Universität Bielefeld

# Lattice calculations of heavy quark correlators


**Olaf Kaczmarek**

University of Bielefeld


**Heavy Flavor and Electromagnetic Probes in Heavy Ion Collisions**

**INT Program INT 14-3**

Seattle

18.09.2014

# Lattice calculations of Heavy Quark correlators

... and how we try to

extract **transport properties** and **spectral properties** from them

**1) Color electric field correlation function**

with A.Francis, M. Laine, M.Müller, T.Neuhaus, H.Ohno

**Heavy quark momentum diffusion coefficient $\kappa$**

**2) Vector meson correlation functions for heavy quarks**

with H-T.Ding, H.Ohno et al.

**Heavy quark diffusion coefficients**

**Charmonium and Bottomonium dissociation patterns**

**3) Vector meson correlation functions for light quarks**

with H-T.Ding, A.Francis, F.Meyer, M.Müller et al.

**Electrical conductivity**

**Thermal dilepton rates**

# Motivation - Transport Coefficients

**Transport Coefficients** are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

**Need to be determined from QCD using first principle lattice calculations!**

here heavy flavour:

Heavy Quark Diffusion Constant D

[H.T.Ding, OK et al., PRD86(2012)014509]

Heavy Quark Momentum Diffusion $\kappa$

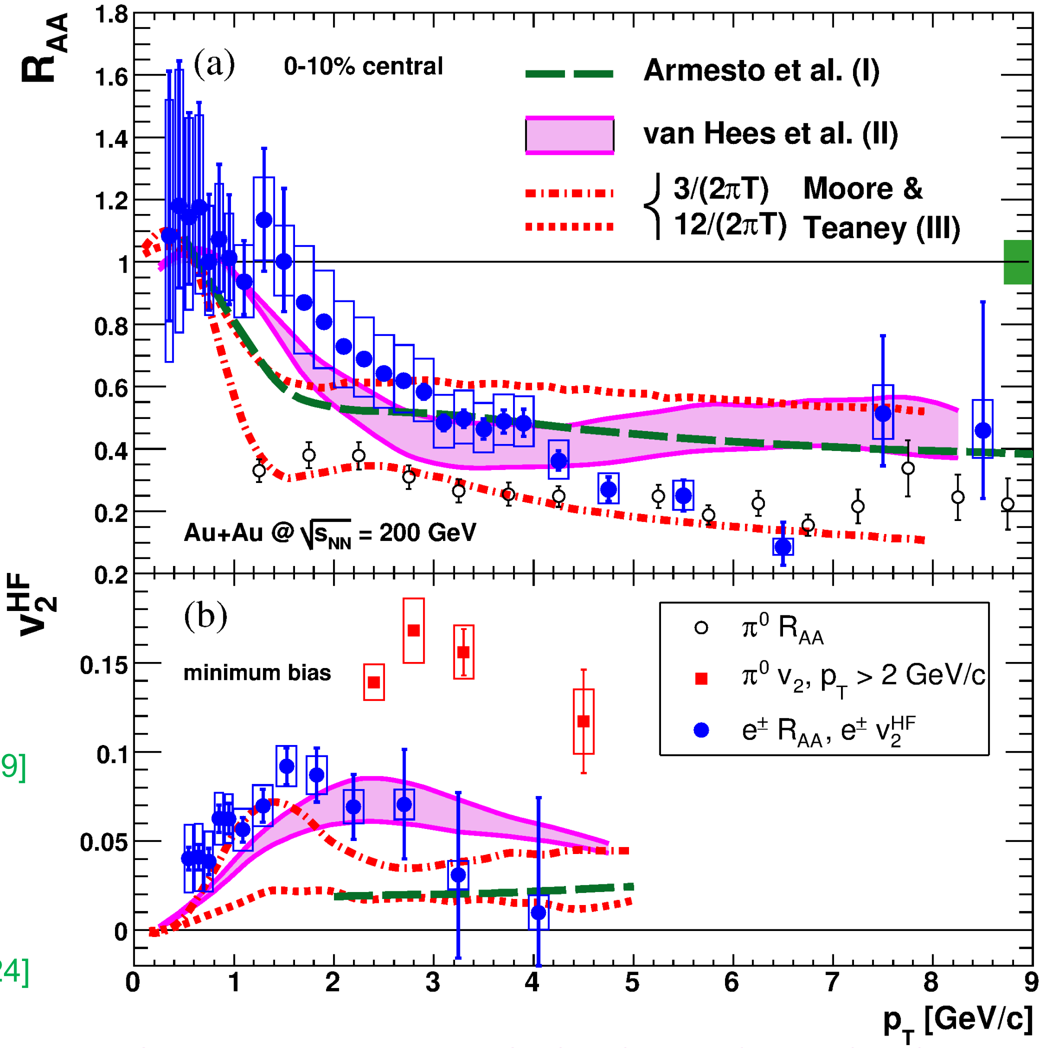
or for light quarks:

[OK, arXiv:1409.3724]

Light quark flavour diffusion

Electrical conductivity

[A.Francis, OK et al., PRD83(2011)034504]




[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

# Transport coefficients from Lattice QCD – Flavour Diffusion

**Transport coefficients usually calculated using correlation function of conserved currents**

$$G(\tau, \mathbf{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \mathbf{p}, T) K(\tau, \omega, T)$$

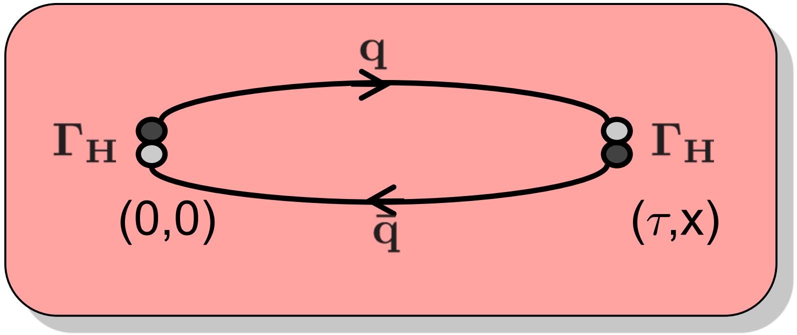
$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

**Lattice observables:**

$$G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle$$

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x})$$

$$G_{\mu\nu}(\tau, \vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$$



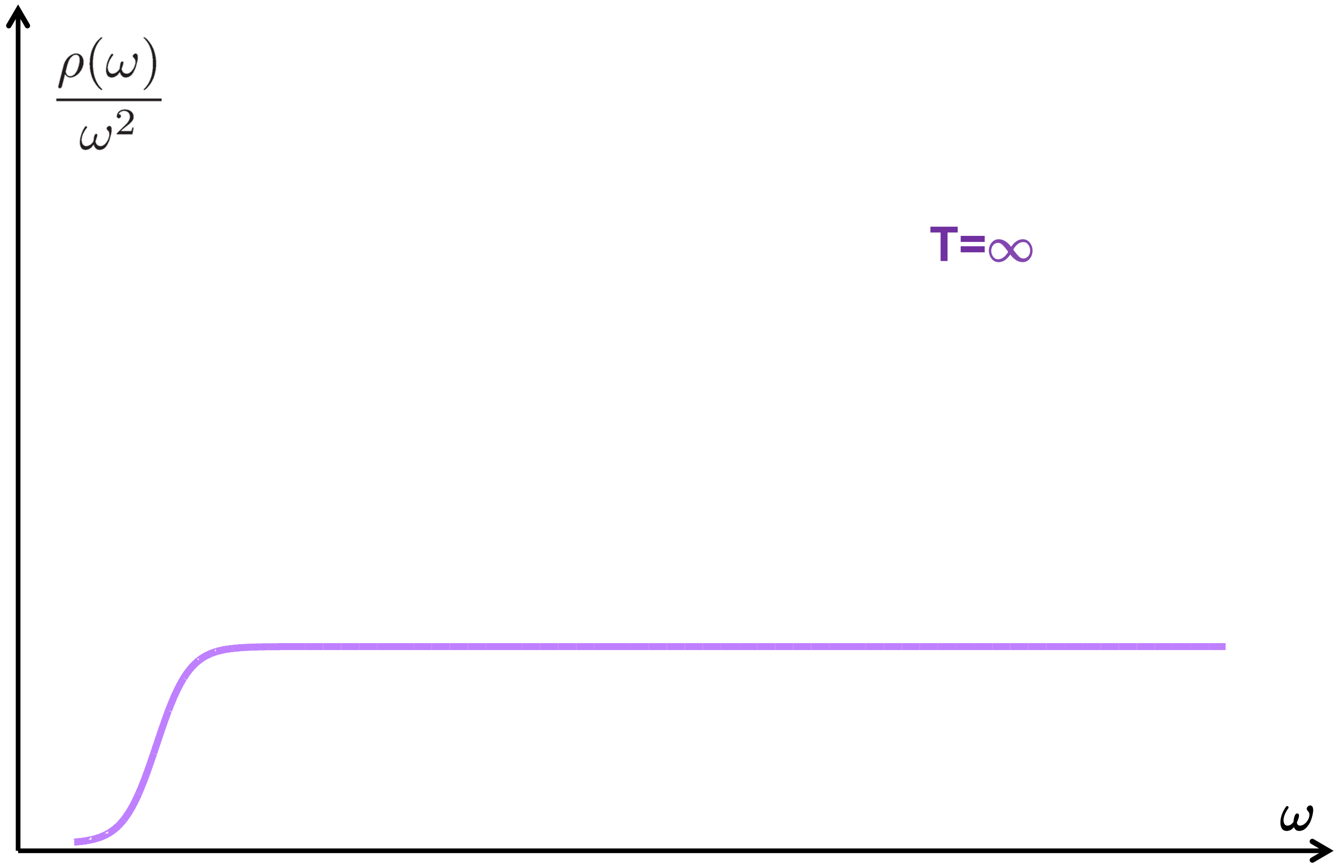

related to a conserved current

only correlation functions calculable on lattice but

**Transport coefficient** determined by slope of spectral function at $\omega$=0 (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

# Quarkonium spectral function – What do we expect!?




+ zero-mode contribution at $\omega$=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

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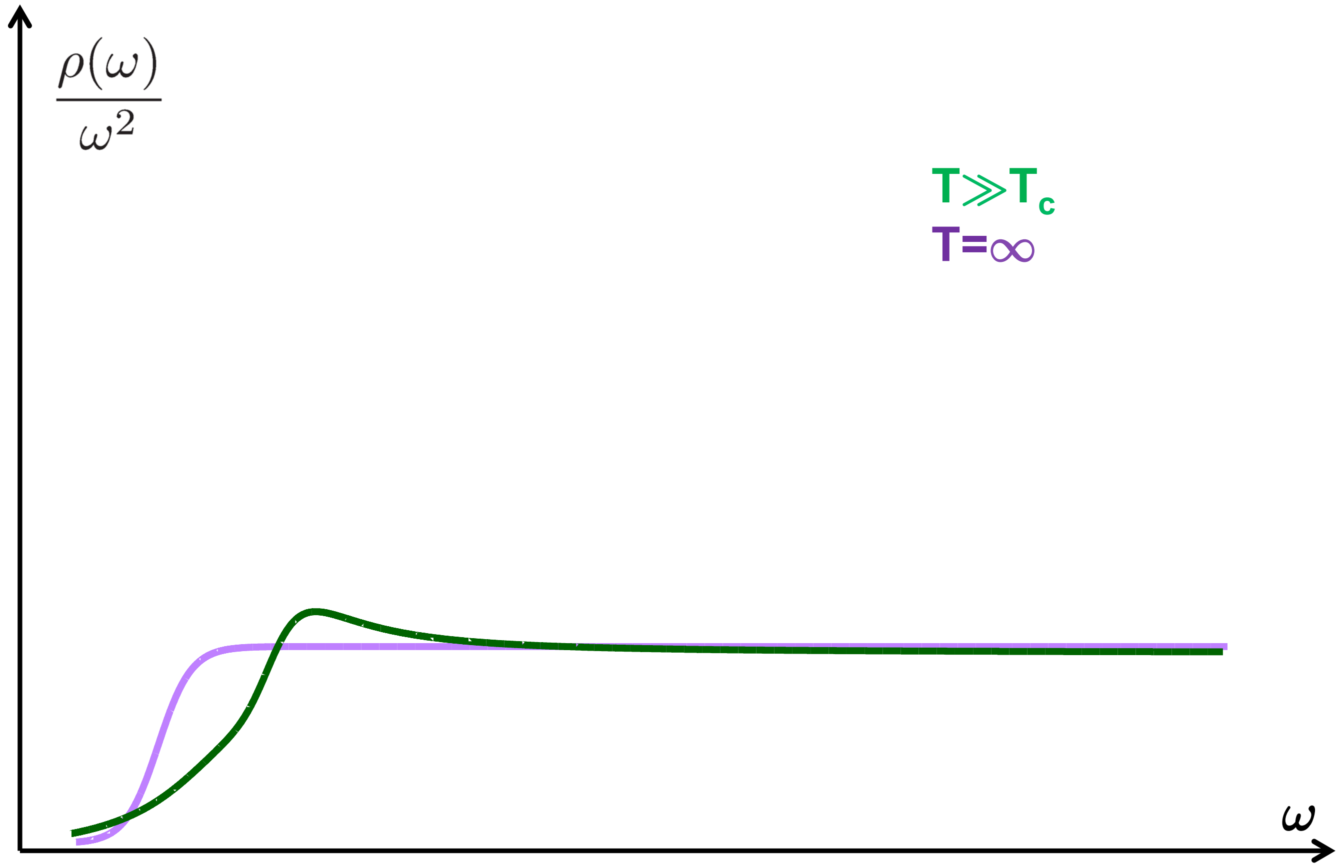


+ zero-mode contribution at $\omega$=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

+ transport peak at small $\omega$: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

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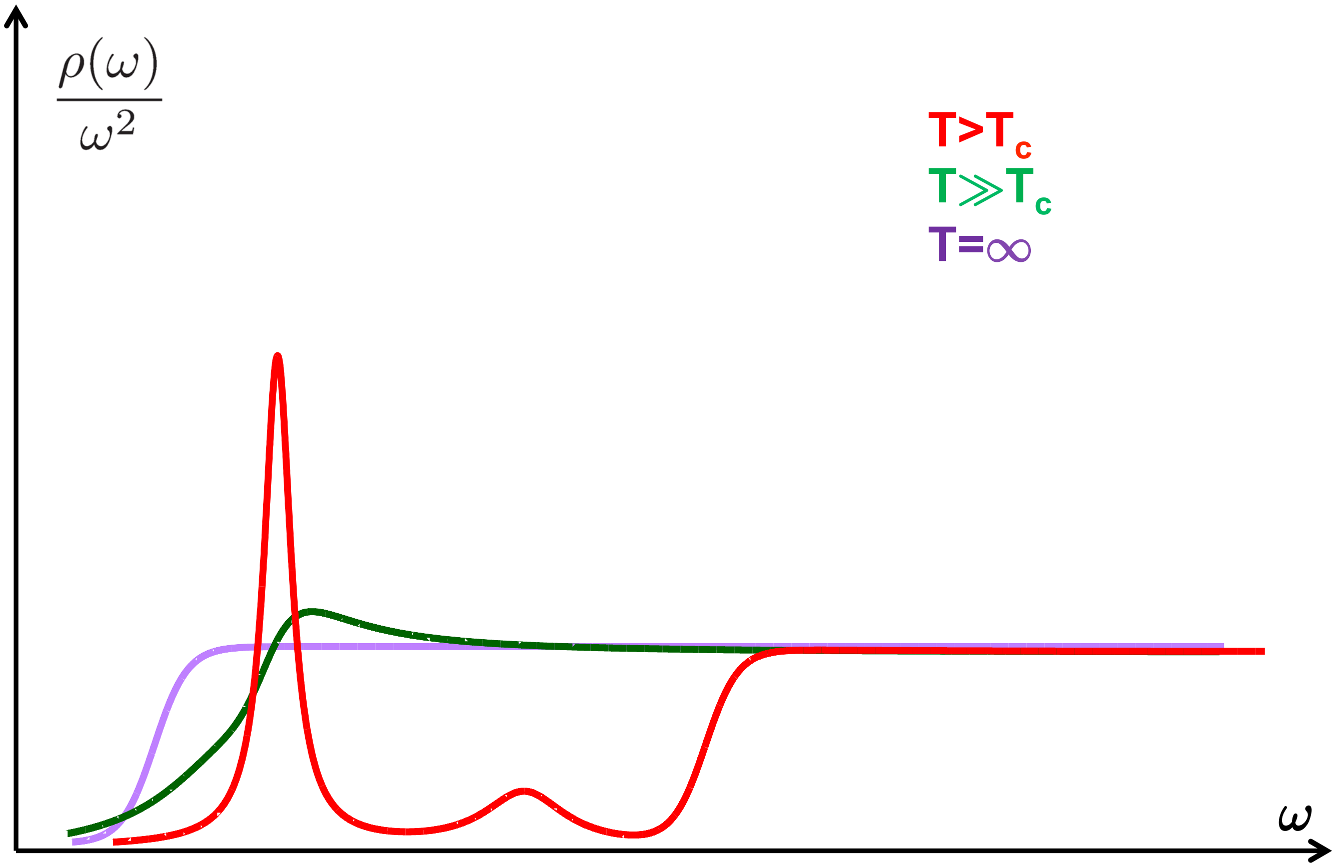


