Where’s the peak??
@Olympic National Park

Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Heavy quark master equations in the Lindblad form

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Refs:
Y.A., arXiv:1403.5783 [hep-ph]
Y.A., PRD87(2013),045016 [arXiv:1209.5068 [hep-ph]]
Y.A. and A.Rothkopf, PRD85(2012),105011 [arXiv:1110.1203 [hep-ph]]

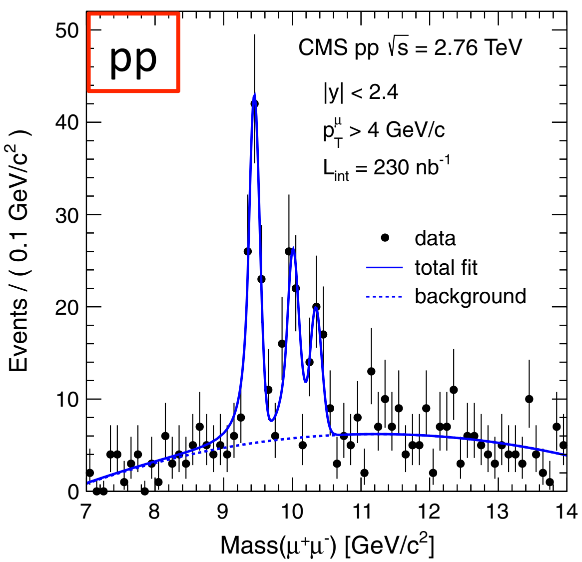
Contents

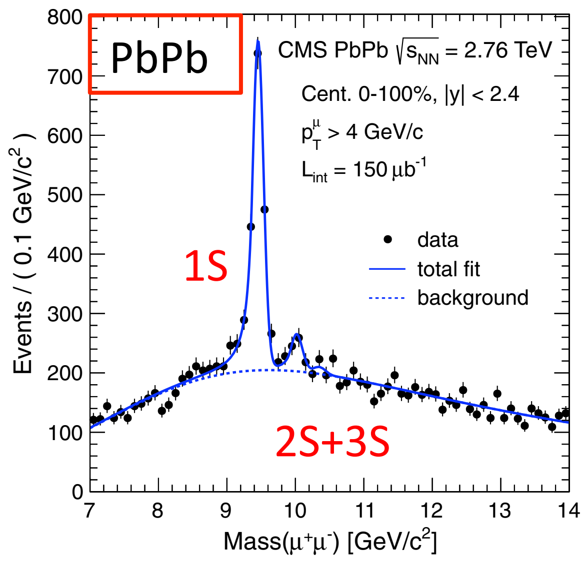
1. Introduction

- Quark-gluon plasma signal in H.I.C.?
  - Quarkonium suppression due to color screening

Matsui, Satz (86)

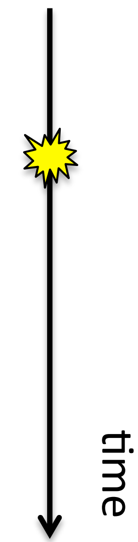
For example, CMS@LHC





More complicated in reality

- More nuclear effects
  - Nuclear wave function
  - Cold nuclear matter effect
  - Thermal QGP effect (← what we study)
  - Hadronization
  - Hadronic interaction
  - Feed-down from excited states



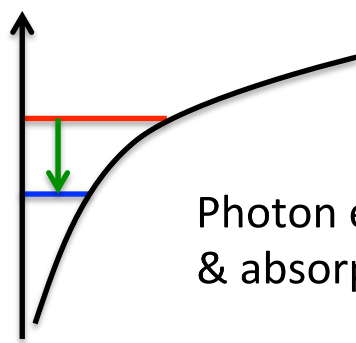
2. Open quantum systems

- System (HQs) + Environment (QGP)
  - Total Hilbert space $\left|\mathcal{H}\right\rangle = \left|qA\right\rangle \otimes \left|Q\right\rangle$
  - Reduced density matrix and master equation

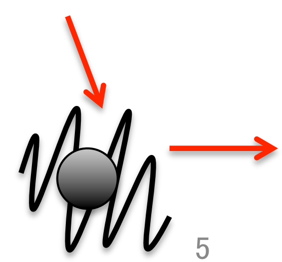
$$\rho_Q(t) \equiv \mathrm{Tr}_{qA}\left[\rho_{\mathrm{tot}}(t)\right] \qquad \dot{\rho}_Q(t) = \mathcal{L}\left[\rho_Q(t)\right]$$

