What can we learn about the neutron-star equation of state from inspiralling binary neutron stars?

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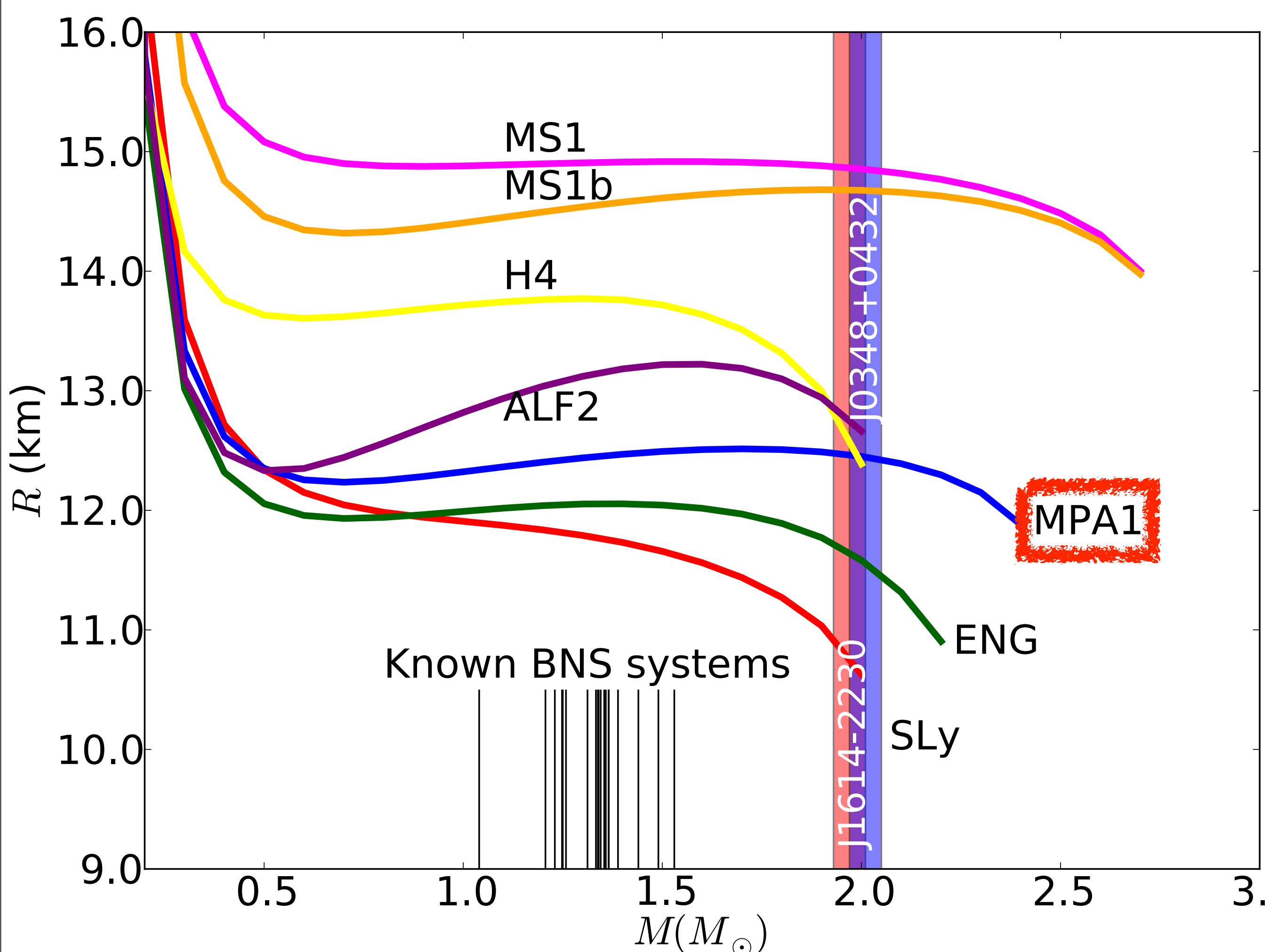
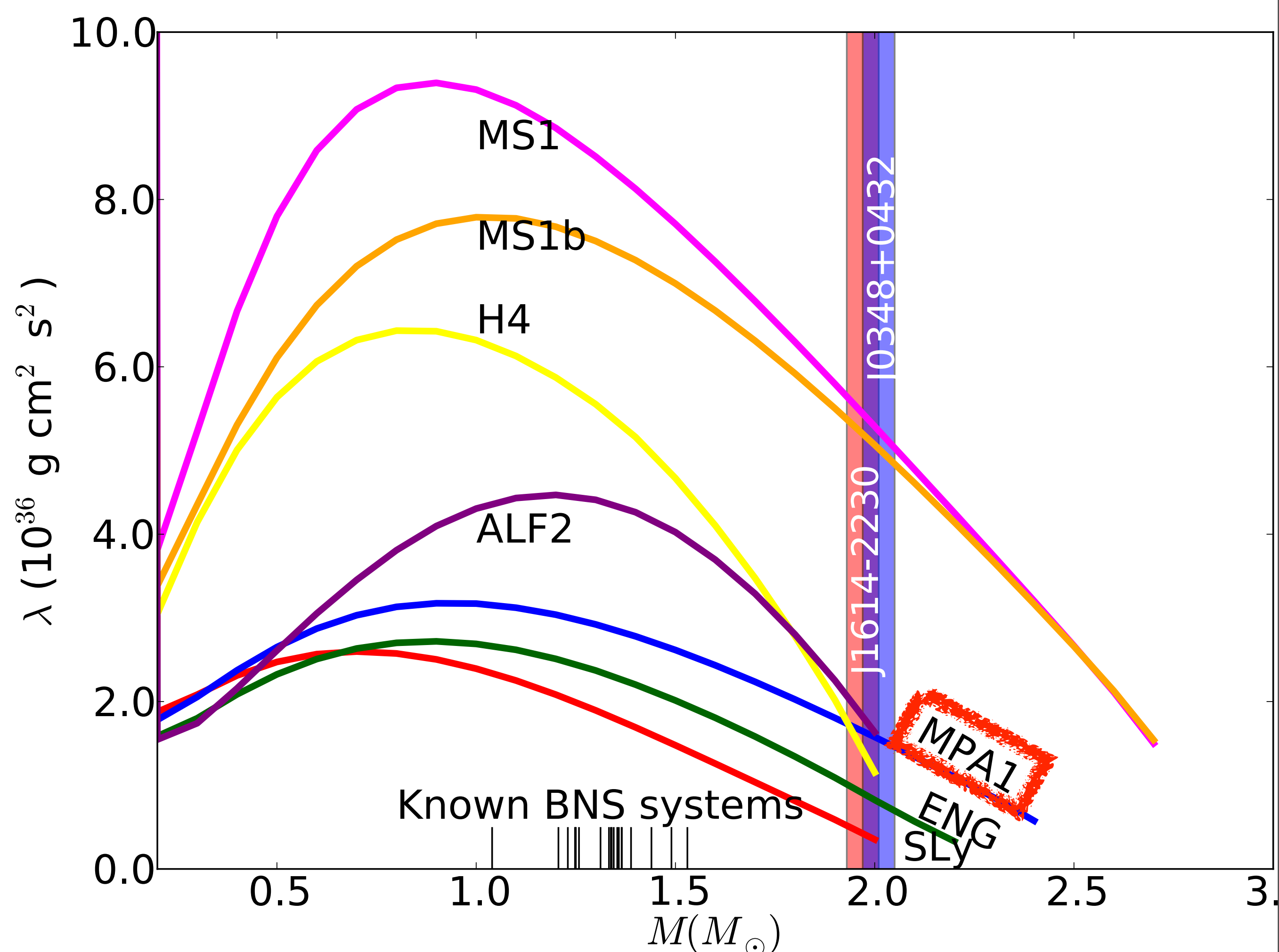
What can we measure from the waveform?

- Phase shift during the inspiral from tidal interactions
  - Tidal field $\mathcal{E}_{ij}$ of one NS induces quadrupole moment Q_{ij} in other NS

$$Q_{ij} = -\lambda(\mathrm{EOS}, M_{\mathrm{NS}})\mathcal{E}_{ij}$$
$$-\Lambda(\mathrm{EOS}, M_{\mathrm{NS}})M_{\mathrm{NS}}^5\mathcal{E}_{ij}$$

  - Increased quadrupole moment leads to more tightly bound system and additional quadrupole radiation





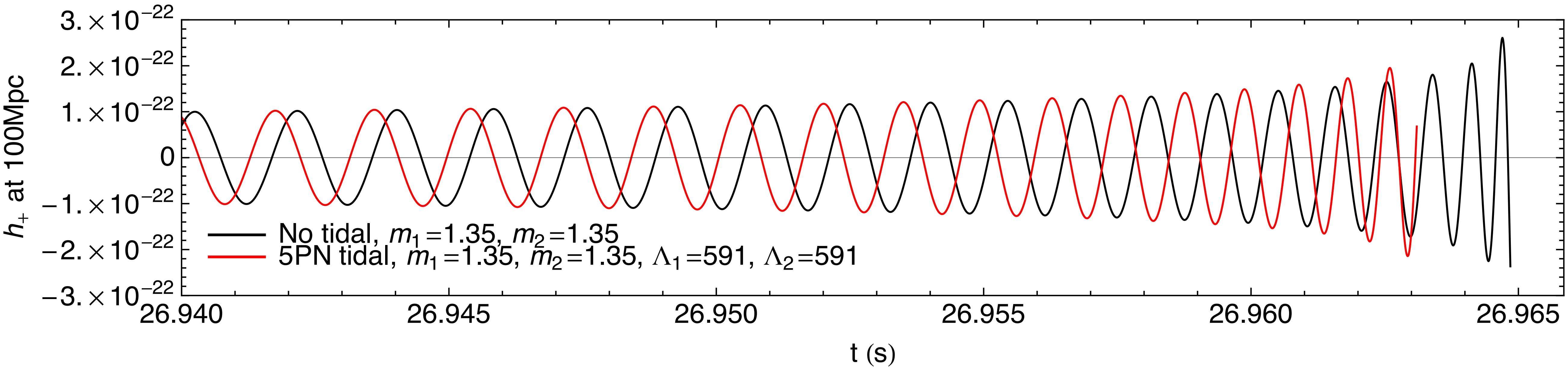
What can we measure from the waveform?

- Tidal effects first appear at same order as 5PN point-particle terms
- Leading term is a linear combination of the tidal deformabilities for each NS
- Results in a phase shift of ~1 cycle up to ISCO depending on the EOS

$$\tilde{h}(f) \propto \frac{\mathcal{M}^{5/6} f^{-7/6}}{D_L} e^{i\psi(f)}$$

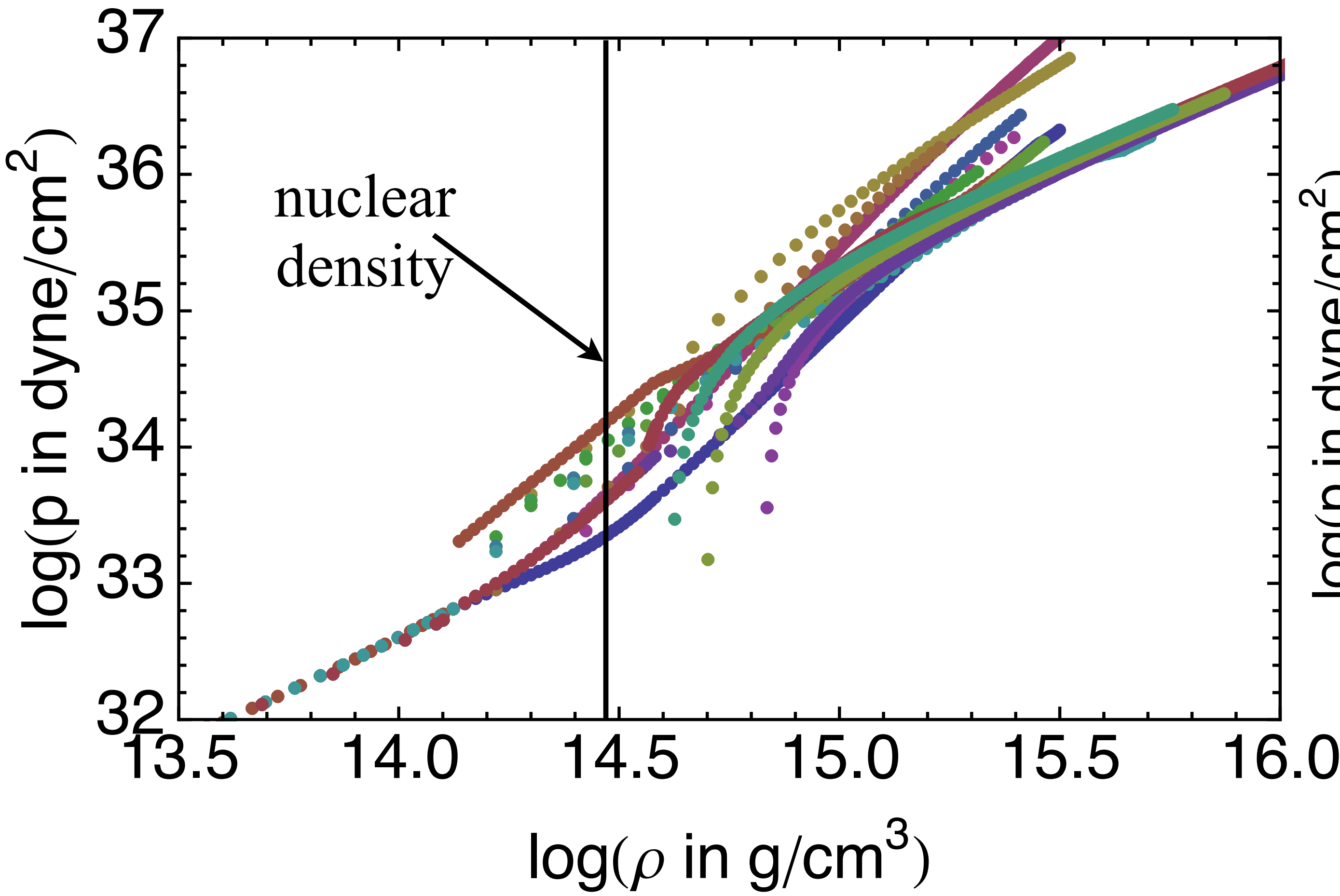
$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta v^5}\left[1 + (\text{PP-PN}) - \frac{39}{2}\tilde{\Lambda}(\mathcal{M}, \eta, \text{EOS}) v^{10}\right]$$

$$\tilde{\Lambda} = \frac{8}{13}\left[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta}(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)\right]$$

