High-Order Order Methods for General-Relativistic Hydrodynamics

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MNRAS 437:L46 (2013), CQG 31:075012 (2014)

Contents

1. Gravitational Waves from Binary Neutron Stars
2. High-Order Methods for GRHD
3. Binary Neutron Stars and Conclusions

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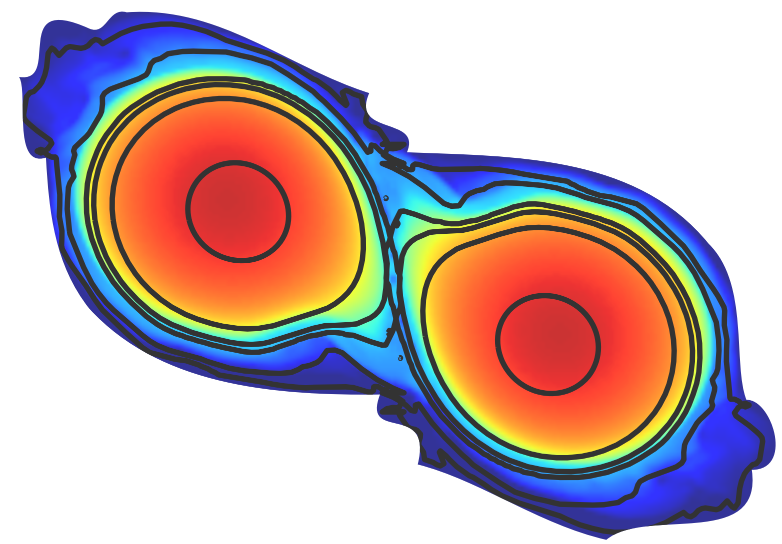
Binary Neutron Stars



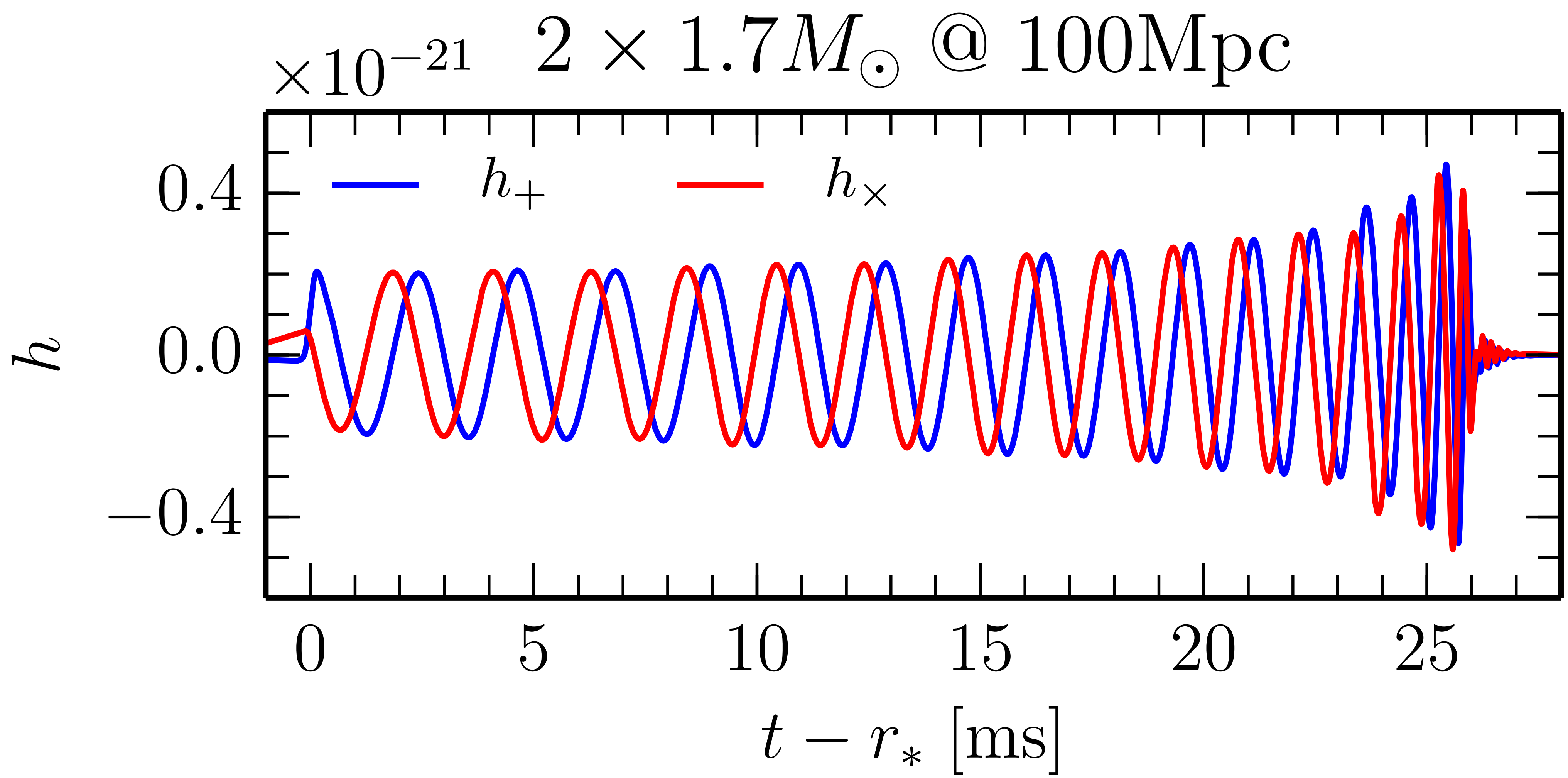
Motivations

- Gravitational waves
- Short gamma ray burst

Dynamics

- Inspiral
- Merger
- Hypermassive NS?
- Black-hole + torus
- Ultra-relativistic jet?