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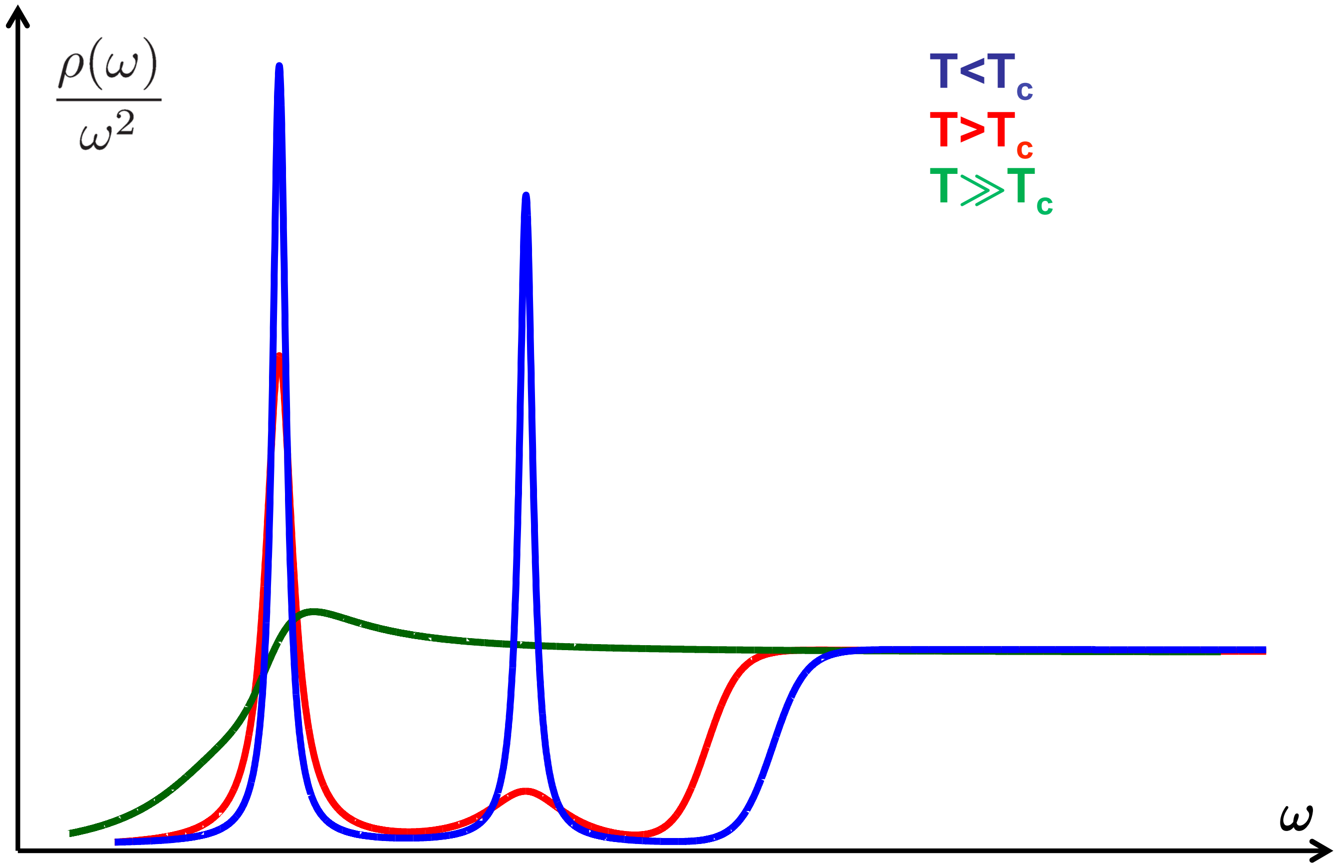


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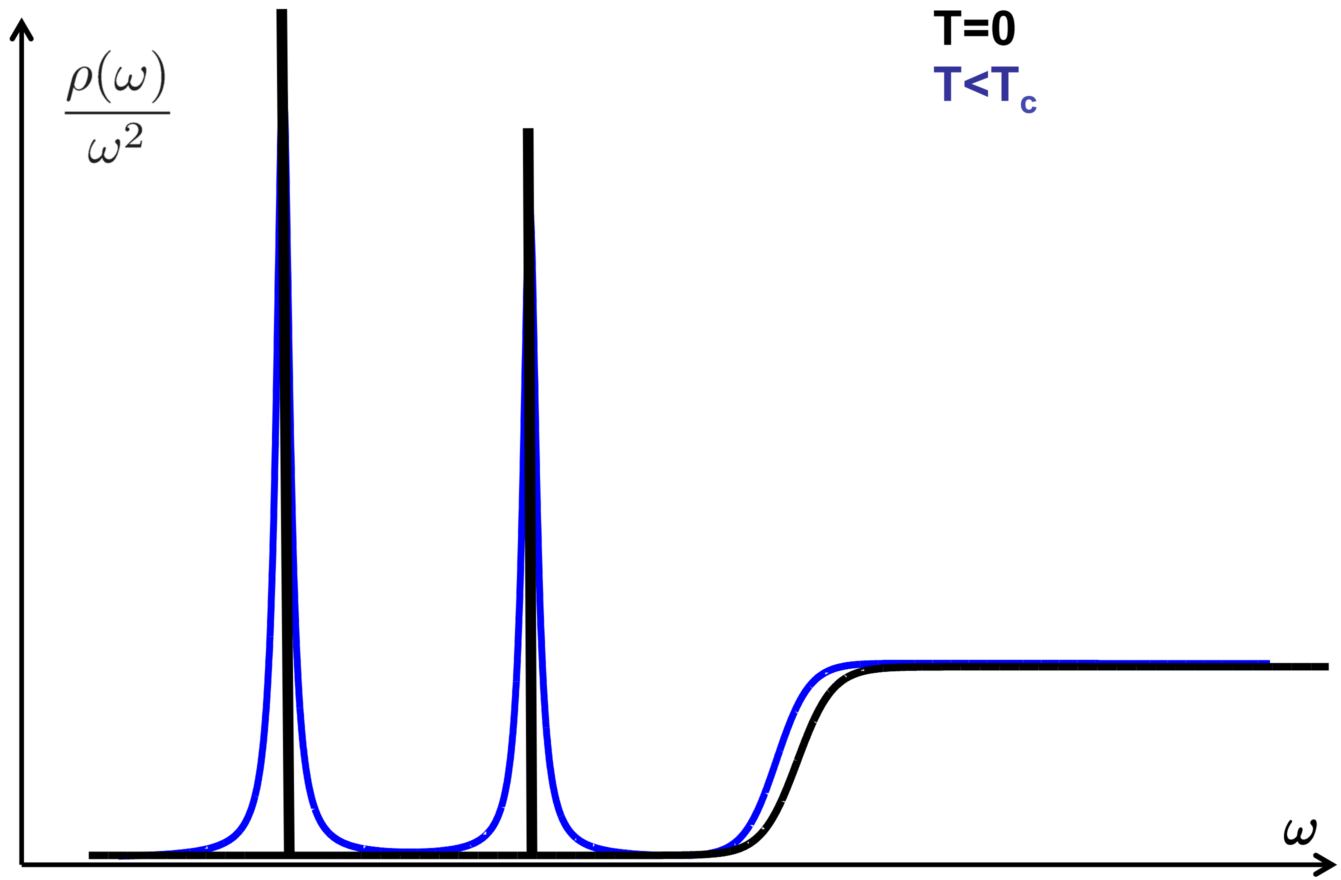


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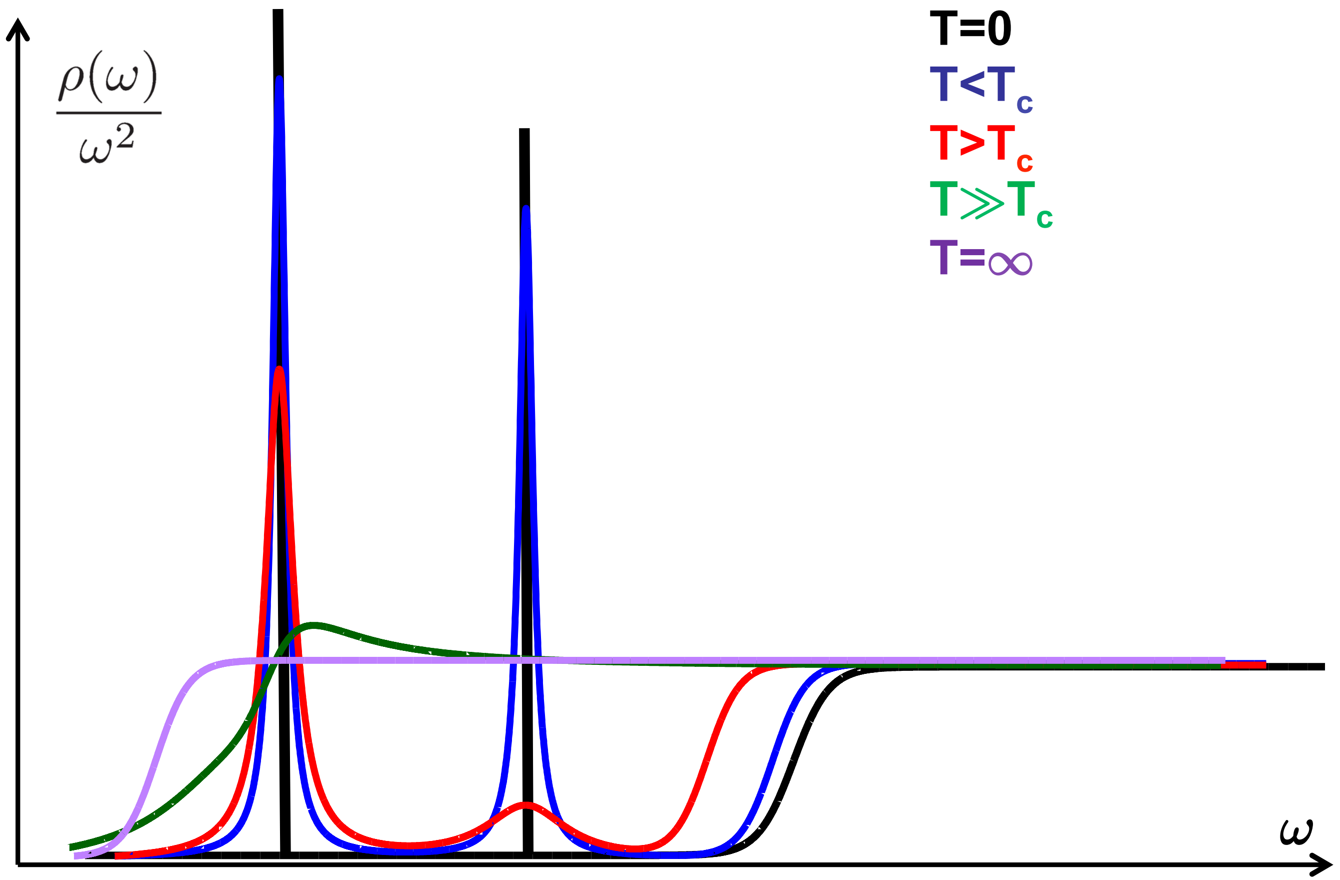


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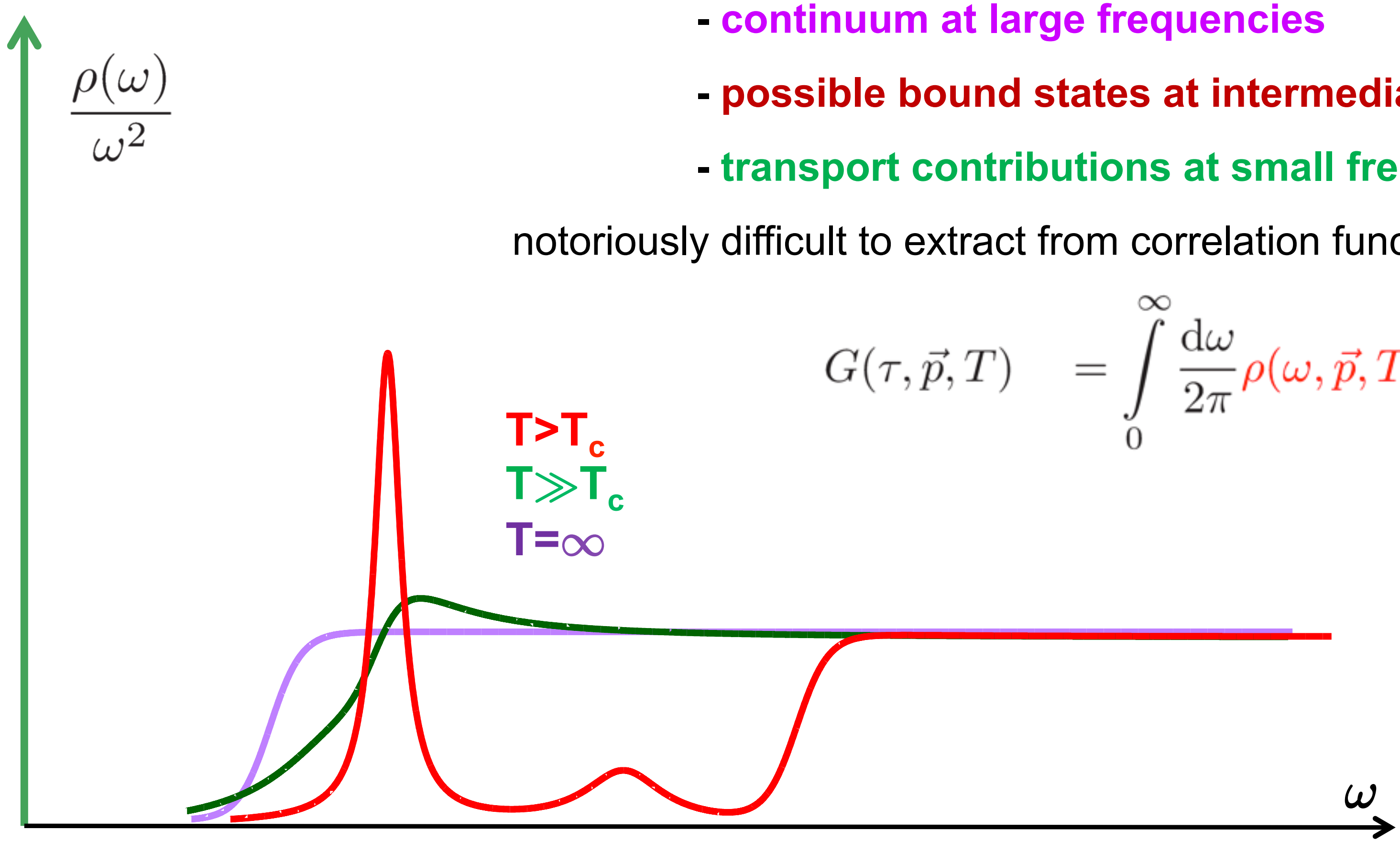
# Quarkonium spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies

notoriously difficult to extract from correlation functions

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$




+ zero-mode contribution at $\omega$=0: $\rho(\omega) = 2\pi\chi_{00}\,\omega\delta(\omega)$

+ (narrow) transport peak at small $\omega$: $\rho(\omega \ll T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

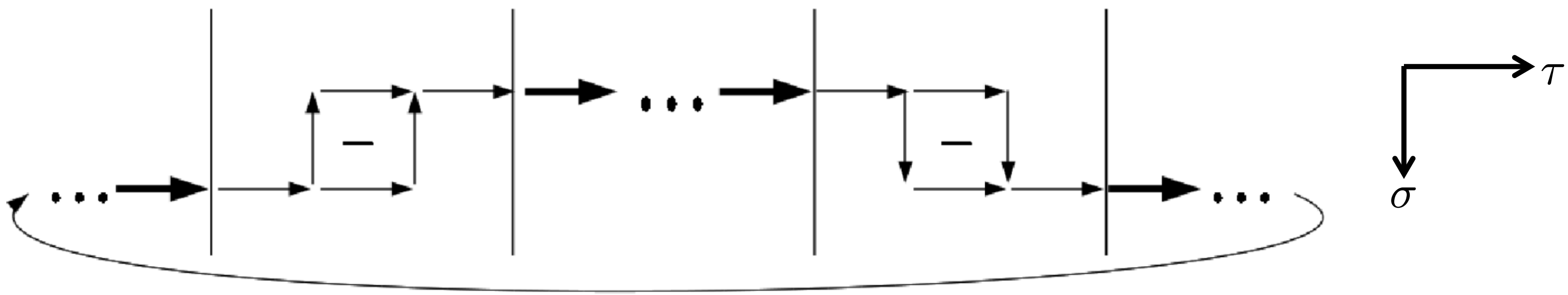
# Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