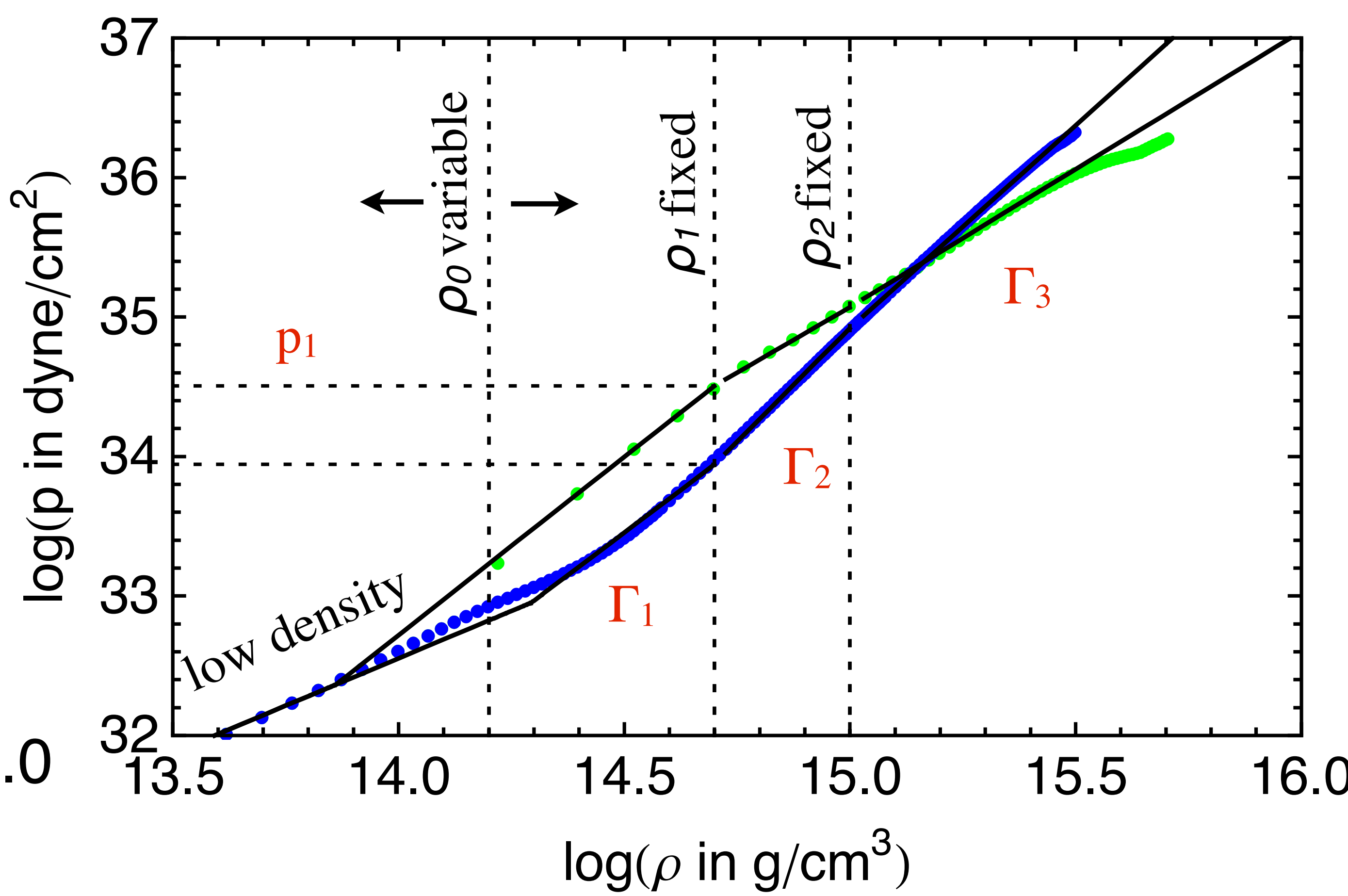


4-parameter EOS fit

Tabulated theoretical models

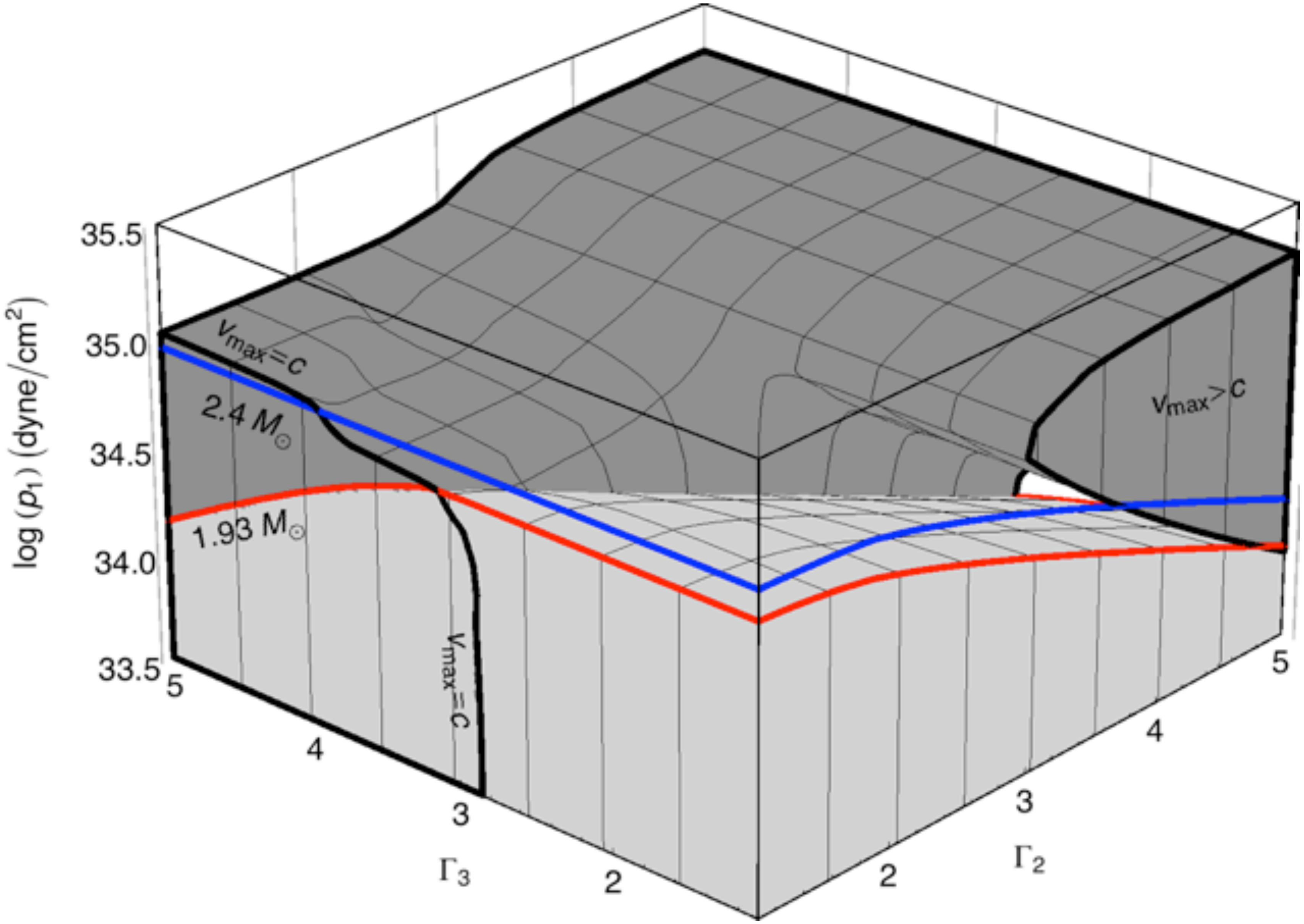


2 examples of fits



$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1}, & \rho_0 < \rho < \rho_1 \\ K_2 \rho^{\Gamma_2}, & \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3}, & \rho > \rho_2 \end{cases}$$

Current EOS constraints



- Causality: Speed of sound must be less than the speed of light in a stable neutron star $v_s = \sqrt{dp/d\epsilon} < c$
- Maximum mass: EOS must be able to support the observed star with mass greater than $1.93 M_\odot$ (lower limit on J0348+0432)

Estimating EOS parameters from LIGO data

- Analogue of 2-step procedure described by Steiner, Lattimer, Brown (Astrophys. J. 722, 33) for mass-radius measurements
  - They combined several mass-radius measurements from accreting neutron stars to estimate EOS parameters
  - We will use estimates of $\mathcal{M}$–η–$\tilde{\Lambda}(\mathcal{M}, \eta, \mathrm{EOS})$ from several BNS inspiral events to estimate EOS parameters

Step 1: Estimate $\mathcal{M}$–η–$\tilde{\Lambda}(\mathcal{M}, \eta, \mathrm{EOS})$

- Can estimate BNS parameters from Bayes theorem:

Posterior, Prior, Likelihood, Evidence

$$p(\vec{\theta}|d_n, \mathcal{H}, \mathcal{I}) = \frac{p(\vec{\theta}|\mathcal{H}, \mathcal{I})p(d_n|\vec{\theta}, \mathcal{H}, \mathcal{I})}{p(d_n|\mathcal{H}, \mathcal{I})}$$

  - $\vec{\theta} = \{\alpha, \delta, \iota, \psi, D_L, t_c, \phi_c, \mathcal{M}, \eta, \tilde{\Lambda}\}$
  - d_n: gravitational wave data from nth BNS system
  - $\mathcal{H}$: waveform model
  - $\mathcal{I}$: prior information about the parameters

- Posterior calculated with MCMC or estimated with Fisher matrix which assumes Gaussian likelihood and prior

- Marginalized distribution trivial to compute with MCMC or Fisher

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n, \mathcal{H}, \mathcal{I}) = \int p(\vec{\theta}|d_n, \mathcal{H}, \mathcal{I}) d\vec{\theta}_{\mathrm{marg}}$$

Step 2: Estimate EOS parameters

- Can estimate EOS parameters from Bayes theorem:

Prior — Likelihood

Posterior

$$p(\vec{x}|d_1 \ldots d_N, \mathcal{H}, \mathcal{I}) = \frac{p(\vec{x}|\mathcal{H}, \mathcal{I}) p(d_1 \ldots d_N|\vec{x}, \mathcal{H}, \mathcal{I})}{p(d_1 \ldots d_N|\mathcal{H}, \mathcal{I})}$$

Evidence

- $\vec{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, \mathcal{M}_1, \eta_1, ..., \mathcal{M}_N, \eta_N\}$
- $d_1 \ldots d_N$: gravitational wave data from all N BNS events
- prior: Flat in EOS parameters except $v_s = \sqrt{dp/d\epsilon} \leq c$ and $M_{\max} \geq 1.93 M_\odot$
- Likelihood:

Marginalized posterior/quasilikelihood from single BNS event

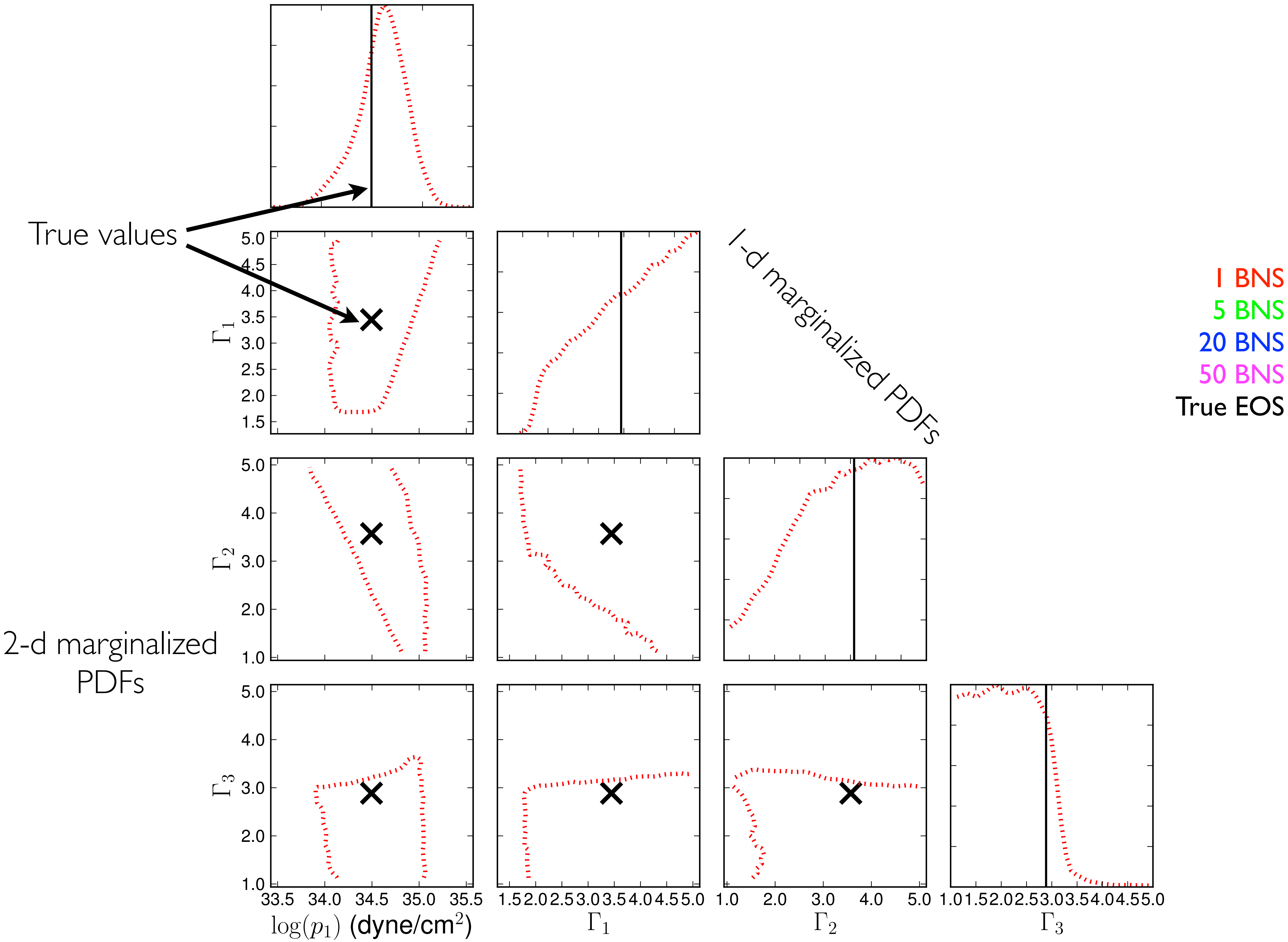
$$p(d_1, \ldots, d_N|\vec{x}, \mathcal{H}, \mathcal{I}) = \prod_{n=1}^{N} p(\mathcal{M}_n, \eta_n, \tilde{\Lambda}_n|d_n, \mathcal{H}, \mathcal{I})|_{\tilde{\Lambda}_n = \tilde{\Lambda}(\mathcal{M}_n, \eta_n, \mathrm{EOS})}$$

- Perform MCMC simulation over the 4+2N parameters, then marginalize over the 2N mass parameters

Simulating a population of BNS events

- We chose the "true" EOS to be MPA1
  - Moderate EOS in middle of parameter space
  - $R(1.4M_{\odot}) \sim 12.5\mathrm{km},$ and $M_{\max} \sim 2.5M_{\odot}$
- Sampled 50 BNS systems with SNR > 8
  - Individual masses distributed uniformly in $(1.2M_{\odot}, 1.6M_{\odot})$
  - Sky position and distance distributed uniformly in volume
  - Orientation distributed uniformly on unit sphere
  - $\tilde{\Lambda}$ then calculated from masses and "true" EOS

EOS Parameters



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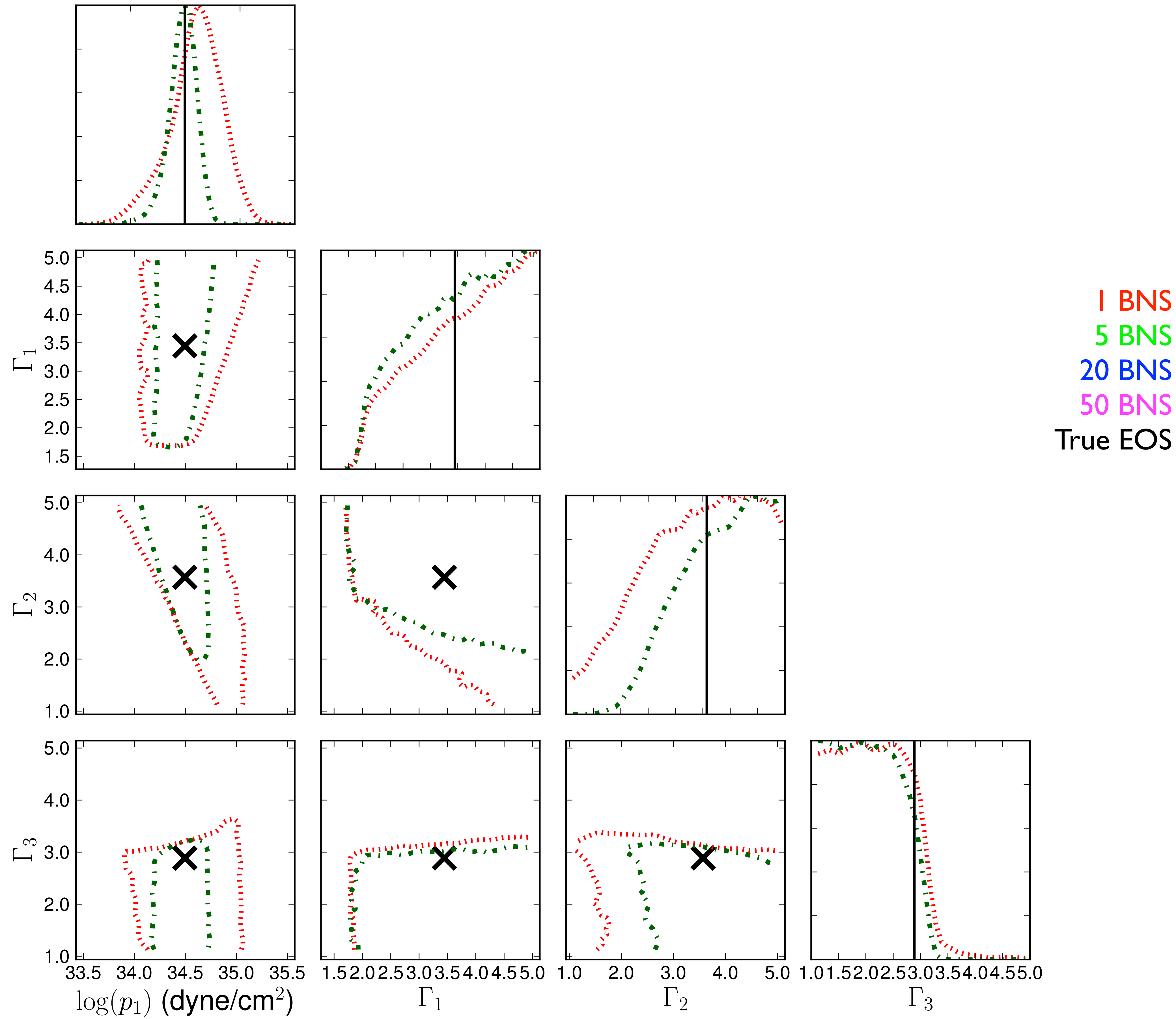

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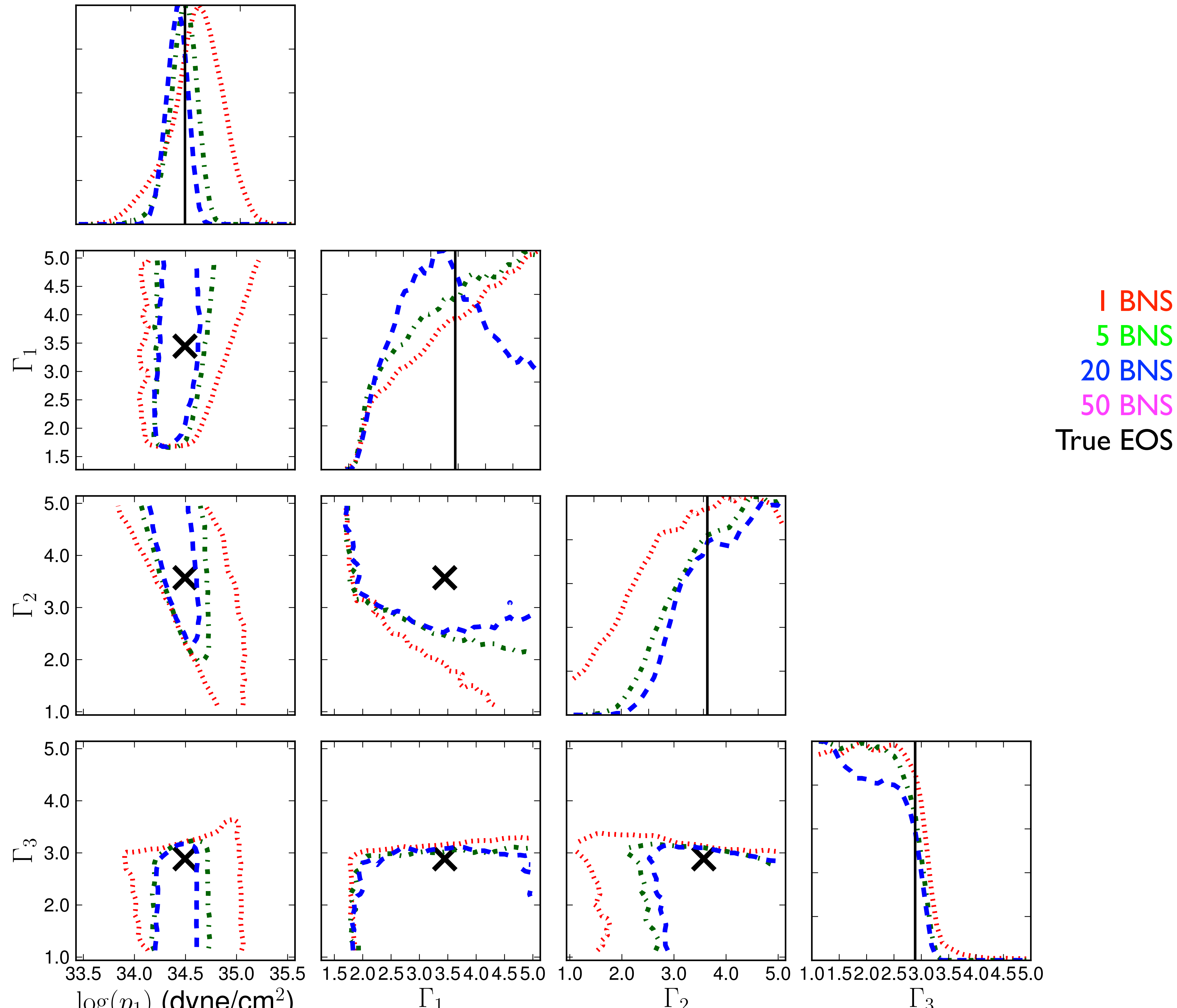