Gravitational Waves Astronomy



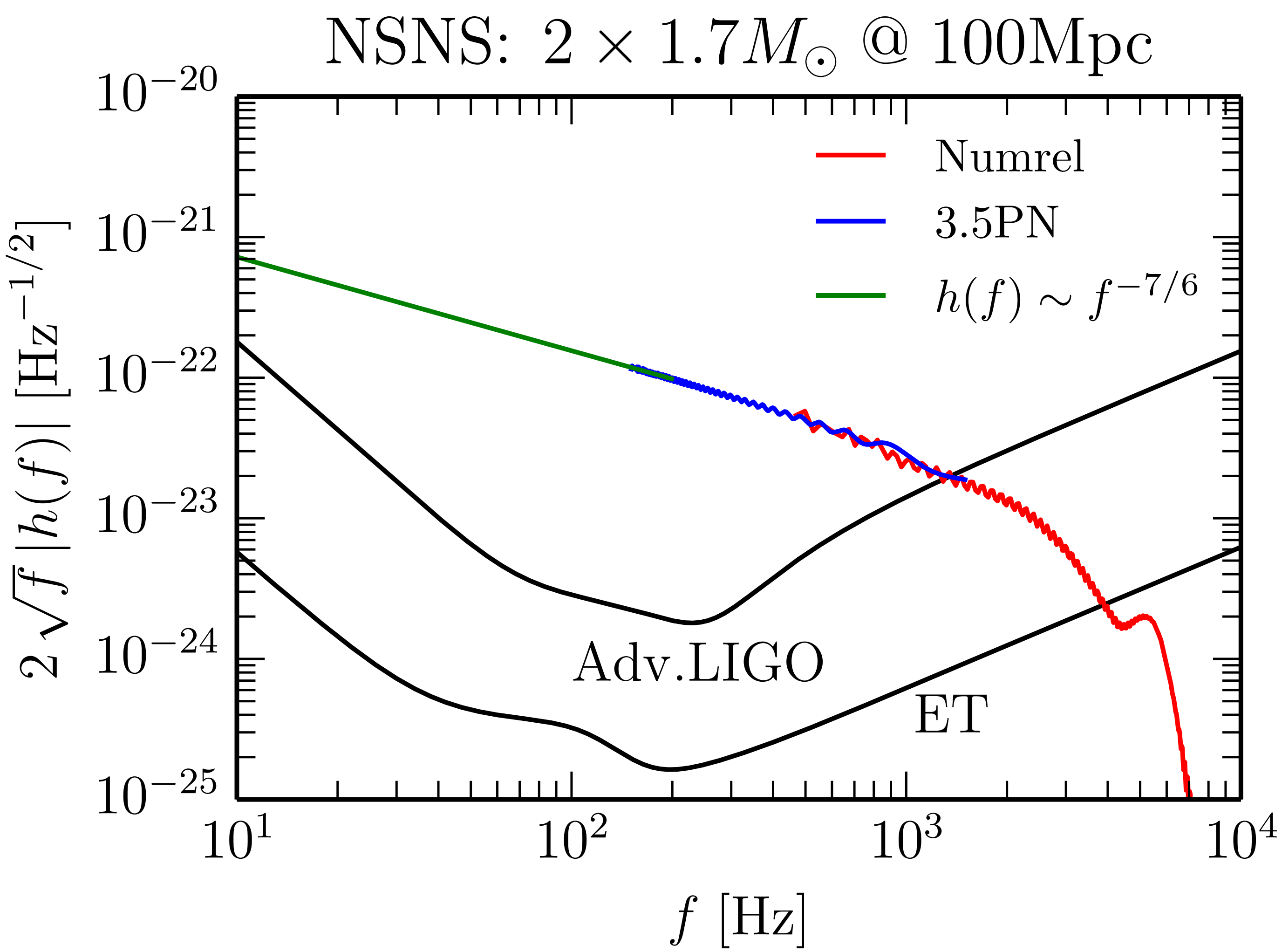
Physics with GWs

- Mass
- Radius
- Equation of state
- …

Challenges

- NSNS mergers are rare
- Signals are weak

GWs from BNS

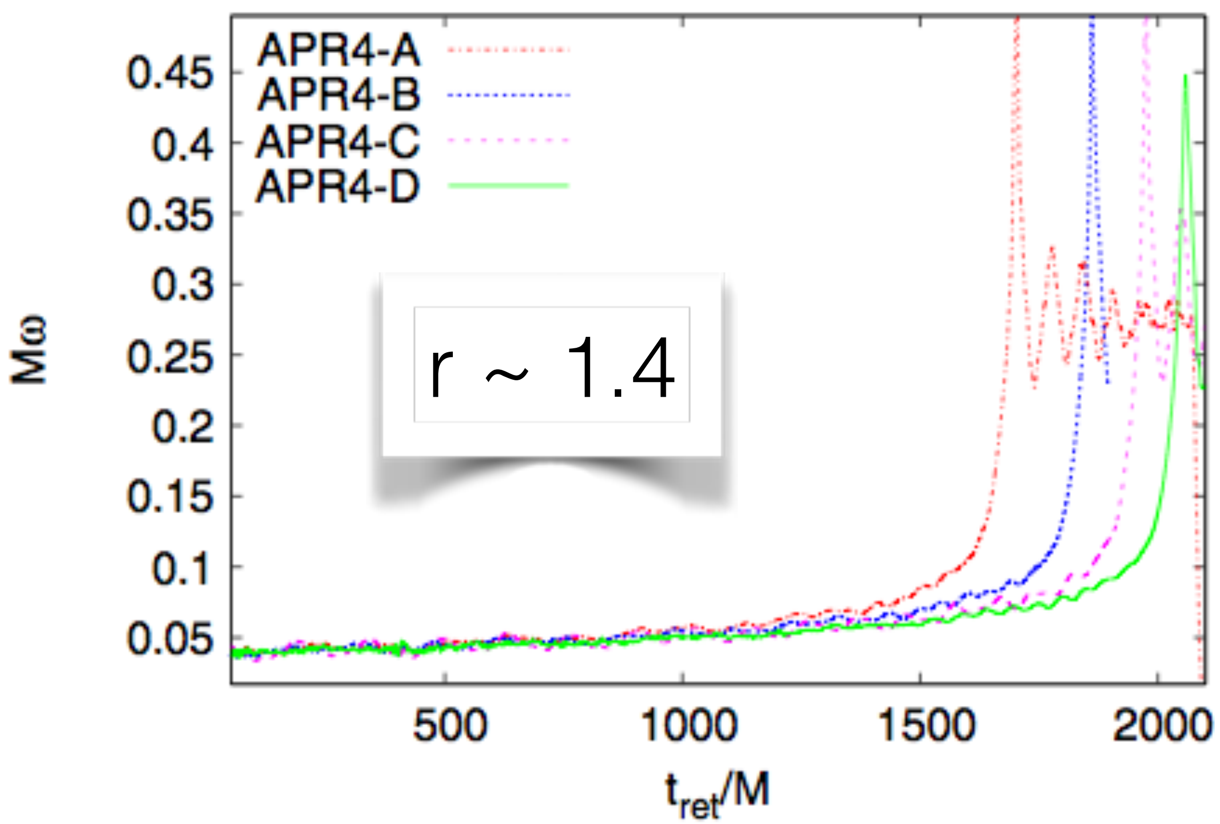


- **Early inspiral:** approximate analytic waveforms
- **Inspiral:** post-Newtonian and effective one-body
- **Late-inspiral and merger:** numerical relativity

Synergy between analytic and numerical relativity

The Need for Higher Accuracy

Problem: phase errors!



From Hotokezaka et al. 2013

Issues

- Phase evolution: most critical quantity
- Numerical inaccuracy hard to quantify
- Increasing the resolution not a solution:

$$\text{Error} \sim h^2$$

but:

$$\text{Costs} \sim h^{-4}$$

Need higher order methods!

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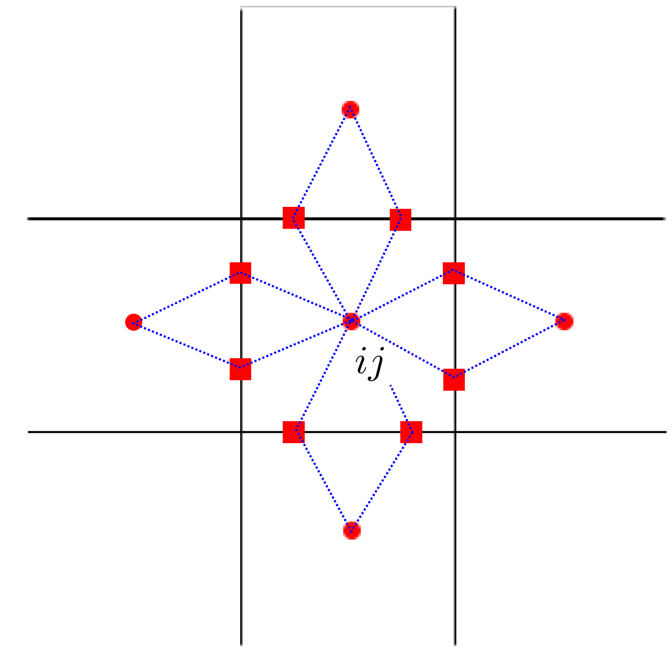
High-Order HRSC Schemes

Finite Volumes

- Complex to implement
- Large comp. costs
- Conservative
- General grids

$$\frac{\mathrm{d}\langle \mathbf{U}\rangle_{ij}}{\mathrm{d}t} = -\frac{1}{V_{ij}}\int_{\partial V_{ij}} \mathbf{F}\cdot \mathrm{d}\mathbf{S}$$

