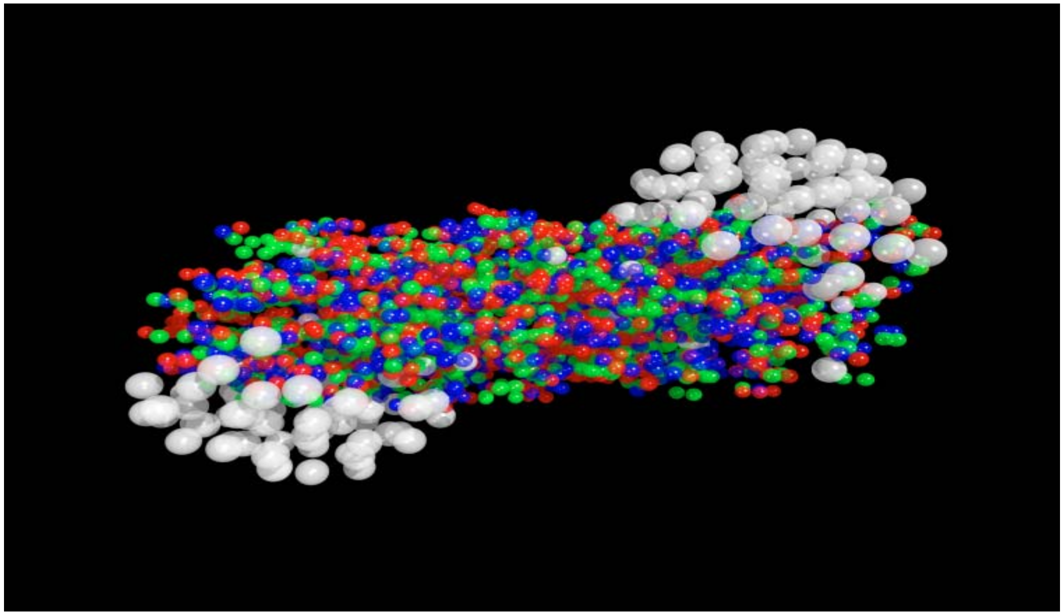
University of Catania
INFN-LNS

Heavy flavor in medium momentum evolution : Langevin vs Boltzmann



Santosh Kumar DaS

In collaboration with: Vincenzo Greco
Francesco Scadina
Salvatore Plumari

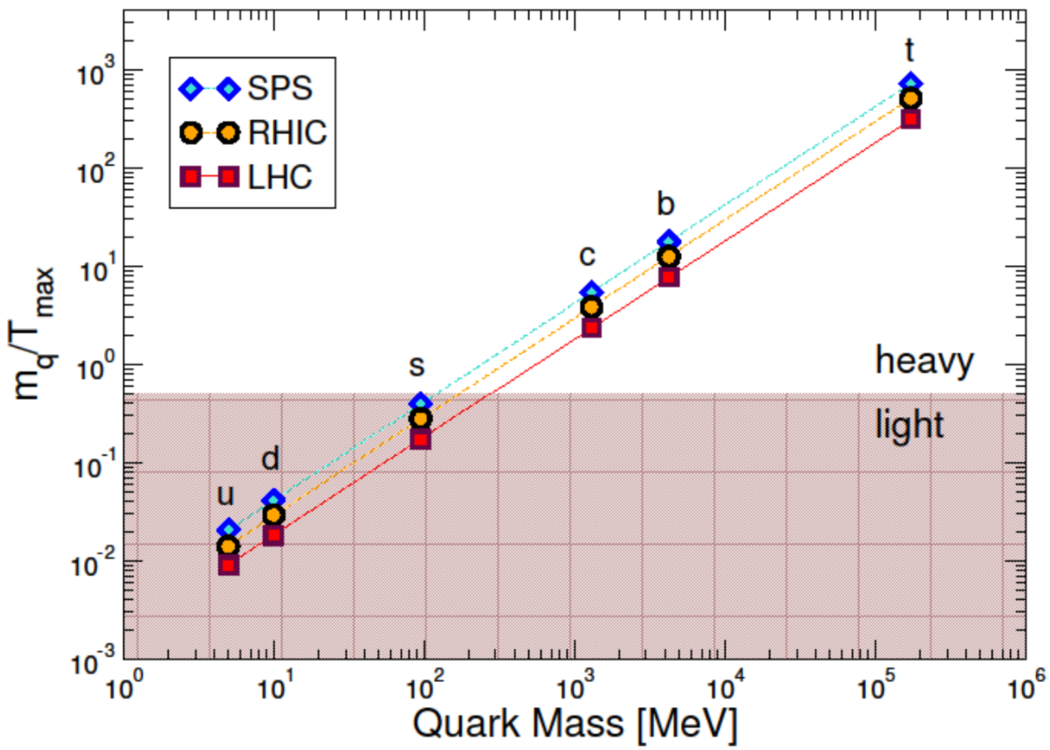
OUTLINE OF MY TALK.............

- **Introduction**
- **Heavy Quarks and Langevin dynamics**
- **Boltzmann approach to heavy quarks dynamics**
- **Similarities and differences between the two approaches in a static medium (Langevin and Boltzmann)**

  1) Spectra
  2) Momentum spreading
  2) Back to back azimuthal correlation

- **Comparison with the experimental observables (RAA and v2)**
- **Effect of hadronic medium on heavy quark observables**
- **Summary and outlook**

Heavy Quark & QGP



SPS to LHC

$\sqrt{s} = 17.3 GeV \ to \ 2.76 TeV$ **~100 times**

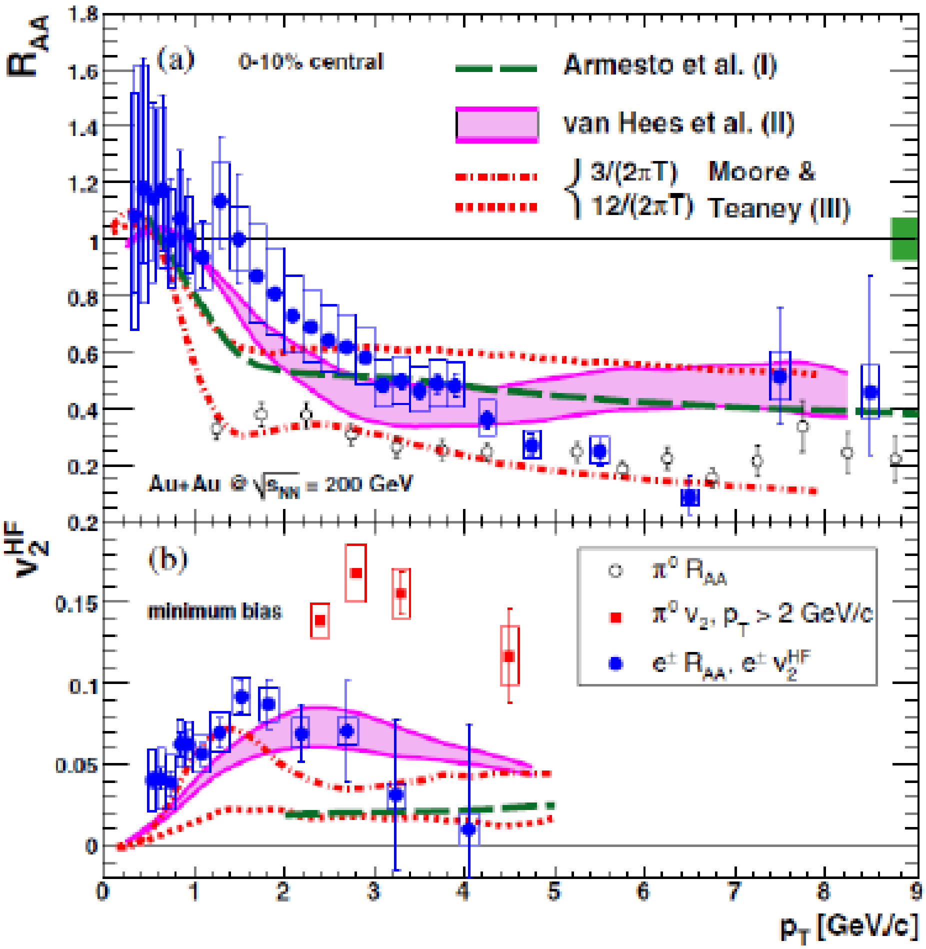
$T_i = 200 \ MeV \ to \ 600 \ MeV$ **~3 times**

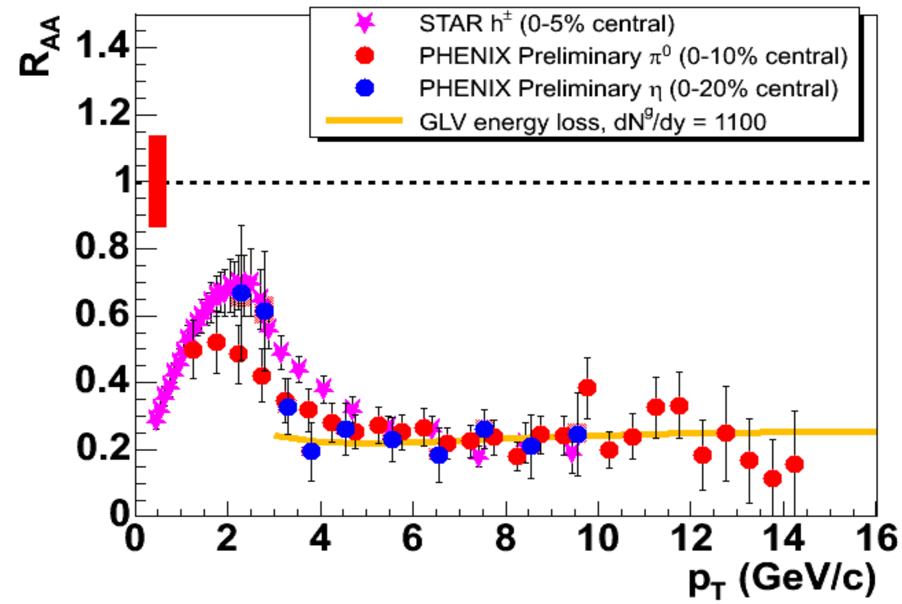
$M_{c,b} >> \Lambda_{QCD}$ **Produced by pQCD process (out of Equil.)**

$\tau_{c,b} << \tau_{QGP}$ **They go through all the QGP life time**

$M_{c,b} >> T_0$ **No thermal production**

Heavy flavor at RHIC

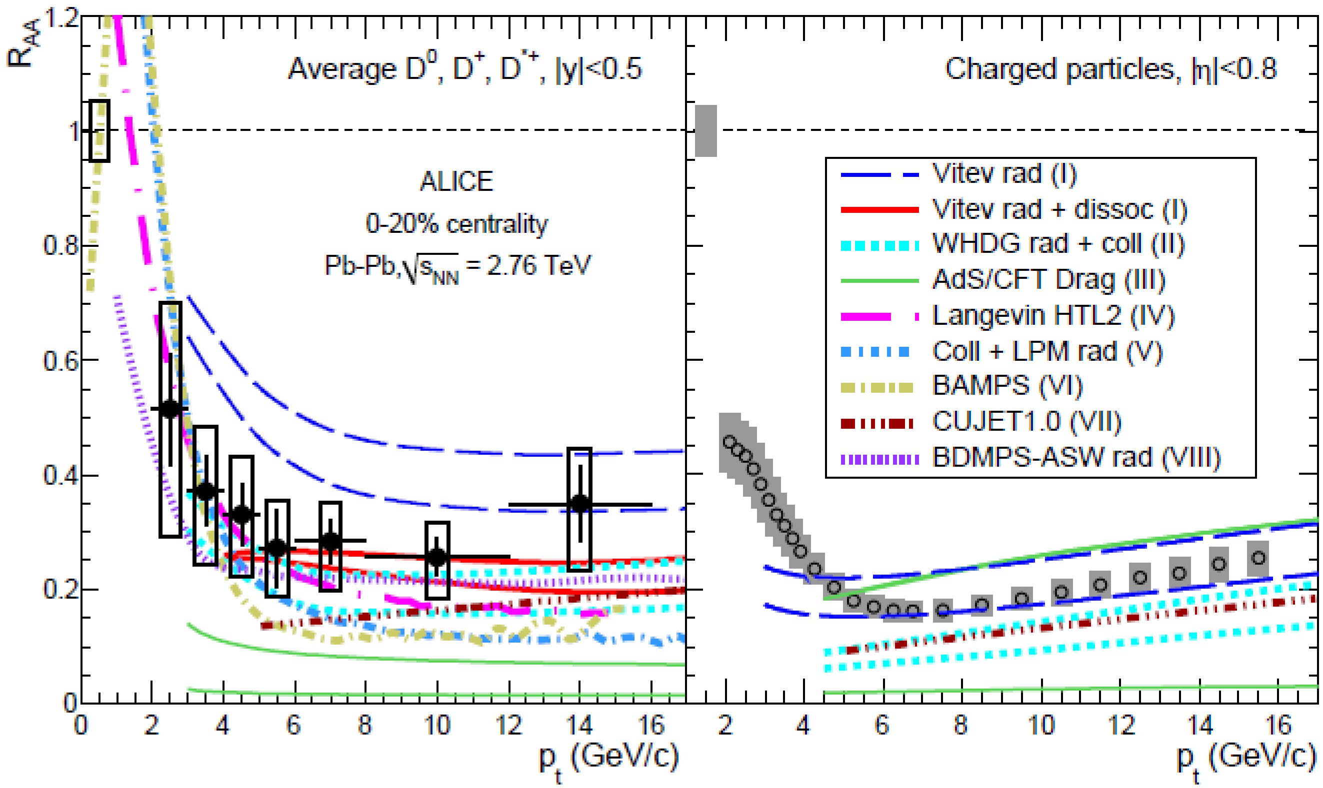




At RHIC energy heavy flavor suppression is similar to light flavor

Simultaneous description of RAA and v2 is a tough challenge for all the models.

Heavy Flavors at LHC



Again at LHC energy heavy flavor suppression is similar to light flavor

Is the momentum transfer really small !

Can one describe both RAA and v2 simultaneously?

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t} + \frac{P}{E}\frac{\partial}{\partial x} + \mathrm{F}.\frac{\partial}{\partial p}\right) f\left(x, p, t\right) = \left(\frac{\partial f}{\partial t}\right)_{col}$$

- The plasma is uniform ,i.e., the distribution function is independent of x.
- In the absence of any external force, F=0

$$R(p,t) = \left(\frac{\partial f}{\partial t}\right)_{col} = \int d^3k \left[\omega(p+k,k) f(p+k) - \omega(p,k) f(p)\right]$$

$$\omega(p,k) = g\int \frac{d^3q}{(2\pi)^3} f'(q) v_{q,p} \sigma_{p,q\to p-k,q+k}$$

→ is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k) f(p+k) \approx \omega(p,k) f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(p) f + \frac{\partial}{\partial p_j}\left[B_{ij}(p) f\right]\right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

$$A_i = \int d^3k\, \omega(p,k) k_i \quad \rightarrow \textbf{Drag Coefficient}$$

$$B_{ij} = \int d^3k\, \omega(p,k) k_i k_j \quad \rightarrow \textbf{Diffusion Coefficient}$$

$$\omega(p+k,k)\,f(p+k) \approx \omega(p,k)\,f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin Equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\, C_{jk}(t, p + \xi dp)\rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent**

Gaussian-normal distributed random variable

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp\left(-\frac{\rho^2}{2}\right)$$

With $<\rho_i(t)\rho_k(t')> = \delta(t-t')\delta_{jk}$

H. v. Hees and R. Rapp
arXiv:0903.1096

$\xi = 0$ **the pre-point Ito**

interpretation of the momentum argument of the covariance matrix.

Langevin process defined like this is equivalent to the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \frac{p_j}{E}\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(p_j\Gamma - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl}f)$$

the covariance matrix is related to the diffusion matrix by

$$C_{jk} = \sqrt{2B_0}P^{\perp}_{jk} + \sqrt{2B_1}P^{||}_{jk}$$

and $$A_i = p_j\Gamma - \xi C_{lk}\frac{\partial C_{ij}}{\partial p_l}$$

With $B_0 = B_1 = D$ $$C_{jk} = \sqrt{2D(E)}\delta_{jk}$$

Relativistic dissipation-fluctuation relation

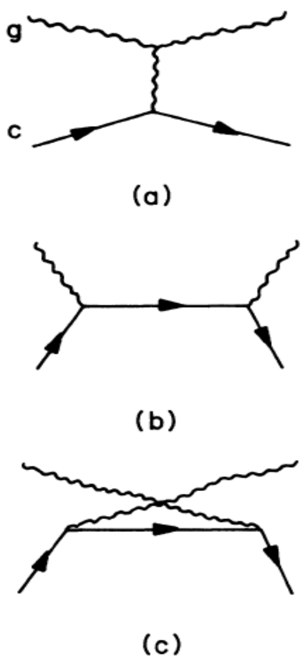
$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

For Collision Process the A_i and B_{ij} can be calculated as following :

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Elastic processes



$$gc \rightarrow gc$$

$$\frac{1}{t} \rightarrow \frac{1}{t-m_D^2}$$

$$m_D = \sqrt{4\pi\alpha_s}T$$

✓ We have introduce a mass into the internal gluon propagator in the t and u-channel-exchange diagrams, to shield the infrared divergence.