**for a single quark in medium**

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]




$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

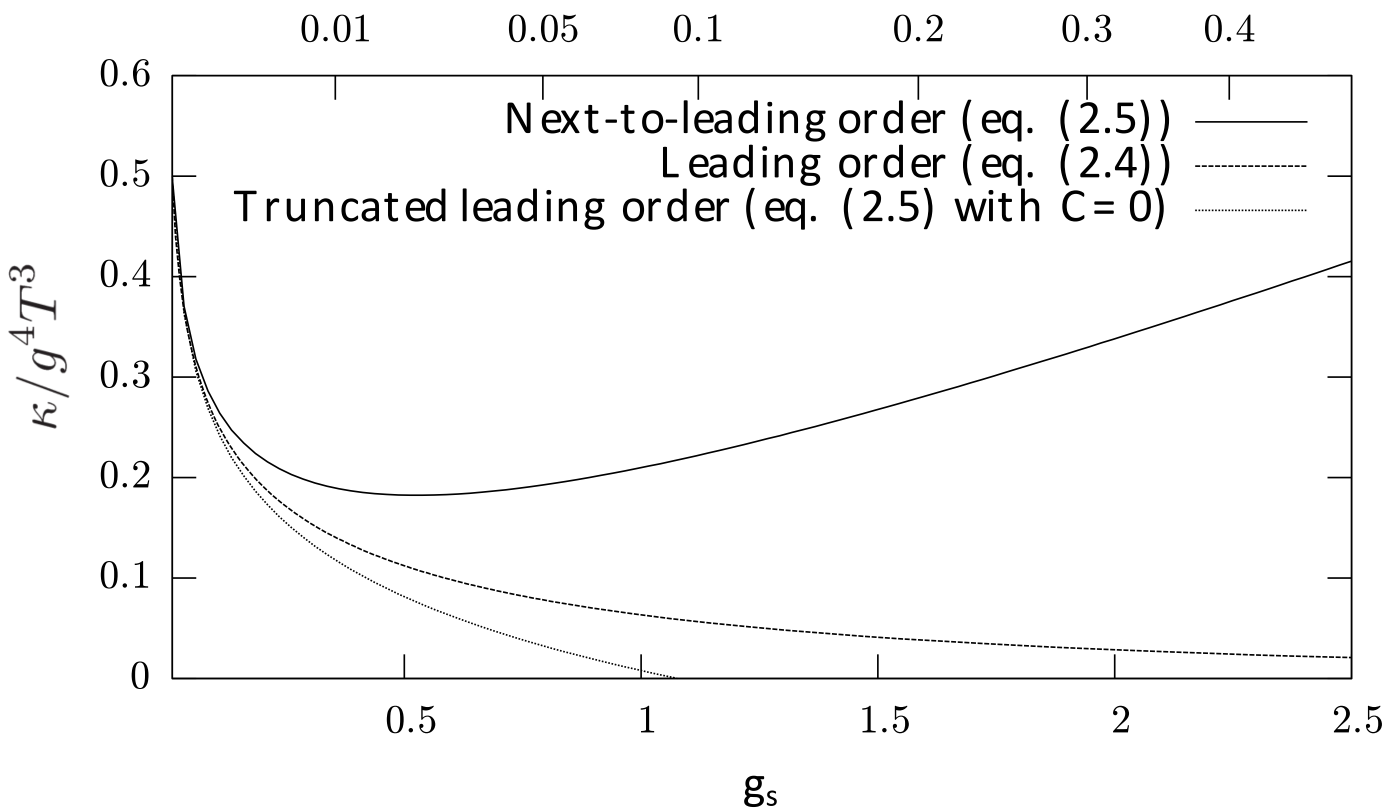
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

# Heavy Quark Momentum Diffusion Constant – Perturbation Theory

can be related to the thermalization rate:
$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

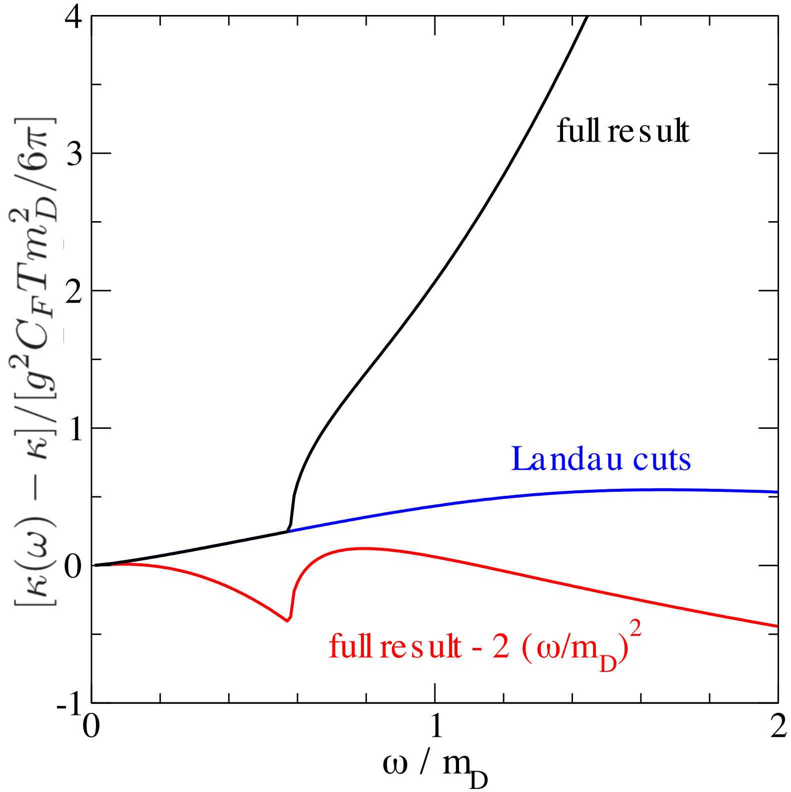



very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

# Heavy Quark Momentum Diffusion Constant – Perturbation Theory

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]




in contrast to a narrow transport peak, from this a smooth limit

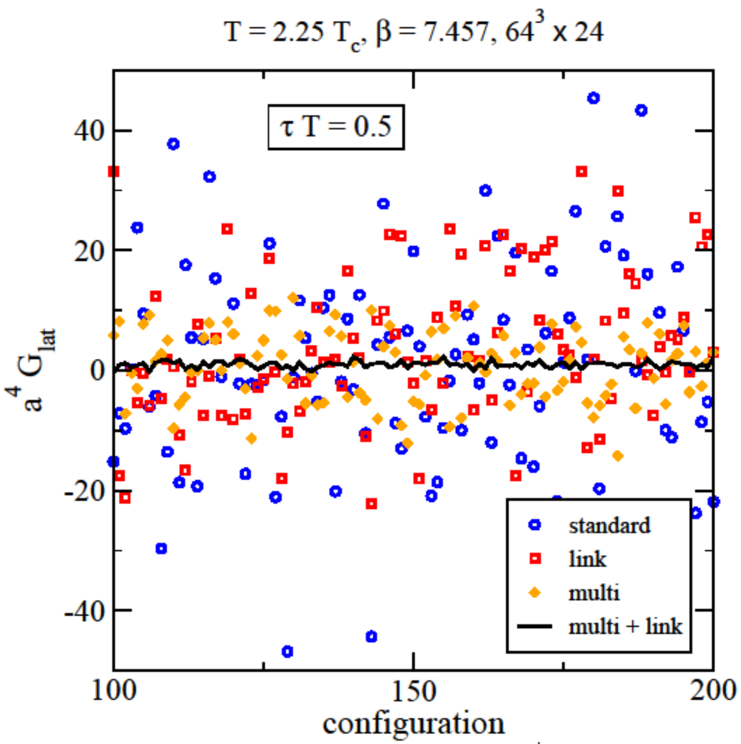
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

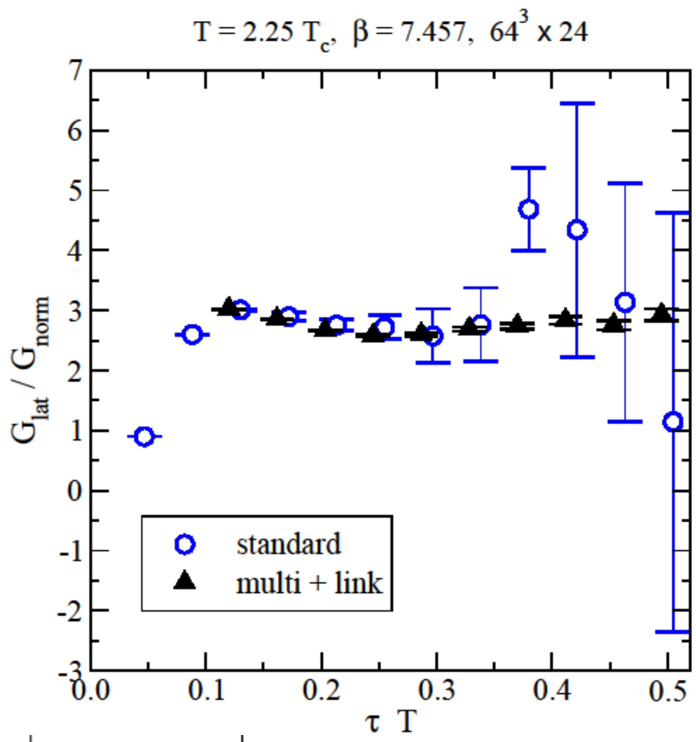
is expected

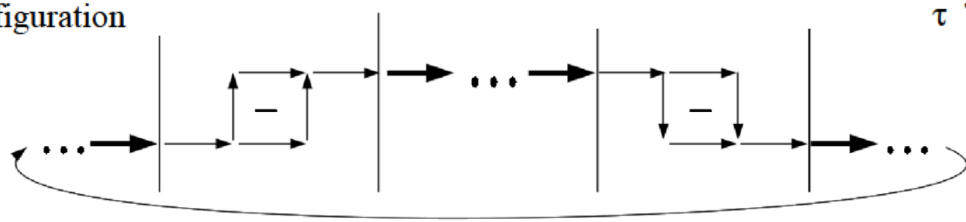
Qualitatively similar behaviour also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

# Heavy Quark Momentum Diffusion Constant – Lattice algorithms

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



due to the gluonic nature of the operator, signal is extremely noisy

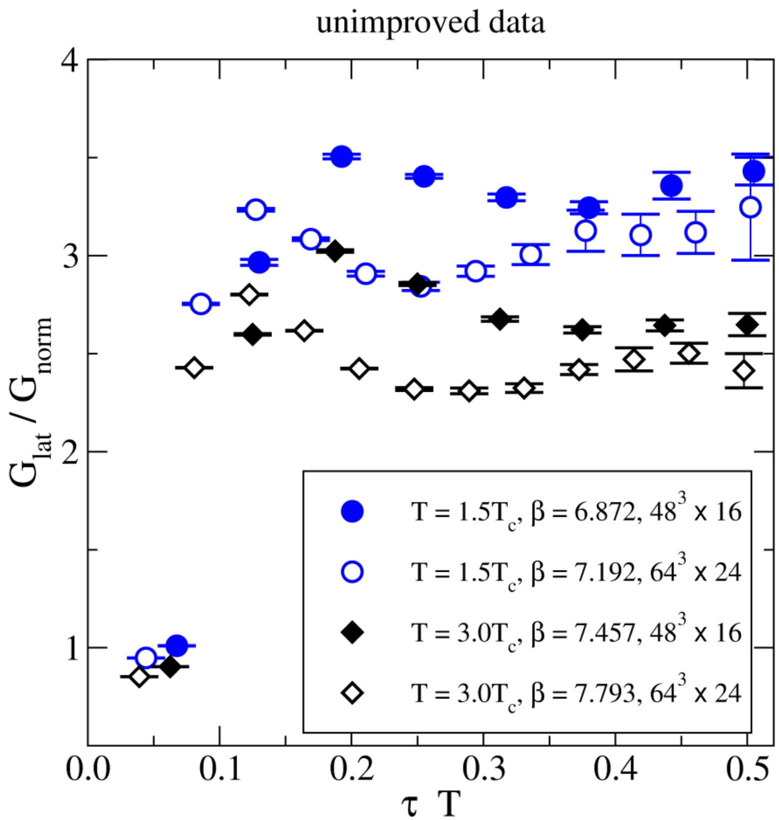
→ **multilevel combined with link-integration techniques to improve the signal**

[Lüscher,Weisz JHEP 0109 (2001)010 and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418, and de Forcrand PLB 151 (1985) 77]

# Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]




normalized by the LO-perturbative correlation function:

$$G_{\text{norm}}(\tau T) \equiv \frac{G^{\text{LO}}_{\text{cont}}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right] \qquad C_F \equiv \frac{N_c^2-1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

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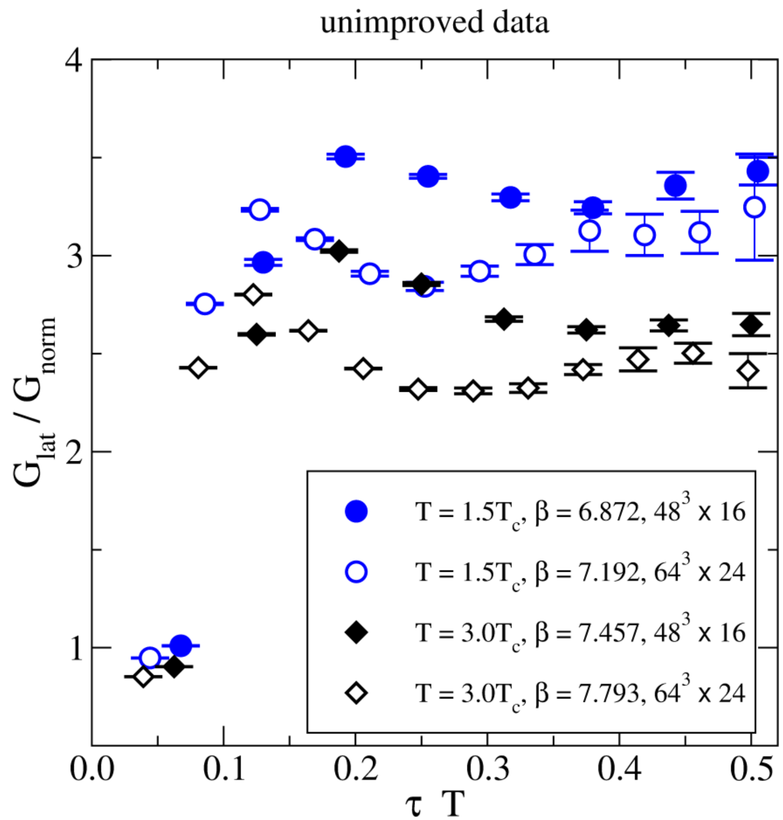
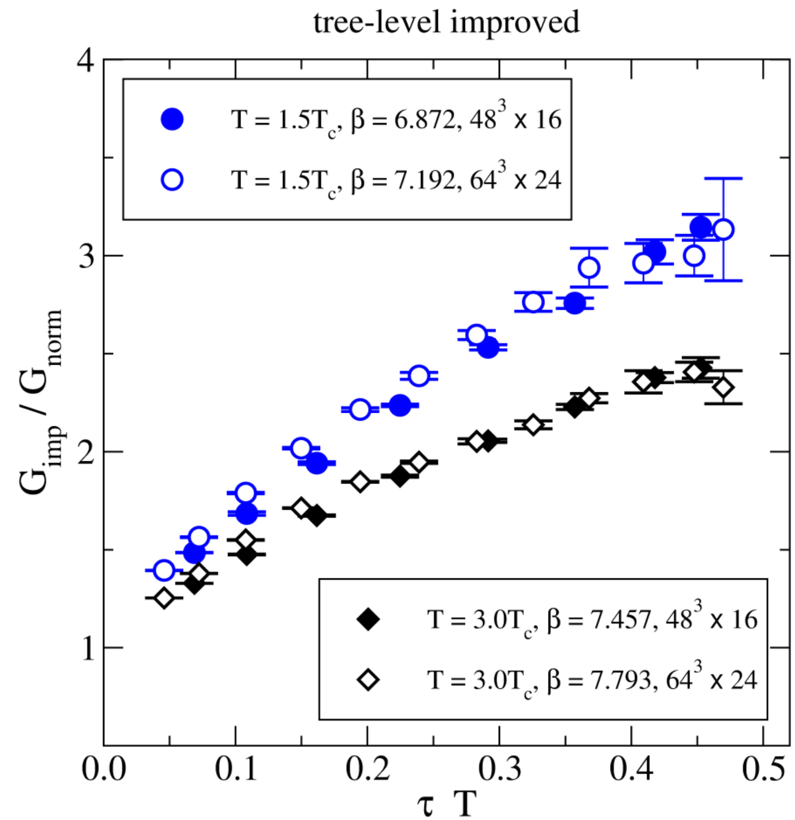



