Transport coefficients, potentials and spectral functions from the lattice

Chris Allton

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Transport coefficients, potentials and spectral functions from the lattice

Gert Aarts, CRA, Alessandro Amato, Wynne Evans, Pietro Giudice, Simon Hands, Aoife Kelly, Seyong Kim, Maria-Paola Lombardo, Dhagash Mehta, Bugra Oktay, Sinead Ryan, Jon-Ivar Skullerud, Don Sinclair, Tim Harris

FASTSUM Collaboration

Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

Lattice Temperatures

1st Generation

2 flavours

smaller volume: (2fm)^3

coarser lattices: $a_s = 0.167$fm

quark mass: $M_\pi/M_\rho =\sim 0.55$

N_s	N_τ	T(MeV)	T/T_c
12	16	460	2.09
12	18	409	1.86
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

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12	80	90	0.42

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3$ – $(4\text{fm})^3$

finer lattices: $a_s = 0.123$fm

quark mass: $M_\pi/M_\rho =\sim 0.45$

N_s	N_τ	T(MeV)	T/T_c
24, 32	16	350	1.90
24	20	280	1.52
24, 32	24	235	1.27
24, 32	28	200	1.09
24, 32	32	175	0.95
24	36	155	0.84
24	40	140	0.76
32	48	115	0.63
16	128	45	0.24

Outline

- Polyakov Loop and T_c
- Light Mesons: Pseudoscalar vs Scalar
- Susceptibilities
- Electrical Conductivity, σ
- Charmonium Potential, $V(r)$

Renormalising the Polyakov Loop

Polyakov Loop, L, related to free energy, F, via:

$$L(T) = e^{-F(T)/T}$$

But F defined up to addivitive constant $\Delta F = f(\beta, \kappa)$.
Imposing renormalisation condition:

$$L_R(T_R) \equiv \text{some number}$$

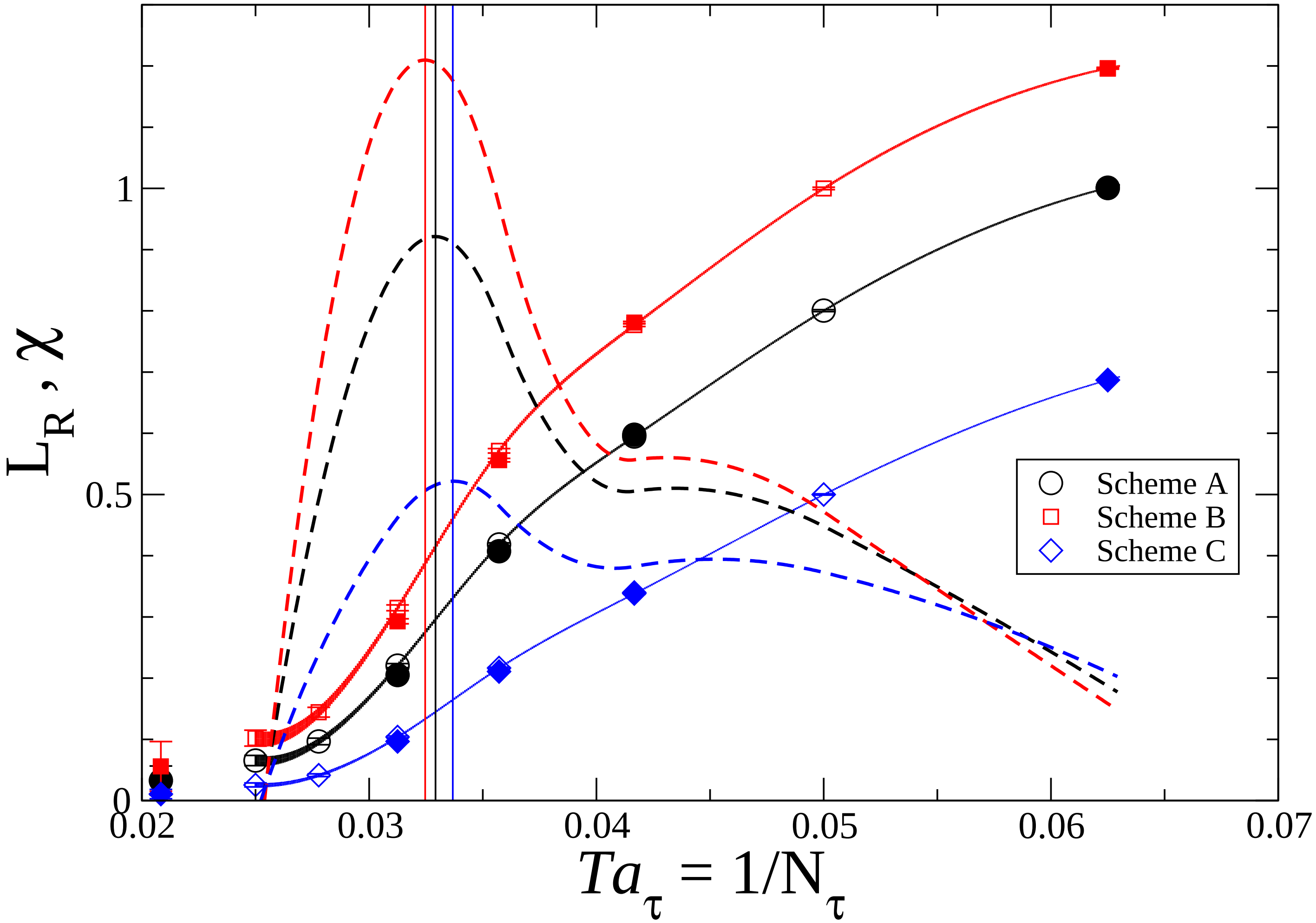
gives us

$$L_R(T) = e^{-F_R(T)/T} = e^{-(F_0(T)+\Delta F)/T} = L_0(T) e^{-\Delta F/T} = L_0(T) {Z_L}^{N_\tau}$$

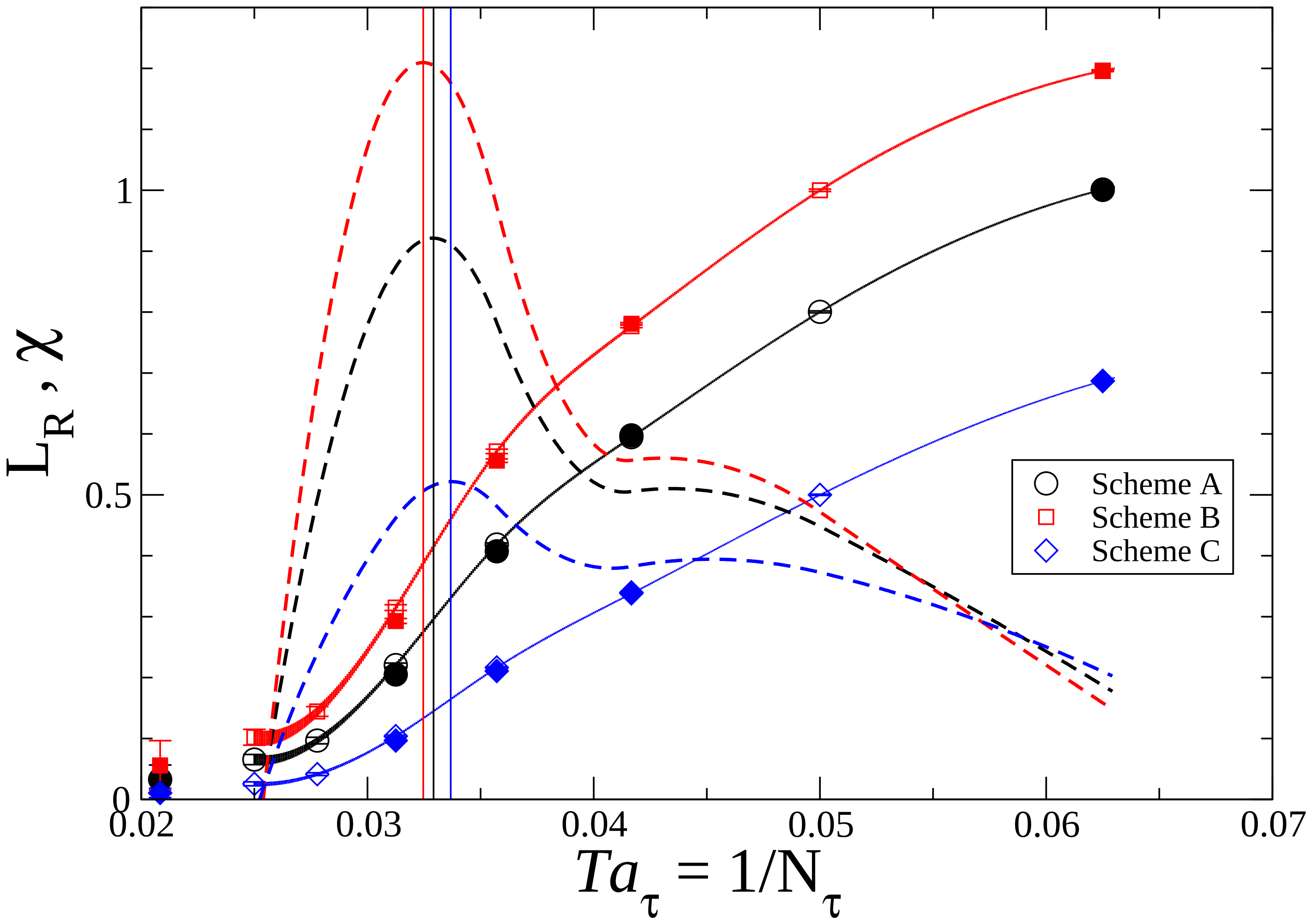
and Z_L defined from renormalisation condition.

Wuppertal-Budapest, PLB713(2012)342 [1204.4089]

Polyakov Loop



Polyakov Loop

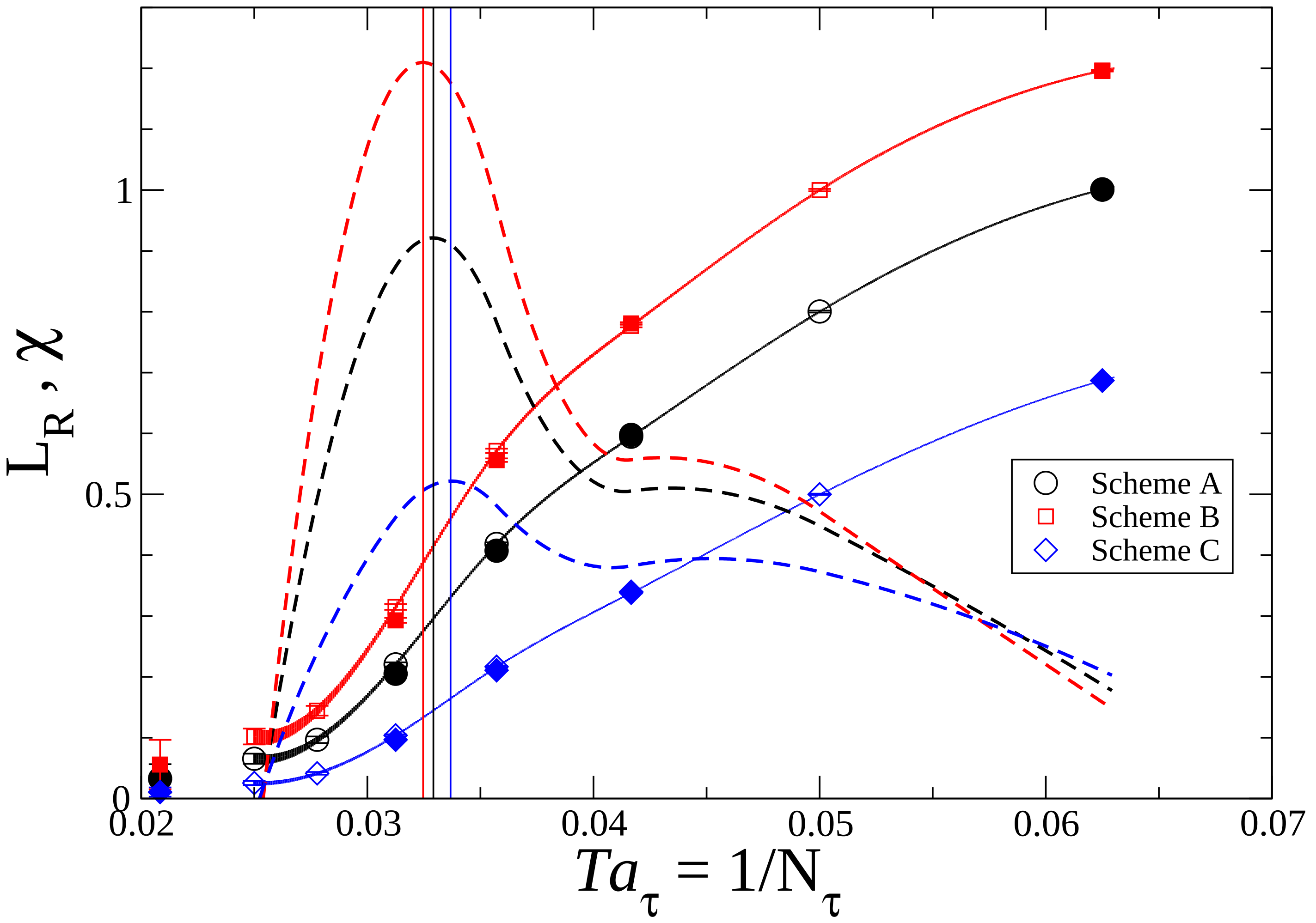


Scheme A: $L_R(Nt = 16) = 1.0$

Scheme B: $L_R(Nt = 20) = 1.0$

Scheme C: $L_R(Nt = 20) = 0.5$

Polyakov Loop



Scheme A: $L_R(Nt = 16) = 1.0$

Scheme B: $L_R(Nt = 20) = 1.0$

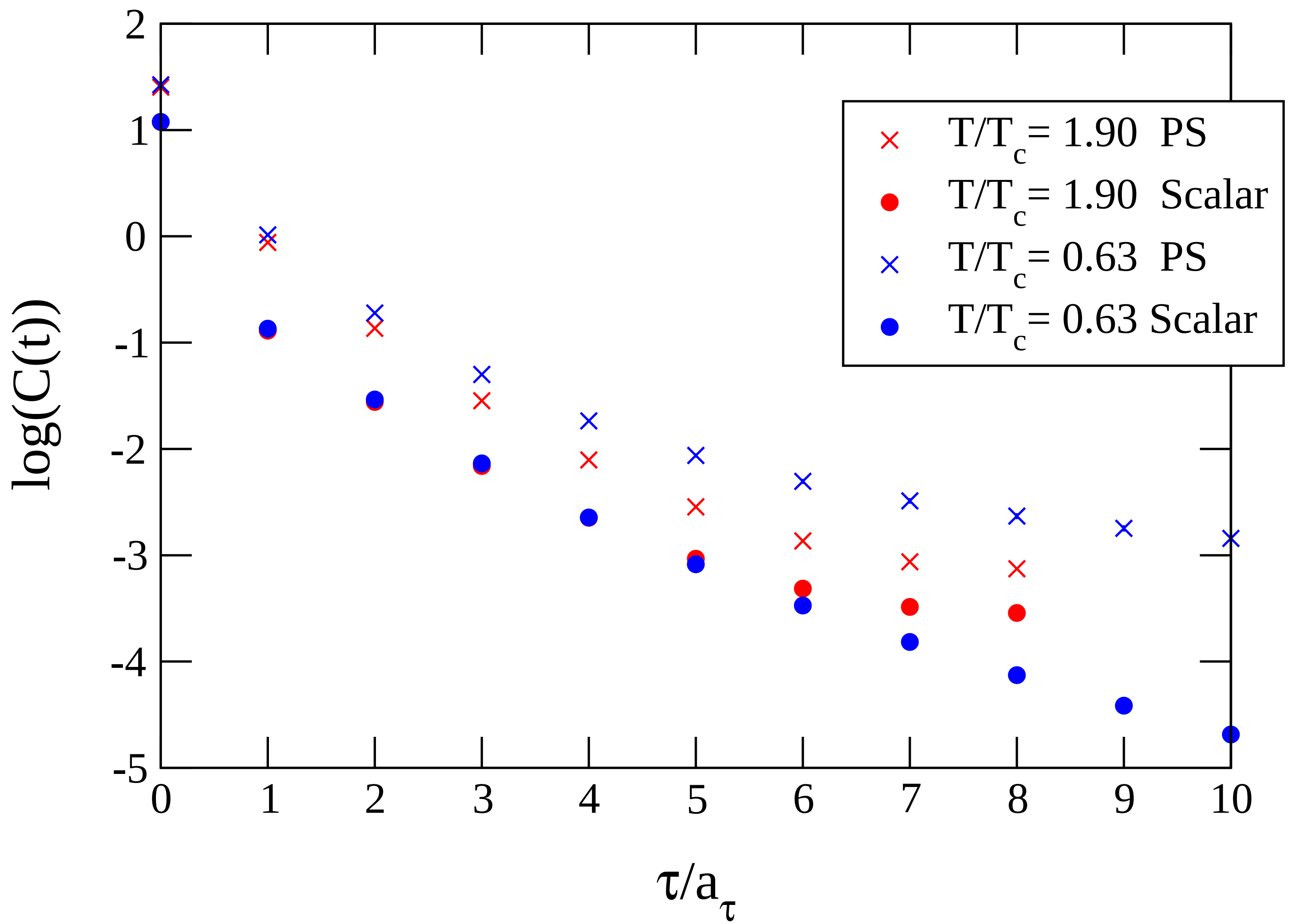
Scheme C: $L_R(Nt = 20) = 0.5$

Cubic spline, solid = 32^3, open = 24^3

$\longrightarrow a_\tau T_c = 0.0329(7)$

i.e. $N_\tau^{\text{crit}} = 30.4(7)$ or $T_c = 171(4)$ MeV

Light mesons & Chiral Symmetry



$\longrightarrow$ (partial) restoration of chiral symmetry at high T

Susceptibilities' Definitions

$$n_i = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_i} \qquad\qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}$$

