Heating up QGP: towards charm quark chemical equilibration

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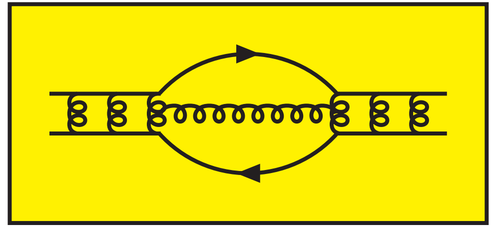
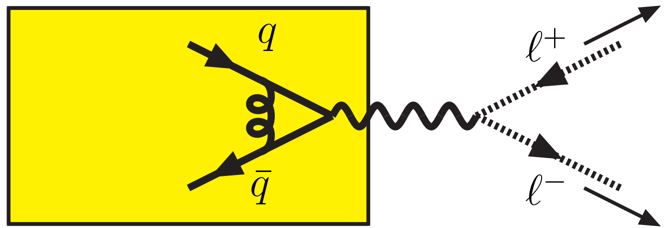
What is it?

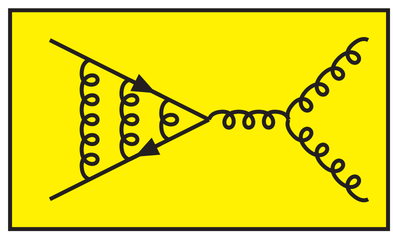
Melting /
recombination:



Leptonic annihilation:



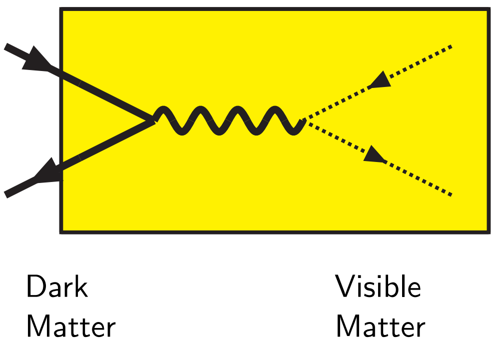
Chemical equilibration:
(in either direction)

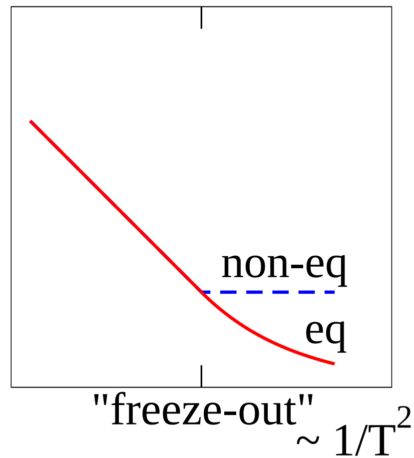


Why is it of wider interest?

Weakly Interacting Massive Particles as dark matter

The system thermalizes after inflation, but then chemically decouples when pair annihilation is not fast enough to track the equilibrium distribution, which is $\sim (\frac{MT}{2\pi})^{3/2} e^{-M/T}$ at $T \ll M$.





Back of the envelope estimate

Equate Hubble rate (H) with annihilation rate ($\Gamma \sim n\sigma v$):

$$
\begin{aligned}
H &\sim n\sigma v \\
\Leftrightarrow \frac{T^2}{m_{\mathrm{Pl}}} &\sim \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T} \frac{\alpha_w^2}{m_W^2} \left(\frac{T}{M}\right)^{1/2} \\
\Rightarrow \frac{M}{T} &\sim \ln\left[\frac{\alpha_w^2\, m_{\mathrm{Pl}}\, M}{m_W^2 (2\pi)^{3/2}}\right] \sim 30 \,.
\end{aligned}
$$

(A real computation gives $M/T \sim 25$.)

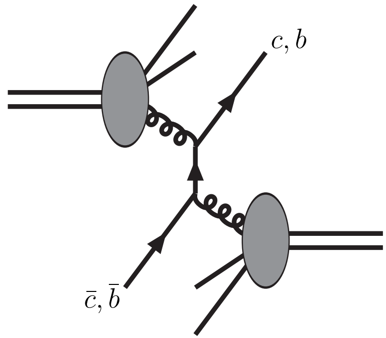
"WIMP miracle": the order of magnitude of the resulting n and energy density $e = Mn$ is correct for $M \sim 1$ TeV.