Markov limit

- Examples
  - Quantum optics
  - Quantum Brownian motion



Photon emission & absorption



Scattering with medium particles

Lindblad form

$$\dot{\rho}_Q(t) = -i\left[H, \rho_Q\right] + \sum_i \gamma_i \left(L_i \rho_Q L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho_Q - \frac{1}{2} \rho_Q L_i^\dagger L_i \right) \quad \left(\gamma_i > 0\right)$$

Lindblad (76)

– General form of Markovian master equation that conserves positivity of density matrix

$$\rho_Q(t) = \sum_{n=1}^{N(t)} \omega_n(t) \left|\psi_n(t)\right\rangle \left\langle \psi_n(t)\right|$$

$$0 < \omega_n(t) \leq 1, \quad \sum_{n=1}^{N(t)} \omega_n(t) = 1$$

⇒ Positive semi-definite

Time scale hierarchies

- 3 time scales
  - Medium correlation time τ_E
  - System relaxation time τ_R ← Describe the system in this time scale
  - System intrinsic time τ_S

- 2 typical regimes of open quantum systems
  - Quantum optical limit $\tau_E << \tau_R, \quad \tau_S << \tau_R$
  - Quantum Brownian motion $\tau_E << \tau_R, \quad \tau_E << \tau_S$

Hierarchy makes things simpler

- Quantum optical limit *When energy levels are gapped*

$\tau_E << \tau_R$ → Markovian approximation

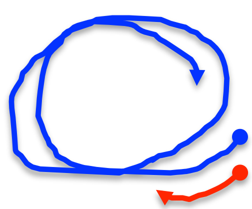
$\tau_S \equiv (\Delta E)^{-1} << \tau_R$ → Rotating wave approximation

Phase gets randomized during τ_R :
Quantum superposition → Statistical ensemble

- Quantum Brownian motion *Wave function approach*

$\tau_E << \tau_R$ → Markovian approximation

$\tau_E << \tau_S$ → Acceleration neglected



Classical motion during τ_E
Trajectories for QBM and QOL

Quarkonium time scales

- When is the QBM approach applicable?

τ_E	τ_R	τ_S
$1/gT$	$1/g^2T,\ M/g^4T^2$	$1/Mg^4$

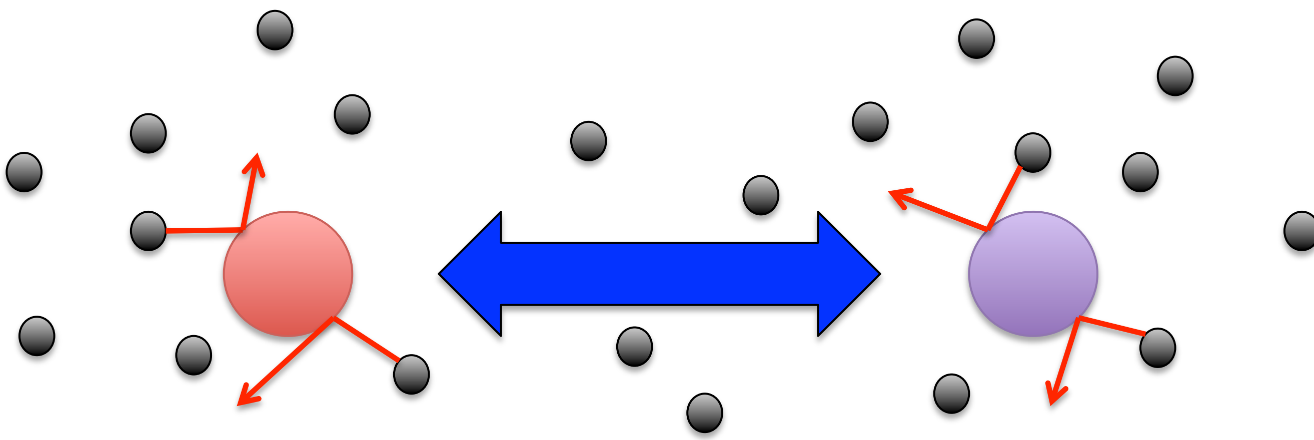
  - τ_E: electric time scale
  - τ_R: color diffusion, kinetic equilibration
  - τ_S: orbital period in Coulomb bound state

$$1/gT << (4\pi)^2/Mg^4 \rightarrow g^3/100 << T/M << 1$$

Classical picture

- Interacting Brownian particles

Young, Shuryak (09)



