Toward a unified description of equilibrium and dynamics of neutron star matter

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Motivation & Outline

- The validity of the models of neutron star matter must be gauged beyond their ability to predict acceptable values of M & R
- Need a *unified* approach providing a *consistent* description of
  - EOS
  - transport properties
  - neutrino interactions
  - supefulid gap
  - …
- The paradigm of nuclear many-body theory
  - *ab initio* approach
  - effective interaction approach
  - bridging the gap: effective interactions from the *ab inito* approach
- Summary & Outlook

The paradigm of nuclear many-body theory

- ⋆ Nuclear matter is described as a collection of pointlike protons and neutrons interacting through the hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}}$$

- ⋆ It has long been realized* that the independent particle – or mean field – approximation, which amounts to replacing

$$\sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}} \to \sum_i U_i \ ,$$

fails to take into account the effects of nucleon-nucleon correlations, which are known to play an important role in determining nuclear structure and dynamics.

*"The limitation of any independent particle model lies in its inability to encompass the correlation between the positions and spins of the various particles in the system" [Blatt & Weiskopf (AD 1952)].

The *ab initio* many-body approach

- ★ The potentials are determined by a fit to the properties of the *exactly solvable* two- and three-nucleon systems
  - v_{ij} strongly constrained by deuteron properties and nucleon-nucleon (NN) scattering data: the ANL v_{18} model, as an example

$$\mathrm{v}_{ij} = \sum_{p=1,18} \mathrm{v}_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^p = [\mathbb{1}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes [\mathbb{1}, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)] \, ,$$

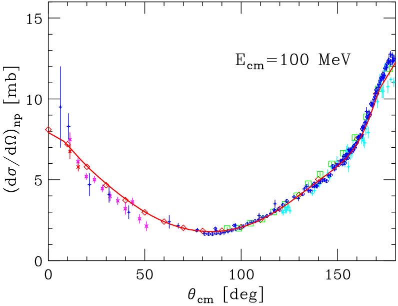
$$[1, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes T_{ij} \quad , \quad (\tau_{zi} + \tau_{zj})$$

  - The three-nucleon potential is determined fitting the properties of the three-nucleon system

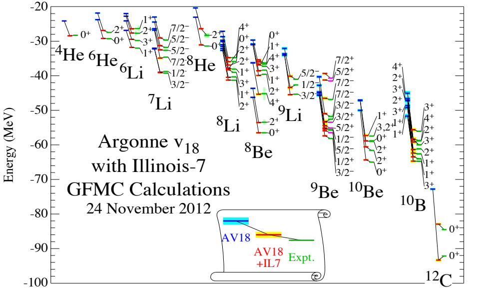
$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{\mathrm{R}}$$

Results of the *ab initio* approach

★ Proton-neutron differential x-section at $E_{\rm cm} = 100$ MeV

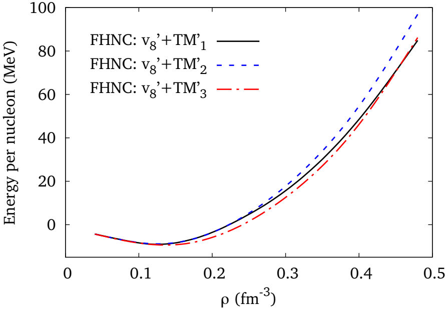


★ Energy level of light nuclei from Green's Function Monte Carlo

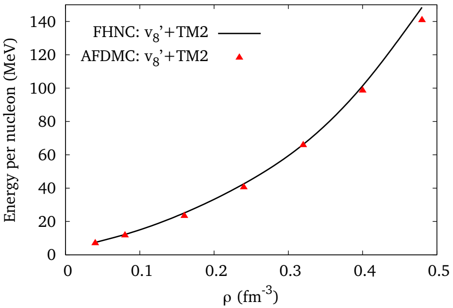


Nuclear matter EOS (no adjustable parameters involved)

★ Binding energy per particle of isospin-symmetric nuclear matter (SNM)



★ Binding energy per particle of pure neutron matter (PNM)



The efective interaction approach

- ⋆ The bare potential is replaced with an *efective* potential, suitable for use within the framework of perturbation theory in the Fermi gas basis. The Skyrme potential, as an example