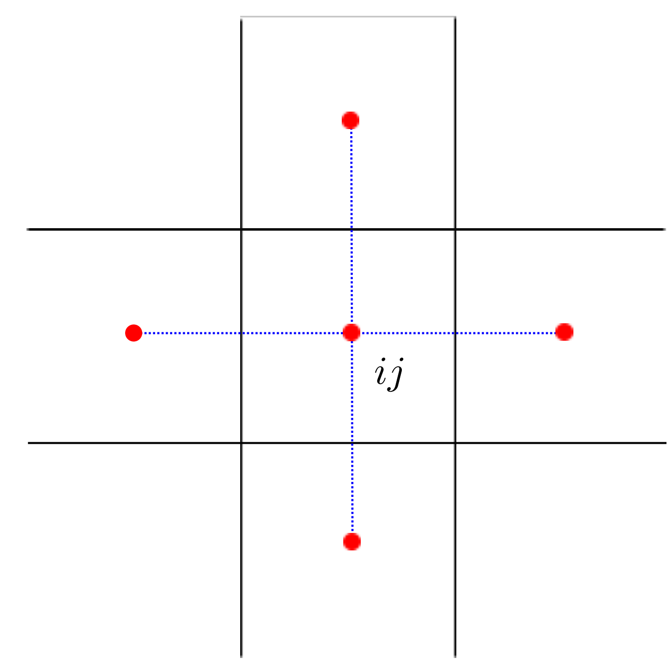


$$\partial_t \mathbf{u} + \nabla \cdot \mathbf{f}(\mathbf{u}) = 0$$

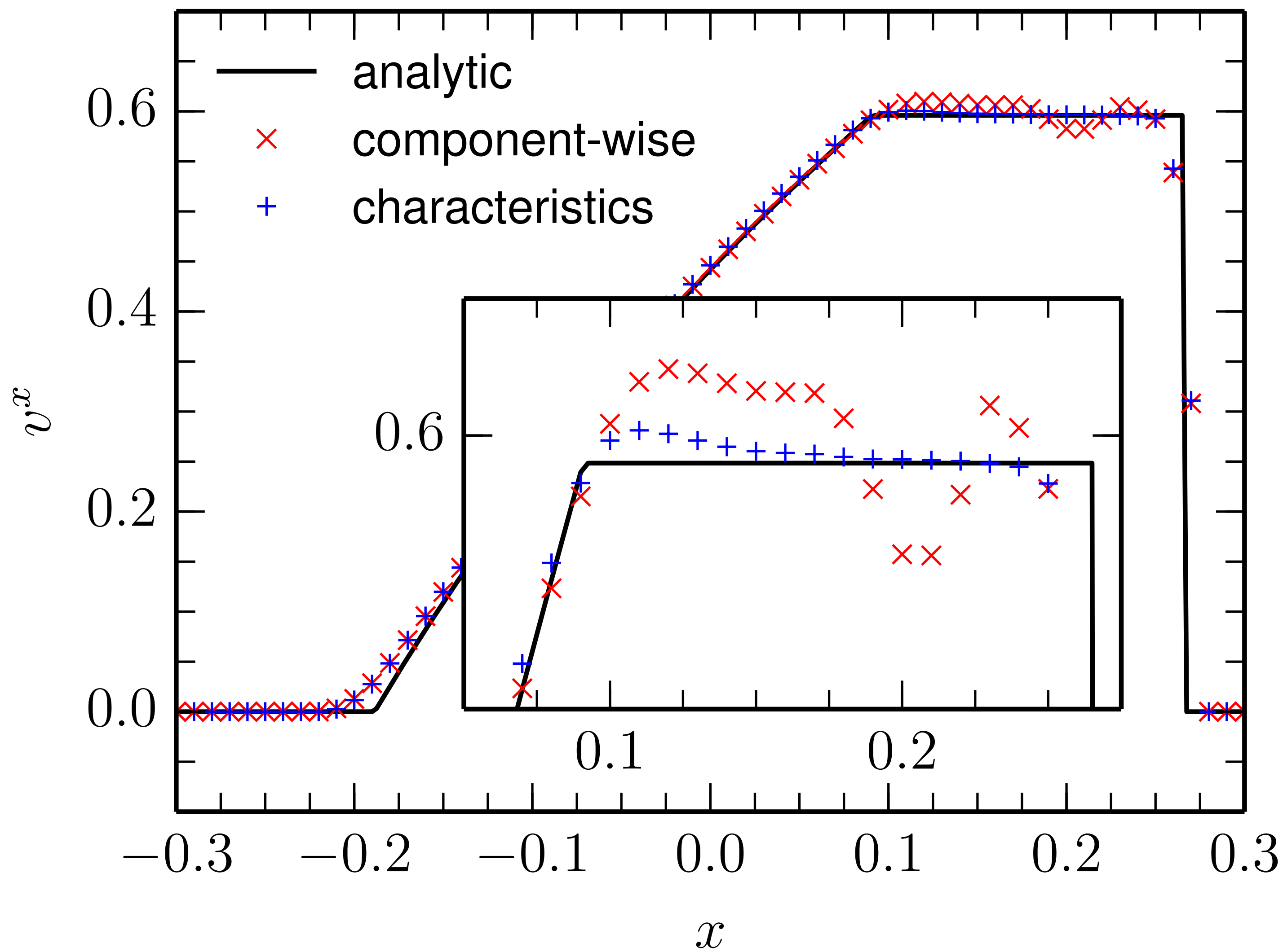
Finite Differences

- Simple to implement
- Low comp. costs
- Discrete conservation
- Tensor product grids

$$\frac{\mathrm{d}\mathbf{U}_{ij}}{\mathrm{d}t} = -\left[D\cdot\mathbf{F}\right]_{ij}$$