B. Svetitsky PRD 37(1987)2484

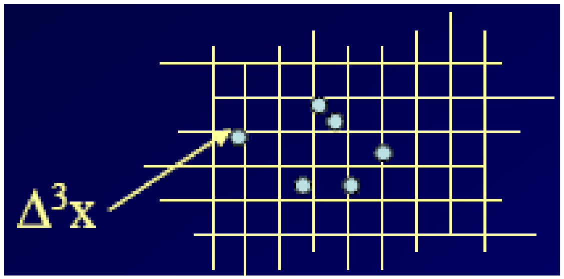
Mustafa, Pal and Srivastava, PRC, 57,889(1998)

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

We consider two body collisions

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2)$$
$$-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2)$$



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

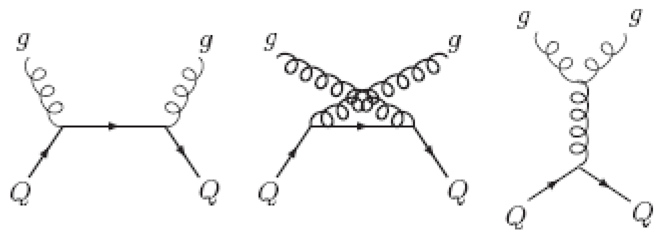
Exact solution

Collision integral is solved with a local stochastic sampling

[Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]

$$P_{22} = \frac{\Delta N_{\text{coll}}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\text{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Cross Section gc -> gc

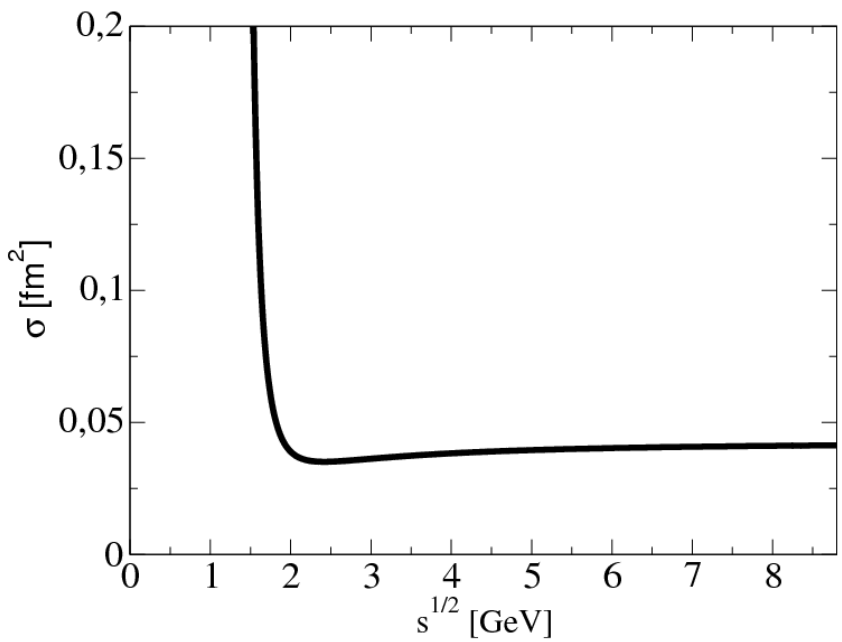


The infrared singularity is regularized introducing a Debye-screaning-mass μ_D

$$\frac{1}{t} \rightarrow \frac{1}{t-m_D^2} \qquad m_D = \sqrt{4\pi\alpha_s}\,T$$

$$\sum|\mathcal{M}|^2 = \pi^2\alpha^2(Q^2)\left[\frac{32(s-M^2)(M^2-u)}{t^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2}\right.$$
$$+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2} + \frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)}$$
$$\left.+16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)}\right]$$

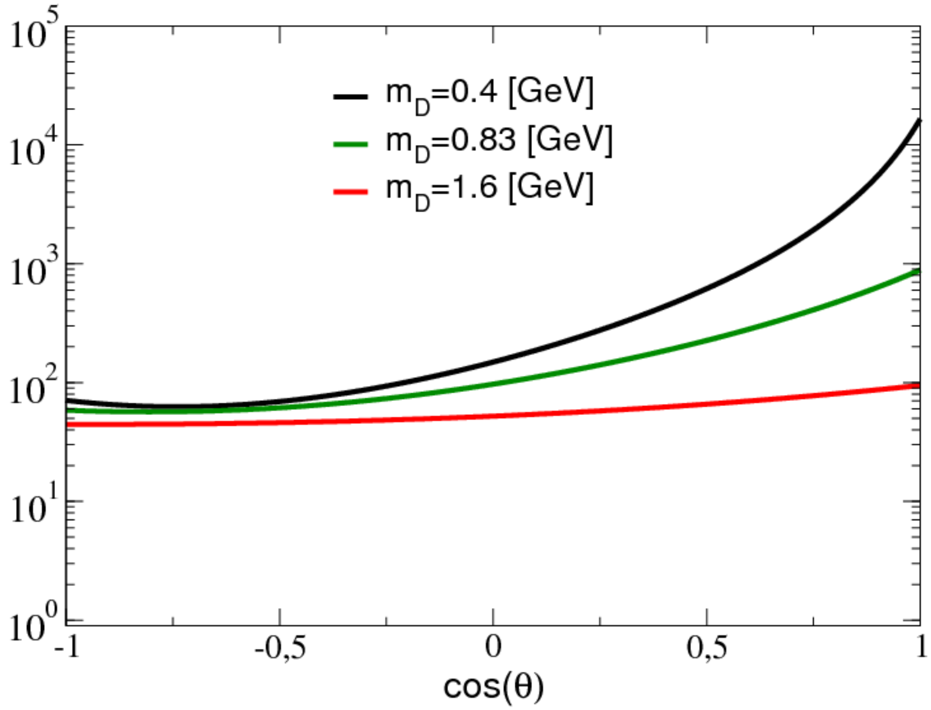
$$\hat{\sigma} = \frac{1}{16\pi(s-M^2)^2}\int_{-(s-M^2)^2/s}^{0} dt \sum|\mathcal{M}|^2$$



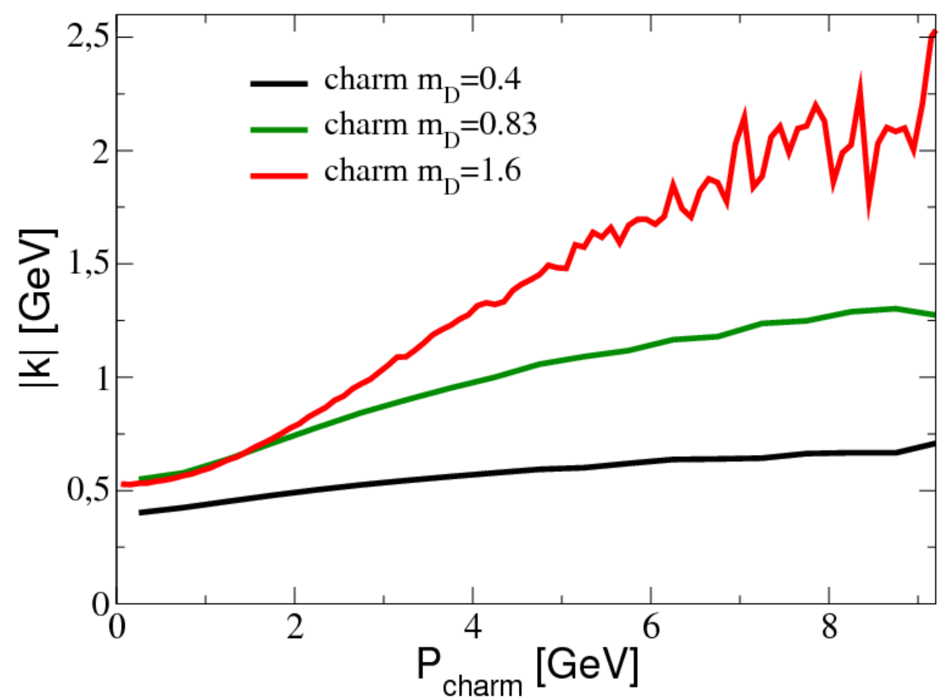
L. Combridge, Nucl. Phys. B151, 429 (1979)]
[B. Svetitsky, Phys. Rev. D 37, 2484 (1988)]

Boltzmann vs Langevin (Charm)

Angular dependence of σ



Mometum transfer vs P

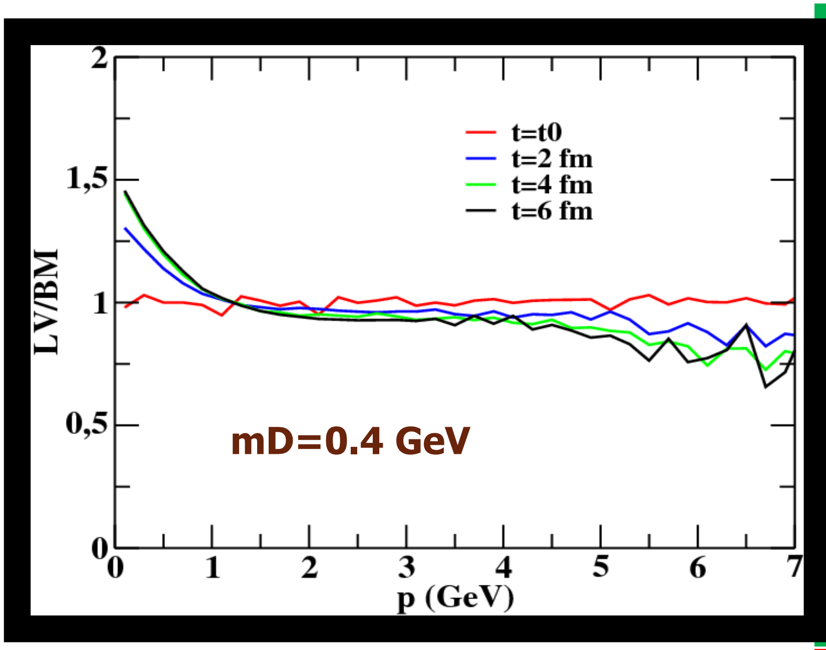


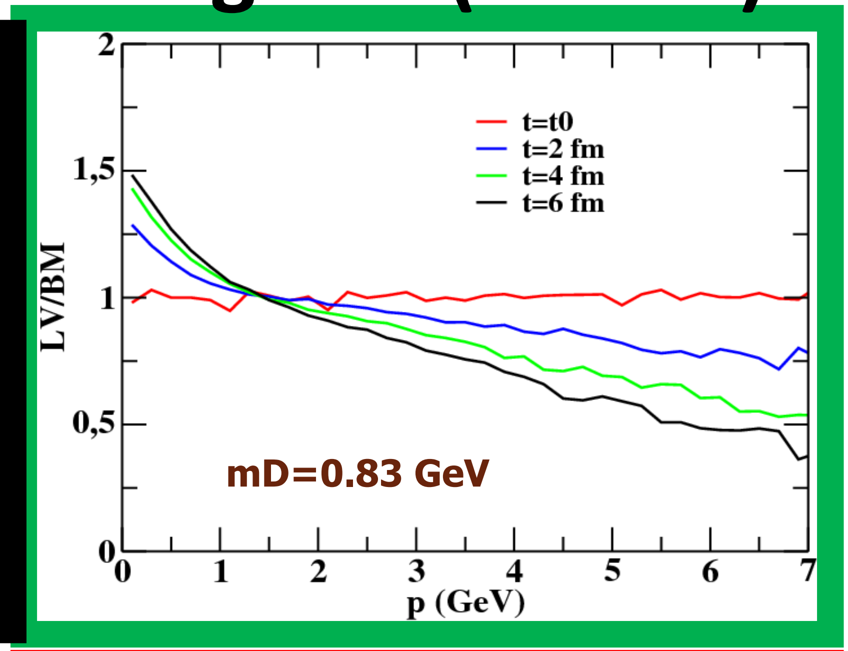
Decreasing m_D makes the σ more anisotropic ➡ **Smaller average momentum transfer**

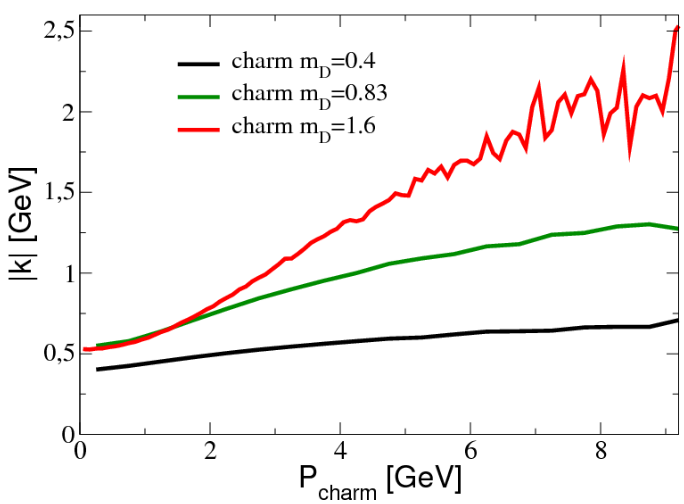
Hees, Mannarelli,Greco,Rapp, PRL100(2008)
Hees,Greco,Rapp. PRC73 (2006) 034913

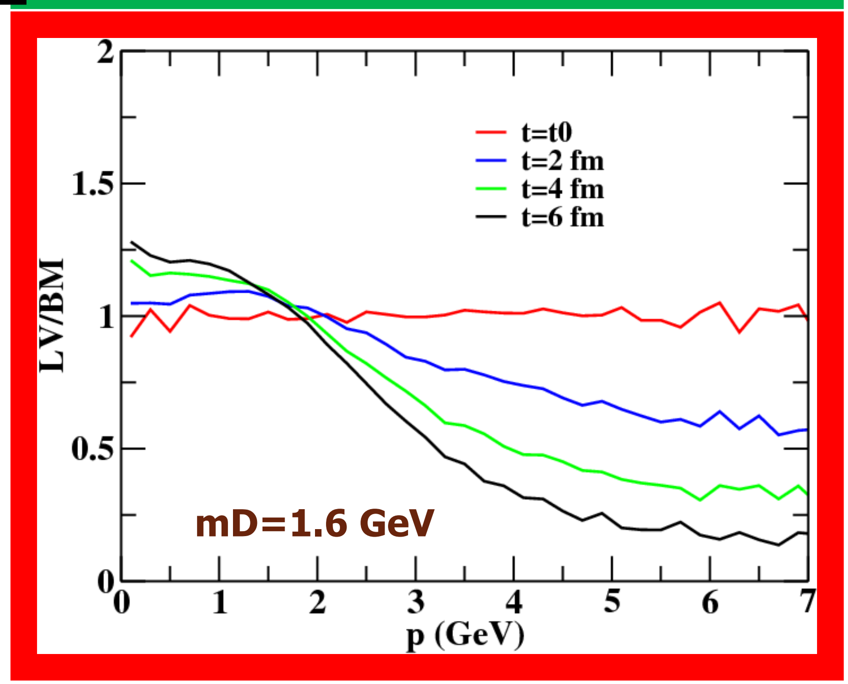
Das, Scardina, Plumari and Greco
arXiv:1312.6857

Boltzmann vs Langevin (Charm)



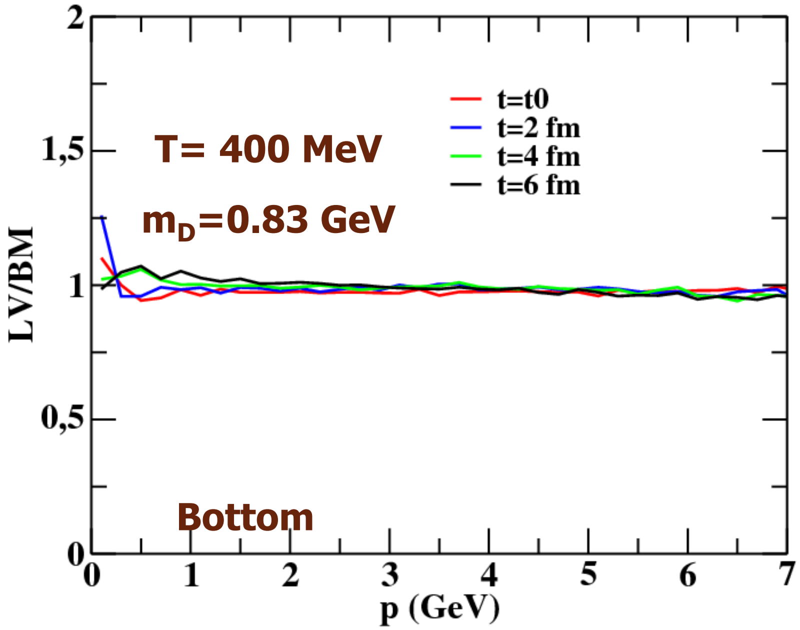


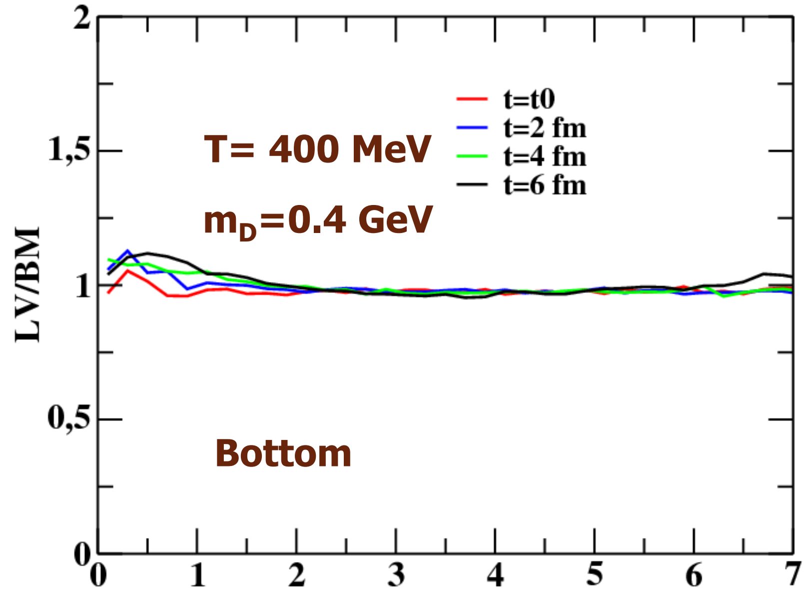


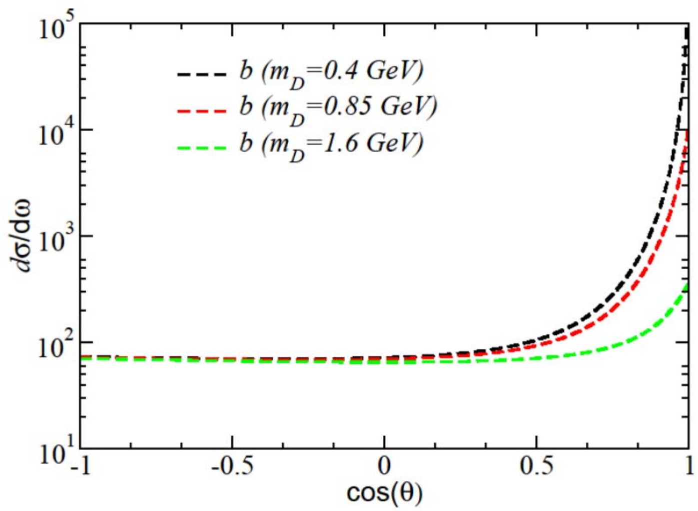


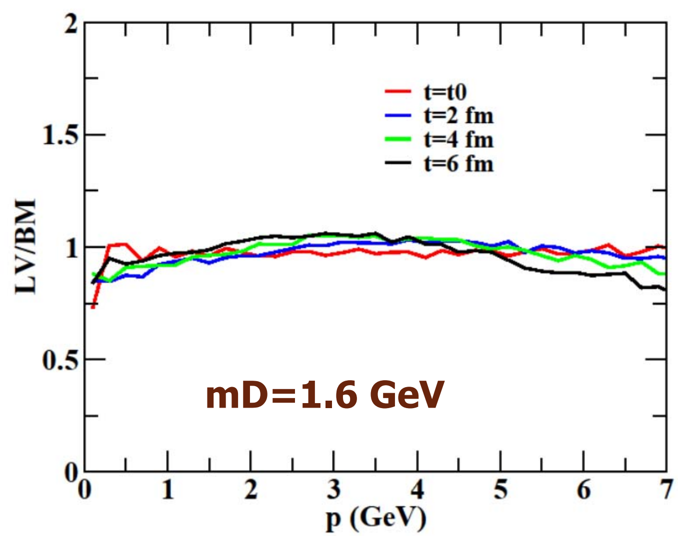
Das, Scardina,Plumari and Greco
arXiv:1312.6857 (PRC, In press)