Can we “simulate” this in QCD? [1]

[1] Based on: ML and Y. Schröder, *Quark mass thresholds in QCD thermodynamics*, Phys. Rev. D 73 (2006) 085009 [hep-ph/0603048]; D. Bödeker and ML, *Heavy quark chemical equilibration rate as a transport coefficient*, JHEP 07 (2012) 130 [1205.4987]; *Sommerfeld effect in heavy quark chemical equilibration*, JHEP 01 (2013) 037 [1210.6153]; Y. Burnier and ML, *Charm mass effects in bulk channel correlations*, JHEP 11 (2013) 012 [1309.1573]; *Temporal mesonic correlators at NLO for any quark mass*, PoS (LATTICE2013) 218 [1310.6124].

(i) Initial production

Initial state is out-of-equilibrium, with a non-thermal abundance of heavy quarks with hard momenta:[2]



If nothing happens afterwards, heavy quarks and antiquarks constitute separate conserved charges.[3]

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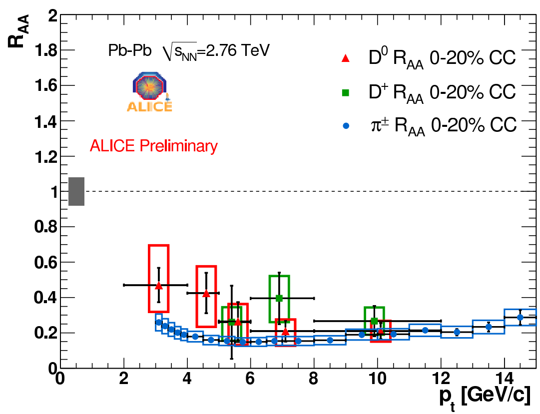
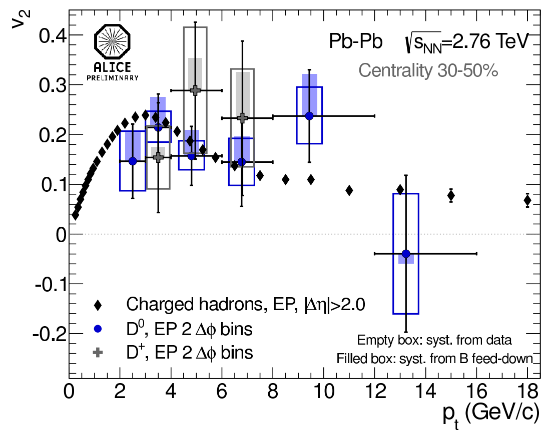
[2] e.g. M. Cacciari *et al*, Phys. Rev. Lett. 95 (2005) 122001 [hep-ph/0502203].
[3] e.g. A. Andronic *et al*, Nucl. Phys. A 789 (2007) 334 [nucl-th/0611023].

(ii) Kinetic equilibration

Charm (and even bottom) do equilibrate kinetically: jets get quenched,[4] quarks adjust their velocities to hydrodynamic flow.[5]

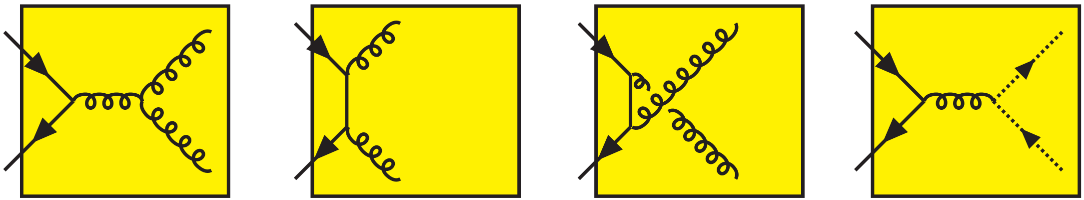




4 e.g. A. Dainese [ALICE Collaboration], 1106.4042.

5 e.g. G. Ortona [ALICE Collaboration], 1207.7239.

