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STRONGLY INTERACTING FERMIONS AND BOSONS IN ONE DIMENSION

EXACT RESULTS AND MAGNETIC CORRELATIONS

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Institute for Nuclear Theory Program

'Universality in Few-Body Systems – Theoretical Challenges and New Directions'

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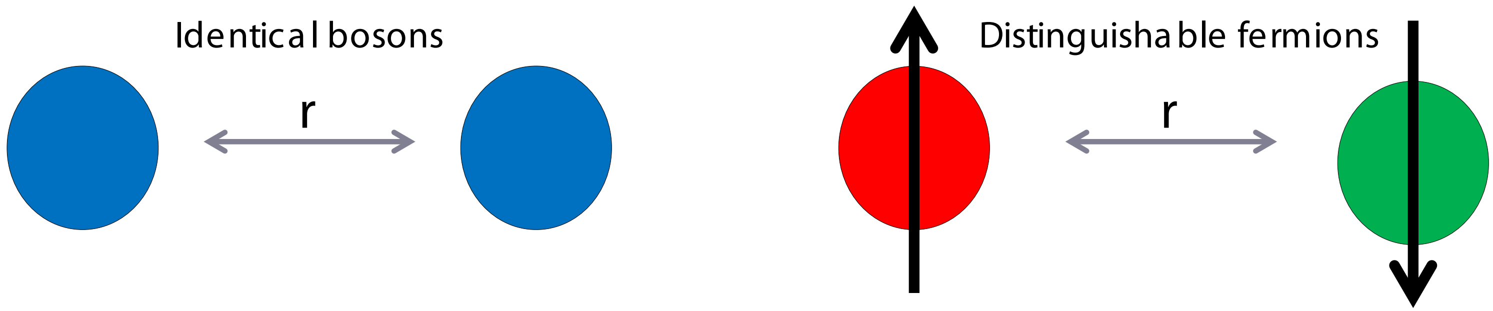
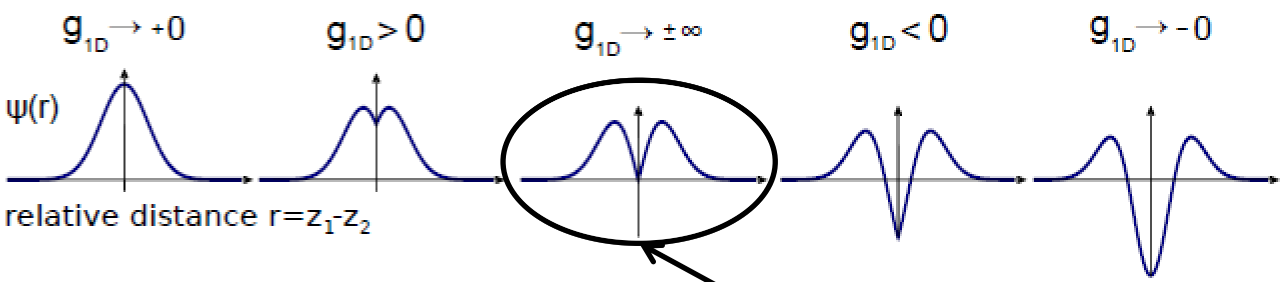
A ONE DIMENSIONAL WORLD



Relative wave function



Interaction

$$g_{1D}\,\delta(r)$$

Source: G. Zürn, thesis

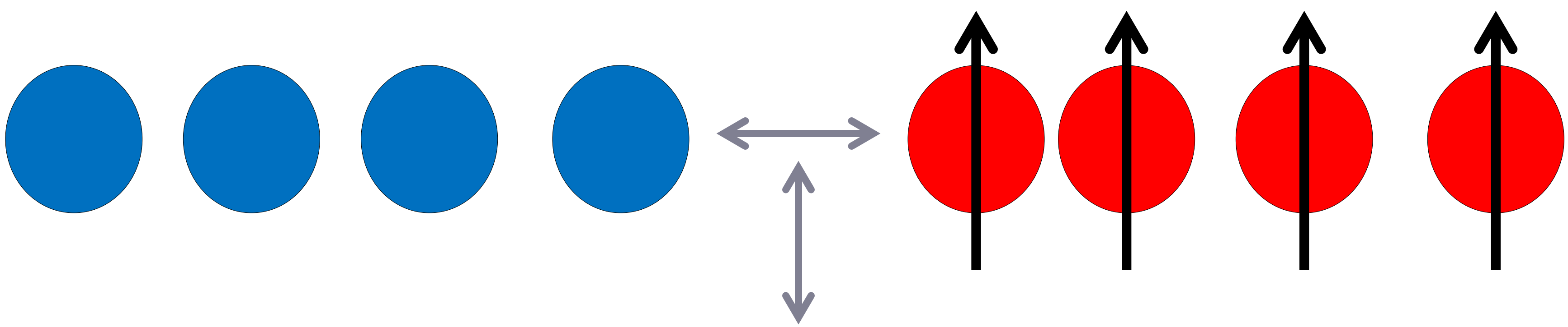
Strong interactions -> Impenetrability!

STRONGLY INTERACTING BOSONS

$|g_{1D}| \rightarrow \infty$ limit

Tonks (1936)-Girardeau (1960) gas
of impenetrable bosons

Mapping identical bosons to spin-polarized fermions. Girardeau (1960).



$$\Psi_B(z_1,\ldots,z_N) = |\Psi_F(z_1,\ldots,z_N)|$$

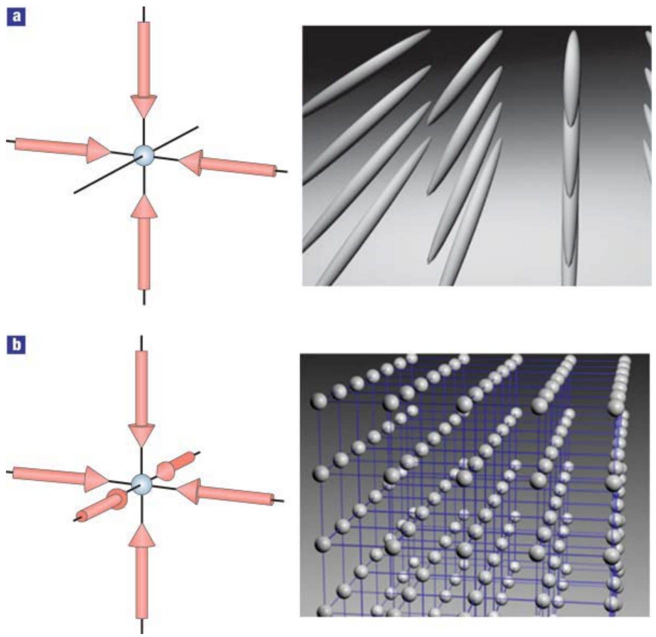
Impenetrable bosons

Antisymmetrized fermions

Lieb-Liniger (1963) used Bethe ansatz to
solve N boson problem for any g>0

EXPERIMENTAL REALIZATION

Optical lattices



I. Bloch, Nature Physics **1**, 23 (2005)

Confinement-induced resonances

Maxim Olshanii
Phys. Rev. Lett. **81**, 938 (1998)

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{m a_\perp^2} \frac{1}{1 - C a_{3D}/a_\perp}$$

Divergent at specific point depending on lattice and 3D Feshbach resonance

EXPERIMENTAL REALIZATION

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

Nature **429**, 277 (2004)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

Science **305**, 1125 (2004)

Experimentally produced and probed the Tonks-Girardeau gas on the repulsive side $g>0$

