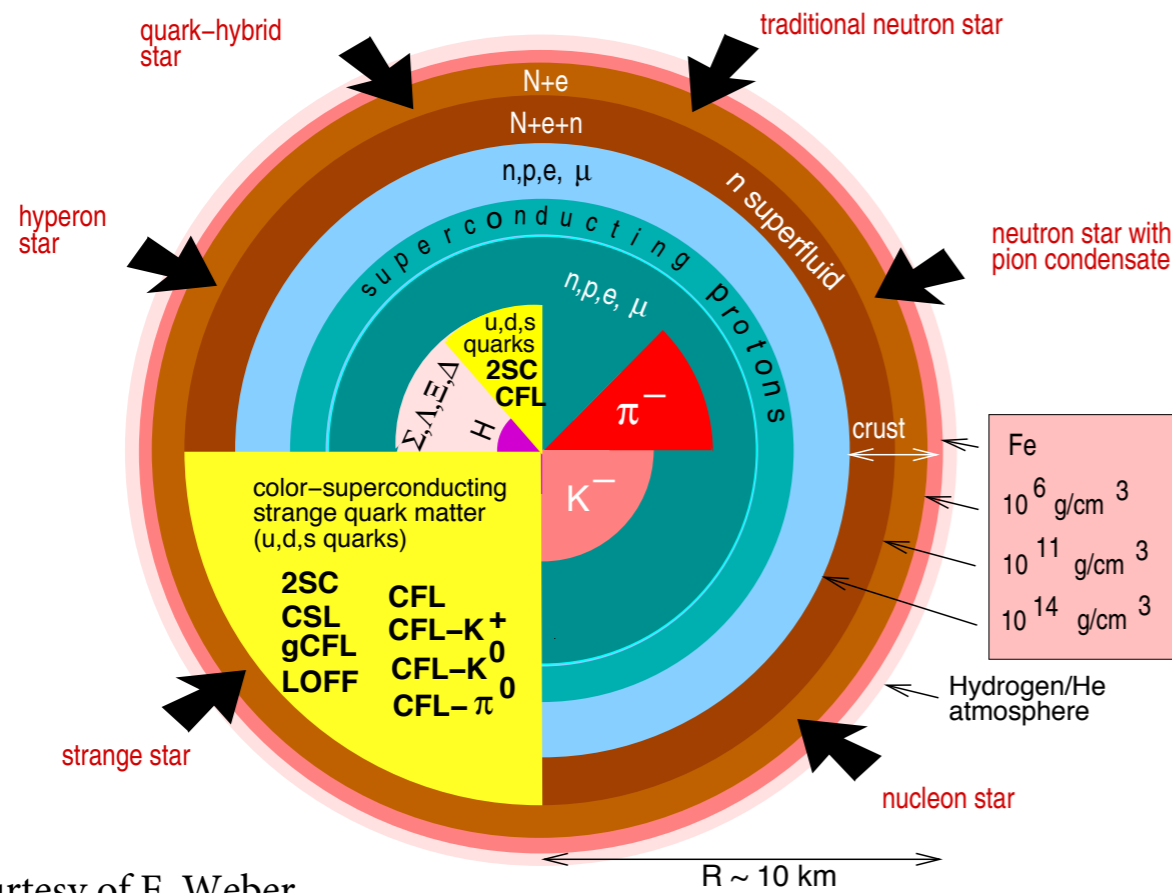


Neutron Star Seismology with Accreting Millisecond Pulsars

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University of Maryland
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T. Strohmayer & S. Mahmoodifar, ApJ 784, 72 (2014) [arXiv:1310.5147 [astro-ph.HE]]
T. Strohmayer & S. Mahmoodifar, Submitted to ApJL (2014)

Compact Stars: The only “laboratory” for the study of cold ultra-dense matter



Courtesy of F. Weber

They may contain exotic forms of matter that are predicted to exist at supranuclear densities.

Dynamic properties of NSs, such as their spin and temperature evolution as well as **stellar oscillations can be powerful probes of NS interiors.**

- **Mass-Radius** depends on the **EoS**
- **Transport properties** (e.g. emissivity, viscosity (shear, bulk), conductivity (electrical, thermal), . . .) depend on the **low energy degrees of freedom**

Low-energy degrees of freedom are very different for different phases, therefore **transport properties** can discriminate between different phases **more efficiently.**

Non-radial oscillations of neutron stars

Pressure modes (p-modes and f-modes): frequencies in the **1000-10 000 Hz** range. Powered by internal pressure fluctuations.

Shear-dominated modes (t-modes and s-modes): frequencies **larger than ~30 Hz**. The overtones in the **kHz range**.

Gravity modes (g-modes): frequencies in the **1-100 Hz** range (in the slow rotation limit), buoyancy as their restoring force.

Inertial modes: Appear in a rotating star. The restoring force of the pulsations is provided by the Coriolis force. They have both significant toroidal (axial) and spheroidal (polar) angular displacements.

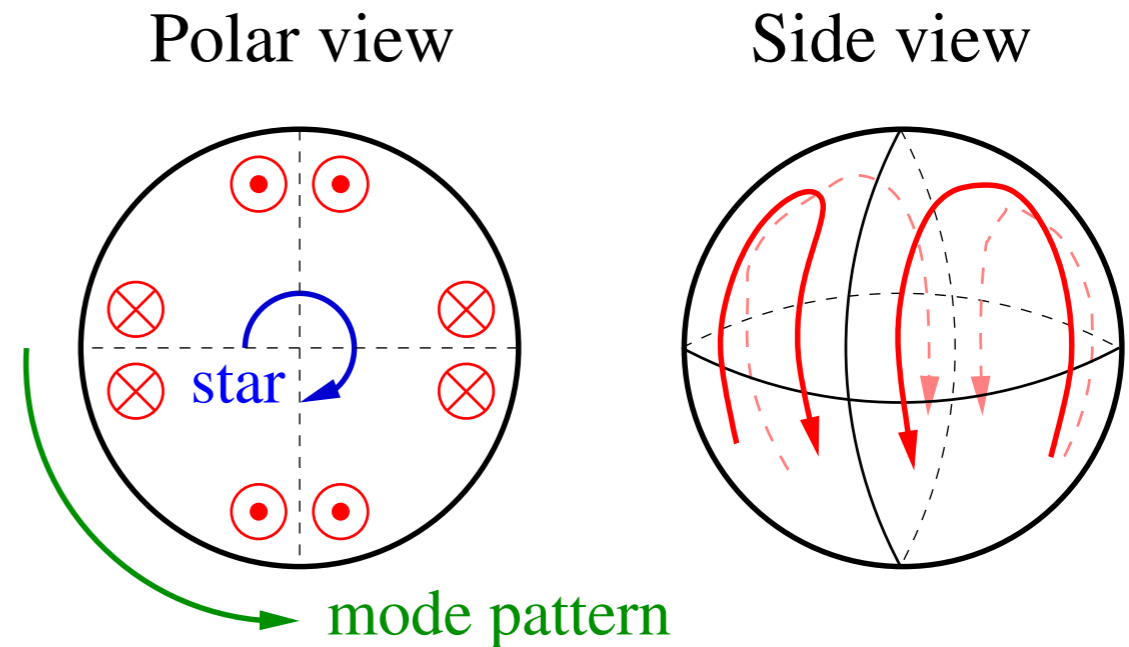
r-modes: A well-known subset of the inertial modes. They are principally toroidal (axial). The pressure and energy density perturbations are small compared to the velocity perturbations.

$$\omega_r = 2m\Omega / (l(l + 1))$$

r-mode oscillations of neutron stars

- Retrograde in the co-rotating frame of the star and prograde according to a distant observer --> they emit positive angular momentum in the form of gravitational waves but since they have negative angular momentum in the co-rotating frame of the star, their amplitude grows due to emission of gravitational radiation.

- Spin-down torque on the star
- Modes damped by viscous processes (shear, bulk), depends on phases of dense matter.
- Viscous damping heats the star.



$$\delta\vec{v} = \alpha\Omega R(r/R)^m \vec{r} \times \vec{\nabla} Y_{mm}(\theta, \phi) + O(\Omega^3),$$

$$\omega_r = \left[\frac{2}{(m+1)} - m \right] \Omega + O(\Omega^3).$$

Damping timescale

The amplitude of the r-mode evolves as $e^{i\omega t - \frac{t}{\tau}}$

$$\frac{1}{\tau} = \frac{-1}{|\tau_{GR}|} + \frac{1}{\tau_S} + \frac{1}{\tau_B}$$

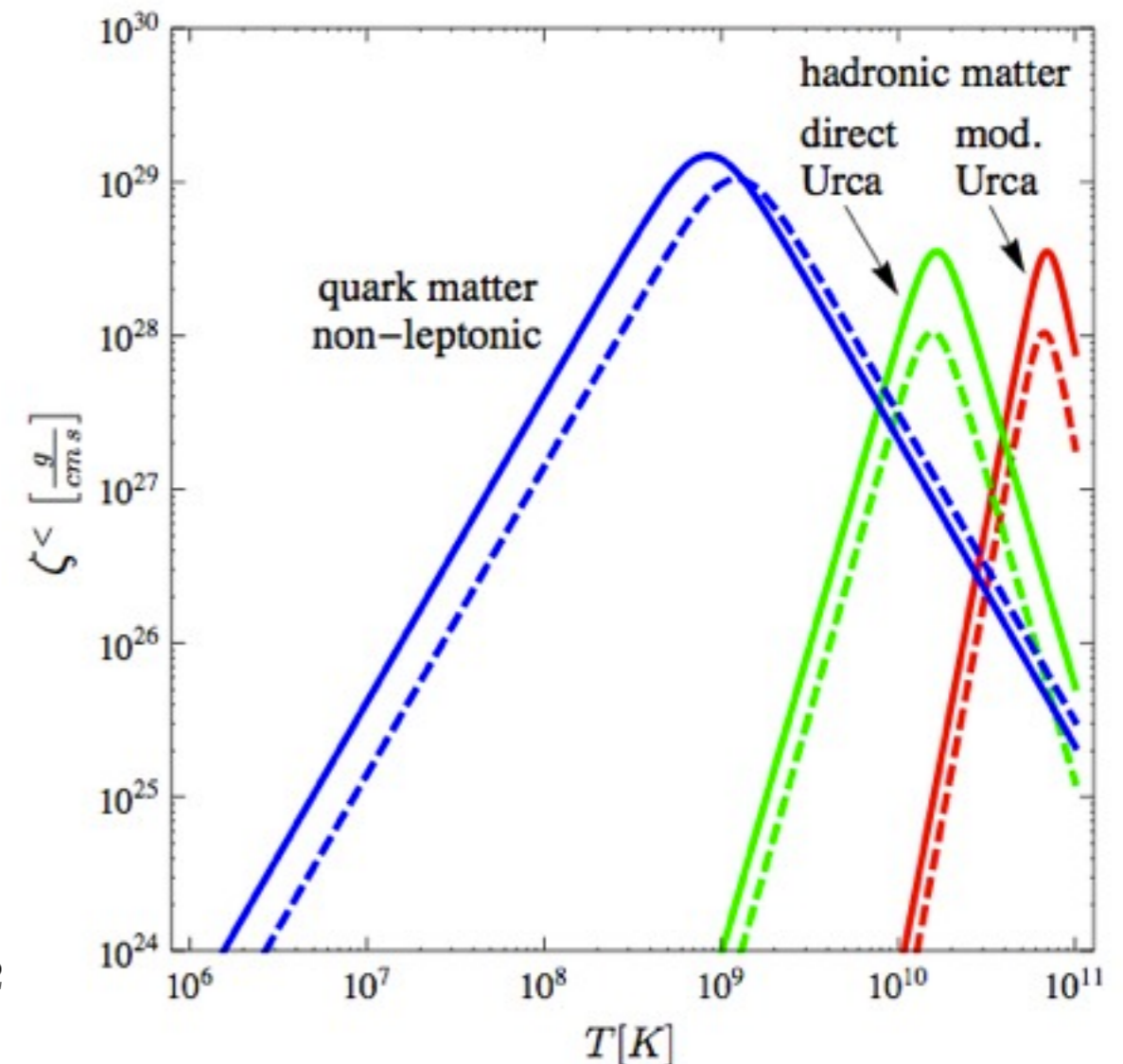
τ_{GR} : Gravitational radiation timescale