(iii) Chemical equilibration: how fast does pair creation or annihilation take place?



The computation is in principle the same as for strangeness,[6] and near equilibrium the answer can be expressed as:

$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2} \left(N_{\rm f} + 2C_F - \frac{N_{\rm c}}{2} \right) \left(\frac{TM}{2\pi} \right)^{\frac{3}{2}} e^{-M/T} \; .$$

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[6] T.S. Biró and J. Zimányi, Phys. Lett. B 113 (1982) 6; J. Rafelski and B. Müller, Phys. Rev. Lett. 48 (1982) 1066 [Erratum-ibid. 56 (1986) 2334]; T. Matsui, B. Svetitsky and L.D. McLerran, Phys. Rev. D 34 (1986) 783 [Erratum-ibid. D 37 (1988) 844].

Numerical estimates:

$$\Gamma_{\text{chem}} \;\simeq\; \frac{2\pi\alpha_s^2 T^3}{9M^2}\left(\frac{7}{6}+N_{\rm f}\right)\frac{\chi_f}{\chi_0}\,,$$

where χ_f, χ_0 are massive and massless quark number susceptibilities. For $N_{\rm f} = 3$, $\alpha_s \sim 0.3$, $M \sim 1.5$ GeV, and χ_f/χ_0 from lattice,[7] yields:

$$\Gamma_{\text{chem}}^{-1} \;\gtrsim\; 60\,\text{fm/c}\,, \quad \text{for } T \sim 400 \text{ MeV}\,,$$

$$\Gamma_{\text{chem}}^{-1} \;\sim\; 10\,\text{fm/c}\,, \quad \text{for } T \sim 600 \text{ MeV}\,.$$

In the current LHC setup: $\Delta t \lesssim 20$ fm/c at $T_{\text{initial}} \lesssim 500$ MeV.

Goal for HIC@FCC: $\Delta t \lesssim 50$ fm/c at $T_{\text{initial}} \lesssim 1$ GeV (?).

[7] H.-T. Ding *et al*, 1011.0695; S. Borsanyi *et al*, 1204.0995.

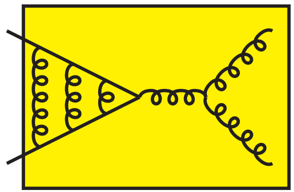
Open questions

- Validity of the weak-coupling expansion?
- Validity of the non-relativistic expansion?
- Non-equilibrium effects beyond linear response?
- Geometry, asymmetries, ...

Issues with perturbation theory (staying non-relativistic)

Sommerfeld effect (i)

Pair-annihilating particles have strong "initial state" interactions; pair-created particles have strong "final state" interactions.



The methods have been elucidated in cosmology, where the "Sommerfeld effect" may also play an important role.[8]

[8] J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter,* Phys. Lett. B 646 (2007) 34 [hep-ph/0610249]; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter,* Phys. Rev. D 82 (2010) 083525 [1005.4678]; A. Hryczuk, R. Iengo and P. Ullio, *Relic densities including Sommerfeld enhancements in the MSSM,* JHEP 03 (2011) 069 [1010.2172]; A. Strumia, *Sommerfeld corrections to type-II and III leptogenesis,* Nucl. Phys. B 809 (2009) 308 [0806.1630].

Sommerfeld effect (ii)

Consider two heavy particles of mass M, interacting through an attractive Coulomb-like potential

$$V(r) = -\frac{g^2 C_{\rm F}}{4\pi r} \; ,$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is the relative distance. Recalling that the reduced mass is $M/2$, and denoting by v the velocity with respect to the center-of-mass frame ($v = v_{\text{rel}}/2$), the stationary Schrödinger equation takes the form

$$\left(-\frac{\nabla^2}{M} + V(r) \right) \psi = M v^2 \psi \; .$$

The probability that the two particles meet, allowing them to co-annihilate, is proportional to $|\psi|^2(0)$.

Sommerfeld effect (iii)

Now, we could first solve the problem with free particles, obtaining a plane-wave solution, and an r-independent $|\psi|^2_{(g^0)}$.