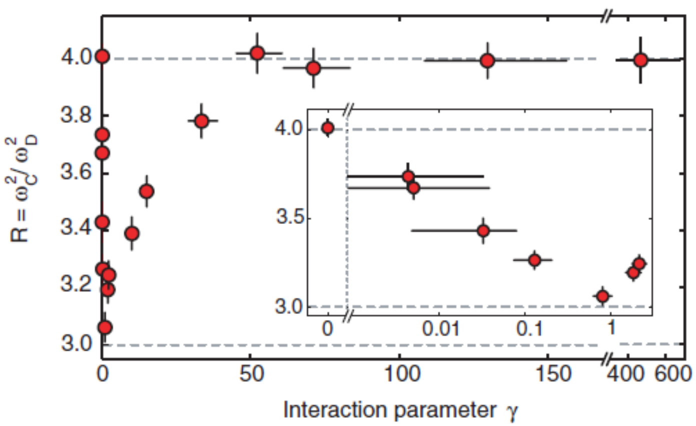
EXPERIMENTAL REALIZATION

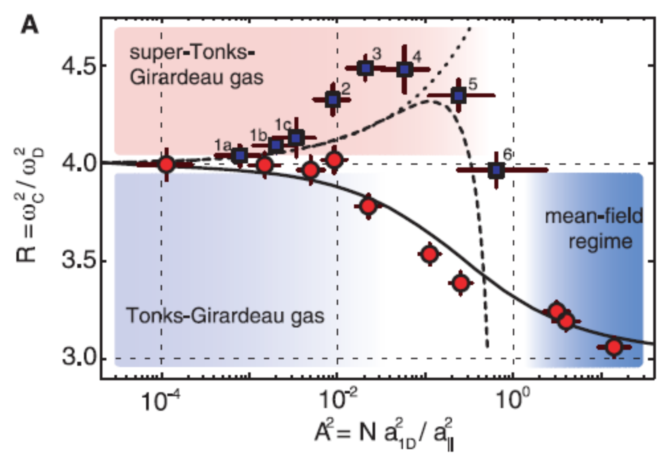
Realization of an Excited, Strongly Correlated Quantum Gas Phase

Elmar Haller,[1] Mattias Gustavsson,[1] Manfred J. Mark,[1] Johann G. Danzl,[1] Russell Hart,[1] Guido Pupillo,[2,3] Hanns-Christoph Nägerl[1]*

Science **325**, 1224 (2009)

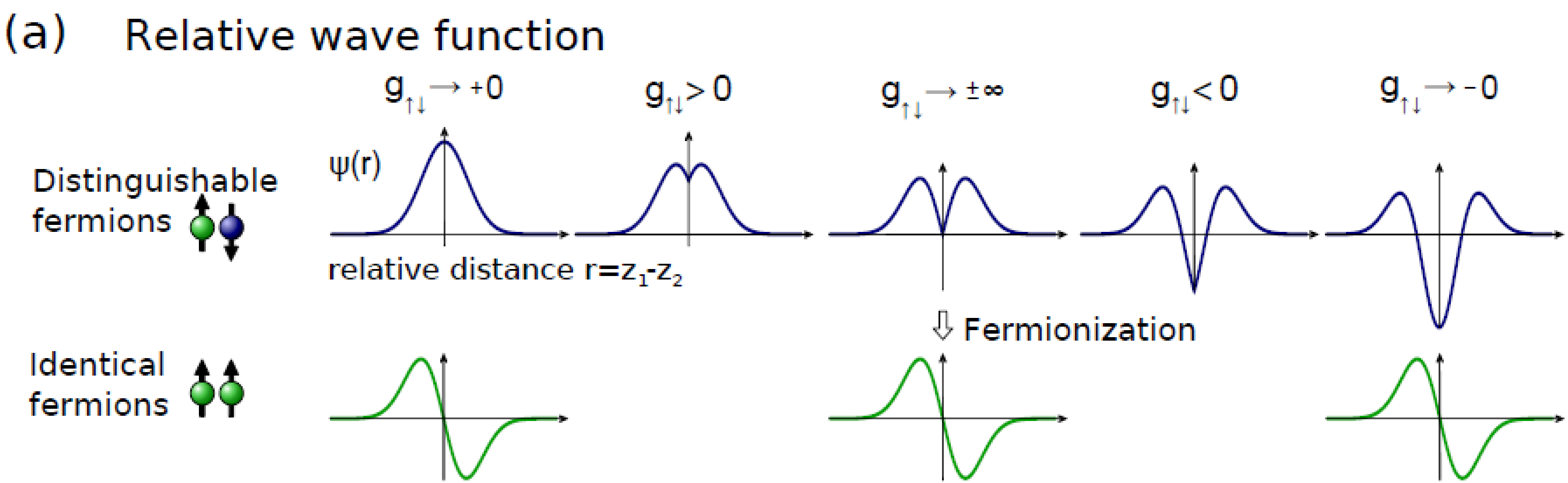
Study of the crossover from g>0 to g<0 in the strongly-interacting regime.





1D FERMIONS – A FRONTIER

Two kinds of relative motion for two-body states!

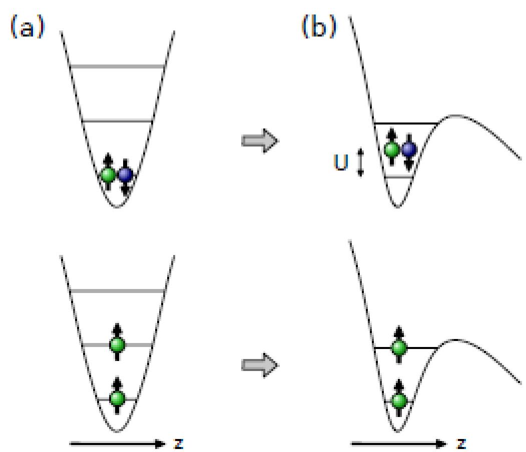


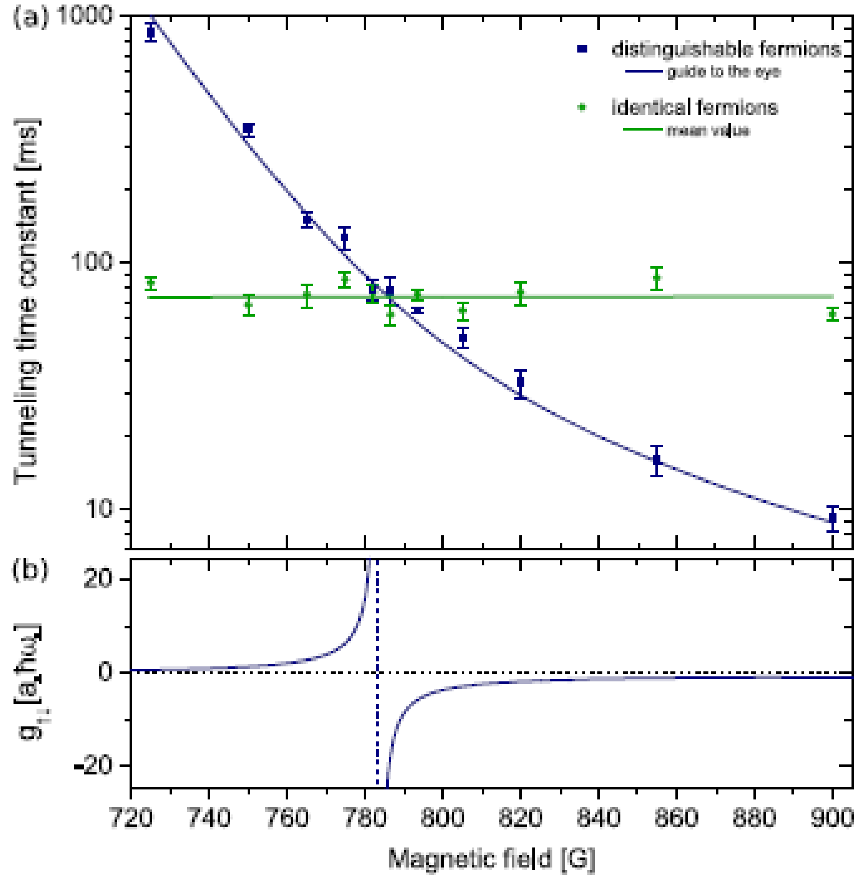
Source: G. Zürn, thesis

Fermionization of two fermions in a 1D harmonic trap:
G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