Characteristic Decomposition

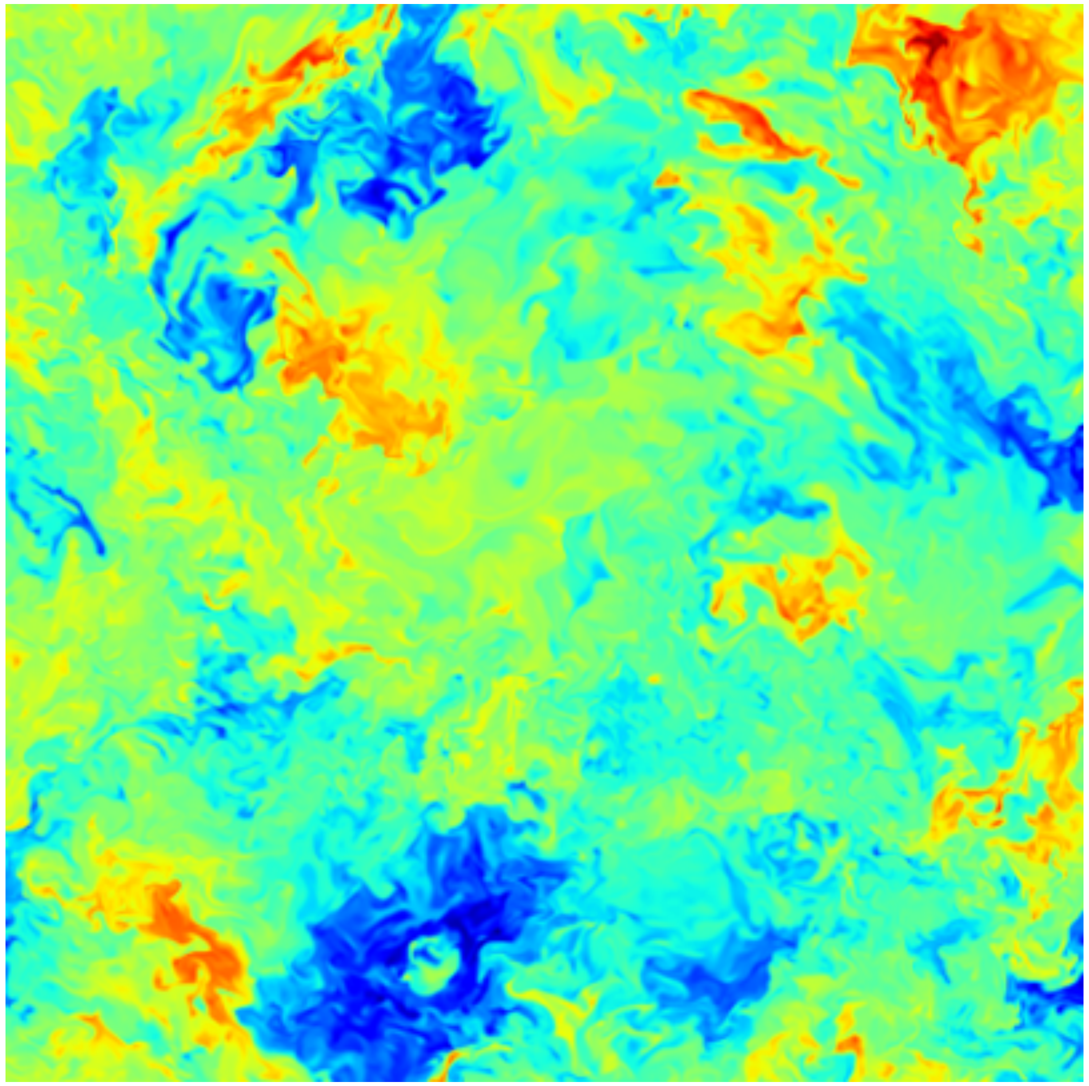


- **What?** Reconstruct fluxes using characteristic variables
- **Why?** Avoid post-shock oscillations
- **Issues**: costs, extension to nuclear hot EOS
- Really needed for binary neutron stars?!? No, but…

Extra equation(s): $u^\mu \partial_\mu Y_e = \dot{Y}_e \implies$ different eigenvalues/vectors!

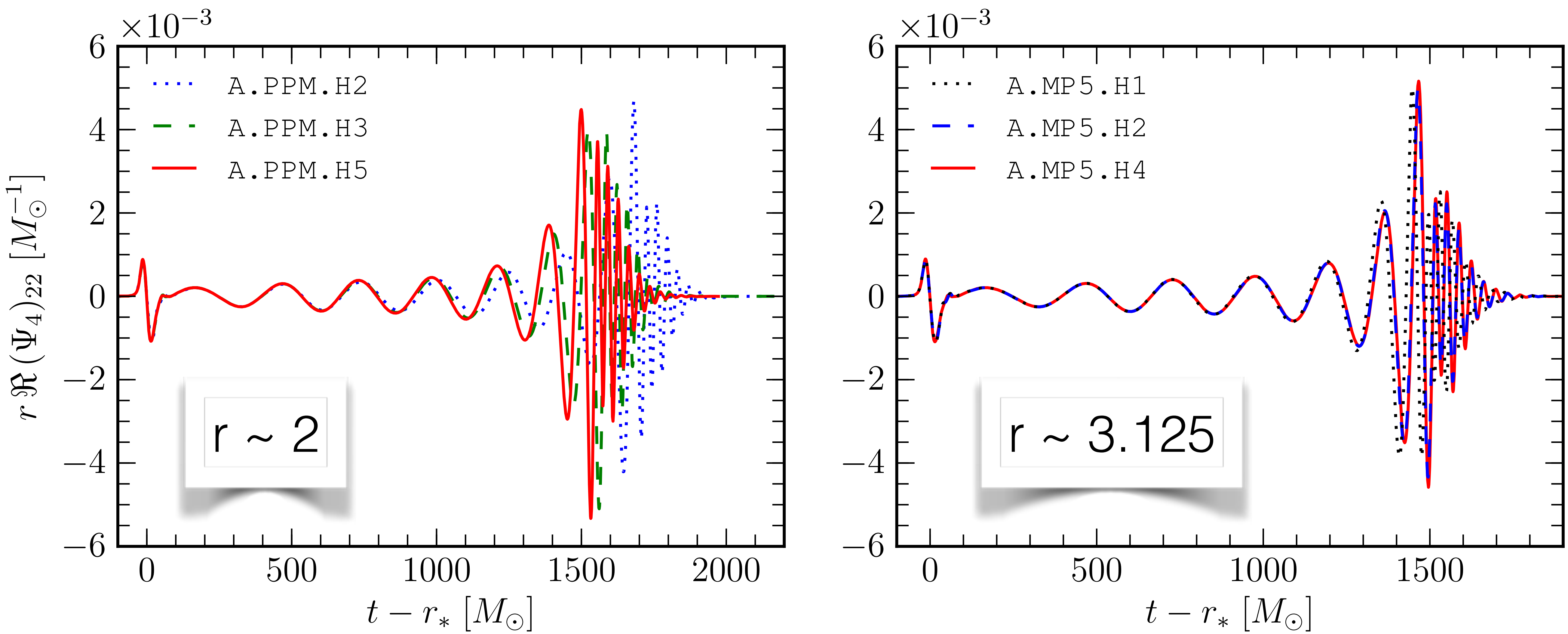
- Use decomposition for the full system? Not known (to me).
- Use approximate decomposition? $\dfrac{\partial p}{\partial Y_e} = 0 \ \ldots$

WhiskyTHC



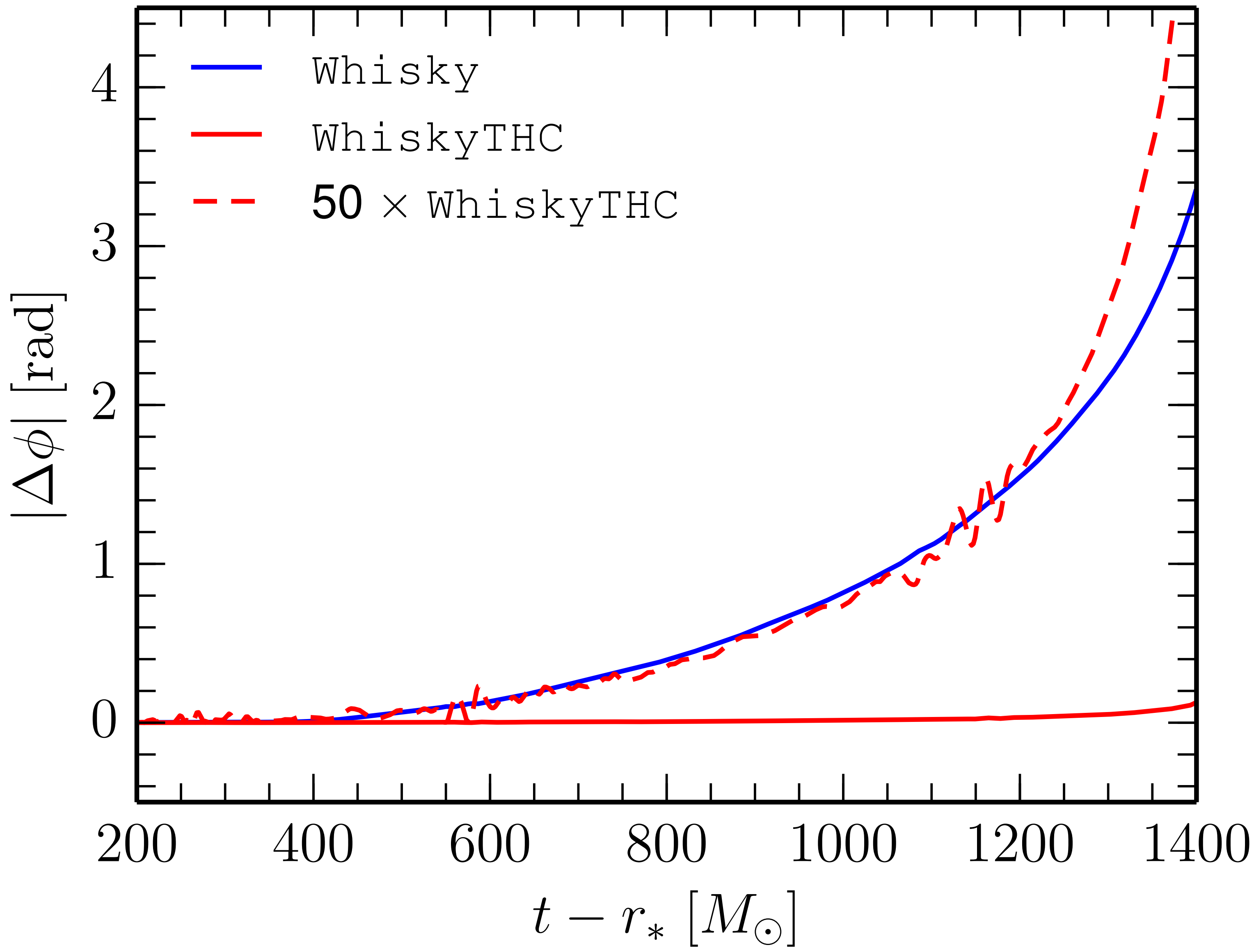
- A new GRHD code
- Spacetime: FD, BSSN
- Nuclear equation of state
- High-order HRSC FD (MP5 + Roe / LF flux-split)
- Central FV+HLLE
- In progress: discontinuous Galerkin methods and FP_N neutrino radiation

