Recent Lattice NRQCD Studies of Bottomonium at Non-Zero Temperature

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in collaboration with
P. Petreczky and A. Rothkopf (arXiv:1409.3630 and Alexander's talk) and G. Aarts, C. Allton, T. Harris, M.P. Lombardo, S.M. Ryan and J.-I. Skullerud, FASTSUM (JHEP1407 (2014) 097)

Outline

Scales at $T = 0$

- Lattice QCD is based on

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \mathcal{O} e^{-\int d^4x \mathcal{L}_E}}{\int D\phi e^{-\int d^4x \mathcal{L}_E}} \tag{1}$$

- Lattice QCD is defined on discrete space-time lattices

$\rightarrow$ various scales

a_τ, a_s (UV cutoff)

$\frac{1}{M_q}$ (Compton wavelength)

$N_s a_s$ (spatial IR cutoff)

$N_\tau a_\tau$ (temporal IR cutoff)

Scales at $T=0$

$$a_\tau << \frac{1}{M_q} << (N_s a_s, N_\tau a_\tau)$$

- for bottomonium, $M_q = M_b (\sim 4.65\mathrm{GeV})$,

  $\frac{1}{M_q} \sim 0.04$ fm and spatial size ~ 1 fm.

  if $a_s \sim 0.01$ fm, $N_s \sim 100$

Scales at $T = 0$

- bound state dynamics in quarkonium $\sim \mathcal{O}(100)$ MeV

$n^{S+1}L_J$	State	$a_\tau M$	$E_0 + M$ (MeV)	$M_{\rm expt}$ (MeV)
$1^1\mathrm{S}_0$	η_b	0.20549(4)	9409(12)	9398.0(3.2)
$2^1\mathrm{S}_0$	η_b'	0.311(3)	10004(21)	9999(4)
$1^3\mathrm{S}_1$	Υ	0.21460(5)	9460*	9460.30(26)
$2^3\mathrm{S}_1$	Υ'	0.318(3)	10043(22)	10023.26(31)
$1^1\mathrm{P}_1$	h_b	0.2963(4)	9920(15)	9899.3(1.0)
$1^3\mathrm{P}_0$	χ_{b0}	0.2921(4)	9896(15)	9859.44(52)
$1^3\mathrm{P}_1$	χ_{b1}	0.2964(4)	9921(15)	9892.78(40)
$1^3\mathrm{P}_2$	χ_{b2}	0.2978(4)	9928(15)	9912.21(40)

Table: comparison from FASTSUM

- large energy scale separation between M_b and binding enery
- sub-percent level accuracy required

Scales at $T = 0$

- Effective Field Theory (EFT) : $M_b,\ M_b v,\ M_b v^2$
- NRQCD : M_b scale is “integrated away”

  bottom quark is “point-like” ($M_b a \sim 1$)
- pNRQCD : $M_b, M_b v$ scales are “integrated away”

  bottom quark is “point-like”
  and bottomonium is also “point-like”
  (“Bohr radius” is also an expansion parameter)
- our choice is lattice NRQCD

Scales at $T \neq 0$

- temperature is an additional scale

$$T = \frac{1}{N_\tau a_\tau} \tag{1}$$

- for consistent lattice NRQCD, $M_b a_\tau \sim 1$
- to keep NRQCD remain valid as an effective field theory, $T << M_b$
- in summary, a consistent lattice NRQCD for bottomonium ($M_b = 4.65$ GeV) requires

$$a_\tau \sim \frac{1}{4.65}(\mathrm{GeV}^{-1}) \tag{2}$$

and

$$T = \frac{1}{N_\tau a_\tau} \sim \frac{4.65\mathrm{GeV}^{-1}}{N_\tau} \tag{3}$$

- if we are interested upto $\sim 2T_c(\sim 300$ MeV for $N_f = 2+1)$,

$N_\tau \sim \mathcal{O}(10)$

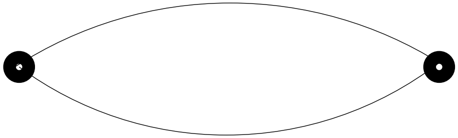
Scales at $T \neq 0$

• for the study of EoS (entropy density, pressure, energy density etc), $N_\tau \sim \mathcal{O}(10)$ doesn't pose a problem

• for the study of in-medium bottomonium, bottomonium correlator is important

$$G(\tau) = \sum_{\vec{x}} \langle \phi^\dagger(\vec{x},\tau;\vec{0},0)\phi(\vec{x},\tau;\vec{0},0)\rangle \quad (1)$$

