# Understanding J/ψ Production
# 40 years after its discovery

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Based on works done with Z.-B. Kang, Y.-Q. Ma, G. Nayak,
G. Sterman, H. Zhang, …

**INT Program (INT-14-3) – Focused workshop on**
***"Heavy Flavor and Electromagnetic Probes in Heavy Ion Collisions"***
**Institute for Nuclear Theory, University of Washington, Seattle, WA 9/15-10/10, 2014**

# November revolution (1974)

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 19

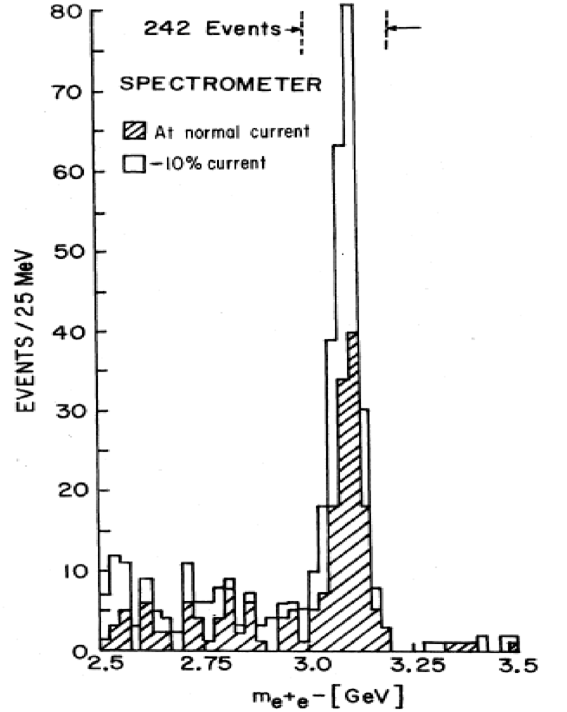
## Experimental Observation of a Heavy Particle $J$†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
*Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

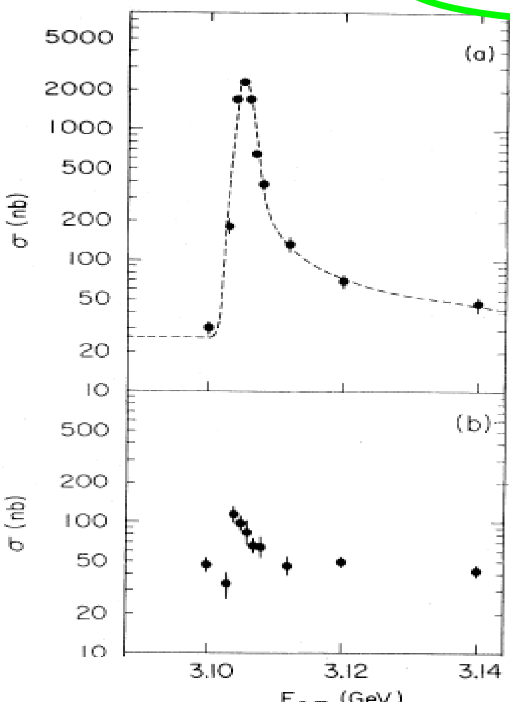
and

Y. Y. Lee
*Brookhaven National Laboratory, Upton, New York 11973*

(Received 12 November 1974)




November, 1974




## Discovery of a Narrow Resonance in $e^+e^-$ Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡
*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse
*Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720*

(Received 13 November 1974)

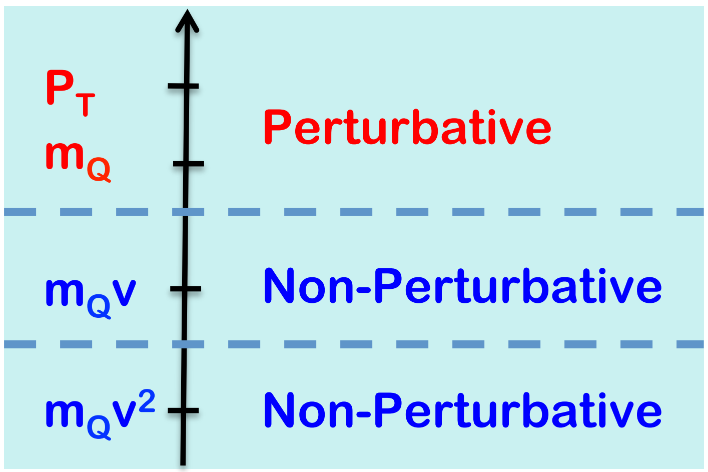
# Heavy quarkonium

- **One of the simplest QCD bound states:**

  Localized color charges (heavy mass), non-relativistic relative motion

  Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

- **Well-separated momentum scales – effective theory:**




Hard — Production of $Q\bar{Q}$ [pQCD]

Soft — Relative Momentum [NRQCD]

$\Lambda_{\text{QCD}}$

Ultrasoft — Binding Energy [pNRQCD]

- **Cross sections and observed mass scales:**

$$\frac{d\sigma_{AB\to H(P)X}}{dy\,dP_T^2} \qquad \sqrt{S}, \qquad P_T, \qquad M_H,$$

PQCD is "expected" to work for the production of heavy quarks

*Difficulty: Emergence of a quarkonium from a heavy quark pair?*

# A long history for the production

- Color singlet model: 1975 –

  Einhorn, Ellis (1975), Chang (1980), Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and $\alpha_s$

- QCD factorization approach: 2005 –

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T >> M_H$: $M_H/P_T$ power expansion + $\alpha_s$ – expansion

  Unknown, but universal, fragmentation functions – evolution

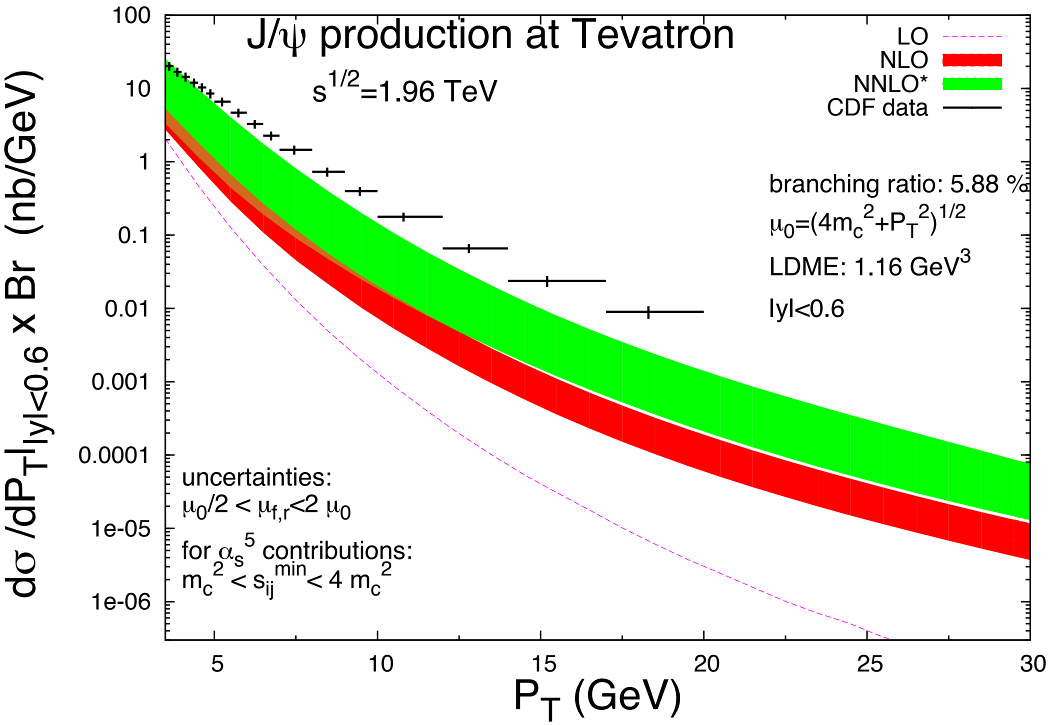
- Soft-Collinear Effective Theory + NRQCD: 2012 –

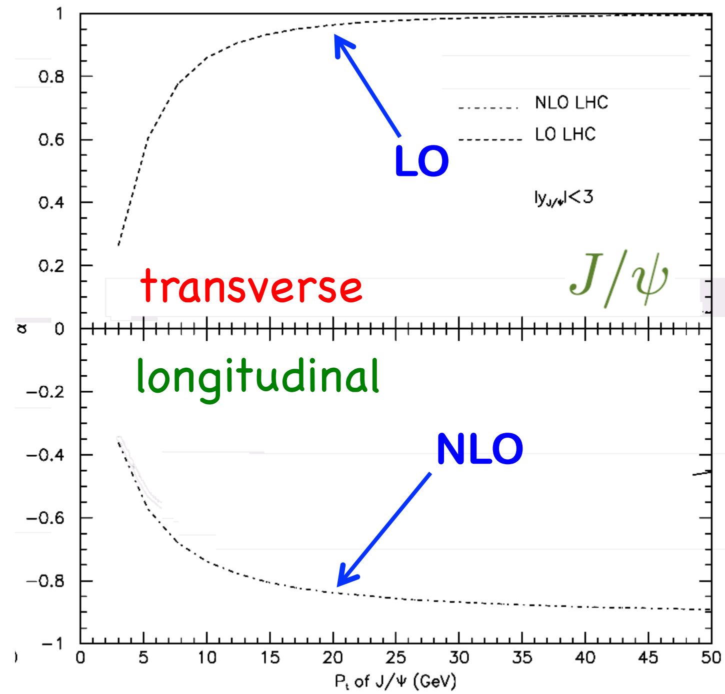
  Fleming, Leibovich, Mehen, …

# Color singlet model (CSM)

- ❑ **Effectively No parameter:**

Campbell, Maltoni, Tramontano (2007),
Artoisenet, Lansburg, Maltoni (2007),
Artoisenet, et al. (2008)