EXPERIMENTAL REALIZATION

Two-body tunneling experiments

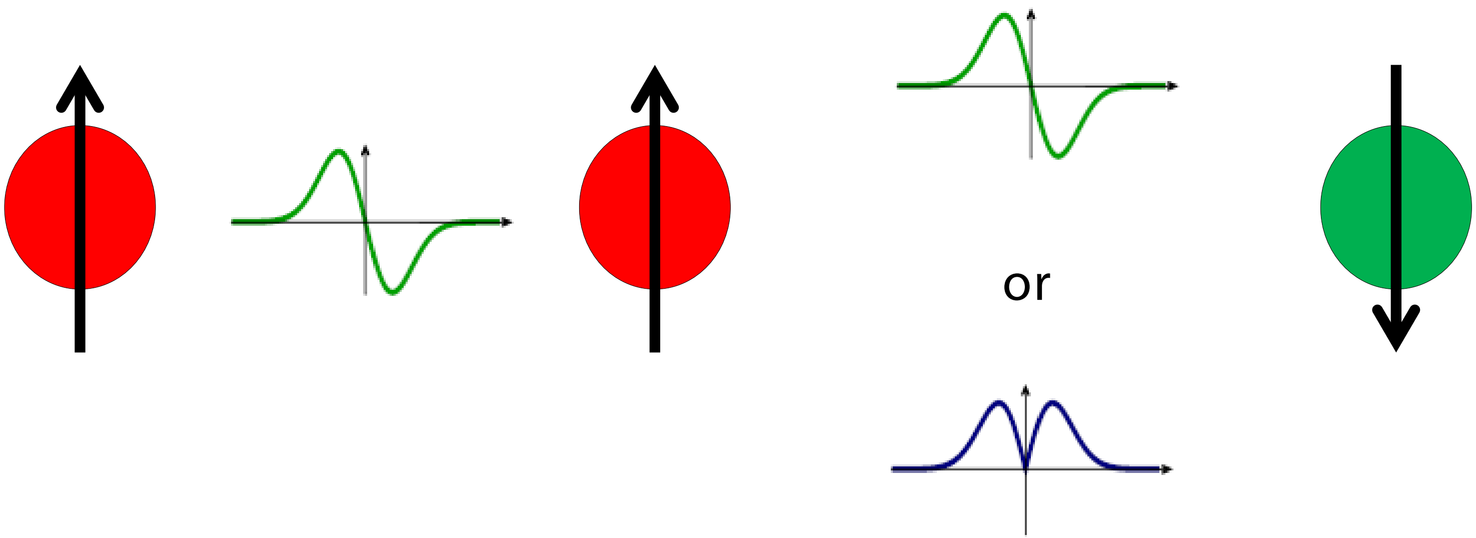




Fermionization of two fermions in a 1D harmonic trap: G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

THREE FERMIONS

Relative wave functions. What should we take?



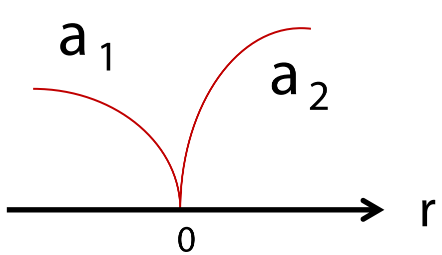
???

Conjecture: Use the symmetric choice for non-identical pairs for any N-body system

M.D. Girardeau, Phys. Rev. A **82**, 011607(R) (2010).

THREE FERMIONS

Let's keep an open mind!



Non-identical relative wave function

Two strict conditions:

1) In limit g_{1D}->∞, relative wave functions have not vanish at zero for identical and non-identical pairs!

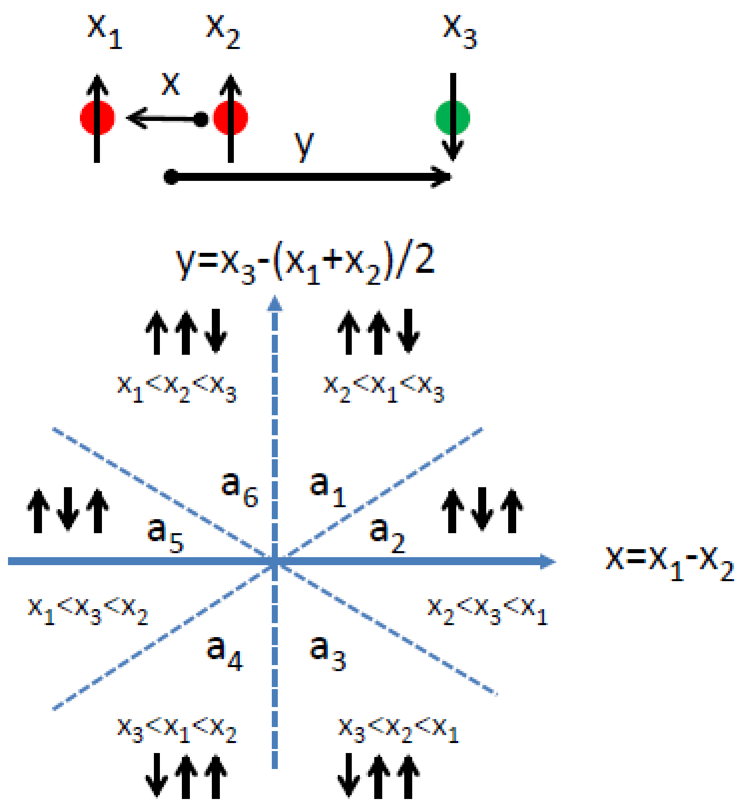
2) Identical fermions must have odd relative wave functions!

IDEA: Keep a_1 and a_2 as free parameters and do a variation!

A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)

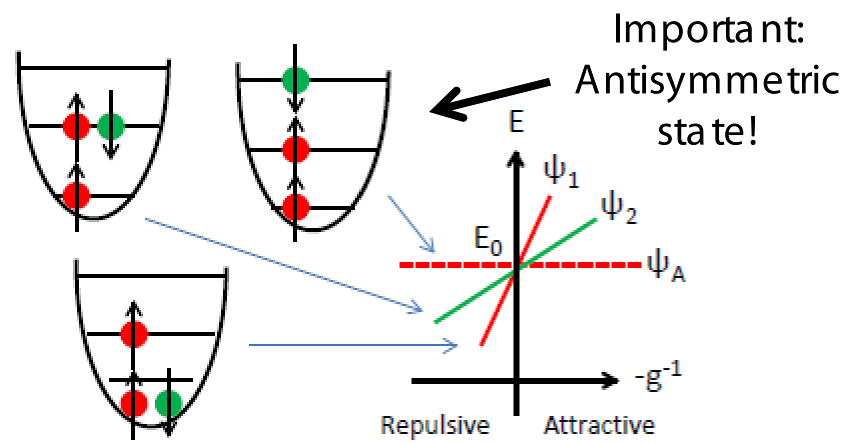
THREE FERMIONS - SOLUTION

Split space in patches



Pauli and parity reduces problem to a_1, a_2, and a_3.

Spectrum on resonance



Optimize derivative!

$$K = -\frac{\partial E}{\partial g^{-1}} = g^2 \frac{\sum_{ij} \int \prod_{k=1}^{N} dx_k |\Psi|^2 \delta(x_i - x_j)}{\langle \Psi | \Psi \rangle}$$

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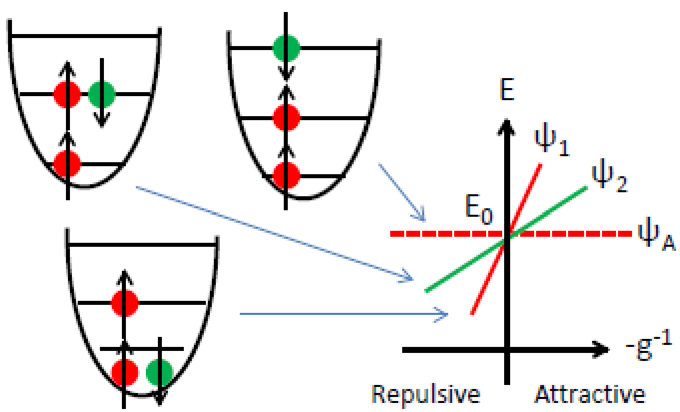
THREE FERMIONS - SOLUTION

$$K = \frac{27}{8\sqrt{2\pi}} \frac{(a_1 - a_2)^2 + (a_2 - a_3)^2}{a_1^2 + a_2^2 + a_3^2}$$

Extremizing solutions are:

$a_1 = a_2 = a_3$	Non-interacting state
$a_1 = a_3$ and $a_2 = 0$	Excited state, even parity
$2a_1 = 2a_3 = -a_2$	Ground state, odd parity

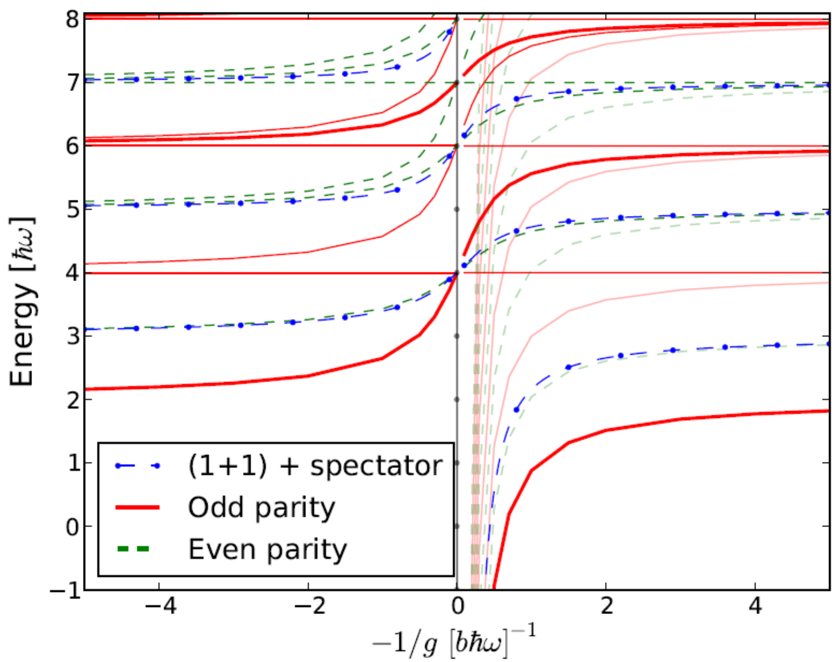


IMPORTANT: Coefficients are generally NOT the same!