• spectral information (mass shift, thermal broadening etc) needs to be obtained from $G(\tau)$ evaluated at $N_\tau \sim \mathcal{O}(10)$ of τ position



bottomonium correlator, $G(\tau,\mathbf{x})$

Scales at $T \neq 0$

$$G(\tau) = \sum_n e^{-E_n\tau} |\langle 0|\phi(0)|n\rangle|^2 \tag{1}$$

• if the states are well defined stationary states,

$$\rightarrow G(\tau) \sim a_0 e^{-E_0\tau} + a_1 e^{-E_1\tau} + a_2 e^{-E_2\tau} + \cdots \tag{2}$$

usual χ^2 fitting is sufficient

• for in-medium bottomonium, the states are no longer narrow

$\rightarrow$ spectral information is needed unless the functional form is known

Scales at $T \neq 0$

• spectral representation

$$G_\Lambda(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Lambda\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Lambda\psi(0,\vec{0})\rangle \quad (1)$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Lambda(\omega,\vec{p}) \quad (2)$$

and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \quad (3)$$

• the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

• numerically ill-posed problem

• Maximum Entropy Method is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)

Scales at $T \neq 0$

$$G_\Lambda(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Lambda\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Lambda\psi(0,\vec{0})\rangle \quad (1)$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Lambda(\omega,\vec{p}) \quad (2)$$

and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \quad (3)$$

- known to have problems (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)

- both the kernel($K(\tau,\omega)$) and the spectral density($\rho_\Gamma(\omega,\vec{p})$) depend on temperature

- constant contribution

Scales at $T \neq 0$

- In NRQCD, with $\omega = 2M + \omega'$ and $T/M << 1$, $K(\tau,\omega) \to e^{-\omega\tau}$

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \qquad (1)$$

- inverse Laplace transform problem
- new improved Bayesian method (Burnier-Rothkopf, PRL111 (2013) 182003, Alexander's talk)

Lattice data used for NRQCD

• FASTSUM anisotropic lattice on $24^3 \times N_t$ (ref. G. Aarts et al, JHEP1407 (2014) 097)

N_s	N_t	a_τ^{-1}	T(MeV)	T/T_c	No. of Conf.
16	128	5.63(4)GeV	44	0.24	499
24	40	5.63(4)GeV	141	0.76	502
24	36	5.63(4)GeV	156	0.84	503
24	32	5.63(4)GeV	176	0.95	998
24	28	5.63(4)GeV	201	1.09	1001
24	24	5.63(4)GeV	235	1.27	1002
24	20	5.63(4)GeV	281	1.52	1000
24	16	5.63(4)GeV	352	1.90	1042

Table: summary for the FASTSUM lattice data set,
$M_b a_s = 2.92, M_b a_\tau = 0.834$

• tadpole- and Symanzik- improved gauge action, tapole-improved Wilson clover quark action ($N_f = 2+1$)

Lattice data used for NRQCD

β	T	T/T_c	a(fm)	u_0	$M_b a$
6.664	140	0.911	0.117	0.87025	2.76
6.700	145	0.944	0.113	0.87151	2.67
6.740	151	0.980	0.109	0.87288	2.57
6.770	155	1.01	0.106	0.87388	2.50
6.800	160	1.04	0.103	0.87485	2.42
6.840	166	1.08	0.0989	0.87612	2.34
6.880	172	1.12	0.0953	0.87736	2.25
6.910	177	1.15	0.0926	0.87827	2.19
6.950	184	1.19	0.0893	0.87945	2.11
6.990	191	1.24	0.086	0.88060	2.03
7.030	198	1.29	0.0829	0.88173	1.96
7.100	211	1.37	0.0777	0.88363	1.84
7.150	221	1.44	0.0743	0.88493	1.75
7.280	249	1.61	0.0660	0.88817	1.56

Table: summary for $N_f = 2+1$ HotQCD $48^3 \times 12$ lattice (A.Bazavov et al, PRD85 (2012) 054503

FASTSUM NRQCD and KPR NRQCD

- fixed lattice scale vs. variable lattice scale
- anisotropic lattices vs. isotropic lattices
- MEM (and BR, cf. T. Harris, Lat2014) vs. BR and MEM
- tuned M_b using kinetic mass vs. $M_b = 4.65$ GeV
- different lattice actions (in lattice NRQCD, we can't take continuum limit)