Susceptibilities' Definitions

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$$Q = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_Q} = \sum_{i=1}^{3} q_i n_i \qquad \chi_Q = \frac{\partial Q}{\partial \mu_Q} = \sum_{i=1}^{3}(q_i)^2 \chi_{ii} + \sum_{i \neq j}^{3} q_i q_j \chi_{ij}$$

$$B = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_B} = \sum_{i=1}^{3} n_i \qquad \chi_B = \frac{\partial B}{\partial \mu_B} = \sum_{i=1}^{3} \chi_{ii} + \sum_{i \neq j}^{3} \chi_{ij}$$

$$\chi_I = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_I^2}$$ with $$\mu_I = \mu_d - \mu_u$$

Susceptibility Terms

Useful to introduce:

$$T_1^i = \langle \frac{T}{V} Tr\left[M^{-1}\frac{\partial M}{\partial \mu_i}\right]\rangle \quad T_2^i = \langle \frac{T}{V} Tr\left[M^{-1}\frac{\partial^2 M}{\partial \mu_i^2}\right]\rangle$$

$$T_3^{ij} = \langle \frac{T}{V} Tr\left[M^{-1}\frac{\partial M}{\partial \mu_i}\right] Tr\left[M^{-1}\frac{\partial M}{\partial \mu_j}\right]\rangle \quad T_4^i = \langle \frac{T}{V} Tr\left[M^{-1}\frac{\partial M}{\partial \mu_i} M^{-1}\frac{\partial M}{\partial \mu_i}\right]\rangle$$

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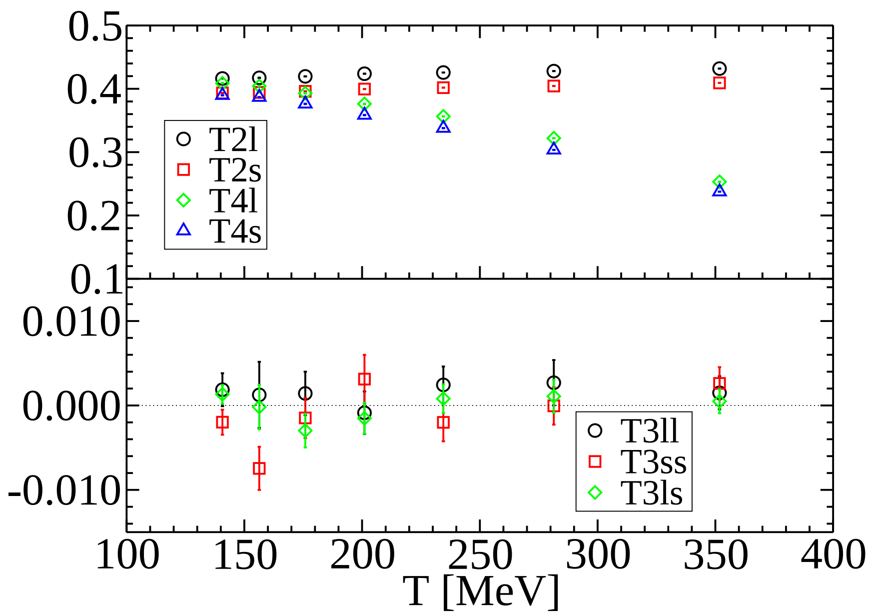
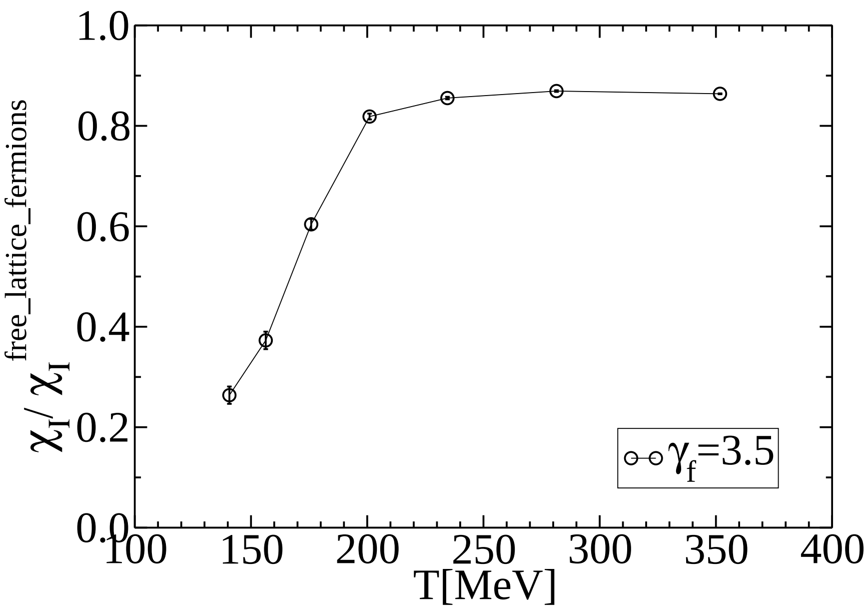
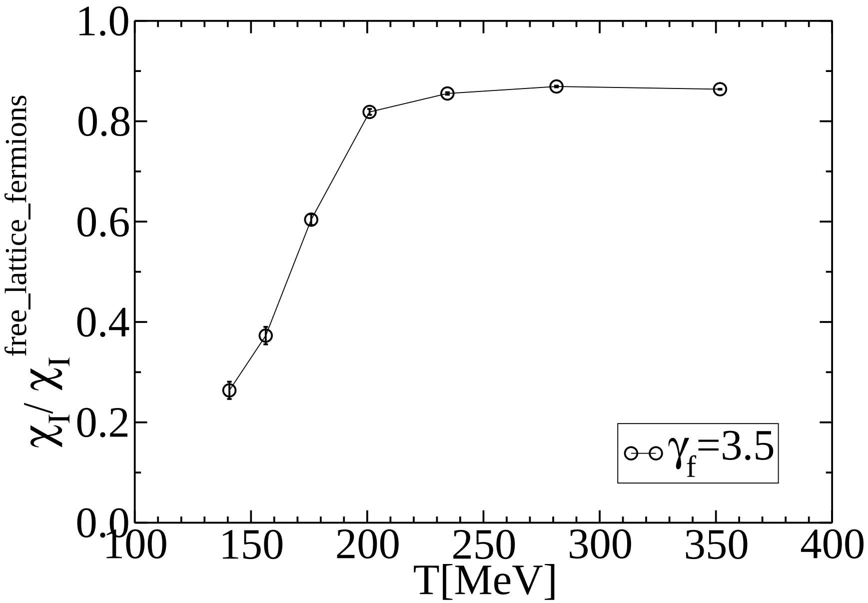


$\chi_{I,Q,B}$



1.0
0.8
0.6
0.4
0.2
0.0
100
150
200
250
300
350
400
T[MeV]
χI/χI free_lattice_fermions
γf=3.5

$\chi_{I,Q,B}$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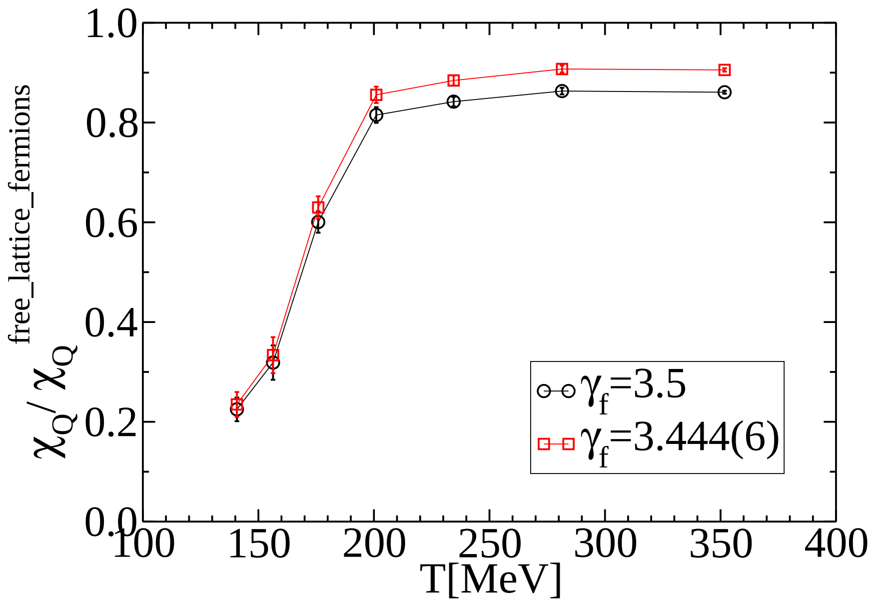
1.0
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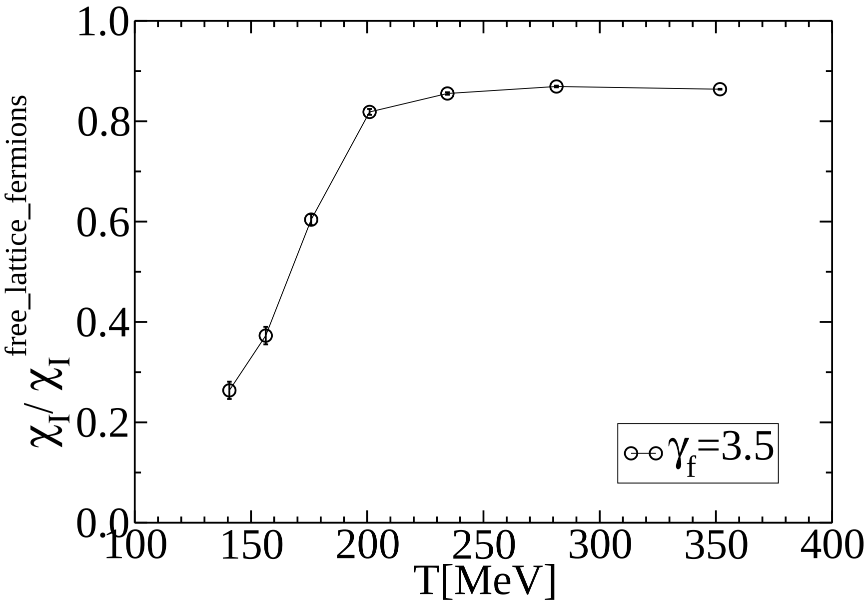
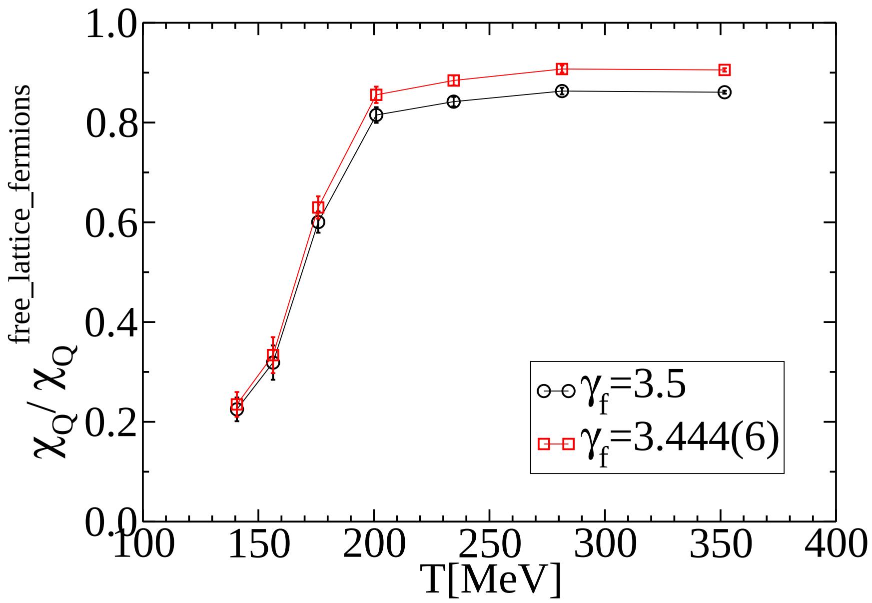
T[MeV]
χQ / χQ free_lattice_fermions
γf=3.5
γf=3.444(6)

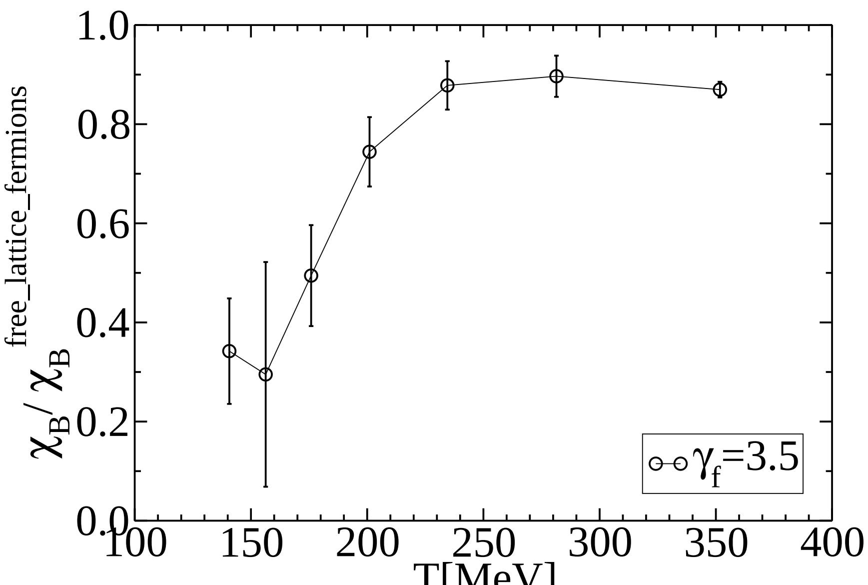
$\chi_{I,Q,B}$



$\chi_I / \chi_I^{\text{free\_lattice\_fermions}}$
T[MeV]
γ_f=3.5

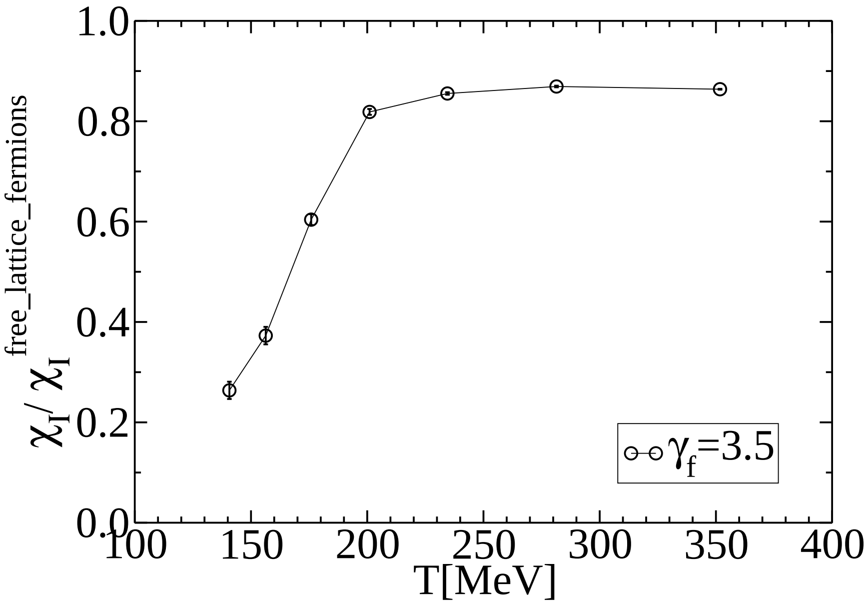


$\chi_Q / \chi_Q^{\text{free\_lattice\_fermions}}$
T[MeV]
γ_f=3.5
γ_f=3.444(6)