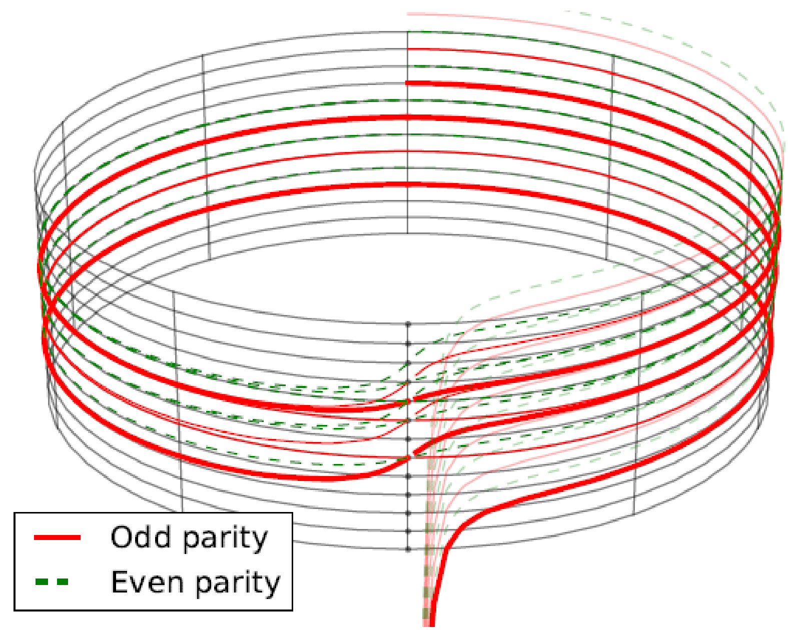
S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013)
A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)

HARMONICALLY TRAPPED SYSTEMS

Standard style



Elegant style



E.J. Lindgren *et al.,* arXiv:1304.2992 (2013).
S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013).

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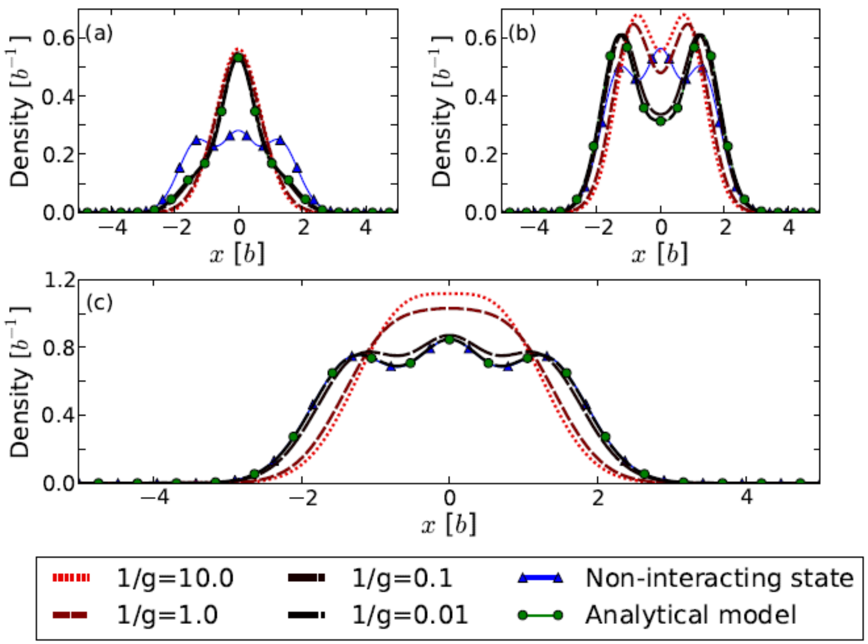
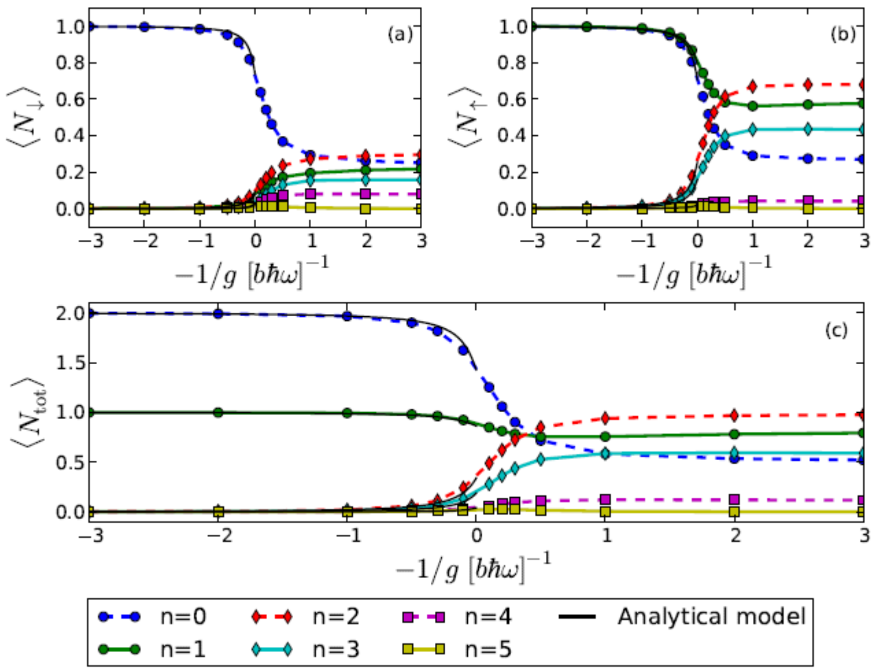
GROUND STATE PROPERTIES

Trap density



Occupation numbers

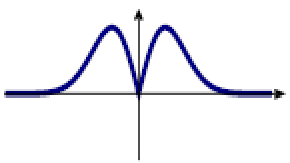


E.J. Lindgren *et al.*, arXiv:1304.2992 (2013).

FERMIONIZATION OF FERMIONS

It is different from identical bosons and spin-polarized fermions!

The 'democratic' solution or trivial Bose-Fermi mapping uses:



between all non-identical pairs.

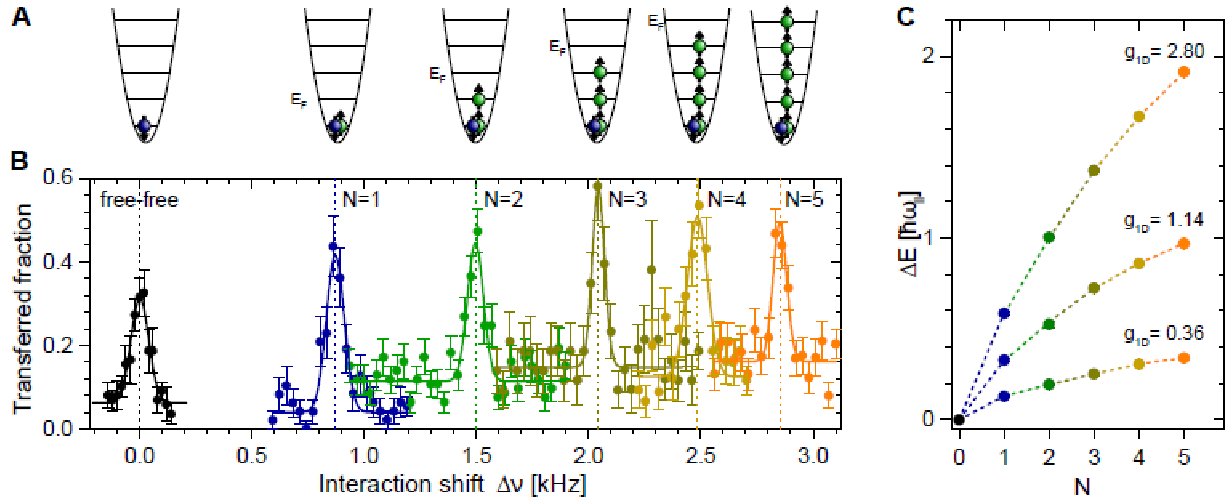
In the 2+1 case it is NOT a relevant eigenstate but rather a linear combination!