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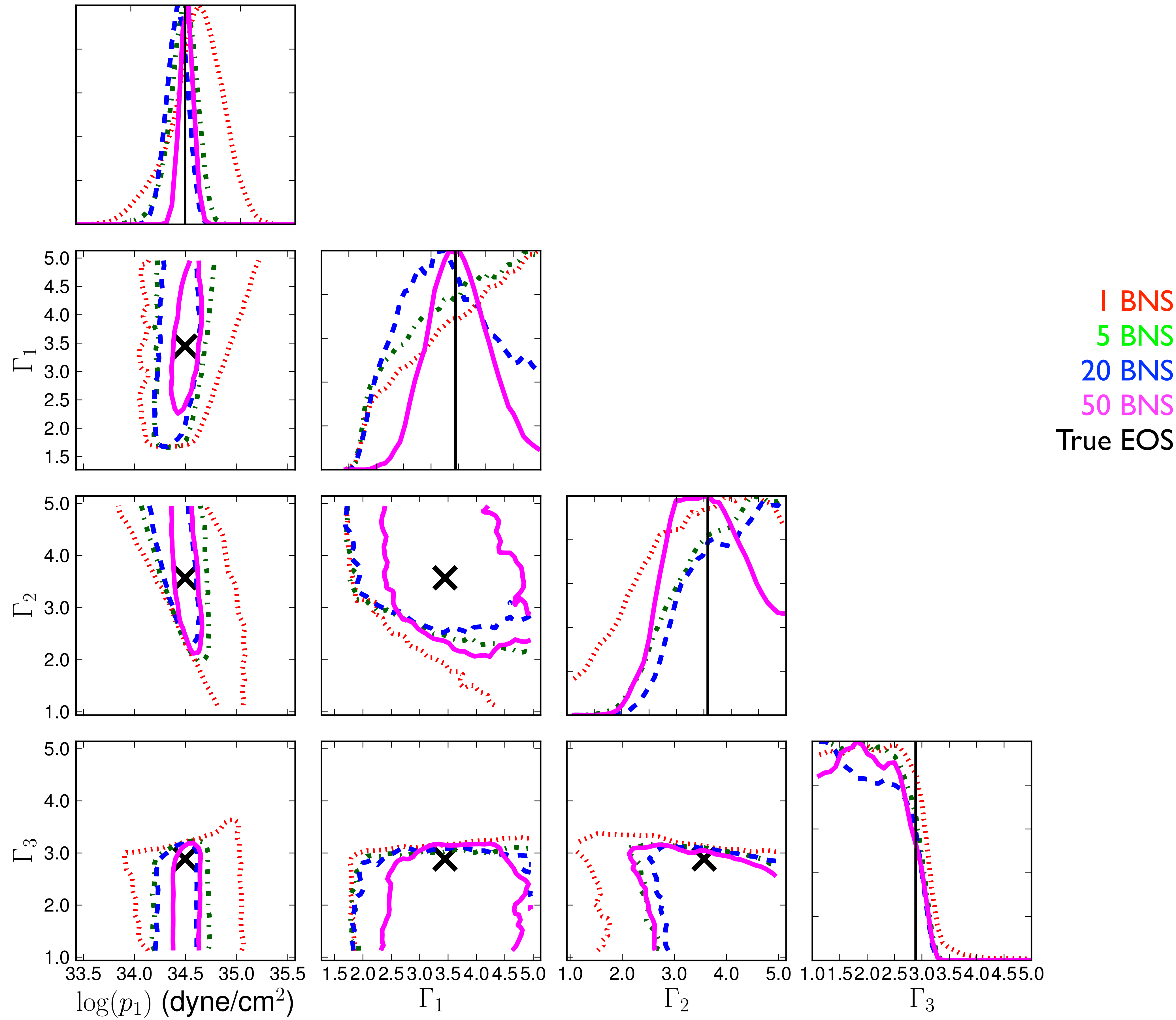
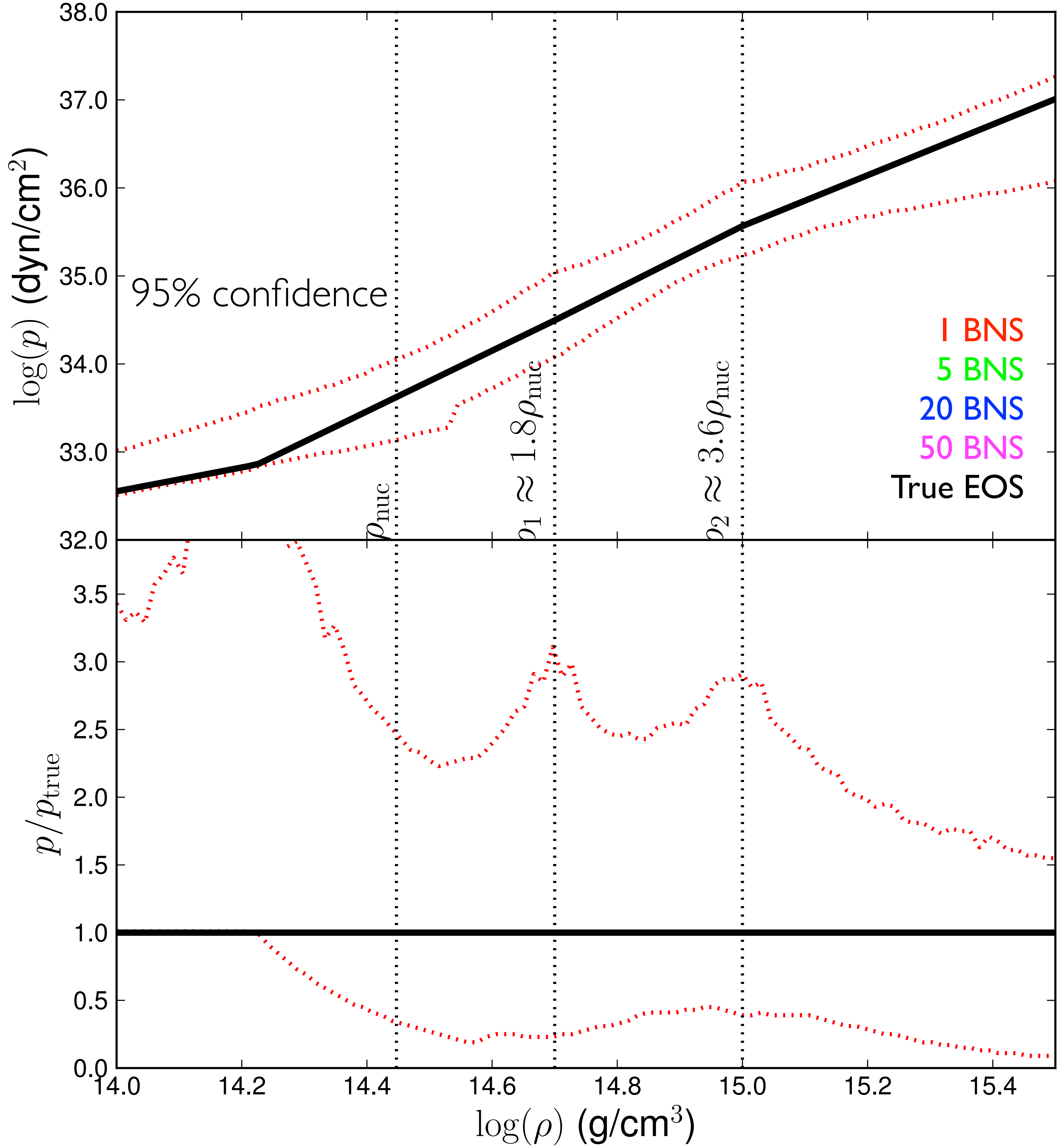


EOS function $p(\rho)$



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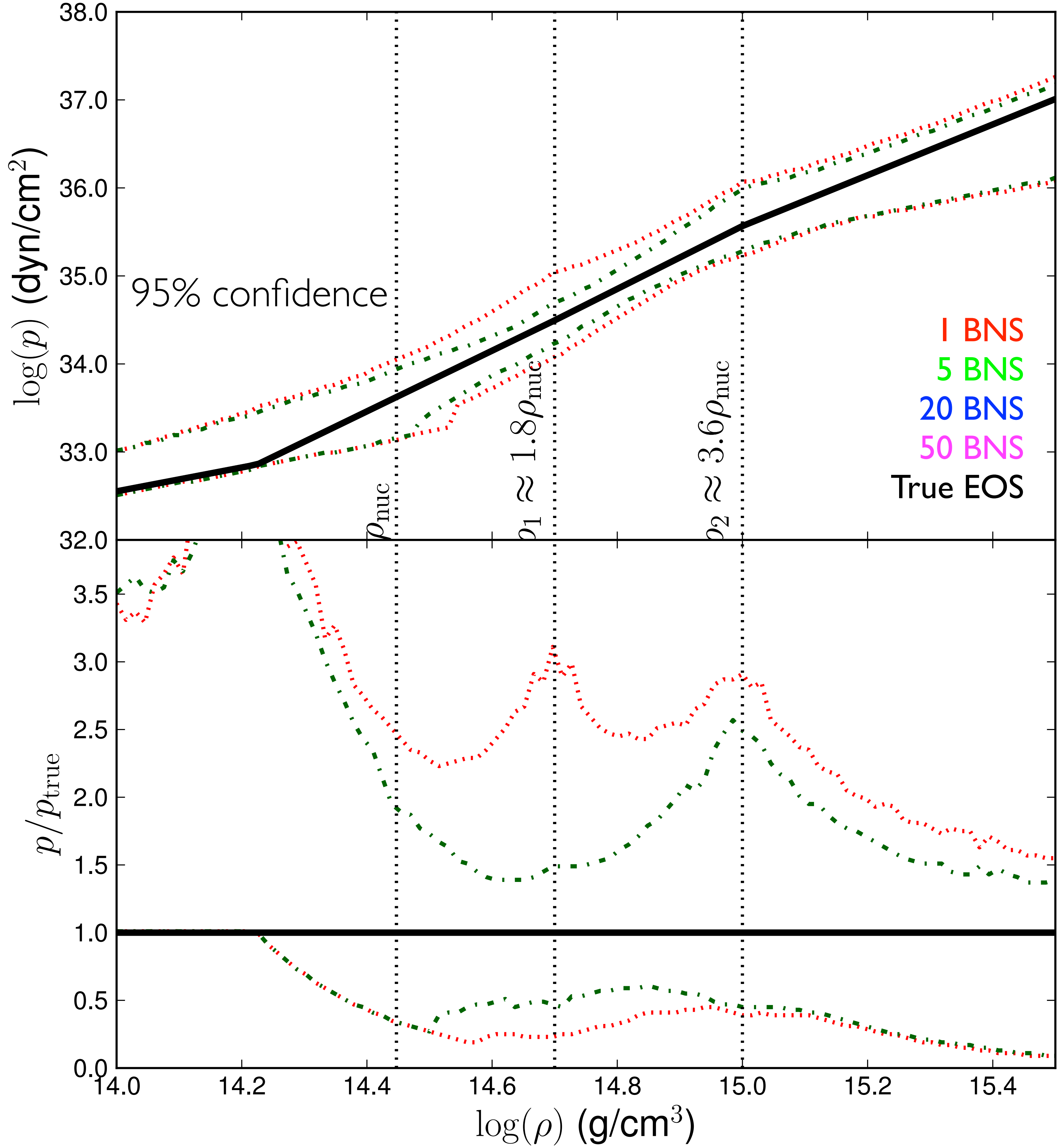

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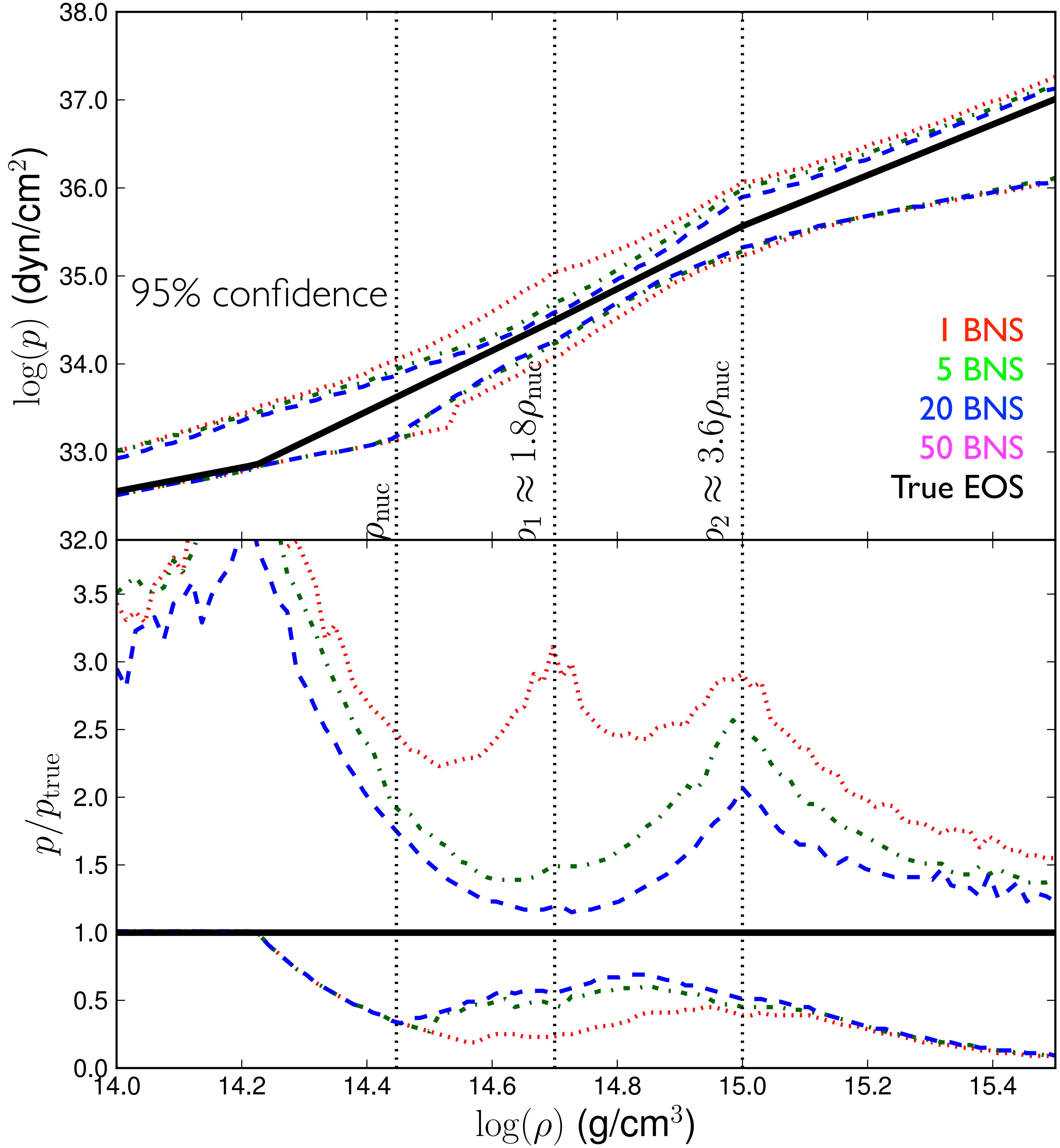