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G_{\rm cont}^{\rm LO}(\overline{\tau T}) = G_{\rm lat}^{\rm LO}(\tau T)$$

**leads to an effective reduction of cut-off effect for all** $\tau$T

# Heavy Quark Momentum Diffusion Constant – Lattice results

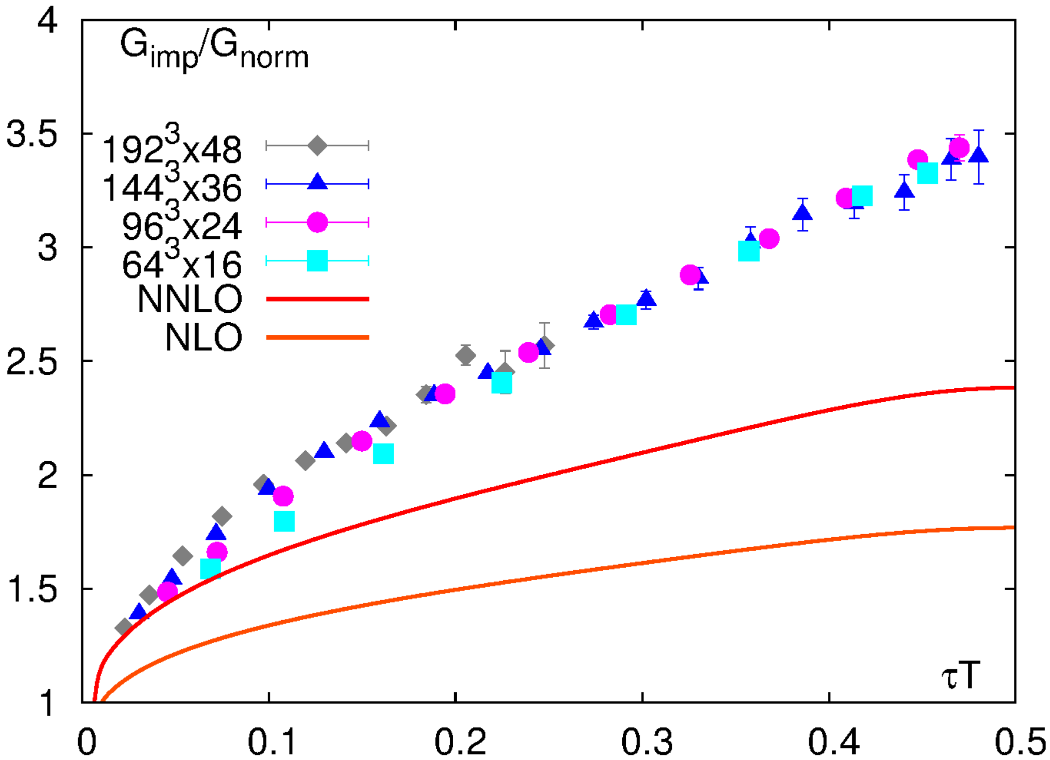
[OK, arXiv:1409.3724]

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.4\ T_c$

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, $a \to 0 \ \leftrightarrow \ N_t \to \infty$
- determine $\kappa$ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

| $N_\sigma$ | $N_\tau$ | $\beta$ | $1/a[\text{GeV}]$ | $a[\text{fm}]$ | #Confs |
|---|---|---|---|---|---|
| 64 | 16 | 6.872 | 7.16 | 0.03 | 100 |
| 96 | 24 | 7.192 | 10.4 | 0.019 | 160 |
| 144 | 36 | 7.544 | 15.5 | 0.013 | 584 |
| 192 | 48 | 7.793 | 20.4 | 0.010 | 223 |

# Heavy Quark Momentum Diffusion Constant – Lattice results




finest lattices still quite noisy at large $\tau$T

but only

**small cut-off effects at intermediate $\tau$T**

cut-off effects become visible at small $\tau$T

need to extrapolate to the continuum

**perturbative behavior in the limit $\tau$T $\rightarrow$ 0**

| $N_\sigma$ | $N_\tau$ | $\beta$ | $1/a$[GeV] | $a$[fm] | #Confs |
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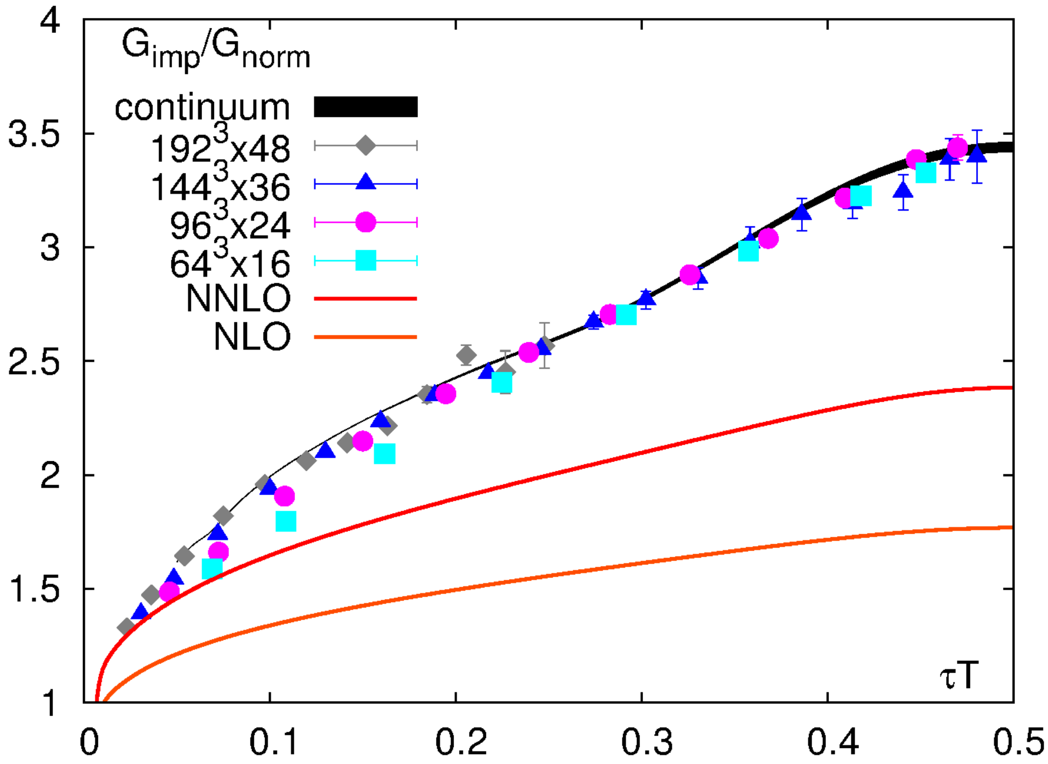
**allows to perform continuum extrapolation,** $a \rightarrow 0 \leftrightarrow N_t \rightarrow \infty$ **, at fixed** $T = 1/a\, N_t$

# Heavy Quark Momentum Diffusion Constant – Continuum extrapolation




finest lattices still quite noisy at large $\tau$T

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**small cut-off effects at intermediate $\tau$T**

cut-off effects become visible at small $\tau$T

need to extrapolate to the continuum

**perturbative behavior in the limit $\tau$T $\rightarrow$ 0**

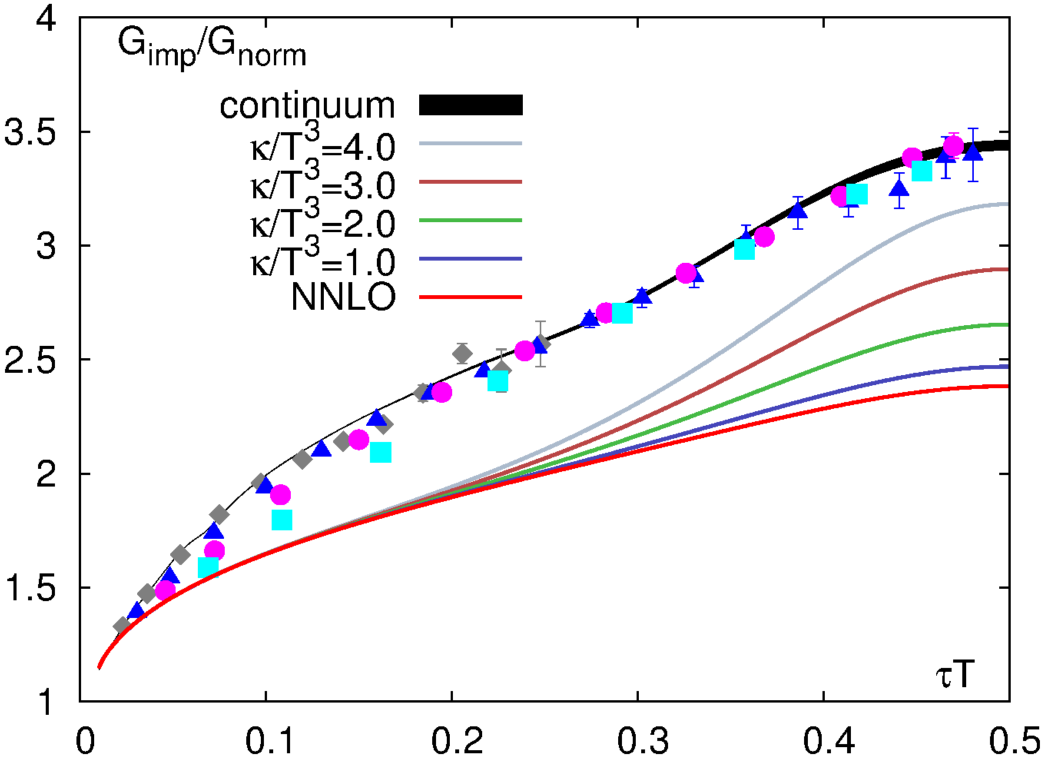
**well behaved continuum extrapolation for 0.05 $\leq$ $\tau$T $\leq$ 0.5**

finest lattice already close to the continuum

coarser lattices at larger $\tau$T close to the continuum

**how to extract the spectral function from the correlator?**

# Heavy Quark Momentum Diffusion Constant – Model Spectral Function




**continuum extrapolation**

$\kappa/T^3 = 4.0$

$\kappa/T^3 = 3.0$

$\kappa/T^3 = 2.0$

$\kappa/T^3 = 1.0$

NNLO perturbation theory

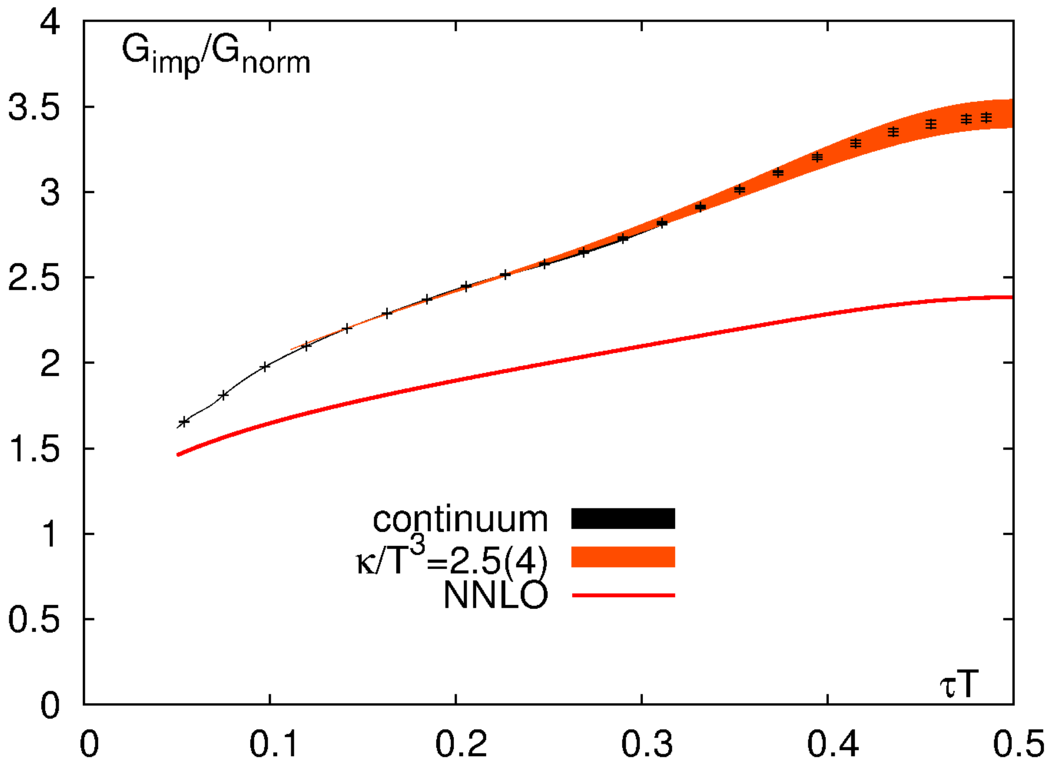
Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{\rho_{\mathrm{NLO}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\mathrm{model}}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

**some contribution at intermediate distance/frequency seems to be missing**

# Heavy Quark Momentum Diffusion Constant – Model Spectral Function




**result of the fit to $\rho_{model}(\omega)$**

with three parameters: $\kappa$,A,B

Model spectral function: transport contribution + NLO + correction

$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm NLO}(\omega) + B\omega^3, \frac{\omega\kappa}{2T} \right\}$$

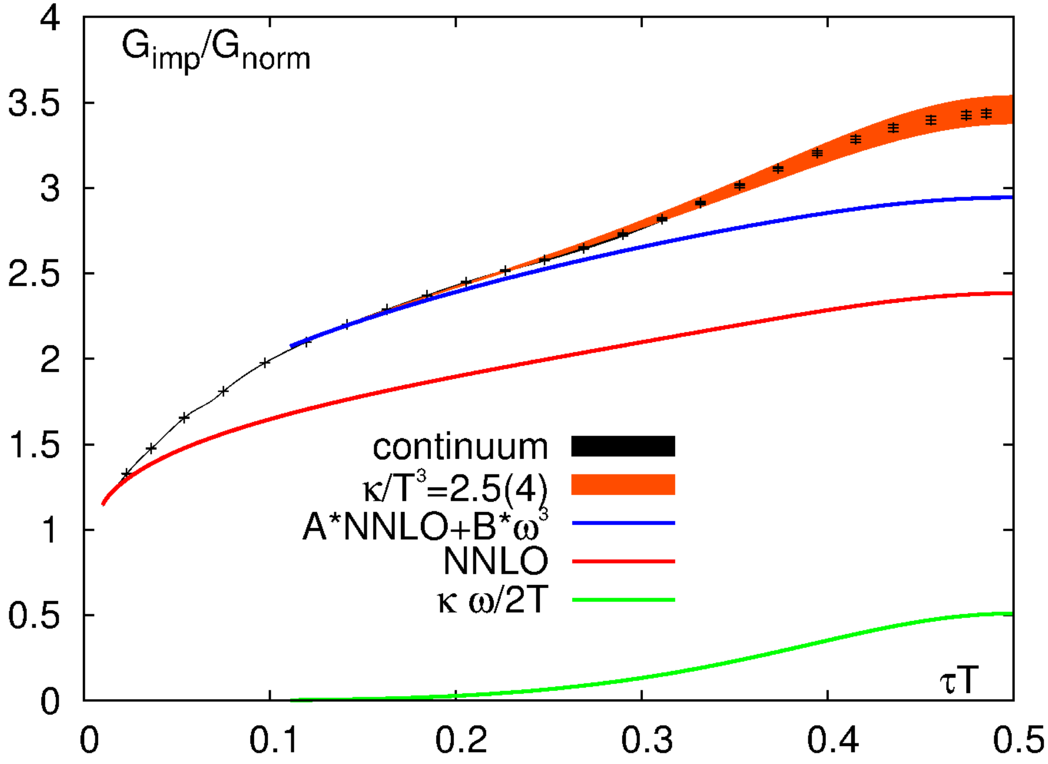
$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh \frac{\omega}{2T}}$$

used to fit the continuum extrapolated data

→ **first continuum estimate of $\kappa$ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.5(4)$$

# Heavy Quark Momentum Diffusion Constant – Model Spectral Function




**result of the fit to $\rho_{model}(\omega)$**

$A\,\rho_{NLO}(\omega)$ + B $w^3$

NNLO perturbation theory

$\frac{\omega\kappa}{2T}$ small but relevant contribution at $\tau T > 0.2$ !