  - Screened force → Screening potential
  - Random force → Random potential
  - Drag force → Dissipation (non-potential)

(Screening potential + Random potential: Stochastic potential)

Quantum description

3. Influence functional for the Lindblad form

- Path-integral formulation for OQS

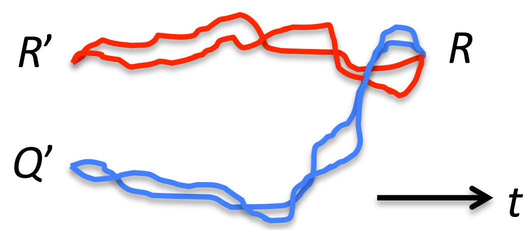
Feynman and Vernon (63)

$$S[x,R] = \int_0^t d\tau L(x,R) = S_A[x] + S_I[x,R] + S_B[R]$$

A: system, B: environment

  - Express $\Psi(x,R)$ and $\Psi^*(y,Q)$ by path integral
  - Path integrate for R and Q under each x and y trajectory (= trace over the environment)

Under each x and y



Influence functional $F[x,y]$

- Density matrix

$$\rho_{\rm red}(t,x,y) = \int dx' dy' J(t,x,y;0,x',y') \underline{\rho_{\rm sys}(0,x',y')}$$

System initial condition

- Propagator

$$J(t,x,y;0,x',y') = \int_{x',y'}^{x,y} D\tilde{x} D\tilde{y} \exp\left[\frac{i}{\hbar}\left(S_{\rm A}[\tilde{x}] - S_{\rm A}[\tilde{y}]\right)\right] F[\tilde{x},\tilde{y}]$$

- Influence functional

$$F[x,y] = \int dR' dQ' dR \underline{\rho_{\rm B}(0,R',Q')}$$

Environment initial condition

$$\times \int_{R',Q'}^{R,R} D[\tilde{R},\tilde{Q}] \exp\left[\frac{i}{\hbar}\left(S_{\rm B}[x] + S_{\rm I}[x,\tilde{R}] - S_{\rm B}[y] - S_{\rm I}[y,\tilde{Q}]\right)\right]$$

Caldeira-Leggett model

- Lagrangian

Caldeira and Leggett (83)

  – Linear coupling

$$S_{\mathrm{A}}[x] = \frac{M\dot{x}^2}{2} - v(x), \quad S_{\mathrm{B}}[\vec{R}] = \frac{m\dot{\vec{R}}^2}{2} - \sum_{k=1}^{N} \frac{m\omega_k^2 R_k^2}{2}, \quad S_{\mathrm{I}}[x,\vec{R}] = -x\sum_{k=1}^{N} C_k R_k$$

- Influence functional

α : two-point functions of environment d.o.f

$$F[x,y] = \exp\left[-\frac{i}{\hbar}\int_0^t d\tau \int_0^\tau ds \left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right] -\frac{1}{\hbar}\int_0^t d\tau \int_0^\tau ds \left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right] \right]$$

Coarse graining

$$\cong \exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau (x\dot{x} - y\dot{y} + x\dot{y} - y\dot{x}) - \frac{\eta k_{\mathrm{B}} T}{\hbar^2}\int_0^t d\tau (x-y)^2 \right]$$

NOT Lindblad form!

Diosi's prescription

- Up to 2nd order derivative in time

Diosi (93)

$$F[x,y] = \exp\left[\begin{array}{l} -\frac{i}{\hbar}\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right] \\ -\frac{1}{\hbar}\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right] \end{array} \right]$$

Lindblad!