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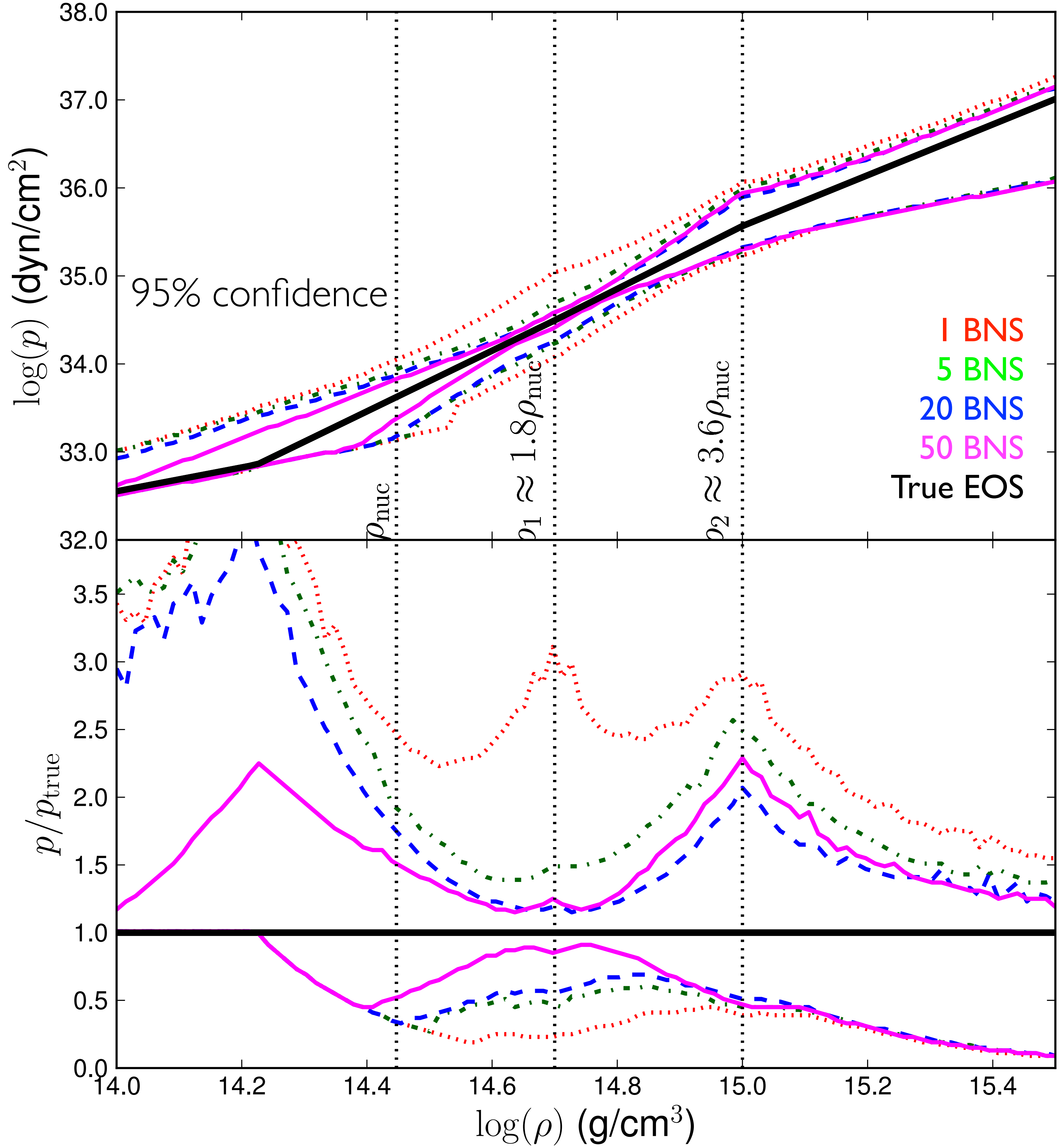
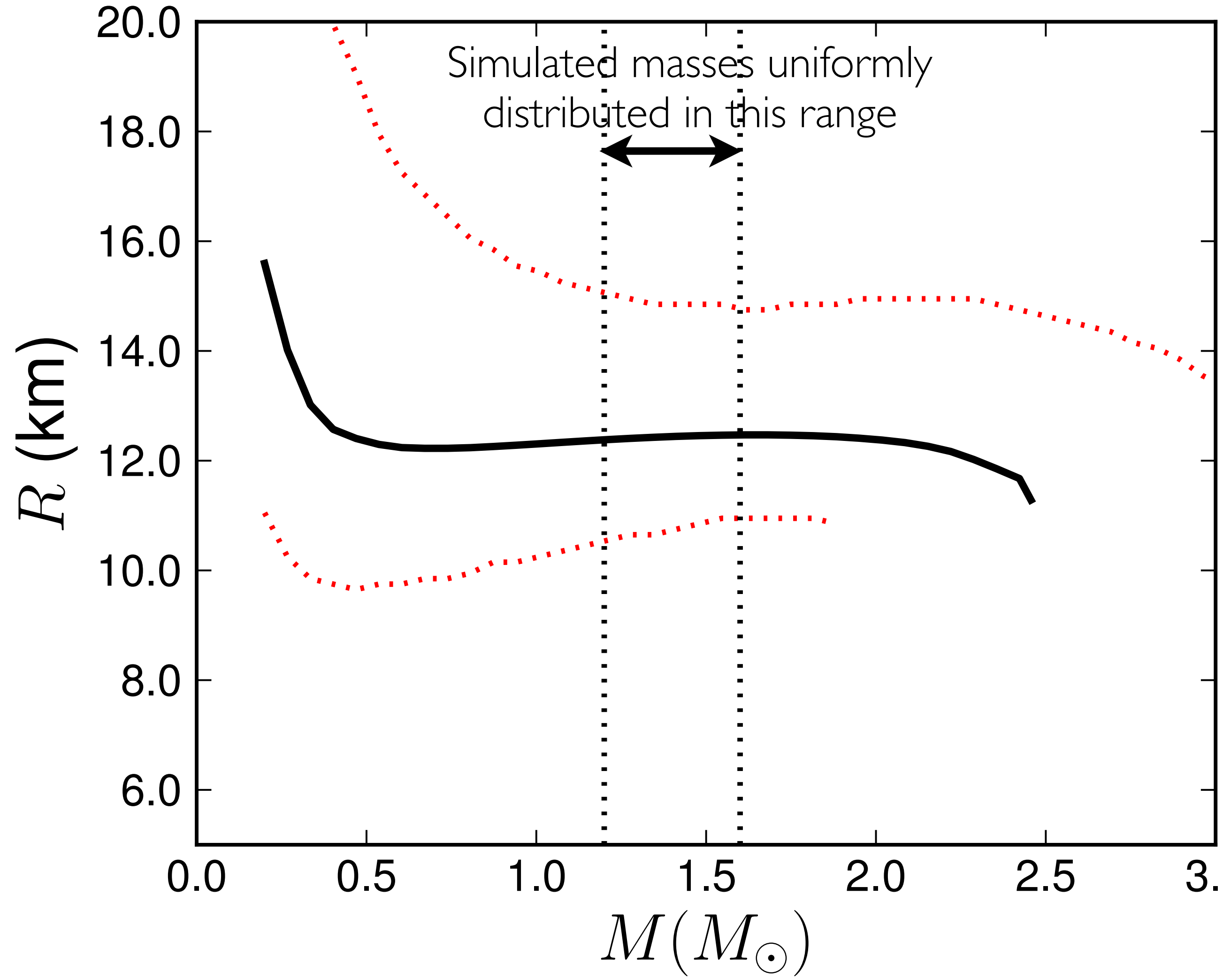
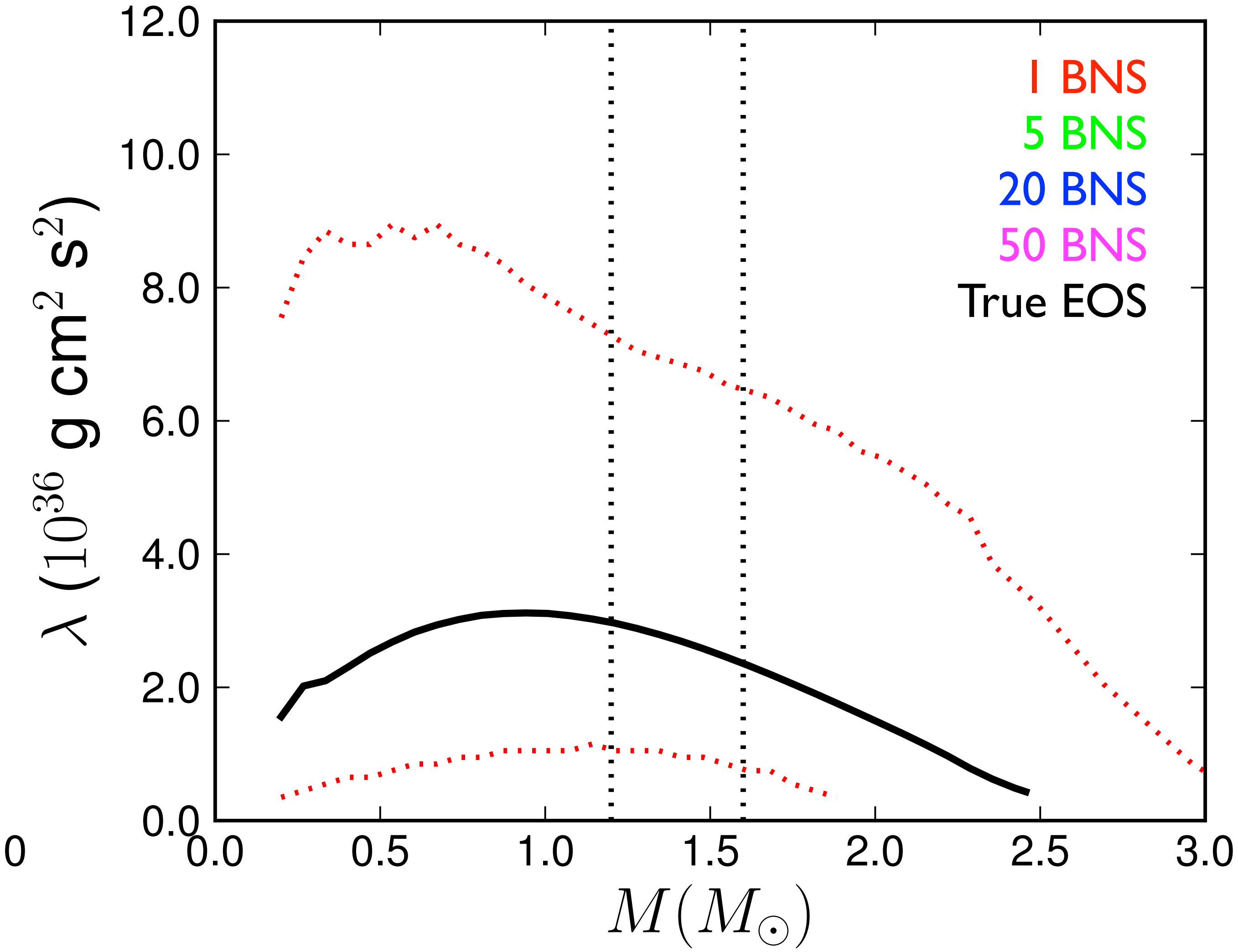


NS Radius and Tidal Deformability

- Del Pozzo et al. found $\lambda(1.4M_{\odot})$ can be measured to +/-10% with 50 sources but the slope of $\lambda(M)$ cannot be measured
- Fitting the EOS function $p(\rho)$ instead of $\lambda(M)$, and using known EOS information dramatically improves the measurement of $\lambda(M)$