Lattice NRQCD Method

- Non-relativistic QCD in FT

$$
\begin{aligned}
G(\vec{x},t=0) &= S(x) \\
G(\vec{x},t=1) &= \left[1+\frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x},t)\left[1+\frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n G(\vec{x},0) \\
G(\vec{x},t+1) &= \left[1+\frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x},t)\left[1+\frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n [1-\delta H]\,G(\vec{x},t)
\end{aligned} \tag{2}
$$

where $S(x)$ is the random source (not smearing) and

Lattice NRQCD Method

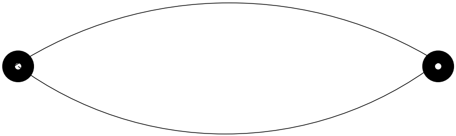
$$\delta H = -\frac{(\vec{D}^{(2)})^2}{8(m_b^0)^3}$$

$$+\frac{ig}{8(m_b^0)^2}(\vec{D}\cdot\vec{E}-\vec{E}\cdot\vec{D})-\frac{g}{8(m_b^0)^2}\vec{\sigma}\cdot(\vec{D}\times\vec{E}-\vec{E}\times\vec{D})-\frac{g}{2m_b^0}\vec{\sigma}\cdot\vec{B}$$

$$+\frac{a^2\vec{D}^{(4)}}{24m_b^0}-\frac{a(\vec{D}^{(2)})^2}{16n(m_b^0)^2}$$

Lattice NRQCD Method

bottomonium correlator, $G(\tau, \mathbf{x})$



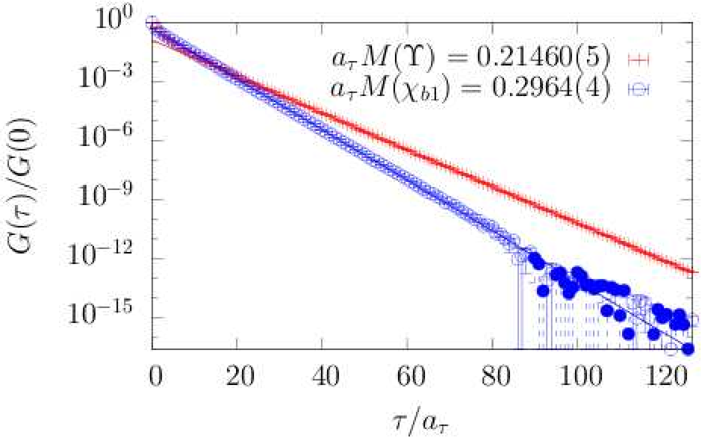
- NRQCD dispersion relation has undetermined zero point energy

$$E_q = \sqrt{M_q^2 + \mathbf{p}^2} \sim M_q + \frac{\mathbf{p}^2}{2M_q} - \frac{\mathbf{p}^4}{8M_q^3} + \cdots$$

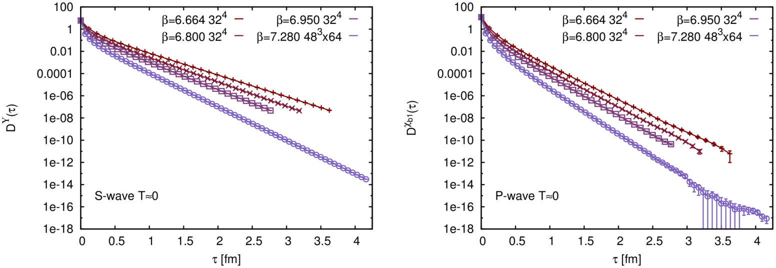
- simulation at zero temperature is required to determine the zero point energy

- FASTSUM NRQCD requires just one $T = 0$ calibration to fix the zero point energy. KPR NRQCD requires $T = 0$ calibration for each lattice spacing