2nd Order vs High Order (I)



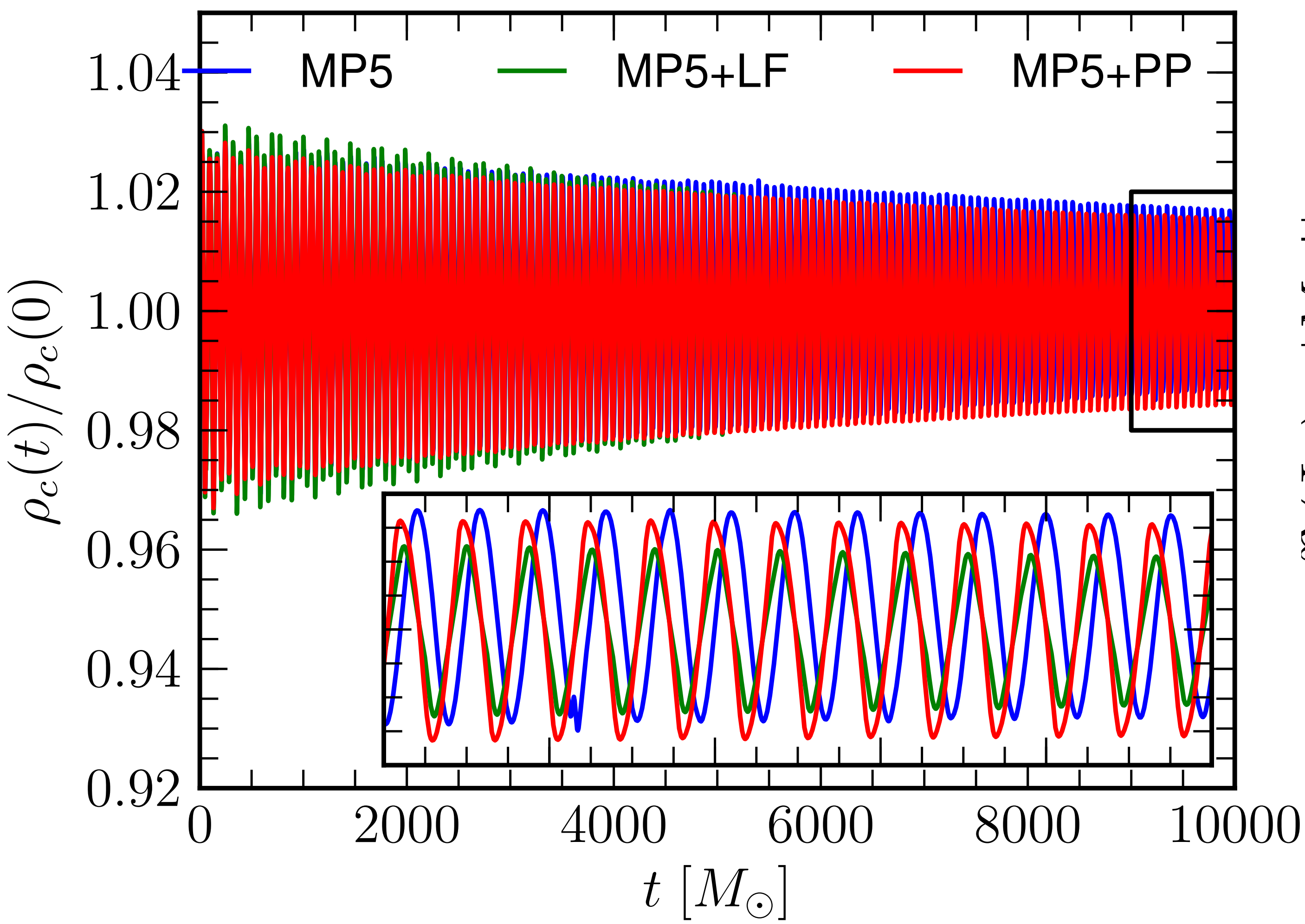
- Better accuracy at lower resolutions
- Smaller de-phasing between different resolutions

2nd Order vs High Order (II)