Bottom: Boltzmann = Langevin









But Larger M_b/T (≈ 10) the better Langevin approximation works

Nuclear Suppression Factor (R_{AA}) :

$$R_{AA} = \frac{\left(\dfrac{dN}{d^2 p_T\, dy}\right)^{Au+Au}}{N_{coll}\left(\dfrac{dN}{d^2 p_T\, dy}\right)^{p+p}}$$

If $R_{AA} = 1$ → No medium

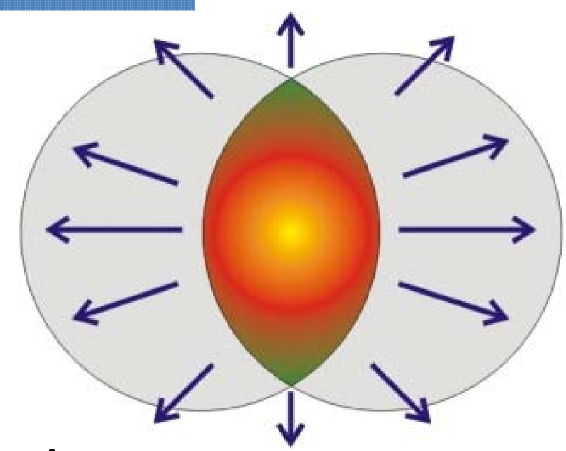
If $R_{AA} < 1$ → Medium

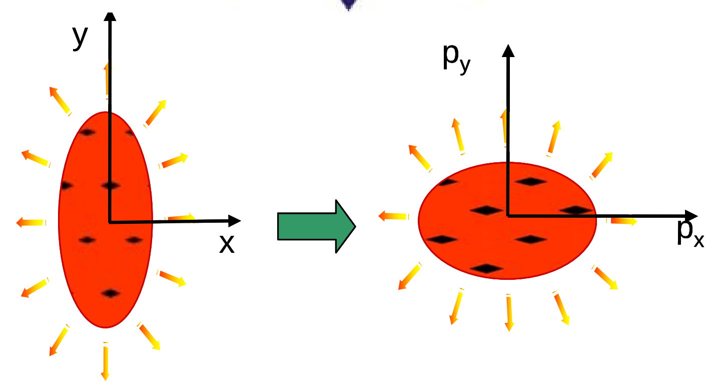
A direct measure of the energy loss

Elliptic Flow :

$$\frac{dN}{p_T\, dp_T\, dy\, d\phi} = \frac{dN}{2\pi p_T\, dp_T\, dy}\left(1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + ...\right)$$

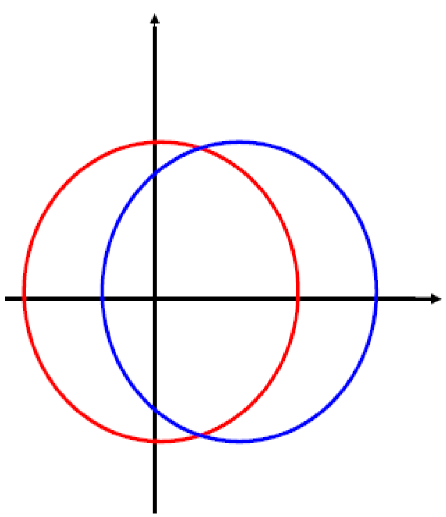
$$v_2^{HF}(p_T) = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$





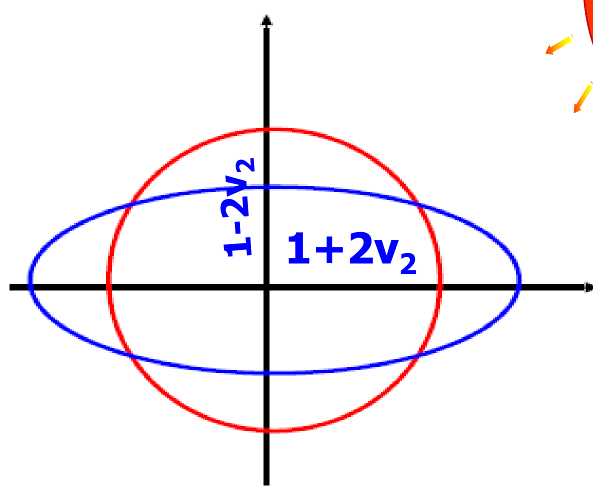
Polar Plots :

$1+2v_1\cos\varphi$



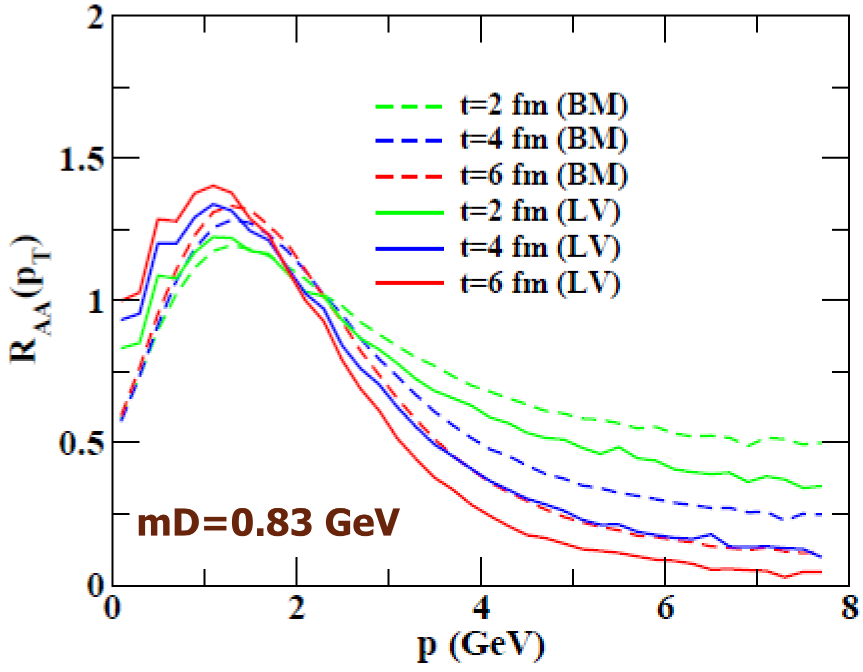
Overall shift

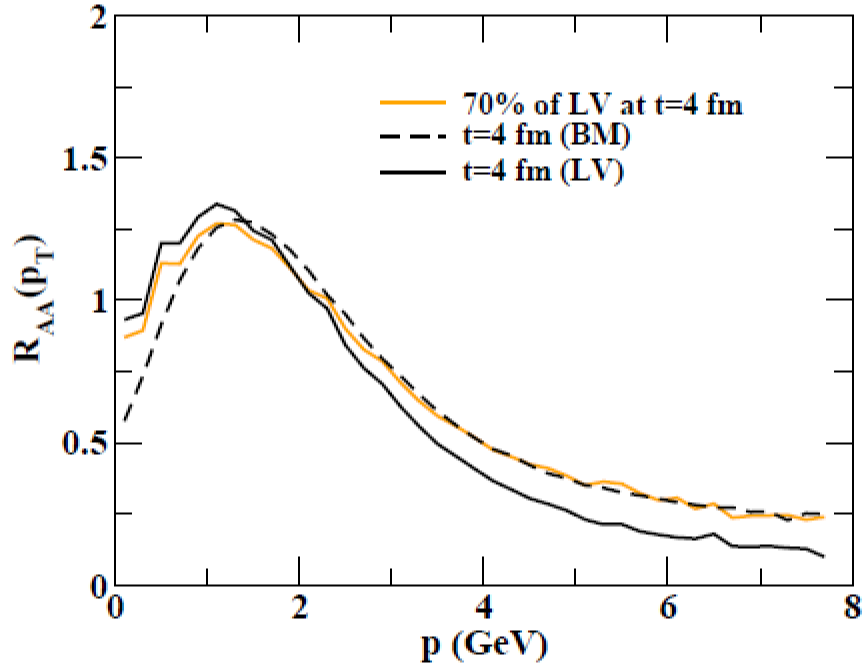
$1+2v_2\cos(2\varphi)$



Major axis = $1+2v_2$
Minor axis = $1-2v_2$

Implication for observable, R_{AA}?





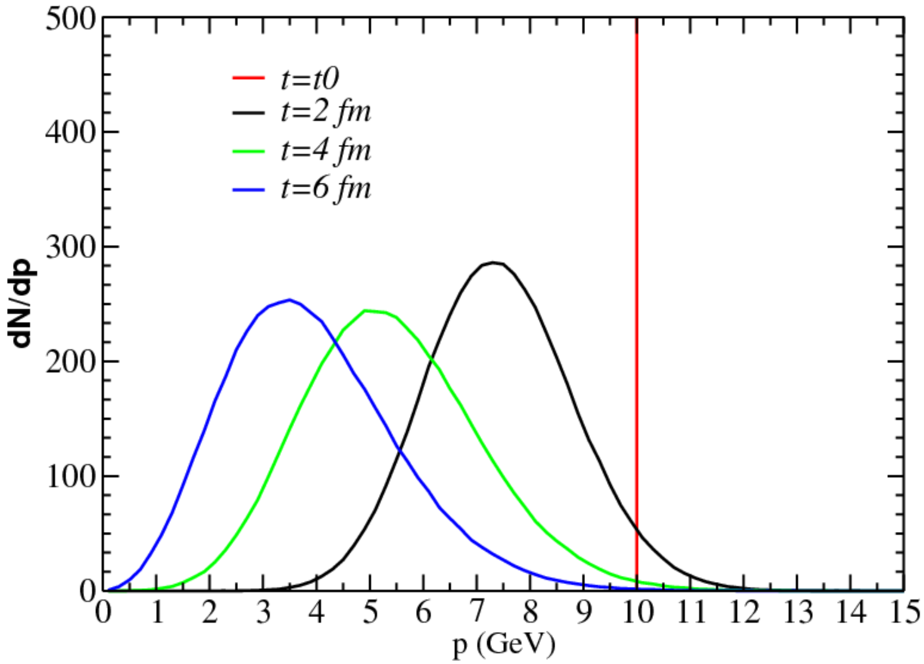
The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

However one can mock the differences of the microscopic evolution and reproduce the same R_{AA} of Boltzmann equation just changing the diffusion coefficient by about a 30 %

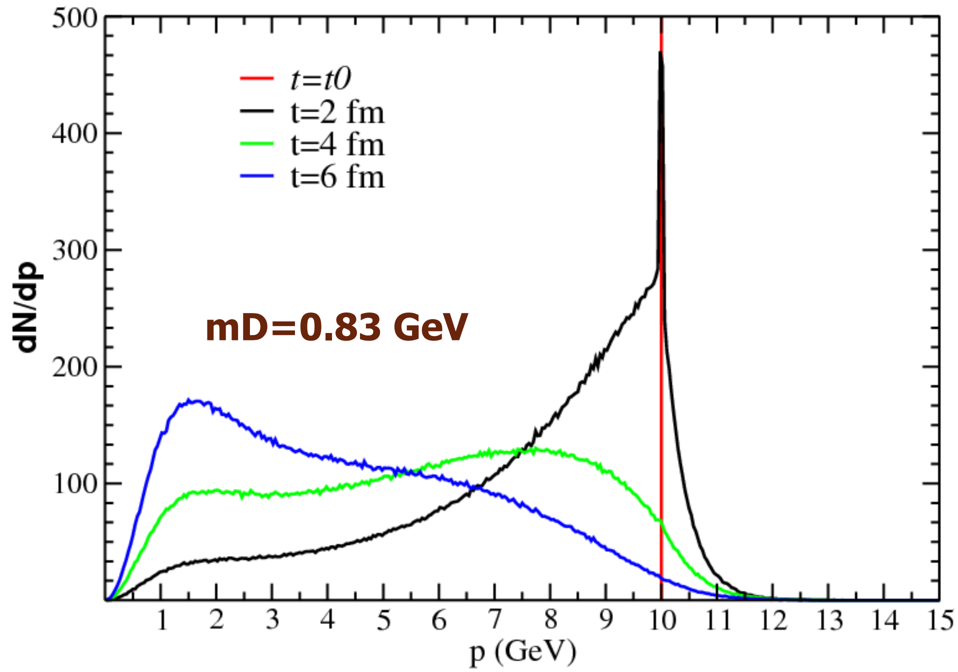
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$

Langevin



Boltzmann



In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks does not follow the Brownian motion

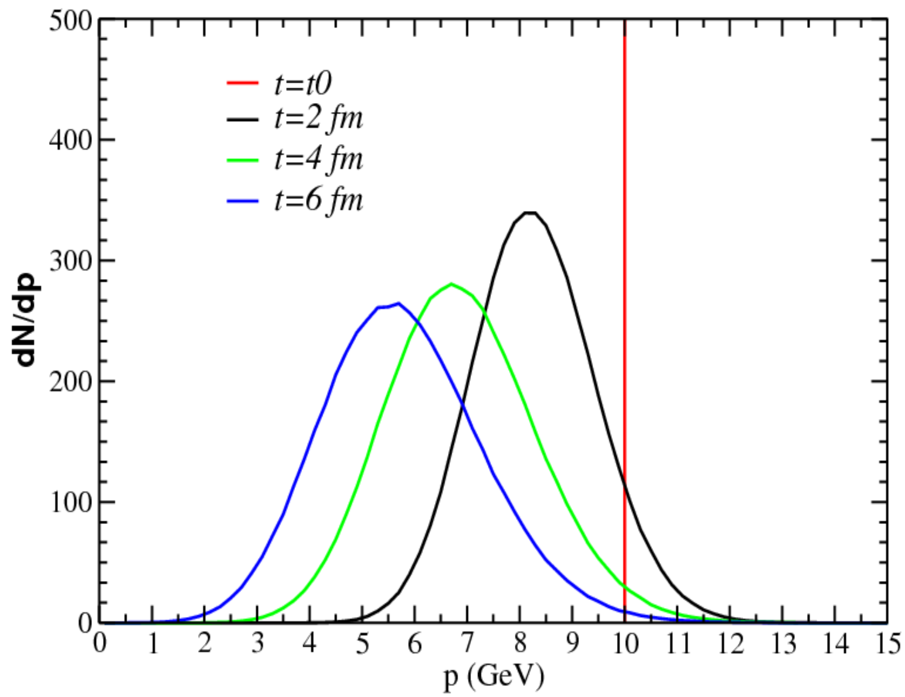
Das, Scardina, Plumari and Greco
arXiv:1312.6857 (PRC, In press)

Momentum evolution starting from a δ (Bottom)