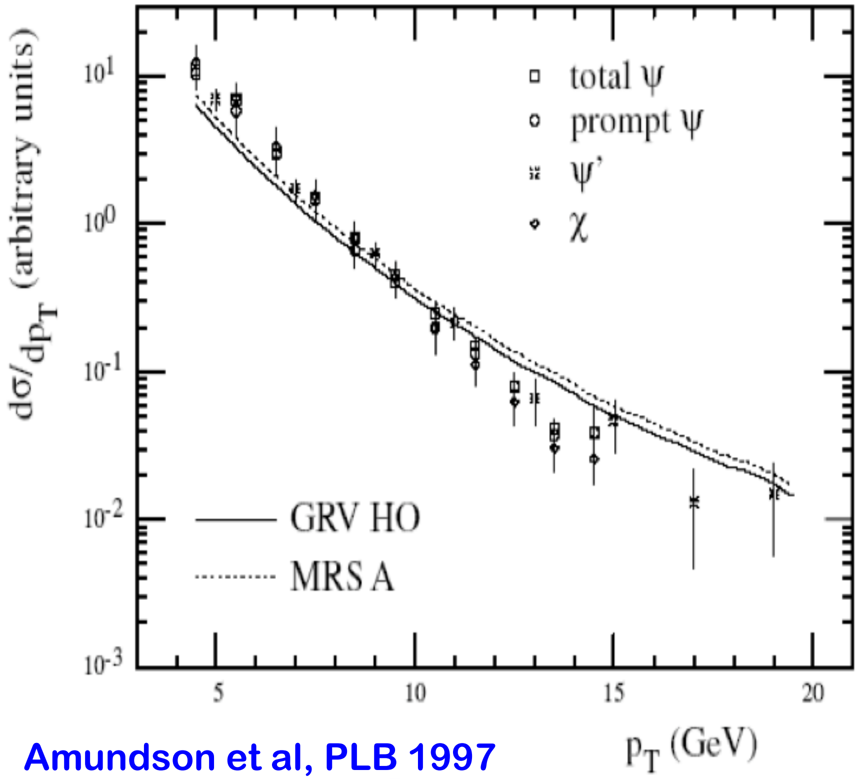




B. Gong et, al. PRL (2008)

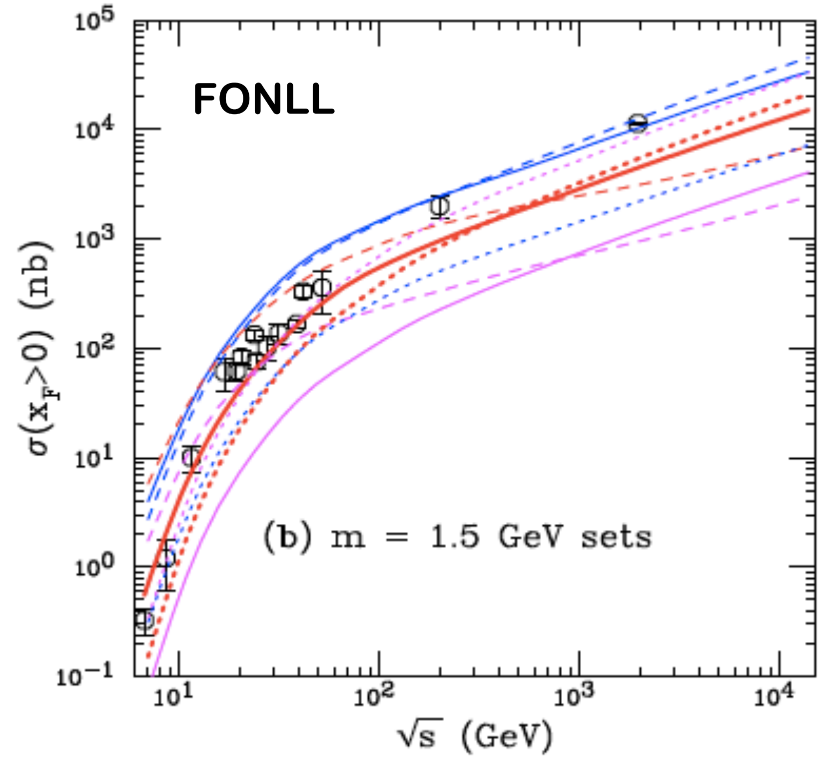
- ❑ **Issues:**
  - ✧ **How reliable is the perturbative expansion?**
  - ✧ **S-wave: large corrections from high orders**
  - ✧ **P-wave: Infrared divergent – CSM is not complete**

# Color evaporation model (CEM)

- One parameter per quarkonium:




Amundson et al, PLB 1997




Nelson, Vogt, Frawley, PRC 2013

- Question:
  - ✧ Better $p_T$ distribution – the shape?
  - ✧ Need intrinsic $k_T$ – its distribution?

# NRQCD – most successful so far

See Kniehl's talk

- NRQCD factorization:

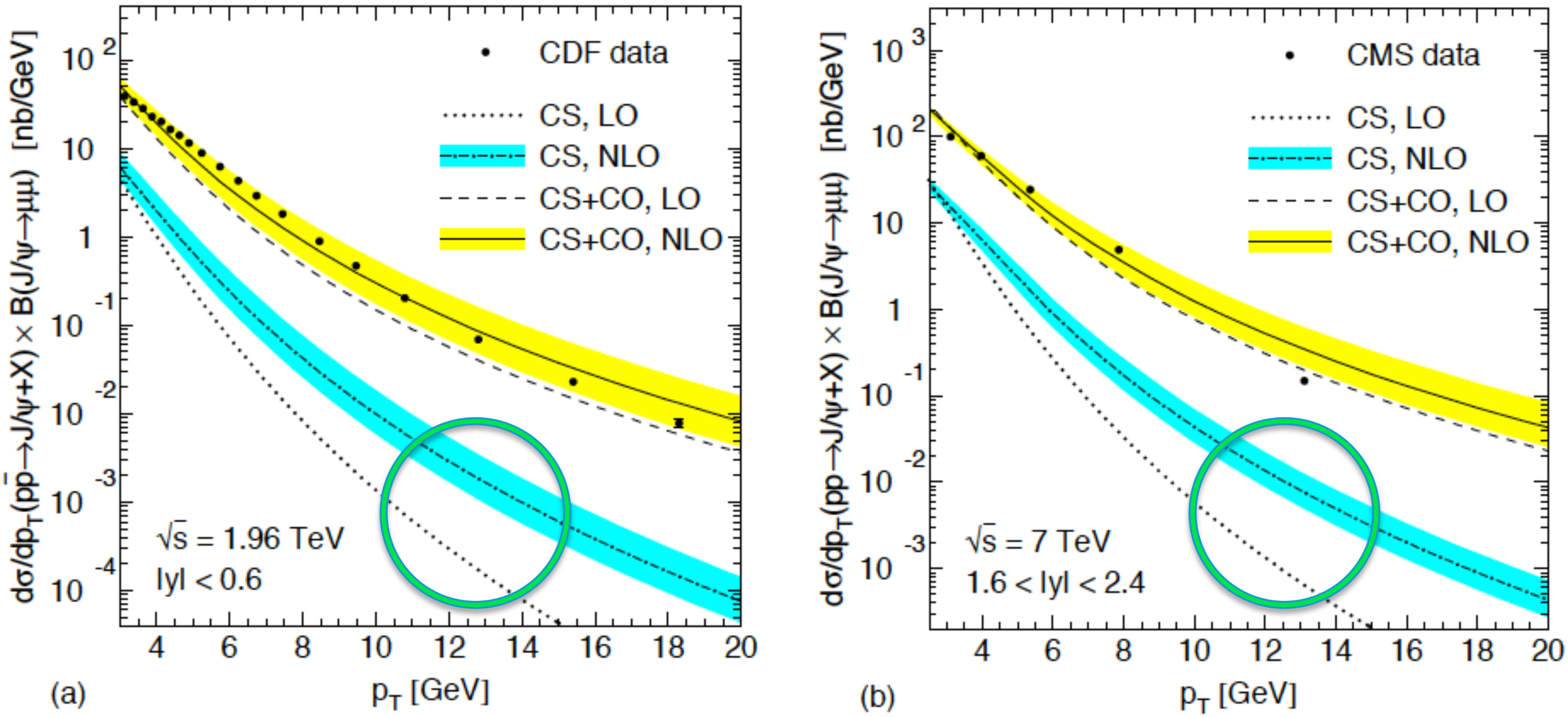
$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle$$

✧ 4 leading channels in v

$${}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

✧ Full NLO in $\alpha_s$

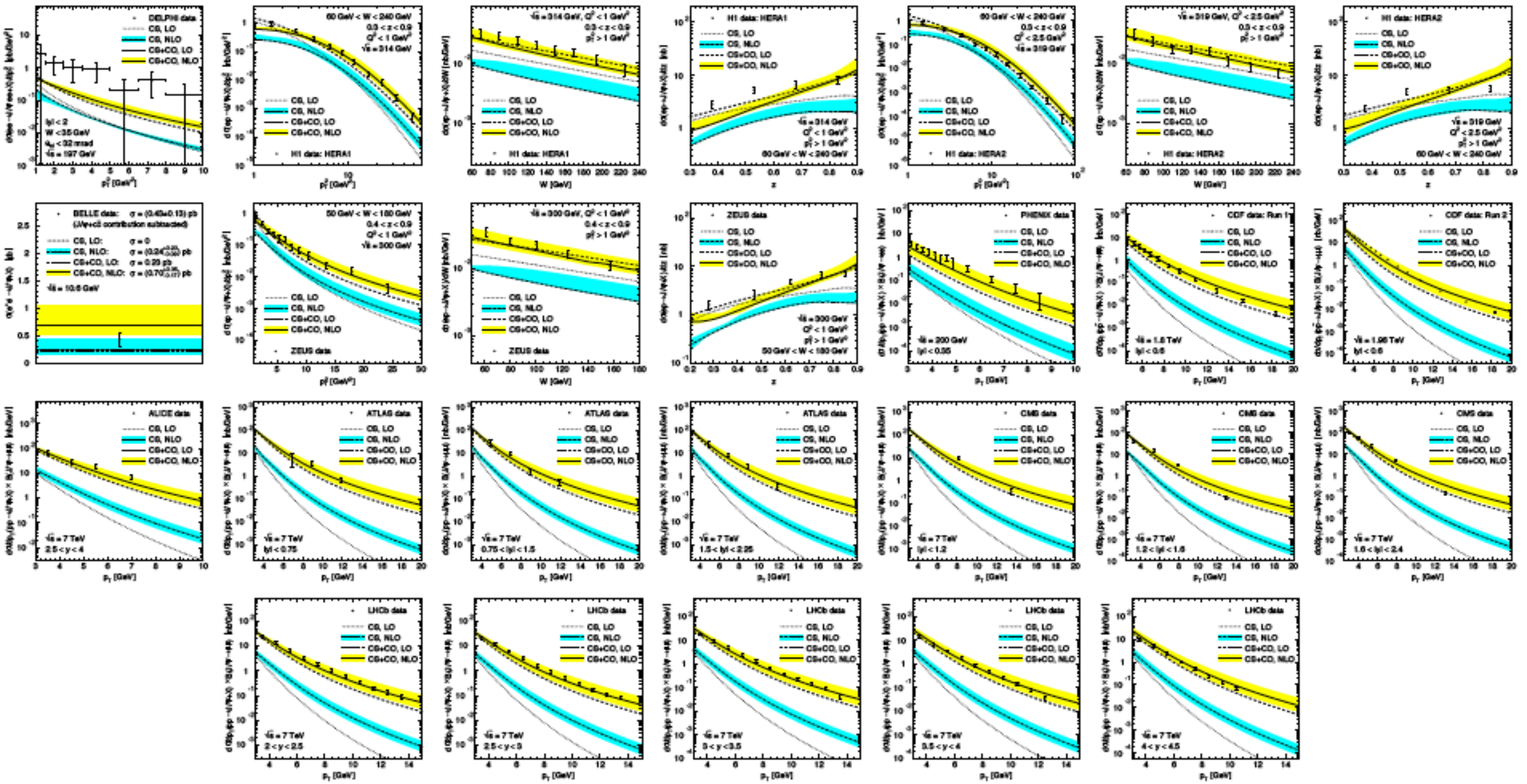
- Phenomenology:




- Fine details – shape – high at large $p_T$?