τ_S : Shear viscosity timescale

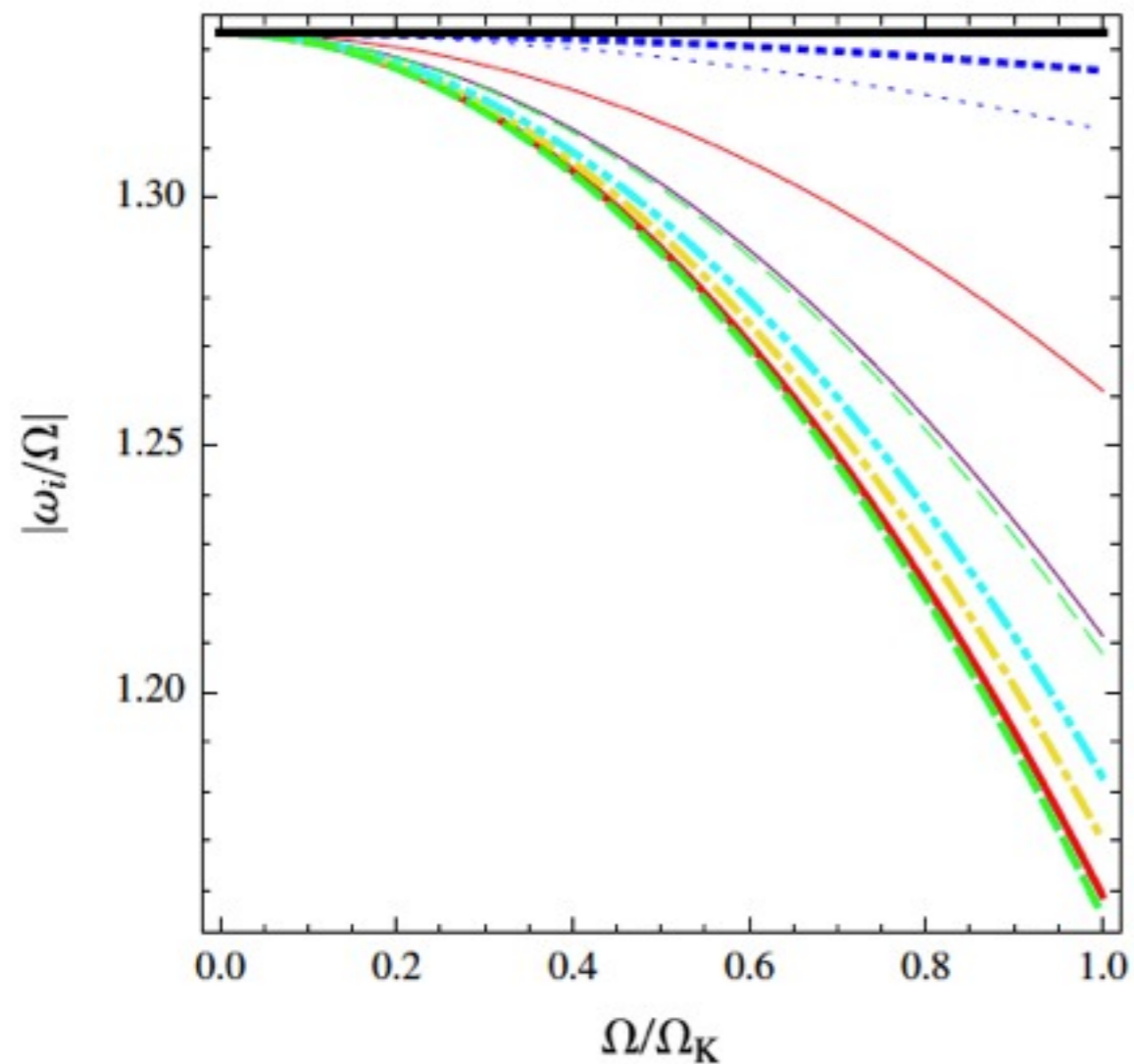
τ_B : Bulk viscosity timescale

The fastest process dominates!

R-mode is **unstable** as long as its growth time is shorter than the damping time due to the viscosity, i.e. when the damping timescale τ is **negative**.



Oscillation frequency



$$\omega_i = m\Omega - \omega_r$$

$$\omega_r \equiv \omega = \kappa(\Omega)\Omega$$

$$\kappa = \kappa_0 + \kappa_2 \frac{\Omega^2}{\pi G \bar{\rho}_0} + \dots$$

$$\kappa_0 = 2/(m+1)$$

Alford, Mahmoodifar, Schwenzer, Phys.Rev.D 85, 024007

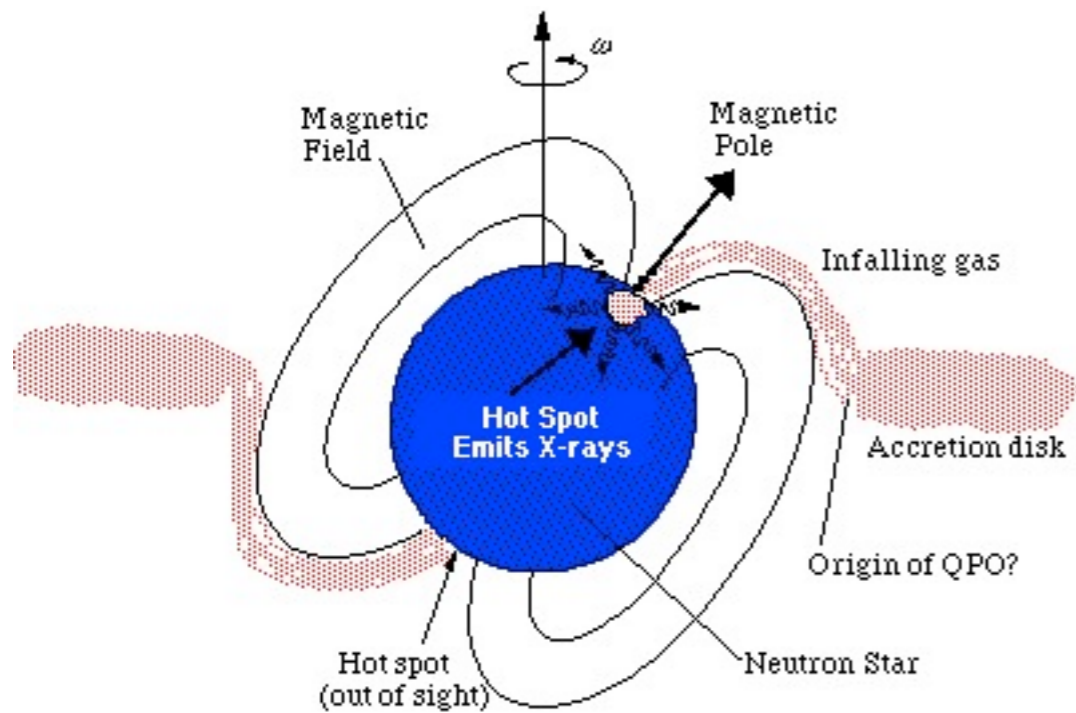
The observations of **global oscillations** of neutron stars can provide a powerful **probe of their interior properties**, similar to helioseismology.

How the presence of non-radial oscillations might be inferred from observations?

- 1) Pulsation modes can **modulate the temperature distribution** across the neutron star surface, coupled with spin can produce flux modulation at mode's **inertial frame** frequency.
- 2) Surface motions induced by a particular oscillation mode can **perturb the X-ray emitting hot-spot**.
 - Since hot-spot rotates with the star, the modulation frequency seen by a distant observer is the **co-rotating frame** frequency.
 - **Most relevant for quasi-toroidal modes** (such as the r- and g-modes) in which the dominant motions are transverse.

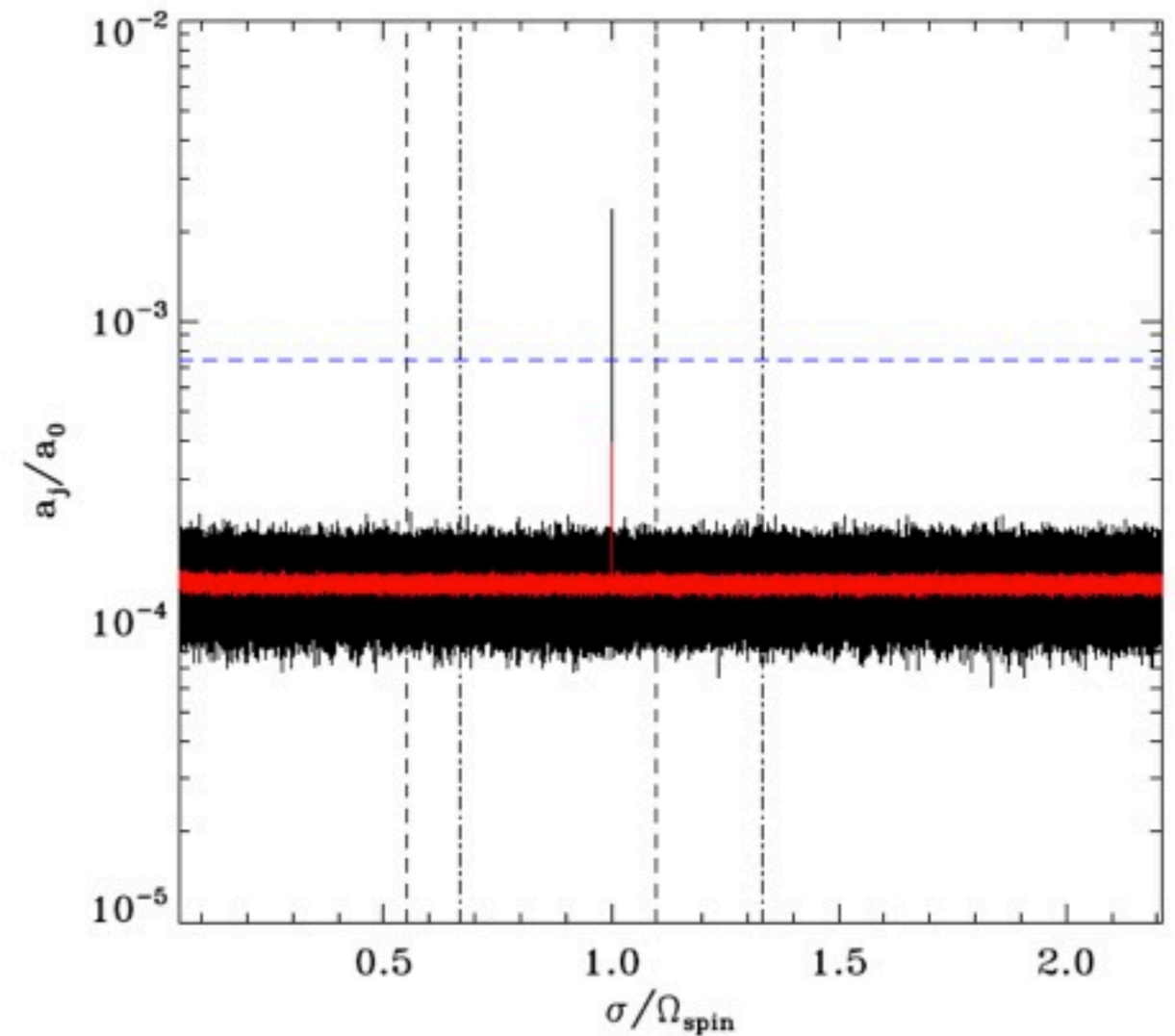
Accreting Millisecond X-ray Pulsars

Accretion stream close to spin axis \rightarrow X-ray hot-spot
Spin modulation \rightarrow pulsations
(Lamb et al. 2009, Patruno & Watts 2011)

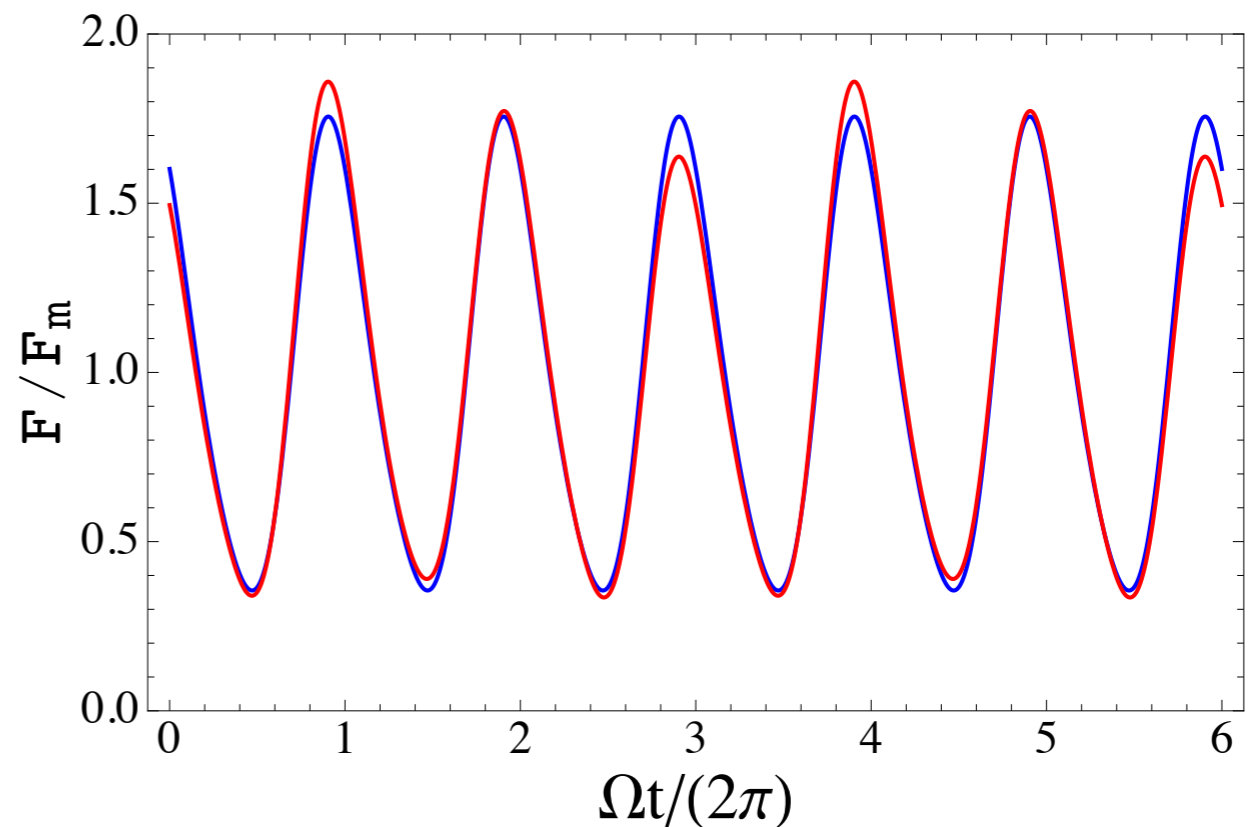
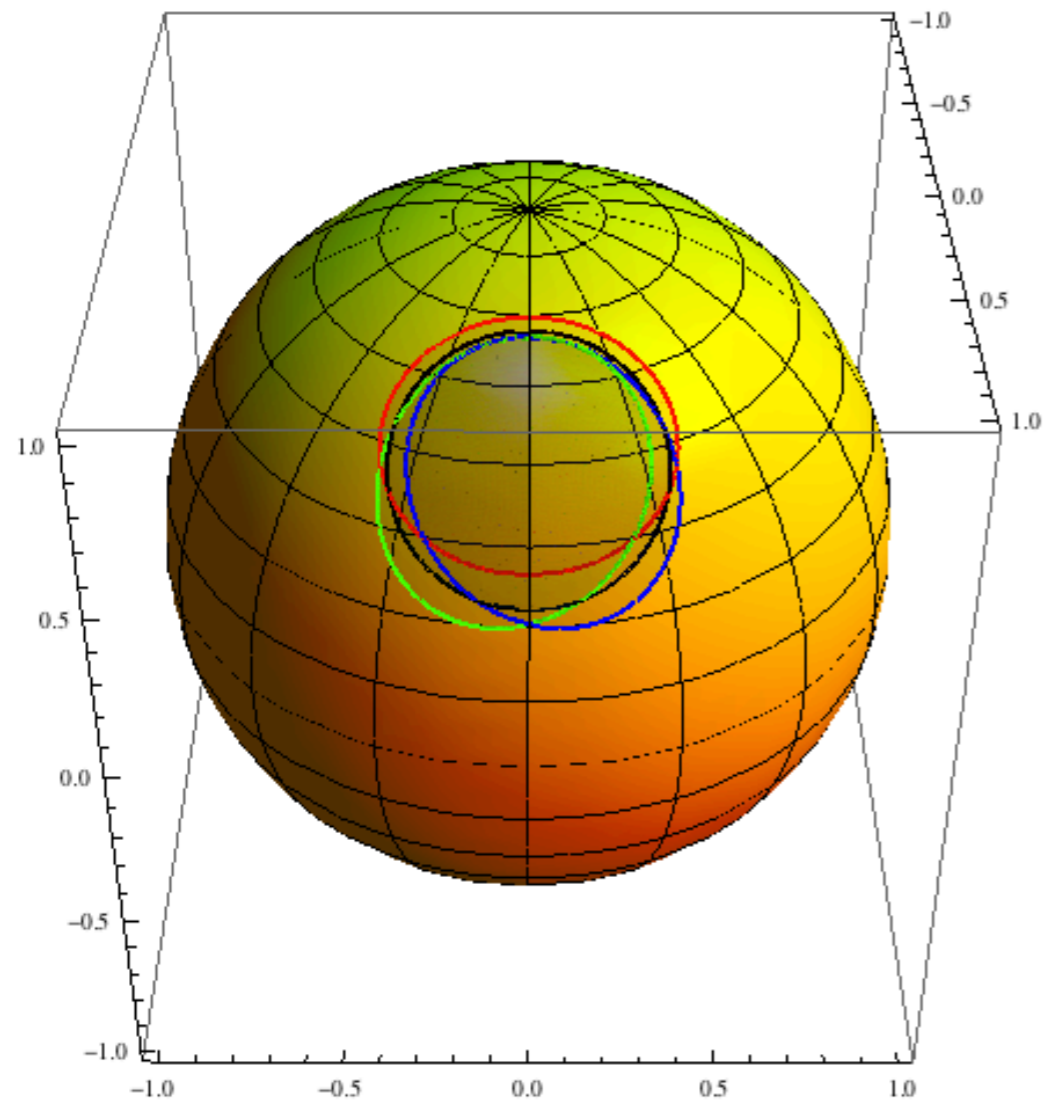