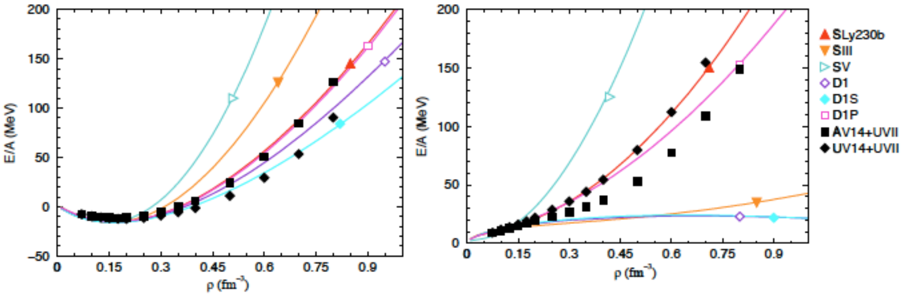
$$v^{\rm eff}_{ij} = \delta(\mathbf{r}_i - \mathbf{r}_j) t(\mathbf{k}, \mathbf{k}')$$

$$\mathbf{k} = \frac{i}{2}(\overrightarrow{\boldsymbol{\nabla}}_1 - \overrightarrow{\boldsymbol{\nabla}}_2) \quad , \quad \mathbf{k}' = \frac{i}{2}(\overleftarrow{\boldsymbol{\nabla}}_1 - \overleftarrow{\boldsymbol{\nabla}}_2)$$

- ⋆ The above definition can be generalized to include spin-dependence. The parameters involved are adjusted in such a way as to reproduce selected nuclear properties, as well as the equilibrium properties of isospin symmetric nuclear matter.
- ⋆ The ground state expectation value of the hamiltonian can be written in the form of a *energy-density functional*

$$\langle H \rangle = \langle \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v^{\rm eff}_{ij} \rangle = \mathcal{E}(\rho_p, \rho_n)$$

Results of the effective interaction approach



- ⋆ Equation of state (EOS) of SNM and PNM computed using different Skyrme- and Gogny-type effective interactions, compared to the variational results obtained from the Argonne-Urbana hamiltonians.
- ⋆ The effective interactions, while being capable to provide a reasonable description of the EOS, are limited by the approximations involved in their definition, lacking a direct connection to the underlying nuclear interactions.

Ab initio effective interaction

- In the *ab initio* approach the uncertainty associated with the dynamical model is decoupled from the approximations involved in many-body calculations
- Once the nuclear hamiltonian is determined, *in principle* its eigenstates can be obtained from the solution of the Schrödinger equation

$$H\,|n\rangle = E_n\,|n\rangle$$

- Calculation of nuclear observables do not involve any additional parameters
- The Schrödinger equation can only be solved for nuclei with mass number $A \leq 12$. Approximations are required for larger A, as well as for uniform nuclear matter in the $A \to \infty$ limit.

Correlated Basis Function (CBF) formalism

- ⋆ The eigenstates of the nuclear hamiltonian are approximated by the set of correlated states, obtained from the eigenstates of the Fermi Gas (FG) model

$$|n\rangle = \frac{F|n_{FG}\rangle}{\langle n_{FG}|F^\dagger F|n_{FG}\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}} F\,|n_{FG}\rangle \quad , \quad F = \mathcal{S} \prod_{j>i} f_{ij}$$

- ⋆ the structure of the two-nucleon correlation operator reflects the complexity of nuclear dynamics

$$f_{ij} = \sum_p f_p(r_{ij}) O^p_{ij}$$

- ⋆ the operators O^n_{ij} are the same as those entering the definition of the NN potential

Cluster expansion and FHNC equations

- ⋆ The ground state expectation value of the hamiltonian is written as a sum of contributions associated with subsystems (clusters) consisisting of an increasing number of particles

$$\langle H \rangle = \frac{\langle 0|H|0\rangle}{\langle 0|0\rangle} = E_{FG} + \sum_{n\geq 2} (\Delta E)_n$$