$\chi_B / \chi_B^{\text{free\_lattice\_fermions}}$
T[MeV]
γ_f=3.5

$\chi_{I,Q,B}$


$\chi_I / \chi_I^{free\_lattice\_fermions}$
T[MeV]
γ_f=3.5
0.0
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0.8
1.0
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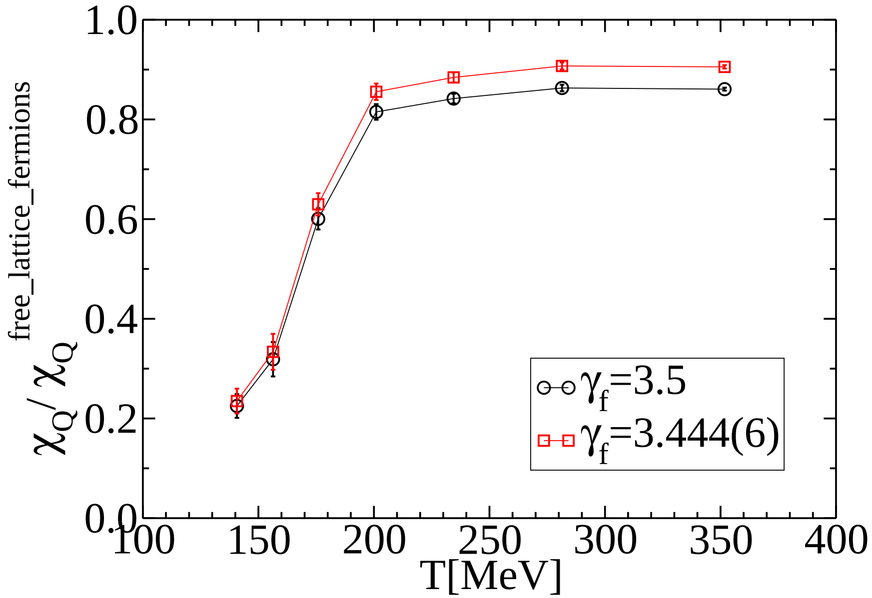
$\chi_Q / \chi_Q^{free\_lattice\_fermions}$
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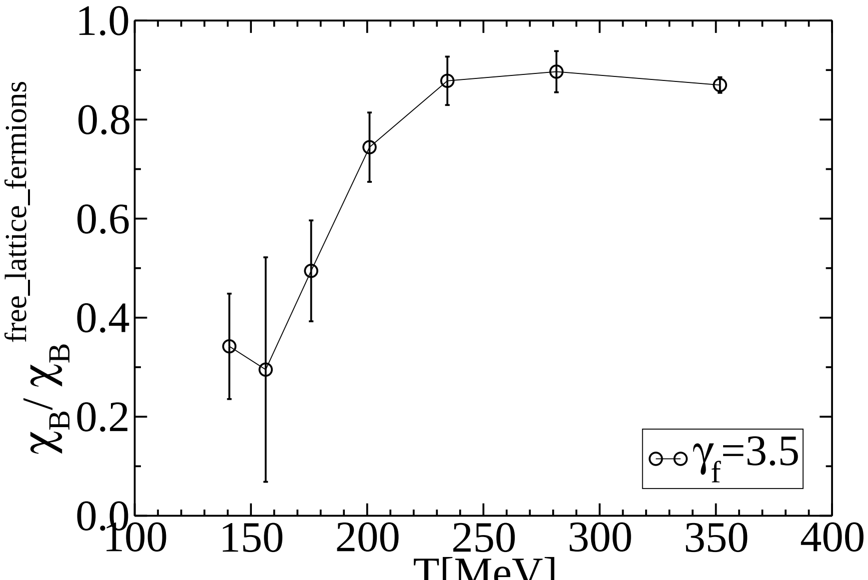
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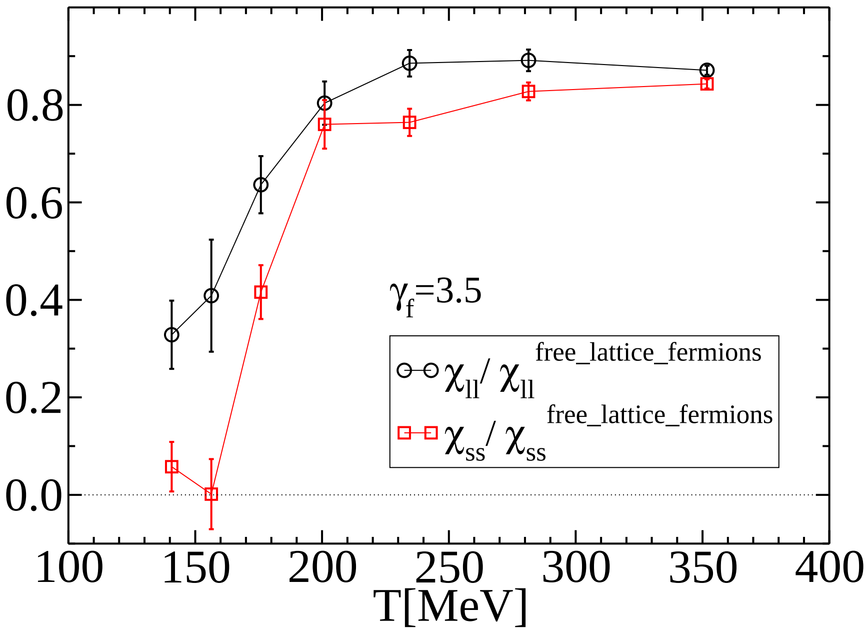
Quark Number Susceptibility



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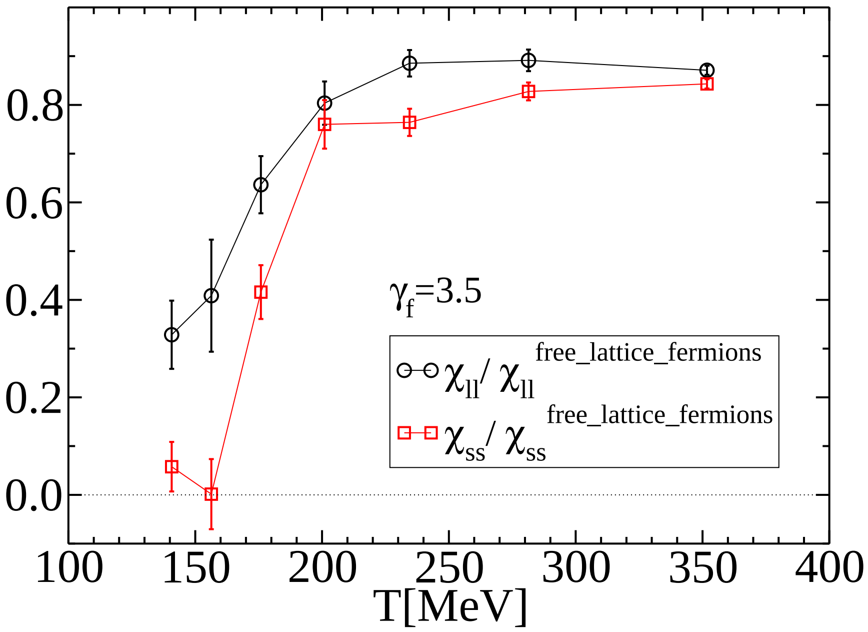

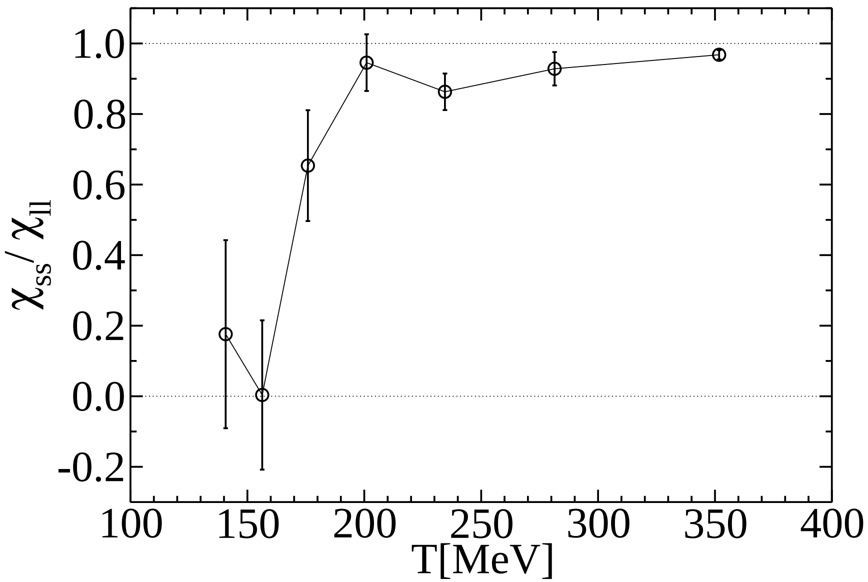

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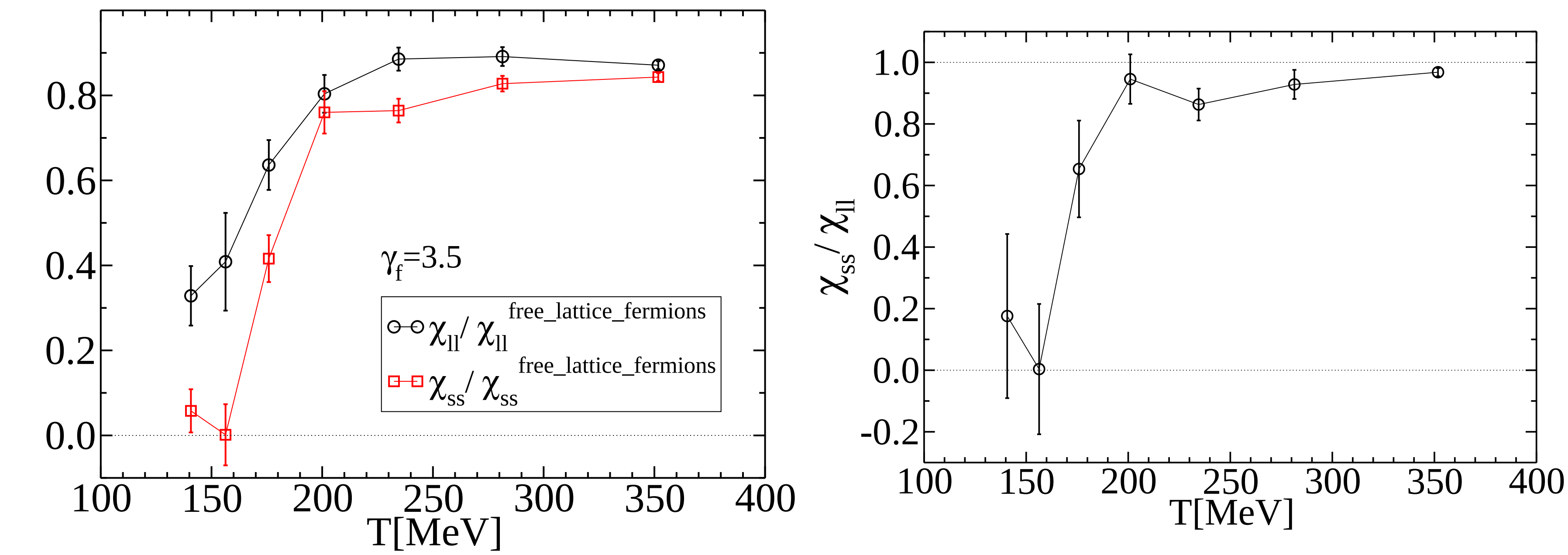

χ_{ll} versus χ_{ss} showing "flavour hierarchy"

Electrical conductivity on the lattice

EM current: $j_\mu^{\rm em} = \frac{2e}{3}\, j_\mu^{\sf u} - \frac{e}{3}\, j_\mu^{\sf d} - \frac{e}{3}\, j_\mu^{\sf s},$

EM Correlator: $G_{\mu\nu}^{\sf em}(\tau) = \int d^3x \, \langle j_\mu^{\rm em}(\tau, \mathbf{x}) j_\nu^{\rm em}(0, \mathbf{0})^\dagger \rangle$

Spectral decomposition:

$$G_{\mu\nu}^{\sf em}(\tau) = \int_0^\infty \frac{d\omega}{2\pi}\, K(\tau,\omega)\, \rho_{\mu\nu}^{\rm em}(\omega) \quad \text{with} \quad K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh[\omega/2T]}$$

Conductivity: $\frac{\sigma}{T} = \frac{1}{6T} \lim_{\omega\to 0} \frac{\rho^{\rm em}(\omega)}{\omega}$

Relationship to Diffusivity: $D\chi_Q = \sigma$

Vector Correlators

Conserved (lattice) vector current used for j_μ^{em}

$$V_\mu^{\mathsf{C}}(x) = \left[\bar{\psi}(x+\hat{\mu})(1+\gamma_\mu)\,U_\mu^\dagger(x)\,\psi(x) - \bar{\psi}(x)(1-\gamma_\mu)\,U_\mu(x)\,\psi(x+\hat{\mu})\right]$$