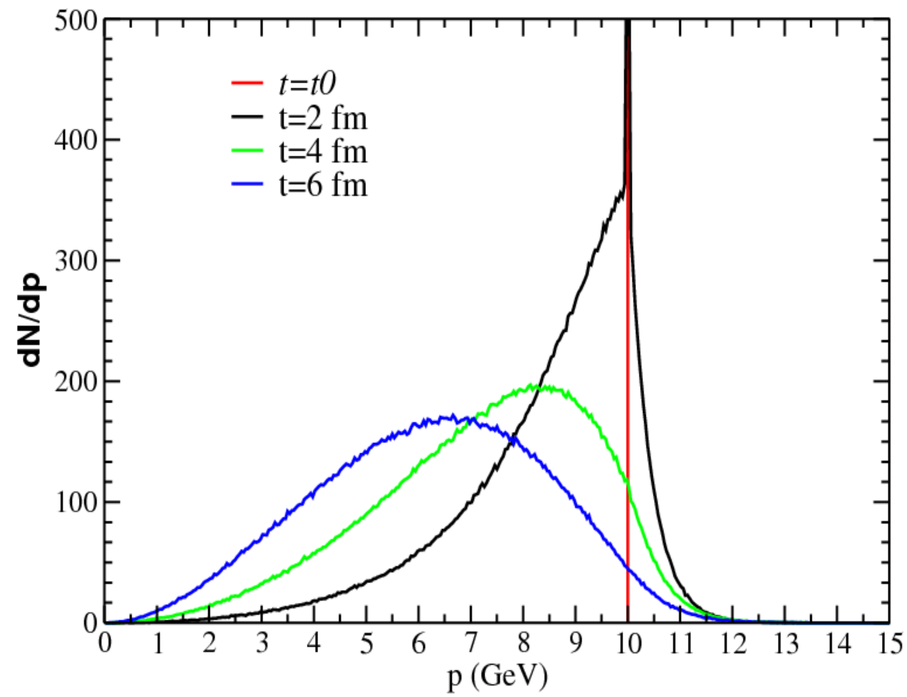
In a Box

$$\frac{dN}{d^3p}_{initial} = \delta\left(p - 10GeV\right)$$

Langevin



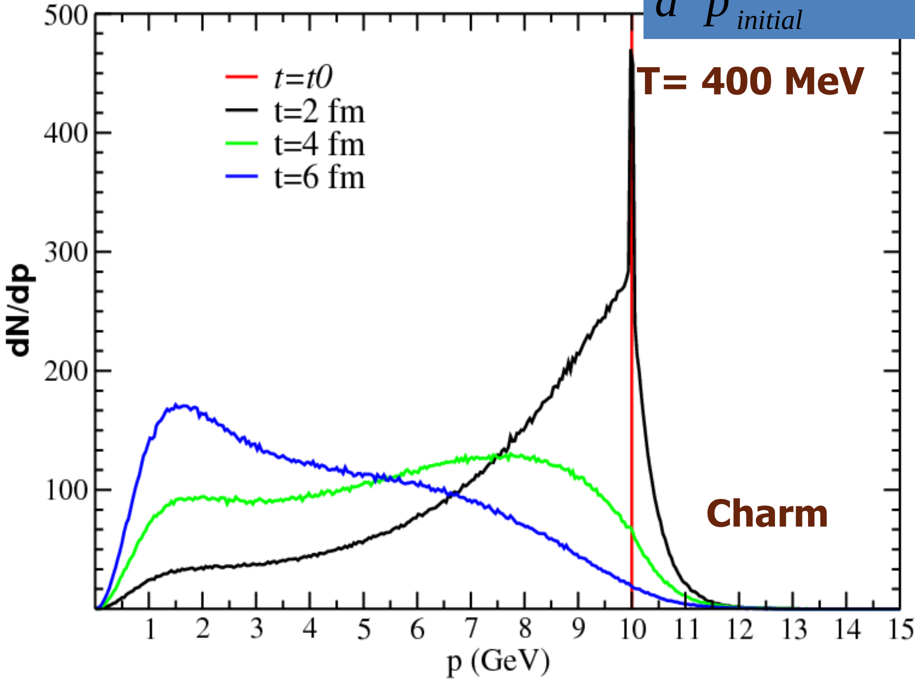
Boltzmann

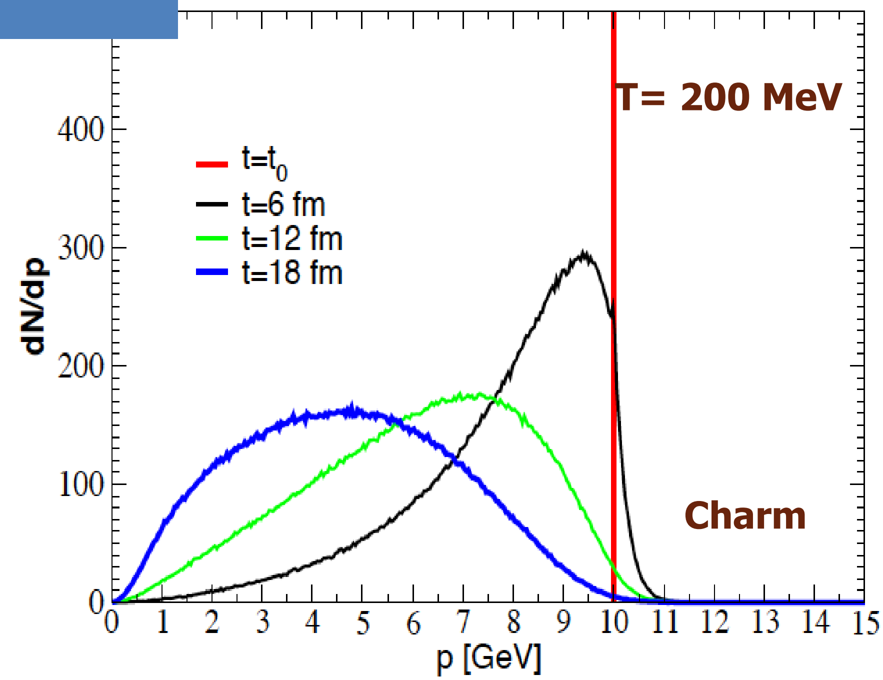


T=400 MeV Mc/T≈3 Mb/T≈10

Momentum evolution for charm vs temperature

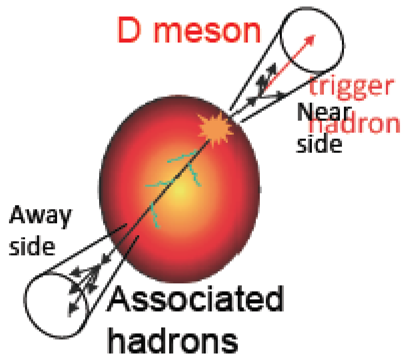
$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$



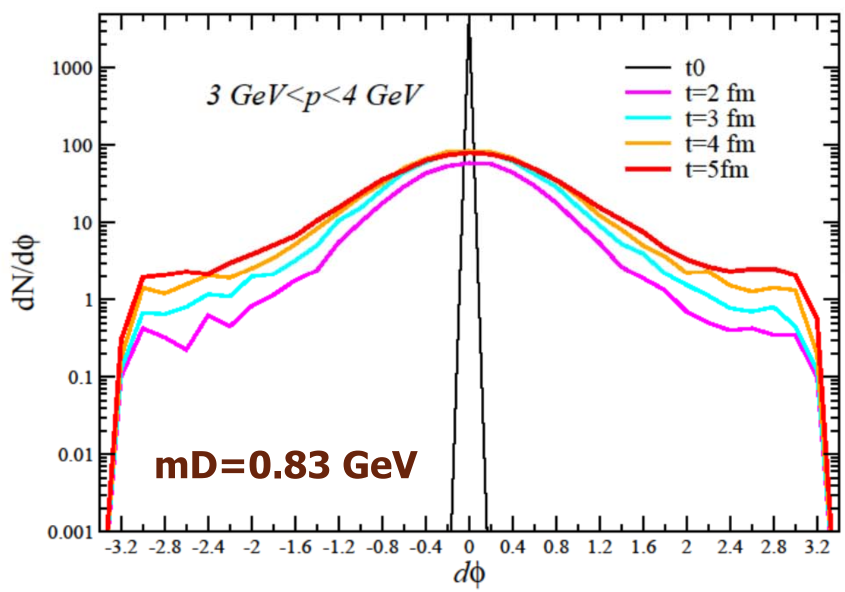


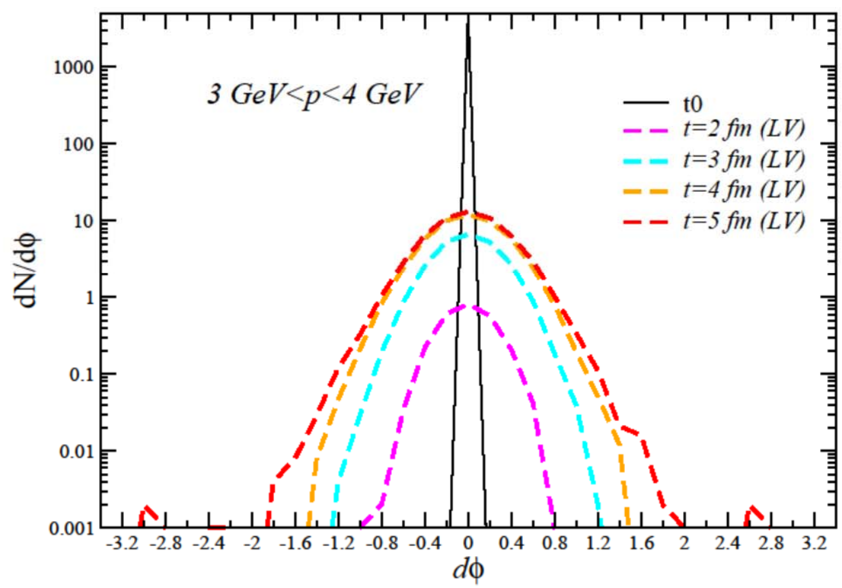
- At 200 MeV Mc/T= 6 -> start to see a peak with a width

Back to Back correlation in a Box



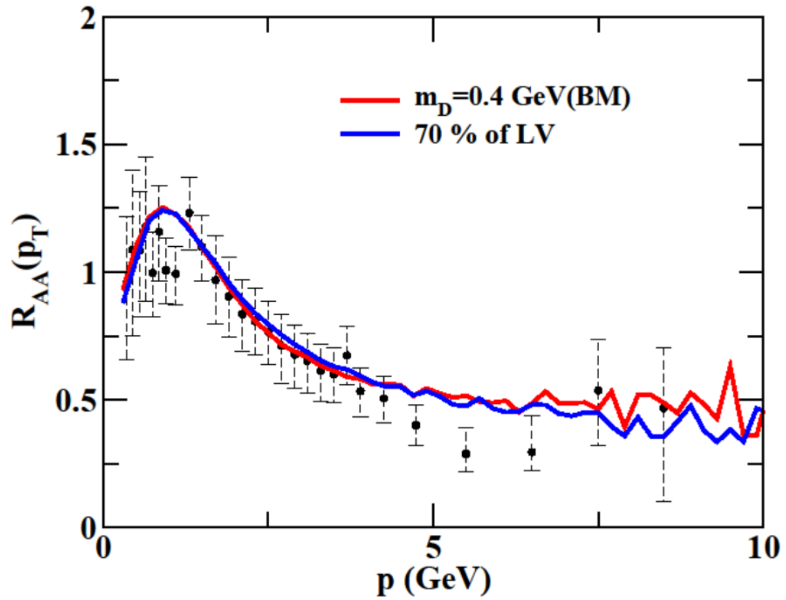
Initialization: px=pz=0, py=10 GeV
x=z=0, y=-2.5 fm

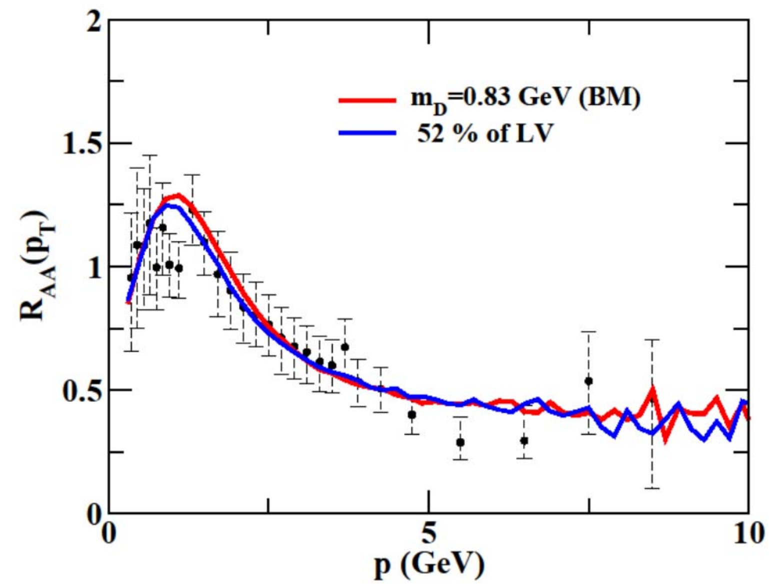


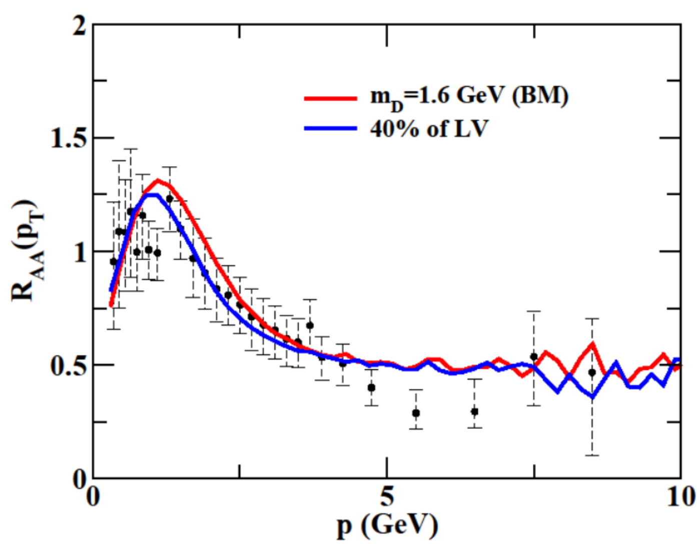


The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm

R_{AA} at RHIC for different <k>





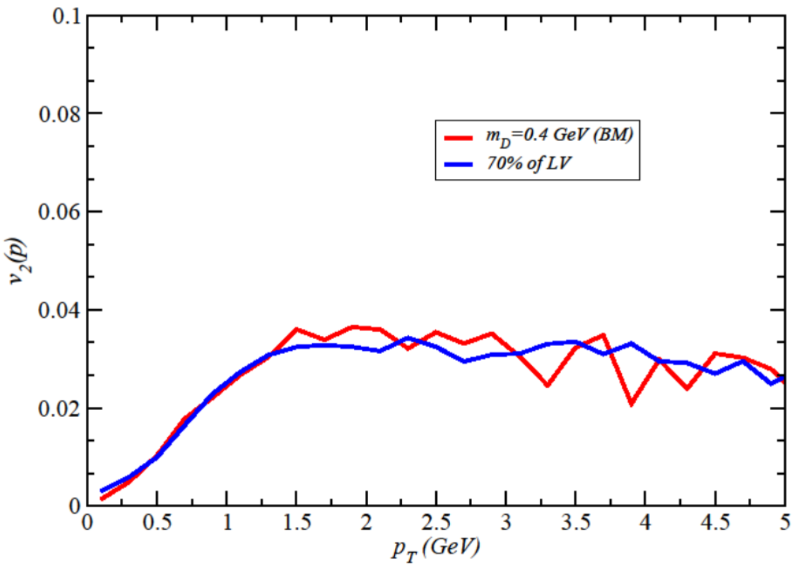


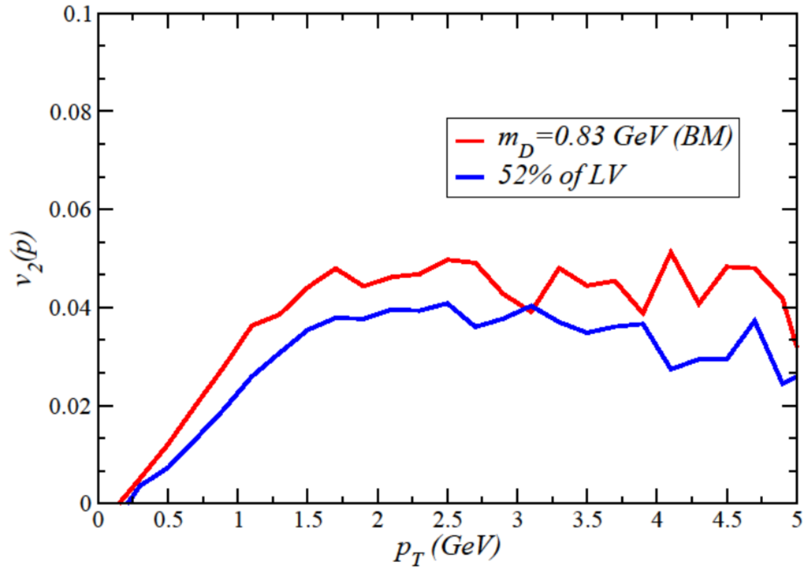
The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

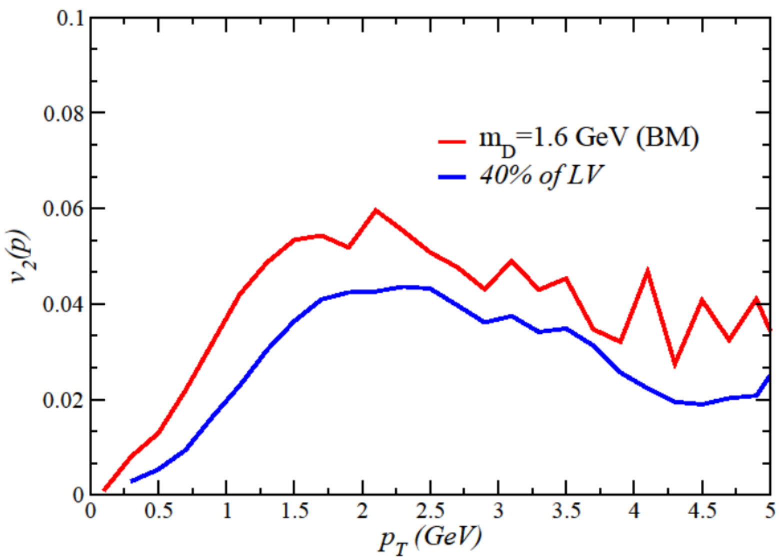
One can get very similar R_{AA} for both the approaches just reducing the diffusion coefficent