- ⋆ The relevant terms of the cluster expansion can be summed up at all orders solving a set of integral equations known as Fermi Hyper-Netted Chain (FHNC) equations
- ⋆ the shapes of the $f_p(r_{ij})$ are determined form the minimization of the ground-state expectation value of the hamiltonian

$$E_0 \geq \min_F \frac{\langle 0_{FG}|F^\dagger H F|0_{FG}\rangle}{\langle 0_{FG}|0_{FG}\rangle}$$

Alternative approach: the CBF effective interaction

- Within CBF, the effective interaction is defined through

$$\langle H \rangle = \frac{\langle 0|F^\dagger (T+V) F|0\rangle}{\langle 0|F^\dagger F|0\rangle} = \langle 0_{FG}|T + V_{\rm eff}|0_{FG}\rangle$$

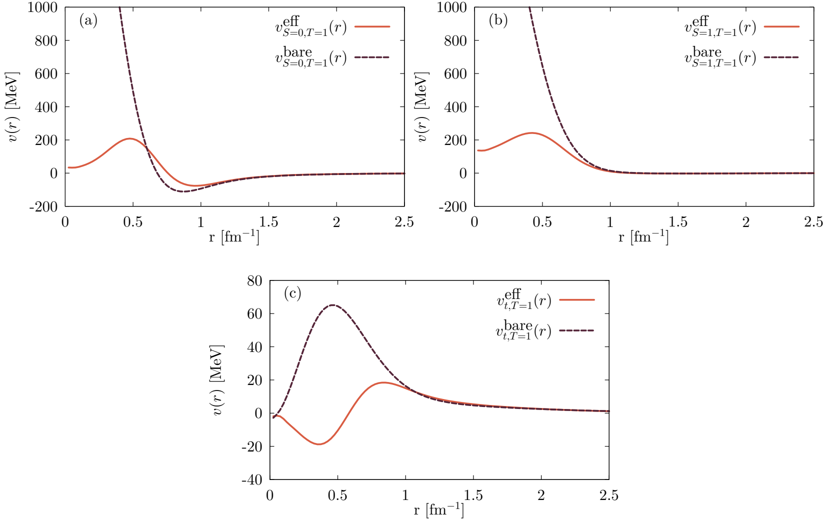
- At two-body cluster level

$$V_{\rm eff} = \sum_{j>i} {\rm v}_{\rm eff}(ij)$$

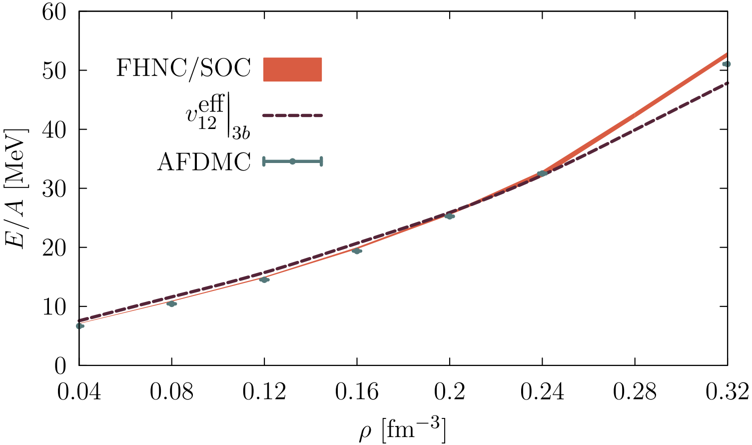
$${\rm v}_{\rm eff}(ij) = f_{ij}^\dagger \left[-\frac{1}{m}(\nabla^2 f_{ij}) - \frac{2}{m}(\boldsymbol{\nabla} f_{ij}) \cdot \boldsymbol{\nabla} + {\rm v}_{ij} f_{ij} \right]$$

- Three-nucleon interactions can be taken into account extending the definition to include three-body cluster contributions

CBF effective interaction at SNM equilibrium density



EOS of PNM obtained using the CBF effective interaction



Transport properties

- ⋆ Landau-Abrikosov-Khalaktnikov formalism: Boltzman equation

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial \epsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial \epsilon}{\partial \mathbf{r}} = I(n)$$

$$n = n_0 + \delta n \quad , \quad n_0 = \{1 + \exp[\beta(\epsilon - \mu)]\}^{-1}$$