$$\cong \exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau (x\dot{x} - y\dot{y} + x\dot{y} - y\dot{x}) - \frac{\eta k_{\mathrm{B}} T}{\hbar^2}\int_0^t d\tau (x-y)^2 \boxed{+ \frac{\hbar^2(\dot{x}-\dot{y})^2}{12(k_{\mathrm{B}}T)^2}} \right]$$

$$\dot{\rho}_{\mathrm{red}} = \frac{i}{\hbar}[H_{\mathrm{R}},\rho_{\mathrm{red}}] - \underbrace{\frac{\eta k_{\mathrm{B}} T}{\hbar^2}[x,[x,\rho_{\mathrm{red}}]]}_{\text{Fluctuation}} - \underbrace{\frac{i\eta}{2M\hbar}[x,\{p,\rho_{\mathrm{red}}\}] - \frac{\eta}{12M^2 k_{\mathrm{B}} T}[p,[p,\rho_{\mathrm{red}}]]}_{\text{Dissipation}}$$

Fluctuation

Dissipation

Heavy quarks in QGP

- Approximations
  - 1/c expansion for HQ action

$$S_{\rm int} = g\int d^4x \rho_a(x) A_a^0(x)$$

  - Perturbative expansion for influence functional

$$F[\rho_1,\rho_2] \equiv \exp\left[iS_{\rm IF}[\rho_1,\rho_2]\right]$$

G: Gluon 2-point functions

$$= \exp\left[-g^2/2\int\int \rho_1 G^{\rm F}\rho_1 + \rho_2 G^{\tilde{\rm F}}\rho_2 - \rho_1 G^{>}\rho_2 - \rho_2 G^{<}\rho_1 + \cdots\right]$$

Real functions $\quad -g^2 G^R_{00,ab}(\omega=0,\vec{r}) \equiv V(\vec{r})\delta_{ab}, \quad -g^2 G^{>}_{00,ab}(\omega=0,\vec{r}) \equiv D(\vec{r})\delta_{ab}$

  - Coarse graining up to 2nd order derivative in time

Influence functional for HQs

- x in CL model ⇔ Color density ρ^a

$$S^{\rm IF}[\rho_1,\rho_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a,\rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})}$$

Complex potential
= Screened potential + fluctuation (→ Stochastic potential)

Lindblad form

$$-\frac{1}{4T}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a,\rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & -D \\ D & D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

Momentum dissipation

$$+\frac{i}{24T^2}\int_{t,\vec{x},\vec{y}} \left(\dot{\rho}_1^a,\dot{\rho}_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & D \\ D & -D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

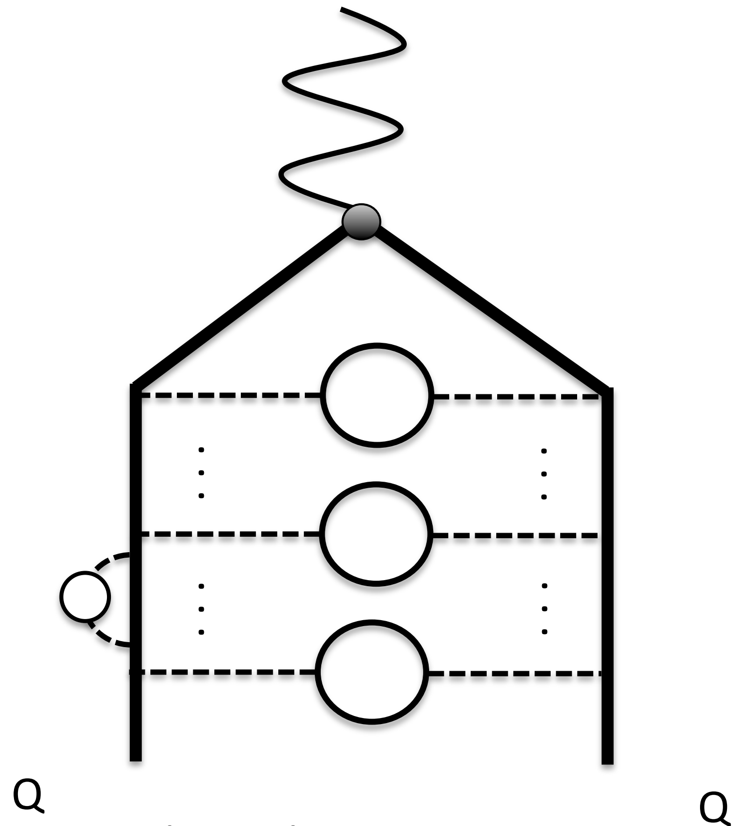
Momentum dissipation

Lindblad form

*For resultant master equations, see my paper in the reference (2014)