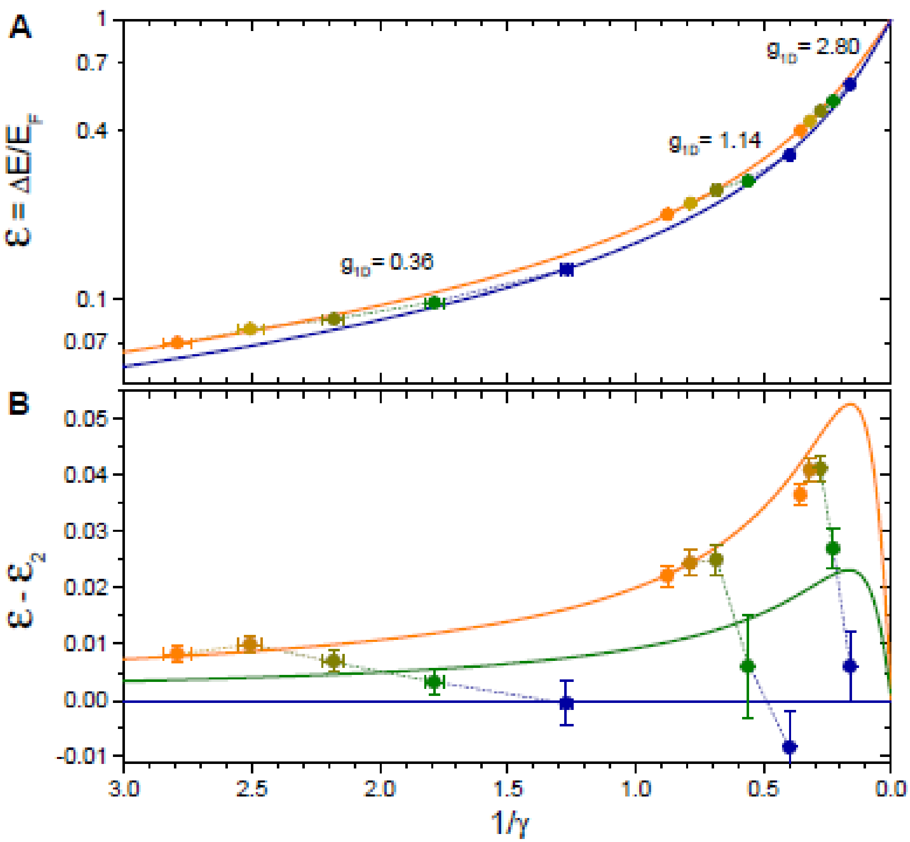
$$\Psi_{BF} = (8^{1/2}\Psi_{gs} + \Psi_{non})/3$$

BUT can we tell the difference in experiments?

S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013)
A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)

Selim Jochim experiments in Heidelberg.





From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz *et al.*, Science **342**, 457 (2013)

Green solid line from
S.E. Gharashi, K.M. Daily, and D. Blume, Phys. Rev. A **86**, 042702 (2012).

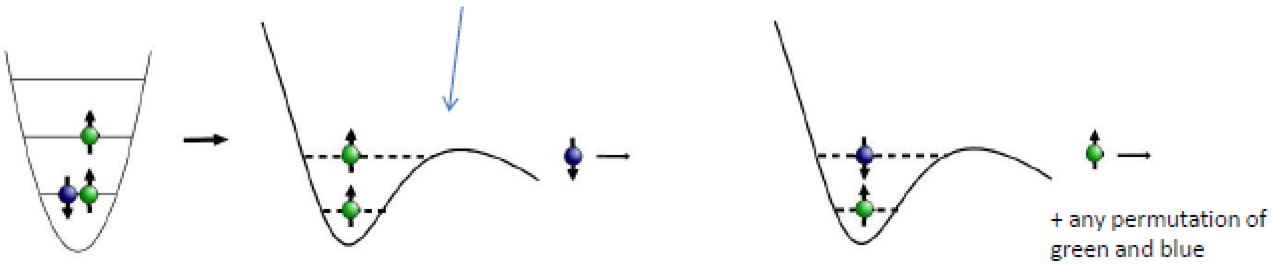
Orange 'many-body' line
J.B. McGuire, J. Math. Phys. **6**, 432 (1965).
G.E. Astrakharchik and I. Brouzos, Phys. Rev. A **88**, 021602 (2013).

EXPERIMENTAL SIGNATURE

Do tunneling experiments!

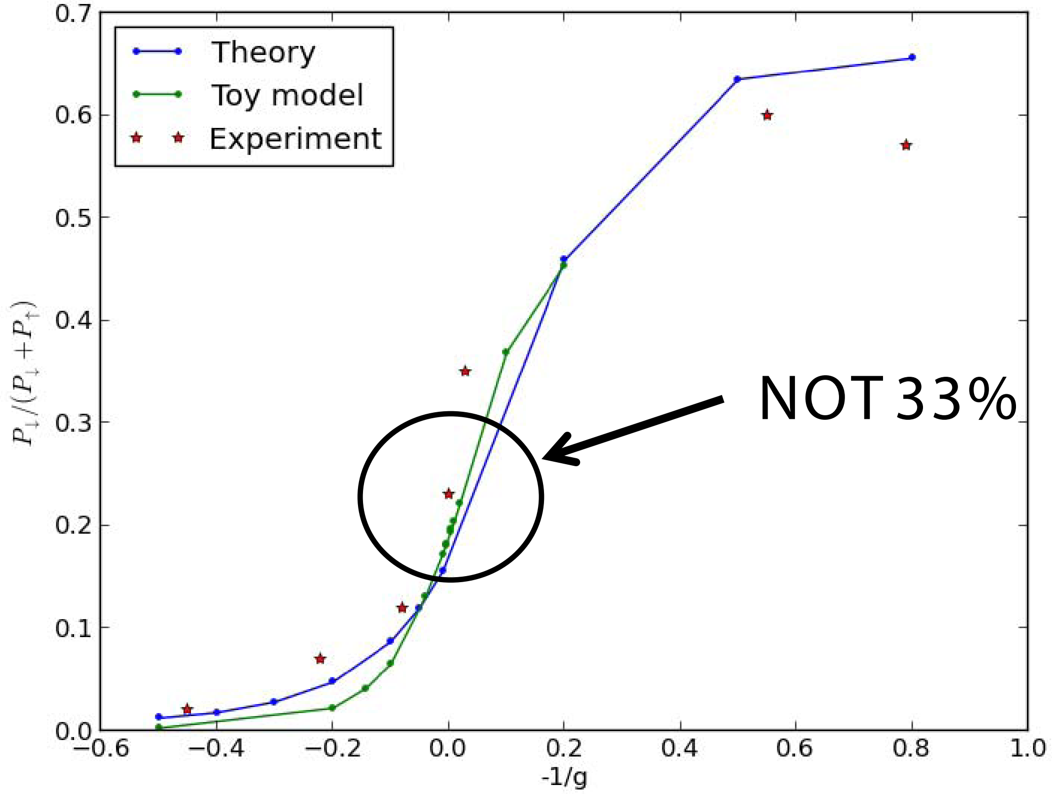
F. Serwane *et al.*, Science **332**, 336 (2011).

G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).