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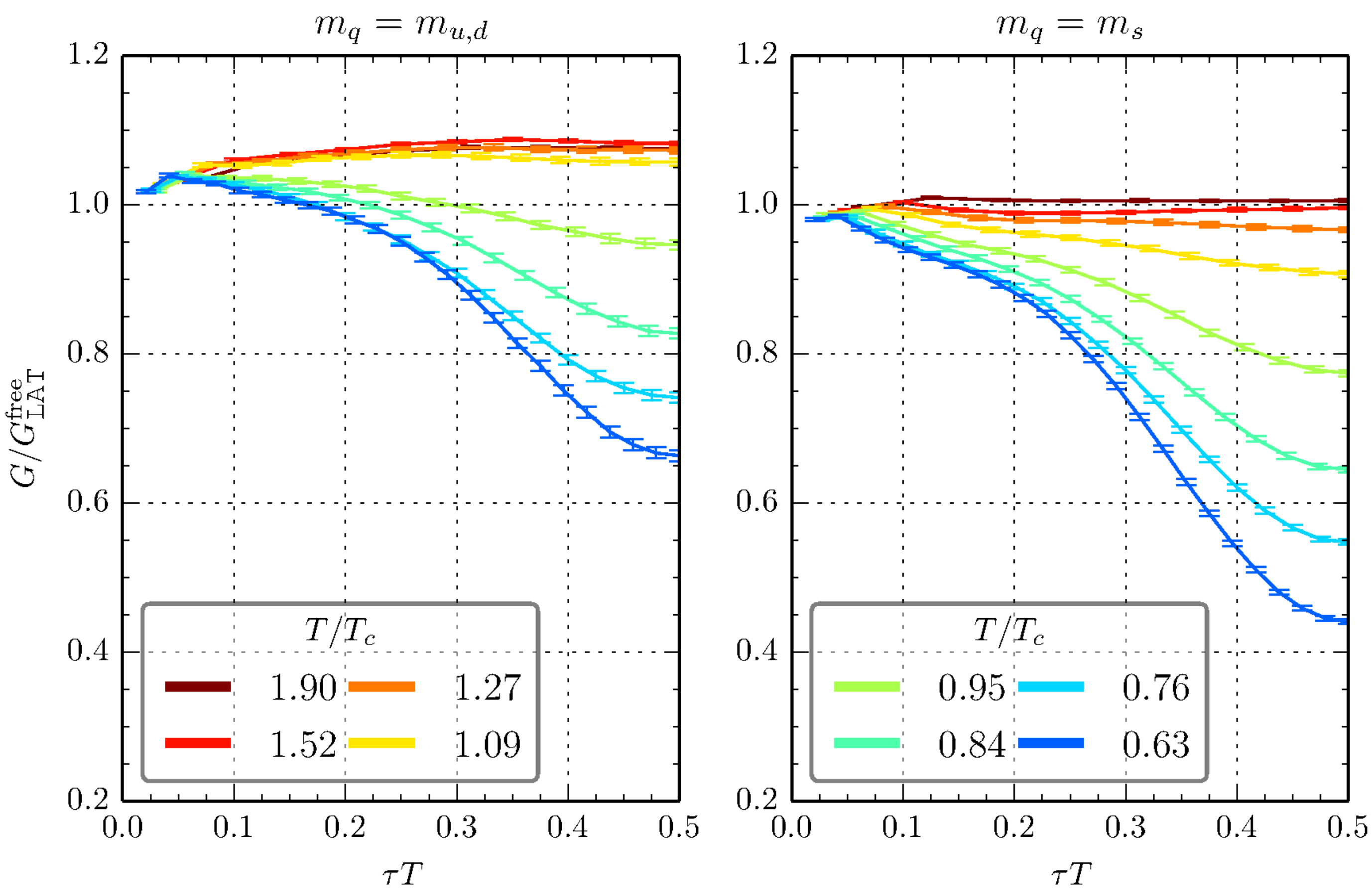
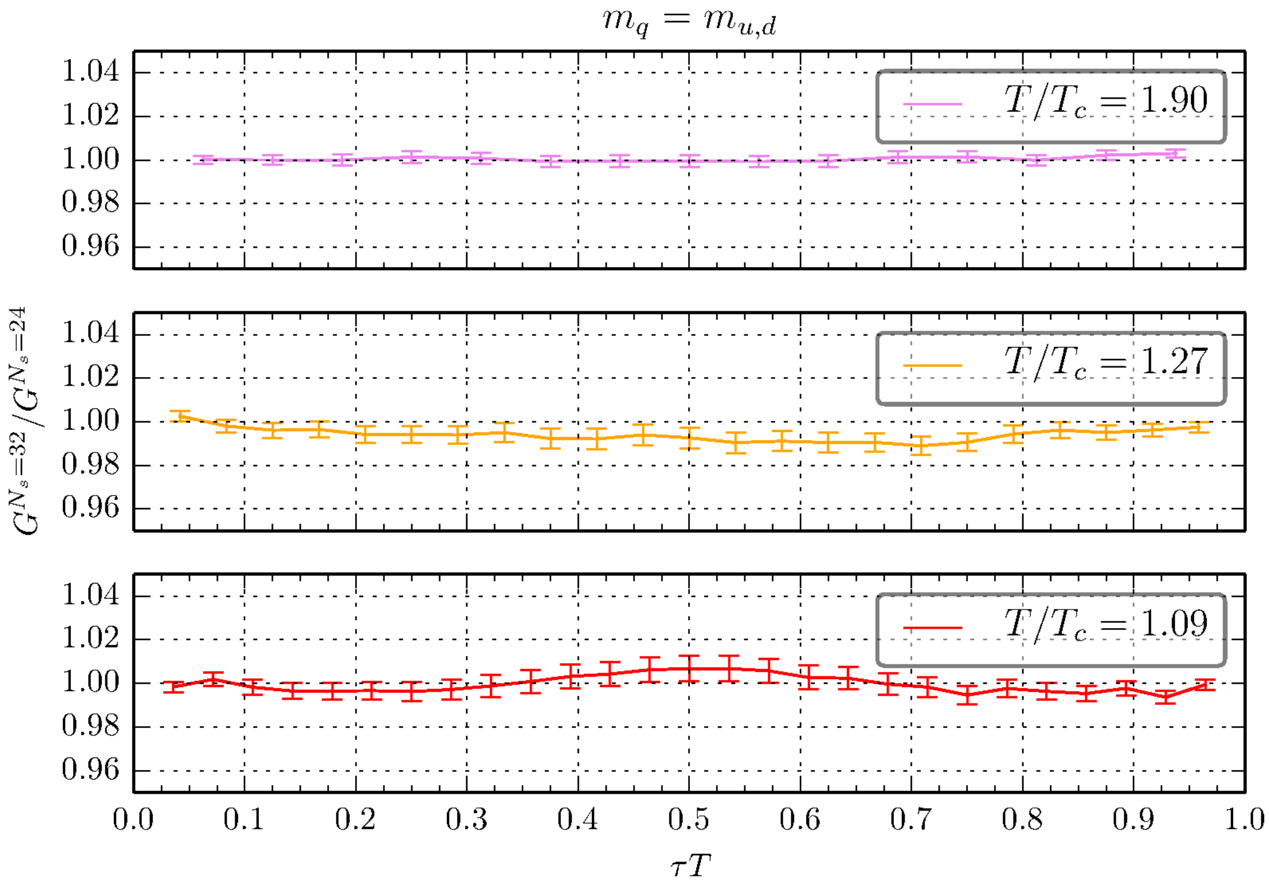


Finite Volume Check



$N_s = 24$ versus $N_s = 32$

Maximum Entropy Method

Recall $$G^{\text{em}}(\tau_i) = \sum_j K(\tau_i, \omega_j)\, \rho^{\text{em}}(\omega_j)$$

Input data: $\tau_i,\ i = \{1, \ldots, \mathcal{O}(10)\}$ Output data : $\omega_j,\ j = \{1, \ldots, \mathcal{O}(10^3)\}$

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$\longrightarrow$ ill-posed

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$\longrightarrow$ ill-posed

$$P[\rho|DH] = \frac{P[D|\rho H] P[\rho|H]}{P[D|H]} \propto \exp(-\chi^2 + \alpha S)$$

$H =$ prior knowledge, $D =$ data

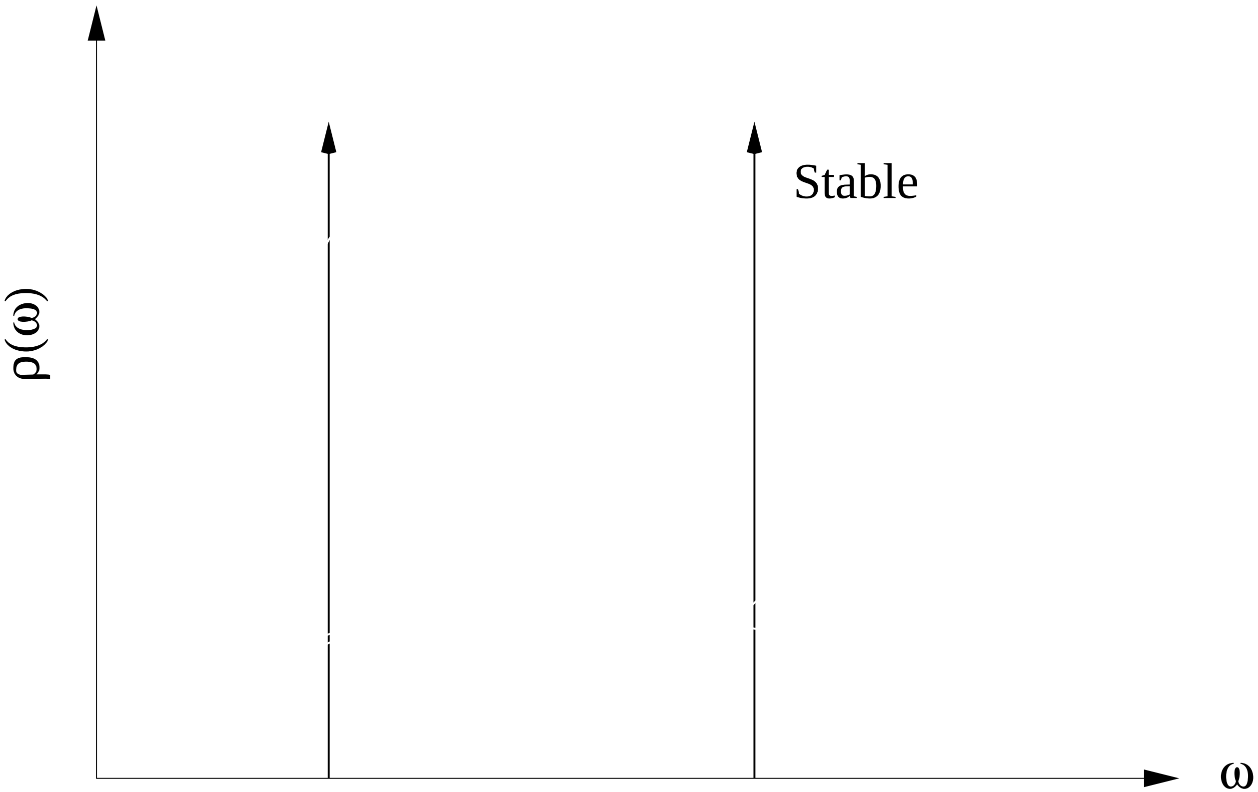
Shannon-Jaynes entropy: $\quad S = \int_0^\infty \frac{d\omega}{2\pi} \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)}\right]$

Default model: $\quad m(\omega) = m_0(b + \omega)\omega$

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

Example Spectral Functions

$$G_2(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$



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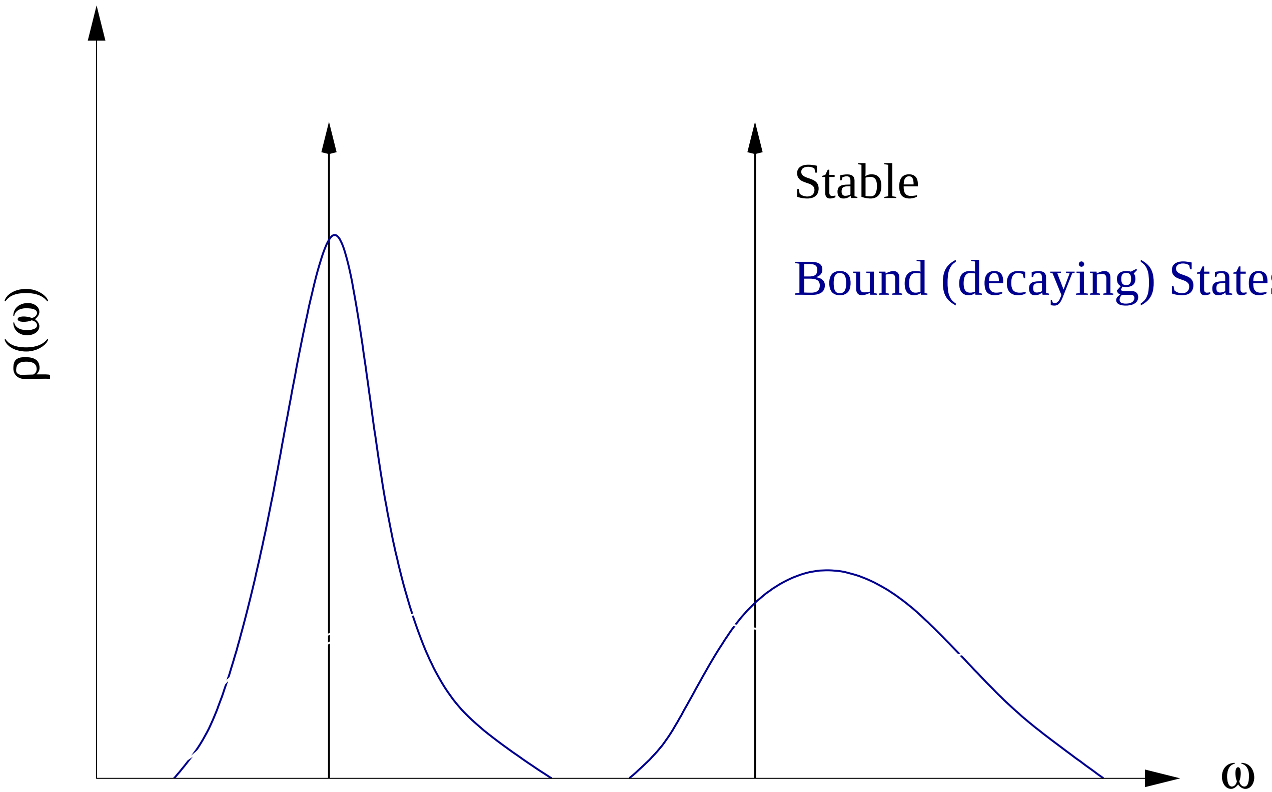

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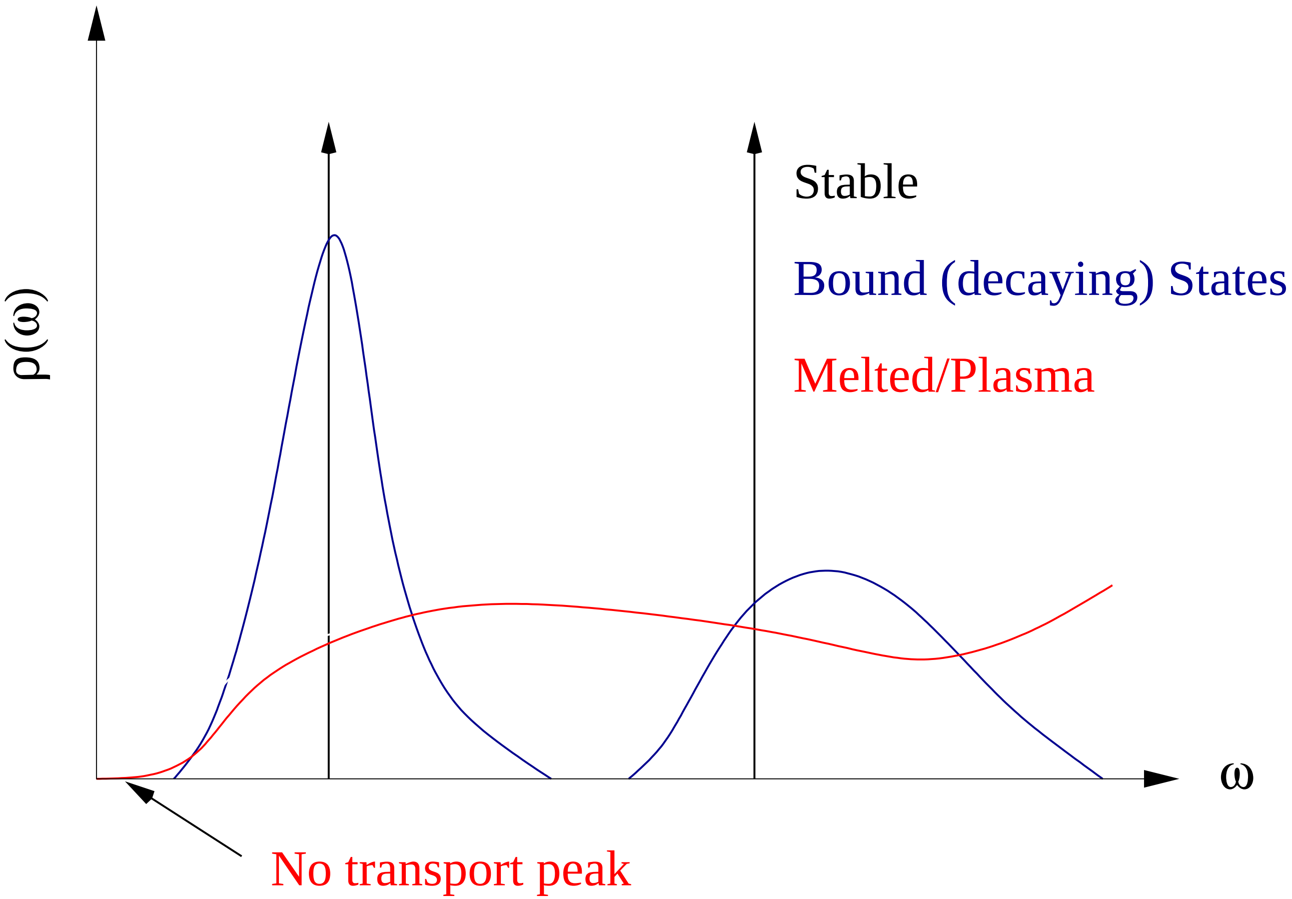