PRL 106, 022003 (2011)

# NRQCD – global analysis



**194 data points from 10 experiments, fix singlet** $\langle O[{}^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

$\langle O[{}^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\quad \langle O[{}^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[{}^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

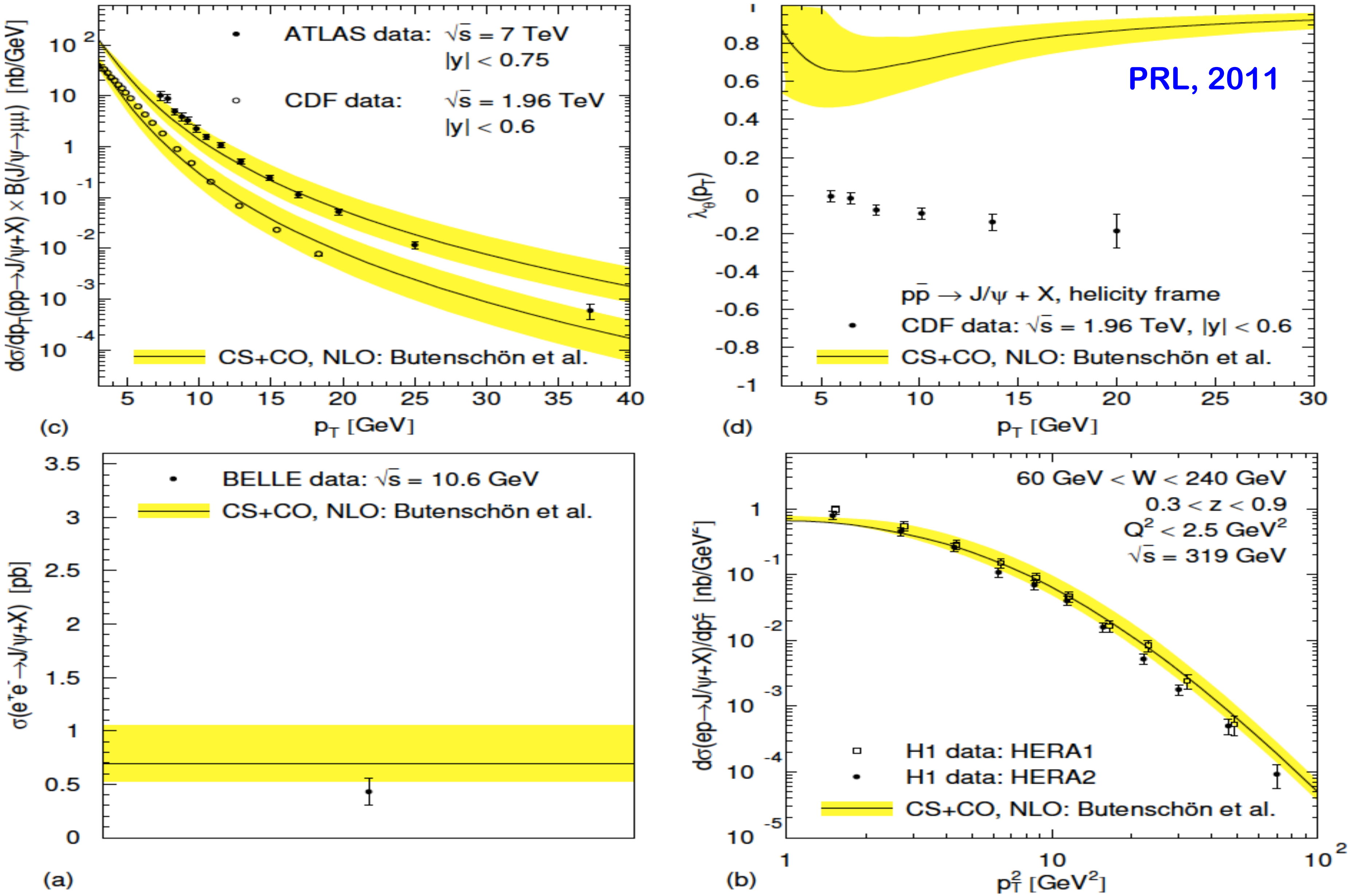
$\chi^2/d.o.f. = 857/194 = 4.42$

Butenschoen and Kniehl, arXiv: 1105.0820

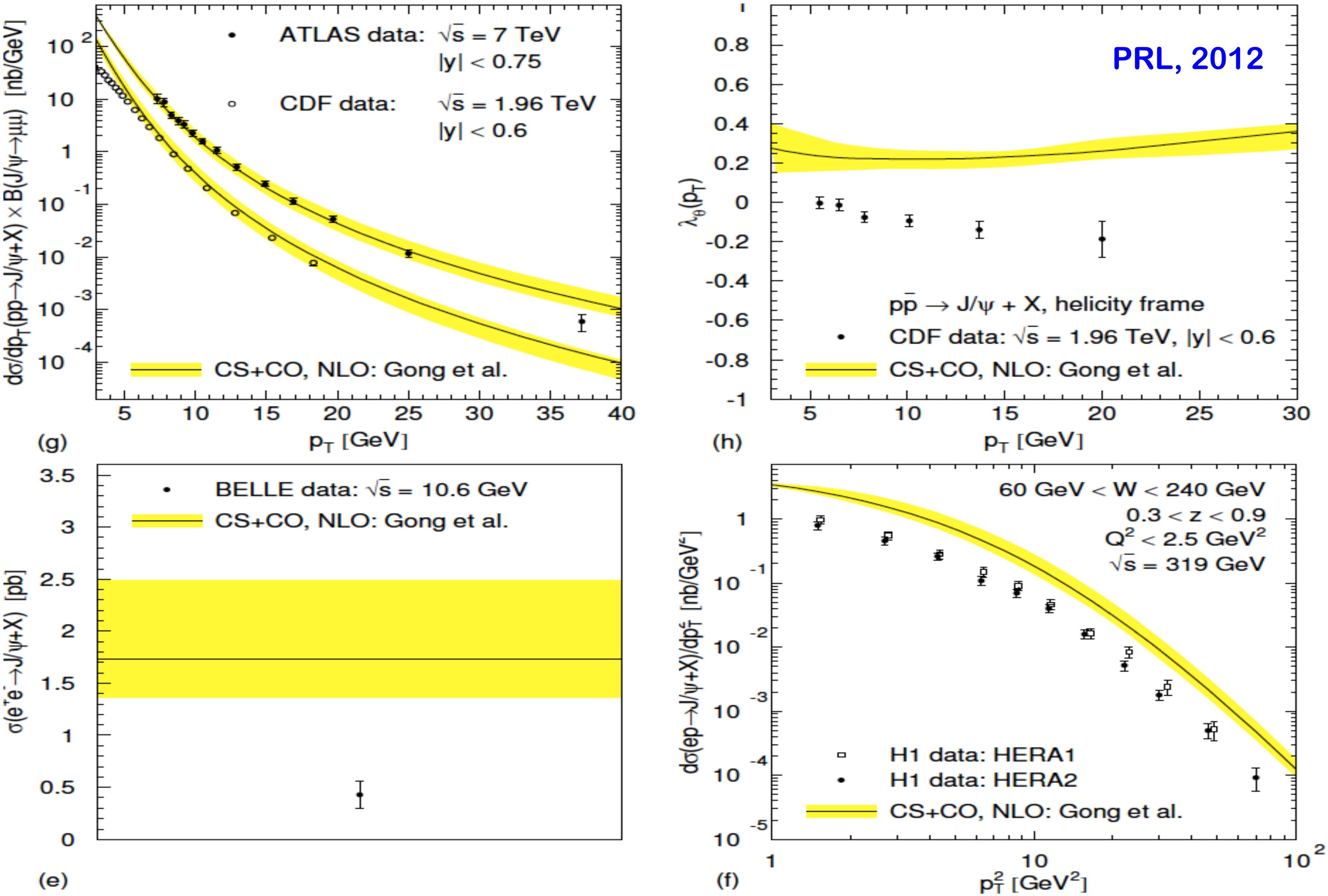
# Anomalies and surprises

- ❑ **Theory – the state of arts – NLO:**
  - ✧ Very difficult to calculate, no analytical expression
    - ➡ hard to obtain a clear physical picture on how various states of heavy quark pair are actually produced?
  - ✧ For some channels, NLO corrections are orders larger than LO
    - ➡ questions whether higher order contributions are negligible, while it is extremely difficult, if not impossible, to go beyond the NLO
- ❑ **Comparison with data:**
  - ✧ Quarkonium polarization – "ultimate" test of NRQCD!
    - ➡ Clear mismatch between theory predictions and data
  - ✧ Universality of NRQCD matrix elements – predictive power!
    - ➡ Clear tension between different data sets, e+e-, ep, pp, …

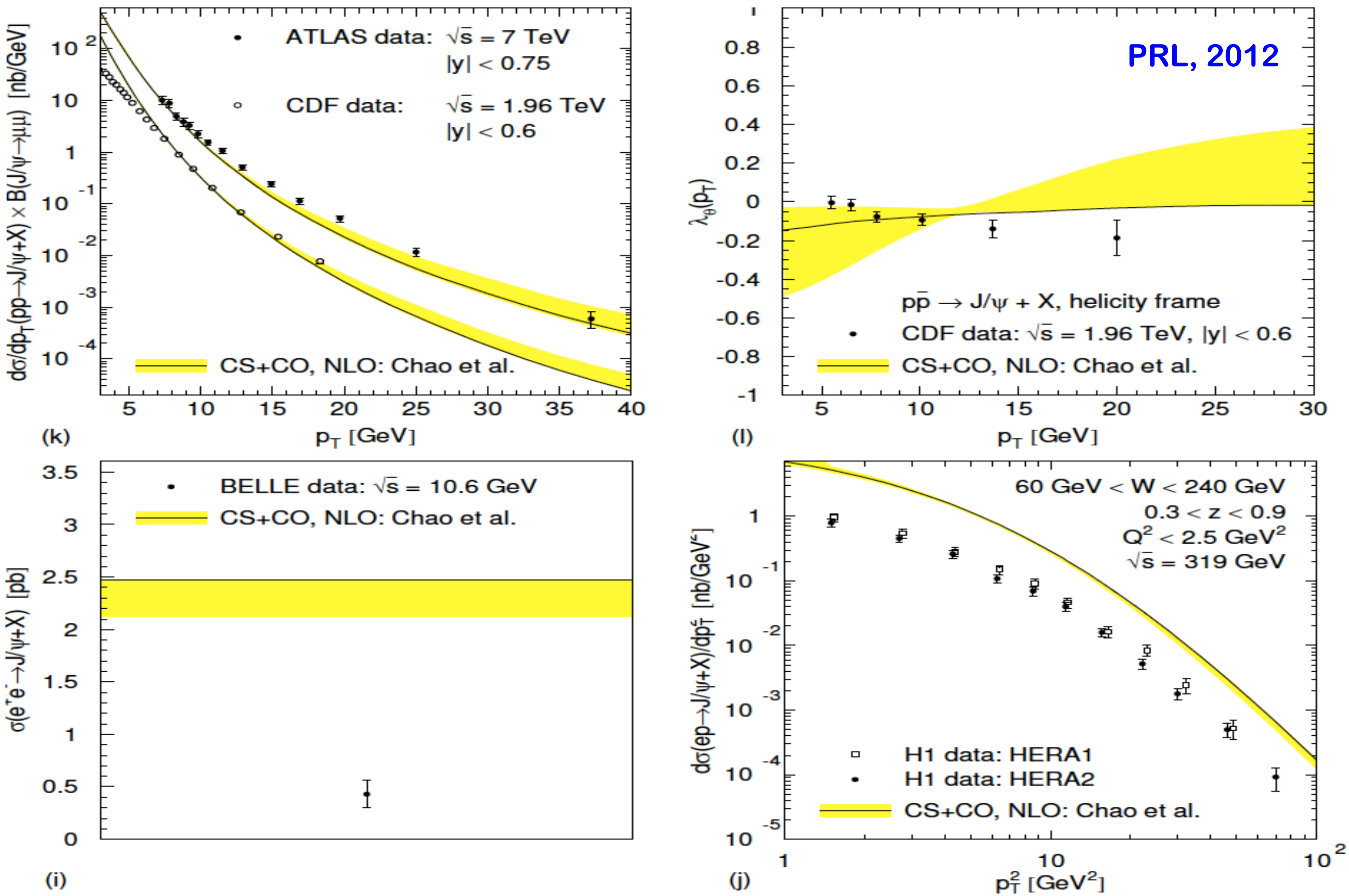
# NLO theory fits – Butenschoen et al.