<http://imagine.gsfc.nasa.gov>

Accretion-powered millisecond X-ray pulsars (AMXPs) show small-amplitude X-ray oscillations with periods equal to their spin periods.



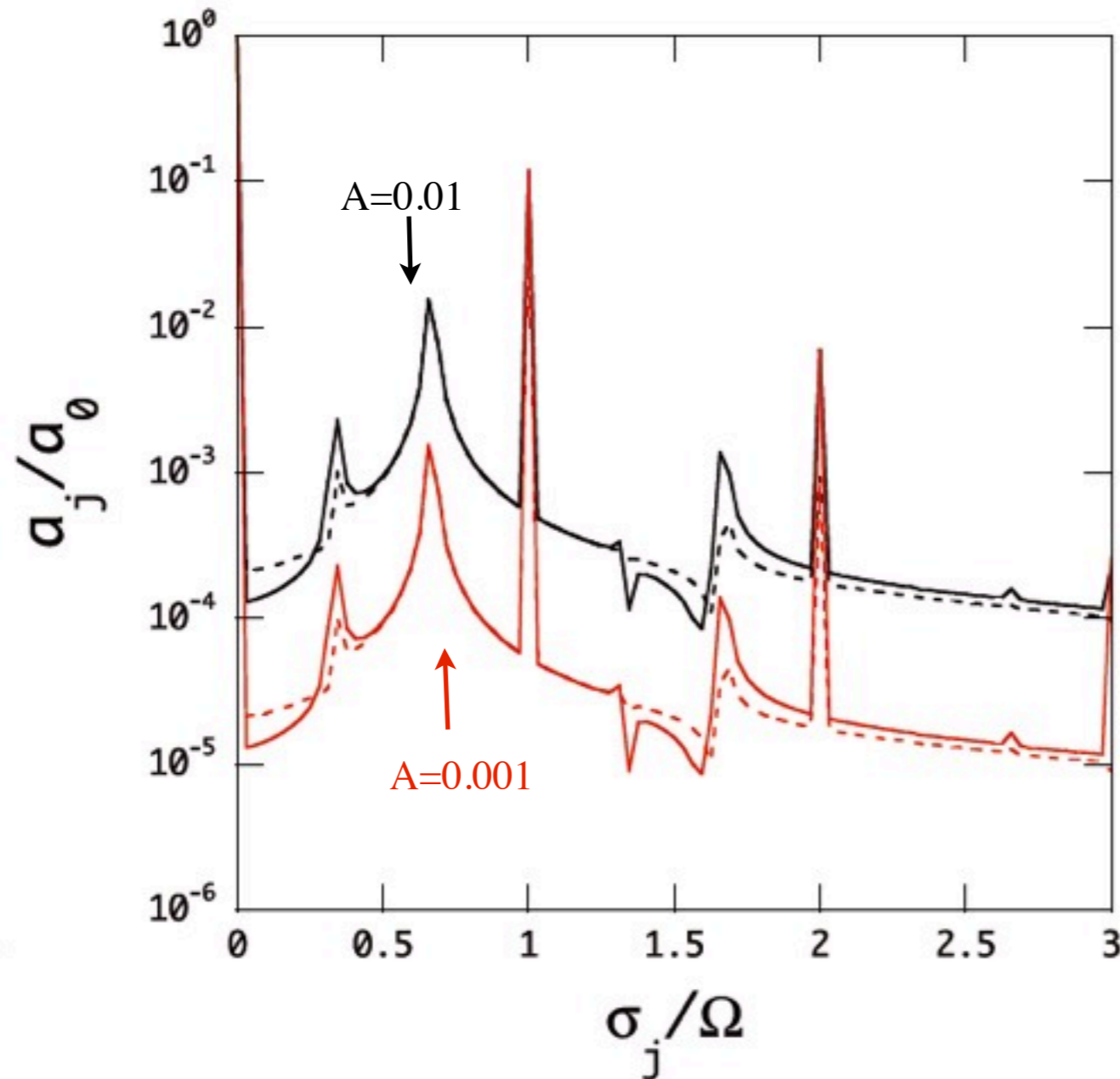
Outer boundary of a deformed hot-spot



- Surface displacements generated by pulsation modes can periodically distort the X-ray emitting hot-spot (Numata & Lee 2010).
- Distortion is maximized for modes with dominant transverse (quasi-toroidal) displacements, such as g-modes and r-modes.

- Light curves produced by an unperturbed (circular) hot-spot (blue) and one perturbed by the $l=m=2$ r-mode (red).
- $\nu_{spin} = 435$ Hz, $M = 1.4M_{\odot}$, $R = 11$ km, Observer's inclination angle = $\pi/3$.

Normalized Fourier amplitudes for light curves from a hot spot perturbed by an $l=m=2$ r-mode



Since the hot-spot rotates with the star, the modulation frequency seen by a distant observer is the **co-rotating frame** frequency of the mode.

Numata & Lee (2010)

$$\frac{Re(\xi)}{R} = [\Xi_r(\theta)\hat{e}_r + \Xi_\theta(\theta)\hat{e}_\theta] \cos(m\phi + \omega t) - \Xi_\phi(\theta)\hat{e}_\phi \sin(m\phi + \omega t)$$

$$A = \max(|\Xi_\theta|, |\Xi_\phi|)$$

ξ : r-mode displacement vector at the surface

Observed modulation, a_j , can constrain A .

Pulsation Searches:

XTE J1751-305

435 Hz spin frequency
42.4 min orbital period

XTE J1814-338

314.36 Hz spin frequency
4.275 hr orbital period

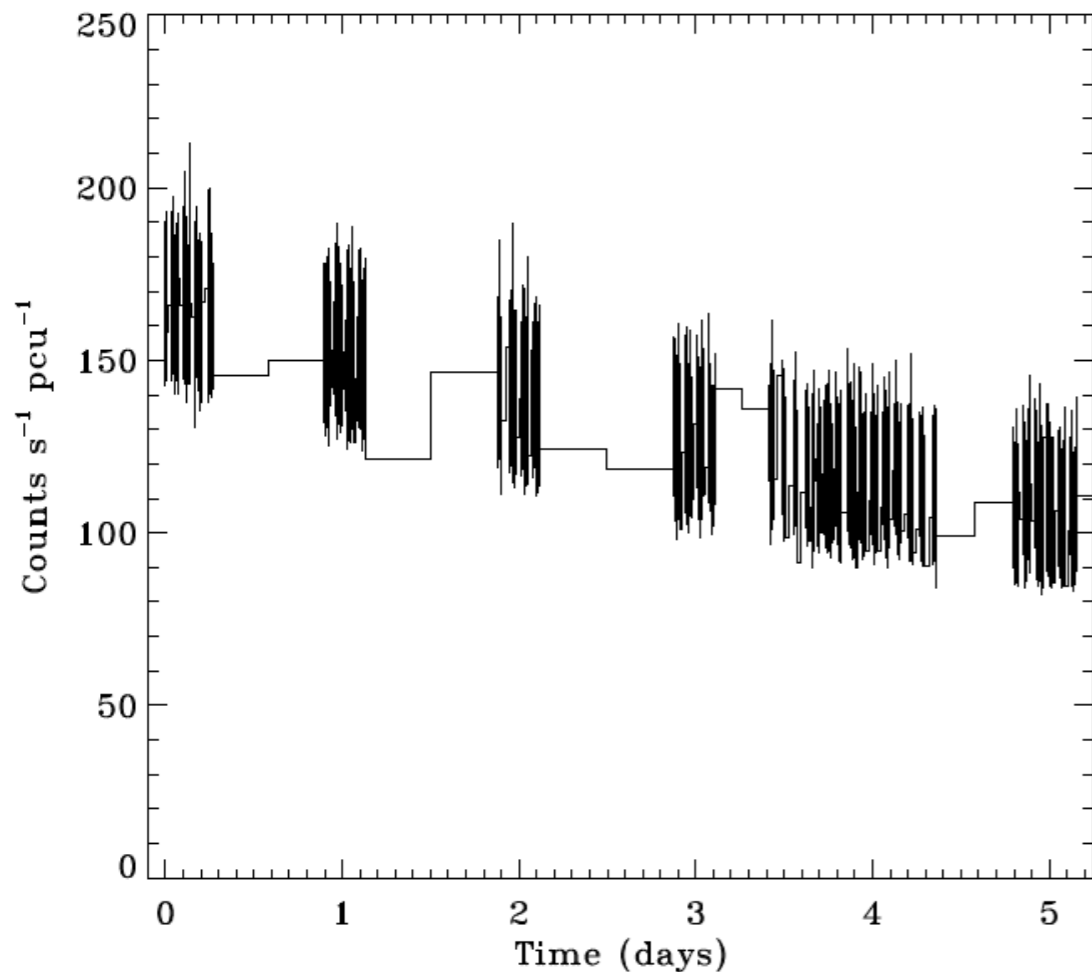
NGC 6440 X-2

205.89 Hz spin frequency
57 min orbital period

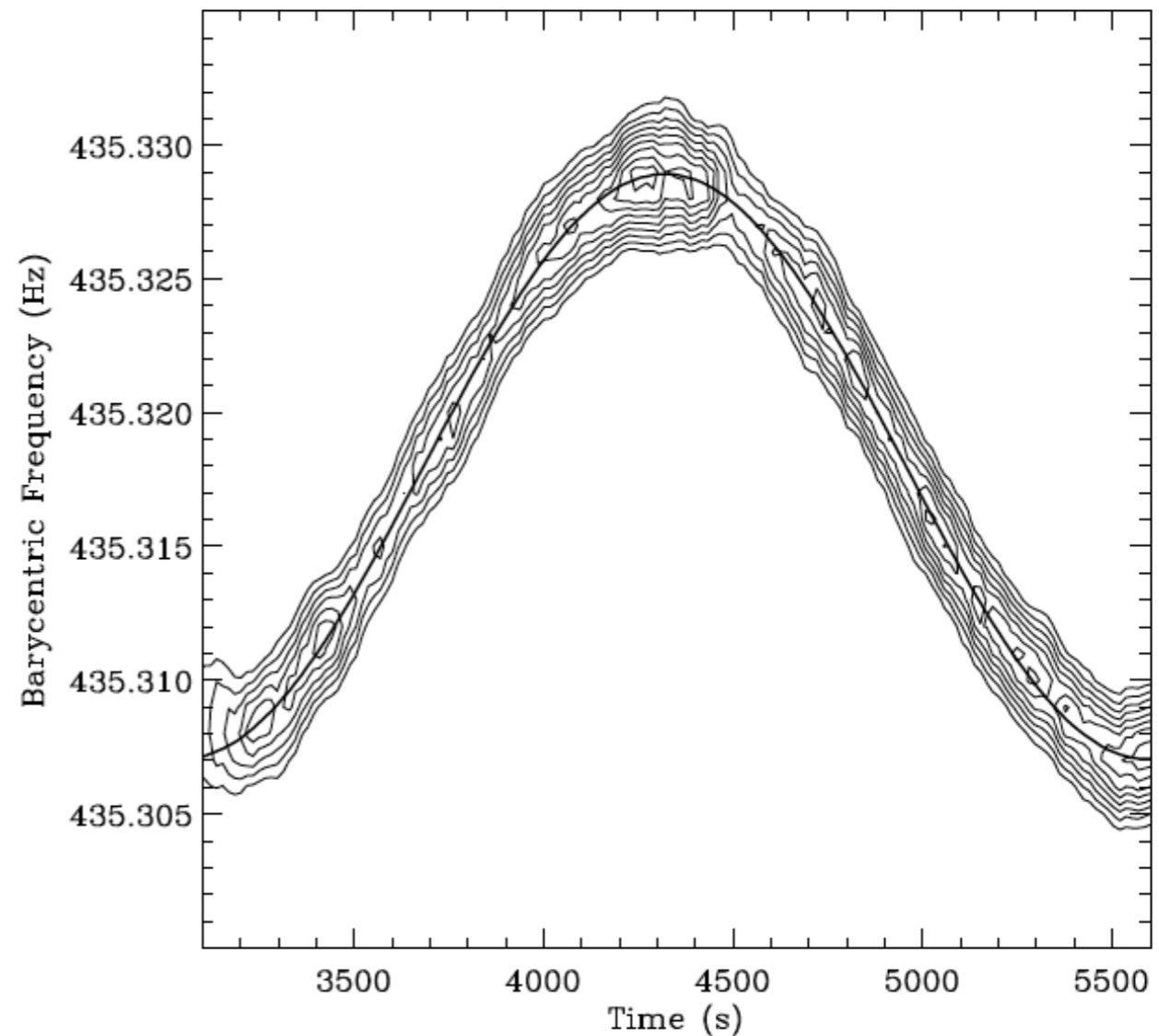
4U 1636-536

582 Hz spin frequency
3.8 hr orbital period

XTE J1751-305

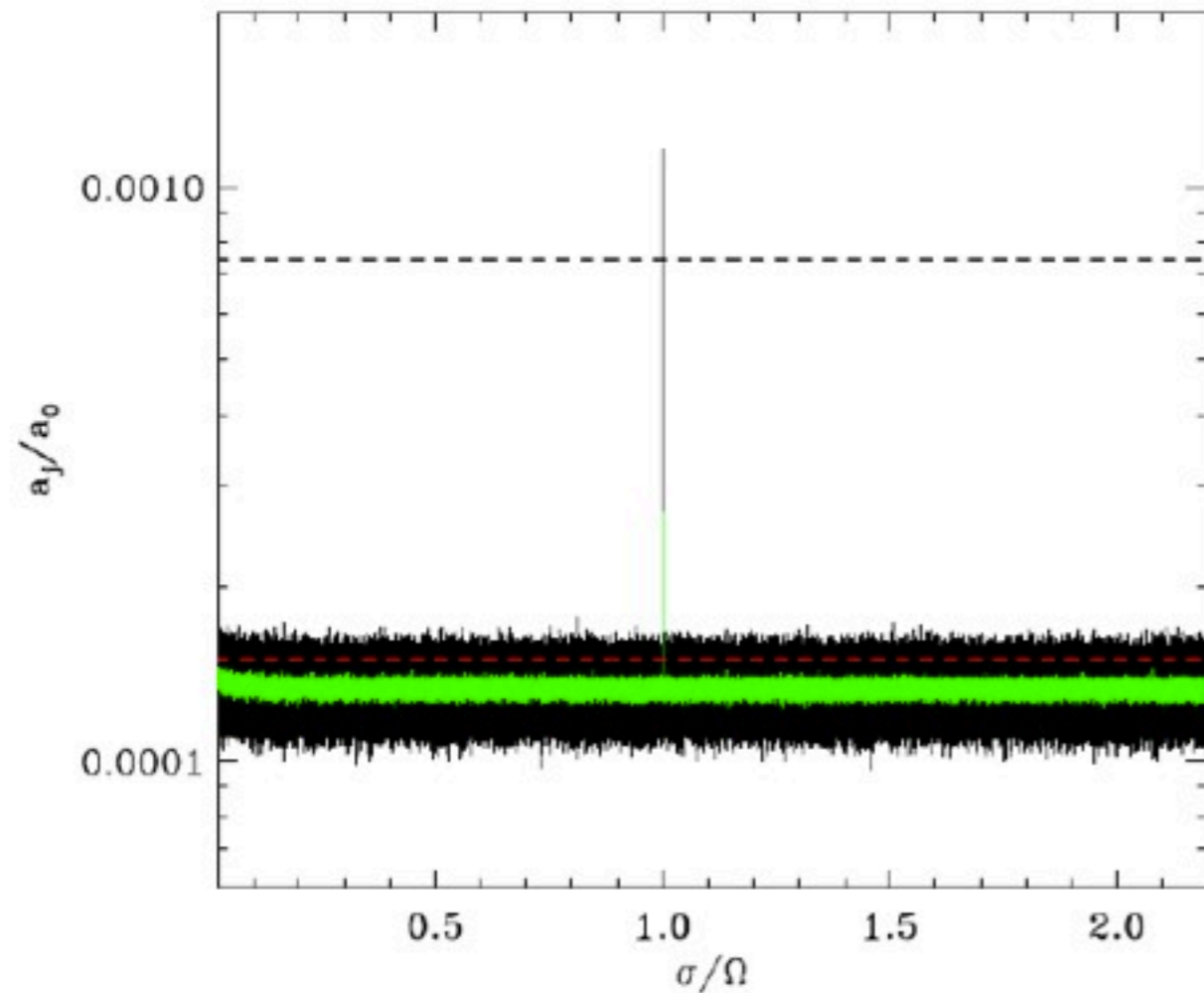


- Light curve in the 2 - 60 keV band from XTE J1751-305 used in our pulsation search.
- These data span the brightest portion of the outburst onset.



- Use orbit model to remove time delays associated with neutron star's orbital motion.
- Observer is effectively at the binary's center of mass.

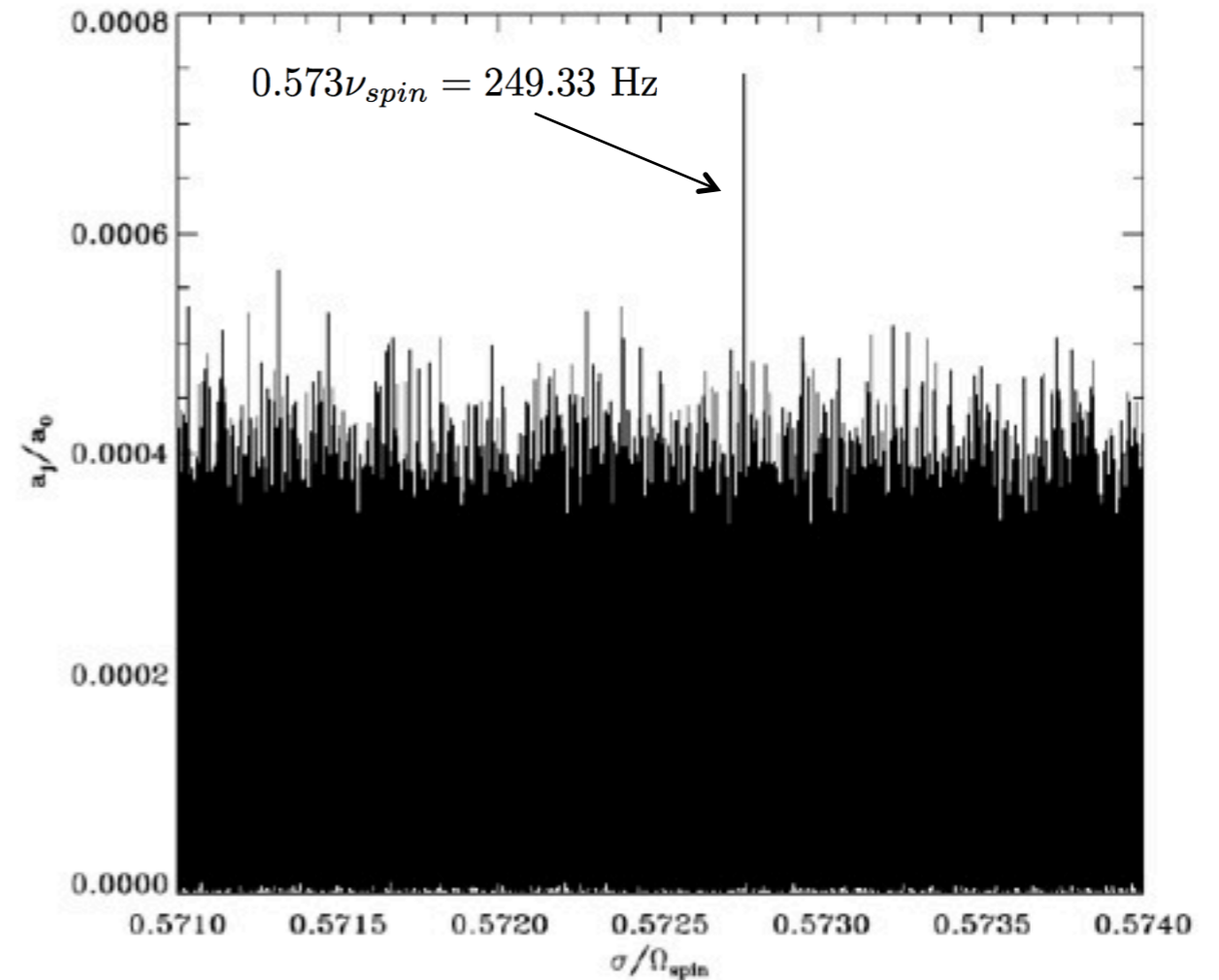
XTE J1751-305



Search for coherent signal, compute single FFT power spectrum, 2^{30} light curve bins.

Targeted search in the frequency range where r-modes (and g-modes) are theoretically expected.

$$(0.4166 \leq \sigma/\Omega \leq 0.7567) \quad (1.243 \leq \sigma/\Omega \leq 1.583)$$



$$0.573\nu_{spin} = 249.33 \text{ Hz}$$

Estimated significance: 1.6×10^{-3}
Slightly better than a 3σ detection!

Implied modulation amplitude of 7.5×10^{-4} ,
signal coherent over ~ 5 day light curve.

Possible mode identifications for J1751:

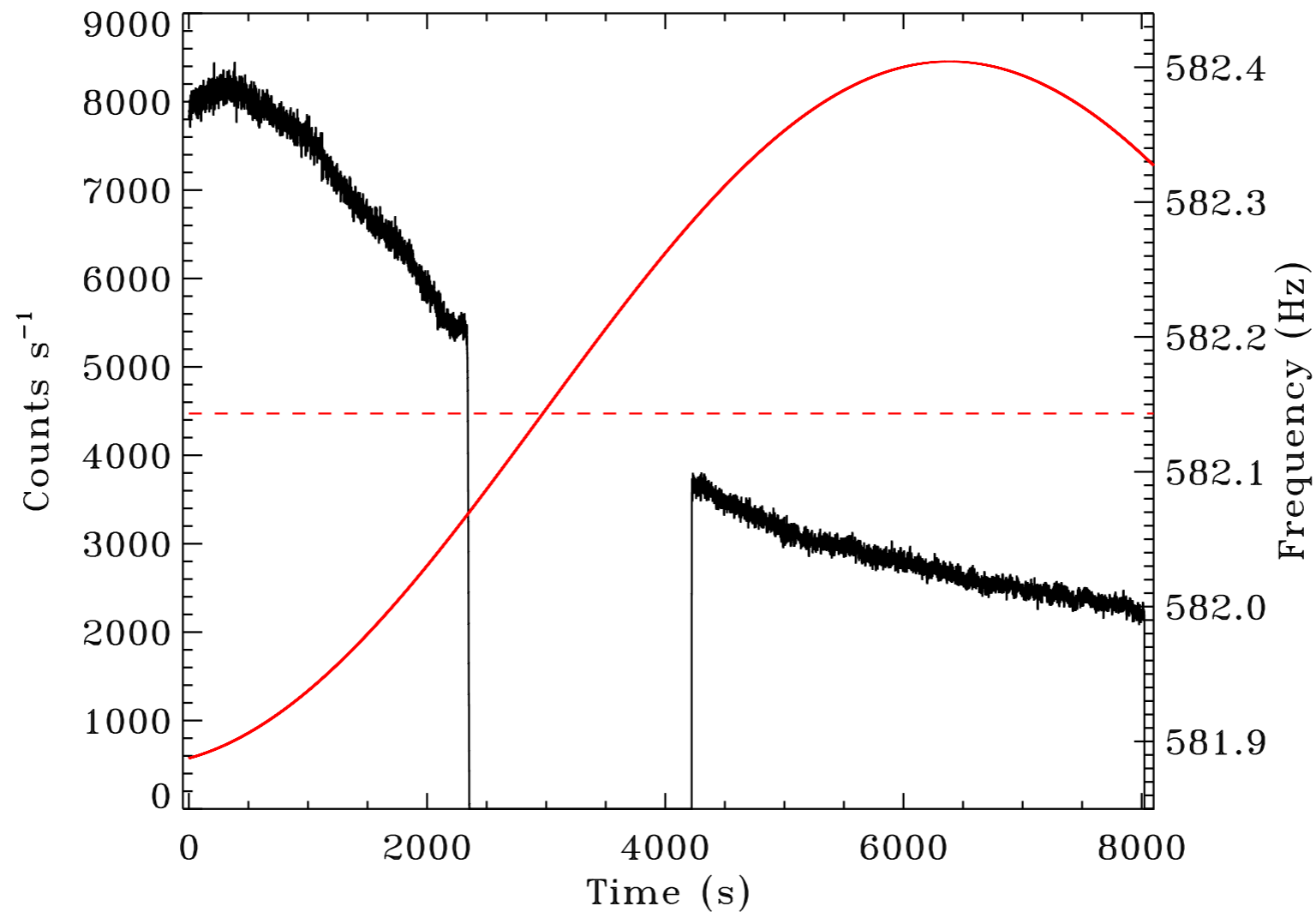
- **Surface g-modes**
- Inertial modes
- **Core r-modes?**

A Coherent Mode Search During the Superburst from 4U 1636-536

Superburst: Rare, energetic X-ray flares observed from LMXBs that are believed to be caused by unstable thermonuclear burning of a carbon-rich layer formed from the ashes of normal Type I X-ray bursts.