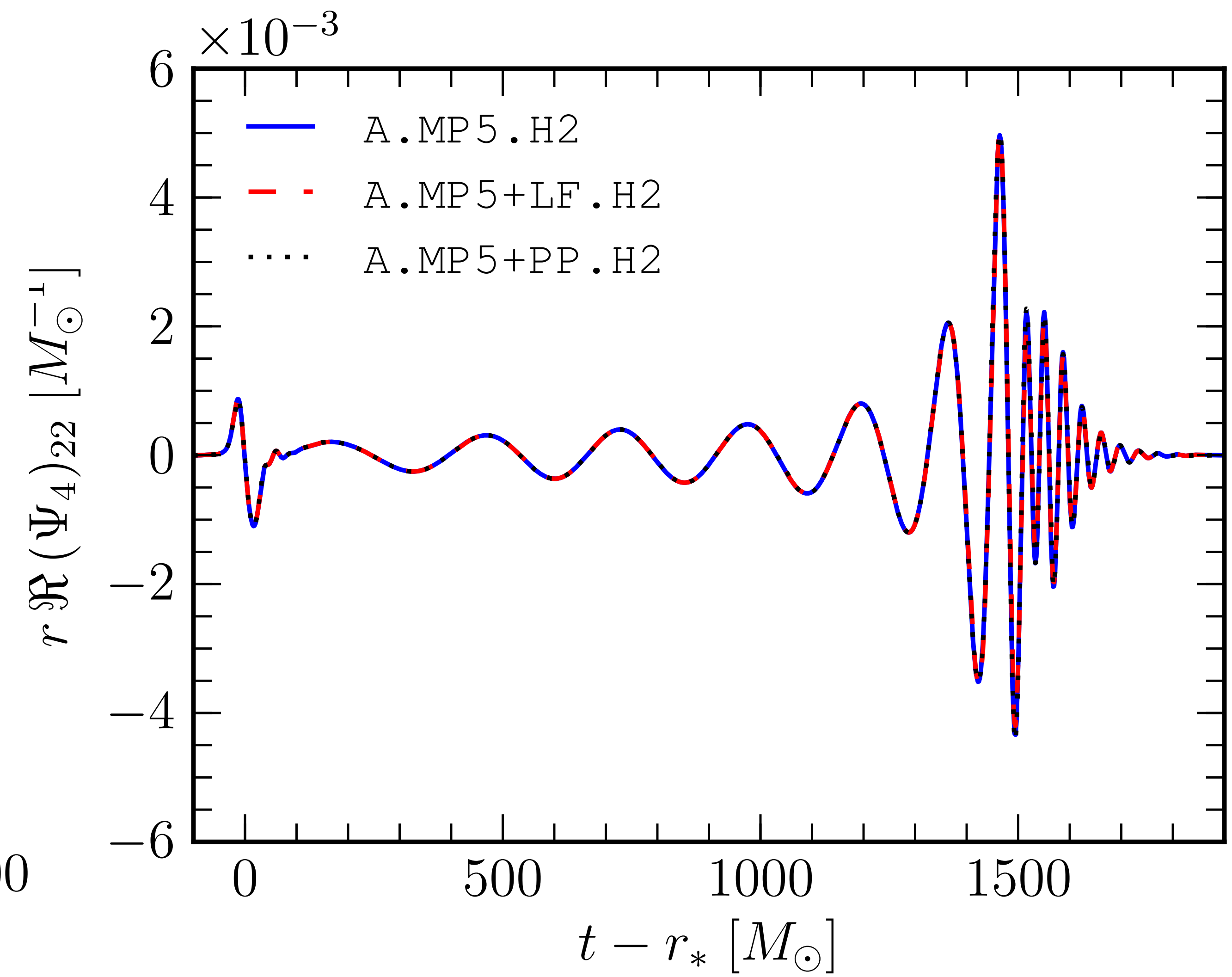


Gain a factor ~50 in phase accuracy at moderate resolution

“Atmosphere” Treatment



TOV: Cowling approximation



Binary neutron stars

Positivity Preserving limiter: code more robust, still not the final solution

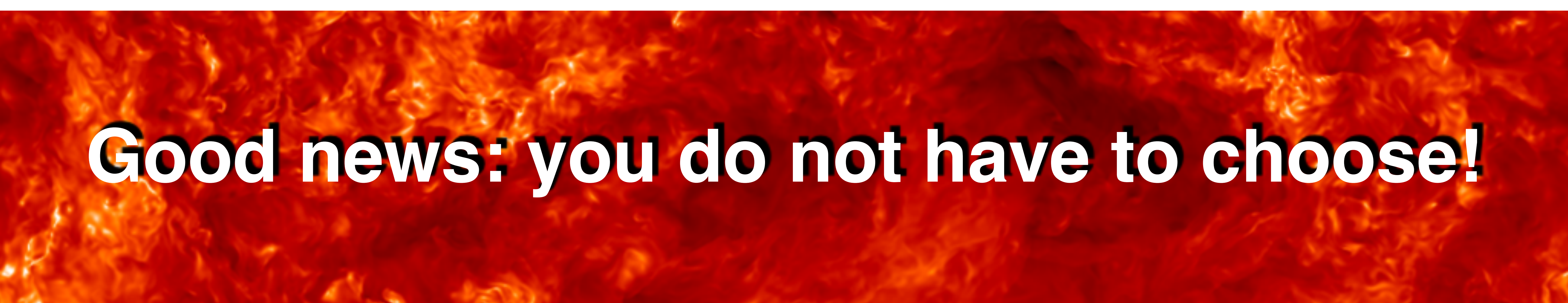
2nd Order vs High Order (III)

2nd Order

- Already available
- Simple and robust
- Efficient
- Good for "messy" situations: shocks, atmosphere, post-merger, …

Higher Order

- Simple to implement
- Complex or less robust
- Cost-competitive
- Good for "clean" situations: inspiral, turbulence, …
- Quantitative results



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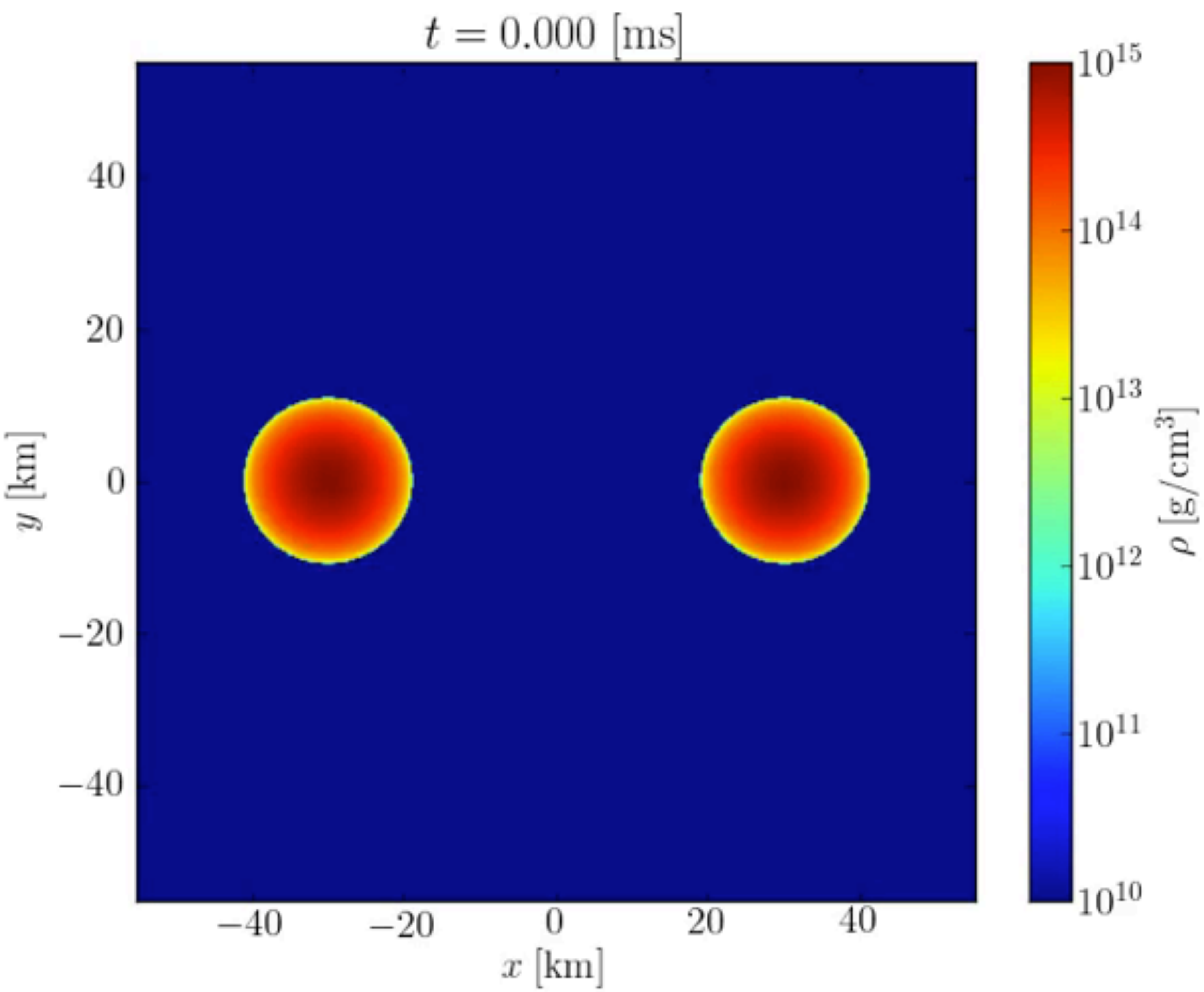
NSNS: Overview