However, because of the attractive force, there is an increased probability for the particles to meet.

This increase constitutes the **Sommerfeld effect**, and is characterized by the coefficient

$$S_1 \equiv \frac{|\psi|^2_{(g^2)}(0)}{|\psi|^2_{(g^0)}(0)} \; .$$

[This can be defined separately for s-wave, p-wave, ...]

Sommerfeld effect (iv)

Remarkably, the value of S_1 can be determined in closed form for the s-wave case:[9]

$$S_1 = \frac{X_1}{1 - e^{-X_1}} \,, \quad X_1 = \frac{g^2 C_\mathrm{F}}{4v} \,.$$

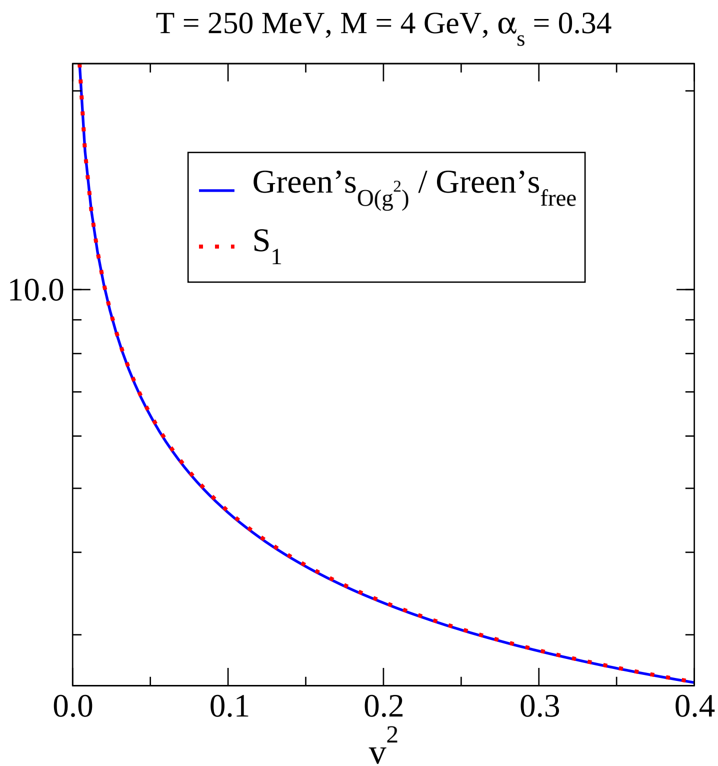
If we then consider a thermal environment, the factor needs to be averaged over the thermal ensemble:

$$\bar{S}_1 \equiv \frac{4}{\sqrt{\pi}} \left(\frac{M}{T} \right)^{3/2} \int_0^\infty \mathrm{d}v \, v^2 e^{-Mv^2/T} \, S_1 \,.$$

[9] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production*, Z. Phys. C 48 (1990) 613.

Sommerfeld effect (v)

Typical values, obtained for QCD-like parameters (here b!):



Sommerfeld effect (vi)

As it happens, in pQCD the process splits up into two parts, the "colour-singlet" discussed here as well as a "colour-octet" one,[10] in which case the interaction is repulsive, and $\bar{S}_8 < 1$.

$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2} \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T} \times \left[\frac{1}{N_c}\bar{S}_1 + \left(\frac{N_c^2 - 4}{2N_c} + N_f\right)\bar{S}_8\right] .$$

The colour-octet channel is weighted more than the colour-singlet channel (with $\bar{S}_1 \simeq 3.4$). So, accidentally, the numerical effect on charm equilibration in QCD is small.

[10] Virtuality $\sim MT \gg k^0 \times$ (width for colour decoherence) $\sim M \times g^2 T/\pi$.

Beyond perturbation theory?

Recall scales:

Extent of imaginary time coordinate: $\frac{1}{T}$.

Expected physical time scale: $\frac{1}{\Gamma_{\text{chem}}} \sim \frac{M^{1/2}}{T^{3/2}}\, e^{M/T} \gg \frac{1}{T}$.

So *even if* managed to shift away the exponential factor, the dynamical time scale is still much larger than the lattice extent, and naive Wick rotation is insufficient.