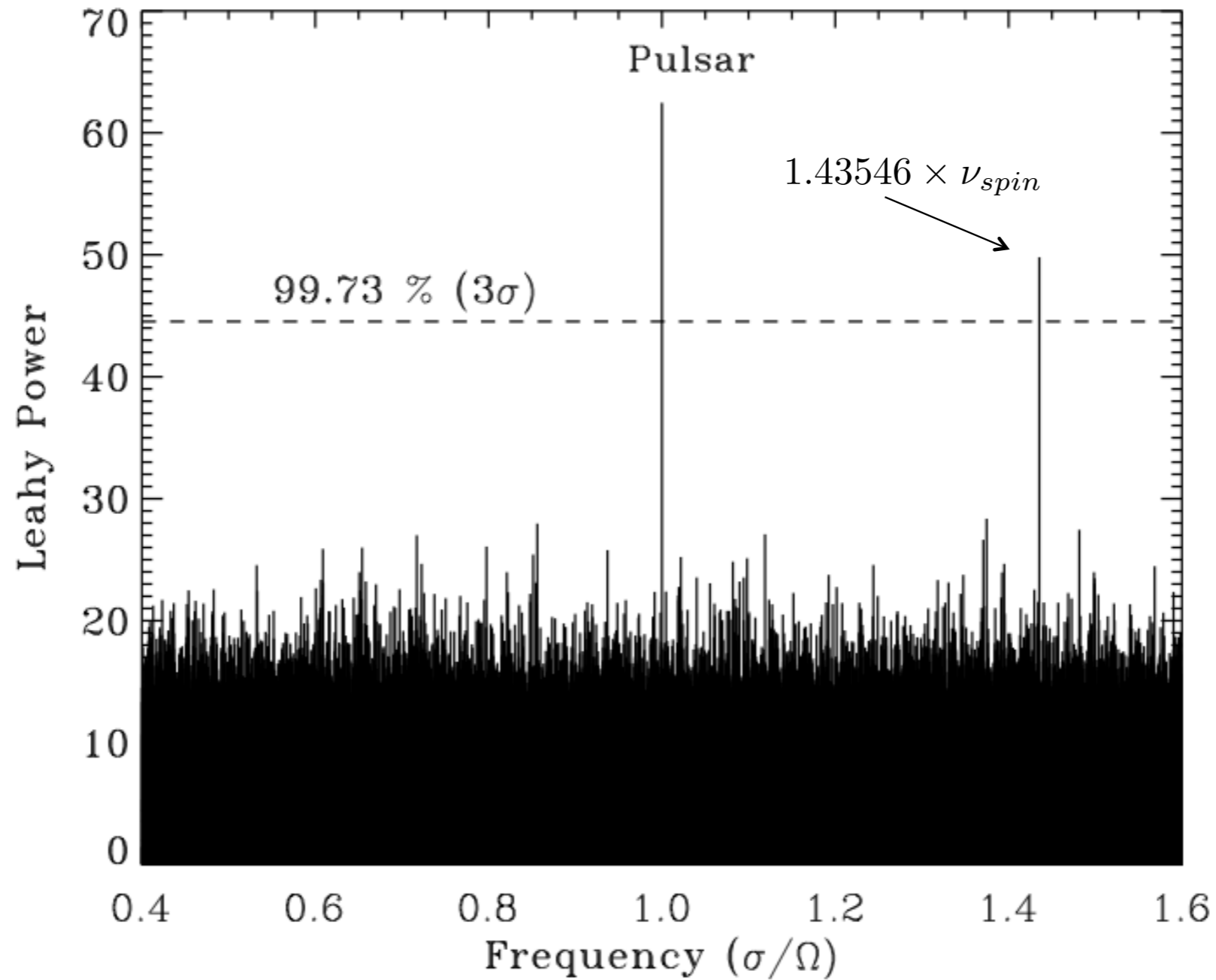
- High luminosity (high counting rate in the PCA) and long duration
- Orbit constraint derived from the 582 Hz pulsations enables a coherent search by removal of the phase delays due to the neutron star's orbital motion
- The thermonuclear burning provides a plausible mechanism for the excitation of modes (the so-called ϵ -mechanism), and the shock wave produced by detonation of the carbon fuel which powers the superburst is also likely to excite wave motions at or near the surface

4U 1636-536



Light curve of the portion of February 22, 2001 thermonuclear superburst from 4U 1636-533 used in our pulsation search, along with the orbital frequency model used to correct the event times (red curve).

4U 1636-536



Estimated significance: 2.5×10^{-4}

$$\exp(-49.3/2) \times N_{trials} = \exp(-49.3/2) \times (4 \times 3.17 \times 10^6)$$

Comparing the frequencies identified in J1751-305 and 4U 1636-536

For the AMXP source the modulation mechanism is likely due to oscillation-induced perturbations to the hot-spot fixed in the rotating frame of the star \longrightarrow modulations at the co-rotating frame frequencies.

For the superburst, where the whole surface is emitting, the stellar surface emission can be modulated by a mode, perhaps due to local variations in the temperature \longrightarrow modulations at the inertial frame frequencies.

$$\omega_i = m\Omega - \omega_r$$

For an m=2 mode: $\omega_r/\Omega = 2 - 1.43546 = 0.56454$

This is very close to the candidate frequency ratio of **0.57276** we identified in the **AMXP XTE J1751-3052**, suggesting that the frequencies identified in the two sources could perhaps be associated with the same oscillation mode

Possible mode identifications for J1751:

- **Surface g-modes**
- Inertial modes
- **Core r-modes?**

Possible mode identifications for J1751:

Surface g-modes

Rotationally-modified **g-modes** associated with a **helium-rich surface layer** or a density discontinuity due to **electron captures on hydrogen** in the accreted ocean.

- Orbital period of J1751 $\sim 42\text{min}$ \longrightarrow Ultracompact system with a likely helium-rich donor
- Non-rotating mode frequencies 20 – 30 Hz, but modes modified by fast rotation (Bildsten et al. 1996; Piro & Bildsten 2004).

$$\omega^2 = 2\Omega\omega_{l,0} \left[\frac{(2l_\mu - 1)^2}{l(l+1)} \right]^{1/2}$$

Consistent with:

($l=m=1$) and ($l=2, m=1$) thermal g-mode in a helium-rich atmosphere

Issues:

Excitation for high accretion rates, but J1751 at $< \dot{M}_{Edd}$.

Fast rotation “squeezes” displacements closer to equator, can mode modulate hot spot closer to pole?

Possible mode identifications for J1751:

Core r-modes

The co-rotating frame frequency of the r-modes in the slow-rotation limit is given by $\omega_0 = \frac{2m\Omega}{l(l+1)}$

$$l = m = 2 \text{ core r-modes (the most unstable ones)} \rightarrow \omega_0 = (2/3)\Omega$$

Larger than the frequency of the candidate signal at $\omega = 0.5727\Omega$.

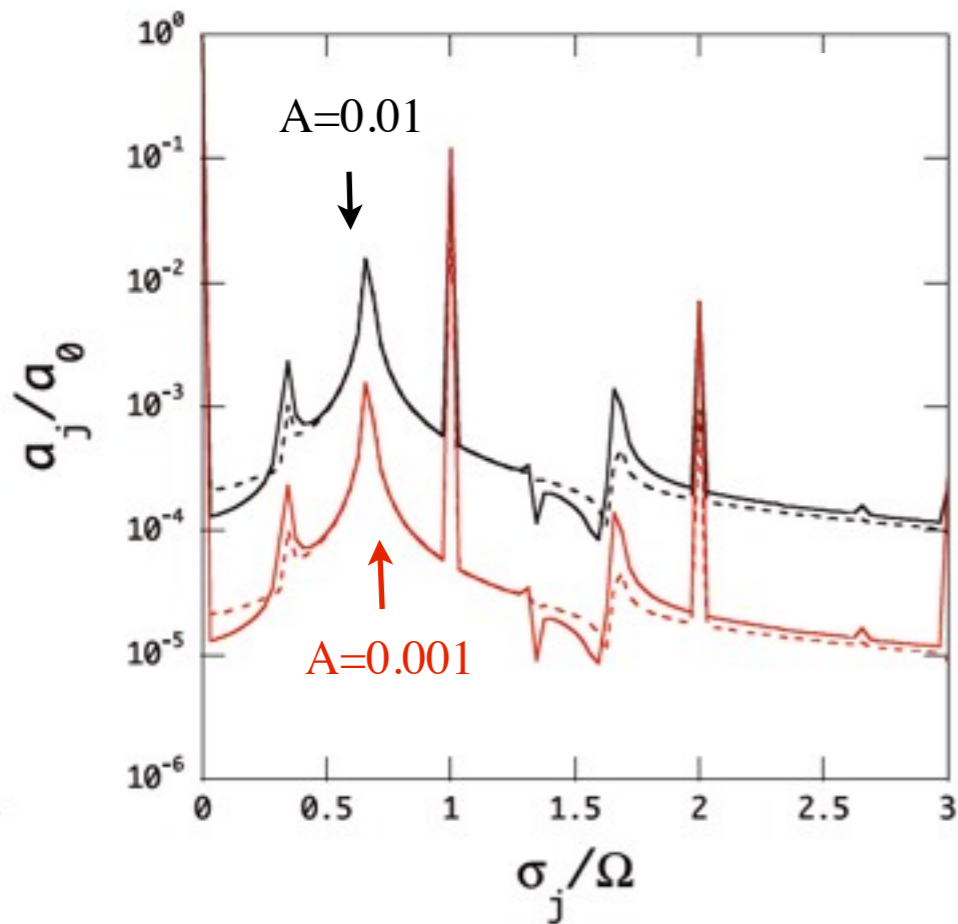
- r-modes are influenced by the solid crust, avoided crossings with torsional modes can push frequency below $2\Omega/3$ (Yoshida & Lee 2001).
- The relativistic effects (gravitational redshift + frame-dragging) tend to lower the rotational frame frequency (Lockitch, Friedman & Andersson 2003).

To first post-Newtonian order $\kappa = \frac{2}{3} \left[1 - \frac{8}{15} \left(\frac{M}{R} \right) \right]$

$$\kappa = \kappa_N + \delta\kappa_{GR} + \delta\kappa_{rot} \text{ (reasonable mass and radius for a neutron star gives } \kappa \sim 0.573)$$

[Andersson, Jones & Ho 2014, MNRAS, 442, 1786](#)

Is the r-mode scenario consistent with the spin evolution of J1751?



$$E_c = \frac{1}{2} \alpha^2 \Omega^2 \tilde{J} M R^2$$

$$A = \max(|\Xi_\theta|, |\Xi_\phi|) \quad A \sim \alpha/2$$

$$A_{J1751} \simeq 7 \times 10^{-4}$$

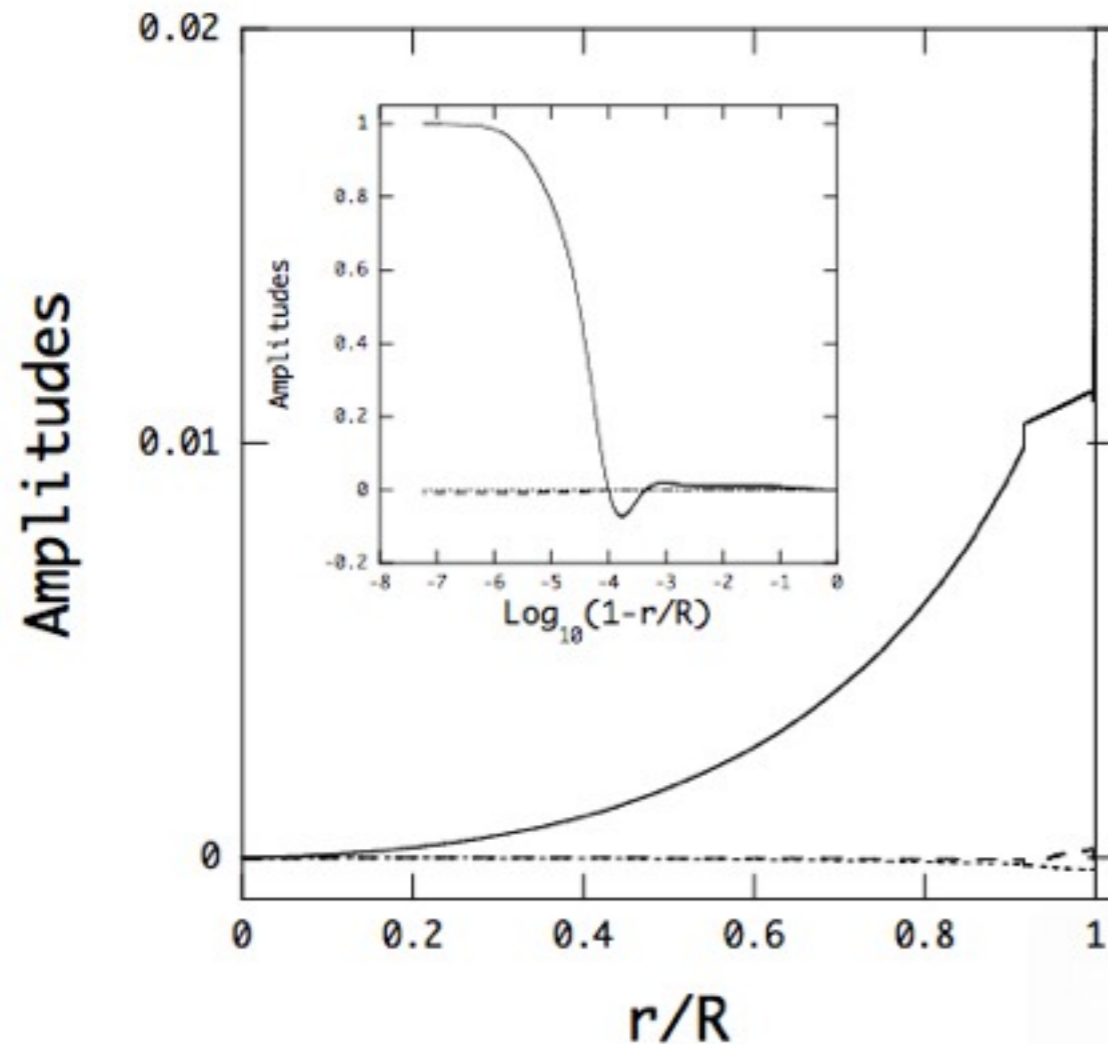
$$\frac{d\Omega}{dt} = -2Q \frac{\Omega \alpha^2}{\tau_V}$$

$$\frac{d\alpha}{dt} = -\alpha \left[\frac{1}{\tau_V} + \frac{1}{\tau_{GW}} \right]$$

$$\left. \frac{d\Omega}{dt} \right|_{J1751} = -1.3 \times 10^{-9}$$

- This would dominate the accretion spin-up rate and therefore is inconsistent with the observations (Patruno & Watts 2012)!
- Candidate oscillation unlikely to be a global r-mode!?