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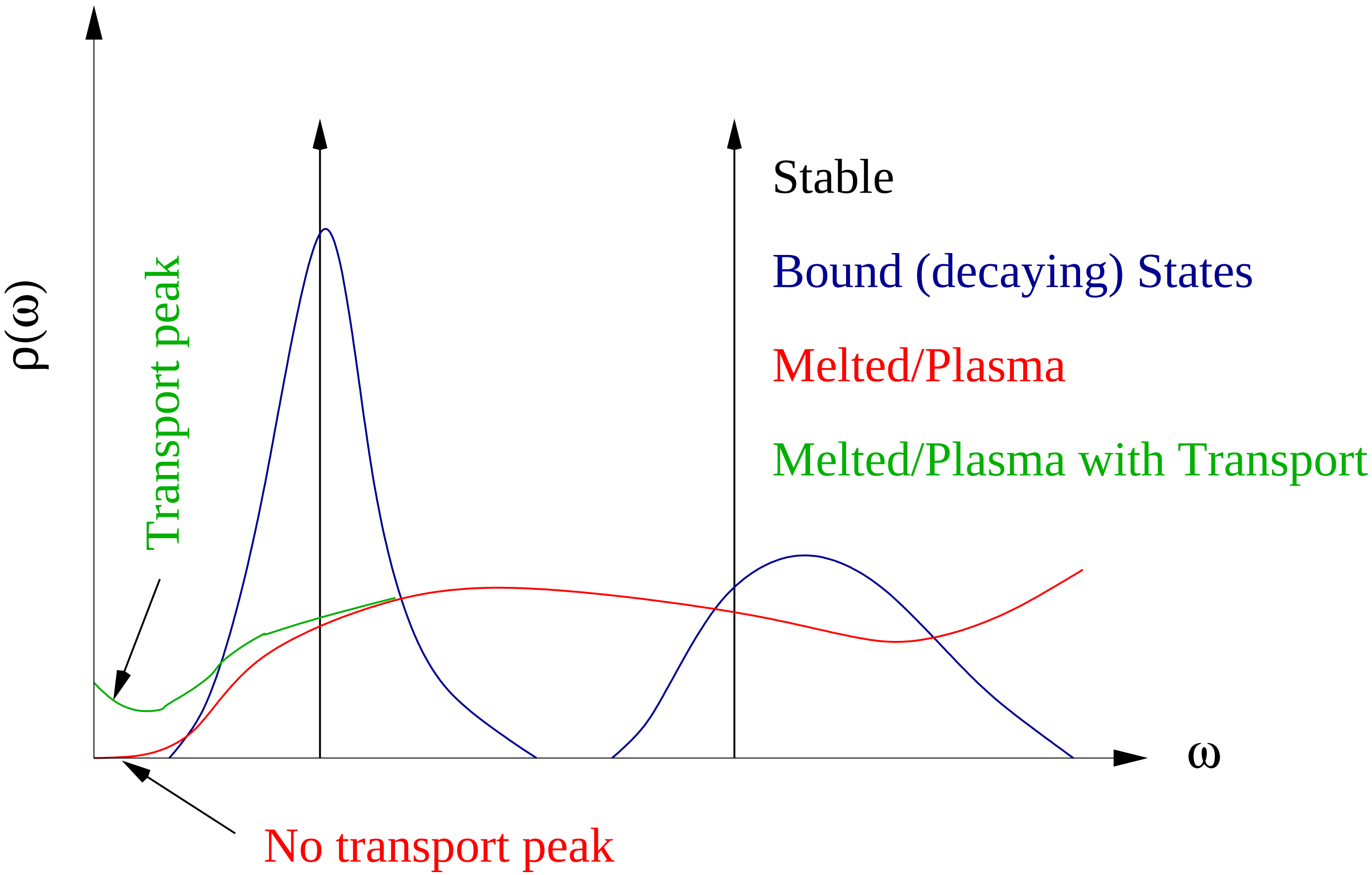
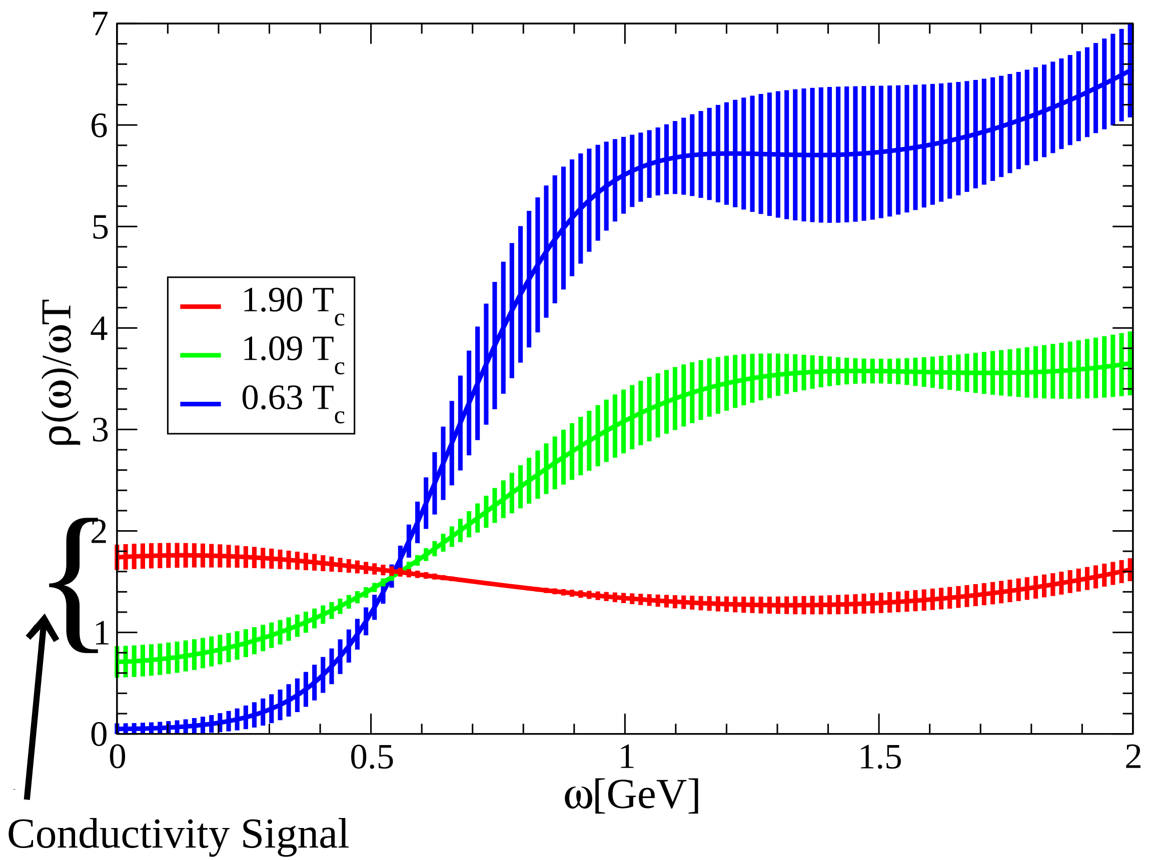
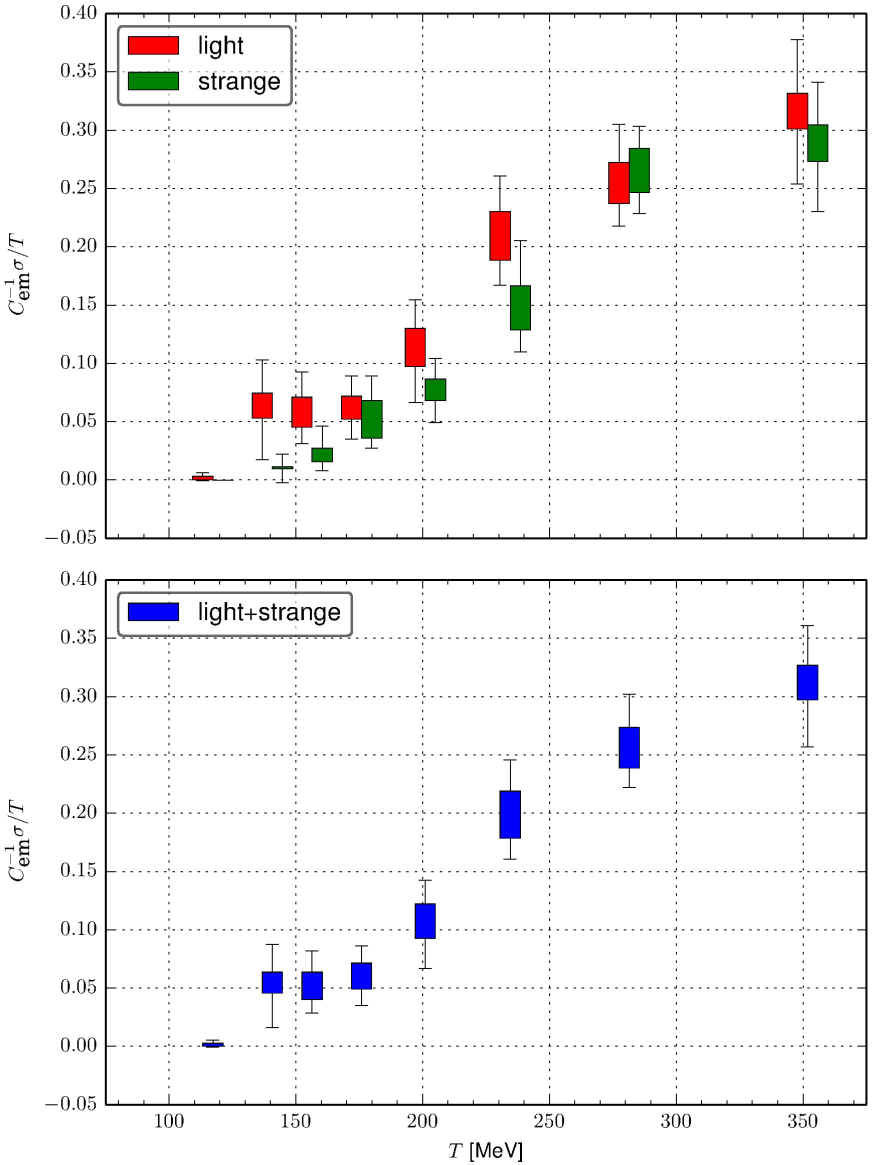


Spectral function results



Recall $\sigma \sim \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$

Conductivity Result σ/T



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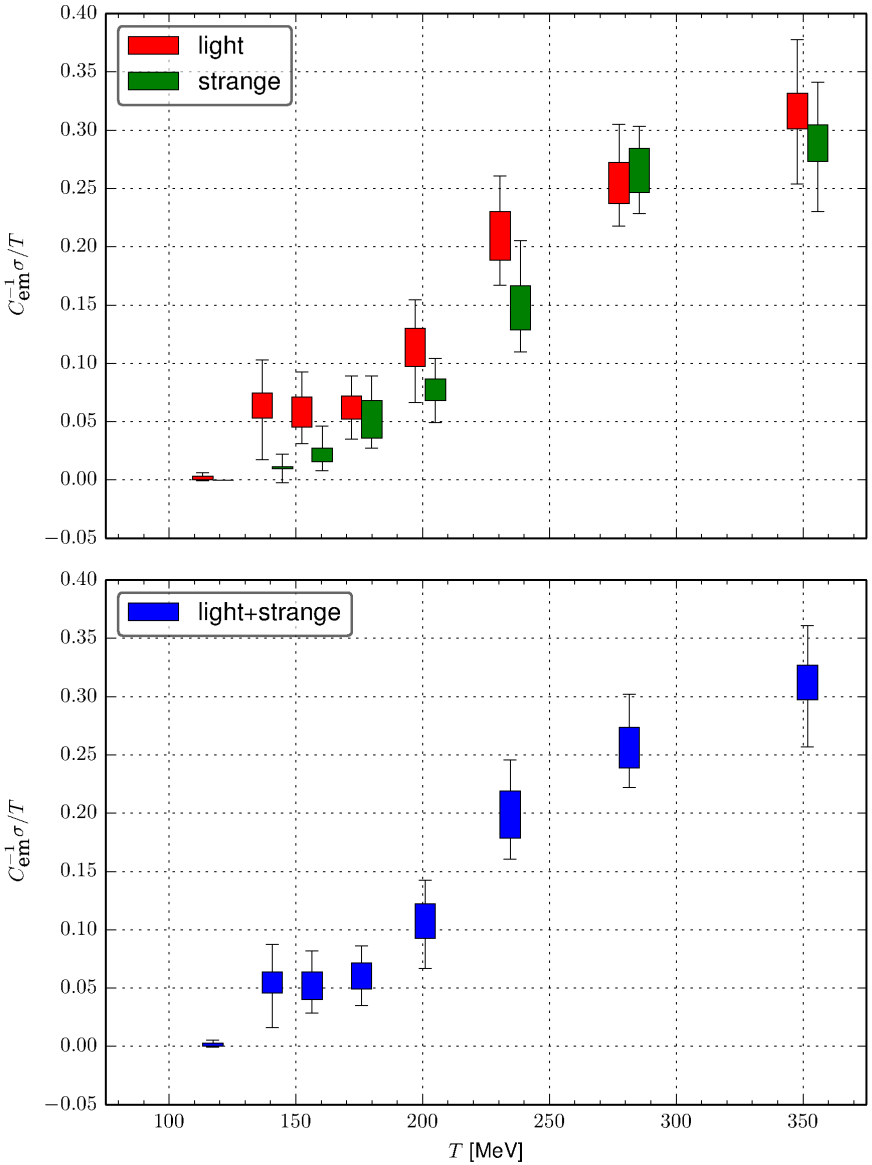


Useful to factor out charge:

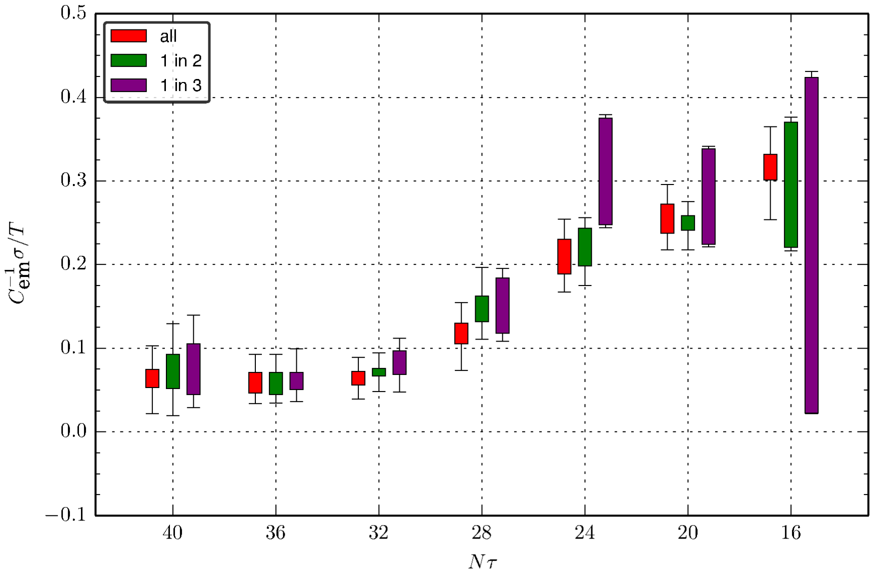
$$C_{\mathrm{em}} = e^2 \sum_f q_f^2$$

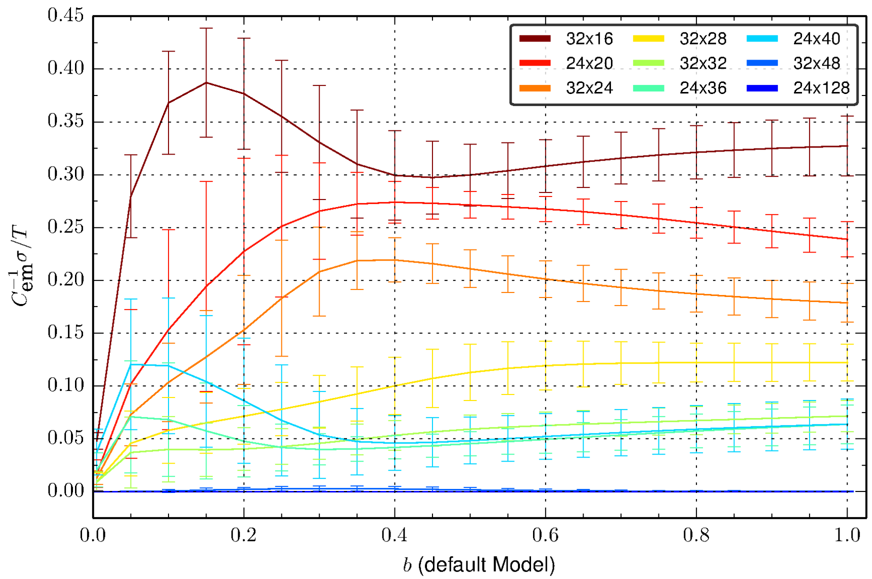
Rectangles =
default model systematic (i.e. b)