The ideal theoretical probe for charm is the trace anomaly.

$$T^{\mu}{}_{\mu} = \underbrace{c_\theta\, g_{\rm B}^2 F^{a\mu\nu} F^a_{\mu\nu}}_{\equiv\,\theta} + \underbrace{\bar\psi M_{\rm B} \psi}_{\equiv\,S}\,, \quad c_\theta = -\frac{b_0}{2} - \frac{b_1 g^2}{4} + \ldots\,.$$

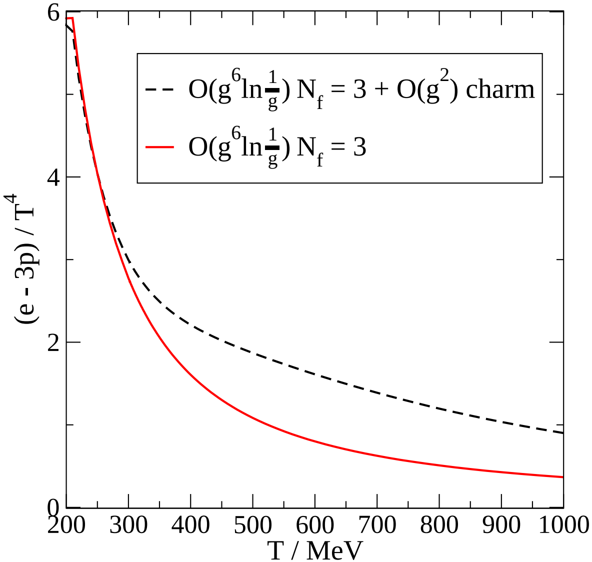
The contribution from S should be small (i) in the chiral limit $M \ll T$, and (ii) for $M \gg T$ when the charm decouples.

Is the relevant comparison $M \leftrightarrow T$, $M \leftrightarrow 3T$, $M \leftrightarrow 2\pi T$, and which mass to use for M (pole, $\overline{\rm MS}$, D^0)?

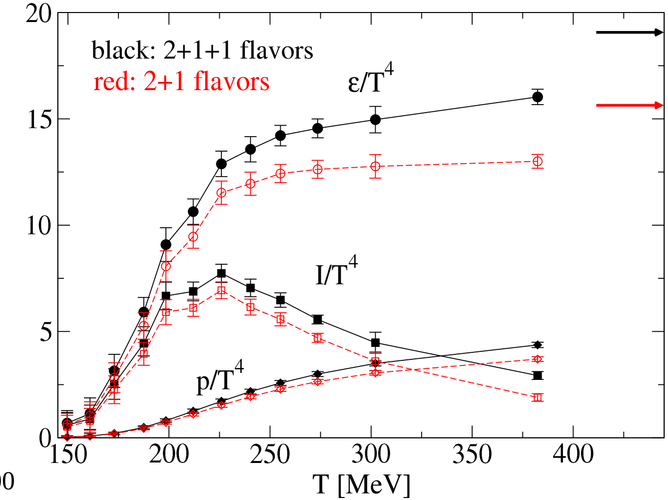
How severe is the exponential suppression?

Measure $\langle T^{\mu}{}_{\mu} \rangle_T$ *assuming* chemical equilibration.[11]

pQCD



lattice (w/o extrapolations)



[11] Lattice: M. Cheng [RBC-Bielefeld Collaboration], PoS LAT2007 (2007) 173 [0710.4357]; C. DeTar *et al*, Phys. Rev. D 81 (2010) 114504 [1003.5682]; S. Borsanyi *et al*, PoS LATTICE 2011 (2011) 201 [1204.0995].

For dynamics: the 2-point correlator of the trace anomaly.