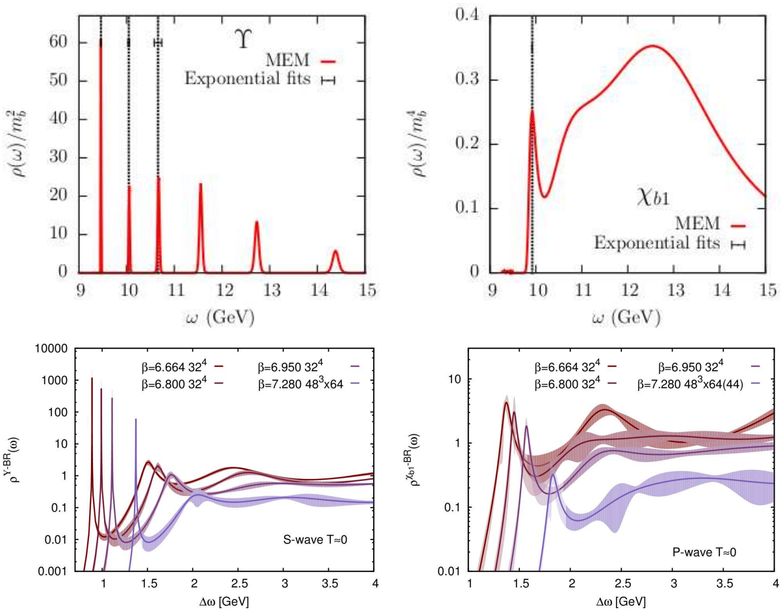
FASTSUM $T = 0$ correlator



KPR $T = 0$ correlator

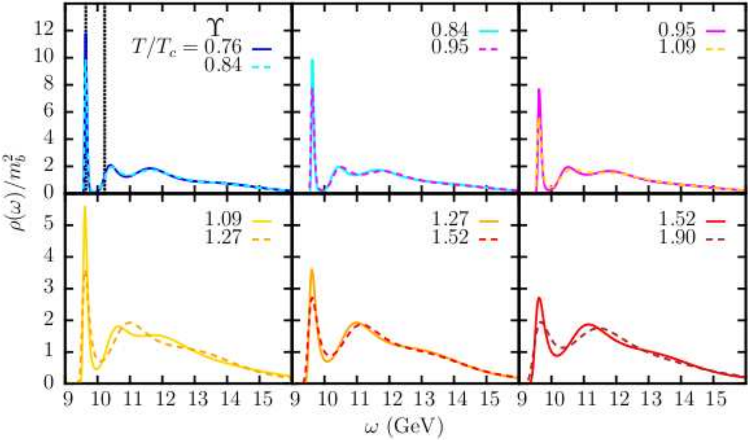


FASTSUM and KPR $T = 0$ spectral function



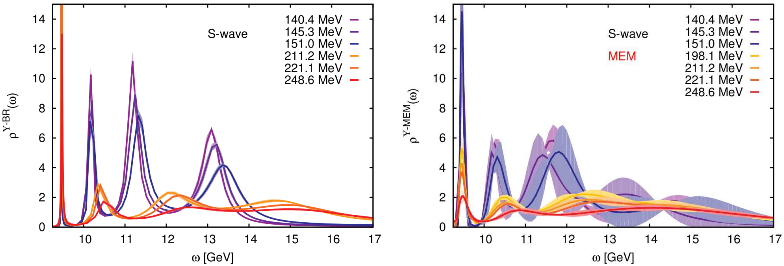
FASTSUM S-wave spectral functions

Υ channel spectral function



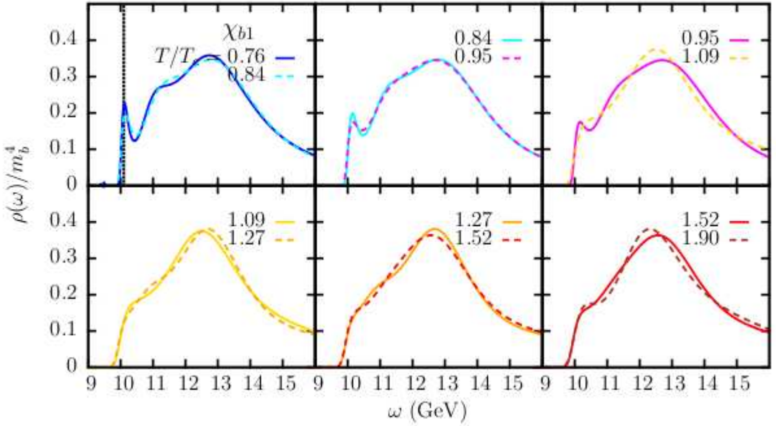
KPR S-wave spectral functions

Υ channel spectral function



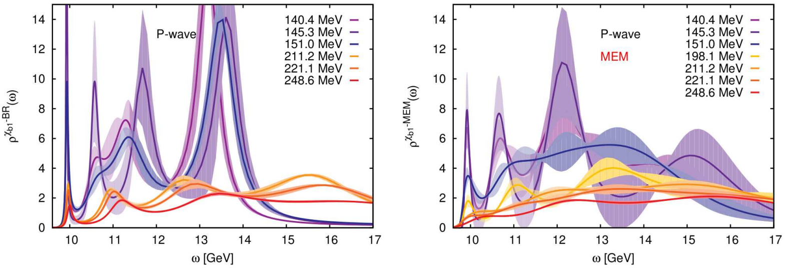
FASTSUM P-wave spectral functions

χ_{b1} channel spectral function



KPR P-wave spectral functions

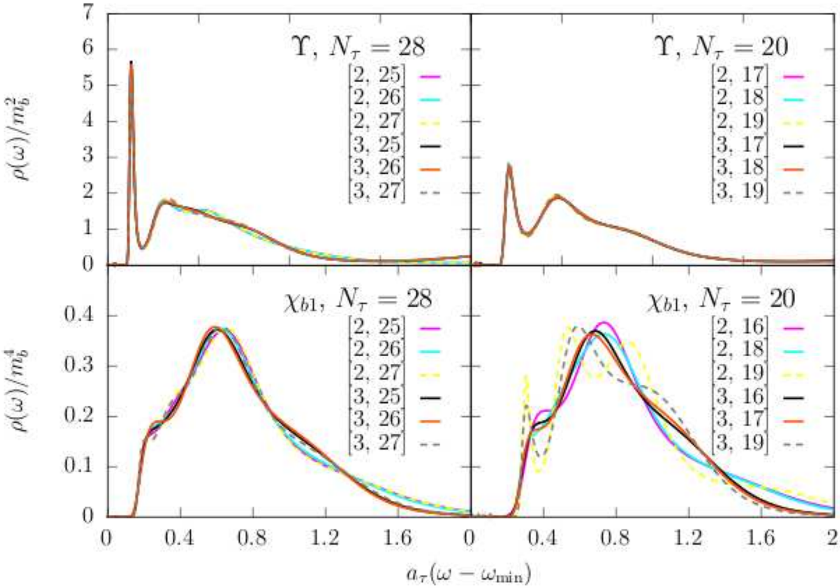
χ_{b1} channel spectral function



Systematics study in FASTSUM

- $\omega_{\min}, \omega_{rmmax}$ range
- default model dependency
- statistical error dependency
- τ range dependency
- comparison with free NRQCD spectral function
- high momentum stability

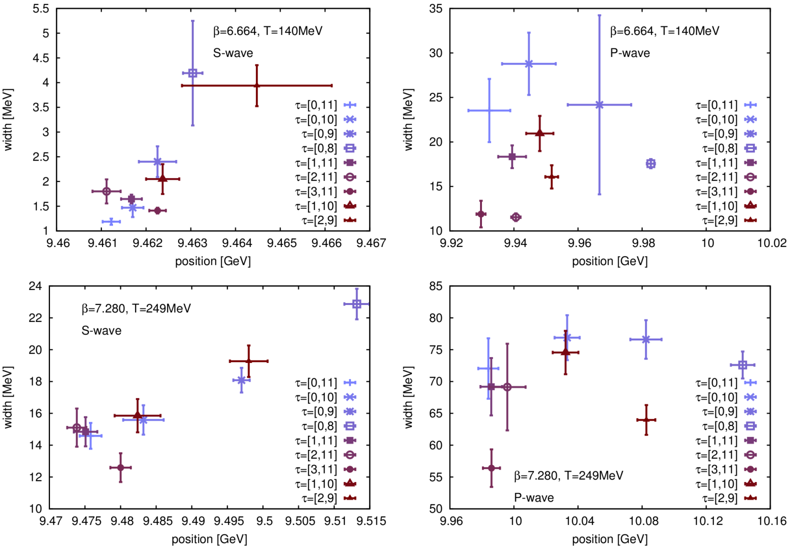
Systematics study in FASTSUM



Systematics study in KPR

- $\omega_{\min}, \omega_{rmmax}$ range
- default model dependency
- statistical error dependency
- τ range dependency
- comparison with free NRQCD spectral function
- high momentum stability
- spectral function reconstruction method dependency

Systematics study in KPR



Conclusion

• on $T = 0$ and $T \neq 0$, lattice NRQCD + new Baysesian Reconstruction (BR) of spectral function on bottomonium, which is systematically improvable and is based on the first principel of quantum field theory (not a model)

• free from known problem in QCD (constant contribution problem) and improvement from MEM

• from both BR and MEM, the ground state of Υ survives but the excited states are suppressed as the temperature increases above T_c. in FASTSUM study, 1S peak of Υ channel remains upto $T = 1.9T_c$ and in KPR study it remains upto $T = 1.6T_c$

• in FASTSUM study, the ground state of χ_{b1} melts above T_c. In KPR study, the ground state of χ_{b1} from BR spectral function retains peak structure even at $1.6T_c$ but that from MEM spectral function shows melting around $1.3T_c$

• further studies are in progress