Recall $\quad m(\omega) = m_0(b + \omega)\omega$

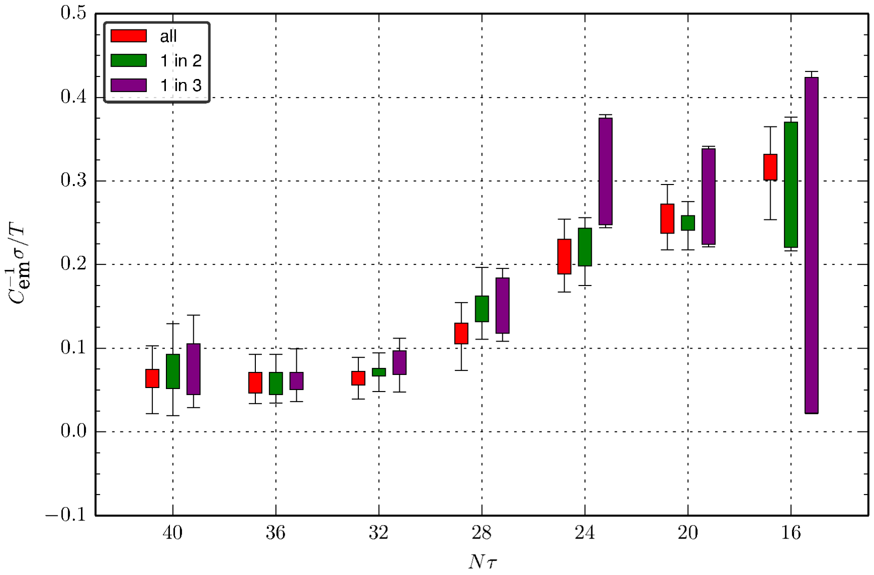
Whiskers = statistical error

MEM Systematics I

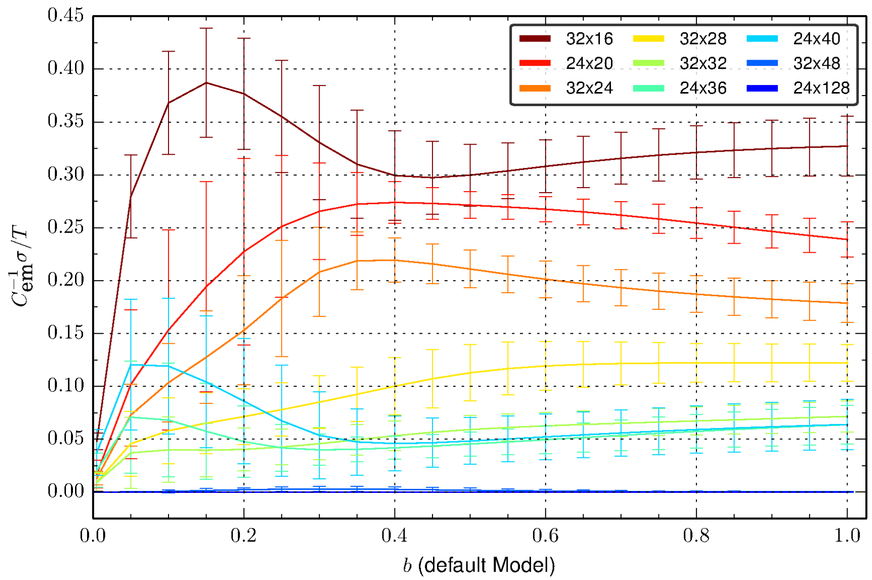




MEM Systematics I



Anisotropy check including:
all or 1 in 2 or 1 in 3
of the τ datapoints



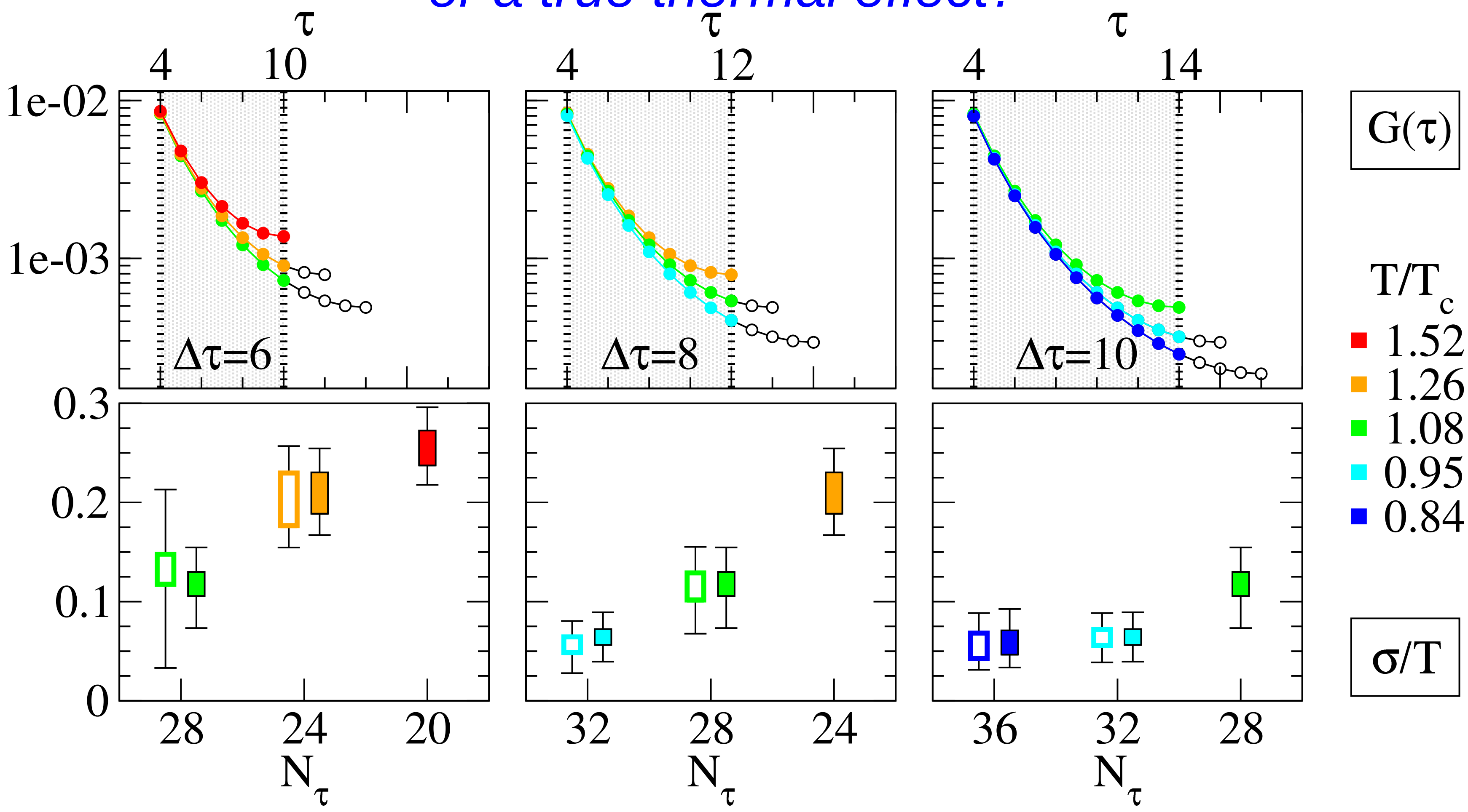
Variation with default model parameter b

Recall $\quad m(\omega) = m_0(b+\omega)\omega$

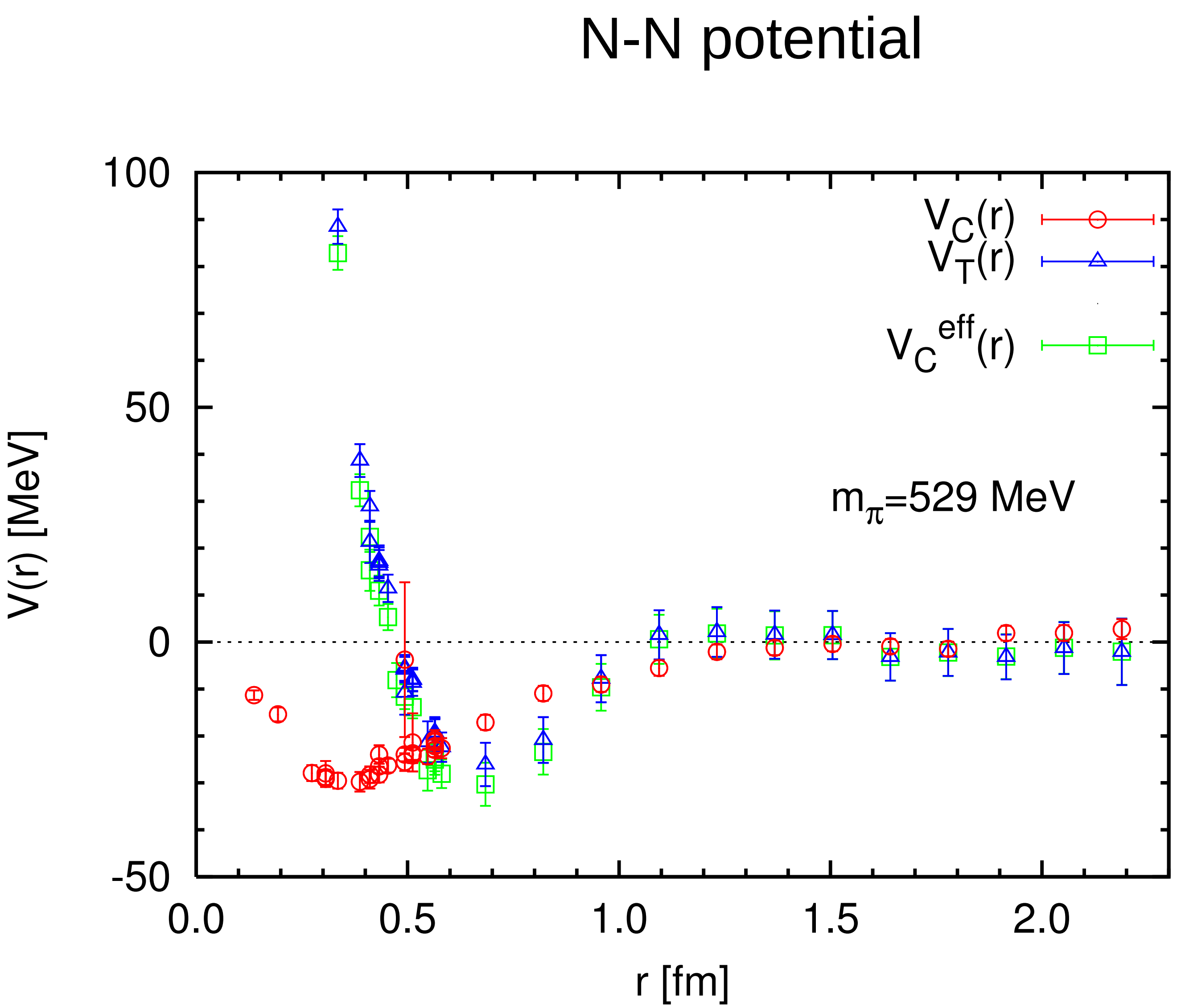
MEM Systematics II

Stability tests discarding the last time slices:

Are we seeing a number-of-datapoints (N_τ) systematic or a true thermal effect?



Lattice goes Nuclear



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068 use the Schrödinger equation to "reverse engineer" the potential, $V(r)$, given the Nambu-Bethe-Salpeter wavefunction, $\psi(r)$:

$$\overset{\text{input}}{}\qquad\overset{\text{input}}{}$$

$$\left(\frac{p^2}{2M} + \underset{\underset{\text{output}}{\downarrow}}{V(r)}\right) \overset{\downarrow}{\psi(r)} = \overset{\downarrow}{E}\ \overset{\downarrow}{\psi(r)}$$

$\psi(r)$ is determined from a lattice simulation from correlators of *non-local* (point-split) operators, $J(x;\vec{r}) = q(x)\,\Gamma\,U(x,x+\vec{r})\overline{q}(x+\vec{r})$

$$\begin{aligned} C(\vec{r},t) &= \sum_{\vec{x}} < J(0;\vec{r})\ J(x;\vec{r}) > \\ &\longrightarrow |\psi(r)|^2\ e^{-Et} \end{aligned}$$

The Method (HAL QCD time-dependent)

Charm treated relativistically

Charmonium Operators: $J_\Gamma(x;\mathbf{r}) = q(x)\,\Gamma\,U(x,x+\mathbf{r})\,\overline{q}(x+\mathbf{r})$

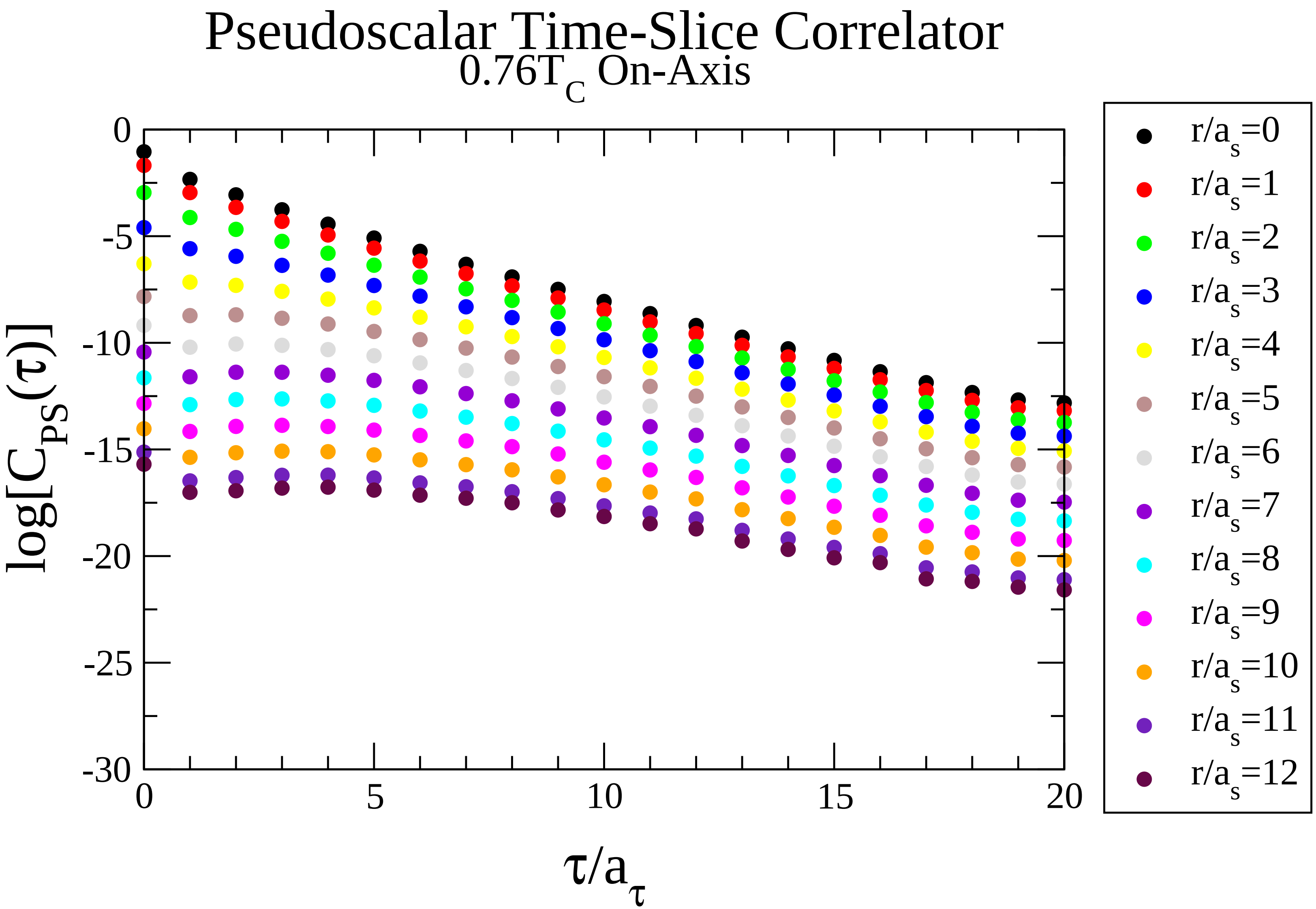
Correlation F'ns: $$C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r})\; J_\Gamma{}^\dagger(0;\mathbf{0})\rangle$$
$$= \sum_j \frac{\psi_j(\mathbf{r})\psi_j^*(\mathbf{0})}{2E_j}\left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right)$$