Trace of the energy-momentum tensor yields "bulk viscosity":

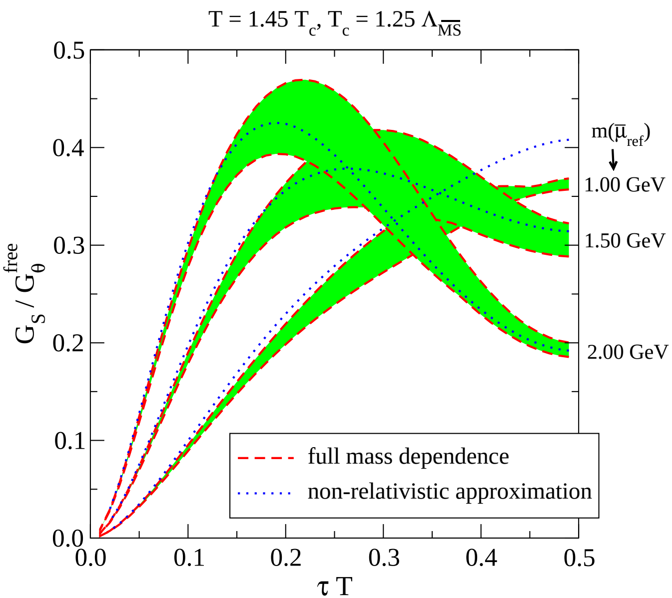
$$\zeta = \frac{1}{9} \lim_{\omega \to 0^+} \left\{ \frac{1}{\omega} \int_{\mathcal{X}} e^{i\omega t} \left\langle \frac{1}{2} [T^{\mu}{}_{\mu}(\mathcal{X}), T^{\mu}{}_{\mu}(0)] \right\rangle_T \right\} .$$

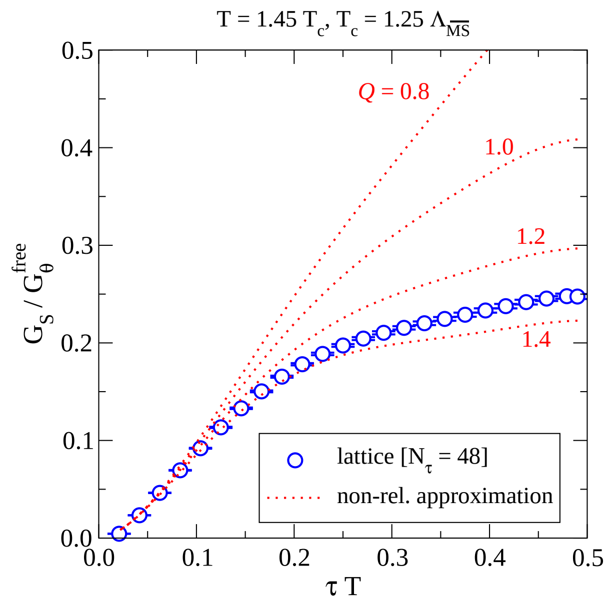
Heavy-quark contribution:

$$\delta\zeta = \frac{1}{18T} \lim_{\omega \to 0^+} \left\{ \frac{2M^2 \chi_f \Gamma_{\rm chem}}{\omega^2 + \Gamma_{\rm chem}^2} \right\} = \frac{M^2 \chi_f}{9T\Gamma_{\rm chem}} .$$

Measure:

$$G_{\rm S}(\tau) = \left\langle \int_{\mathbf{x}} S(\tau, \mathbf{x})\, S(0) \right\rangle_T .$$





$\Rightarrow$ Here charm has 25-30% influence even at $T \sim 300$ MeV. There is a strong mass dependence.

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[12] H.-T. Ding *et al*, 1204.4945 (quenched). In the simulations, $m_c(\bar{\mu}_{\text{ref}}) \approx 0.97$ GeV. In the plot, $Q \sim M/m_c(\bar{\mu}_{\text{ref}}) = 1 + 4g^2(\bar{\mu}_{\text{ref}})C_F/(4\pi)^2 + \mathcal{O}(g^4) \simeq 1.2$.

Summary

With increasing energy, it may become possible to “simulate” the non-equilibrium thermodynamics of WIMP freeze-out in future Heavy Ion Collision experiments.

In this case the weak interactions of WIMPs are replaced by the strong interactions of charm quarks, but this change is compensated for by the much faster expansion rate.

For a quantitative determination of the charm quark chemical equilibration rate as a function of temperature, further work is needed both in perturbation theory (e.g. NLO) and on the lattice.