Physical process

- Color density interaction



Independent scatterings

Interaction + self energy
- Screening
- Fluctuation
- Dissipation

4. Stochastic potential with color

- Screened potential and fluctuation

$$S^{\mathrm{IF}}[\rho_1,\rho_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a,\rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})}$$

↓ Rewrite using Gaussian noise with nonlocal correlation

$$e^{iS_{\mathrm{IF}}} = \exp\left[-\frac{i}{2}\int_{t,\vec{x},\vec{y}} \boxed{V(\vec{x}-\vec{y})}\rho_1^a(t,\vec{x})\rho_2^a(t,\vec{y})\right]$$

Stochastic potential

$$\times\left\langle \exp\left[-i\int_{t,\vec{x}} \boxed{\xi^a(t,\vec{x})}\left(\rho_1^a(t,\vec{x})-\rho_2^a(t,\vec{x})\right)\right]\right\rangle_\xi$$

$$\left\langle \xi^a(t,\vec{x})\xi^b(s,\vec{y})\right\rangle = -D(\vec{x}-\vec{y})\delta^{ab}\delta(t-s)$$

(D: Negative definite)

QM in stochastic potential

- Stochastic Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi(t,\vec{r}) = \left[\begin{array}{l} -\frac{\nabla_r^2}{M} + iC_{\rm F}D(0) + \left(V(r) + i\,D(r)/2\right)\left[t^a \otimes (-t^{*a})\right] \\ +\xi^a(t,\vec{r}/2)\left[t^a \otimes 1\right] + \xi^a(t,-\vec{r}/2)\left[1 \otimes (-t^{*a})\right] \end{array}\right]\Psi(t,\vec{r})$$

$$(\Psi : 3 \otimes 3^*, \quad \vec{r} = \vec{x} - \vec{y})$$

  - Color rotation by fluctuation : singlet ⇄ octet
  - Typical correlation length of fluctuation : $l_{\rm fluct} \sim 1/gT$
  - Bound state size : $l_{\rm coh}$ (coherence length)

Decoherence time scales

- Color projected density matrix

$$\rho_{1,8}(t,\vec{r},\vec{s}) \equiv \mathrm{Tr}_{\mathrm{color}}\left[P_{\substack{\mathrm{singlet,}\\ \mathrm{octet}}} \left\langle \Psi(t,\vec{r})\Psi^*(t,\vec{s}) \right\rangle_\xi \right]$$

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix}_{(t,\vec{r},\vec{s})} = \cdots + \mathcal{D}(\vec{r},\vec{s})\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix}_{(t,\vec{r},\vec{s})}$$

Decoherence time scale can be estimated using *D*(*r*,*s*)

– Decoherence takes place

- $t_{\mathrm{dec}} \sim 1/g^2T$ ($l_{\mathrm{coh}} >> l_{\mathrm{fluct}}$) ← Soft scattering time interval
- $t_{\mathrm{dec}} \sim 1/g^4T^3 l_{\mathrm{coh}}^2$ ($l_{\mathrm{coh}} << l_{\mathrm{fluct}}$) ← Takes longer time

4. Summary & outlook

- Quarkonium in QGP as open quantum systems: Influence functional and Lindblad form
- Finite-temperature potential model consists not only of screened potential but also of fluctuation (stochastic potential)
- Decoherence time scale depends on bound state size
- Phenomenology: Stochastic Schrödinger equation on hydro background

Backup slides

Subtle issues (personal views)

- Time ordering in coarse graining

Later Earlier

$$-\frac{i}{\hbar}\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right] \cong -\frac{i\eta}{2\hbar}\int_0^t d\tau\,(x\dot{x} - y\dot{y} + x\dot{y} - y\dot{x})$$

Later Earlier

$$-\frac{1}{\hbar}\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right] \cong -\frac{\eta k_{\mathrm{B}} T}{\hbar^2}\int_0^t d\tau\left[(x-y)^2 + \frac{\hbar^2(\dot{x}-\dot{y})^2}{12(k_{\mathrm{B}}T)^2}\right]$$

CL model Diosi's term

- Time differentiation in coarse graining
  - Should not be treated as one of the kinetic terms
  - If treated so, master equation does not change in CL model while it becomes different with Diosi's term.