Schrödinger Eq'n $$\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + V_\Gamma(r)\right]\psi_j(r) = E_j\psi_j(r)$$

Apply this to C_Γ : $$\frac{\partial C_\Gamma(r,\tau)}{\partial\tau} = \sum_j \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)\frac{\psi_j^*(0)\psi_j(r)}{2E_j}e^{-E_j\tau}$$
$$= \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)C_\Gamma(r,\tau)$$

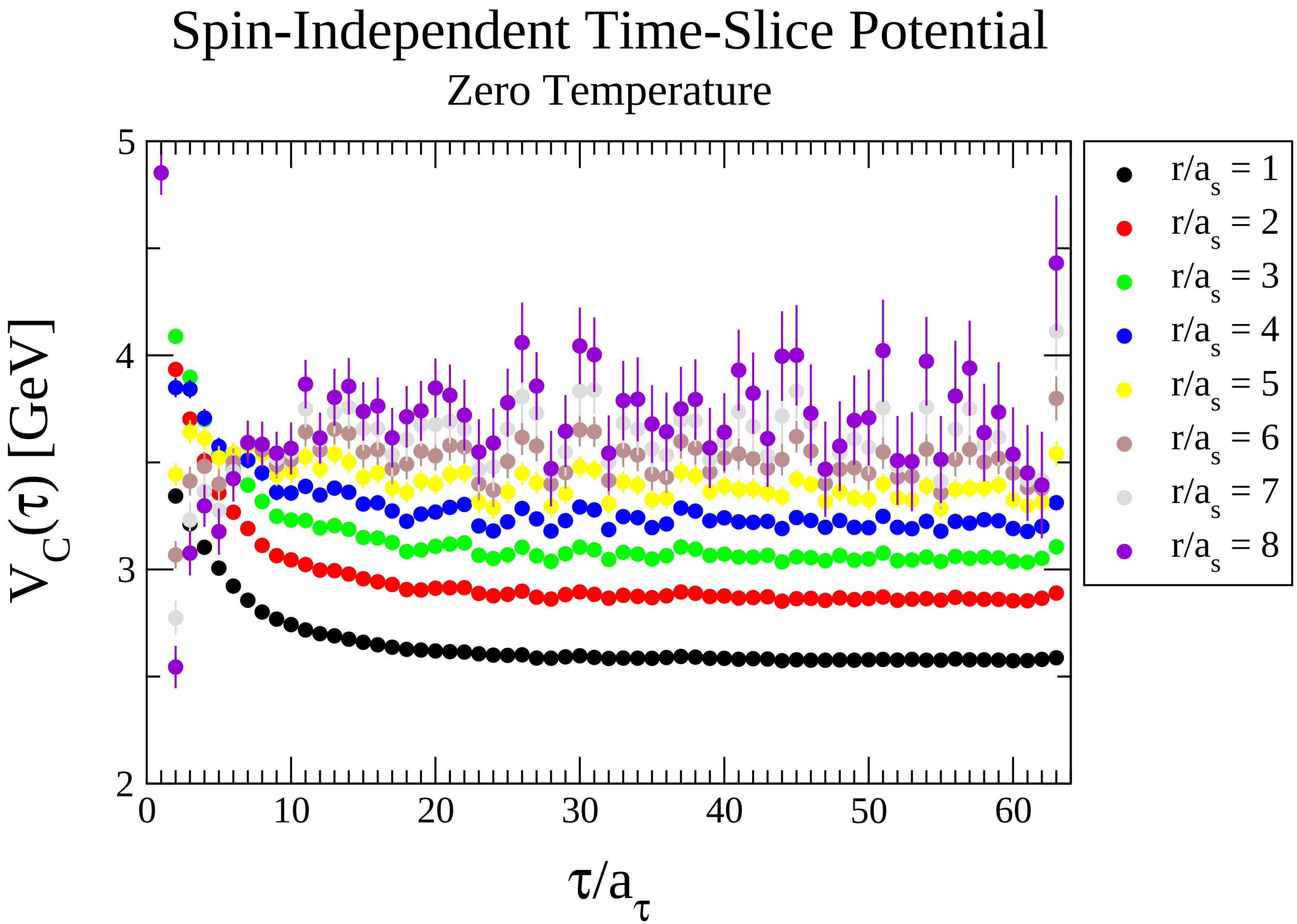
This gives an algebraic equation for $V_\Gamma(r,\text{“}\tau\text{”})$