Source: G. Zürn

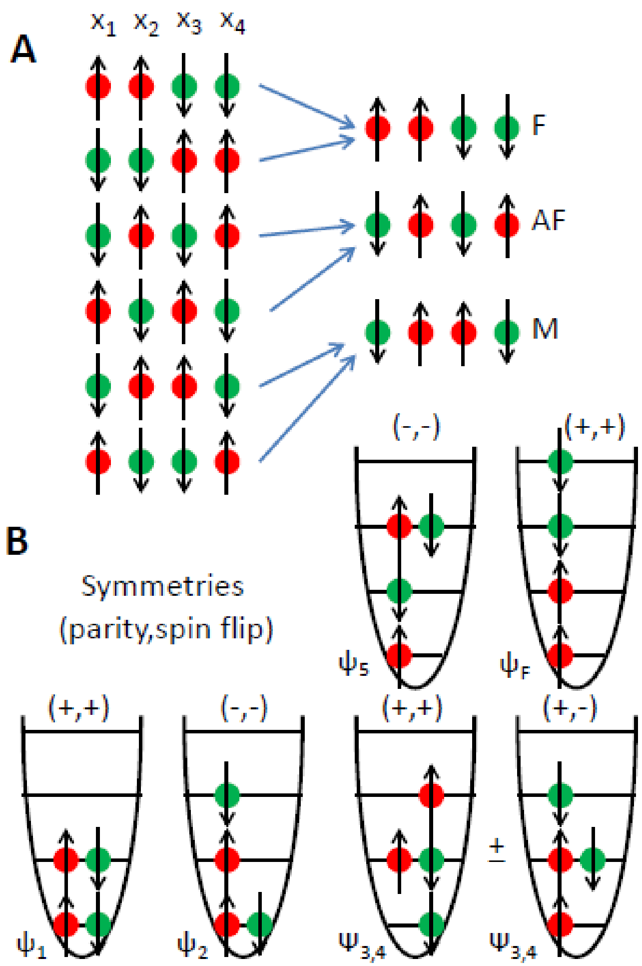
THEORY VS. EXPERIMENT

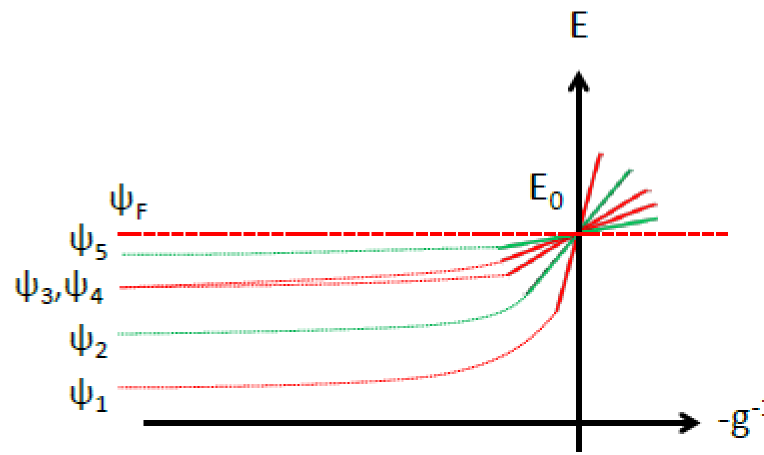


Data from Jochim group (G. Zürn)

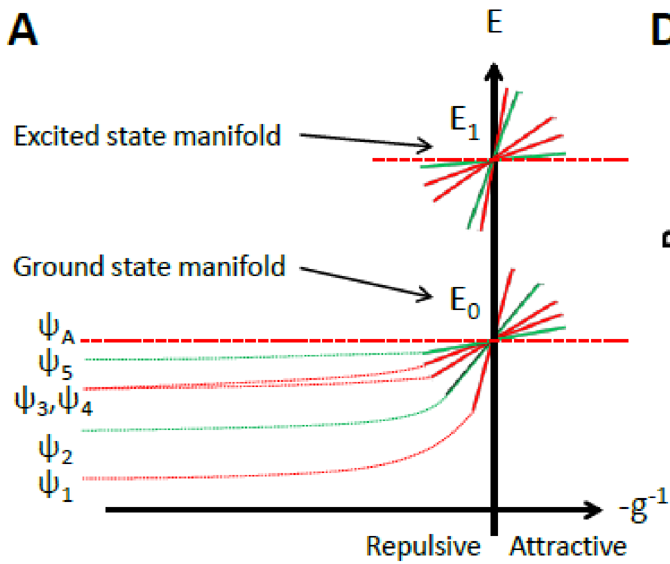
FOUR-BODY SYSTEMS

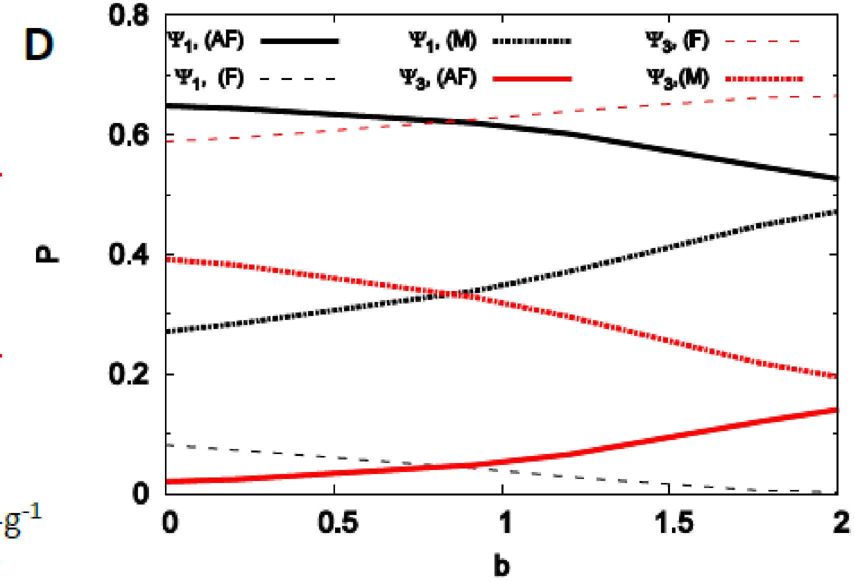
A.G. Volosniev *et al.*, arXiv:1306.4610v1 (2013)

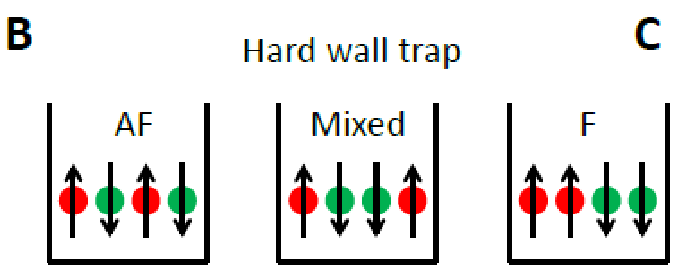




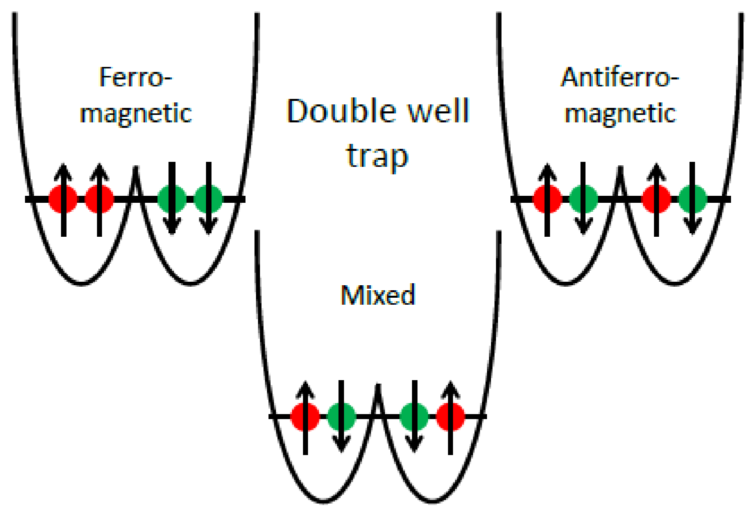
$$V(x) = \frac{1}{2} m\omega^2 (|x| - b)^2$$





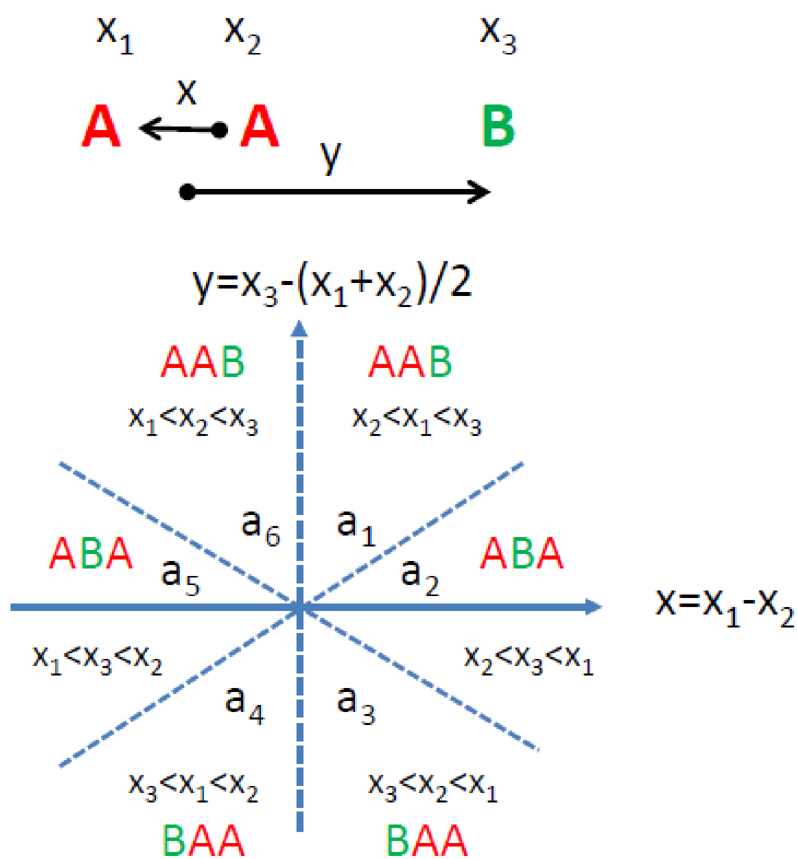


	AF	Mixed	F
P(ψ1)	0.622	0.333	0.045
P(ψ4)	0.045	0.333	0.622



A.G. Volosniev *et al.*, arXiv:1306.4610v2 (2013)