The smaller averege transfered momentum the better Langevin works

v_2 at RHIC centrality 20-30 %



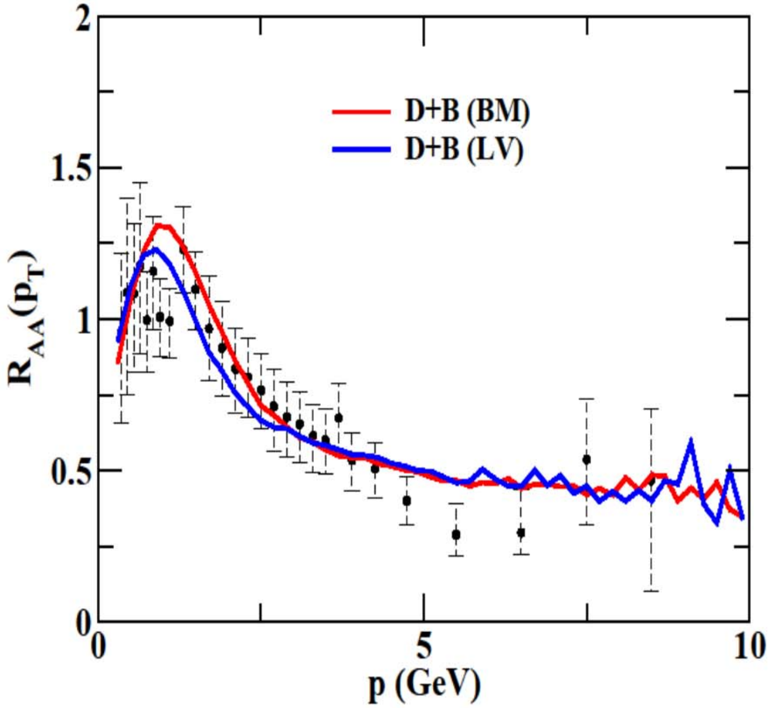


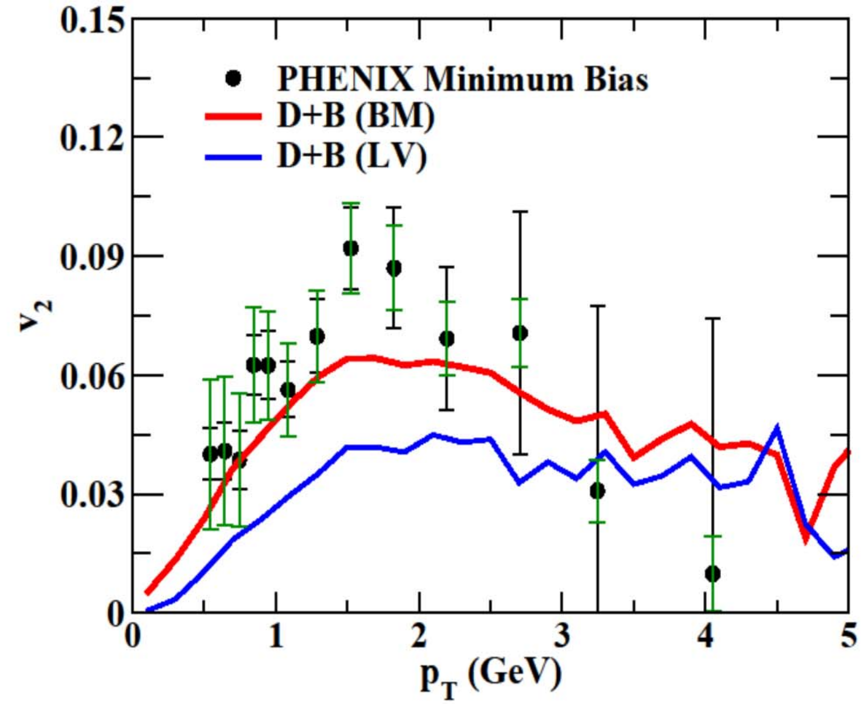


Also for v_2 the smaller averege transfered momentum the better Langevin works

Boltzmann is more efficient in producing v_2 for fixed R_{AA}

R_{AA} and v2 at RHIC at mD=1.6 GeV



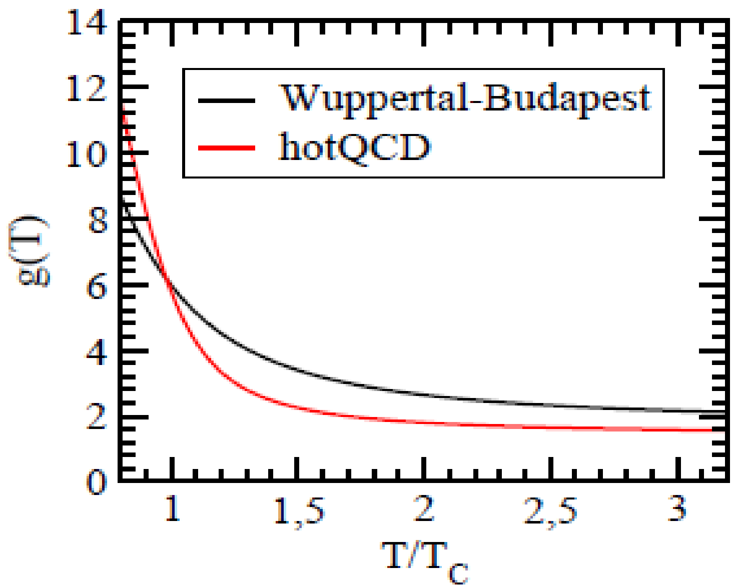


Das, Scadina,Plumari and Greco
arXiv:1312.6857

Our results can be further improved by implementing Coalescence + Fragmentation for hadronisation.

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach !

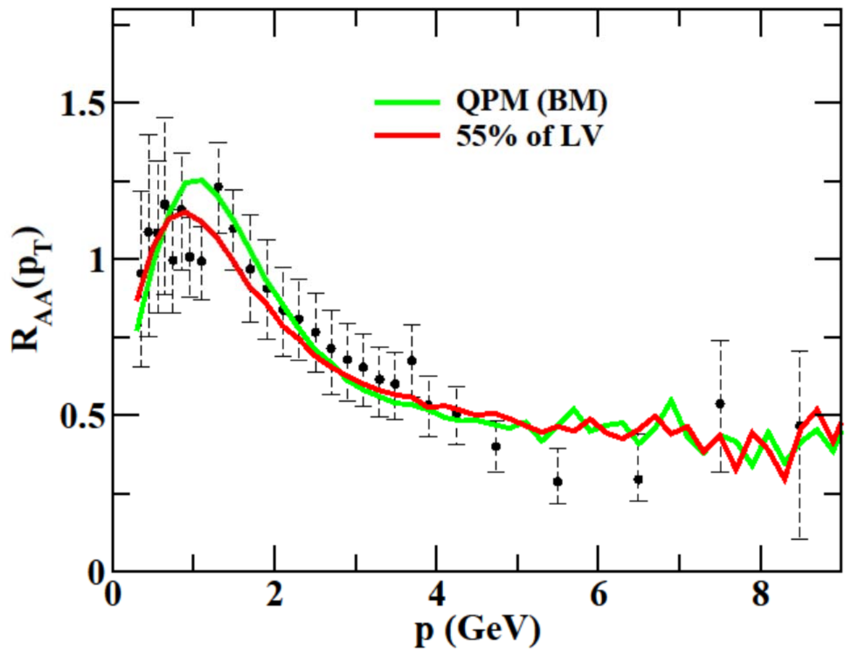
Using inputs from quasi particle model

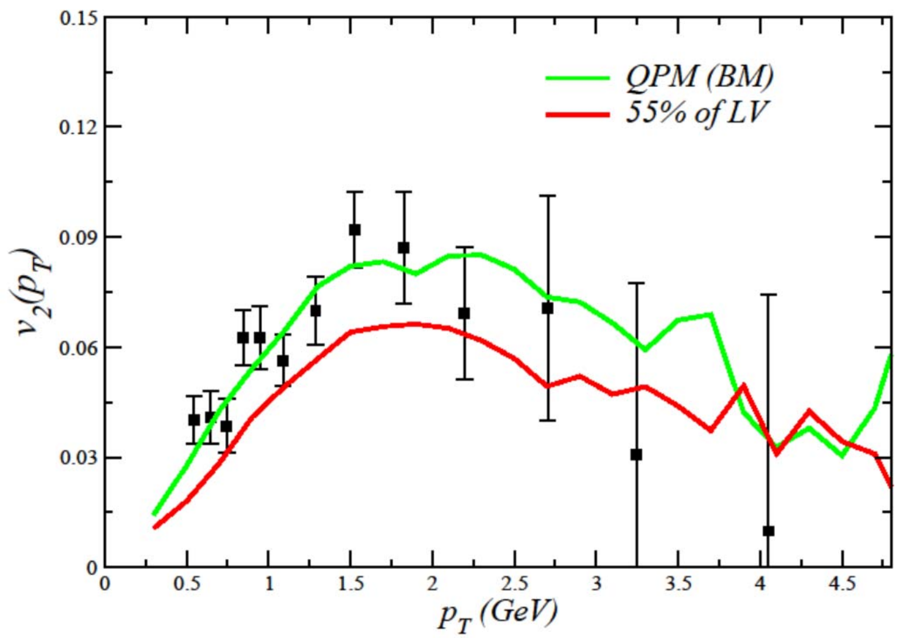


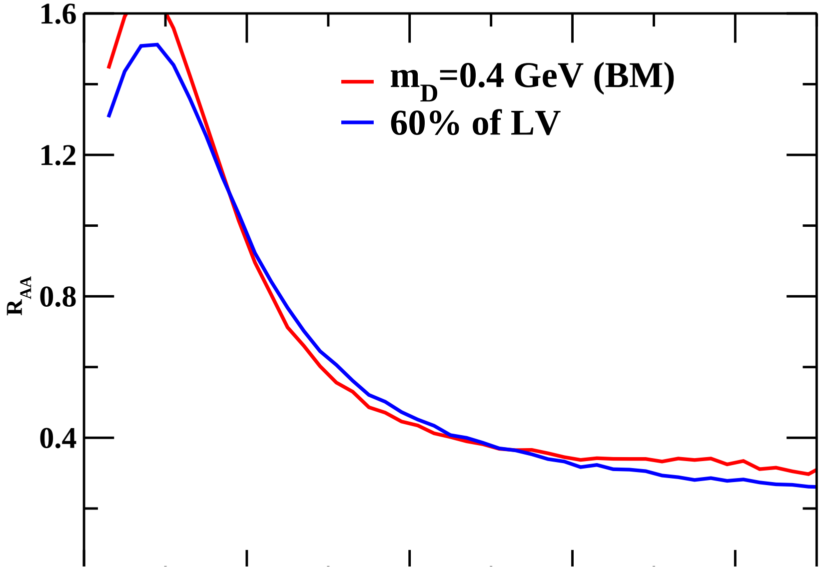
Plumari, Alberico, Greco and Ratti
PRD,84,094004 (2011)

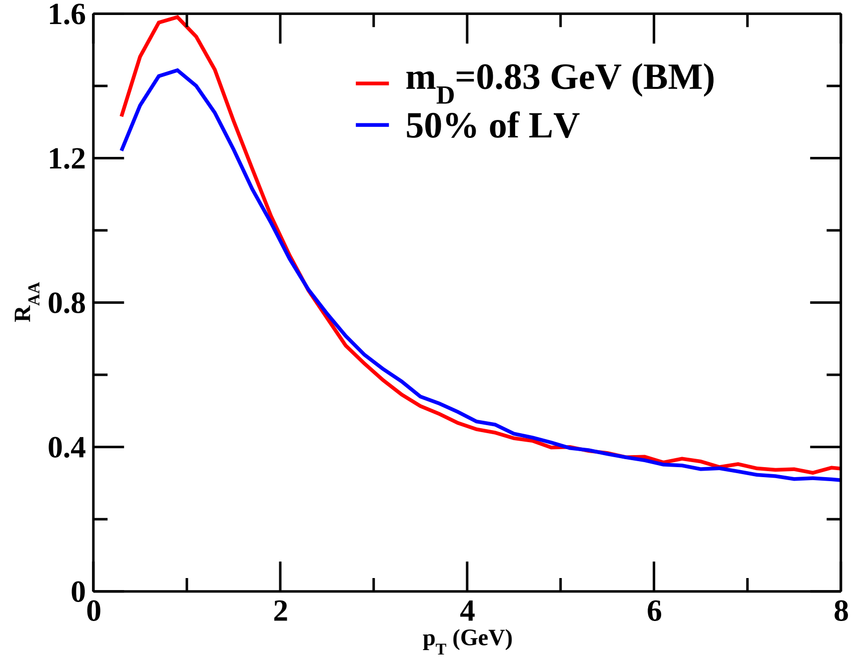
Berrehrah,Bratkovskaya,Cassing,Gossiaux, Aichelin, Bleicher
PRC,89,054901 (2014)

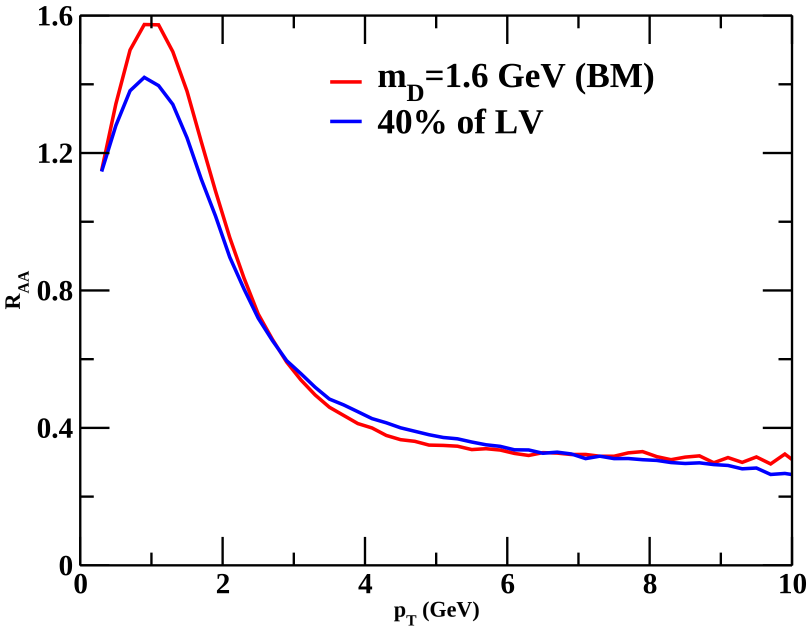
Berrehrah, Gossiaux, J. Aichelin, W. Cassing, Bratkovskaya
arXiv:1405.3243





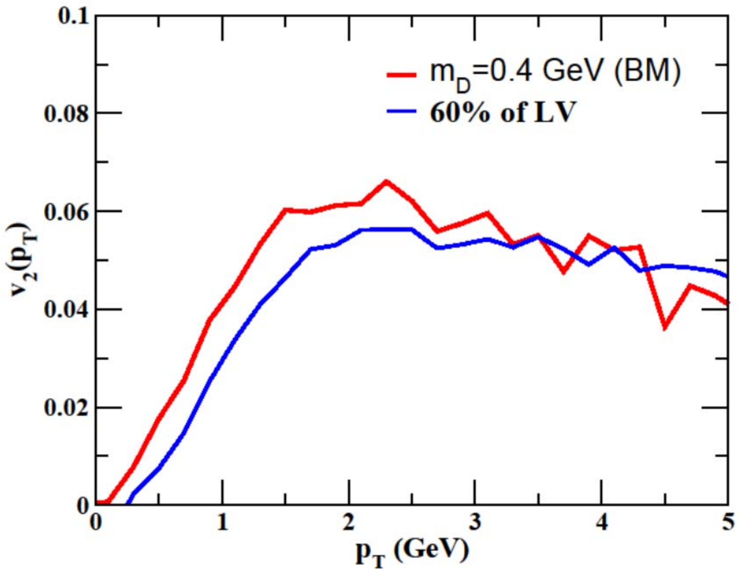


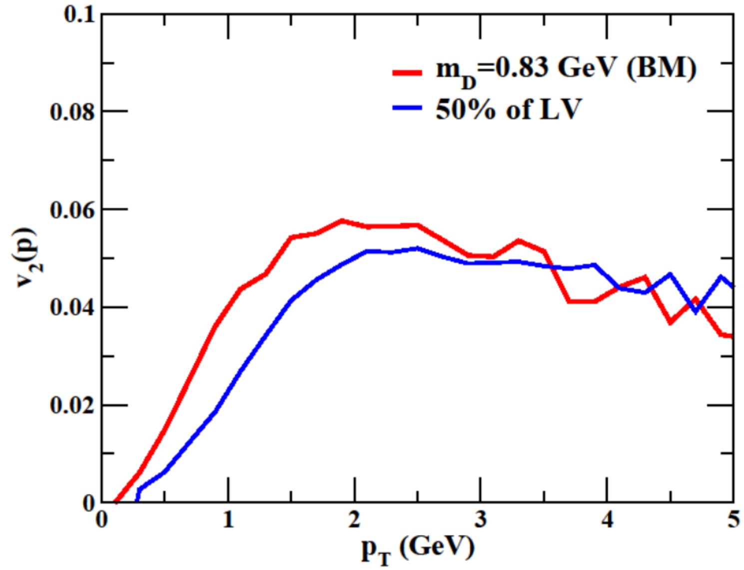


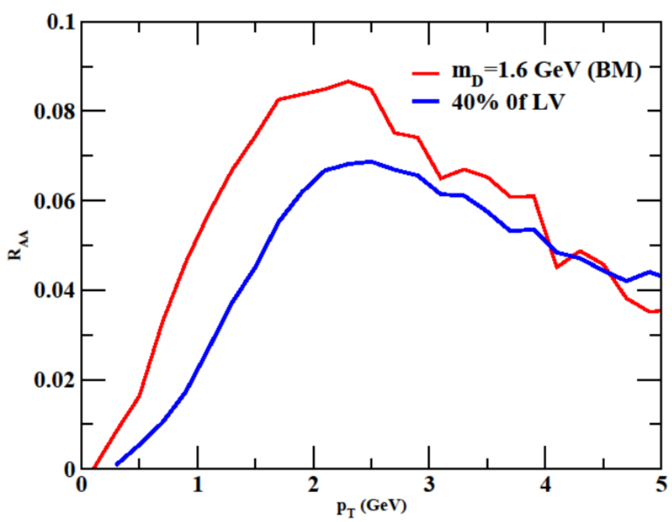


One can get very similar R_{AA} for both the approaches just reducing the diffusion coefficent