# NLO theory fits – Gong et al.



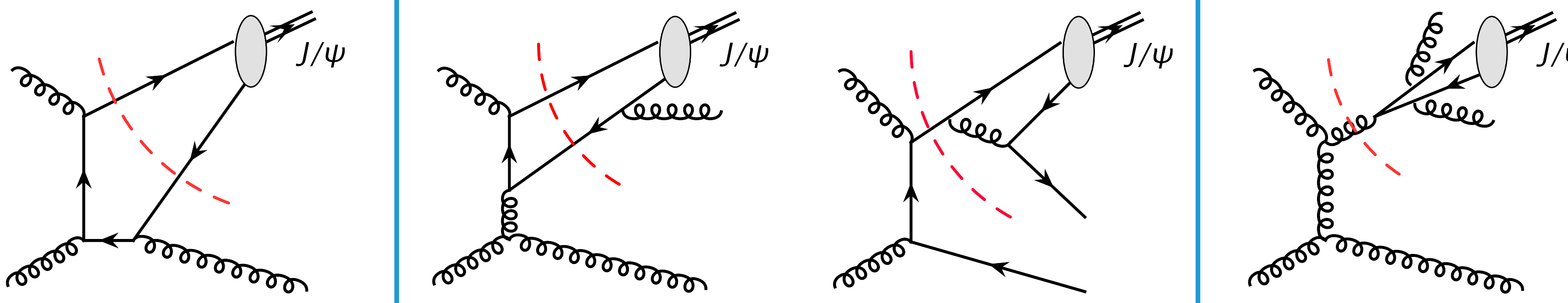
# NLO theory fits – Chao et al.



# Why high orders in NRQCD are so large?

Kang, Qiu and Sterman, 2011

## ❑ Consider J/$\psi$ production in CSM:




**LO in $\alpha_s$**

**NNLP** $\propto \alpha_s^3 \frac{m_Q^4}{p_T^8}$

**NLO in $\alpha_s$**

**NLP in 1/P$_T$** $\propto \alpha_s^4 \frac{m_Q^2}{p_T^6}$

**NNLO in $\alpha_s$**

**LP:** $\propto \alpha_s^5 \frac{1}{p_T^4}$

✧ **High-order correction receive power enhancement**

✧ **Expect no further power enhancement beyond NNLO**

✧ $[\alpha_s \ln(p_T^2/m_Q^2)]^n$ **ruins the perturbation series at sufficiently large p$_T$**

***Leading order in $\alpha_s$-expansion =\= leading power in 1/p$_T$-expansion!***

***At high p$_T$, fragmentation contribution dominant***

# QCD factorization approach

Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010, …

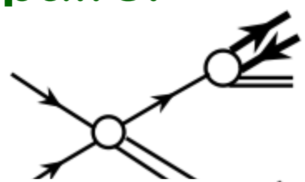
## ❑ Factorization formalism:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to [Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$

## ❑ Production of the pairs:

$$\hat{p}_Q = \frac{1+\zeta}{2z}\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\hat{p}$$

✧ at $1/m_Q$:

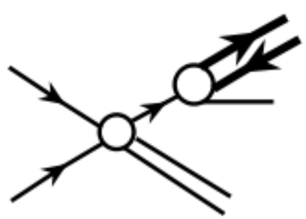


$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to [Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa), \mu)$$

✧ between: [ $1/m_Q$ , $1/P_T$]



$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots$$
$$+ \frac{m_Q^2}{\mu^2}\Gamma(z) \otimes D_{[Q\bar{Q}(\kappa)\to H}(\{z_i\}, m_Q, \mu)$$

# Evolution of fragmentation functions

Kang, Ma, Qiu and Sterman, 2013

- **Independence of the factorization scale:**

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$$

- **at Leading power in $1/P_T$:** **DGALP evolution**

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$

- **next-to-leading power in $1/P_T$ – New non-linear evolution!**

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

- **Evolution kernels are perturbative:**

- **Set mass:** $m_Q \to 0$ **with a caution**

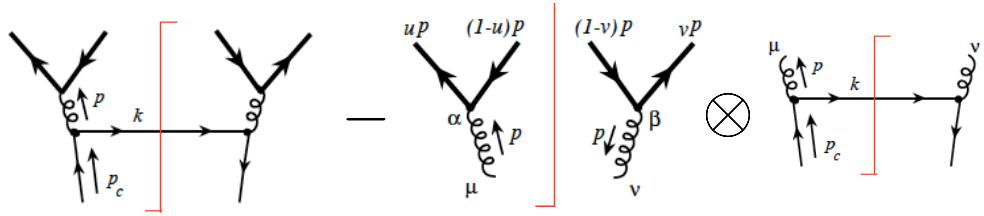
# Evolution kernels

Kang, Ma, Qiu and Sterman, 2013

## ❑ Kernel for 1 ➔ $Q\bar{Q}$ at ( $\alpha_s^2$ ):

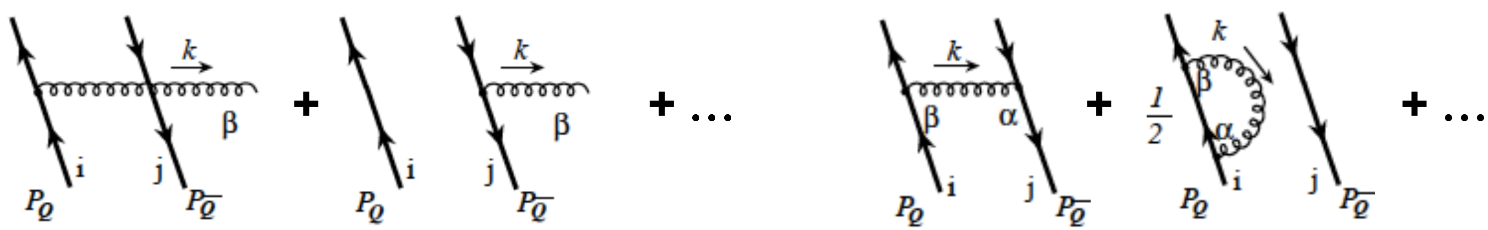
$$\frac{1}{\mu^2}\Gamma^{(2)}_{f\to[Q\bar{Q}(sI)]}(z,u,v) = \frac{\partial}{\partial \ln \mu^2}\mathcal{D}^{(2)}_{f\to[Q\bar{Q}(sI)]}(z,u,v;\mu^2) - \int_z^1 \frac{dz'}{z'}\mathcal{D}^{(1)}_{g\to[Q\bar{Q}(sI)]}(z',u,v)\gamma^{(1)}_{f\to g}(\frac{z}{z'})$$

**Example:** "$q \to [Q\bar{Q}(v8)]$"




$$\Gamma^{(2)}_{q\to[Q\bar{Q}(v8)]}(z,u,v) = \alpha_s^2\left[\frac{N_c^2-1}{4N_c}\right]\left(\frac{64(1-z)}{z^2}\right)$$

## ❑ Kernel for $Q\bar{Q}$ ➔ $Q\bar{Q}$ at ( $\alpha_s$ ):




**Example:** " $[Q\bar{Q}(v8)] \to [Q\bar{Q}(v1)]$ "

$$K^{(1)}_{v8\to v1}(z,u,v;u'v') = \frac{\alpha_s}{2\pi}\left[\frac{1}{2N_c}\right]\frac{z}{2(1-z)}\left(\frac{u}{u'}+\frac{\bar{u}}{\bar{u}'}\right)\left(\frac{v}{v'}+\frac{\bar{v}}{\bar{v}'}\right)$$
$$\times\left[\delta(u-zu')-\delta(\bar{u}-z\bar{u}')\right]\left[\delta(v-zv')-\delta(\bar{v}-z\bar{v}')\right]$$

***All channels are calculated***

# Short-distance hard parts

Kang, Ma, Qiu and Sterman, 2014

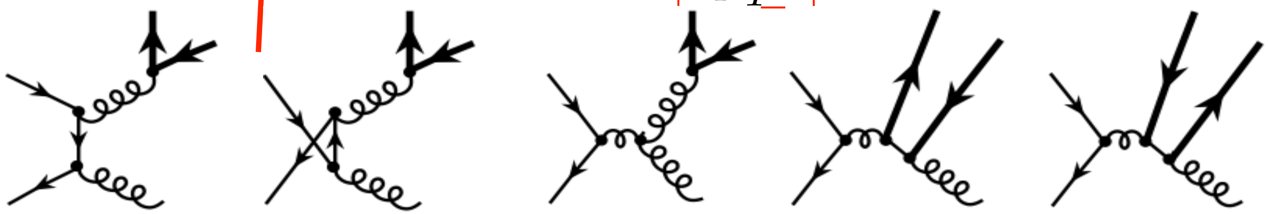
## ❑ Separation of different powers:

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

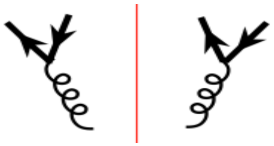
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$\frac{\alpha_s^3(\mu)}{p_T^6}$ $\qquad$ $\frac{\alpha_s^2(\mu)}{p_T^4}$ $\qquad$ $\frac{\alpha_s(2m_Q)}{(2m_Q)^2}$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$ :



$D^{(1)}_{g\to[Q\bar{Q}]}$ :



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$E_p \frac{d\hat{\sigma}^{(3)}_{q+\bar{q}\to[Q\bar{Q}(n)](p)}}{d^3p} \equiv \left[\frac{4\pi\alpha_s^3}{\hat{s}}\right] \frac{1}{\bar{u}u\bar{v}v} H_{q\bar{q}\to[Q\bar{Q}(n)]}(\hat{s},\hat{t},\hat{u})\,\delta(\hat{s}+\hat{t}+\hat{u})$$

$$H_{q\bar{q}\to[Q\bar{Q}(a8)]}(\hat{s},\hat{t},\hat{u}) = 2\left[\frac{N_c^2-1}{8N_c}\right]\left[1+\zeta_1\zeta_2-\frac{4}{N_c^2}\right]\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\right]$$

***All channels are calculated***

# Predictive power and status

- **Calculation of short-distance hard parts in pQCD:**

  Power series in $\alpha_s$, without large logarithms

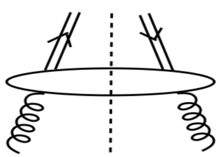
  LO is now available for all partonic channels — Kang, Ma, Qiu and Sterman, 2014

- **Calculation of evolution kernels in pQCD:**

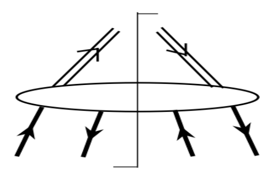
  Power series in $\alpha_s$, without large logarithms

  LO is now available for both mixing kernels and pair evolution kernels of all spin states of heavy quark pairs — Kang et al. 2013, Fleming et al. 2013

- **Input FFs at $\mu_0$ – non-perturbative, but, universal**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- **Physics of the input scale: $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

  ***Different quarkonium states require different input distributions!***

# Non-perturbative input distributions

- **Sensitive to the properties of quarkonium produced:**

  **Should, in principle, be extracted from experimental data**

- **Large heavy quark mass and clear scale separation:**

  $\mu_0 \sim m_Q \gg m_Q\, v$ ➡ **Apply NRQCD to the FFs** – ***a conjecture!***

  ✧ **Single parton FFs – valid to two-loops:**

  Nayak, Qiu and Sterman, 2005

  $$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

  Braaten, Yuan, 1994
  Ma, 1995, …
  Braaten, Chen, 1997
  Braaten, Lee, 2000,
  Ma, Qiu, Zhang, 2013
  …