- ⋆ The collision integral $I(n)$ depends on the probability of the *in medium NN* scattering process

$$W = \frac{16\pi^2}{m^{\star 2}} \left(\frac{d\sigma}{d\Omega}\right)$$

- ⋆ The description of transport properties require dynamical models providing an accurate description of NN scattering in the nuclear medium, constrained by the available data in the zero-density limit

Shear Viscosity of pure neutron matter

- ⋆ Abrikosov-Khalatnikov (AK) estimate of the shear viscosity in the low-temperature limit

$$\eta_{AK} = \frac{1}{5}\rho m^{\star} \mathrm{v}_F^2 \tau \, \frac{2}{\pi^2(1-\lambda_\eta)}$$

- ⋆ Quasiparticle lifetime

$$\tau T^2 = \frac{8\pi^4}{m^{*3}} \, \frac{1}{\langle W \rangle} \; ,$$

- ⋆ Angle-averaged collision probability

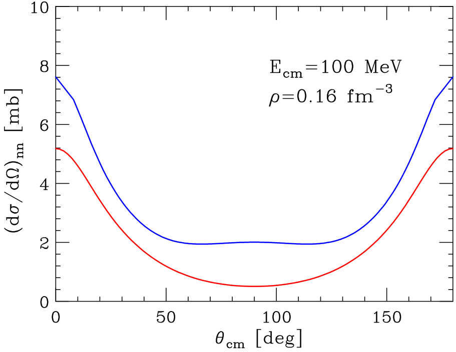
$$\langle W \rangle = \int \frac{d\Omega}{2\pi} \, \frac{W(\theta,\phi)}{\cos(\theta/2)} \quad , \quad \lambda_\eta = \frac{\langle W[1 - 3\sin^4(\theta/2)\sin^2\phi] \rangle}{\langle W \rangle}$$

In medium neutron-neutron cross section

- ⋆ From Fermi's golden rule

$$W(\mathbf{p},\mathbf{p}') = 2\pi \left|\hat{v}_{\text{eff}}(\mathbf{p}-\mathbf{p}')\right|^2 \rho(\mathbf{p}')$$

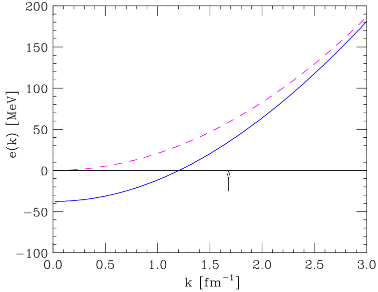
$$\frac{d\sigma}{d\Omega_{\mathbf{p}'}} = \frac{m^{\star 2}}{16\pi^2} \left|\hat{v}_{\text{eff}}(\mathbf{p}-\mathbf{p}')\right|^2$$

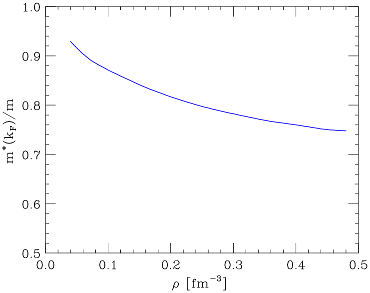


Single particle spectrum and effective mass

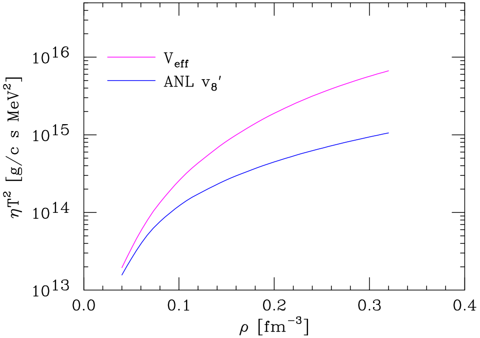
- ⋆ Calculations carried out within the Hartree-Fock approximation using the CBF effective interaction

$$e(k) = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | \mathrm{v}_{\mathrm{eff}} | \mathbf{k}\mathbf{k}' \rangle_a \quad , \quad \frac{1}{m^\star} = \frac{1}{k} \, \frac{de(k)}{dk}$$