V_2 @ LHC centrality 30-50%







Also for v_2 the smaller averege transfered momentum the better Langevin works

Boltzmann is more efficient in producing v_2 for fixed R_{AA}

Hadronic Phase

$D\pi \rightarrow D\pi$

$DK \rightarrow DK$

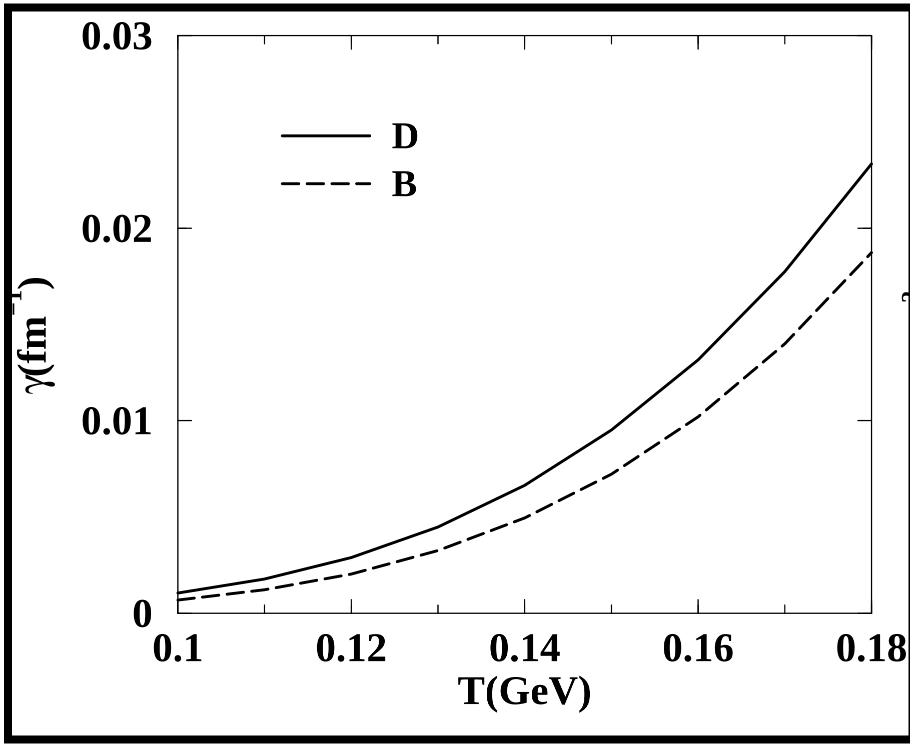
$D\eta \rightarrow D\eta$

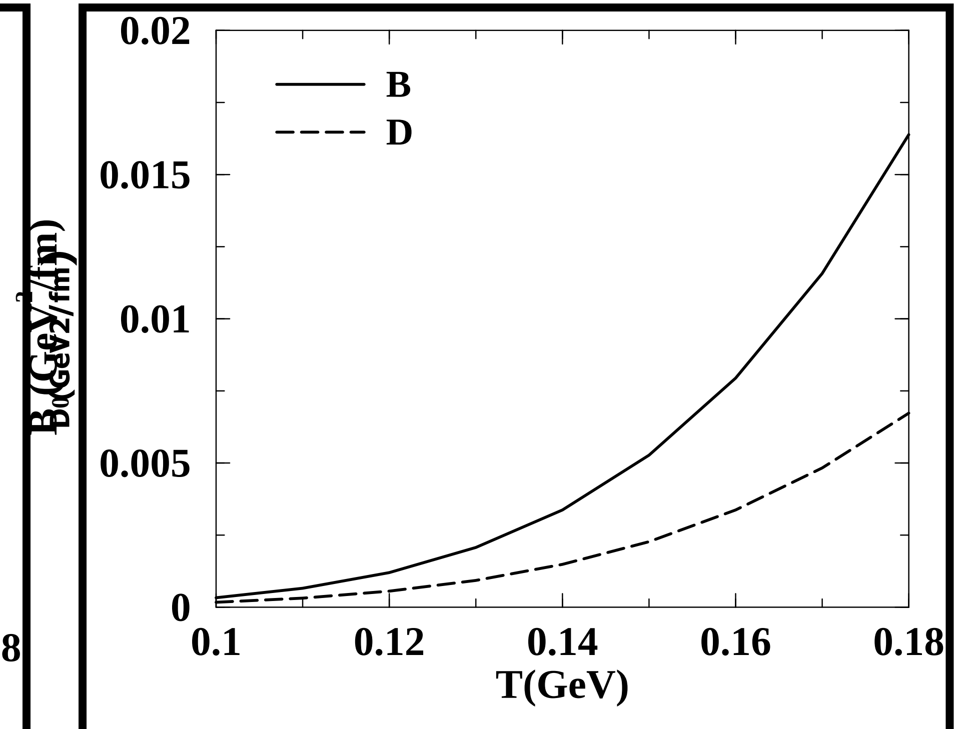
Using scattering length obtained from Heavy Meason Chiral Perturbation Theory

$B\pi \rightarrow B\pi$

$BK \rightarrow BK$

$B\eta \rightarrow B\eta$

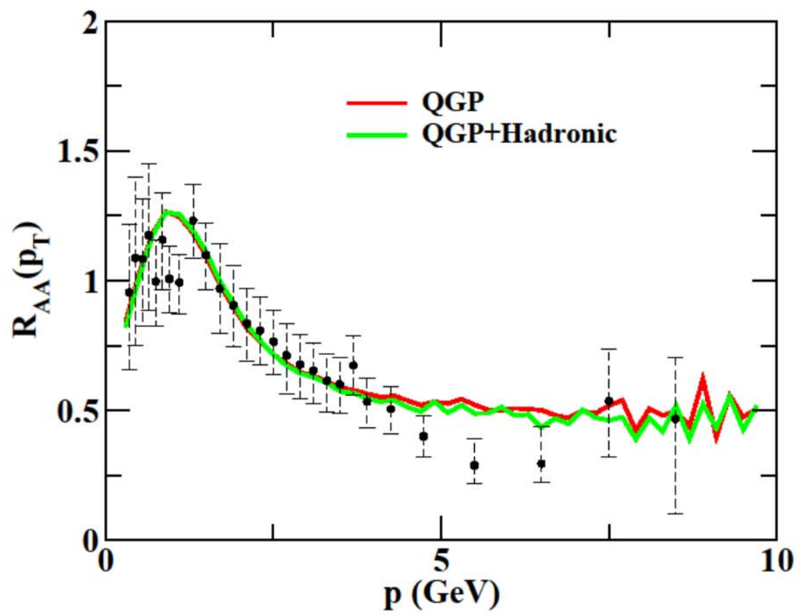


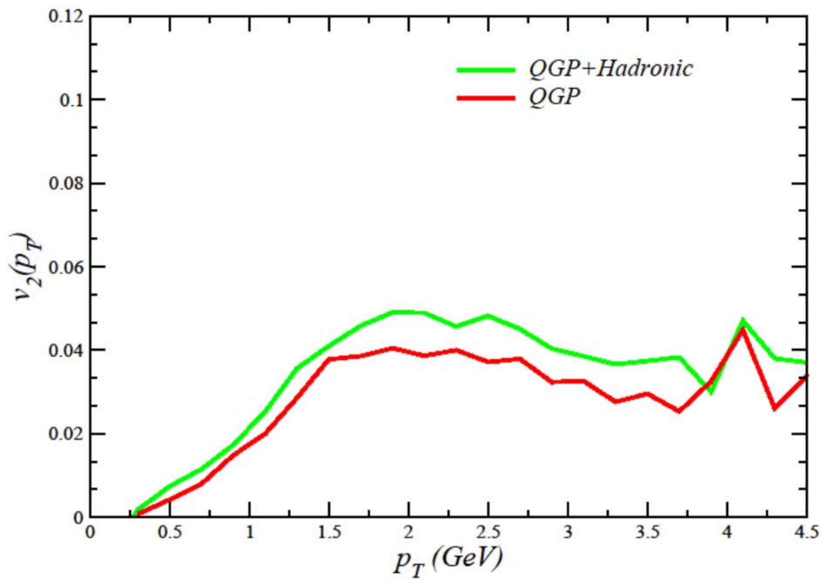


Das,Ghosh , Sarkar and Alam
Phys Rev D 85,074017 (2012)

M. Laine,JHEP,04, 124 (2011)
He, Fries ,Rapp, Phys. Let.B701, 445(2012)
Ghosh, Das, Sarkar , Alam, PRD84,011503 (2011)
Abreu, Cabrera, Llanes-Estrada,Torres-Rincon, Annals Phys. 326, 2737 (2011)
Torres-Rincon, Tolos , Romanets, PRD,89, 074042 (2014)

Hadronic Phase





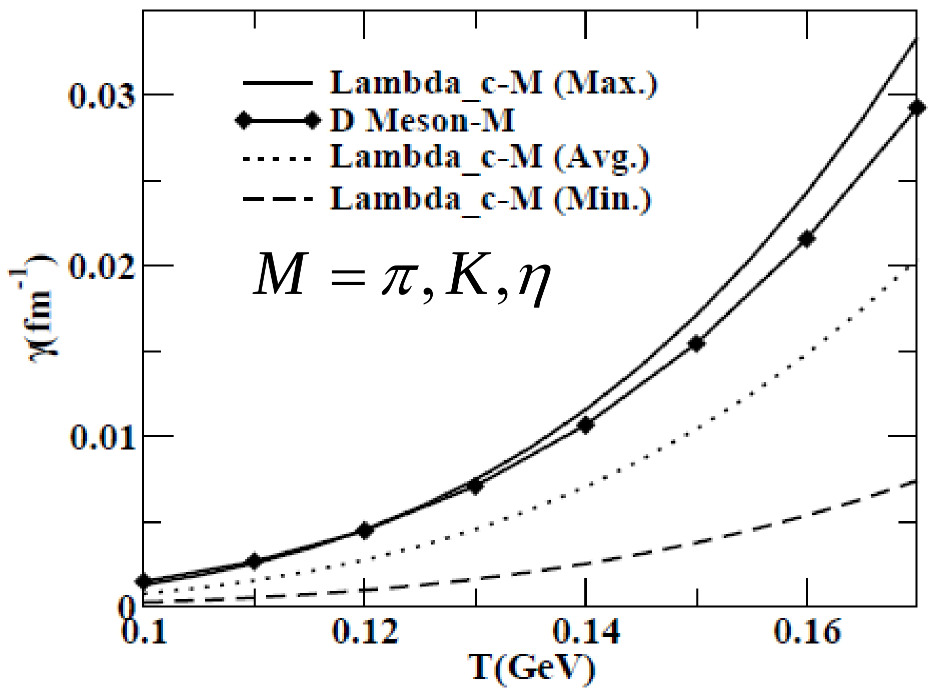
Das, Scardina, Greco, Alam
Under Preparation

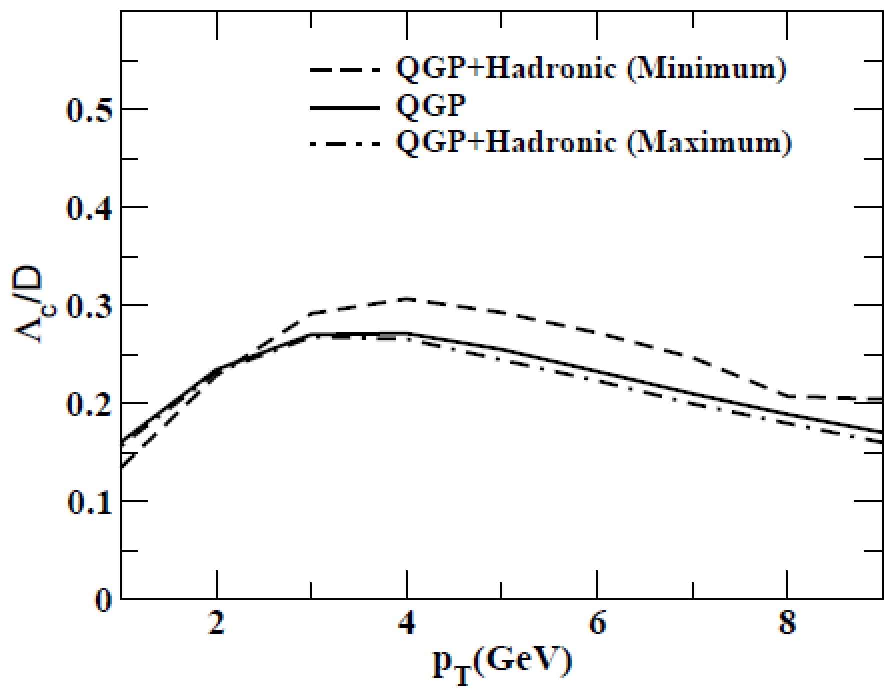
Das, Ghosh,Sarkar,Alam, PRD,88,017501, (2013)

He, Fries , Rapp , arXiv:1401:3817

Ozvenchuk, Torres-Rincon, Gossiaux, Aichelin, Tolos, arXiv:1408.4938

Heavy Baryon to Meson Ratio





Ghosh, Das, Greco, Sarkar and Alam
PRD,90, 054018 (2014)

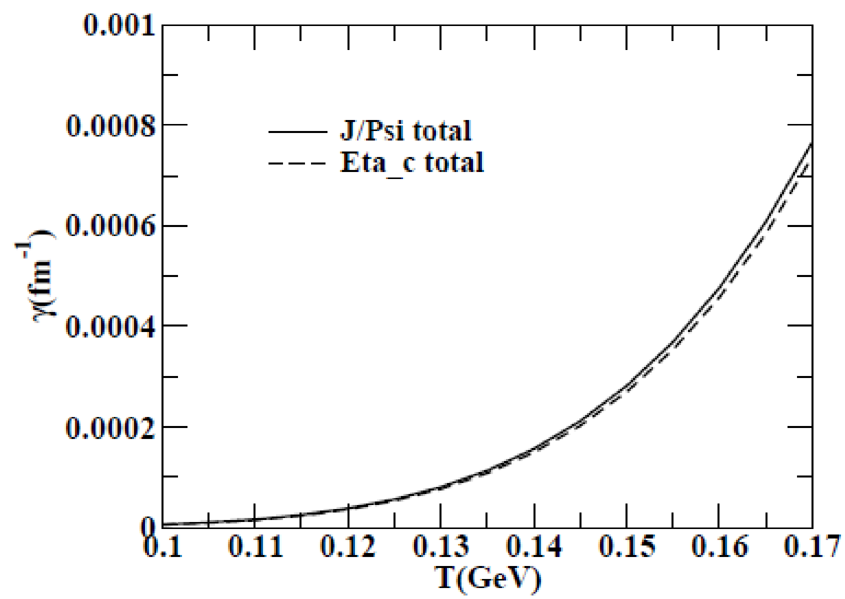
Z. Liu, S. Zhu, PRD 86, 034009 (2012);
NPA 914,494 (2013

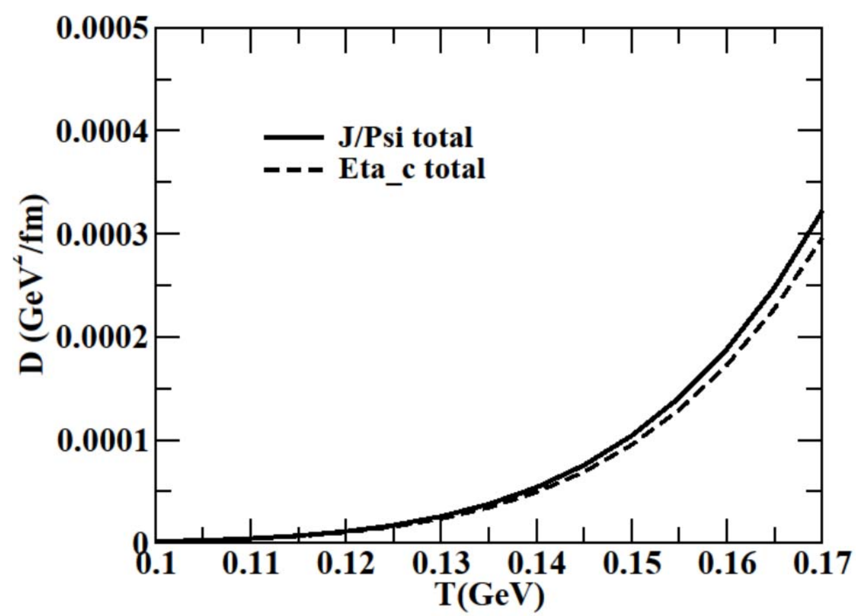
J/Psi in Hadronic Phase

$$J/\psi\pi \rightarrow J/\psi\pi$$
$$J/\psi\rho \rightarrow J/\psi\rho$$
$$J/\psi N \rightarrow J/\psi N$$

$$\eta_c\pi \rightarrow \eta_c\pi$$
$$\eta_c\rho \rightarrow \eta_c\rho$$
$$\eta_c N \rightarrow \eta_c N$$

Yokokawa, Sasaki, Hatsuda and Hayashigaki
PRD 74, 034504 (2006)





Mitra, Ghosh, Das, Sarkar and Alam
arXiv:1409.4652

Summary & Outlook ……

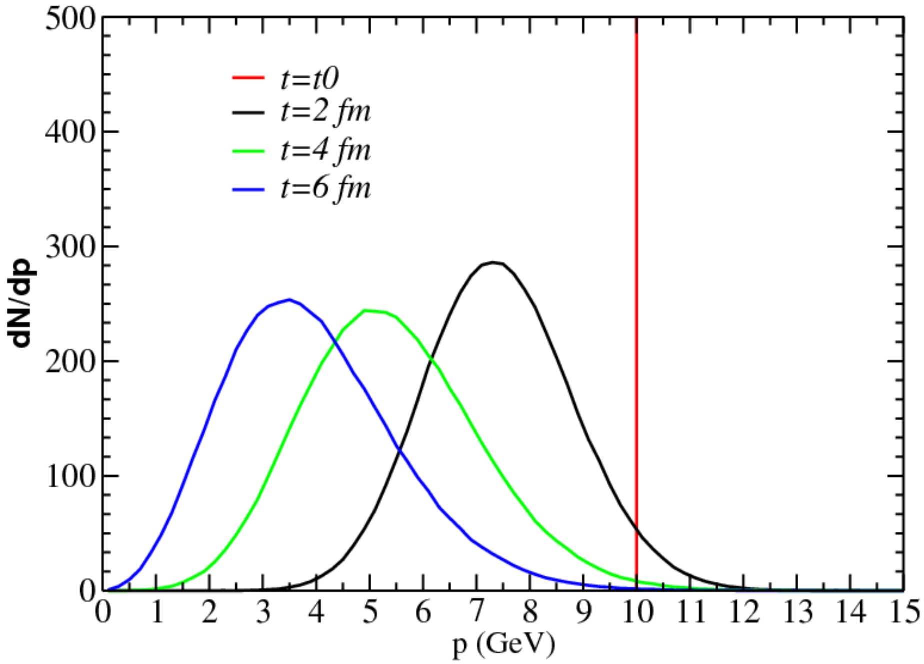
- Both Langevin and Boltzmann equation has been solved in a box for heavy quark propagating in a thermal bath composed of gluon at T= 400 MeV as well as for a expanding medium at RHIC and LHC energies.
- We found charm quarks does not follow the Brownian motion at RHIC and LHC energies.
- Langevin dynamics overestimate the suppression than the Boltzmann approach.
- For a given RAA Boltzmann approach develop larger v2.
- With isotropic cross-section one can reproduce RAA and v2 simultaneously within the Boltzmann approached at RHIC energy.
- We have also highlighted the significance of the hadronic phase.
- Implementation of radiative process is under progress.


Thank You . . . .