  ***Complete LO+NLO for S, P states & NNLO for singlet S state***

  ✧ **Heavy quark pair FFs – valid to one-loop:**

  $$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

  Kang, Ma, Qiu and Sterman, 2014

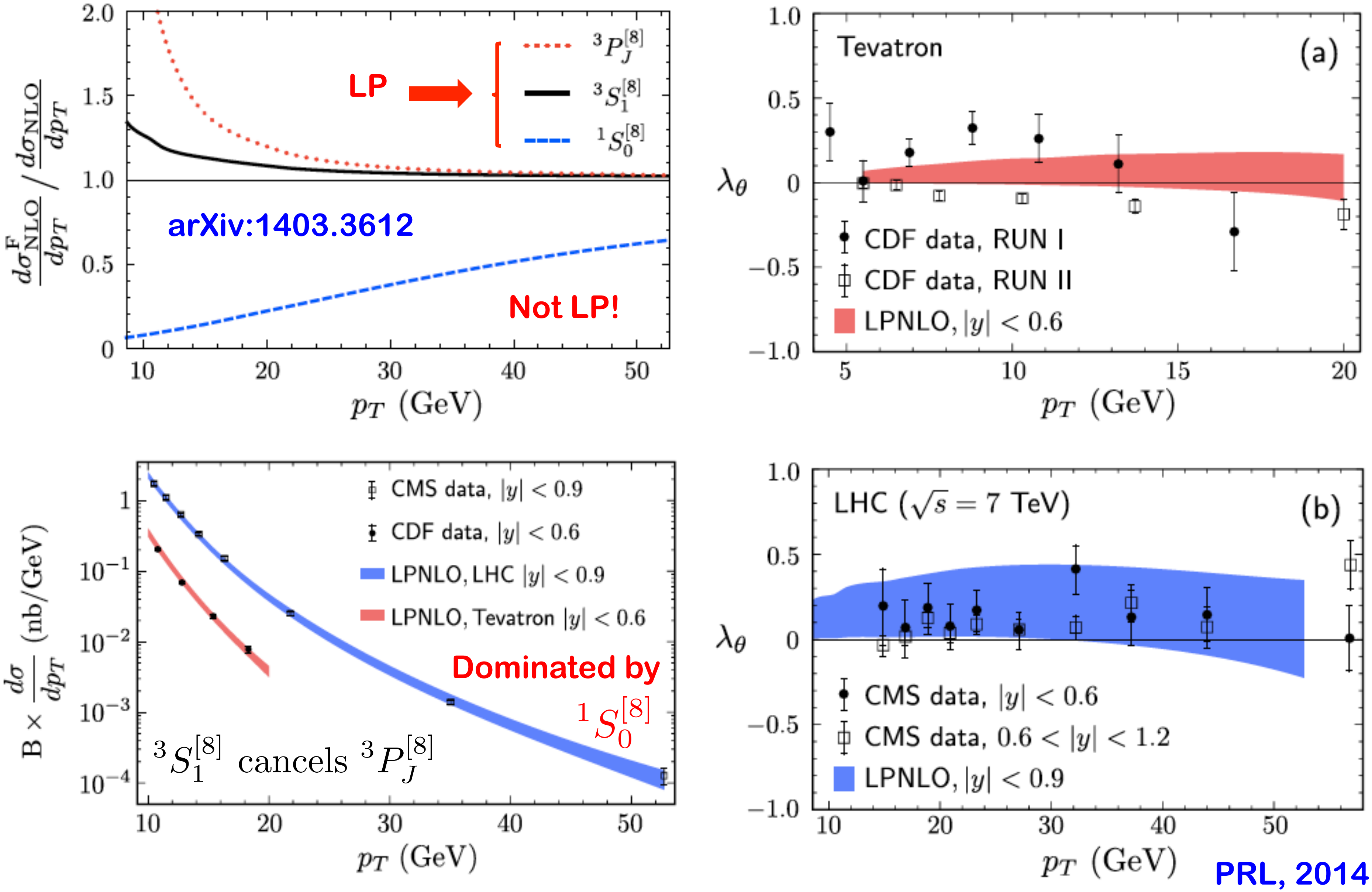
  ***Full LO+NLO for S, P states is now available***

  Ma, Qiu, Zhang, 2013

- **No all-order proof of such factorization yet!**

  ***Reduce "many" unknown FFs to a few universal NRQCD matrix elements!***

# Leading power fragmentation – Bodwin et al.



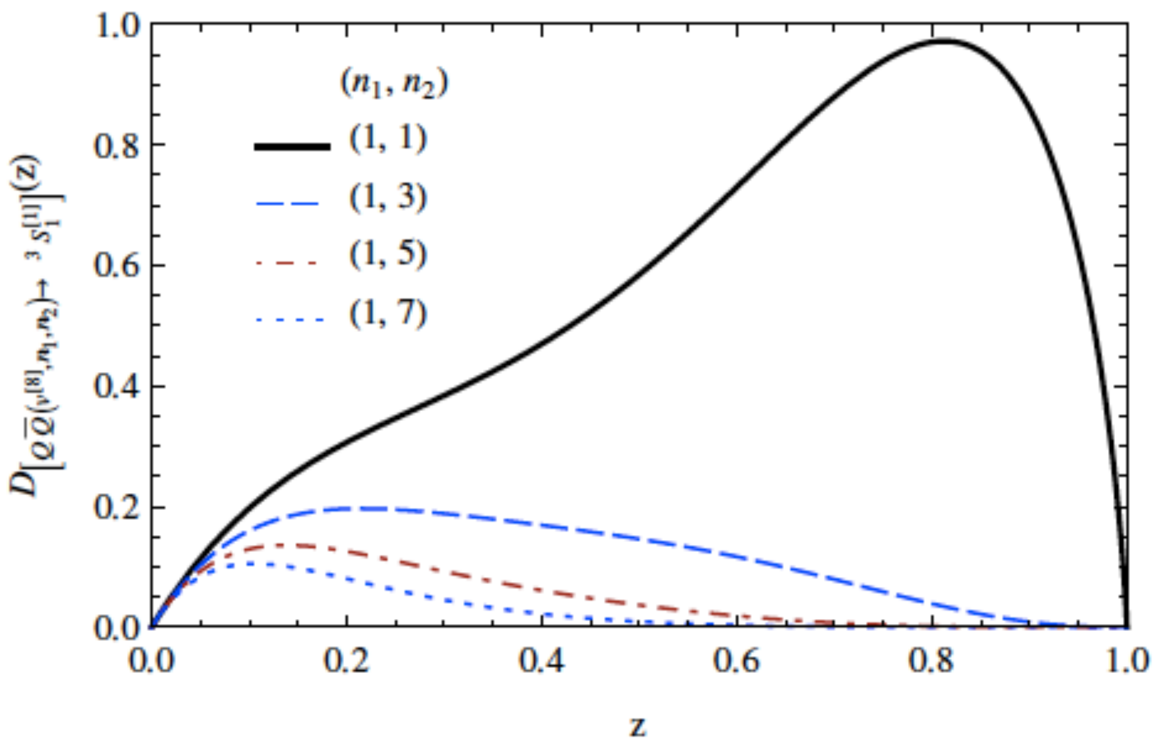
# Next-to-leading power fragmentation – Ma et al.

Ma, Qiu, Zhang, 2013

- **Heavy quark pair FFs:**

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \Big\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \Big\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

- **Moment of the FFs:** $\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$

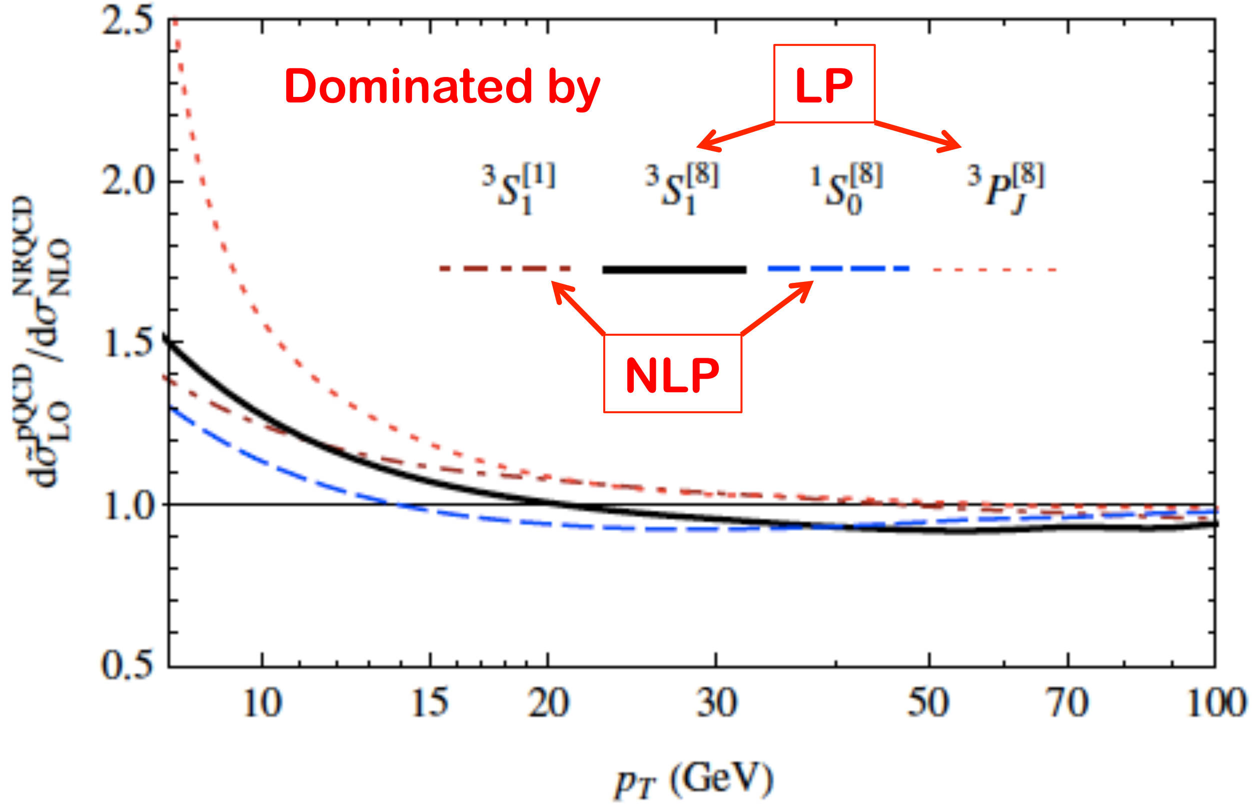


# Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- Channel-by-channel comparison:




independent of NRQCD matrix elements

LO analytical results reproduce NLO NRQCD calculations (numerical)

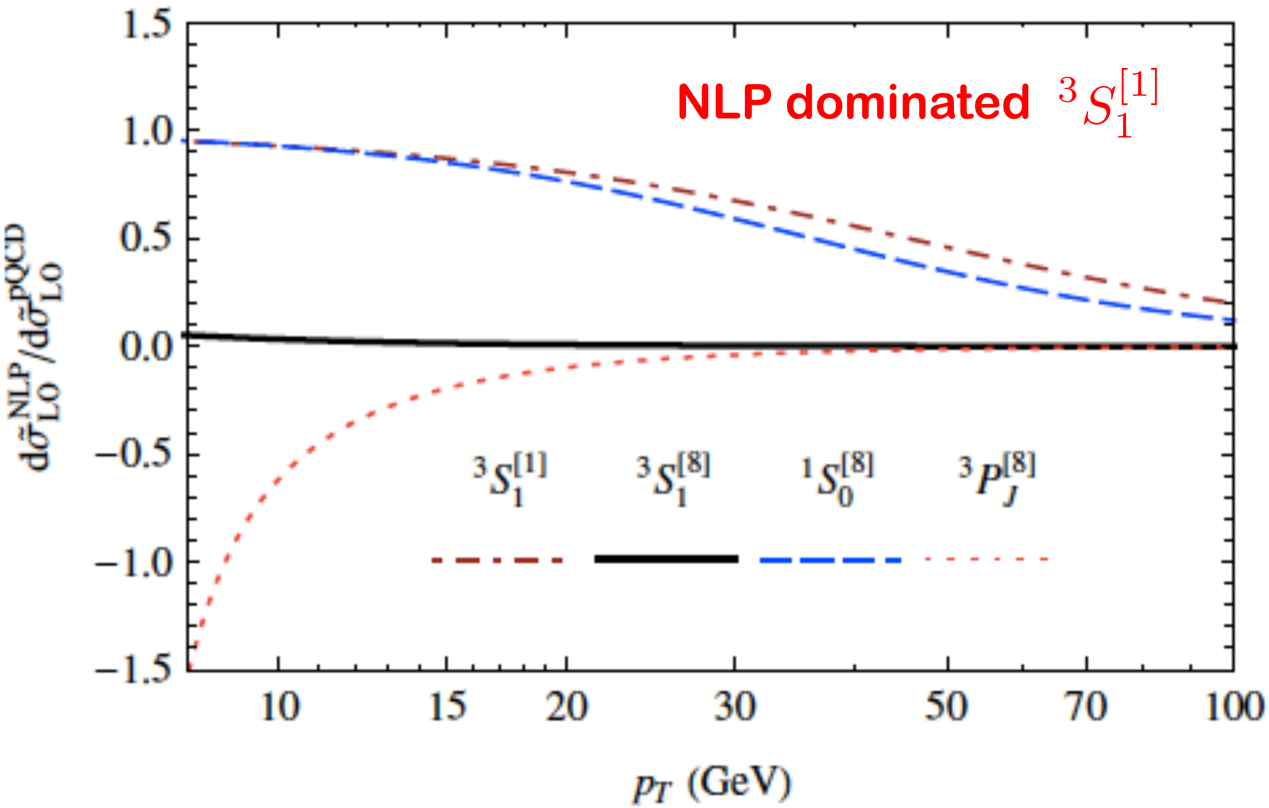
PRL, 2014

# Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- LP vs. NLP (both LO):