THREE TWO-COMPONENT BOSONS



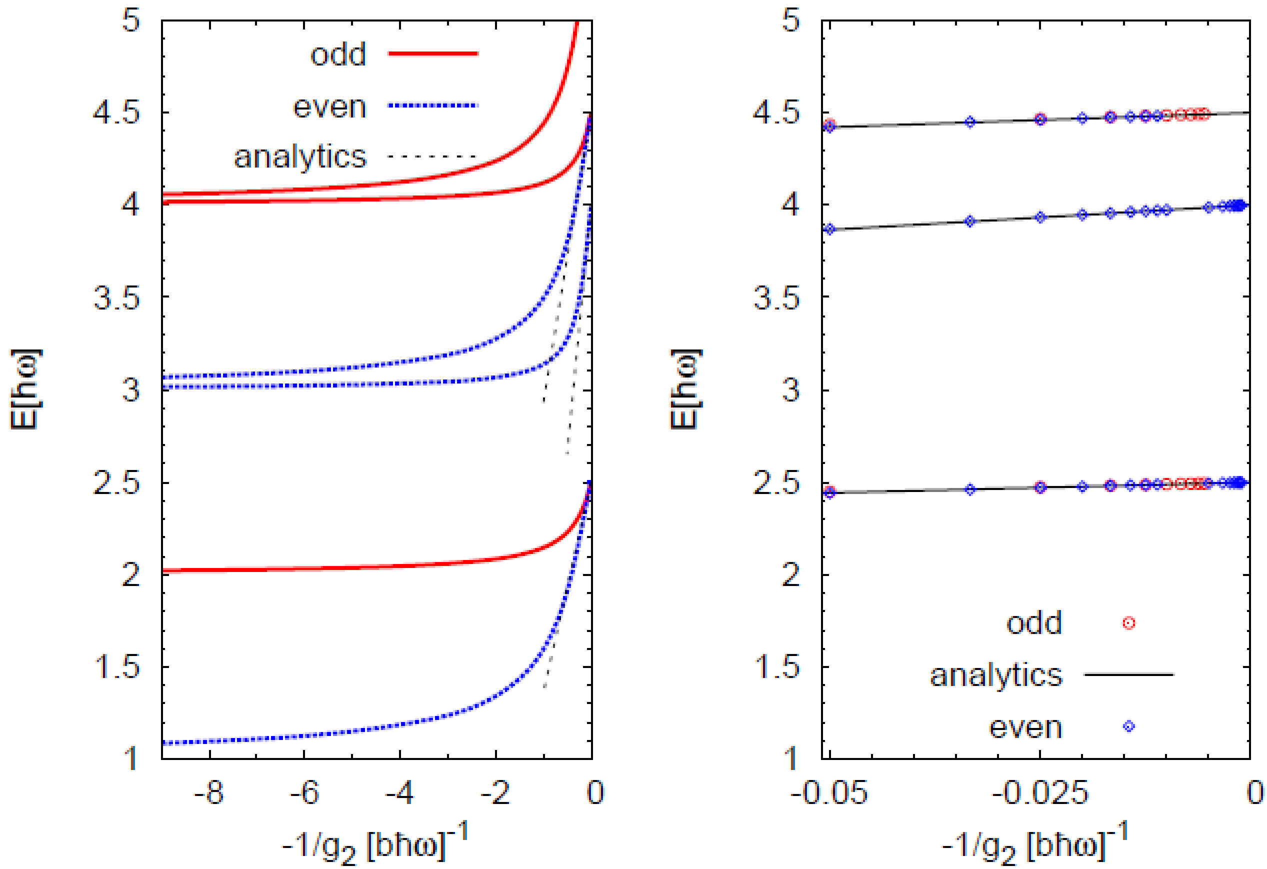
N.T. Zinner *et al.*, arXiv:1309.7219 (2013).

Stochastic variational calculations

x_1 x_2 x_3

A **A** **B**

$$H = \sum_{i=1}^{3} \frac{p_i^2}{2m} + g_2\delta(x_1 - x_3) + g_2\delta(x_2 - x_3)$$



N.T. Zinner *et al.*, arXiv:1309.7219 (2013).

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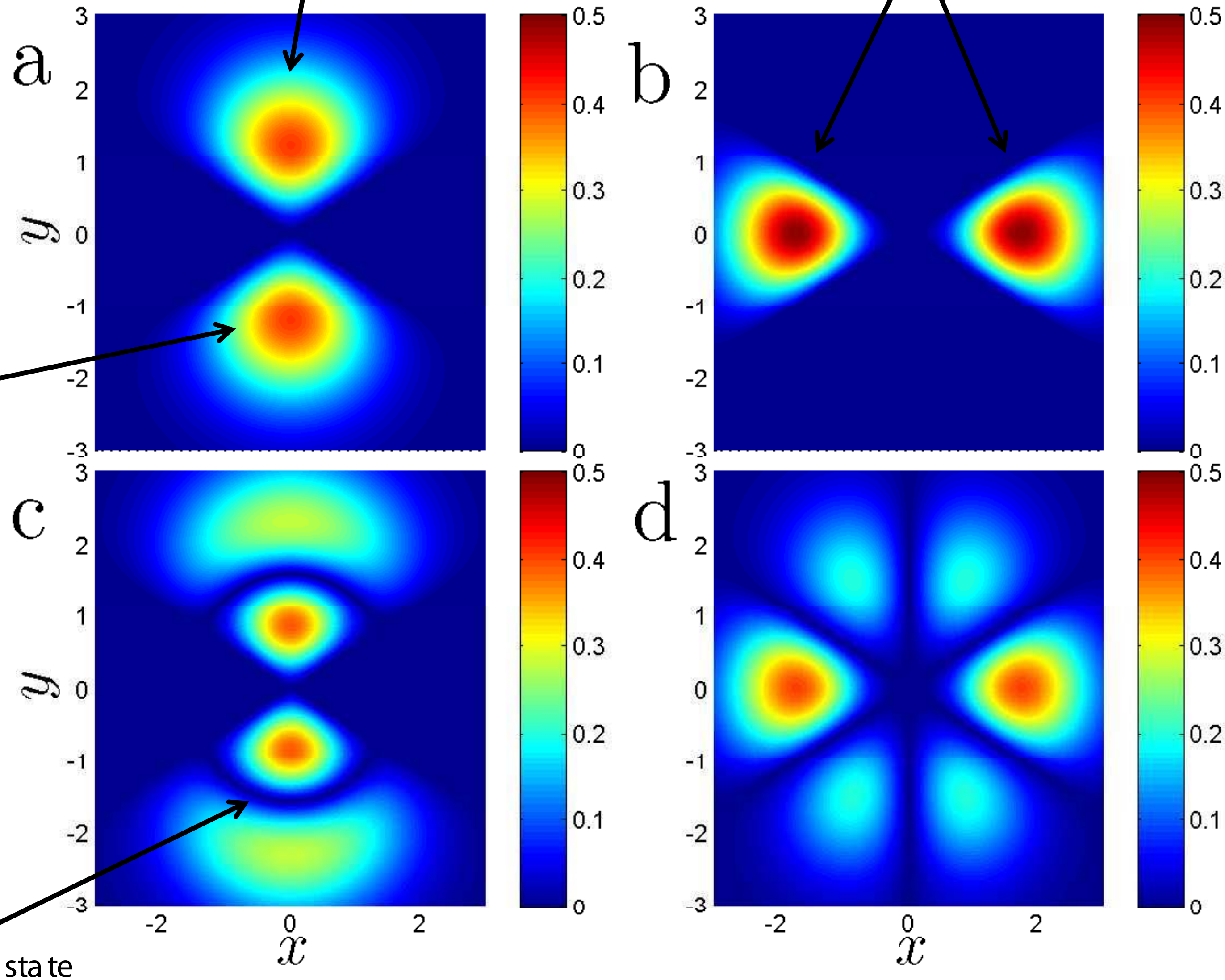
AAB

Perfect ferromagnet?