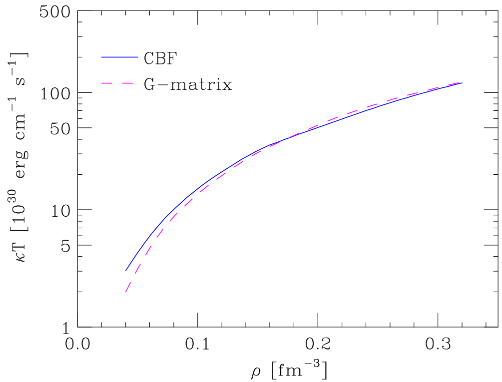


★ Density dependence of ηT^2 n pure neutron matter



★ Note: the SLya effective interaction, adjusted to reproduce the the microscopic EOS, predicts $\eta T^2 \sim 6 \times 10^{13}$ g cm^{-1} s^{-1} MeV^2 at nuclear matter equilibrium density, to be compared with the result obtained from the CBF effective interaction $\eta T^2 \sim 1.4 \times 10^{15}$ g cm^{-1} s^{-1} MeV^2 .

Thermal conductivity of pure neutron matter



- ⋆ The transport coefficients computed using the CBF effective interaction is remarkably close to the result obtained within the G-matrix approach using the same bare NN potential. Note: three-body interactions are not taken into account.

Neutrino interactions in nuclear matter

- Neutral current interactions in neutron matter

$$J_Z^\mu = \sum_i j_i^\mu \quad , \quad j_i^\mu = \overline{\psi}_{n_i} \gamma^\mu (1 - C_A \gamma^5) \psi_{n_i}$$

- In the non relativistic limit

$$J_Z^0 \to \hat{O}_{\mathbf{q}}^\rho = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \quad , \quad \mathbf{J}_Z \to \hat{O}_{\mathbf{q}}^{\boldsymbol{\sigma}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \boldsymbol{\sigma}_i$$

- Neutrino scattering rate and response functions

$$W(\mathbf{q},\omega) = \frac{G_F^2}{4\pi^2} \left[(1 + \cos\theta) S^\rho(\mathbf{q},\omega) + \frac{C_A^2}{3} (3 - \cos\theta) S^\sigma(\mathbf{q},\omega) \right] ,$$

$$S^\rho(\mathbf{q},\omega) = \frac{1}{N} \sum_n |\langle n|\hat{O}_{\mathbf{q}}^\rho|0\rangle|^2 \delta(\omega + E_0 - E_n) \quad , \quad S^\sigma(\mathbf{q},\omega) = \sum_\alpha S_{\alpha\alpha}^\sigma(\mathbf{q},\omega)$$

$$S_{\alpha\beta}^\sigma(\mathbf{q},\omega) = \frac{1}{N} \sum_n \langle n|\hat{O}_{\mathbf{q}}^{\sigma_\alpha}|0\rangle\langle 0|\hat{O}_{\mathbf{q}}^{\sigma_\beta}|n\rangle \delta(\omega + E_0 - E_n)$$

Density and spin responses of pure neutron matter

- The target response tensor

$$W_A^{\mu\nu} = \sum_n \langle 0 | J_Z^{\mu\,\dagger} | n \rangle \langle n | J_Z^{\nu} | 0 \rangle \delta(\omega + E_0 - E_n)$$

must be computed using correlated initial and final states, which amounts to compute the transition matrix element of the *effective operator*

$$\widetilde{J}_A^{\mu} = \frac{1}{\sqrt{\mathcal{N}_0 \mathcal{N}_n}} F^{\dagger} J_A^{\mu} F$$

between FG states $|n)$

- In the one particle-one hole sector

$$|n\rangle = \frac{1}{\sqrt{\mathcal{N}_{ph}}} F|ph) \quad , \quad \langle n | J_A^{\nu} | 0 \rangle \to (ph | \widetilde{J}_A^{\mu} | 0)$$