NLP dominated $^1S_0^{[8]}$ for wide pT

LP dominated $^3S_1^{[8]}$ and $^3P_J^{[8]}$

PT distribution is consistent with distribution of $^1S_0^{[8]}$

PRL, 2014

# QCD factorization vs NRQCD factorization

## ❑ QCD factorization – not always true:

- ✧ Expand physical cross section in powers of $1/p_T$
- ✧ Expand the coefficient of each term in powers of $\alpha_s$
- ✧ Factorization is valid for all powers of $\alpha_s$ of the $1^{st}$ two terms in $1/p_T$

## ❑ NRQCD factorization – conjectured:

- ✧ Expand physical cross section in powers of relative velocity of HQ
- ✧ Expand the coefficient of each term in powers of $\alpha_s$
- ✧ Verified to NNLO in $\alpha_s$ for the leading power term in the *v*-expansion

## ❑ Connection:

If NRQCD factorization for fragmentation functions is valid,

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$
$$+ E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

Mass effect + connection to lower $p_T$ region

# Heavy quarkonium polarization

Ma et al. 2014

- **Polarization = input fragmentation functions:**
  - ✧ Partonic hard parts and evolution kernels are perturbative
  - ✧ Insensitive to the properties of produced heavy quarkonia

- **Projection operators – polarization tensors:**

$$\mathcal{P}^{\mu\nu}(p) \equiv \sum_{\lambda=0,\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}$$

Unpolarized quarkonium

$$\mathcal{P}_T^{\mu\nu}(p) \equiv \frac{1}{2}\sum_{\lambda=\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = \frac{1}{2}\left[-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n}\right]$$

Transversely polarized quarkonium

$$\mathcal{P}_L^{\mu\nu}(p) \equiv \mathcal{P}^{\mu\nu}(p) - 2\mathcal{P}_T^{\mu\nu}(p) = \frac{1}{p^2}\left[p^\mu - \frac{p^2}{2p\cdot n}n^\mu\right]\left[p^\nu - \frac{p^2}{2p\cdot n}n^\nu\right]$$

Longitudinally polarized quarkonium

for produced the quarknium moving in +z direction with

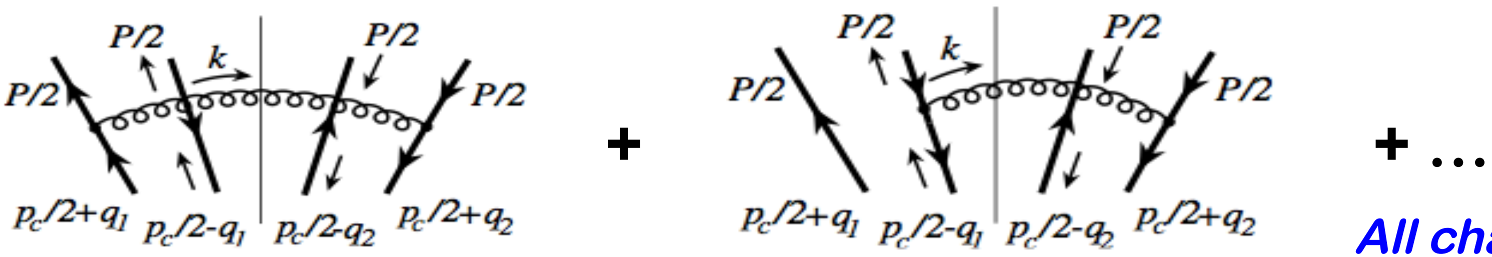
$$p^\mu = (p^+, p^-, p_\perp) = p^+(1, 0, \mathbf{0}_\perp) \qquad p^2 = n^2 = 0$$

$$n^\mu = (n^+, n^-, n_\perp) = (0, 1, \mathbf{0}_\perp) \qquad p\cdot n = p^+$$

# Polarized fragmentation functions

Kang, Ma, Qiu and Sterman, 2014
Zhang, Ph.D. Thesis, 2014

## ❑ Color singlet as an example:




*All channels are calculated*

### ✧ A axial vector pair to a singlet NRQCD pair:

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

### ✧ A vector pair to a singlet NRQCD pair:

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}\frac{z}{1-z}\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

**where**

$$\Delta_+(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)+\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)+\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$

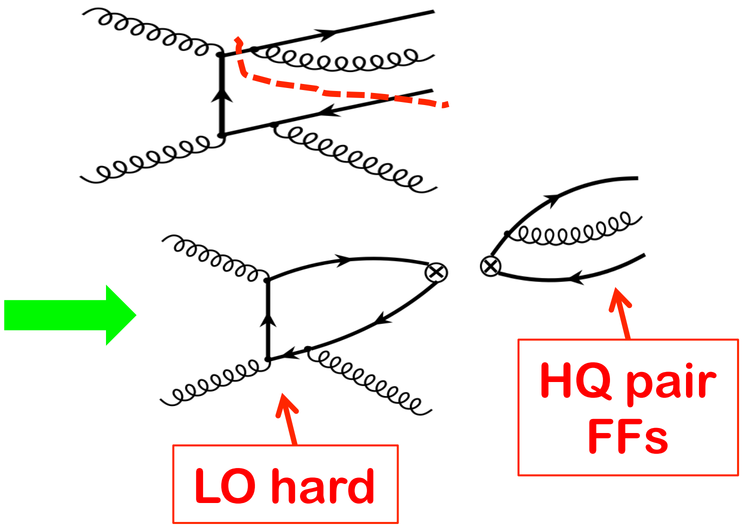
$$\Delta_-(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)-\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)-\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$

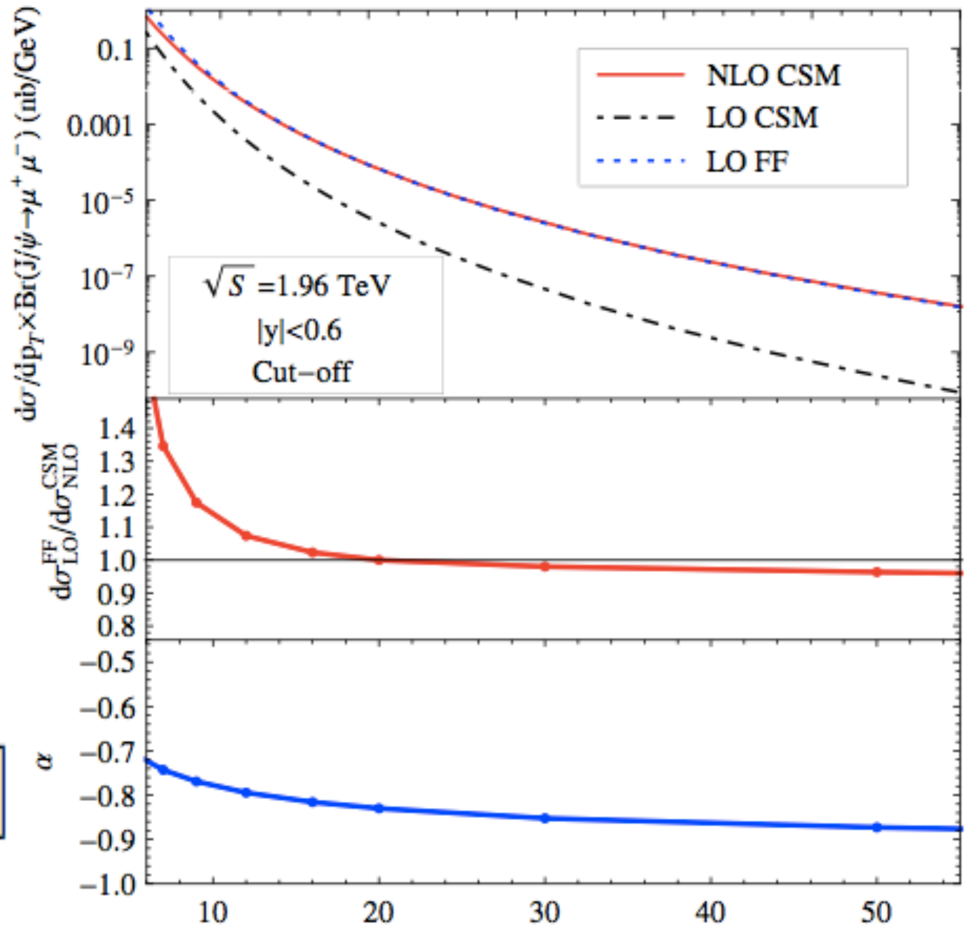
$$r(z) \equiv \frac{z^2\mu^2}{4m_c^2(1-z)^2}$$

# Production and polarization

Kang, Ma, Qiu and Sterman, 2014

## ❑ Color singlet as an example:




$$\sigma^{(\mathrm{NLO})}_{\mathrm{NRQCD}} \propto \left[ d\hat{\sigma}^{A(\mathrm{LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(\mathrm{LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \right]$$

Reproduce NLO CSM for $p_T$ > 10 GeV!

Cross section + polarization

*QCD Factorization = better controlled HO corrections!*

# Summary

- It has been almost 40 years since the discovery of J/Ψ
- When $p_T >> m_Q$ at collider energies, earlier models calculations for the production of heavy quarkonia are not perturbatively stable

  LO in $\alpha_s$-expansion may not be the LP term in $1/p_T$-expansion

- QCD factorization works for both LP and NLP ($\alpha_s$ for each power)
  - LP dominates: ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels
  - NLP dominates: ${}^1S_0^{[8]}$ and ${}^3S_1^{[1]}$ channels
  - From current data: ${}^3P_J^{[8]}$ likely to cancel ${}^3S_1^{[8]}$
    the production dominated by ${}^1S_0^{[8]}$
- A full global analysis, based on QCD factorization formalism including NLP and evolution, is needed!