Model spectral function: transport contribution + NLO + correction

$$\rho_{\text{model}}(\omega) \equiv \max\left\{A\rho_{\text{NLO}}(\omega) + B\omega^3, \frac{\omega\kappa}{2T}\right\}$$

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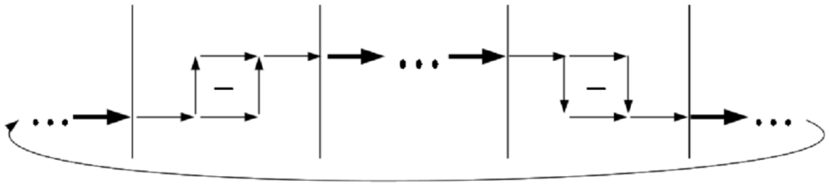
# Conclusions and Outlook

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(\frac{1}{T};\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U(\frac{1}{T};0)]\right\rangle}$$



**Continuum extrapolation for the color electric correlation function**

**extracted from Quenched Lattice QCD**

- **using noise reduction techniques to improve signal**
- **and an Ansatz for the spectral function**

**→ first continuum estimate for the Heavy Quark Momentum Diffusion Coefficient $\kappa$**

**More detailed analysis of the systematic uncertainties needed**

- **Different Ansätze for the spectral function**
- **Other techniques to extract the spectral function**

**Other Transport coefficients from Effective Field Theories?**

**In the following:** **Vector Meson Correlation Functions**

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega,\mathbf{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

**Lattice observables:**

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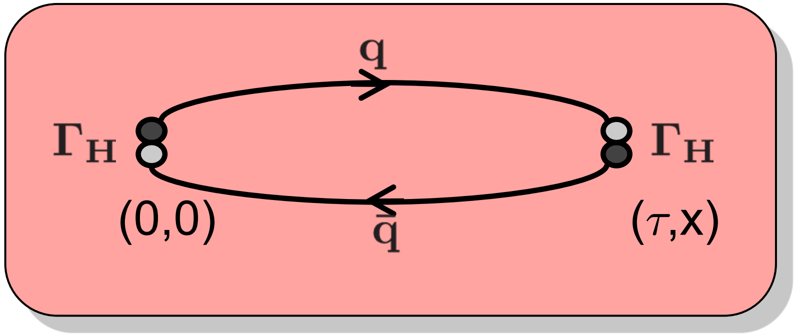


related to a conserved current

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# Lattice set-up

[H.T.Ding, OK et al., PRD86(2012)014509]

Quenched SU(3) gauge configurations (separated by 500 updates) at 4 temperatures

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 128$
$N_\tau = 16, 24, 32, 48, 96$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to charm quark mass

| | Mass in GeV | | | |
|---|---|---|---|---|
| $\beta$ | $J/\psi$ | $\eta_c$ | $\chi_{c1}$ | $\chi_{c0}$ |
| 6.872 | 3.1127(6) | 3.048(2) | 3.624(38) | 3.540(25) |
| 7.457 | 3.147(1)(25) | 3.082(2)(21) | 3.574(8) | 3.486(4) |
| 7.793 | 3.472(2)(114) | 3.341(2)(104) | 4.02(2)(23) | 4.52(2)(37) |

| $\beta$ | $a$[fm] | $a^{-1}$[GeV] | $L_\sigma$[fm] | $c_{SW}$ | $\kappa$ | $N_\sigma^3 \times N_\tau$ | $T/T_c$ | $N_{conf}$ |
|---|---|---|---|---|---|---|---|---|
| 6.872 | 0.031 | 6.432 | 3.93 | 1.412488 | 0.13035 | $128^3 \times 32$ | 0.74 | 126 |
| | | | | | | $128^3 \times 16$ | 1.49 | 198 |
| 7.457 | 0.015 | 12.864 | 1.96 | 1.338927 | 0.13179 | $128^3 \times 64$ | 0.74 | 179 |
| | | | | | | $128^3 \times 32$ | 1.49 | 250 |
| 7.793 | 0.010 | 18.974 | 1.33 | 1.310381 | 0.13200 | $128^3 \times 96$ | 0.73 | 234 |
| | | | | | | $128^3 \times 48$ | 1.46 | 461 |
| | | | | | | $128^3 \times 32$ | 2.20 | 105 |
| | | | | | | $128^3 \times 24$ | 2.93 | 81 |

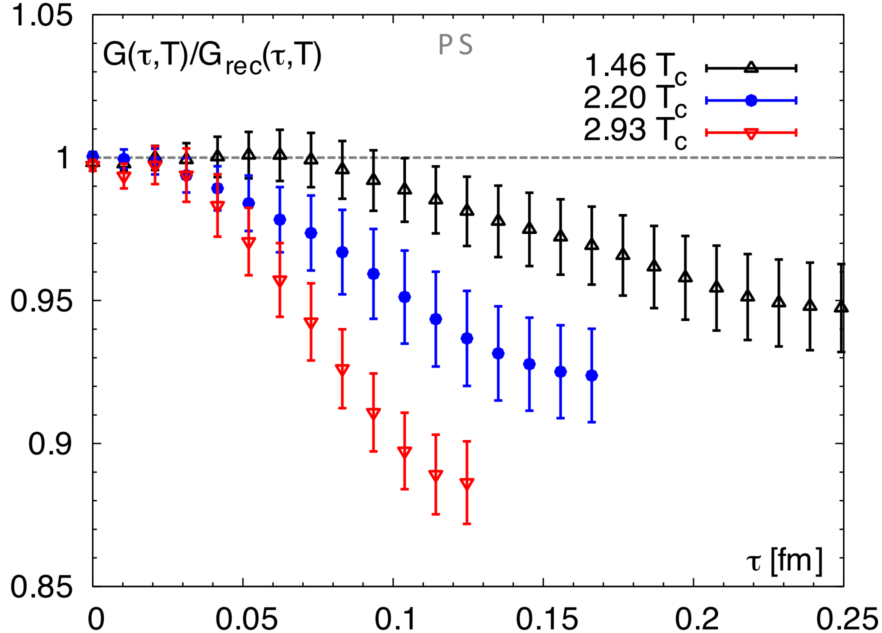
cut-off dependence

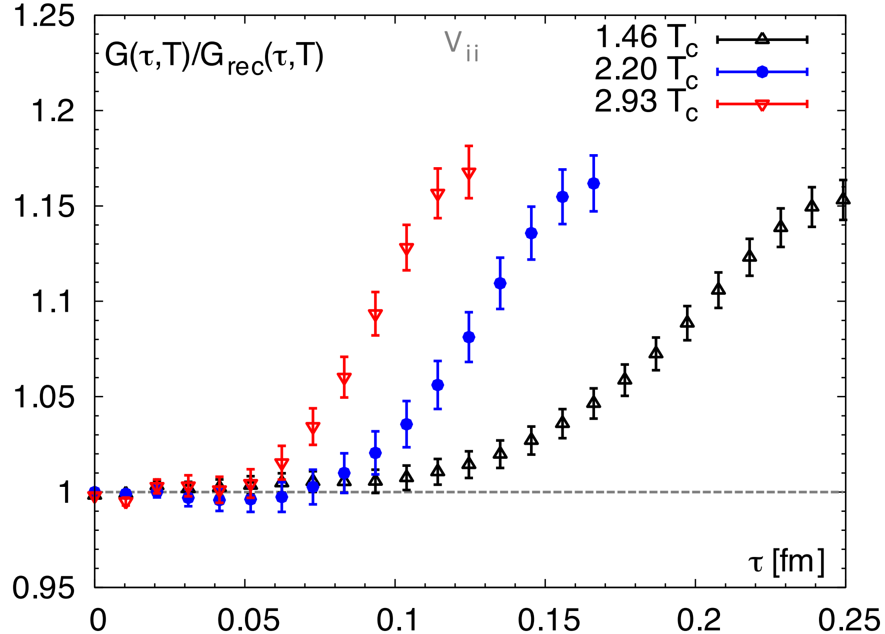
volume dependence

close to continuum ($m_c a \ll 1$)

Temperature dependence

# Charmonium Correlators vs Reconstructed Correlators




$$G_{rec}(\tau,T) = \int \sigma_0(\omega, 0.75T_c) K(\omega,\tau,T)$$

$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{00}(\tau,T)$$

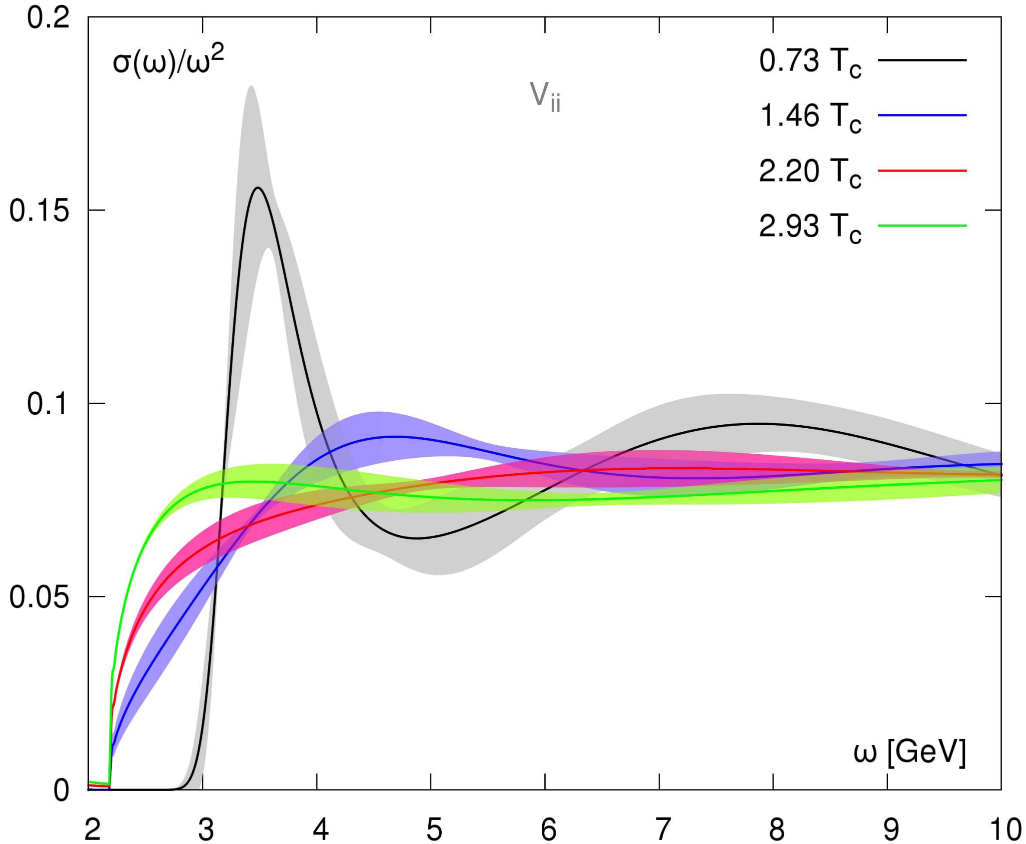
- main T-effect due to transport contribution
- well described by small $\omega$-part of $\sigma_T(\omega,\text{T})$
- explains the rise in the vector channel
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

# Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from Maximum Entropy Method analysis on a fine but finite lattice:

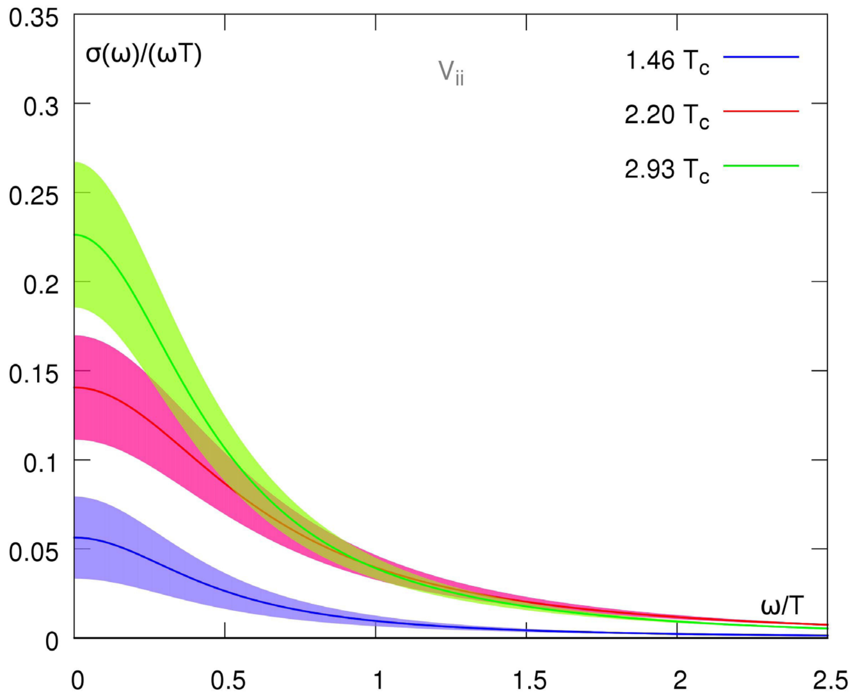



**statistical error band from Jackknife analysis**

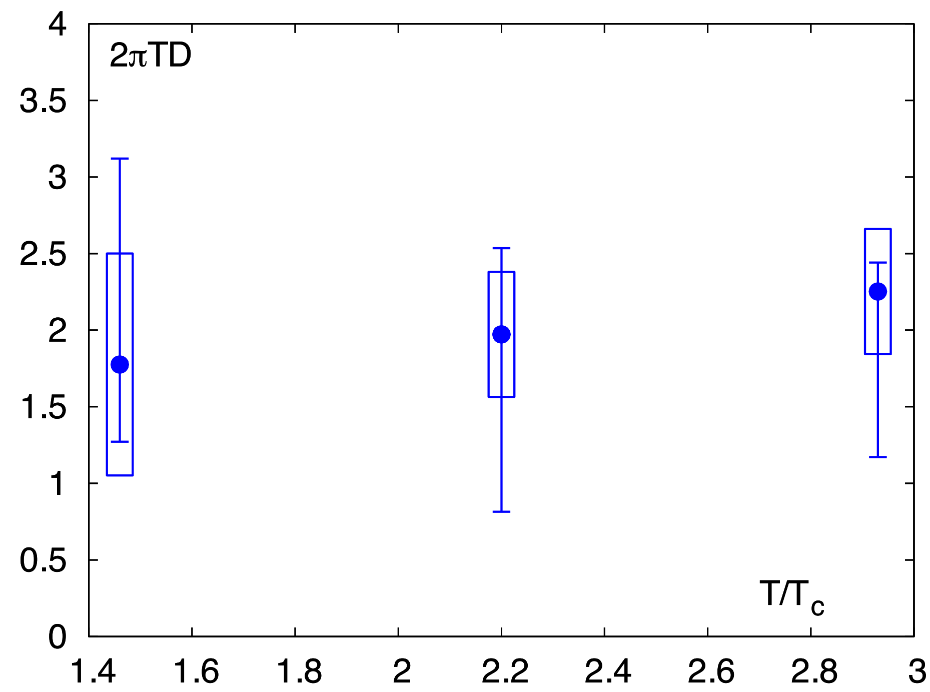
**no clear signal for bound states at and above 1.46 $T_c$**

**study of the continuum limit and quark mass dependence on the way!**

# Charmonium Spectral function – Transport Peak




[H.T.Ding, OK et al., PRD86(2012)014509]




$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate ($\alpha_s$~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$

NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

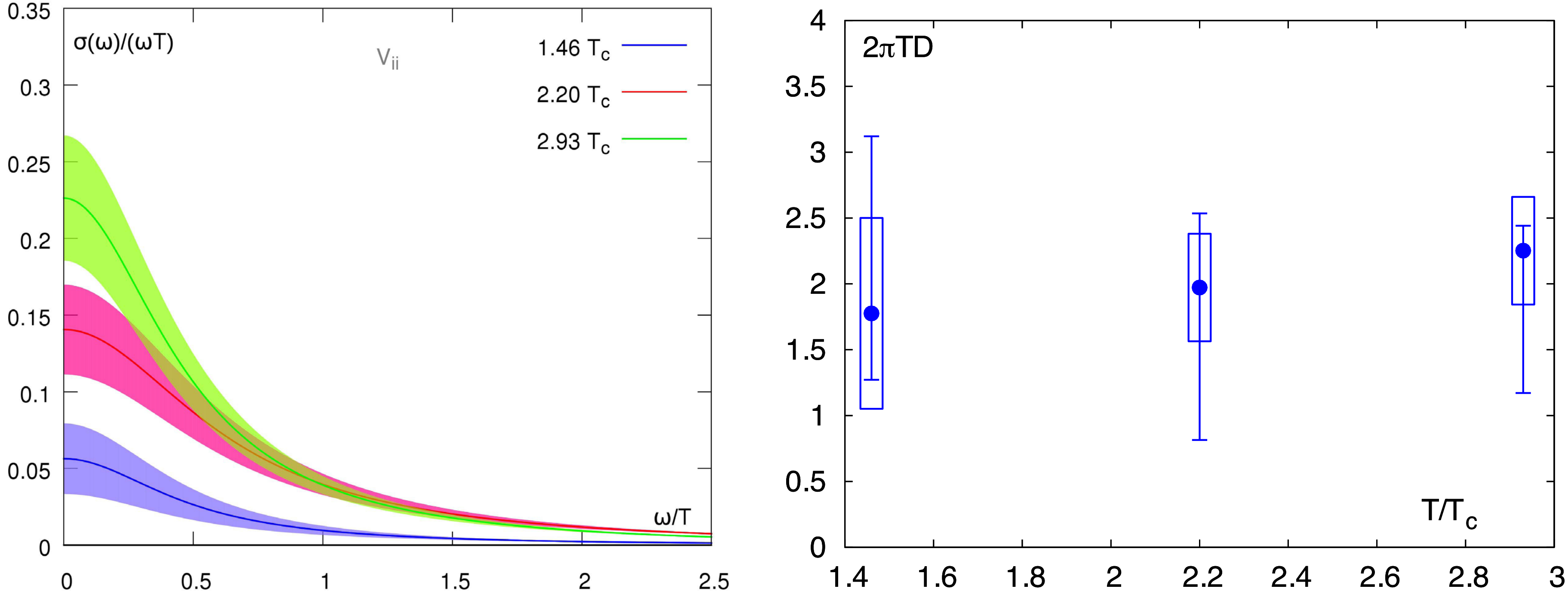
Strong coupling limit:

$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

# Charmonium Spectral function – Transport Peak

[H.T.Ding, OK et al., PRD86(2012)014509]




**Still large systematic uncertainties**

- how to extract the spectral function
- cut-off effects become larger with increasing $m_q$
- quark mass dependence → bottomonium
- continuum limit needed

# Charmonium and Bottomonium correlators

[H.T.Ding, H.Ohno, OK, work in progress]

- Standard plauette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On fine and large isotropic lattices
- 2 different cutoff
- $T = 0.7 - 1.4 T_c$
- Both charm & bottom
- Computing meson correlation functions

| $\beta$ | $N_\sigma$ | $N_\tau$ | $T/T_c$ | # confs. |
|---|---|---|---|---|
| 7.192 | 96 | 48 | 0.7 | 259 |
| | | 32 | 1.1 | 476 |
| | | 28 | 1.2 | 336 |
| | | 24 | 1.4 | 336 |
| 7.793 | 192 | 96 | 0.7 | 36 |
| | | 48 | 1.4 | 49 |

| $\beta$ | $a$ [fm] | $a^{-1}$ [GeV] | $\kappa_{\mathrm{charm}}$ | $\kappa_{\mathrm{bottom}}$ | $m_{J/\Psi}$ [GeV] | $m_\Upsilon$ [GeV] |
|---|---|---|---|---|---|---|
| 7.192 | 0.0190 | 10.4 | 0.13194 | 0.12257 | 3.105(3) | 9.468(3) |
| 7.793 | 0.00968 | 20.4 | 0.13221 | 0.12798 | 3.089(6) | 9.437(6) |

Experimental values: $m_{J/\psi}$ = 3.096.916(11) GeV, $m_\Upsilon$ = 9.46030(26) GeV

J. Beringer *et al*. [PDG], PRD 86 (2012) 010001

# Spatial Correlation Function and Screening Masses

["Signatures of charmonium modification in spatial correlation functions", F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, (2012) arXiv:1203.3770]

Correlation functions along the **spatial direction**

$$G(z,T) = \int dxdy \int_0^{1/T} d\tau \langle J(x,y,z,\tau) J(0,0,0,0) \rangle$$

are related to the meson spectral function at **non-zero spatial momentum**

$$G(z,T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega,p_z,T)}{\omega}$$

exponential decay defines **screening mass $M_{scr}$ :** $\mathbf{G(z,T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr} z}}$

bound state contribution

$$\sigma(\omega,p_z,T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

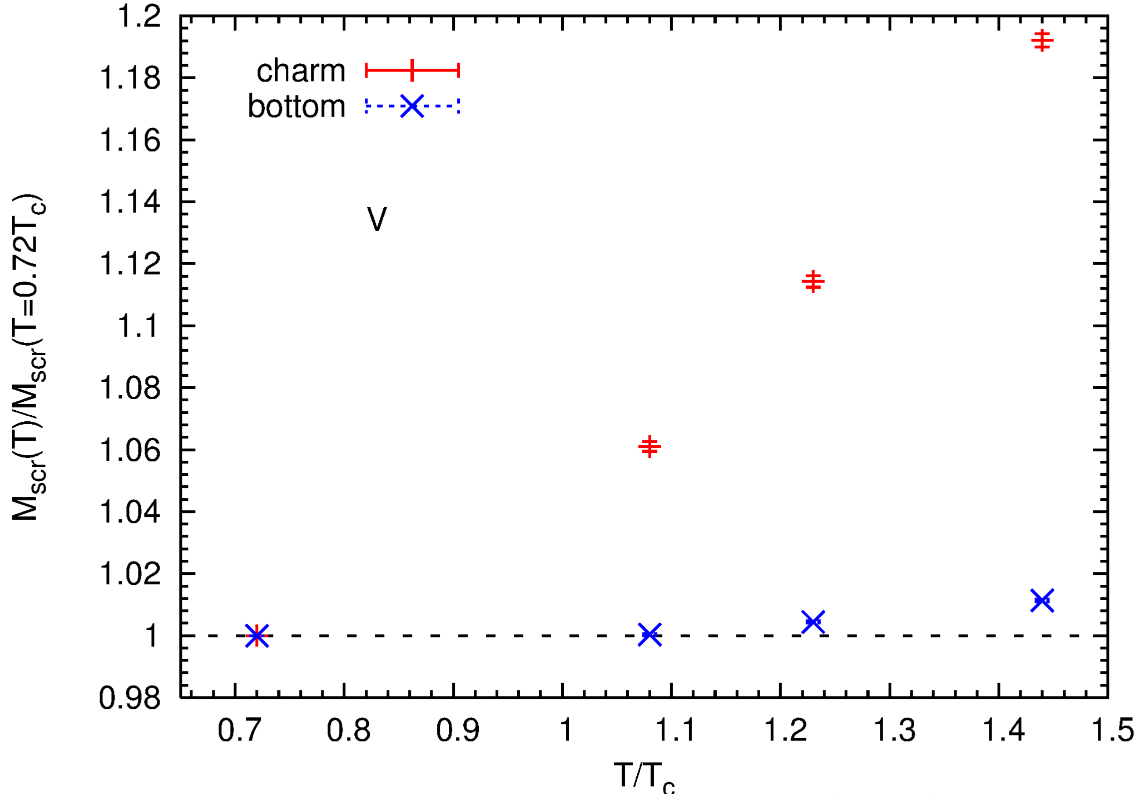
high-T limit (non-interacting free limit)

$$\sigma(\omega,p_z,T) \sim \sigma_{free}(\omega,p_z,T)$$

$M_{scr} = M$ ⟶ indications for medium modifications/dissociation ⟵ $M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$

# Spatial Correlation Function and Screening Masses

[H.T.Ding, H.Ohno, OK, work in progress]




exponential decay defines **screening mass $M_{scr}$ :** $\mathbf{G(z,T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr}z}}$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

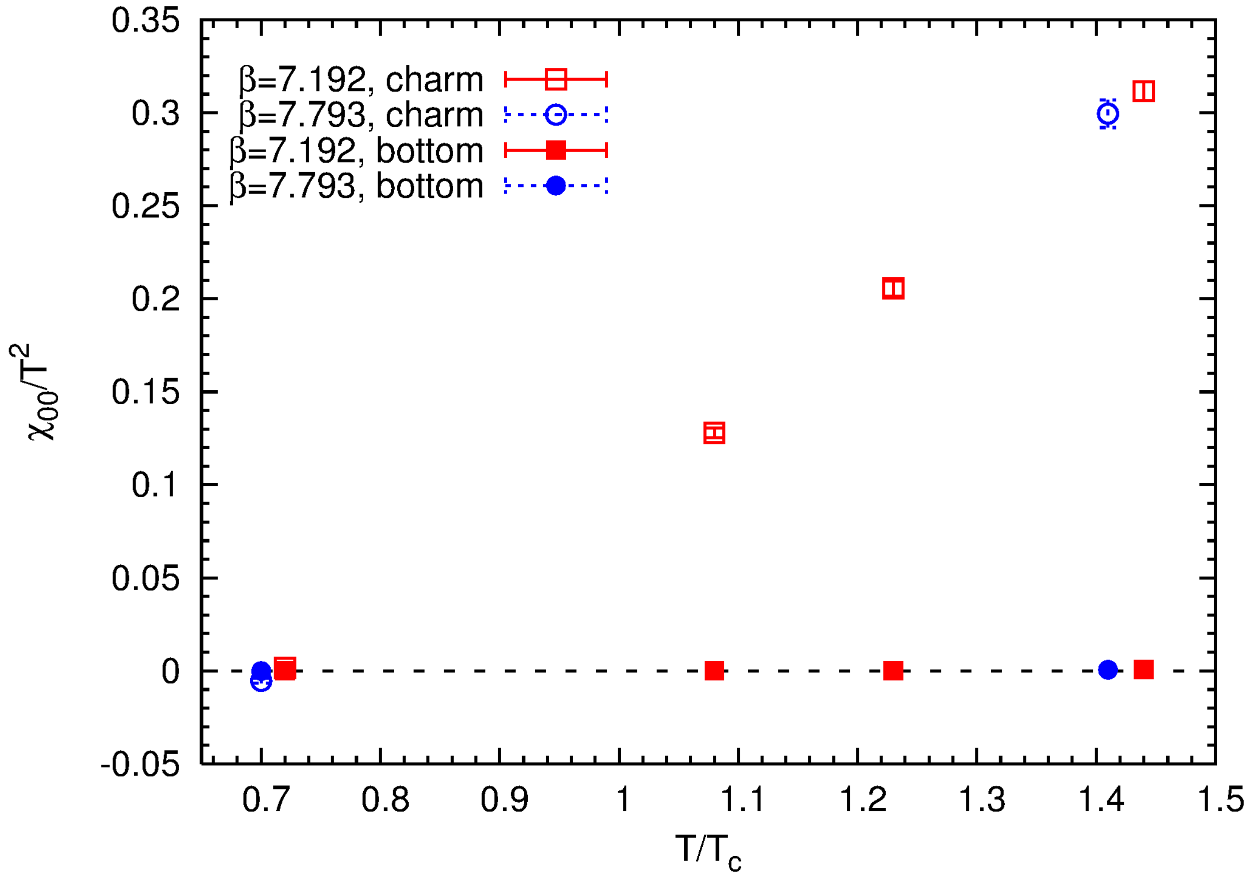
$M_{scr} = M$ → indications for medium modifications/dissociation ← $M_{scr} = 2\sqrt{(\pi T)^2 + m_c{}^2}$

# Charmonium and Bottomonium correlators

[H.T.Ding, H.Ohno, OK, work in progress]

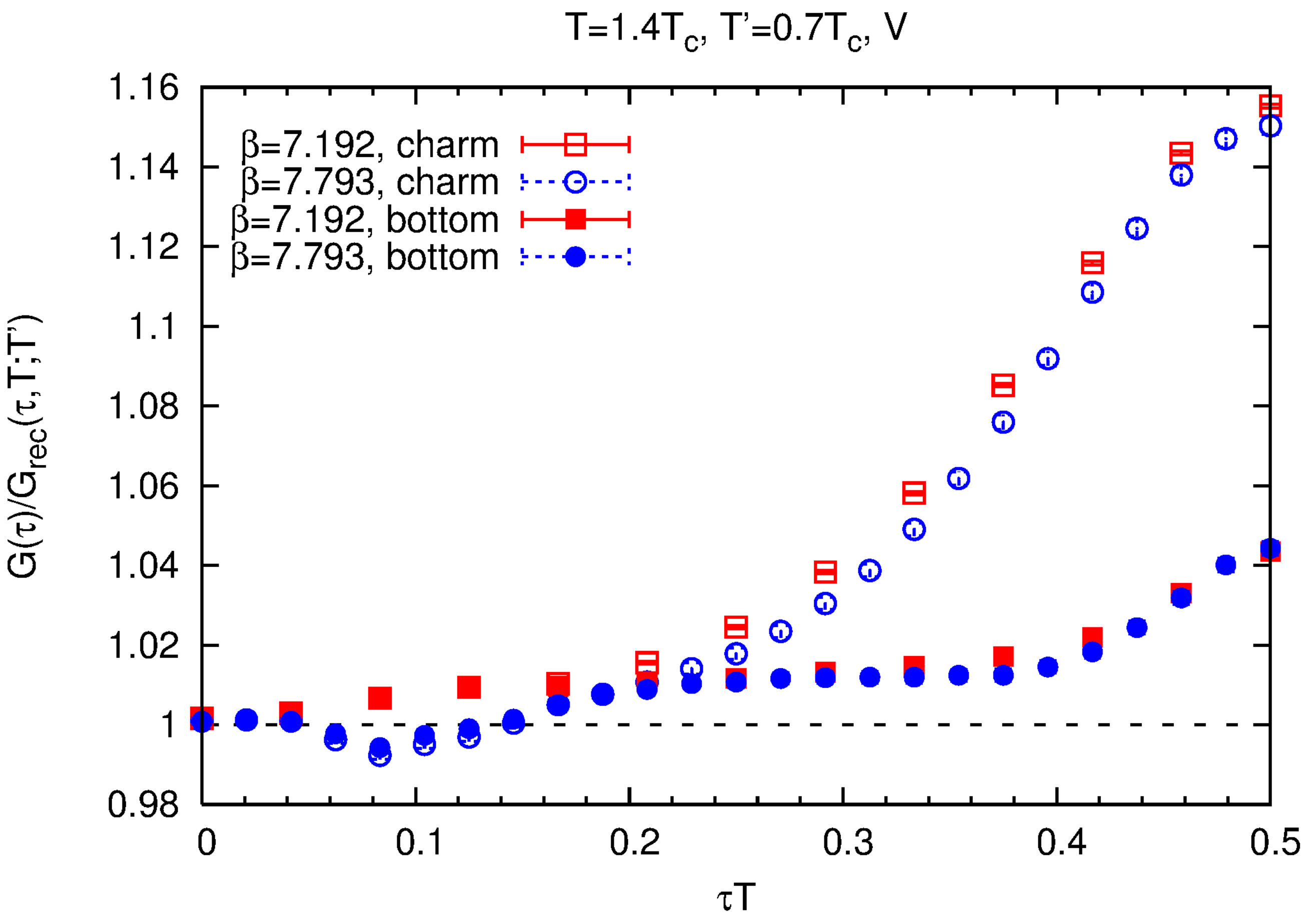
**Quark number susceptibility:**

$$\rho^V_{00}(\omega) = 2\pi\chi_{00}\omega\delta(\omega)$$
$$G^V_{00}(\tau) = T\chi_{00}$$



# Charmonium and Bottomonium correlators

[H.T.Ding, H.Ohno, OK, work in progress]



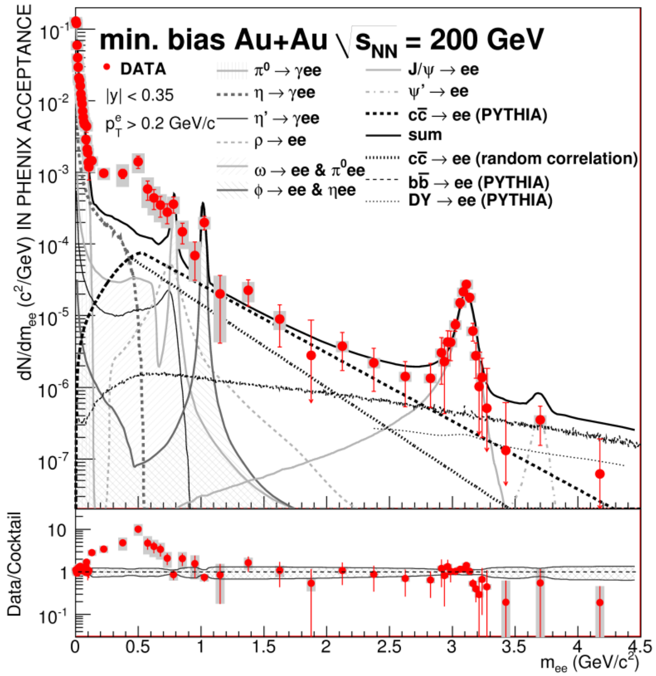
# Light Quark correlators – Motivation: Low-Mass Dilepton rates

pp-data well understood by hadronic cocktail

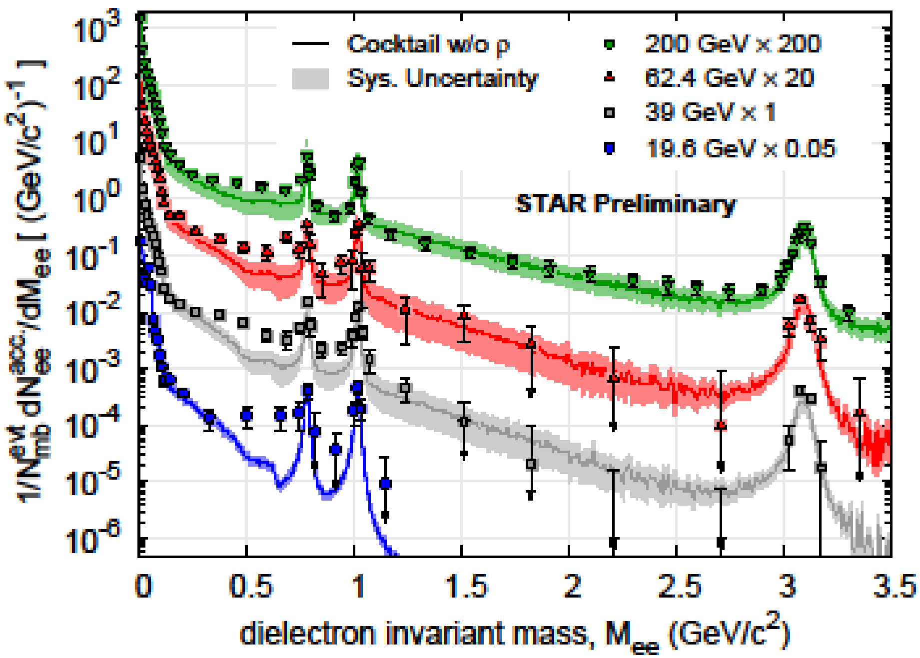
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP→ spectral functions from lattice QCD