ABA

Perfect antiferromagnet?

Ground state



First excited state

BAA

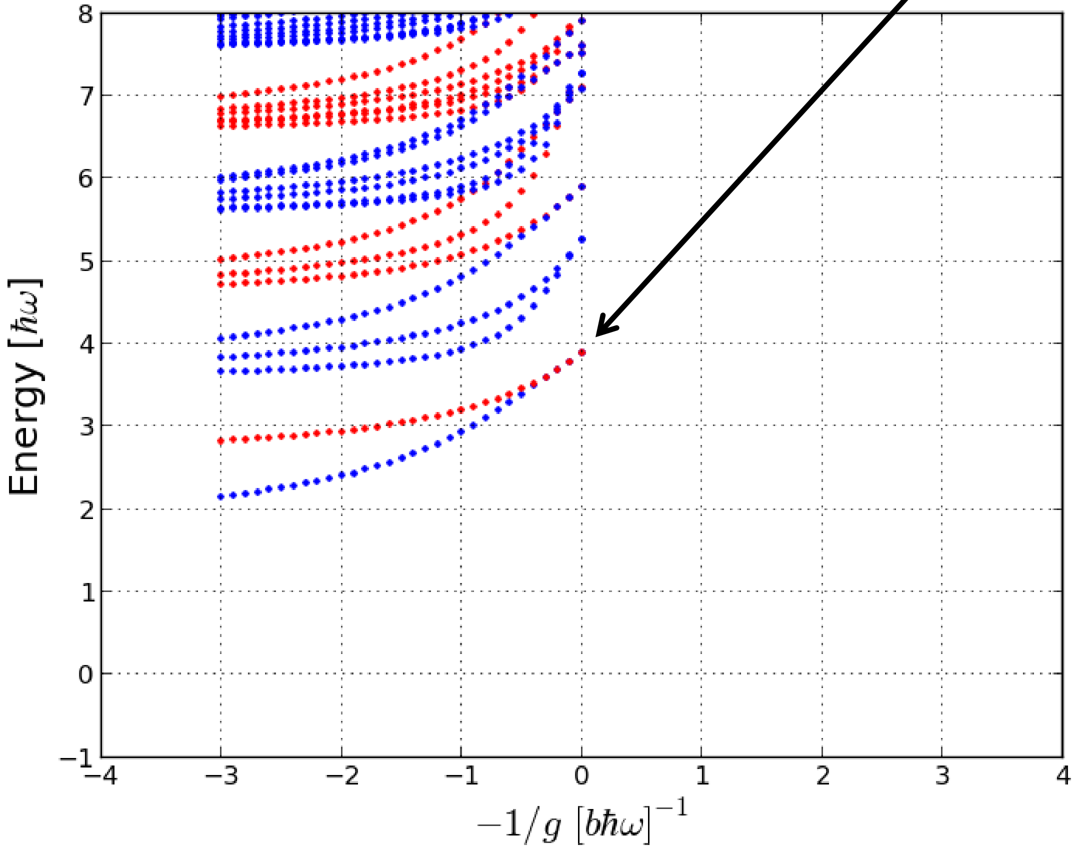
Ground state for 2+1 fermions

2nd excited state with node!

FOUR-BODY BOSONS

Ground state structure

AABB+BBAA ??



MAIN MESSAGES

- Complete theory goes beyond Bose-Fermi mapping
- Must connect states to eigenstates in the spectrum
- 'Magnetic' correlations are accessible
- Good agreement with experimental data
- Fermions and bosons can be VERY different even in the hard-core limit!
- Engineering of ferro- and antiferromagnetic states!
- Wave functions and not energies are the most important objects!

GENERALIZATIONS

› More particles – also Bose-Fermi mixtures
› Excited states
› Impurity Problems
› Multi-well setups
› Dynamics!

ACKNOWLEDGEMENTS

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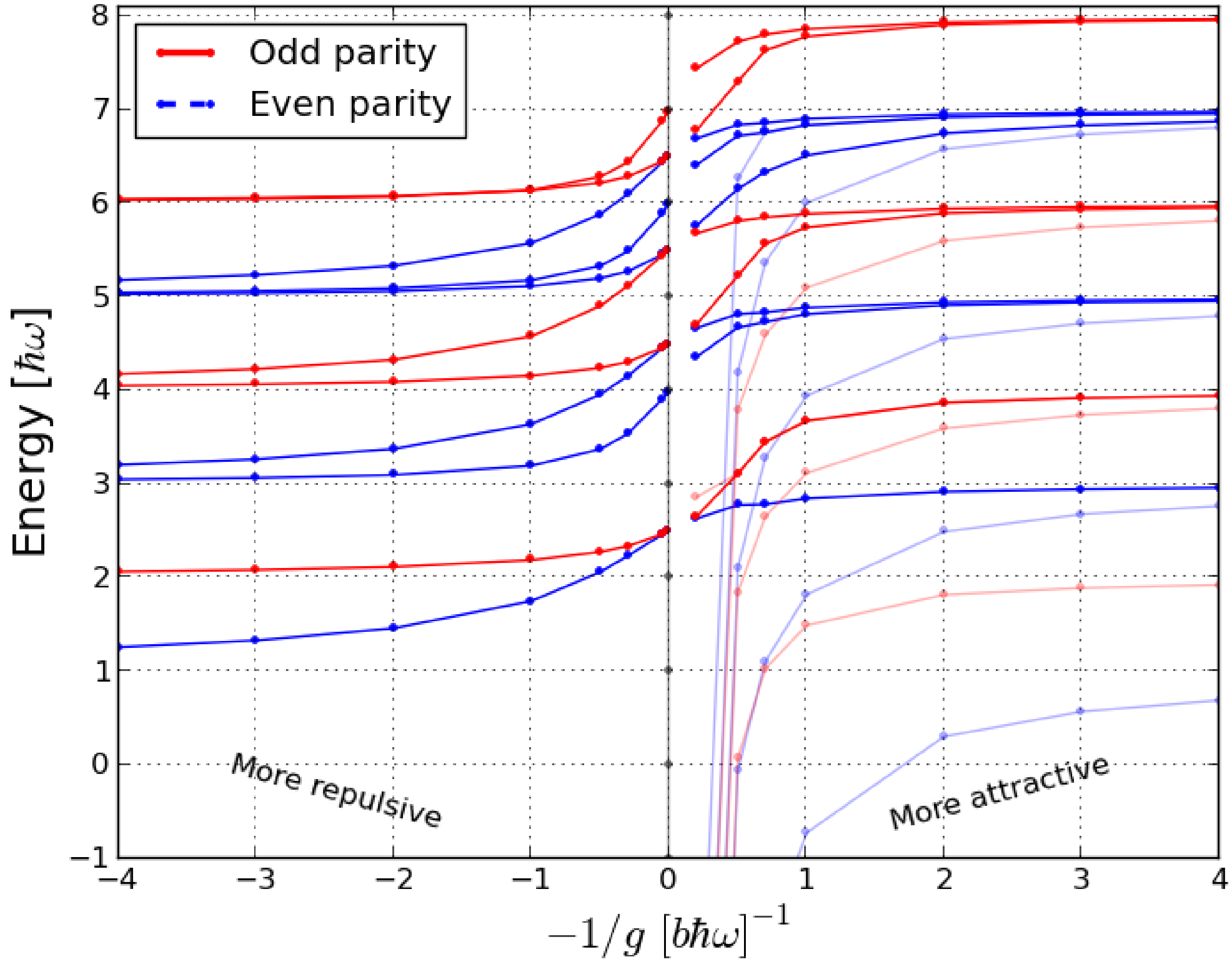
- Jochim group in Heidelberg: Selim Jochim, Gerhard Zürn, Thomas Lompe, Simon Murmann, Andre Wenz

Thank you for your attention!

2+1 bosons


Odd parity
Even parity
Energy [$\hbar\omega$]
$-1/g\ [b\hbar\omega]^{-1}$
More repulsive
More attractive
8
7
6
5
4
3
2
1
0
−1
−4
−3
−2
−1
0
1
2
3
4