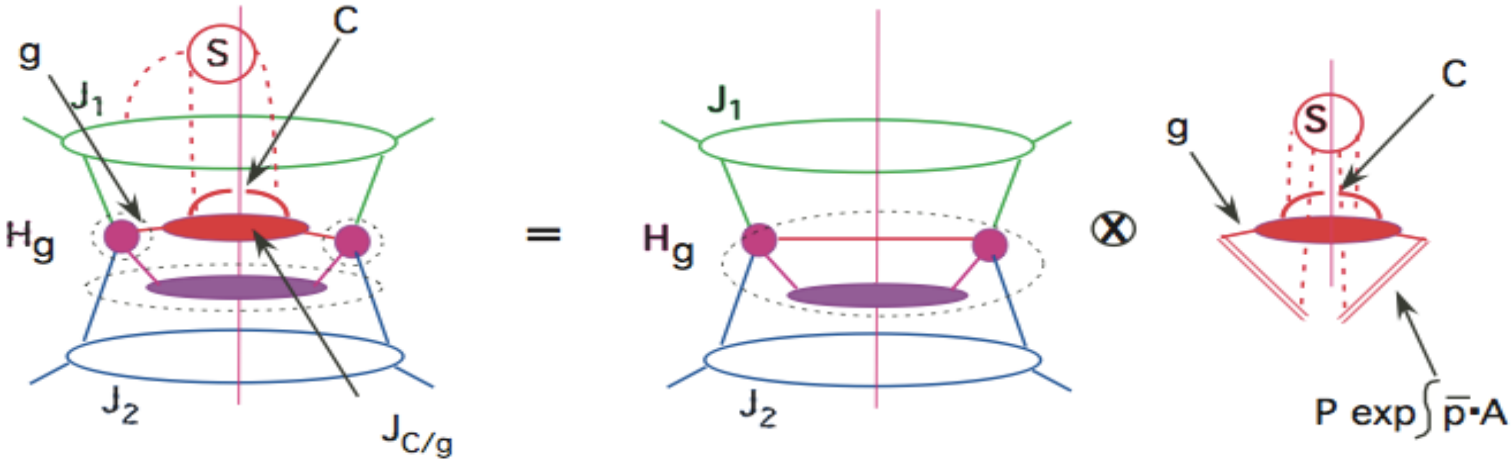
**Thank you!**

# Backup slides

# PQCD Factorization

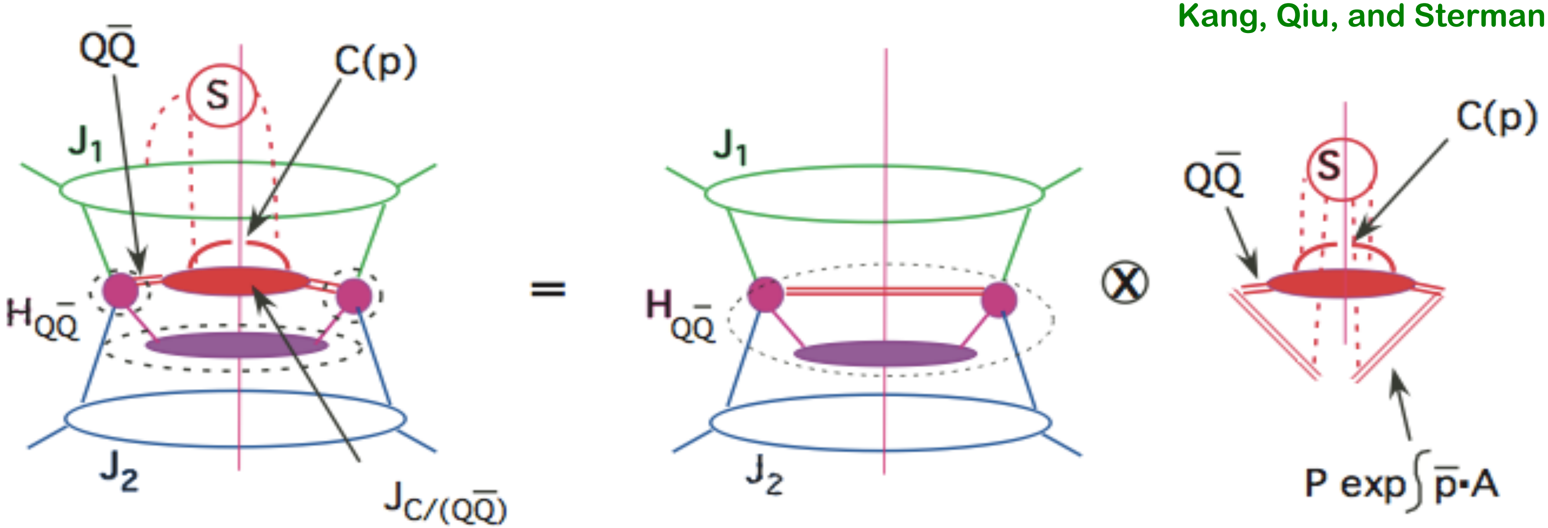
Nayak, Qiu, and Sterman, 2005

- Leading power – single hadron production



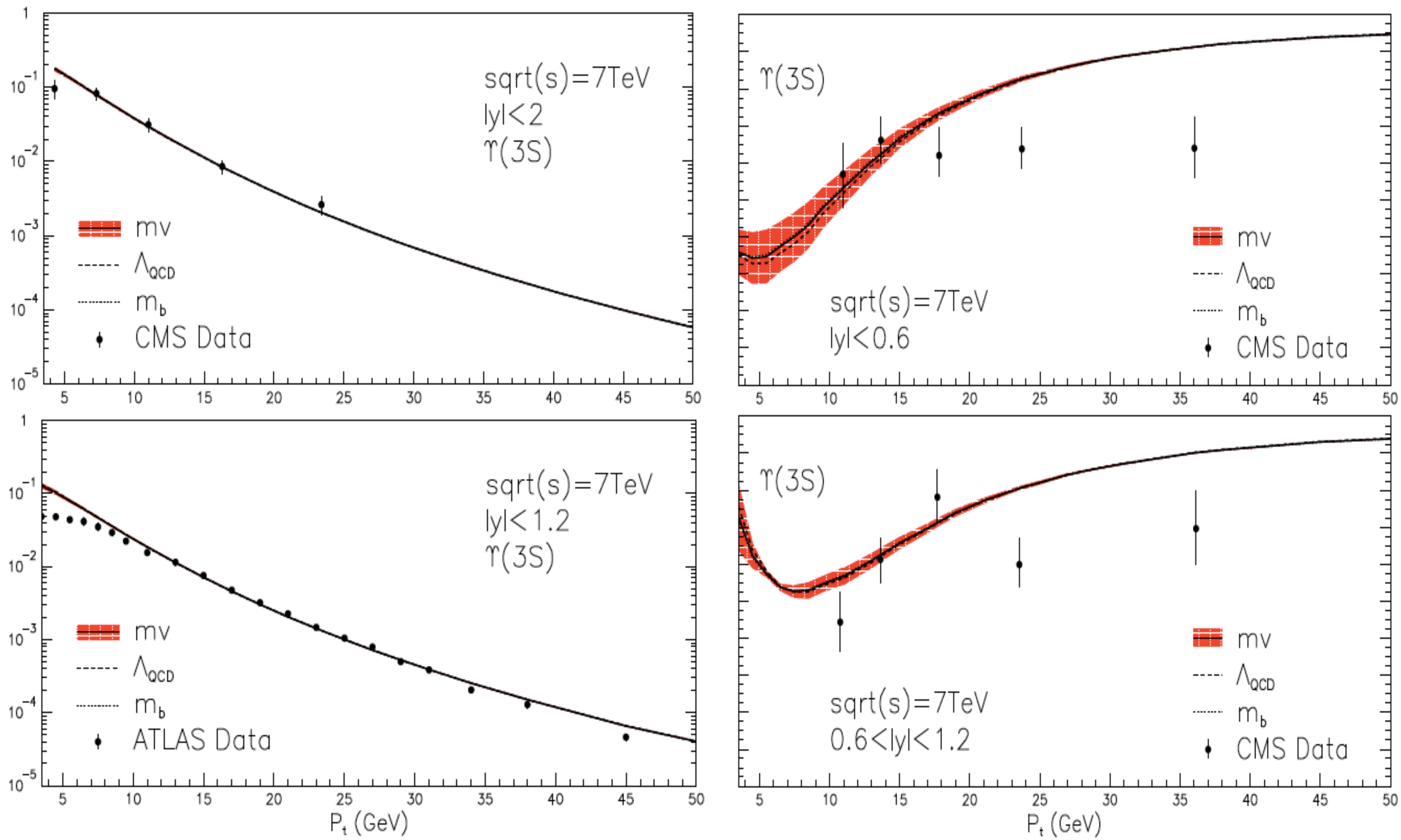

- Next-to-leading power – $Q\bar{Q}$ channel:

Qiu, Sterman, 1991
Kang, Qiu, and Sterman, 2010



# NLO theory fits – Υ production

**Cross section** | **Polarization**

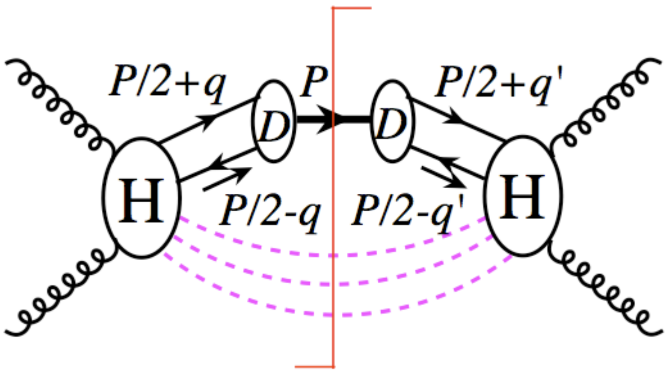



Gong et al. PRL, 2013

# Production of heavy quark pairs

Kang, Ma, Qiu and Sterman, 2013

## ❑ Perturbative pinch singularity:




$$P^\mu = (P^+, 4m^2/2P^+, 0_\perp)$$

$$q^\mu = (q^+, q^-, q_\perp)$$

$$q \neq q'$$

$$D_{ij}(P,q) \propto \langle \mathrm{J}/\psi | \psi_i^\dagger(0)\, \chi_j(y) | 0 \rangle$$

### ✧ Scattering amplitude:

$$\mathcal{M} \propto \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr}\left[ \hat{H}(P,q,Q) \frac{\gamma\cdot(P/2-q)+m}{(P/2-q)^2 - m^2 + i\epsilon} \hat{D}(P,q) \frac{\gamma\cdot(P/2+q)+m}{(P/2+q)^2 - m^2 + i\epsilon} \right]$$

### ✧ Potential poles:

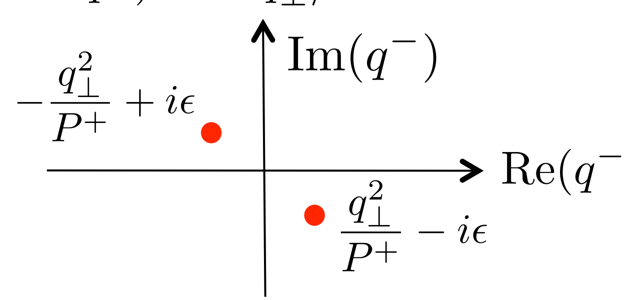
$$q^- = [q_\perp^2 - 2m^2(q^+/P^+)]/(P^+ + 2q^+) - i\epsilon\theta(P^+ + 2q^+) \rightarrow q_\perp^2/P^+ - i\epsilon$$

$$q^- = -[q_\perp^2 + 2m^2(q^+/P^+)]/(P^+ - 2q^+) + i\epsilon\theta(P^+ - 2q^+) \rightarrow -q_\perp^2/P^+ + i\epsilon$$