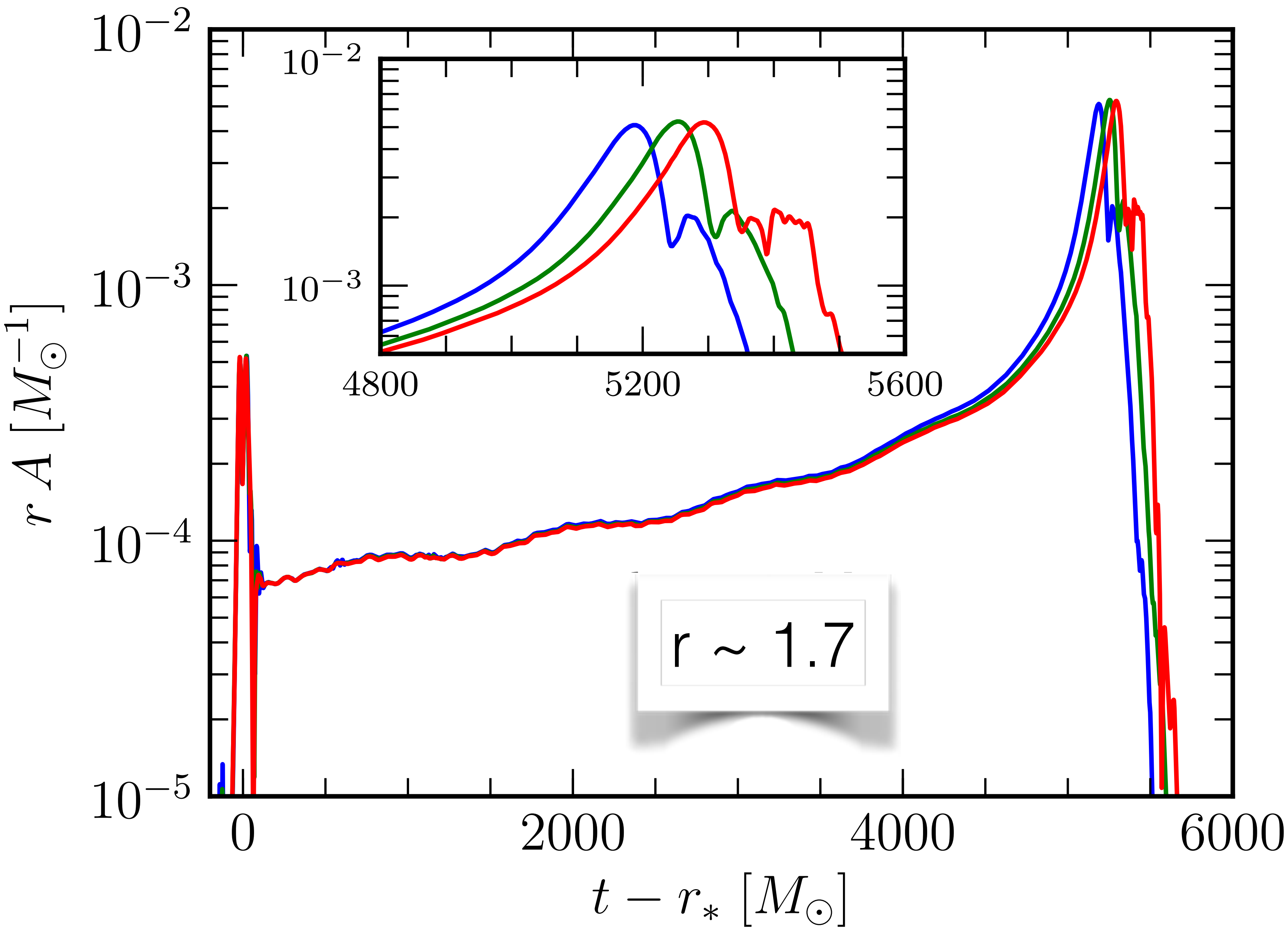
Initial data

- Quasi-circular orbit
- $p = (\Gamma - 1)\rho\epsilon, \ \Gamma = 2$
- $M/R = 0.18$
- $M/2 = 1.72\, M_\odot$
- Separation 60 km

Evolution

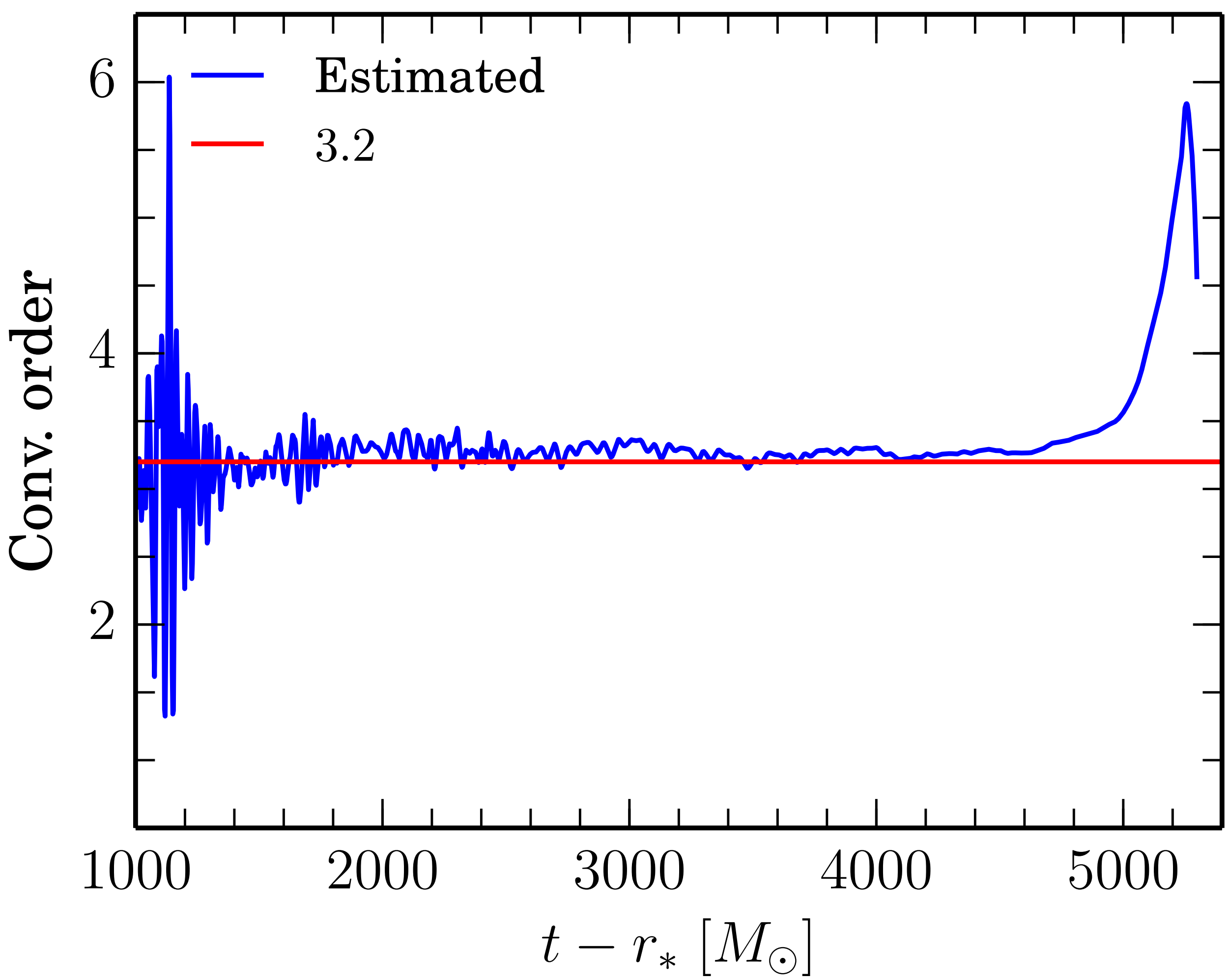
- 6.5 orbits up to contact
- 16 GW cycles