[PHENIX PRC81, 034911 (2010)]




[STAR preliminary, arXiv:1210.5549]

**Dileptonrate directly related to vector spectral function:**

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \vec{\mathbf{p}}, \mathbf{T})$$

# Transport coefficients from Lattice QCD – Electrical conductivity

**Transport coefficients usually calculated using correlation function of conserved currents**

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\mathbf{p},T)K(\tau,\omega,T)$$

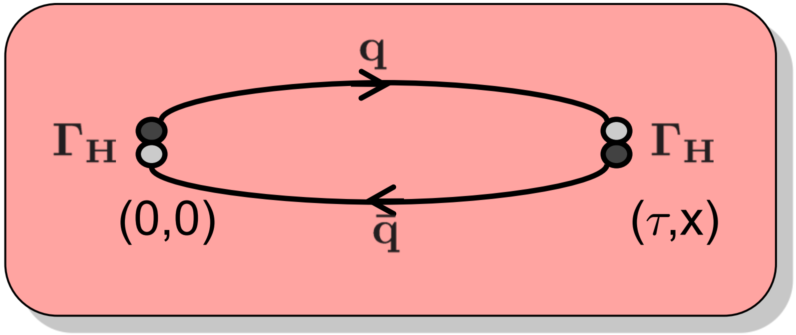
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

**Lattice observables:**

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$




related to a conserved current

only correlation functions calculable on lattice but

**Transport coefficient** determined by slope of spectral function at $\omega$=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

# Spectral functions at high temperature

## Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\rho_{00}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega)$$
$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)$$

$\delta$-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

## With interactions (but without bound states):

while $\rho_{00}$ is protected, the $\delta$-funtion in $\rho_{ii}$ gets smeared:

$\kappa = \frac{\alpha_s}{\pi}$ at leading order

**Ansatz**:

$$\rho_{00}(\omega) = 2\pi \chi_q \omega \delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

# Electrical Conductivity

**Electrical Conductivity** ⟷ slope of spectral function at $\omega$=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 = \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW}\ C_{em}$$

# Vector correlation function on large & fine lattices

[H.T.-Ding, OK et al., PRD83 (2011) 034504]

**Quenched SU(3) gauge configurations** at $T/T_c$=1.5 (separated by 500 updates)

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma$ = 32 – 128
$N_\tau$ = 16, 24, 32, 48

Temperature: $T = \frac{1}{aN_\tau}$

**Non-perturbatively O(a) clover improved Wilson fermions**

Non-perturbative renormalization constants

**Quark masses close to the chiral limit,** $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

Volume dependence

| $N_\tau$ | $N_\sigma$ | $\beta$ | $c_{SW}$ | $\kappa$ | $Z_V$ | $1/a$[GeV] | $a$[fm] | #conf |
|---|---|---|---|---|---|---|---|---|
| 16 | 32 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 60 |
| 16 | 48 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 62 |
| 16 | 64 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 77 |
| 16 | 128 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 129 |
| 24 | 128 | 7.192 | 1.3673 | 0.13440 | 0.842 | 9.65 | 0.020 | 156 |
| 32 | 128 | 7.457 | 1.3389 | 0.13390 | 0.851 | 12.86 | 0.015 | 255 |
| 48 | 128 | 7.793 | 1.3104 | 0.13340 | 0.861 | 19.30 | 0.010 | 431 |

cut-off dependence & continuum extrapolation

close to continuum

# New results at 1.1 and 1.2 $T_c$

[M.Müller et al., arXiv:1301.7436]

**PRACE-Project:**
Thermal Dilepton Rates and
Electrical Conductivity in the QGP

(JUGENE Bluegene/P in Jülich)

| | $1.1\ T_c$ | $1.2\ T_c$ | | | | | |
|---|---|---|---|---|---|---|---|
| $N_\sigma$ | $N_\tau$ | $N_\tau$ | $\beta$ | $\kappa$ | $1/a$[GeV] | $a$[fm] | #Confs |
| 96 | 32 | 28 | 7.192 | 0.13440 | 9.65 | 0.020 | 250 |
| 144 | 48 | 42 | 7.544 | 0.13383 | 13.21 | 0.015 | 300 |
| 192 | 64 | 56 | 7.793 | 0.13345 | 19.30 | 0.010 | 240 |

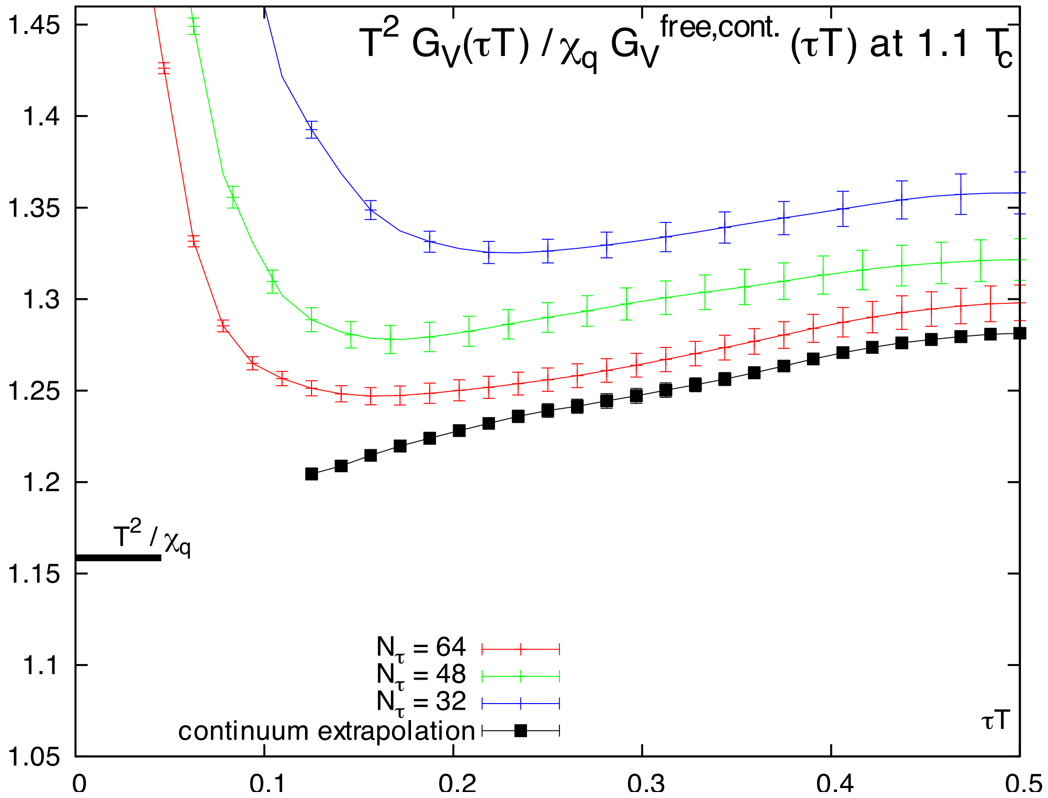
**study of T-dependence of dilepton rates and electrical conductivity**

**fixed aspect ratio $N_\sigma/N_\tau = 3$ to allow continuum limit at finite momentum:**

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

**constant physical volume** $(1.9\text{fm})^3$

# Continuum extrapolation



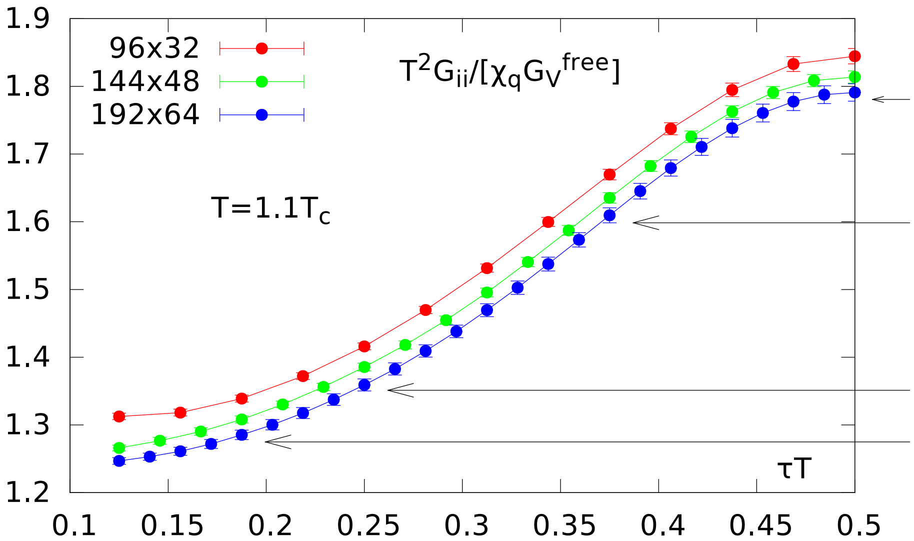

**cut-off effects visible at all distances but**

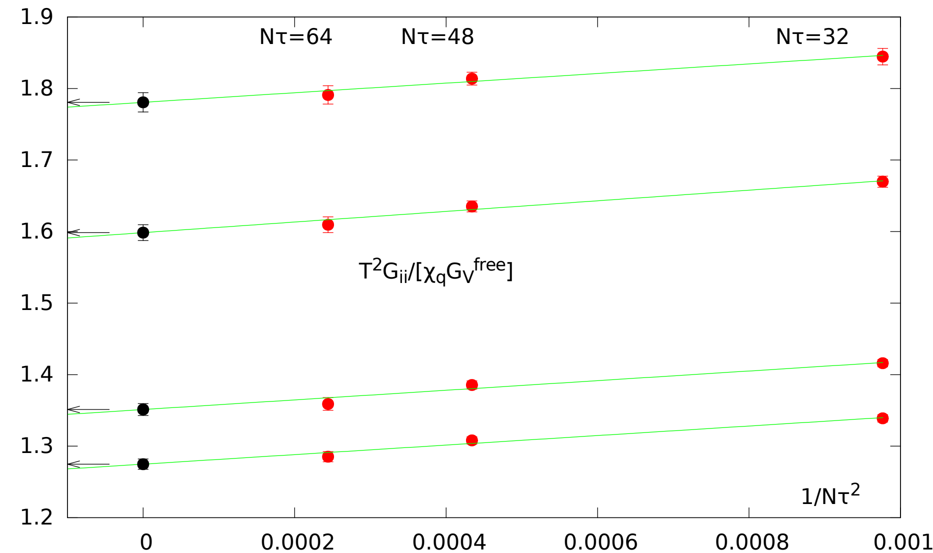
**well defined continuum limit on the correlator level**

**well behaved continuum correlator down to small distances**

**approaching the correct asymptotic limit for $\tau \rightarrow 0$**

# Continuum extrapolation

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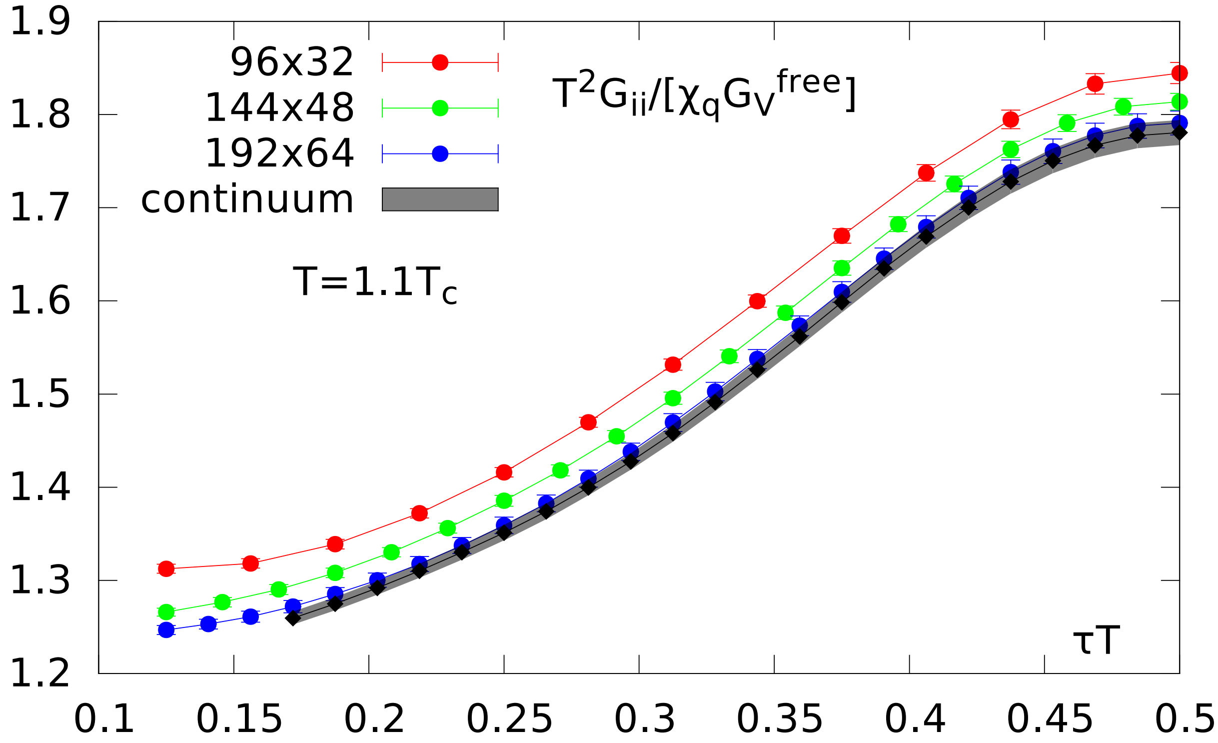
# Continuum extrapolation




**cut-off effects visible at all distances but**

**well defined continuum limit on the correlator level**

**well behaved continuum correlator down to small distances**

**approaching the correct asymptotic limit for $\tau \to 0$**

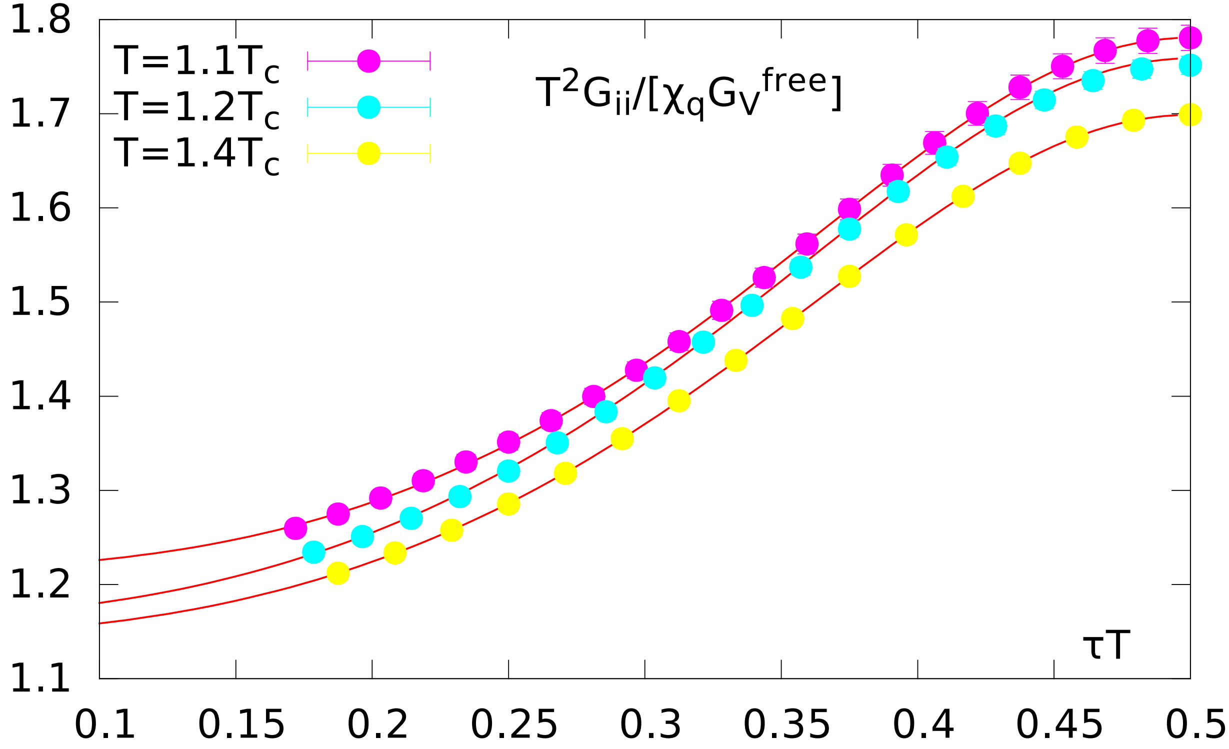
# Spectral function and electrical conductivity