Eigenfunctions of an unstable $l=m=2$ r-mode in a fast rotating NS with a solid crust



A strong amplification of the amplitudes between the fluid core and the surface fluid ocean,

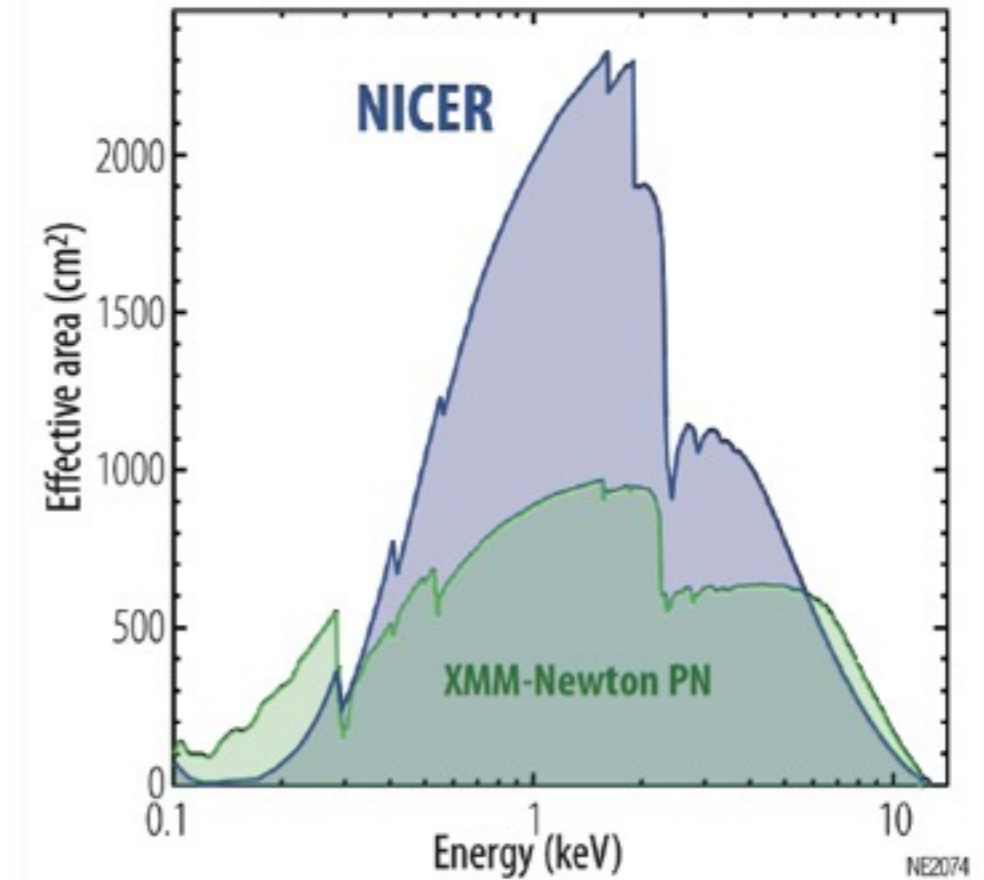
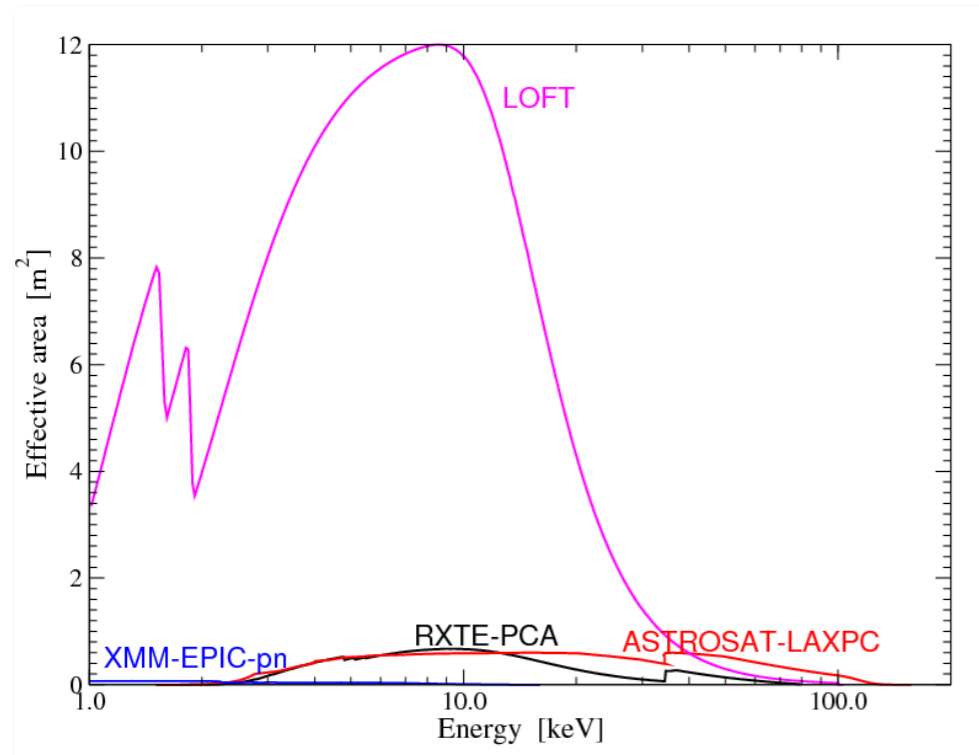
$$f_{amp} \equiv \alpha_{surface} / \alpha_{core} \sim 10^2$$

U. Lee 2014 (arXiv:1403.3476)

Eigenfunctions of an unstable $l' = m = 2$ r-mode with the ratio $\kappa \cong 0.6567$ at $\nu_{spin} = 435\text{Hz}$ for a $1.4M_{\odot}$ neutron star model with the shear modulus $\mu_{crust} = \mu_0$.

Future capabilities: NICER and LOFT

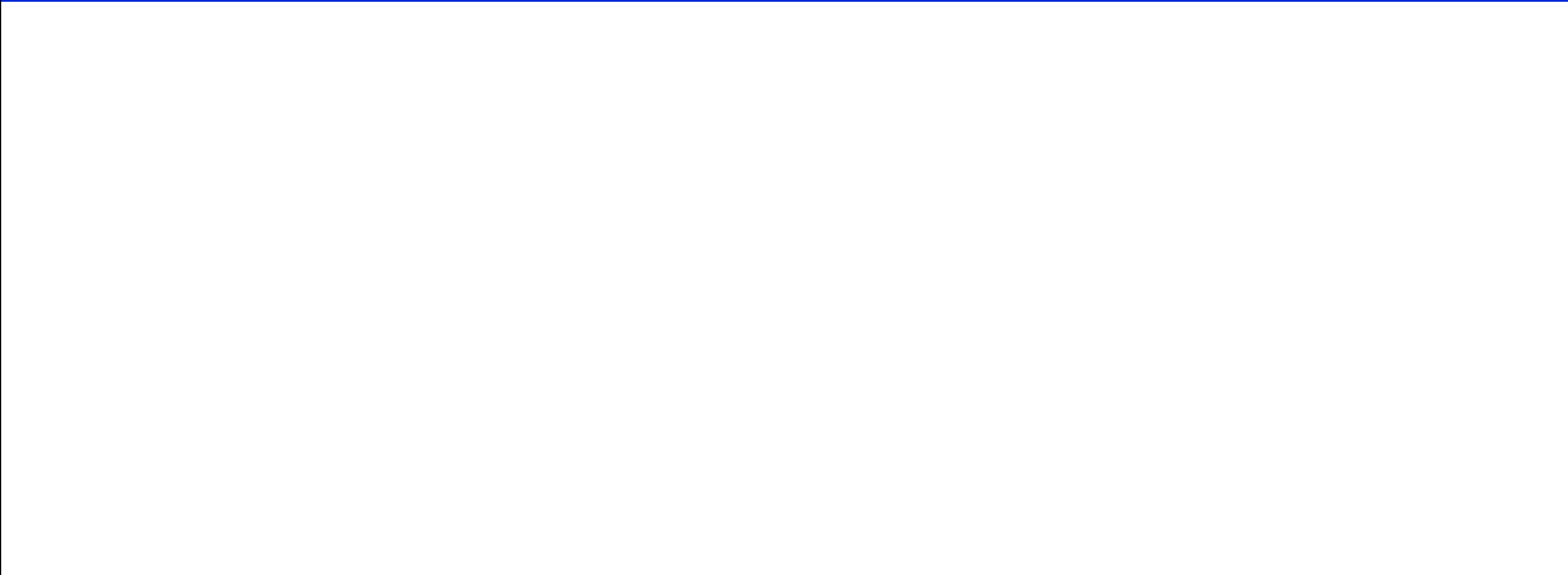
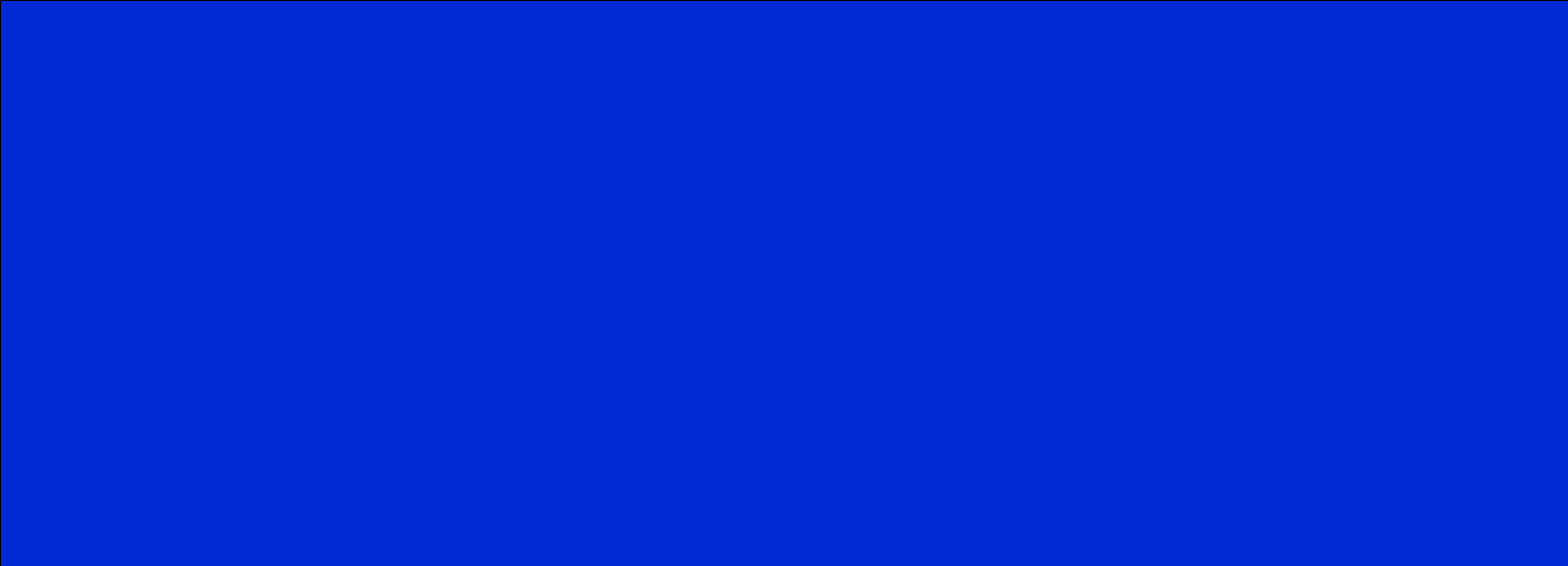
- Sensitivity for coherent search scales $\sim 1 / (N_{\text{tot}})^{1/2}$.
- LOFT/LAD 10-12 m² (3 – 15 keV)
- 20-30 x count rate of RXTE/PCA
- Likely can reach $a_{\text{amp}} \sim 1 - 2 \times 10^{-5}$