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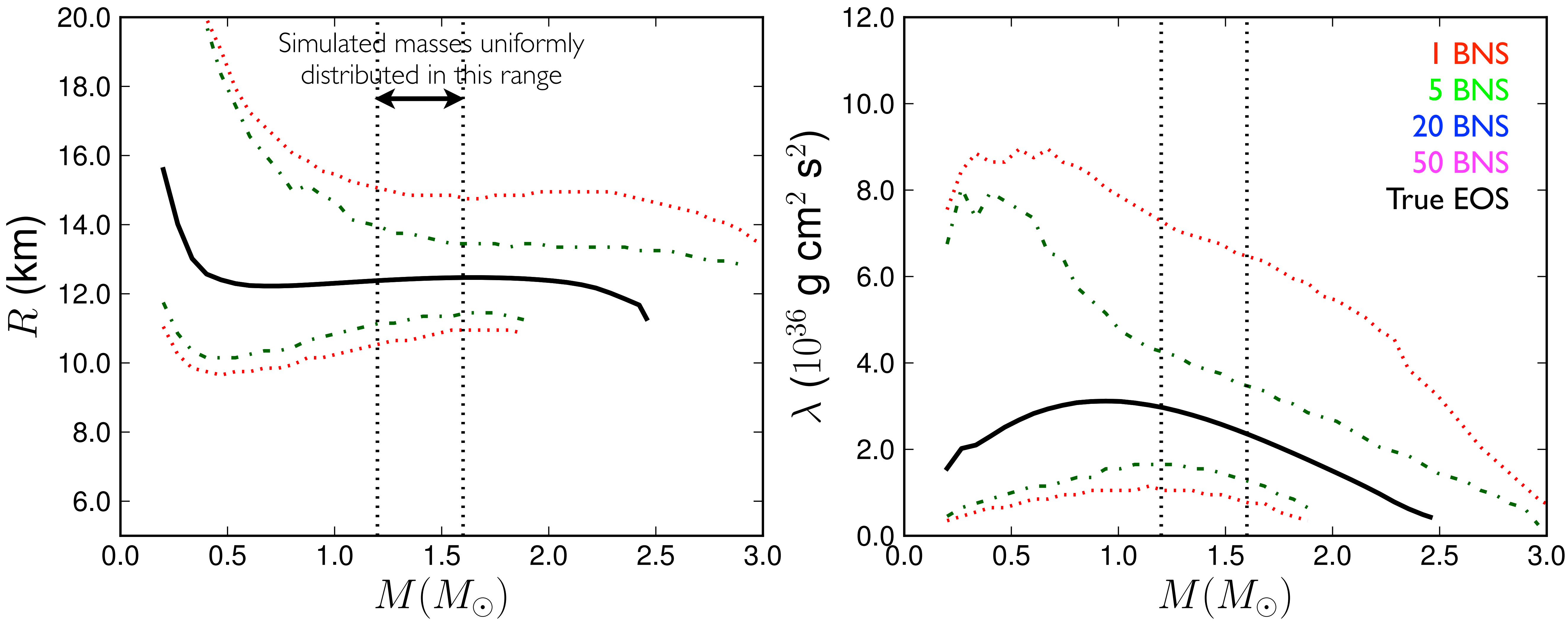

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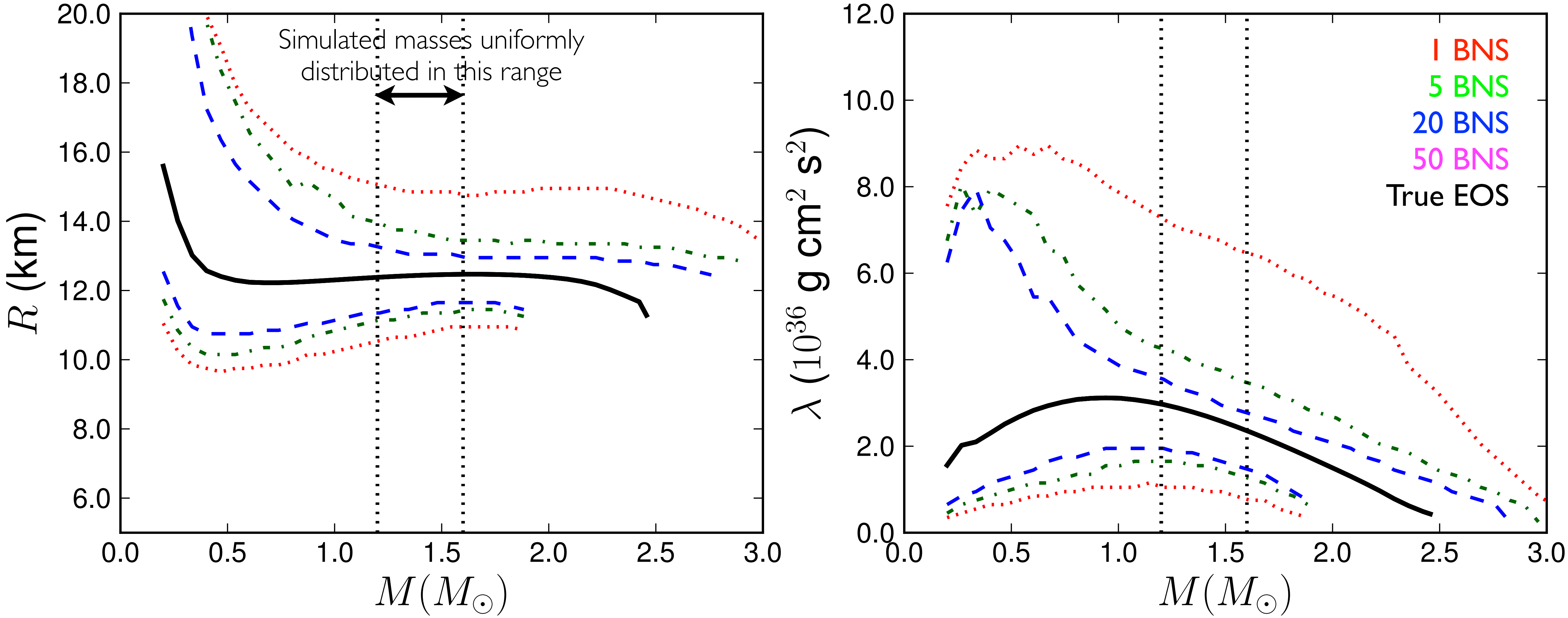

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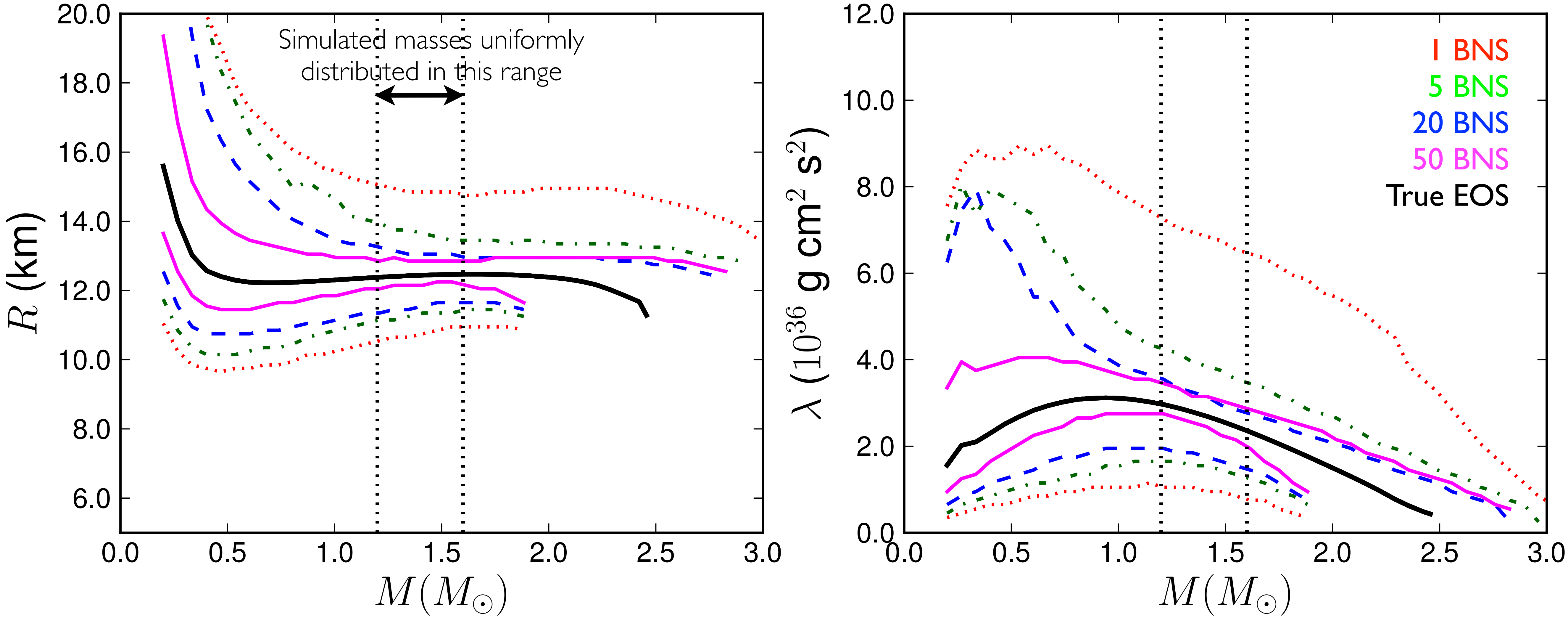
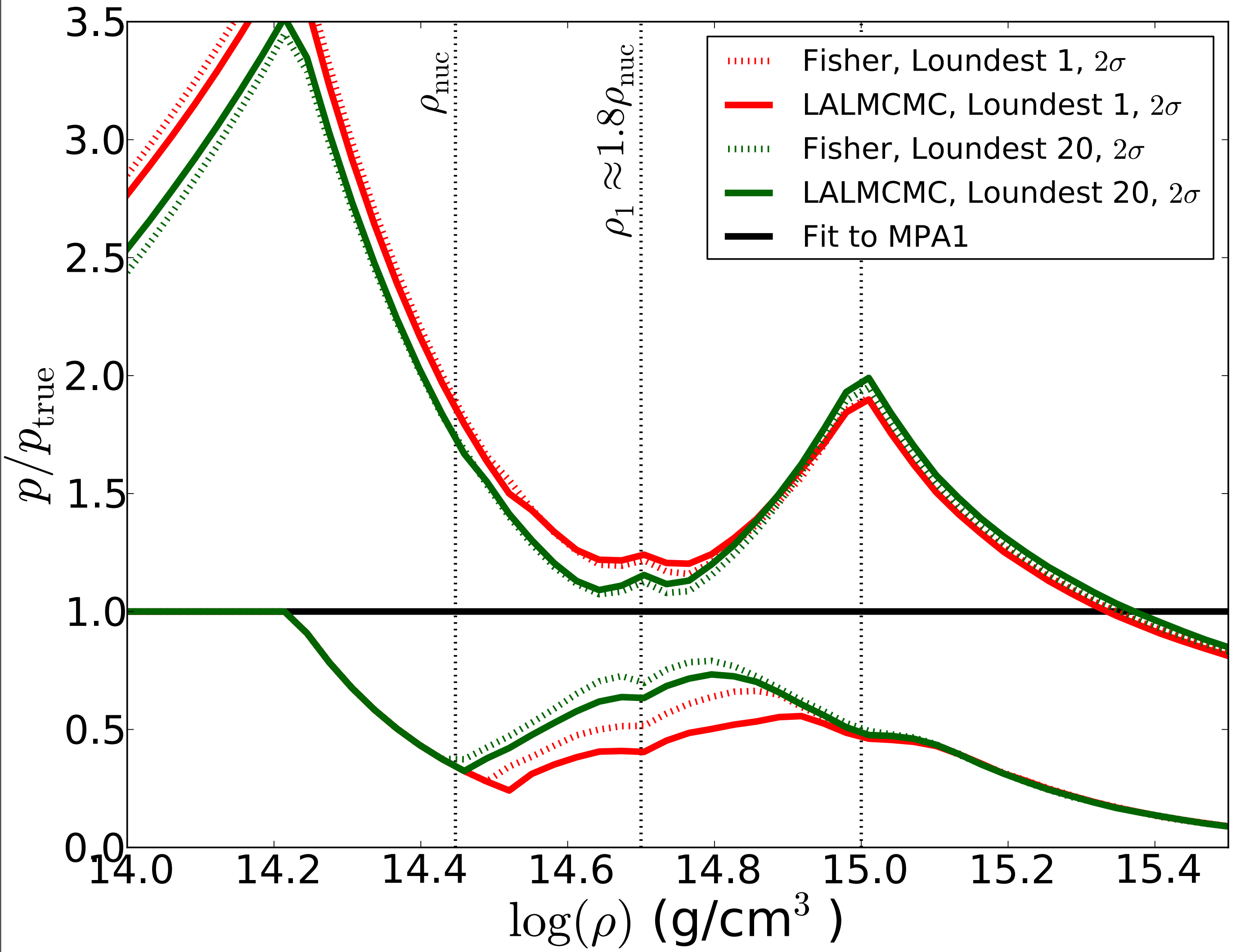
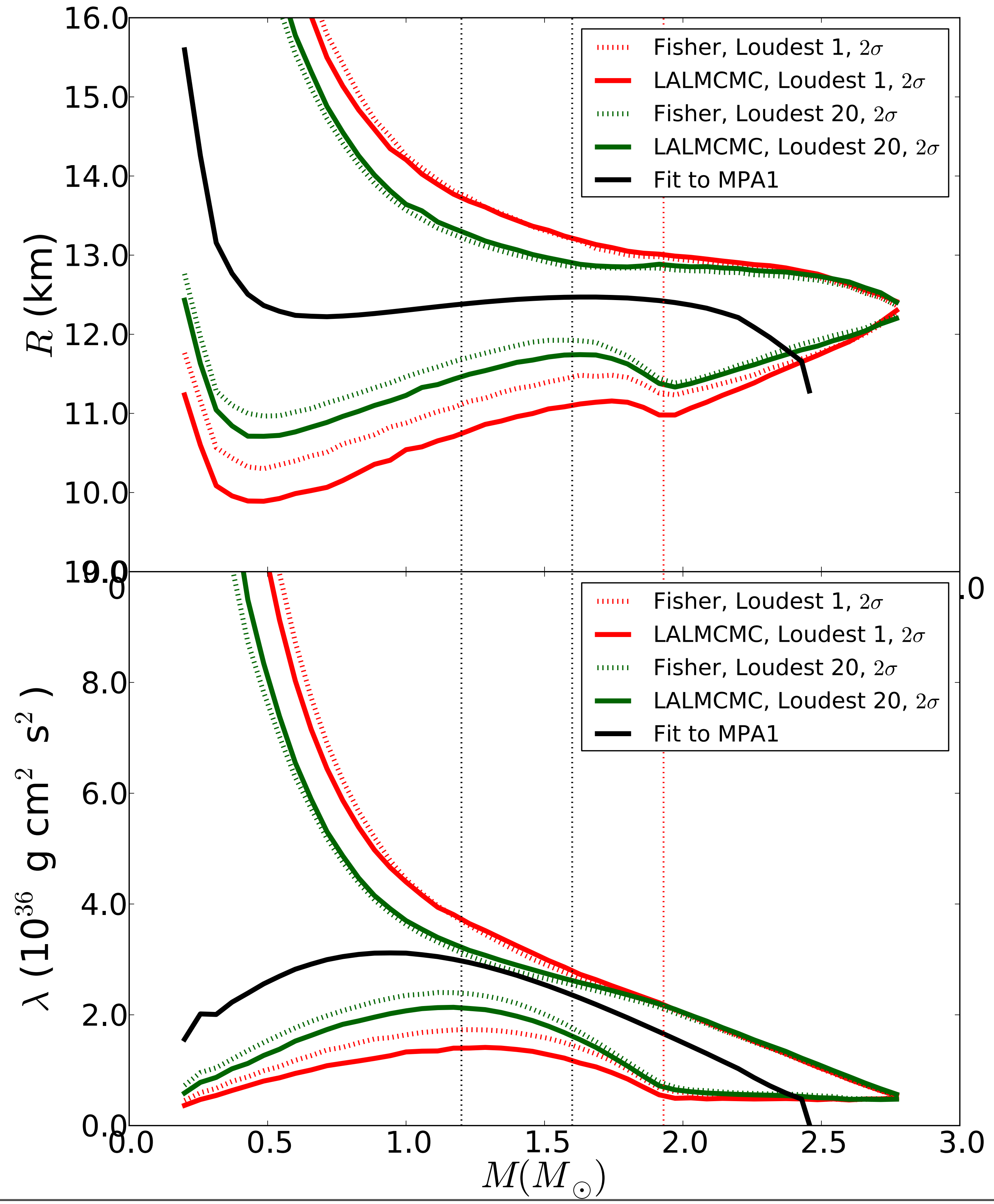