Momentum evolution starting from a δ (Charm) in a Box

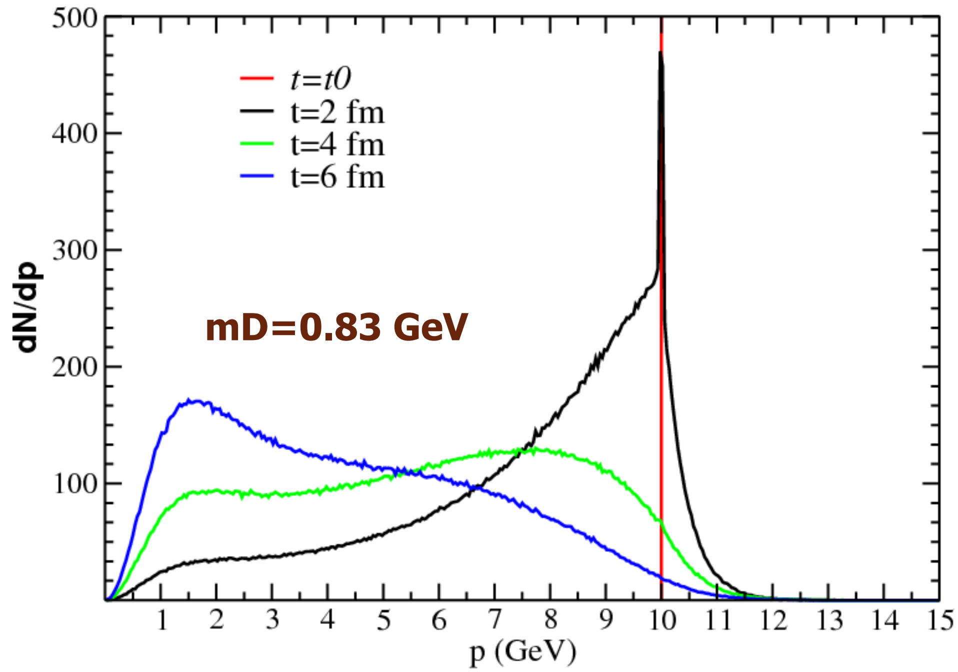
$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$

Langevin



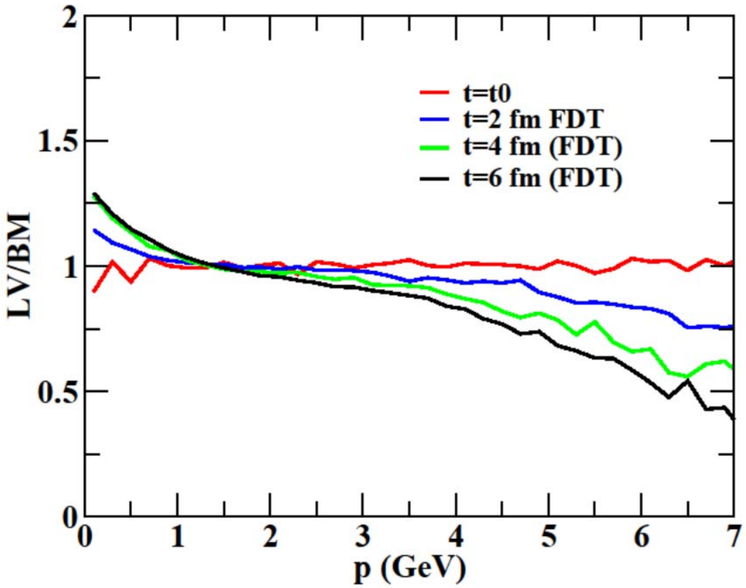
In case of Langevin the distributions are Gaussian as expected by construction

Boltzmann

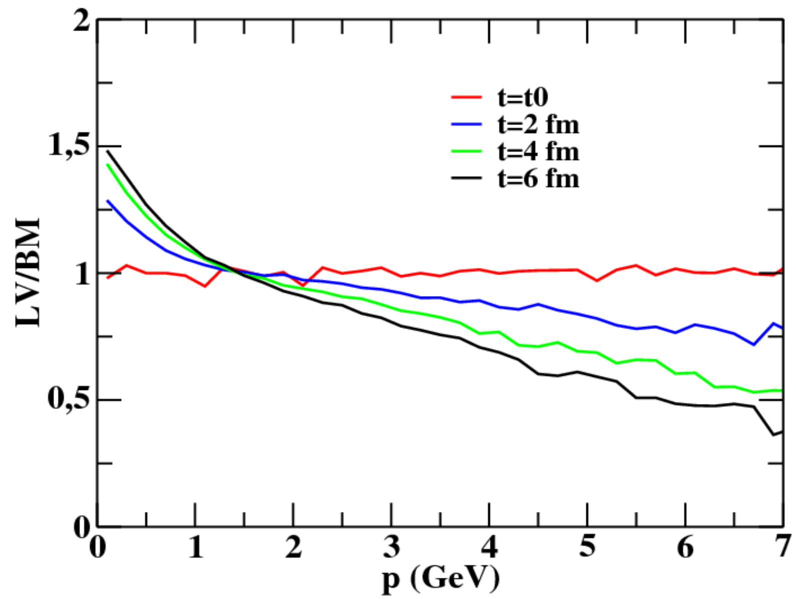


In case of Boltzmann the charm quarks does not follow the Brownian motion

Das, Scadina,Plumari and Greco
arXiv:1312.6857



With FDT



With pQCD

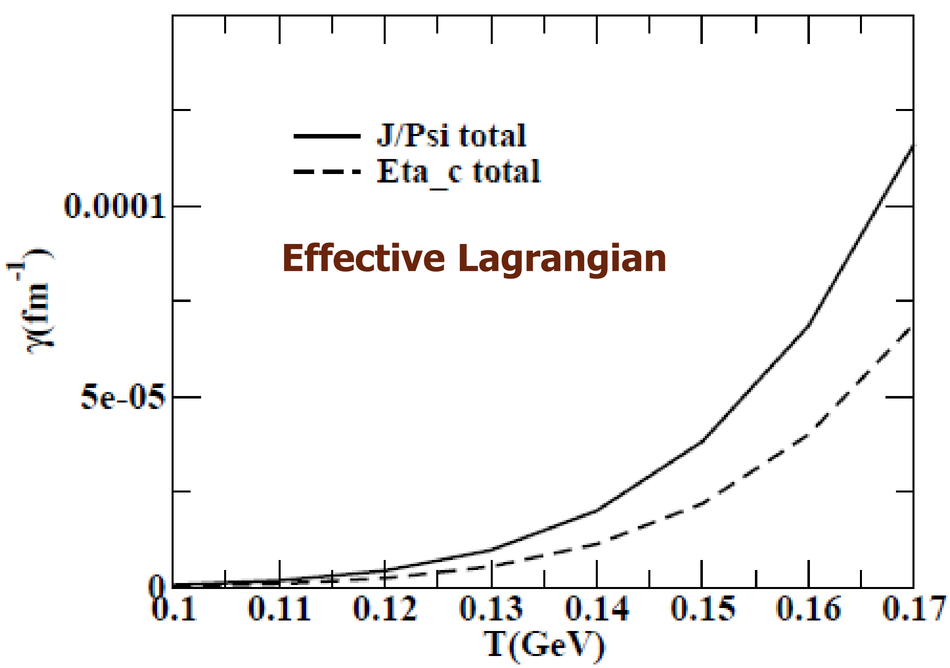
J/Psi in Hadronic Phase

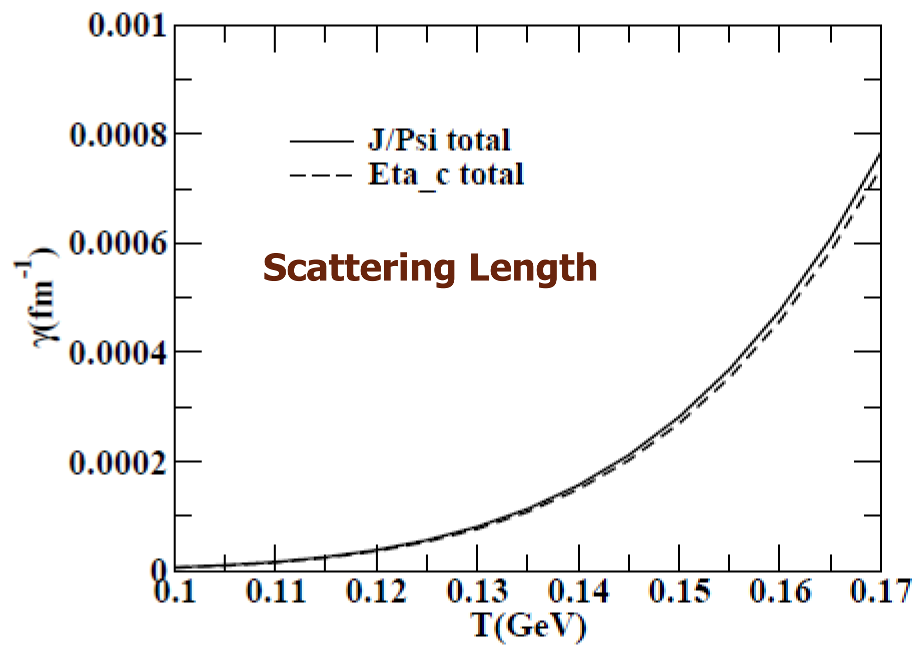
$$J + V \rightarrow \eta_c \rightarrow J/\Psi + V$$
$$\eta_c + V \rightarrow J/\psi \rightarrow \eta_c + V$$
$$V = \rho, \omega, \phi$$

Haglin ,Gale, PRC 63, 065201(2001).

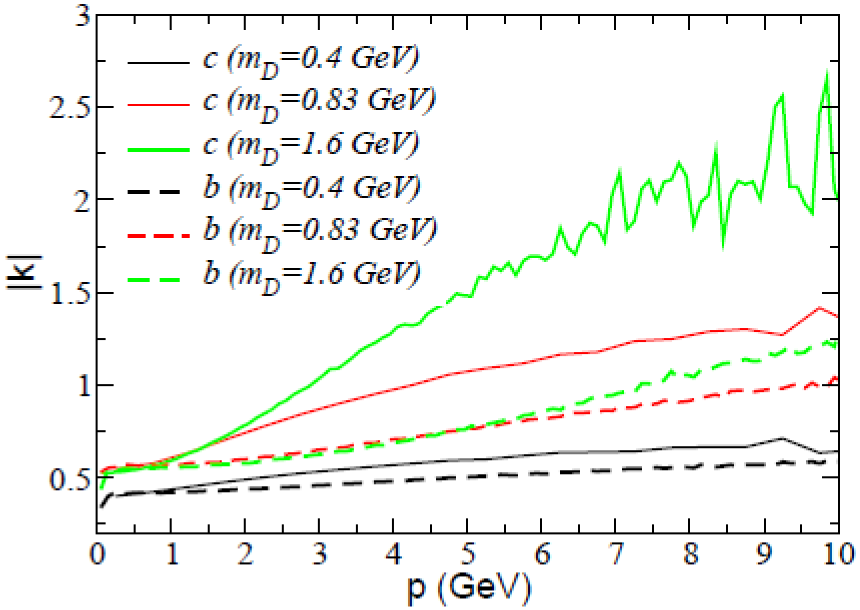
$$J/\psi\pi \rightarrow J/\psi\pi \qquad \eta_c\pi \rightarrow \eta_c\pi$$
$$J/\psi\rho \rightarrow J/\psi\rho \qquad \eta_c\rho \rightarrow \eta_c\rho$$
$$J/\psi N \rightarrow J/\psi N \qquad \eta_c N \rightarrow \eta_c N$$

Yokokawa, Sasaki, Hatsuda and Hayashigaki
PRD 74, 034504 (2006)

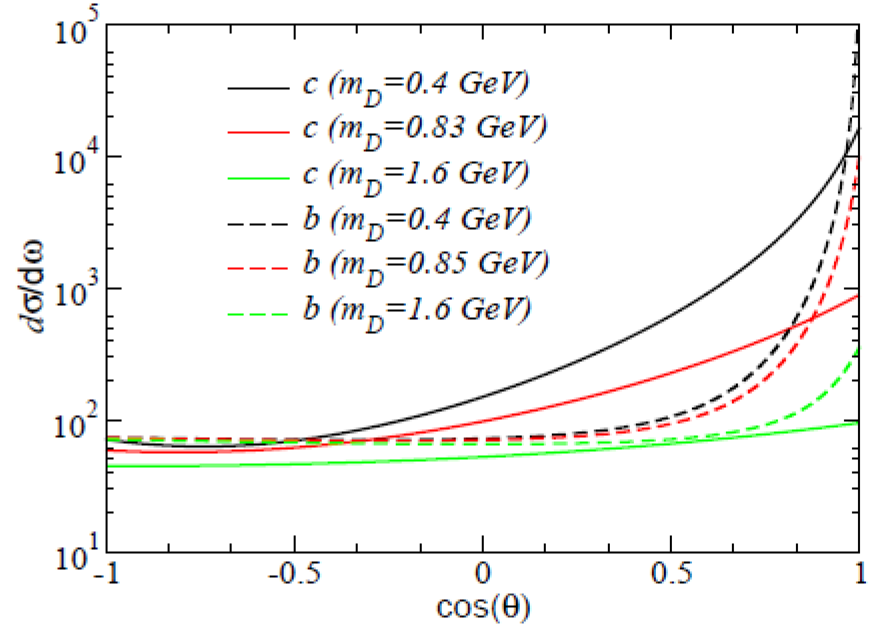




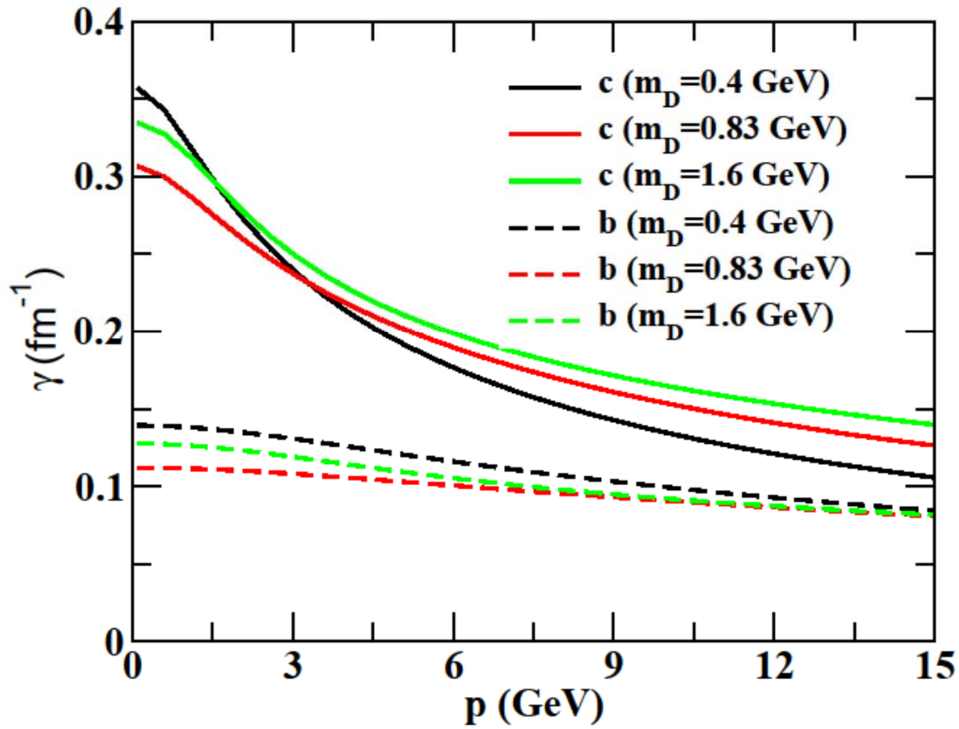
Mitra, Ghosh, Das, Sarkar and Alam
arXiv:1409.4652



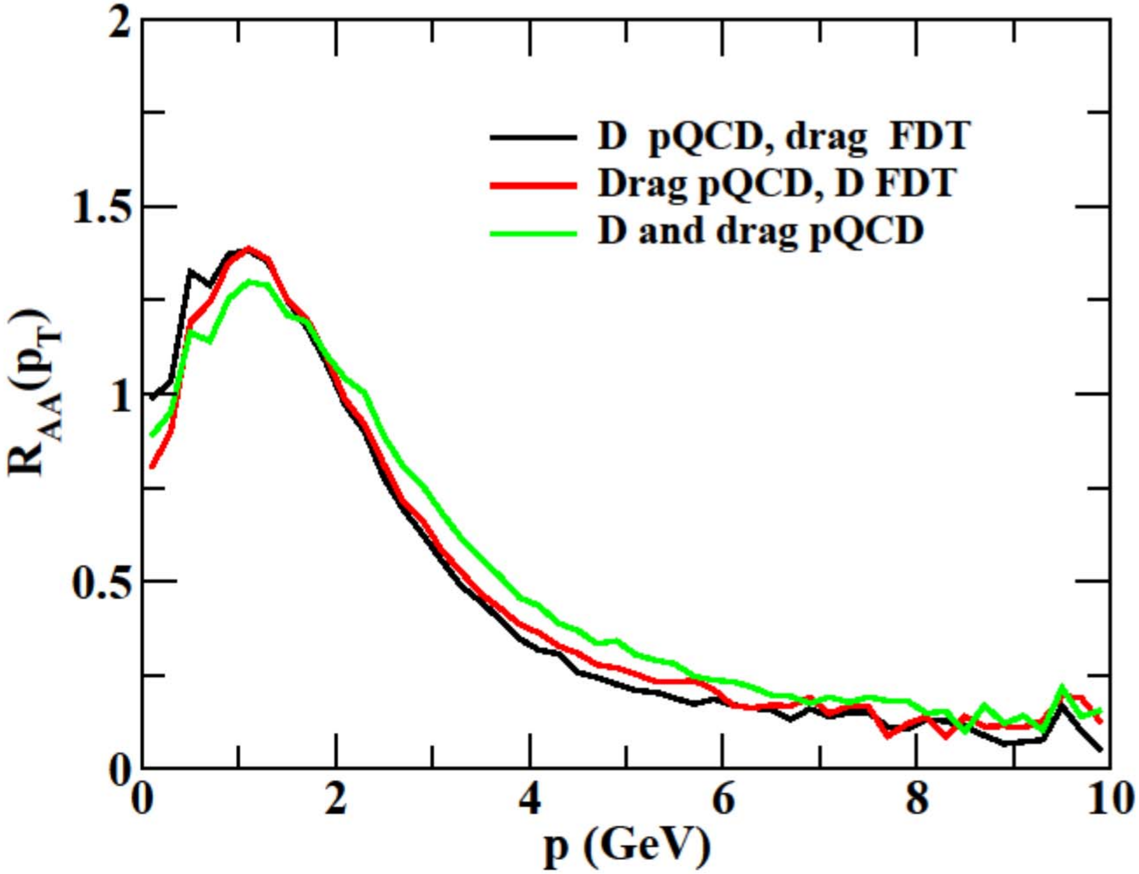
c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.83 GeV)
b (m_D=1.6 GeV)
|k|
p (GeV)