“Extended” Correlation Functions

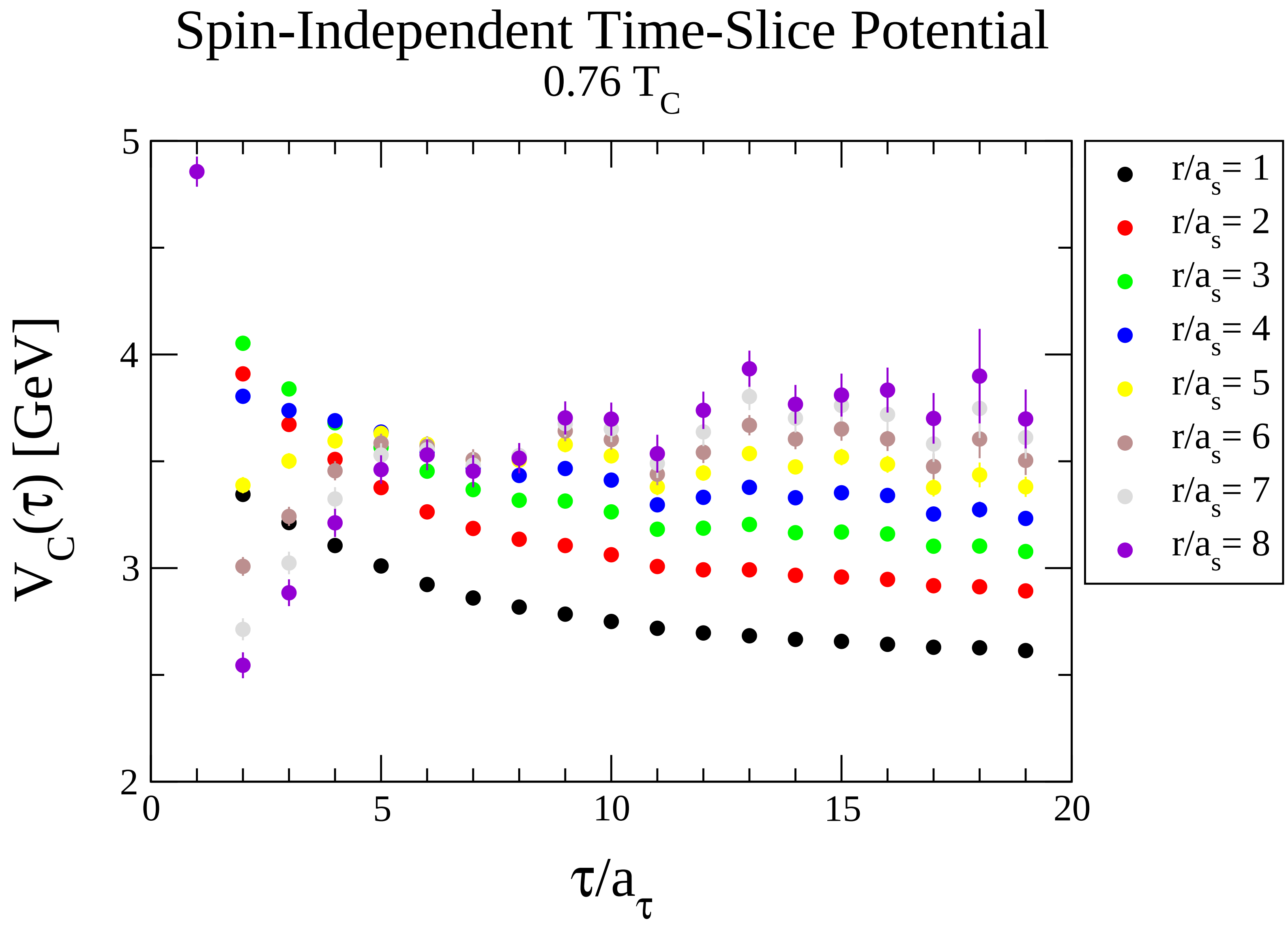


Local-extended charmonium correlators for $N_\tau = 40$

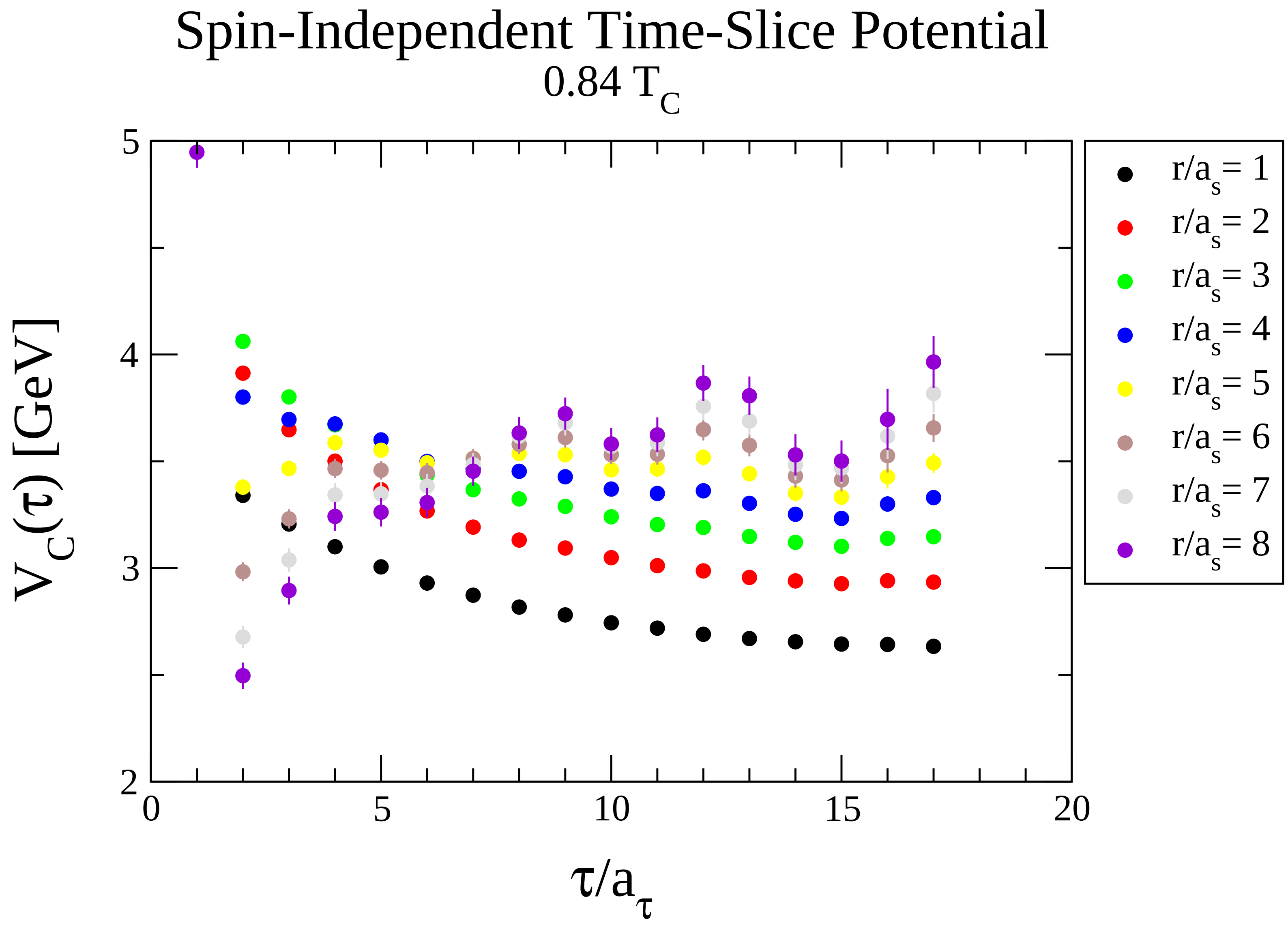
"$\tau-$dependent" Potential $T = 0$



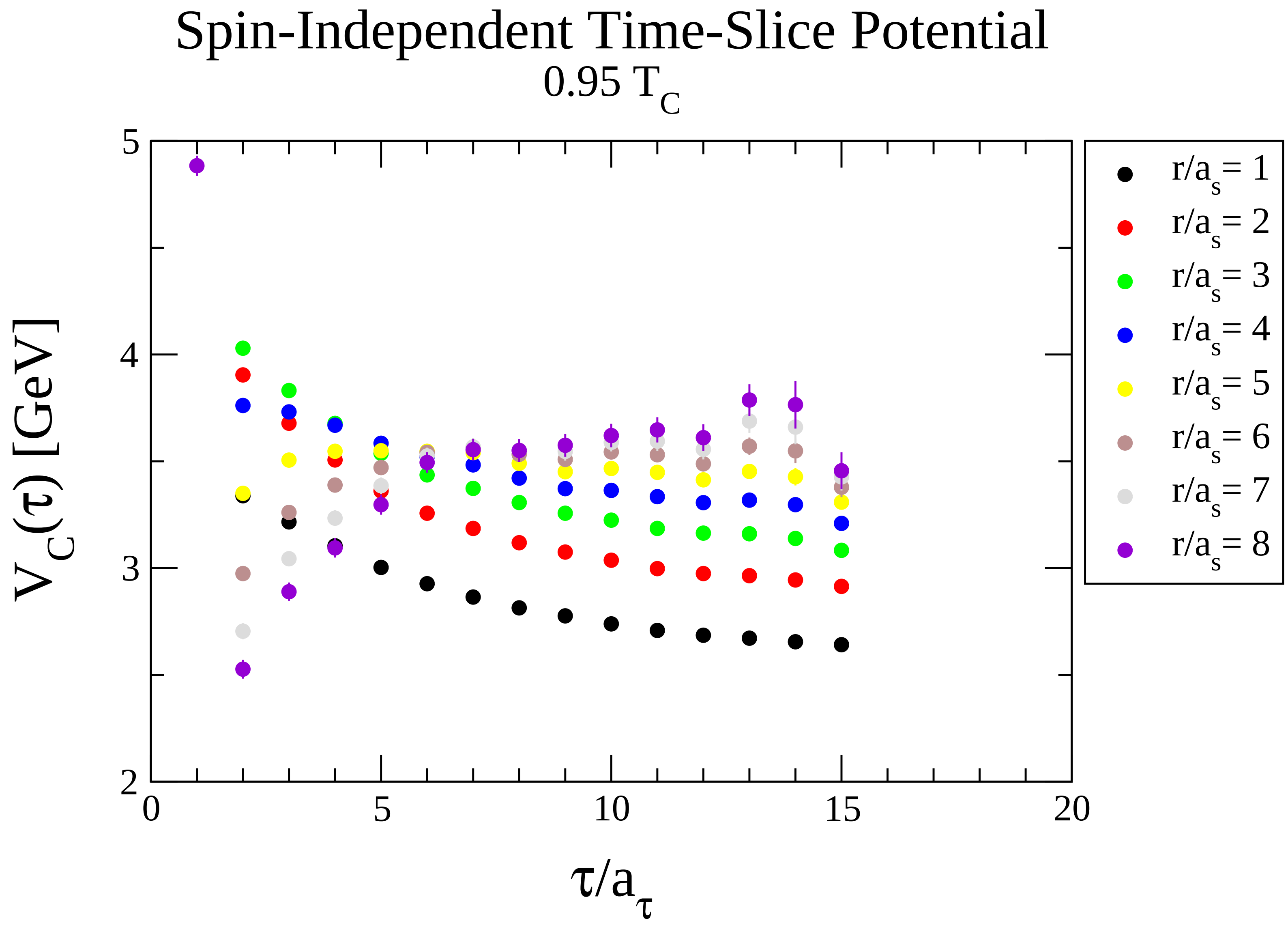
“$\tau-$dependent” Potential $0.76T_c$



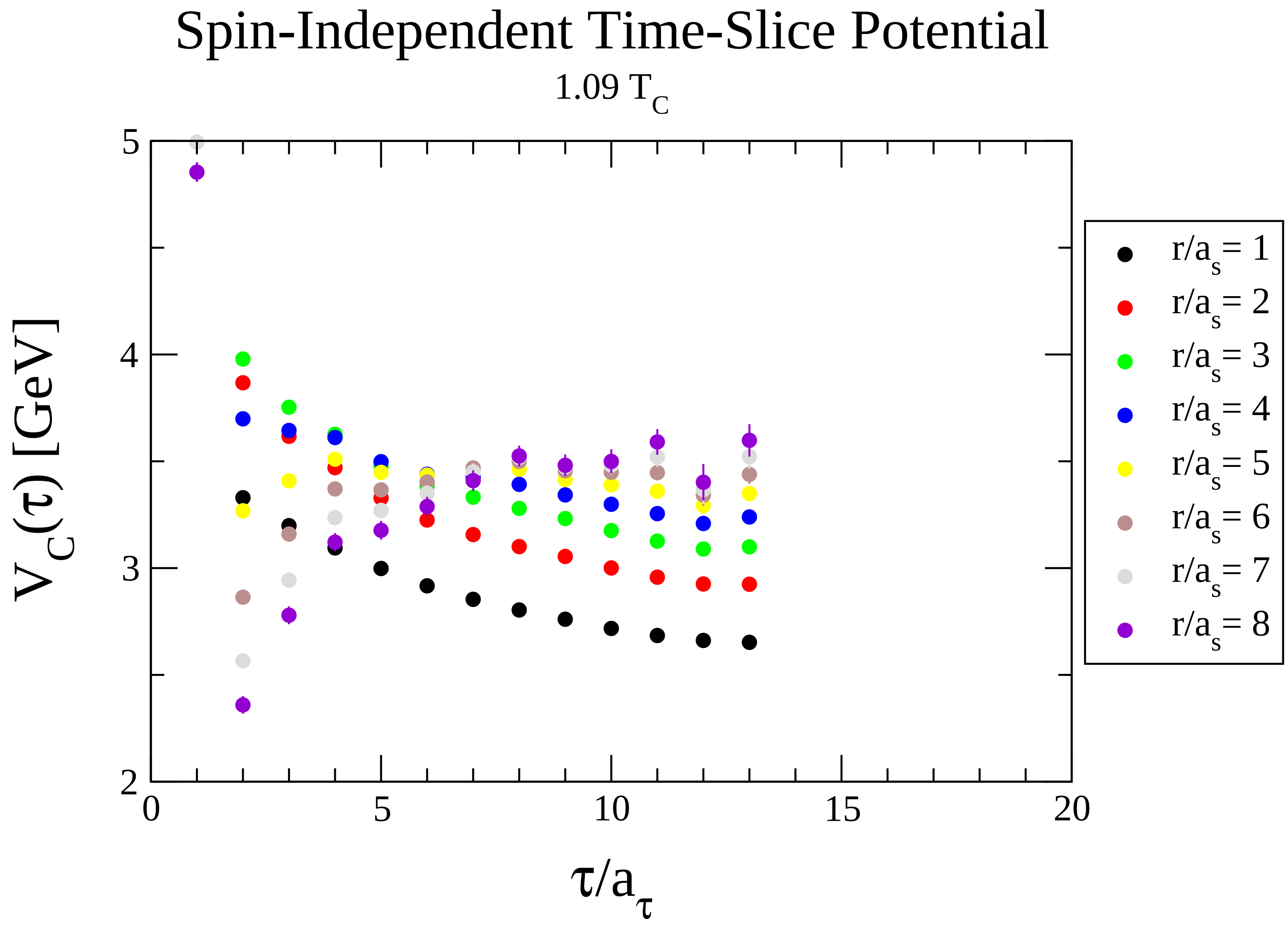
"$\tau-$dependent" Potential $0.84T_c$



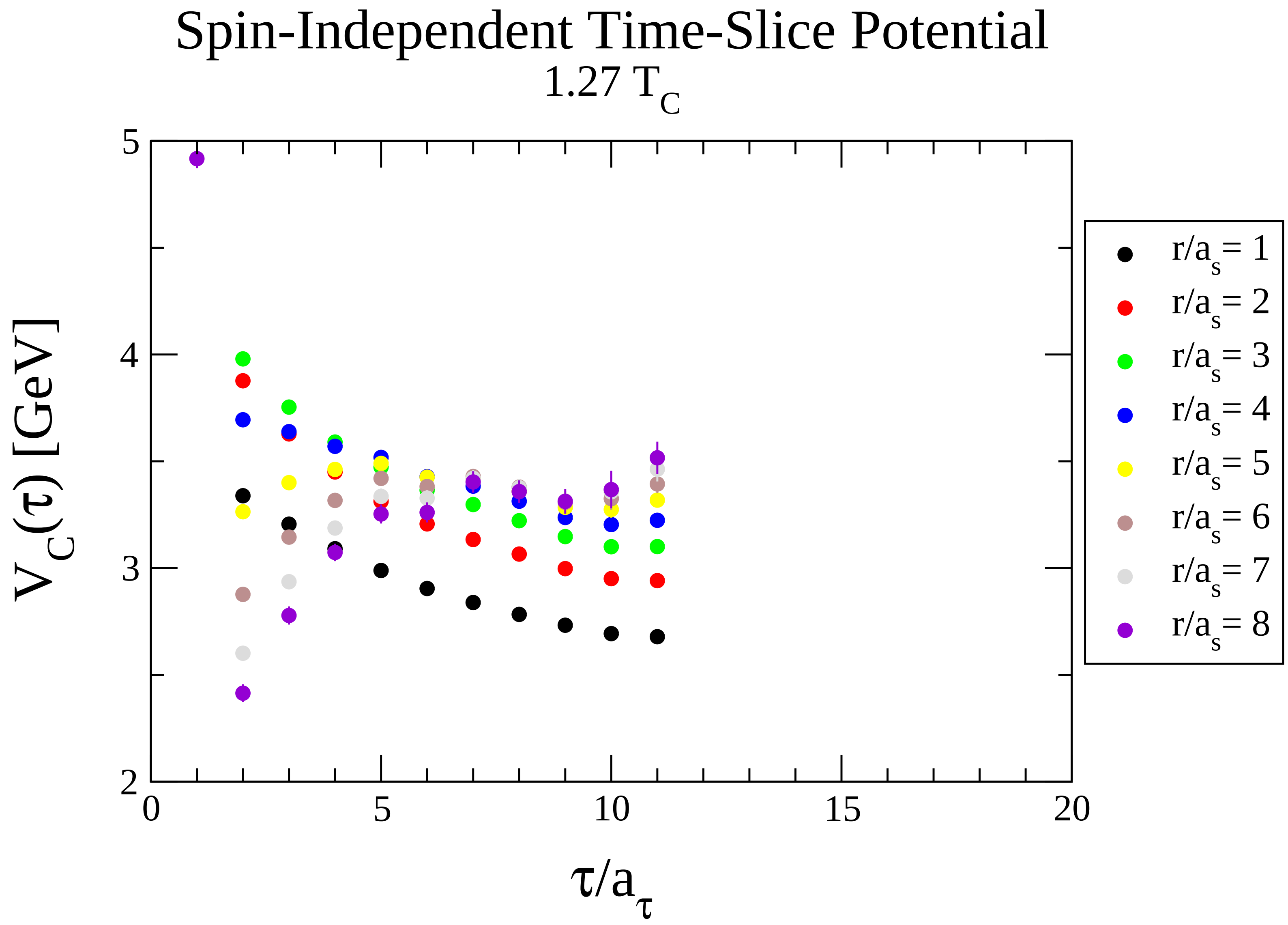
“$\tau-$dependent” Potential $0.95T_c$



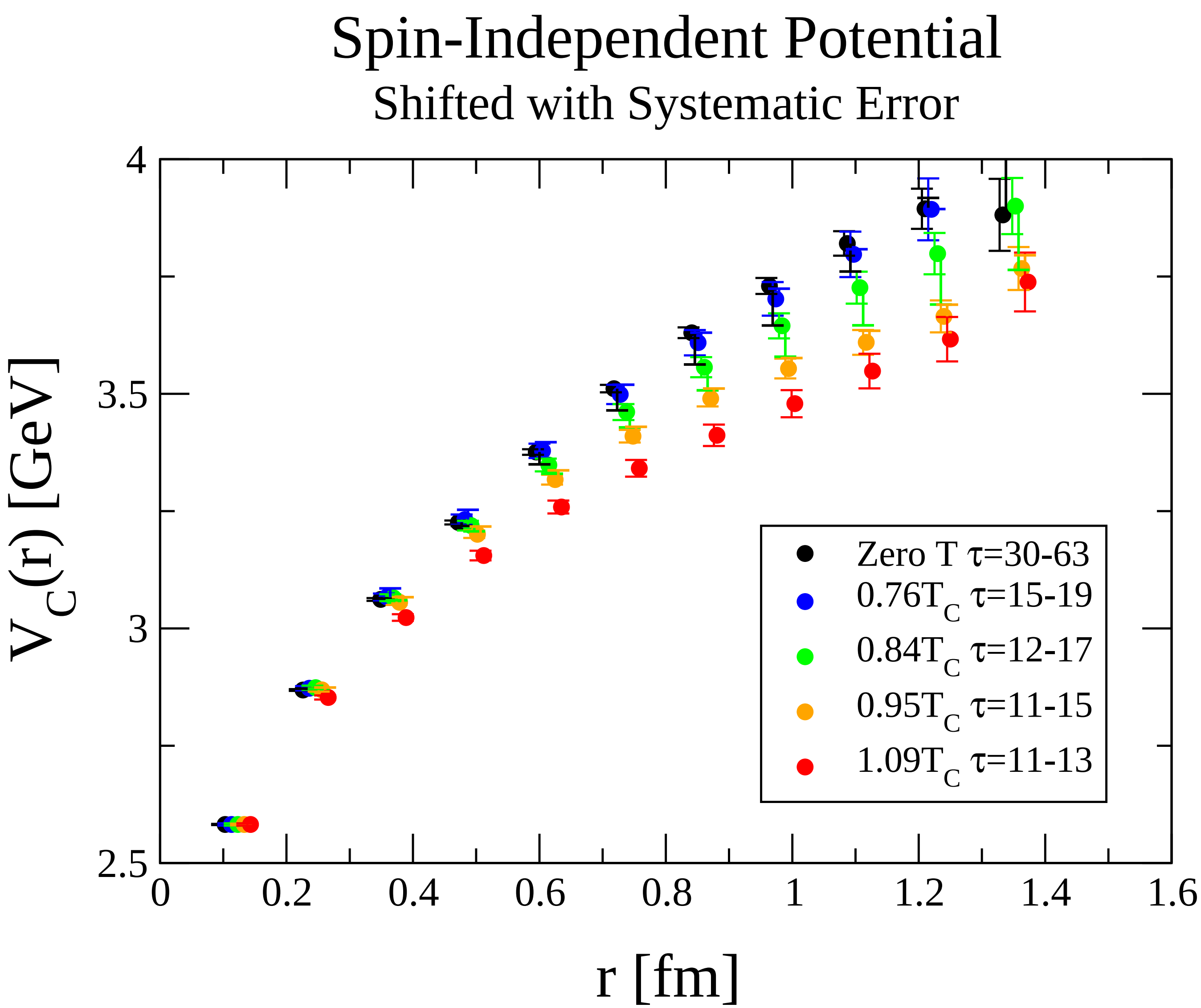
"τ−dependent" Potential $1.09T_c$



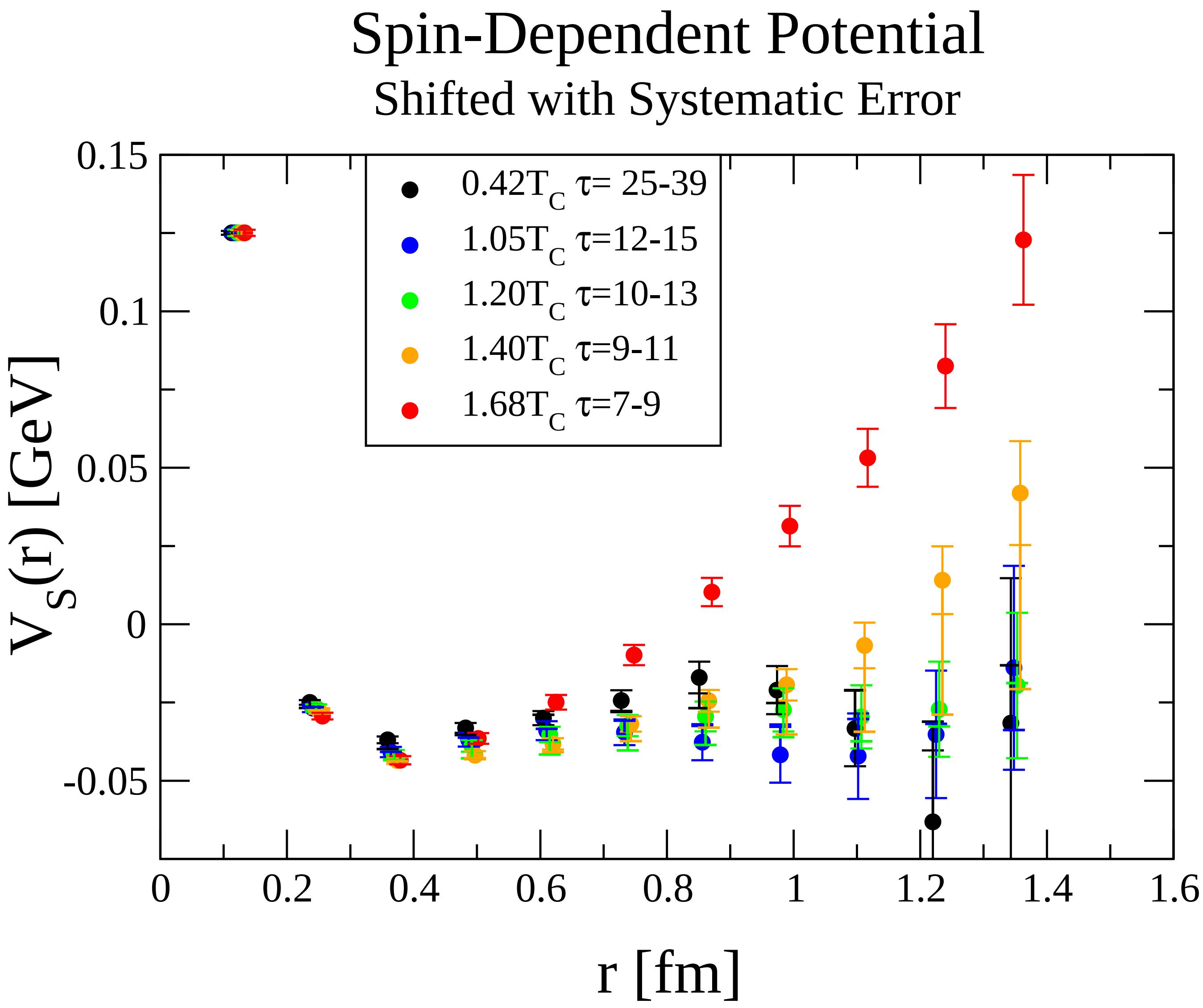
“$\tau-$dependent” Potential $1.27T_c$



Spin-Independent (i.e. Central) Potential



Spin-Dependent Potential



Summary

Electrical Conductivity

- First time the temperature dependency has been uncovered on lattice
- Results compatible with previous determinations

Inter-quark potential in charmonium at finite temperature

First time this was done with:

- relativisitc quarks rather than static quarks
- finite temperature rather than $T = 0$

EXTRA SLIDES

Lattice Parameters

	1st Generation	2nd Generation (HSC parameters)
Flavours	2	2+1
Volume(s)	(2fm)^3	(3fm)^3 & (4fm)^3
a_s [fm]	0.167	0.123
a_t [fm]	0.028	0.035
anisotropy	6	3.5
M_π / M_ρ	~ 0.55	~ 0.45
Action	Gauge: Symanzik Improved Fermion: fine-Wilson, coarse-Hamber-Wu stout-link	Gauge: Symanzik Improved Fermion: Clover, Tadpole Improved