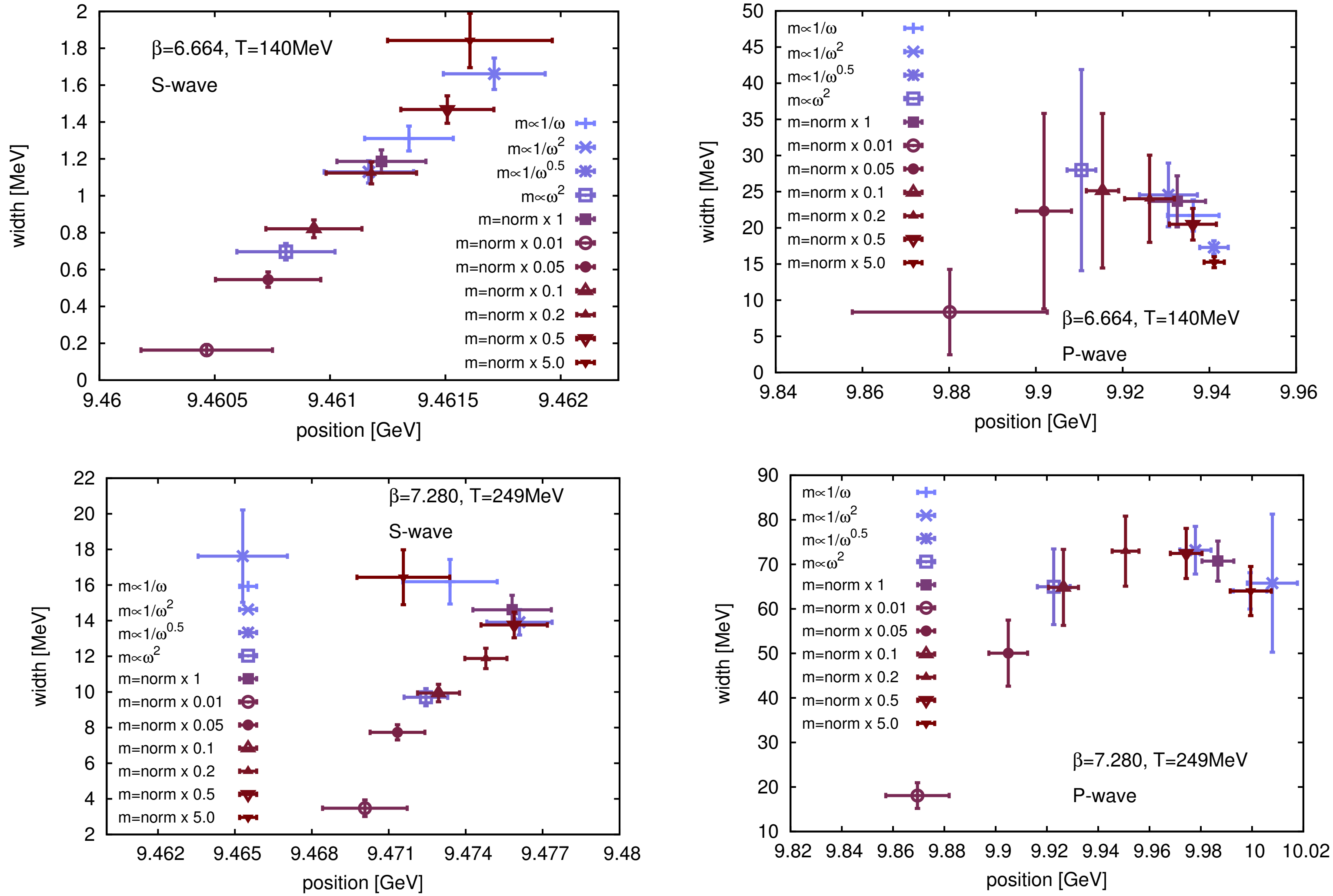

Free Spectra: Default Model Dependence