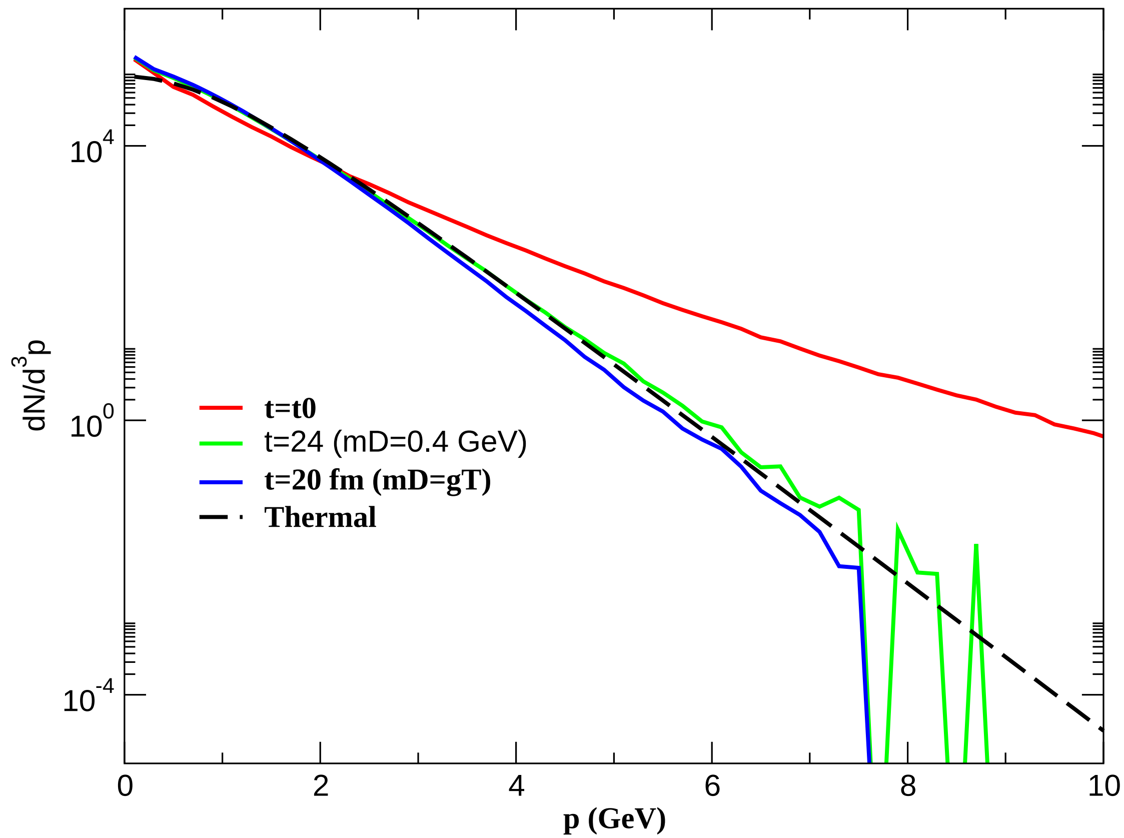
c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.85 GeV)
b (m_D=1.6 GeV)
dσ/dω
cos(θ)



c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.83 GeV)
b (m_D=1.6 GeV)
γ (fm^{-1})
p (GeV)
0
0.1
0.2
0.3
0.4
0
3
6
9
12
15

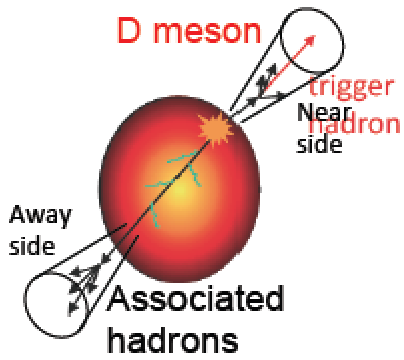


D pQCD, drag FDT
Drag pQCD, D FDT
D and drag pQCD
$R_{AA}(p_T)$
p (GeV)
0
0.5
1
1.5
2
0
2
4
6
8
10

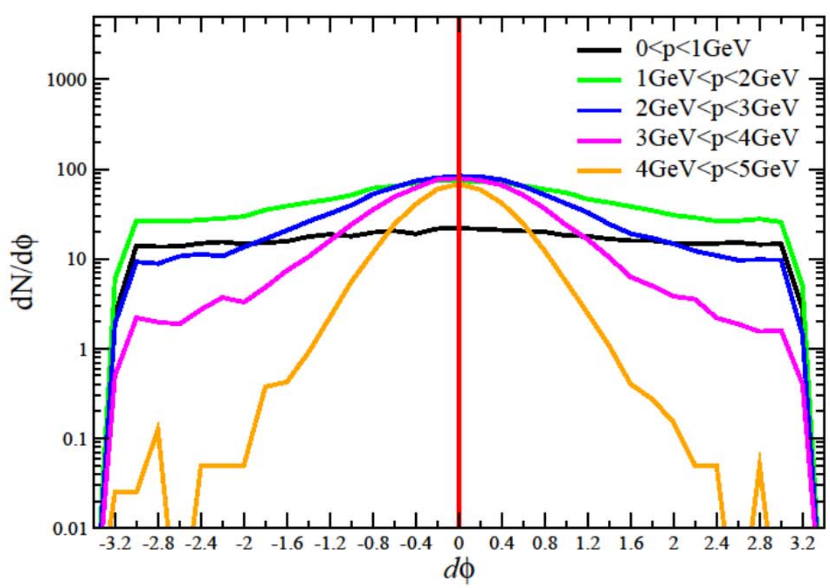


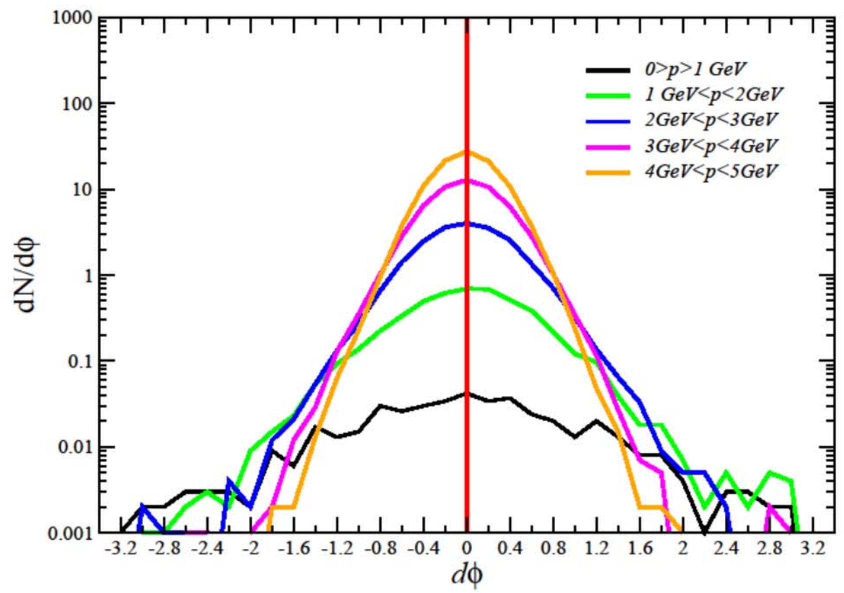
dN/d³p
10⁴
10⁰
10⁻⁴
0
2
4
6
8
10
p (GeV)
t=t0
t=24 (mD=0.4 GeV)
t=20 fm (mD=gT)
Thermal

Back to Back correlation



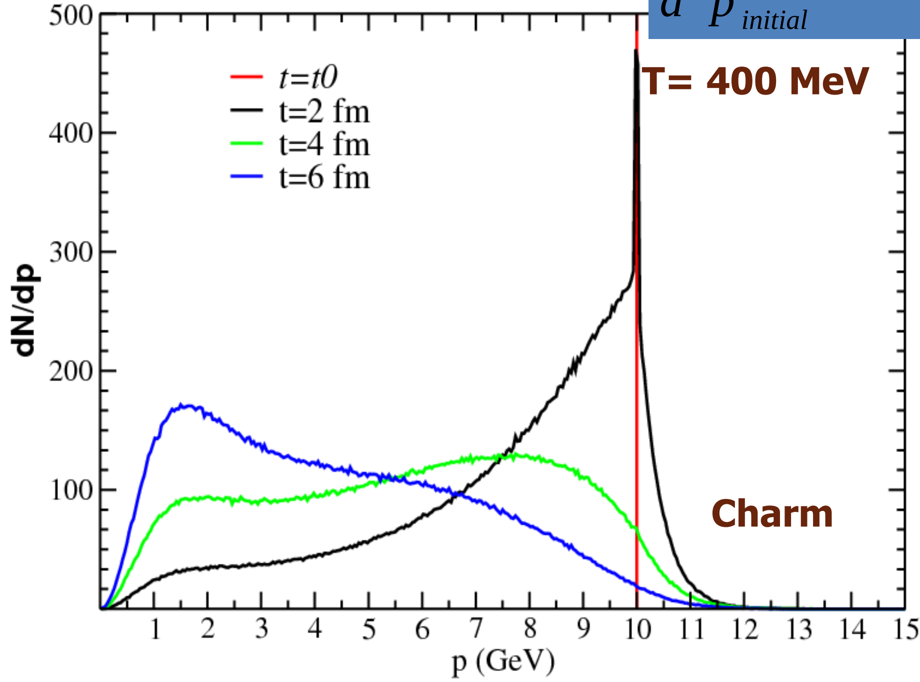
The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm

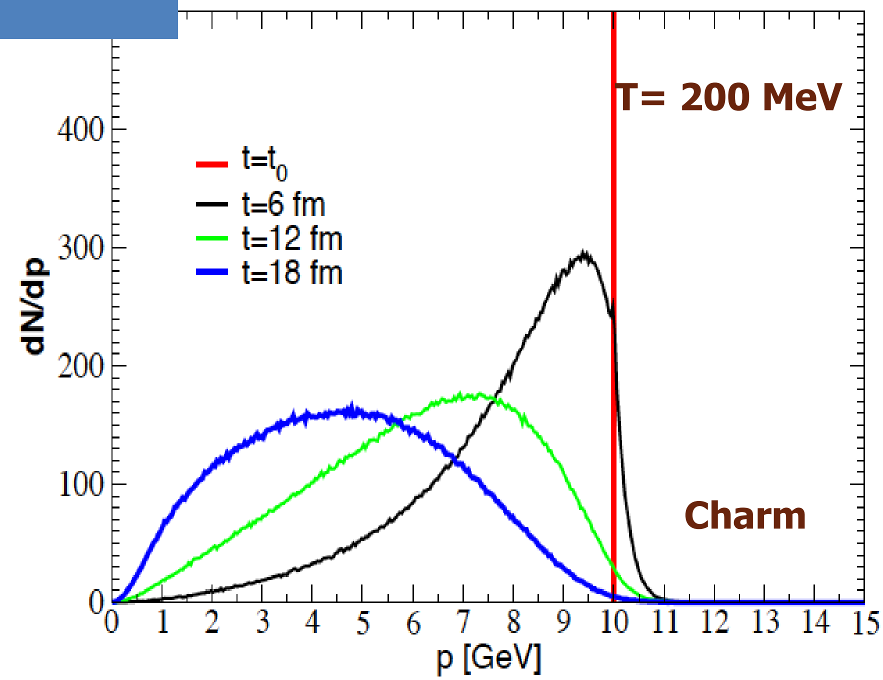


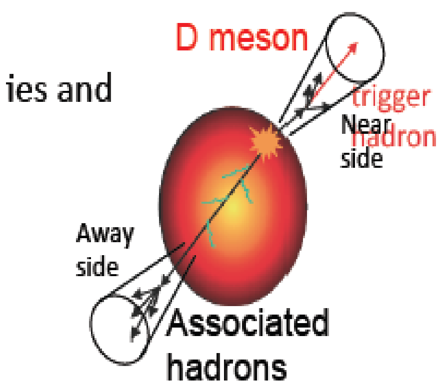


Momentum evolution for charm vs temperature

$$\frac{dN}{d^3p}_{initial} = \delta\left(p - 10GeV\right)$$



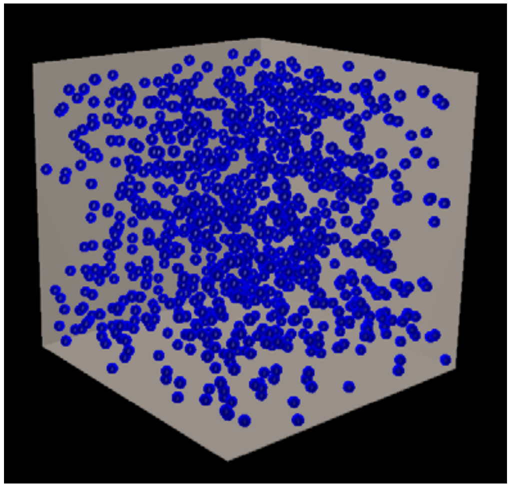




- **At 200 MeV Mc/T= 6 -> start to see a peak with a width**

Such large spread of momentum implicates a large spread in the angular distributions that could be experimentally observed studying the back to back **Charm-antiCharm** angular correlation

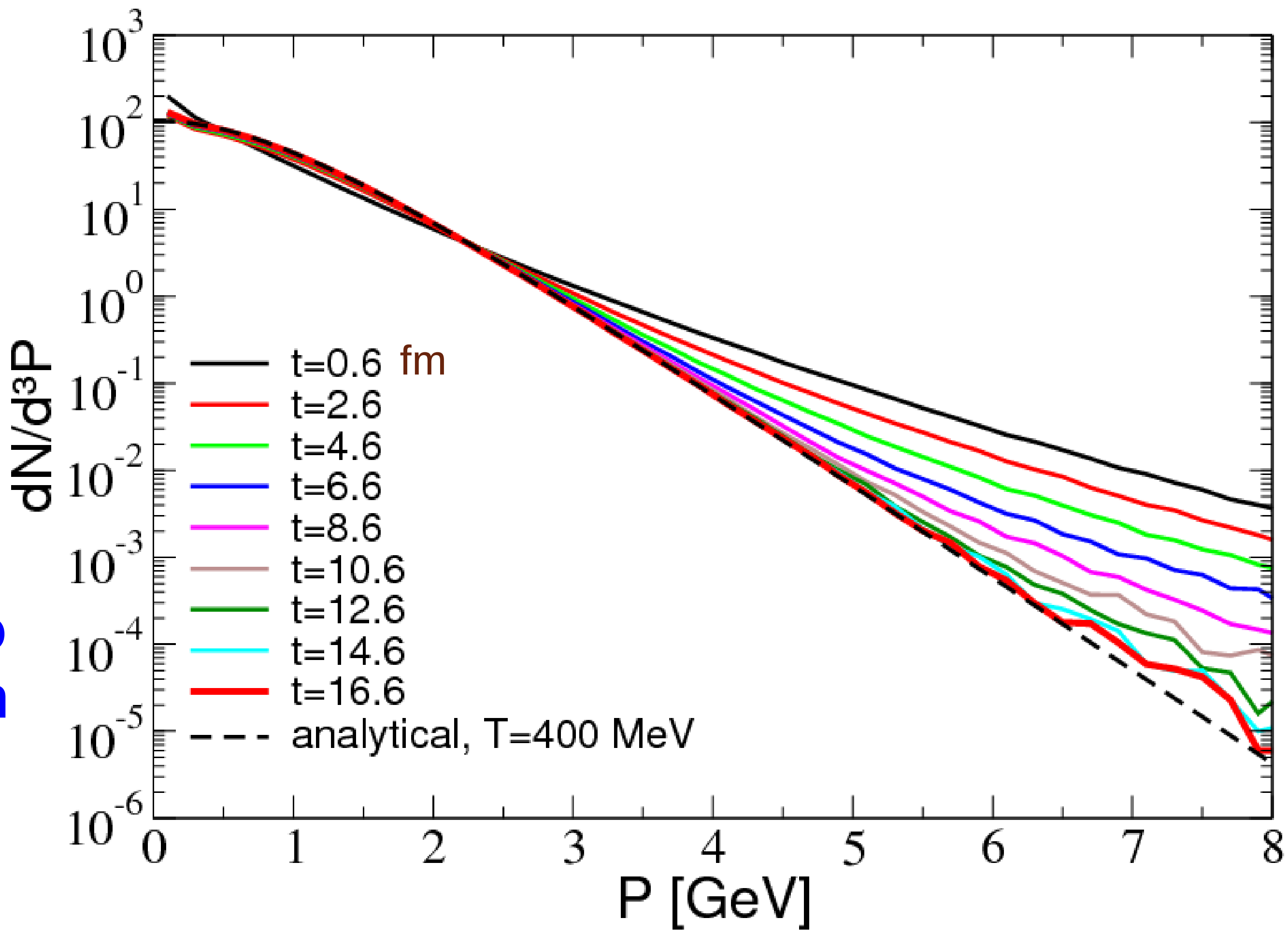
Charm evolution in a static medium



Simulations in which a particle ensemble in a **box** evolves dynamically

Bulk composed only by **gluons in thermal equilibrium** at **T=400 MeV**

C and Cbar are initially distributed: uniformly in r-space, while in p-space



Due to collisions charm approaches to thermal equilibrium with the bulk

I) LPM effect : Suppression of bremsstrahlung and pair production.

Formation length ($l_f = \frac{\hbar}{q_\perp}$) : The distance over which interaction is spread out

1) It is the distance required for the final state particles to separate enough that they act as separate particles.

2) It is also the distance over which the amplitude from several interactions can add coherently to the total cross section.

As $q_\perp$ increase → l_f reduce → Radiation drops proportional

S. Klein, Rev. Mod. Phys **71** (1999)1501

(II) Dead cone Effect : Suppression of radiation due to mass

$$\frac{1}{\sigma}\frac{d^2\sigma}{dzd\theta^2} \sim C_F \frac{\alpha_s}{\pi}\frac{1}{z}\frac{\theta^2}{(\theta^2 + 4\gamma)^2}$$

where $z = 2 - x_1 - x_2$ *and* $\gamma = \frac{m^2}{s}$

Where $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ $\longrightarrow$ the energy fraction of the final state quark and anti-quark.

Radiation from heavy quarks suppress in the cone from θ =0 (minima) to θ=2 √γ (maxima)