**Use our Ansatz for the spectral function**

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\;\tanh(\omega/4T)$$

**and fit to the continuum extrapolated correlators**

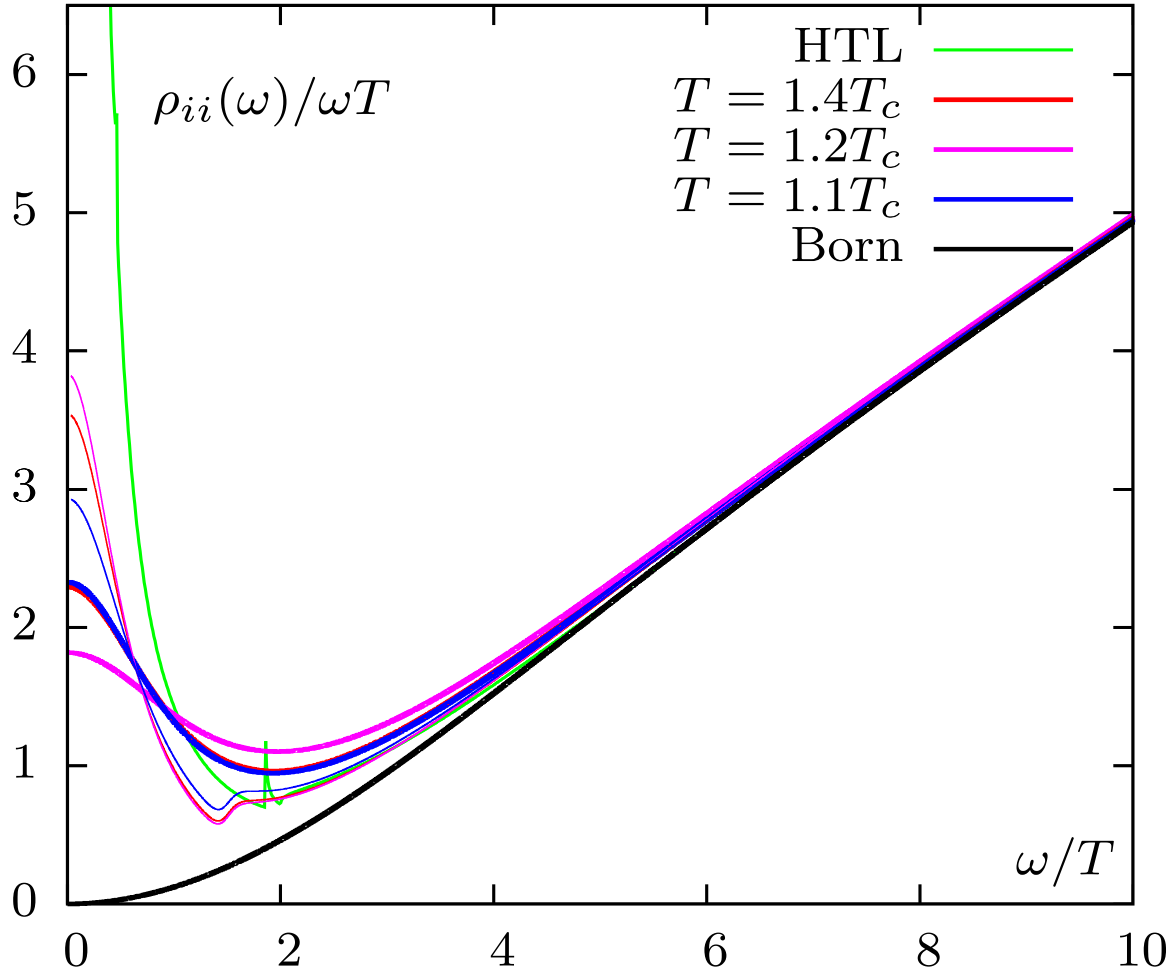



**all three temperatures are well described by this rather simple Ansatz**

# Spectral function and electrical conductivity

## Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$




**Analysis of the systematic errors**

using truncation of the large $\omega$ contribution

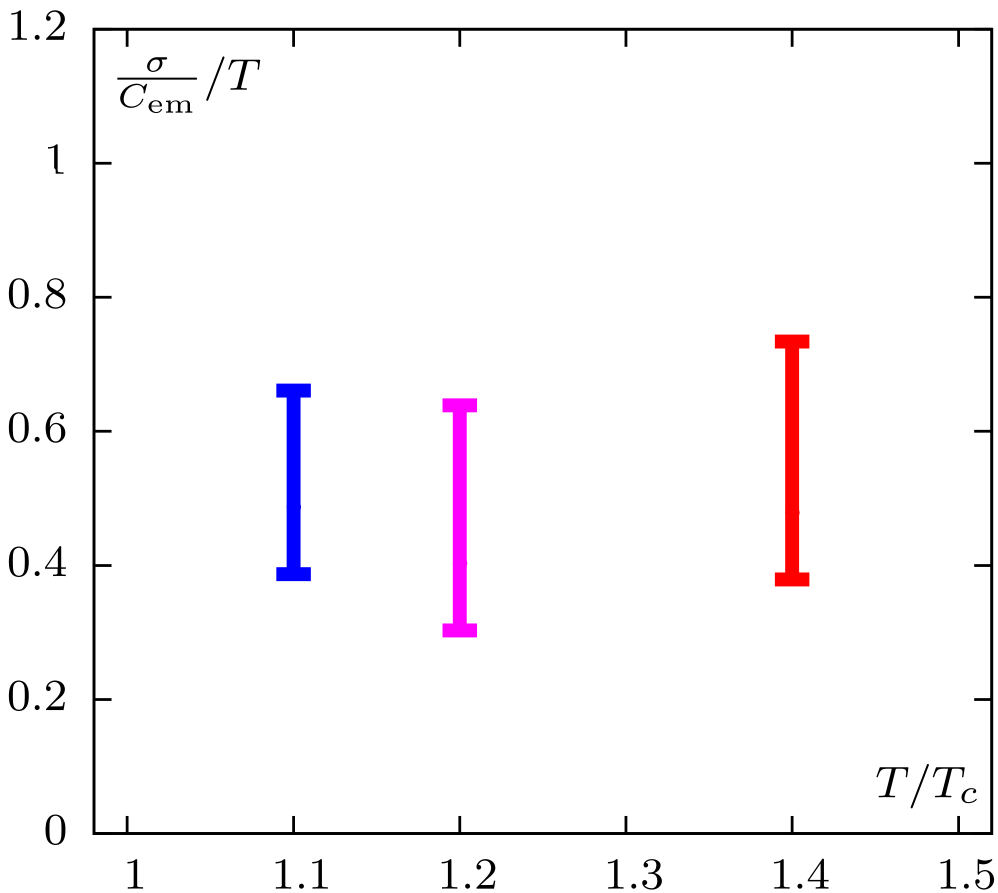
$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

**electrical conductivity**

**T-dependence of the electrical conductivity:**

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

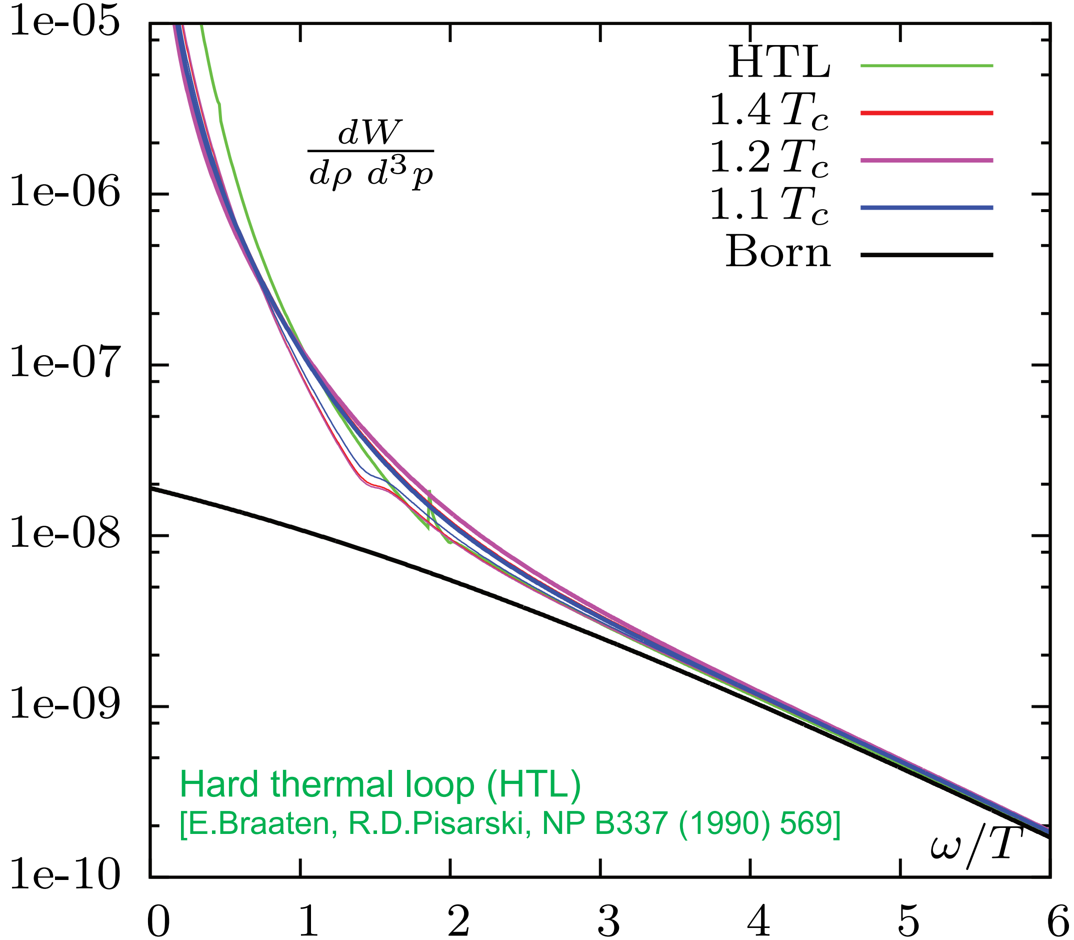



similar studies using dynamical clover Wilson (w/o continuum limit):
A.Amato et al., arXiv:1307.6763
B.B.Brandt et al., JHEP 1303 (2013) 100
previous studies using staggered fermions (need to distinguish $\rho_{even}$ and $\rho_{odd}$):
S.Gupta, PLB 597 (2004) 57
G.Aarts et al., PRL 99 (2007) 022002

# Dilepton rates

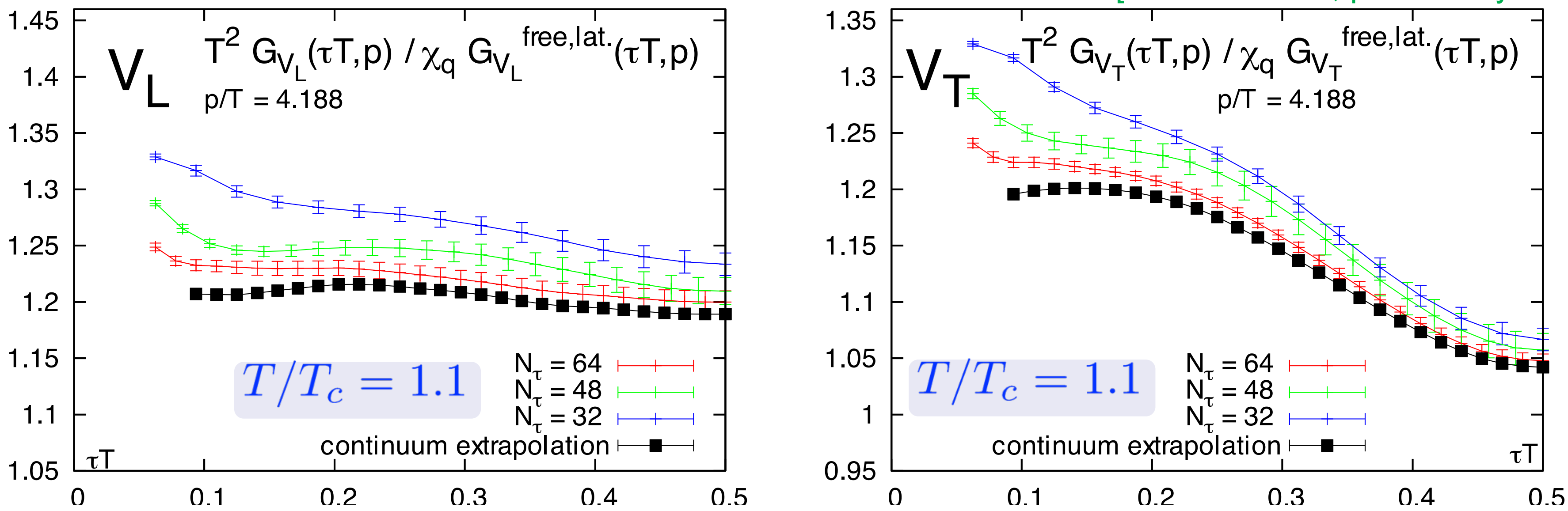
**Dileptonrate directly related to vector spectral function:**

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

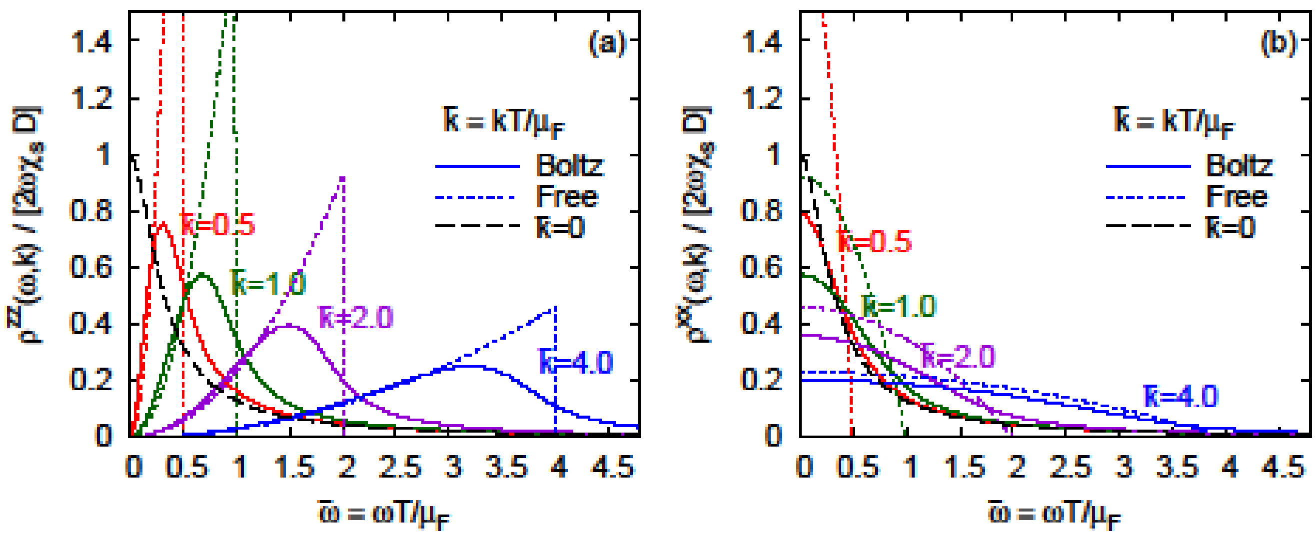


# Non-zero momentum

[M.Müller et al., preliminary 2012]




**indications for non-trivial behavior of spectral functions at small frequencies:**




[Hong+Teaney, PRC82 (2010)044908]

# Conclusions and Outlook

**Conclusions:**

Detailed knowledge of the **vector correlation function** in the region $1.1 \leq T/T_c \leq 1.5$

→ **continuum extrapolation** of correlation function and thermal moments

continuum $G_V(\tau T)$ well reproduced by **Breit-Wigner plus continuum** Ansatz for $\sigma_V(\omega)$ in the temperature region $1.1 \leq T/T_c \leq 1.5$

→ **electrical conductivity** $\sigma/T$ shows small temperature effects

→ **Dilepton rate** approaches leading order Born rate for $\omega/T \geq 4$
enhancement at small $\omega/T$

**Outlook:**

include HTL result for $\sigma_V(\omega)$ at large $\omega/T$ in the Ansatz

vector correlation function at **non-zero momentum**

especially close to $T_c$ effects of dynamical quarks need to be included