Including long range correlations

- ⋆ Allow for propagation of the particle-hole pair, giving rise to the excitation of collective modes. Replace

$$|n\rangle \to \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the $N \times N$ hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i | \mathrm{v}_{\mathrm{eff}} | h_j p_j)$$

with the CBF effective interaction and the Hartree-Fock spectrum

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | \mathrm{v}_{\mathrm{eff}} | \mathbf{k}\mathbf{k}' \rangle_a$$

Alternative approach: Landau theory

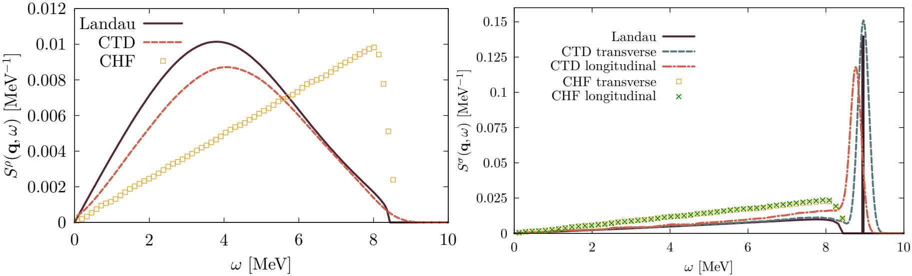
- ⋆ Landau theory of normal Fermi liquids can also be employed to obtain the density and spin responses of pure neutron matter
- ⋆ the value of the Landau parameters can be obtained from the quasiparticle interaction, which can be in turn expressed in terms of matrix elements of the effective interaction

$$
\begin{aligned}
f_{\sigma\sigma'\mathbf{p}\mathbf{p}'} &= f_{\mathbf{p}\mathbf{p}'} + g_{\mathbf{p}\mathbf{p}'}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}') + f_{\mathbf{p}\mathbf{p}'}S_{12}(\mathbf{p}-\mathbf{p}') \\
&= \langle \mathbf{p}\sigma\ \mathbf{p}'\sigma' | \mathrm{v}_{\rm eff} | \mathbf{p}\sigma\ \mathbf{p}'\sigma' \rangle - \langle \mathbf{p}\sigma\ \mathbf{p}'\sigma' | \mathrm{v}_{\rm eff} | \mathbf{p}'\sigma'\ \mathbf{p}\sigma \rangle
\end{aligned}
$$

- ⋆ this formalism can be easily extended to non zero temperatures, in the range $T << T_F$

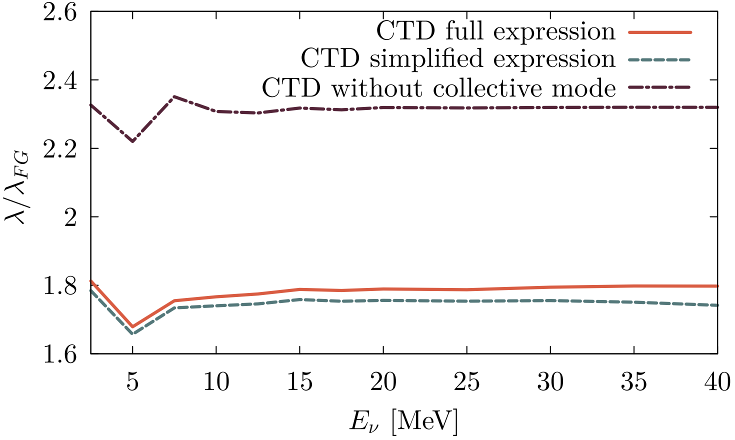
Charged current interactions at low-momentum transfer

- ⋆ Fermi (density, left) and Gamow-Teller (spin, right) contributions to the response of pure neutron matter at nuclear matter equilibrium density ($\rho_0 = 0.16$ fm^{-3}) and momentum transfer $|\mathbf{q}| = 0.1$ fm^{-1}