Fisher matrix versus MCMC

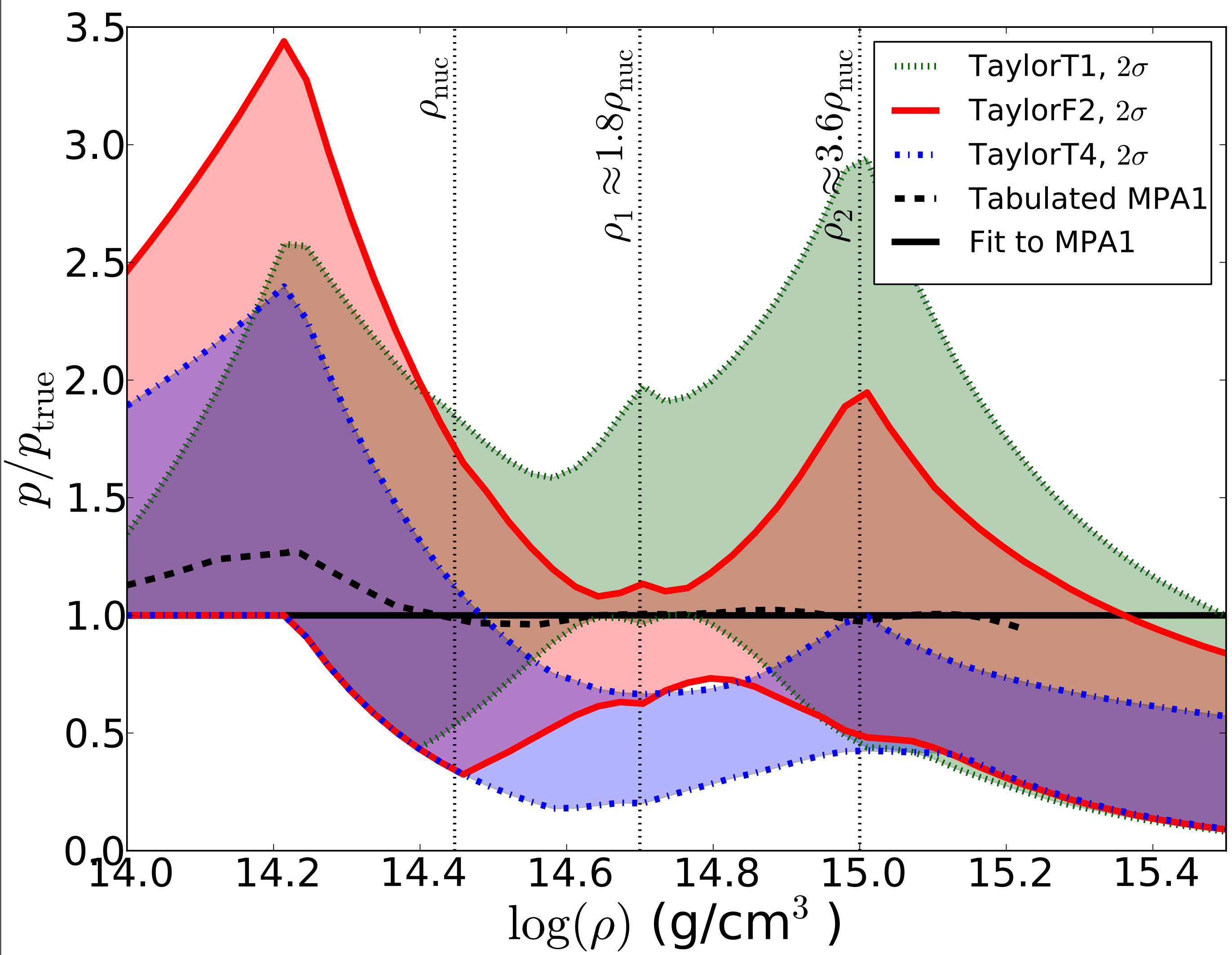
- Fisher matrix gives comparable results to lalinference_mcmc for loudest signals

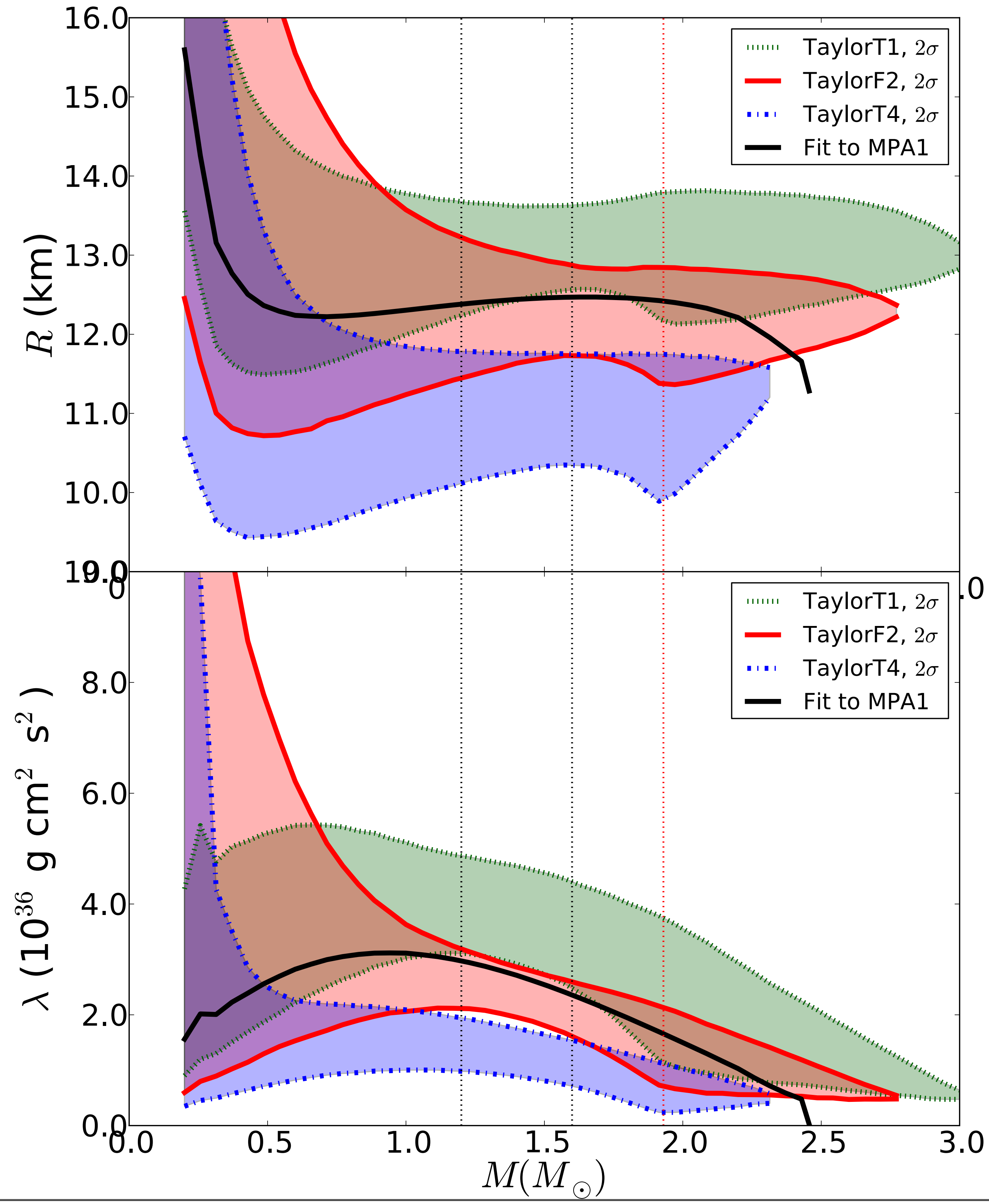




Systematic errors in point particle waveform

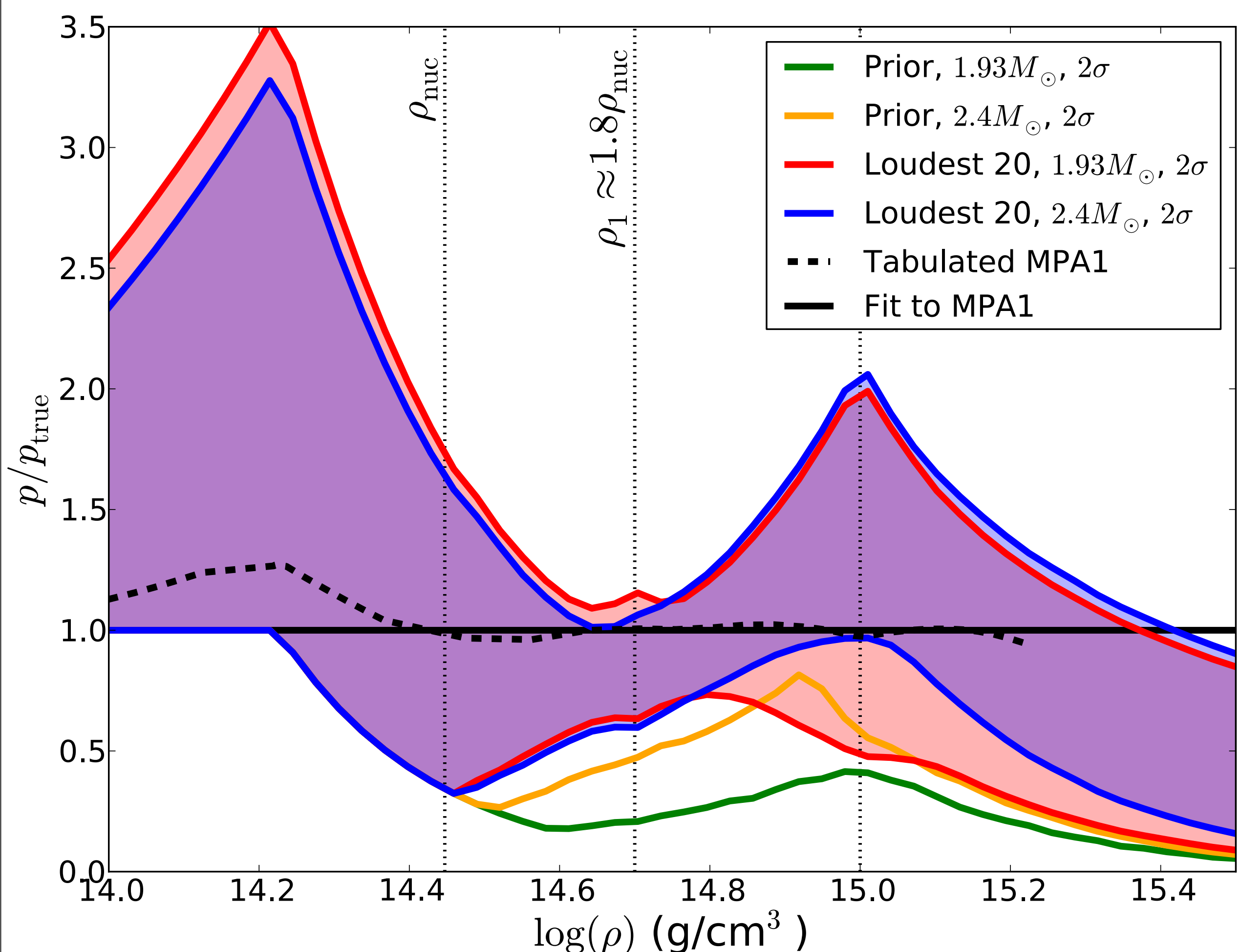
- Injected TaylorF2, TaylorT1, TaylorT4 waveforms and used TaylorF2 as template