Subtle issues (personal views)

- Integration by parts?

$$F[x,y]=\exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})-\frac{\eta k_{\mathrm{B}}T}{\hbar^2}\int_0^t d\tau(x-y)^2\right]$$

$$\text{or}\ \ \tilde{F}[x,y]=\exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(-\dot{x}x+\dot{y}y-\dot{x}y+\dot{y}x)-\frac{\eta k_{\mathrm{B}}T}{\hbar^2}\int_0^t d\tau(x-y)^2\right]$$

$$\Rightarrow \tilde{\rho}_{\mathrm{red}}=\exp\left[i\eta\hat{x}^2/2\right]\rho_{\mathrm{red}}\exp\left[-i\eta\hat{x}^2/2\right]$$

  - Influence functional should be determined without ambiguity with total derivative terms.

(partial) gauge invariance

- Gauge transformation (local only in time)

$$\because S_{\text{int}} = g\int d^4x\rho_a(x)A_a^0(x)$$

  – Density matrix

$$\rho_Q(t,\vec{x},\vec{y}) \to \tilde{\rho}_Q(t,\vec{x},\vec{y}) = U(t)\rho_Q(t,\vec{x},\vec{y})U^{-1}(t)$$

$$\dot{\rho}_Q = \mathcal{L}\left[\rho_Q\right] \to \dot{\tilde{\rho}}_Q = U\mathcal{L}\left[\rho_Q\right]U^{-1} + \left(\dot{U}U^{-1}(t)\tilde{\rho}_Q + \tilde{\rho}_Q U\dot{U}^{-1}\right)$$

  – Physical observable (singlet)

$$\left\langle O_{\text{singlet}}\right\rangle_\rho(t) = \int d^3xd^3y\text{Tr}_{\text{color}}\left[\rho_Q(t,\vec{x},\vec{y})\left\langle\vec{y}\middle|O_{\text{singlet}}\middle|\vec{x}\right\rangle\right]$$

$$\frac{d}{dt}\left[\left\langle O_{\text{singlet}}\right\rangle_\rho(t) - \left\langle O_{\text{singlet}}\right\rangle_{\tilde{\rho}}(t)\right] = 0 \quad \because \dot{U}(t)U^{-1}(t) + U(t)\dot{U}^{-1}(t) = 0$$

U and singlet observable *O* commute

Color singlet and octets

- Projected density matrix

$$\rho_1(t,\vec{r},\vec{s}) = \mathrm{Tr}_{\mathrm{color}}\left[P_1\rho_{QQ_c}(t,\vec{r},\vec{s})\right], \quad \rho_8(t,\vec{r},\vec{s}) = \mathrm{Tr}_{\mathrm{color}}\left[P_8\rho_{QQ_c}(t,\vec{r},\vec{s})\right]$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_1\\ \rho_8\end{pmatrix}_{(t,\vec{r},\vec{s})} = i\frac{\nabla_r^2-\nabla_s^2}{M}\begin{pmatrix}\rho_1\\ \rho_8\end{pmatrix} + i\left(V(\vec{r})-V(\vec{s})\right)\begin{bmatrix}C_\mathrm{F} & 0\\ 0 & -1/2N_\mathrm{c}\end{bmatrix}\begin{pmatrix}\rho_1\\ \rho_8\end{pmatrix} + \mathcal{D}(\vec{r},\vec{s})\begin{pmatrix}\rho_1\\ \rho_8\end{pmatrix}$$

Kinetic | Potential | Decoherence

$$\mathcal{D}(\vec{r},\vec{s}) = 2C_\mathrm{F}D(\vec{0}) - \left(D(\vec{r})+D(\vec{s})\right)\begin{bmatrix}C_\mathrm{F} & 0\\ 0 & -1/2N_\mathrm{c}\end{bmatrix}$$

$$-2D(\vec{r}-\vec{s})\begin{bmatrix}0 & 1/2N_\mathrm{c}\\ C_\mathrm{F} & C_\mathrm{F}-1/2N_\mathrm{c}\end{bmatrix} + 2D(\vec{r}+\vec{s})\begin{bmatrix}0 & 1/2N_\mathrm{c}\\ C_\mathrm{F} & -1/N_\mathrm{c}\end{bmatrix}$$

Color singlet and octets are mixed by decoherence.