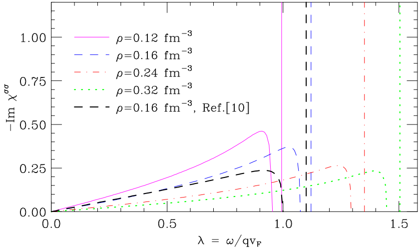


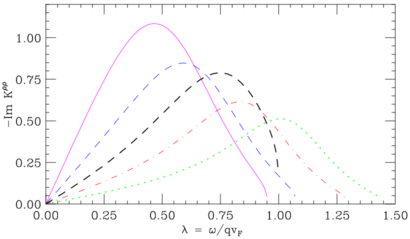
- ⋆ the collective mode is only excited in the spin channel

Neutrino mean free path in neutron matter at $\rho = \rho_0$



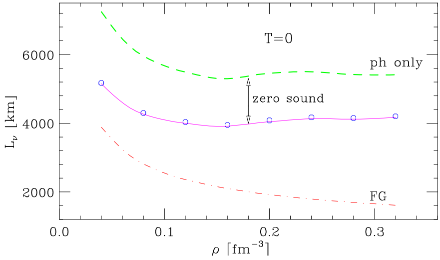
★ Dependence on momentum transfer at $\rho = \rho_0$

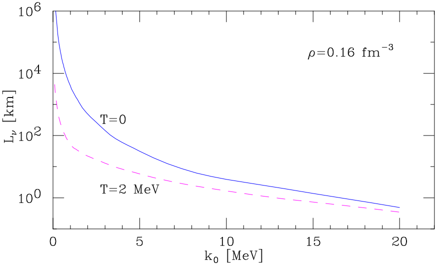




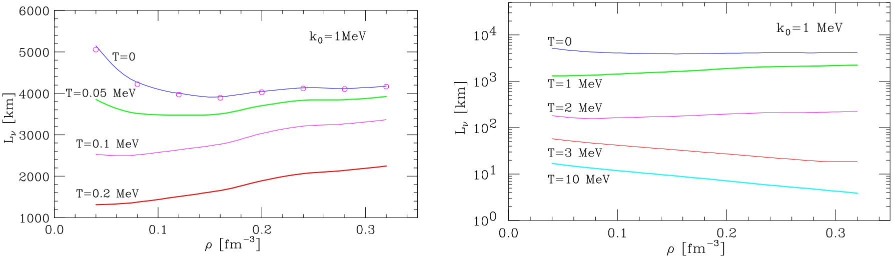
★ Mean free path of a non degenerate neutrino in neutron matter

- ▹ Left: density-dependence at $k_0 = 1$ MeV and $T = 0$
- ▹ Right: energy dependence at $\rho = 0.16$ fm^{-3} and $T = 0, 2$ MeV





★ Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm^{-3}

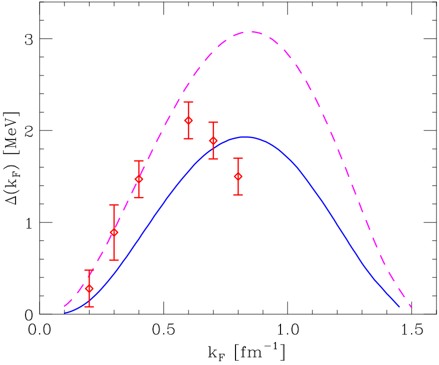


Neutron pairing in the $^1S_0$ channel

★ Gap equation

$$\Delta(k) = -\frac{1}{\pi} \int k'^2 dk' \frac{\mathrm{v}(k,k')\Delta(k')}{[(e(k') - \mu)^2 + \Delta^2(k')]^{1/2}}$$

$$\mathrm{v}(k,k') = \int r^2 dr j_0(kr) \mathrm{v}_{\mathrm{eff}}(r) j_0(k'r)$$



Summary & Outlook

- ⋆ Resolving the degeneracy associated with models of the EOS providing similar values of neutron stars' mass and radius will require the study of different properties
- ⋆ This analysis will in turn require the development of *novel approaches*, allowing for a *consistent* description based on a *unified dynamical model*
- ⋆ Effective interactions *obtained from realistic nuclear hamiltonians* provide a powerful tool to carry out calculations of a number of different quantities, ranging from the EOS to single particle properties and in medium scattering probabilities
- ⋆ The model dependence associated with the many-body approach employed to obtain the effective interaction apperas to be remarkably weak