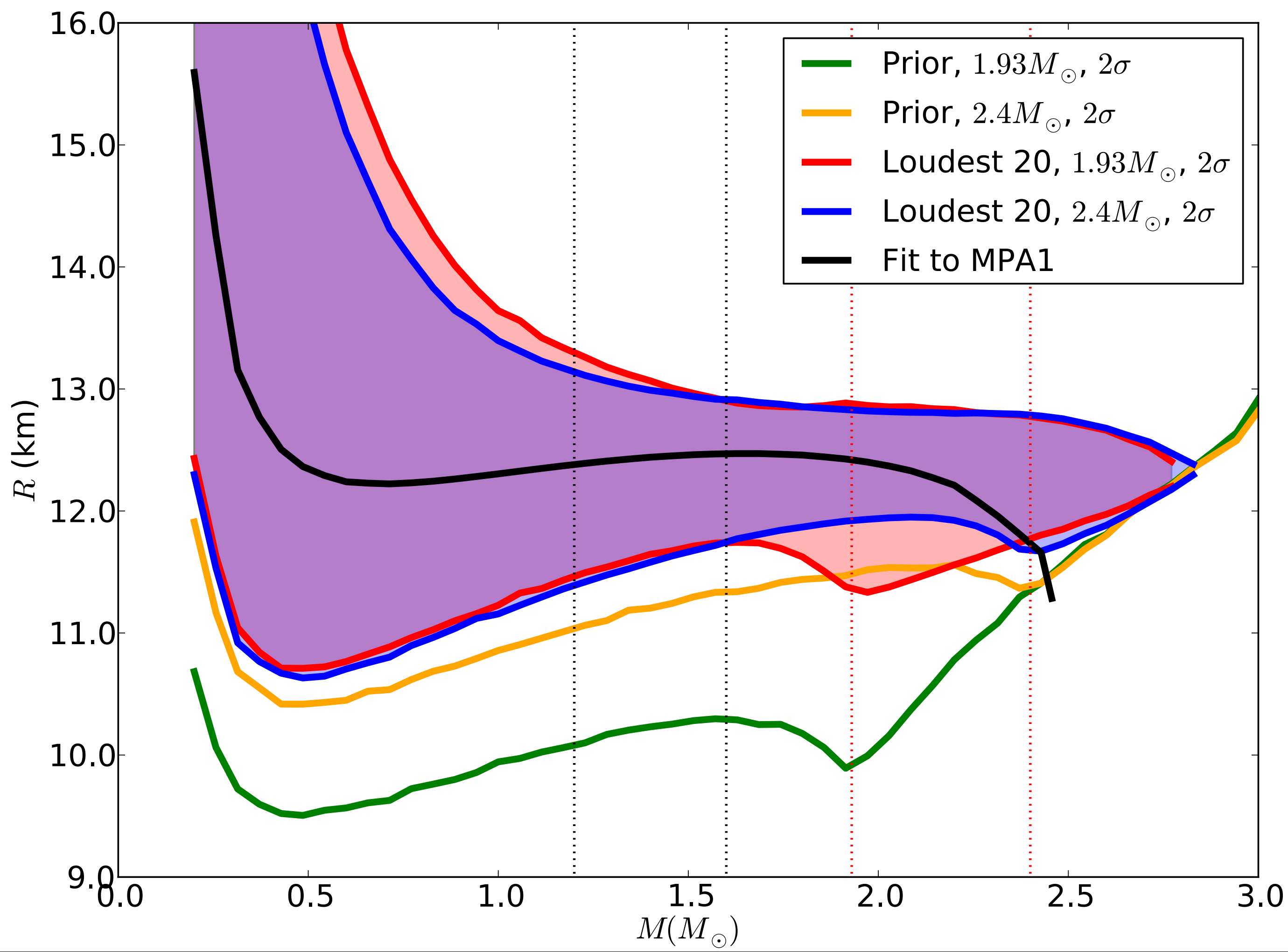




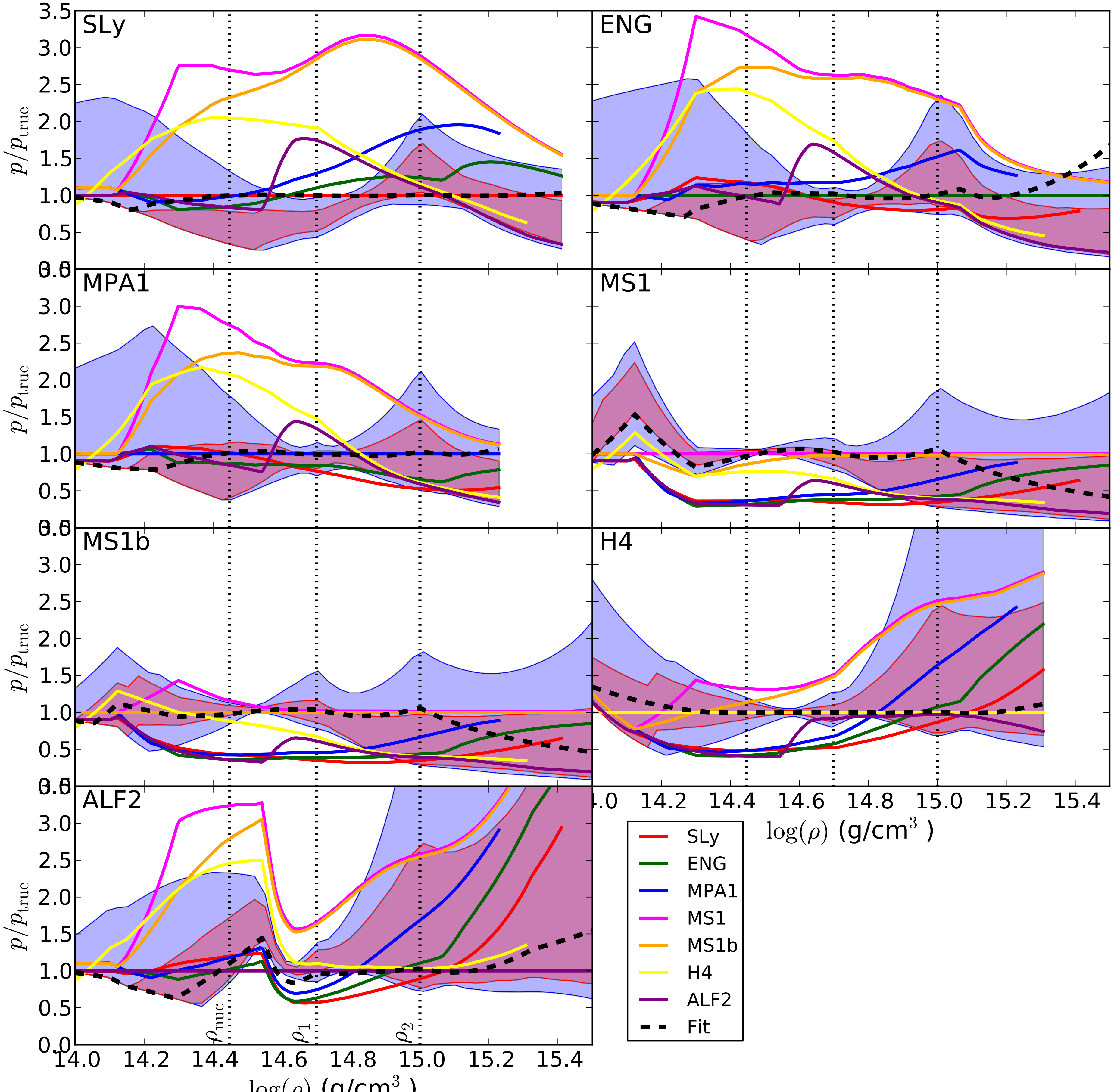
Higher mass NS observations

- Black-widow pulsars may have particularly high masses
  - PSR B1957+20: $2.40 \pm 0.12 M_{\odot}$ with lower limit $1.66 M_{\odot}$
  - PSR J1311-3430: $2.68 \pm 0.14 M_{\odot}$ with lower limit $2.1 M_{\odot}$





Other EOS models



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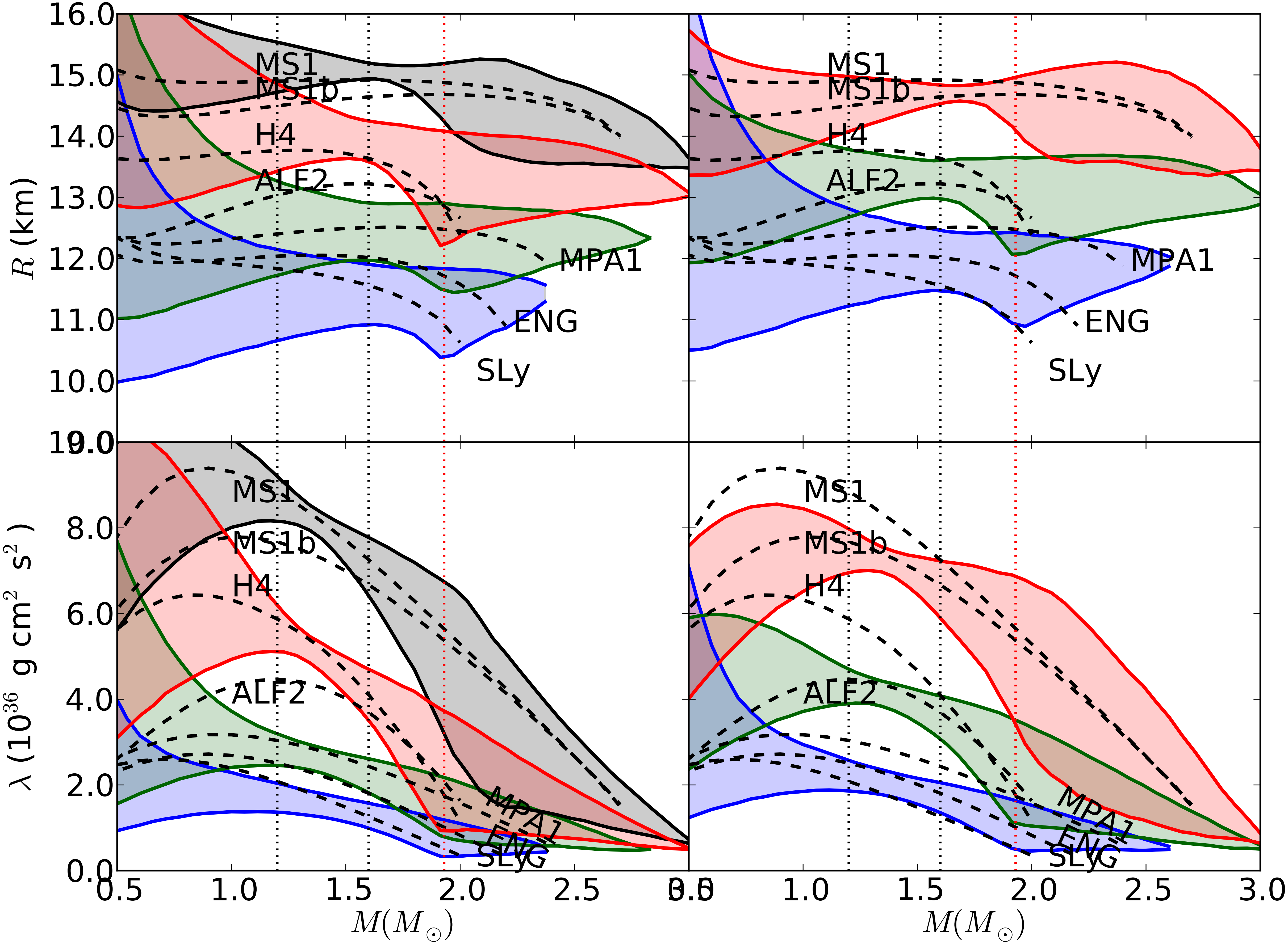

Conclusion

- Detailed EOS information can be found from the inspiral of BNS systems with aLIGO
    - Results are roughly equivalent to mass-radius observations
- Systematic errors from uncertainty in point-particle waveform are significant
    - Unrelated to those in mass-radius observations