### ✧ Condition for pinched poles:

$$P^+ \gg q^+(2m^2/q_\perp^2) \geq 2m$$

**At High $P_T$**

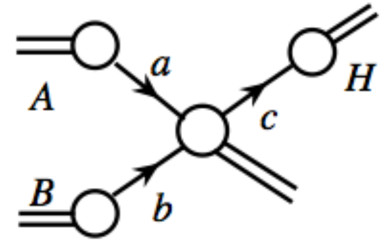


# Why such power correction are important?

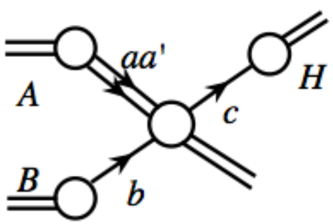
- Leading power in hadronic collisions:

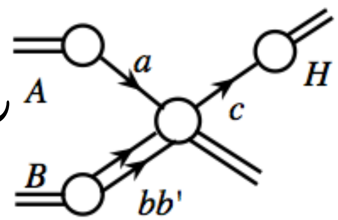
Kang, Ma, Qiu and Sterman, 2013

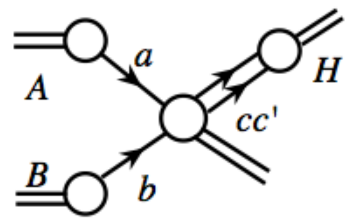
$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$




- 1st power corrections in hadronic collisions:



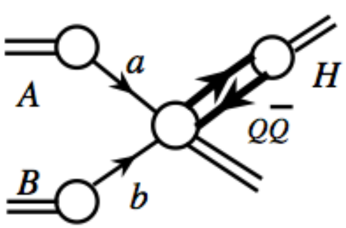
$\sim$


$\sim$



$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

or $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$

- Dominated 1st power corrections:




$$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$$

**Key: competition between** $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

# Evolution kernels

Kang, Ma, Qiu and Sterman, 2013

## ❑ Evolution equation:

$$\kappa, \kappa' = v, a, t$$

$$\frac{\partial}{\partial \ln \mu^2} \mathcal{D}_{Q\bar{Q}[\kappa]\to J/\psi}(z_h, \zeta_1, \zeta_2, \mu^2)$$

$$= \frac{\alpha_s}{2\pi} \int_{z_h}^{1} \frac{dz}{z} \int_{-1}^{1} d\zeta_1' \int_{-1}^{1} d\zeta_2' P_{\kappa\to\kappa'}(\zeta_1, \zeta_2, \zeta_1', \zeta_2', z) \mathcal{D}_{Q\bar{Q}[\kappa']\to J/\psi}(z_h/z, \zeta_1', \zeta_2', \mu^2)$$

## ❑ Evolution kernels:

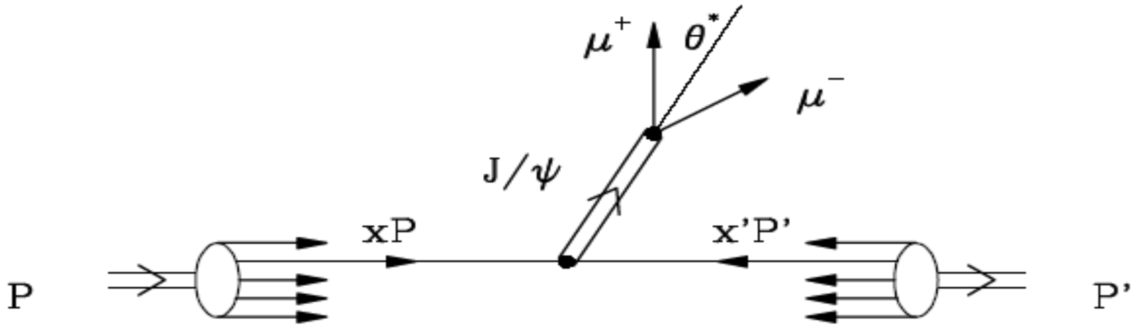
$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} \mathcal{K}_1 & \mathcal{T}_1 & \mathcal{K}_2 & \mathcal{T}_2 & 0 & 0 \\ \mathcal{R}_1 & \mathcal{S}_1 & \mathcal{R}_2 & 0 & 0 & 0 \\ \mathcal{K}_2 & \mathcal{T}_2 & \mathcal{K}_1 & \mathcal{T}_1 & 0 & 0 \\ \mathcal{R}_2 & 0 & \mathcal{R}_1 & \mathcal{S}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{K}_1' & \mathcal{T}_1' \\ 0 & 0 & 0 & 0 & \mathcal{R}_1' & \mathcal{S}_1' \end{pmatrix} \otimes \begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix}$$

**Example:** $\mathcal{K}_1 = P_{v8\to v8} = P_{a8\to a8}$

**NOTE:** Our results are consistent with those by Fleming et al. [arXiv: 1301.3822], but, a difference in logarithms

# Heavy quarkonium polarization

- Measure angular distribution of $\mu^+\mu^-$ in J/ψ decay




- Normalized distribution – integrate over φ:

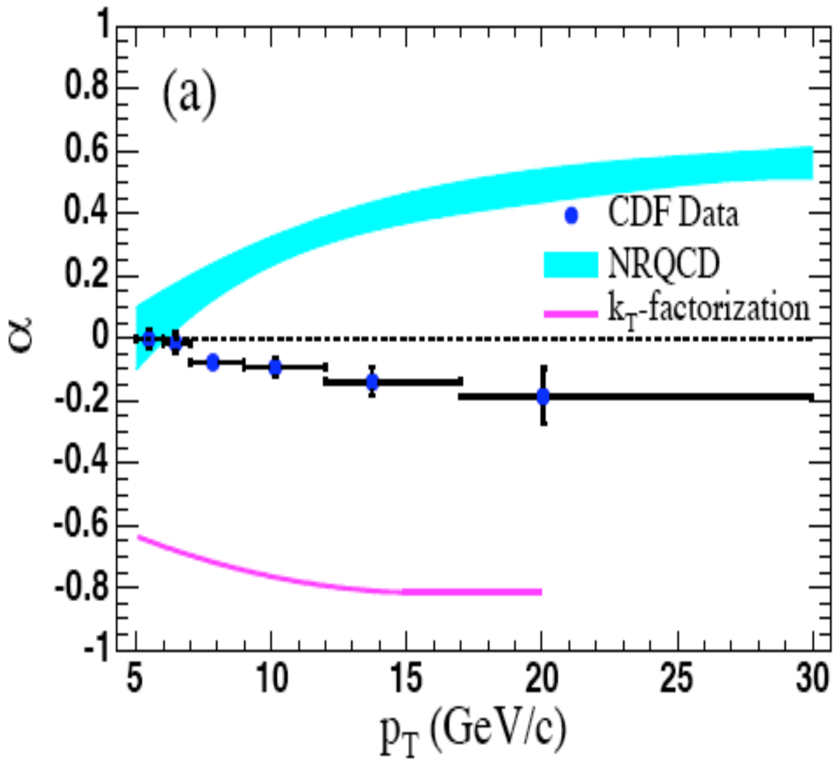
$$I(\cos\theta^*) = \frac{3}{2(\alpha+3)}\left(1+\alpha\cos^2\theta^*\right)$$

$$\alpha = \begin{cases} +1 & \text{fully transverse} \\ 0 & \text{unpolarized} \\ -1 & \text{fully longitudinal} \end{cases}$$

**Also referred as $\lambda_\theta$ by LHC experiments**

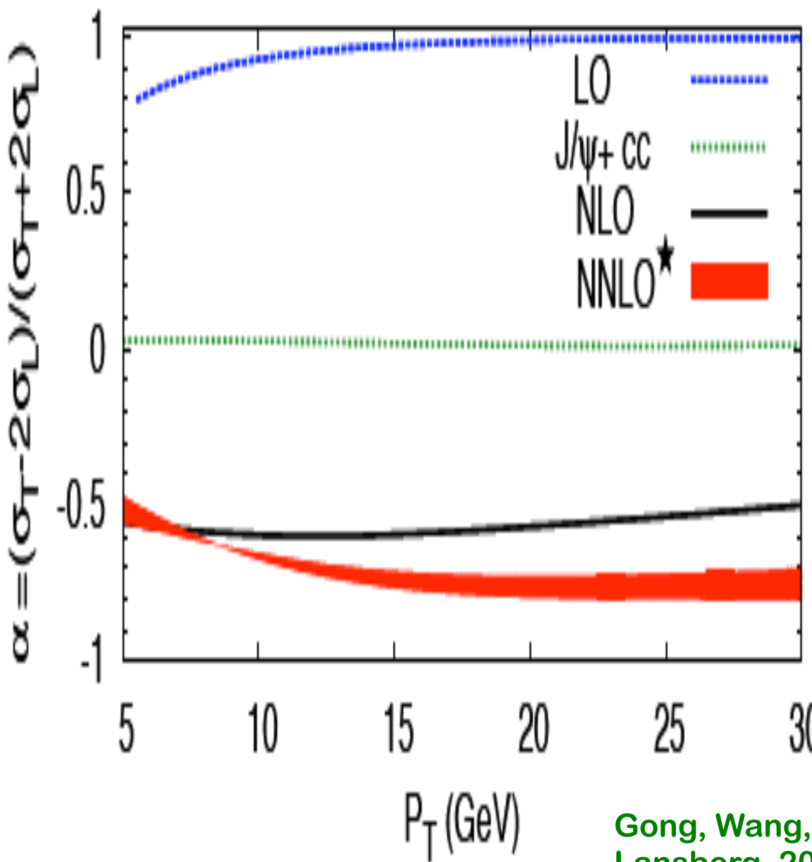
# Theory predictions on J/ψ polarization

**NRQCD**




Cho & Wise, Beneke & Rothstein, 1995, …

**CSM**




Gong, Wang, 2008
Lansberg, 2009

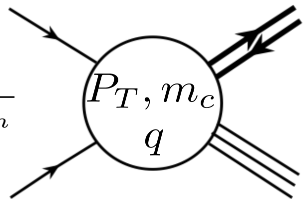
- ✧ **NRQCD: Dominated by color octet – NLO is not a huge effect**
- ✧ **CSM: Huge NLO – change of polarization?**

# Relativistic corrections

## ❑ Leading $v^2$ relativistic correction:

Fan, Ma, Chao, 2009

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$




$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

$$R^{(1)}({}^3S_1^{[1]}) = \left.\frac{G({}^3S_1^{[1]})}{F({}^3S_1^{[1]})}\right|_{p_T \gg m} = \frac{1}{6},$$

$$R^{(1)}({}^1S_0^{[8]}) = \left.\frac{G({}^1S_0^{[8]})}{F({}^1S_0^{[8]})}\right|_{p_T \gg m} = -\frac{5}{6},$$

$$R^{(1)}({}^3S_1^{[8]}) = \left.\frac{G({}^3S_1^{[8]})}{F({}^3S_1^{[8]})}\right|_{p_T \gg m} = -\frac{11}{6},$$

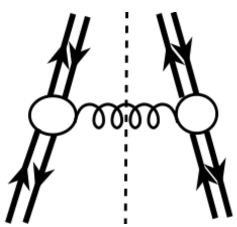
$$R^{(1)}({}^3P^{[8]}) = \left.\frac{G({}^3P^{[8]})}{F({}^3P^{[8]})}\right|_{p_T \gg m} = -\frac{31}{30},$$

## ❑ All order $v^2$ corrections:

Ma, Qiu, 2013

$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$R({}^1S_0^{[8]}) = 1 - \frac{5}{6}\delta + \frac{259}{360}\delta^2 - \frac{3229}{5040}\delta^3 + \cdots.$$

$$R({}^3S_1^{[8]}) = 1 - \frac{11}{6}\delta + \frac{191}{72}\delta^2 - \frac{167}{48}\delta^3 + \cdots,$$

$$R({}^3P^{[8]}) = \frac{2R_a({}^3P^{[8]}) + R_v({}^3P^{[8]})}{3} = 1 - \frac{31}{30}\delta + \frac{4111}{4200}\delta^2 - \frac{4631}{5040}\delta^3 + \cdots$$