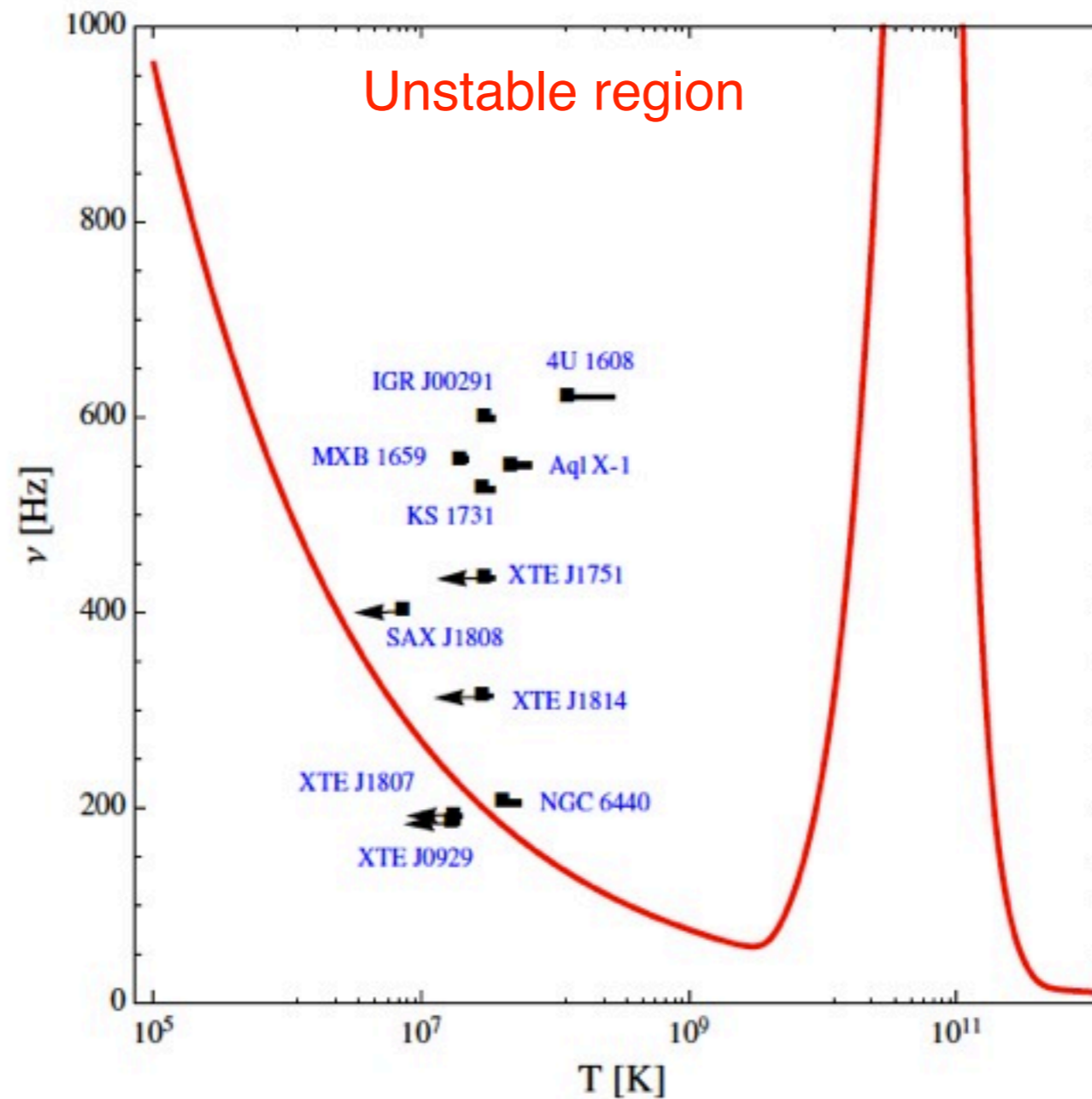
- NICER has smaller effective area than RXTE, but it may detect a brighter source or a longer observation time
- NICER enables searches for oscillation modes in rotation-powered pulsars.

Conclusion and Outlook

- Detection of global neutron star oscillation modes would open a new window on neutron star interiors.
- Even small mode amplitudes may influence the X-ray flux at measurable levels.
- Confirmation of candidate in J1751, and other source detections would be important.
- Additional searches using the RXTE data in more AMXPs underway (including SAX J1808).
- Light curve calculations for g-modes, to explore “visibility” of modes squeezed closer to equator.
- Larger collecting areas, more photons needed to improve sensitivities, LOFT, and NASA’s NICER can contribute.



LMXBs inside the r-mode instability region of normal hadronic stars with APR EoS!



Mahmoodifar & Strohmayer, *ApJ* **773**, 140 (2013)

Upper bounds on the r-mode amplitude

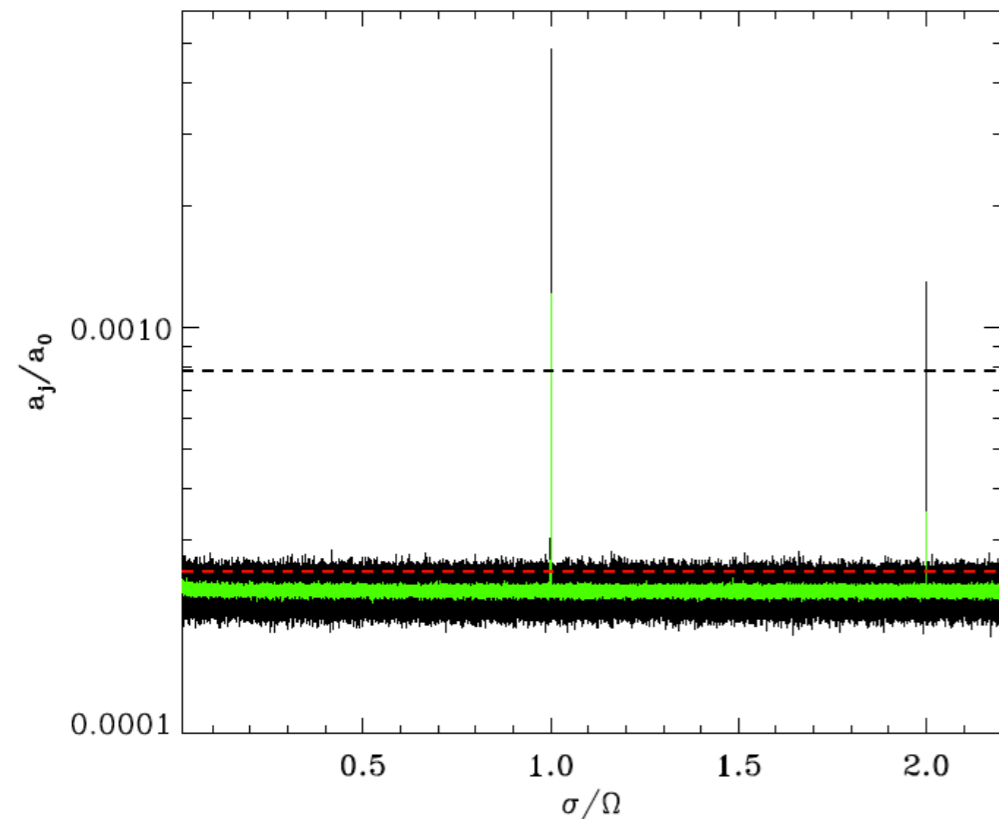
We assume that in **thermal equilibrium**, the quiescent NS luminosity is powered by dissipation from a steady-state r-mode.

$$W_d(\alpha) = L_\nu(T) + L_\gamma(T)$$

$$W_d(\alpha) = \frac{-2E_{rmode}}{\tau_{GR}} \quad \text{Reheating due to r-mode dissipation}$$

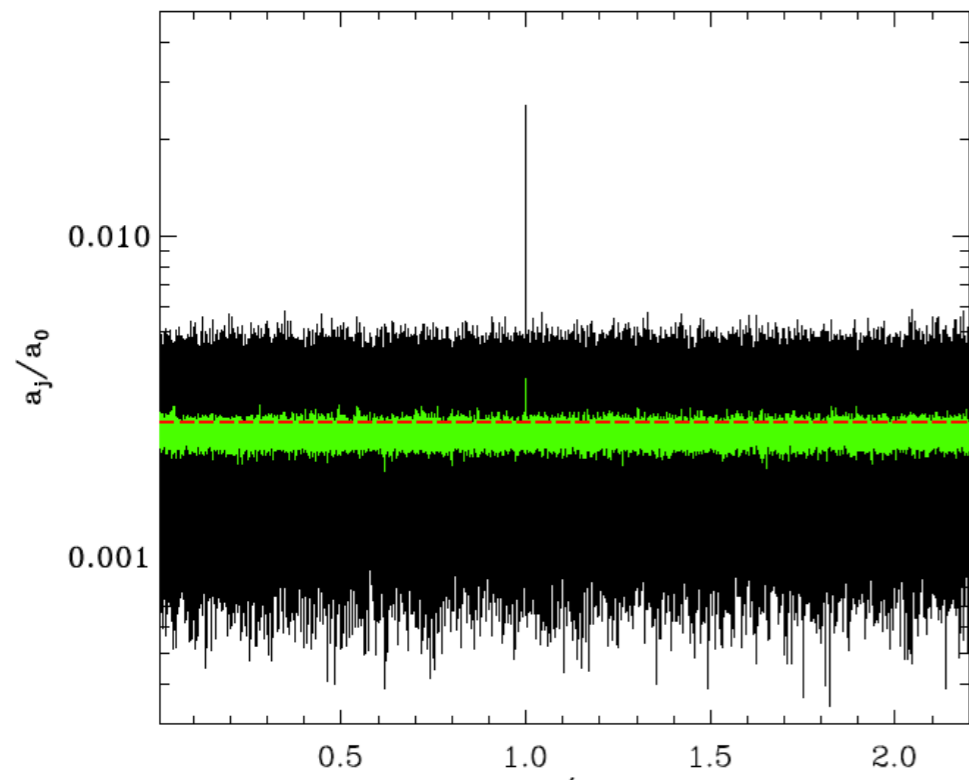
Source	$\alpha_{th.eq}$ ($1.4 M_\odot$)	$\alpha_{th.eq}$ ($2.0 M_\odot$)	$\alpha_{th.eq}$ ($2.21 M_\odot$)	$\dot{\nu}$ (Hz s $^{-1}$) ($1.4 M_\odot$)
4U 1608–522	7.15×10^{-8}	6.60×10^{-8}	2.61×10^{-5}	-1.44×10^{-15}
IGR J00291+5934	1.41×10^{-8}	1.32×10^{-8}	3.99×10^{-7}	-4.42×10^{-17}
MXB 1659–29	1.16×10^{-8}	1.07×10^{-8}	1.49×10^{-7}	-1.78×10^{-17}
Aql X-1	3.49×10^{-8}	3.27×10^{-8}	2.26×10^{-6}	-1.49×10^{-16}
KS 1731–260	2.35×10^{-8}	2.20×10^{-8}	6.44×10^{-7}	-4.81×10^{-17}
XTE J1751–305	5.09×10^{-8}	4.76×10^{-8}	1.44×10^{-6}	-6.13×10^{-17}
SAX J1808–3658	1.28×10^{-8}	1.19×10^{-8}	3.30×10^{-8}	-2.19×10^{-18}
XTE J1814–338	1.76×10^{-7}	1.67×10^{-7}	4.49×10^{-6}	-7.49×10^{-17}
NGC 6440	1.54×10^{-6}	1.45×10^{-6}	8.03×10^{-5}	-2.90×10^{-16}

Mahmoodifar & Strohmayer, ApJ **773**, 140 (2013)



XTE J1814-338

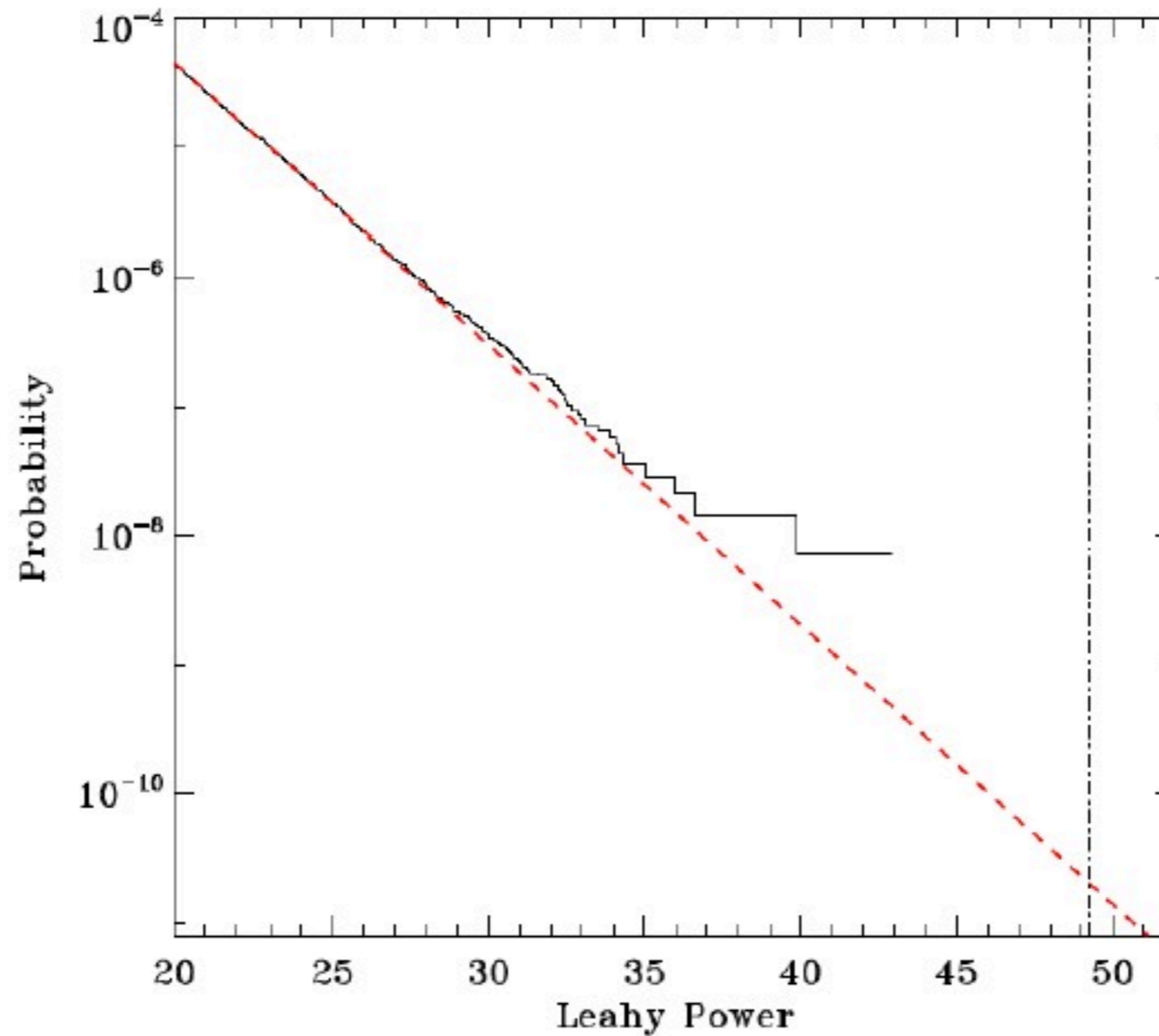
- Carried out similar search as for J1751. No candidate signals found. Set upper limit on amplitude of $\sim 8 \times 10^{-4}$.
- Pulsar has strong first harmonic.