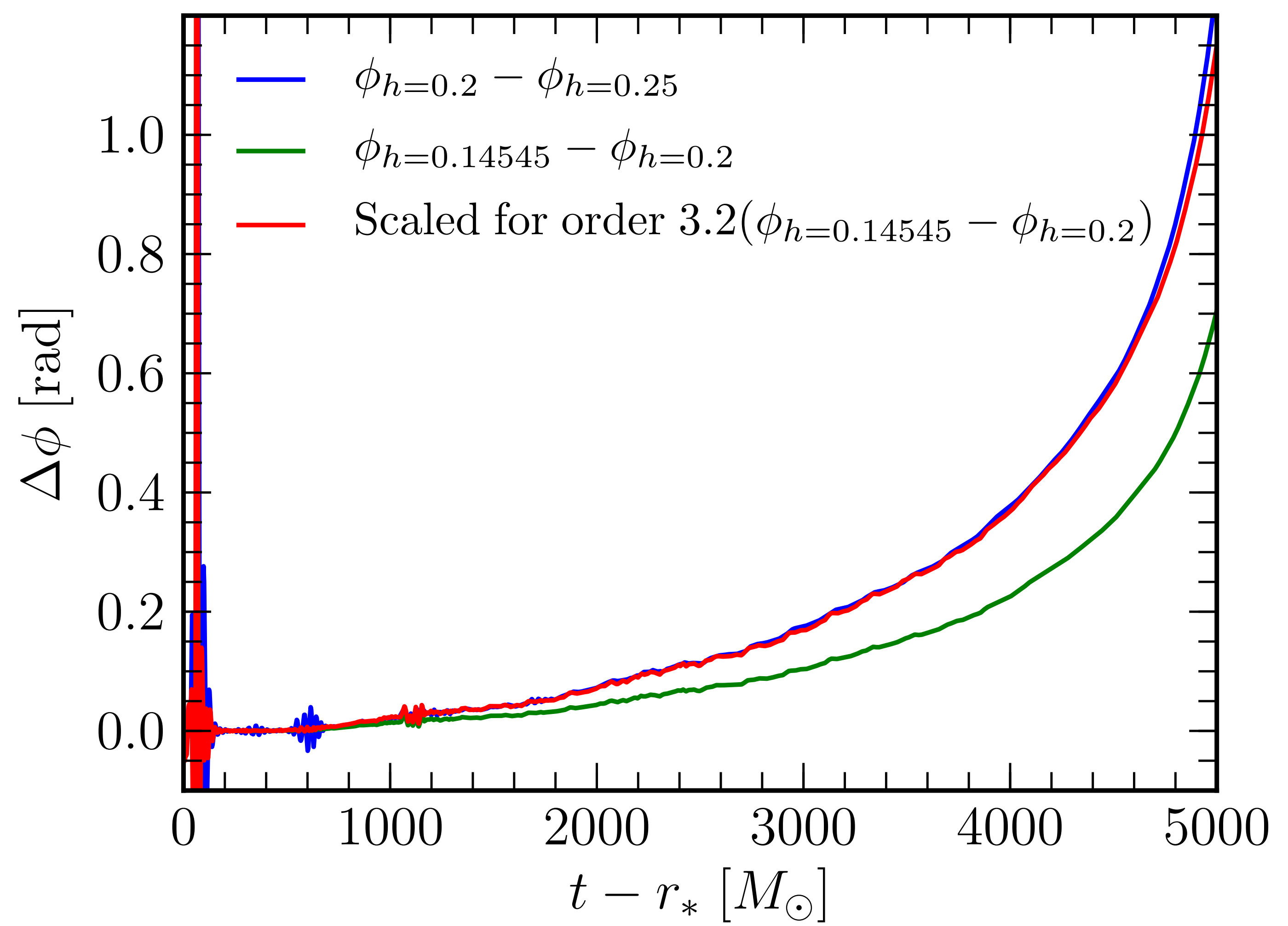
Gravitational Waves



- Merger time consistent between different resolutions
- Main source of error is now the initial data

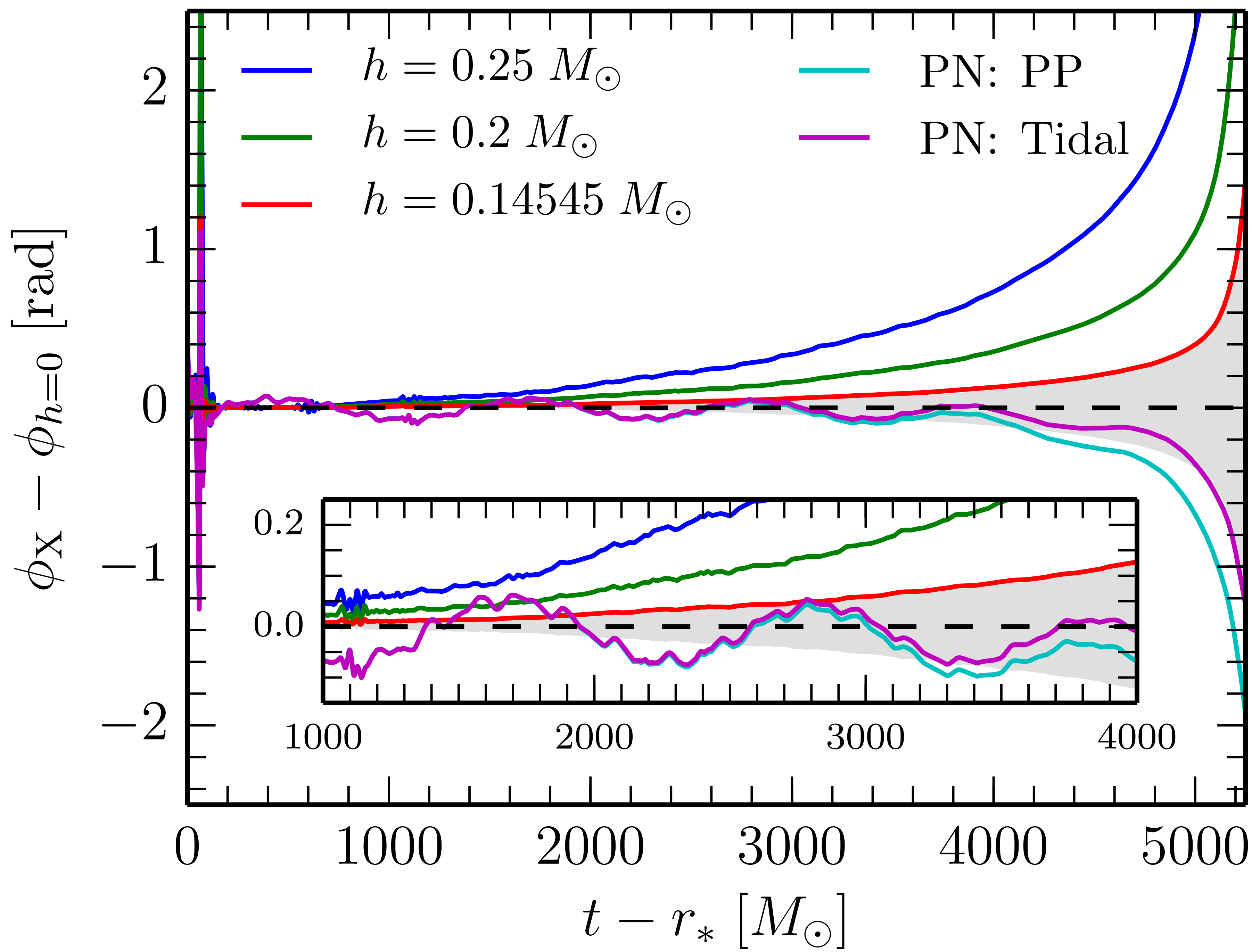
Convergence





- “Clean” convergence at ~ 3rd order until contact
- No need for alignment / rescaling of the waveforms

Comparison with Analytic Models



Very good agreement with PN theory up to contact

Conclusions

- The accuracy of numrel GWs for NSNS greatly improved with higher-order methods
- WhiskyTHC: first higher-order GRHD code
- Tidally corrected Taylor-T4 PN found to be accurate up to contact*

* at least for compact binaries