NGC 6440 X-2

- Pulsar observed in only 4 RXTE orbits (not as much data as for J1751 and J1814).
- No candidate signals detected, with upper limits on the modulation amplitude of $\sim 3 \times 10^{-3}$.

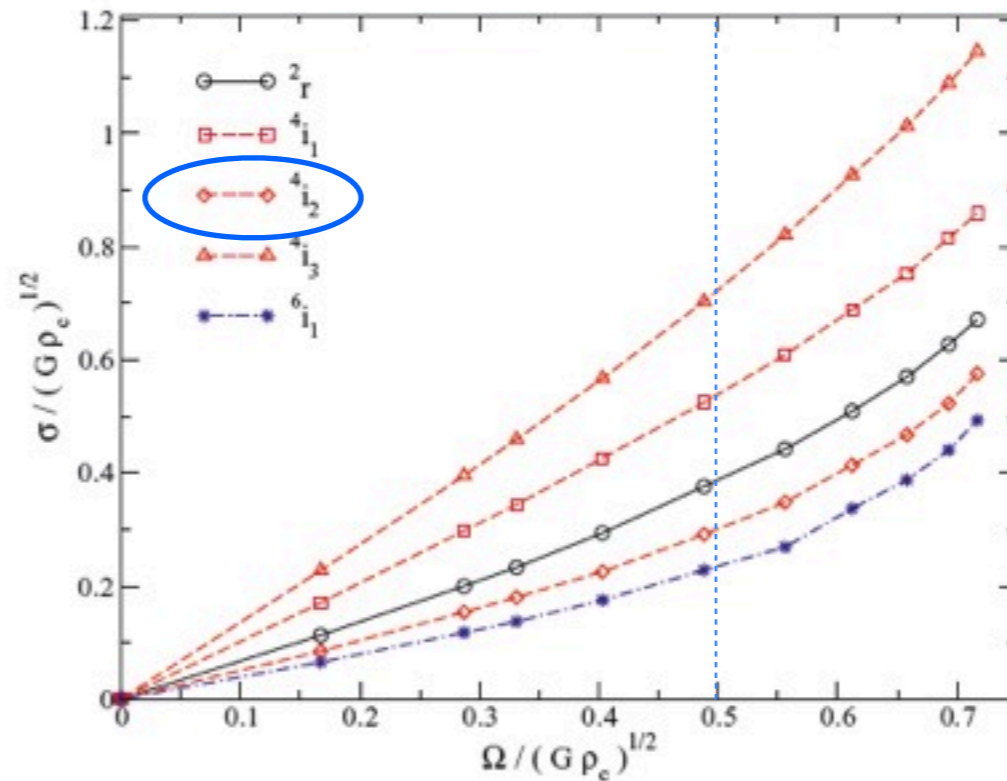
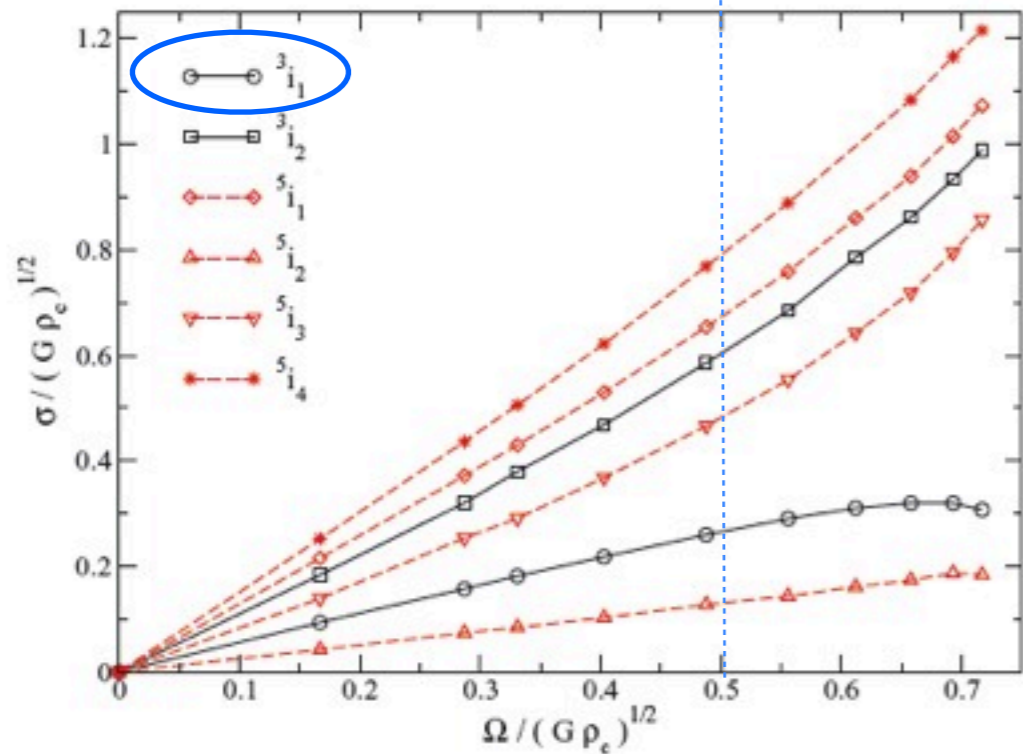
Probability to exceed a given Leahy-normalized Fourier power in a single trial.



Estimated significance: 1.6×10^{-3}

Slightly better than a 3σ detection!

Possible mode identifications for J1751: Inertial modes

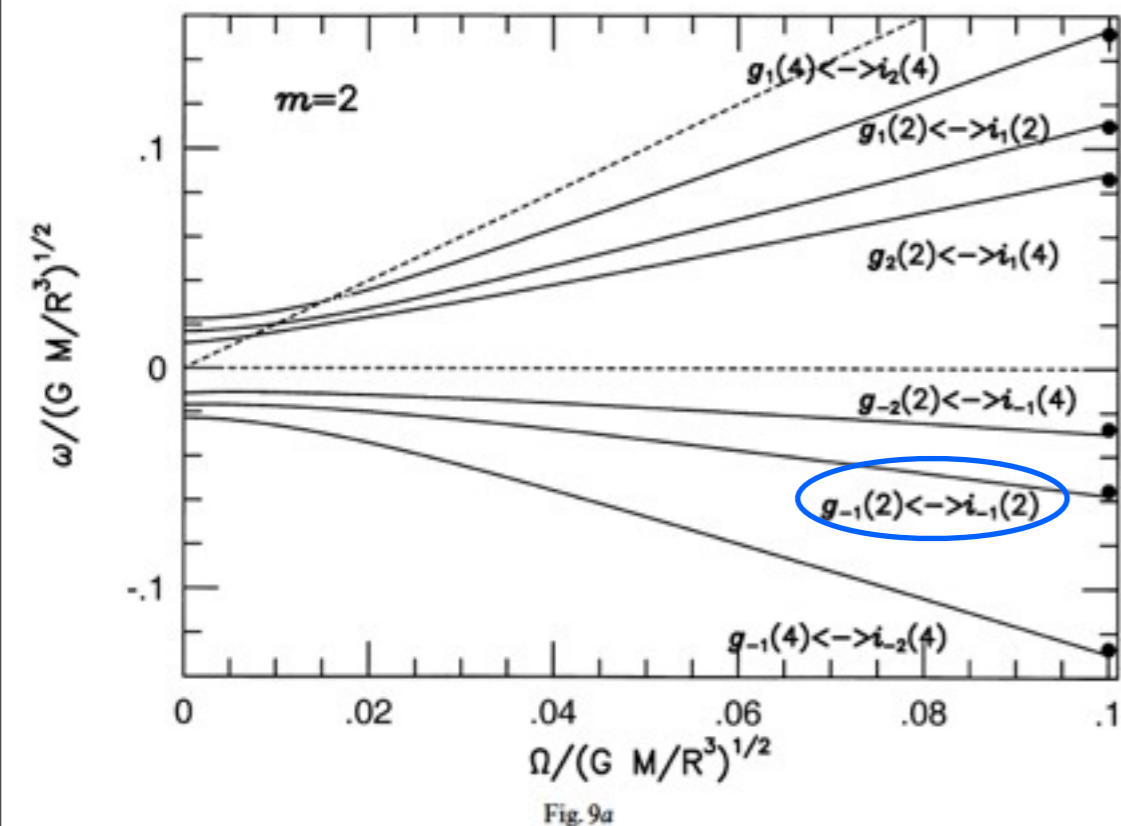


Passamonti et al. (2009)

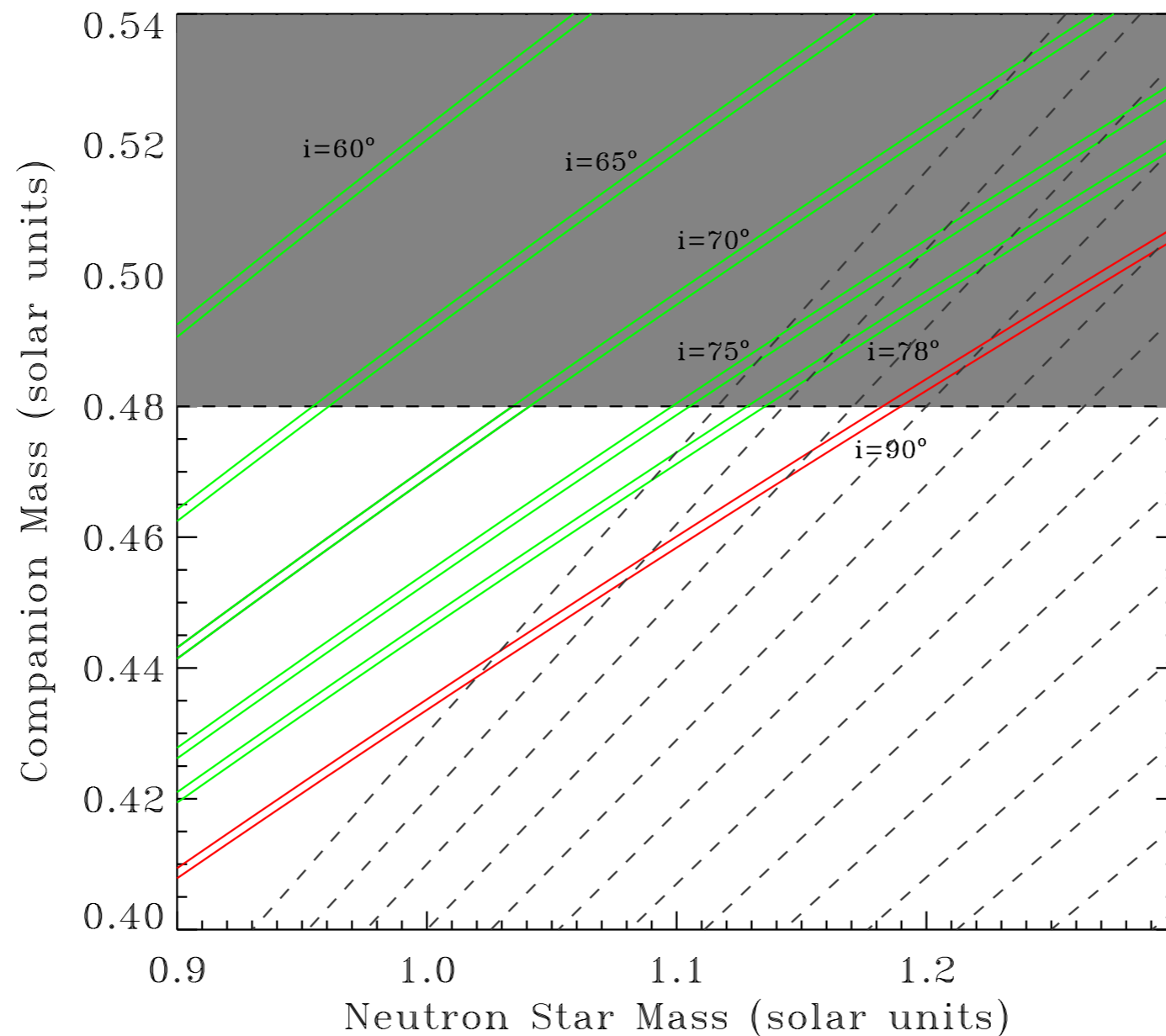
- More realistic equations of state
- Solid crust
- Higher spin rates

Similar to the global r-modes these inertial modes will likely have appreciable surface amplitudes closer to the rotational poles than the surface based, rotationally modified g-modes

Yoshida & Lee (2000)



Constraints on the component masses of 4U1636-536



Constraints on the component masses of 4U 1636-536 assuming a projected neutron star velocity of $134.5 \pm 0.2 \text{ km s}^{-1}$ (inferred from the coherent mode frequency detection).