The running of couplings and anomalous thermodynamics in Bose gases near resonance

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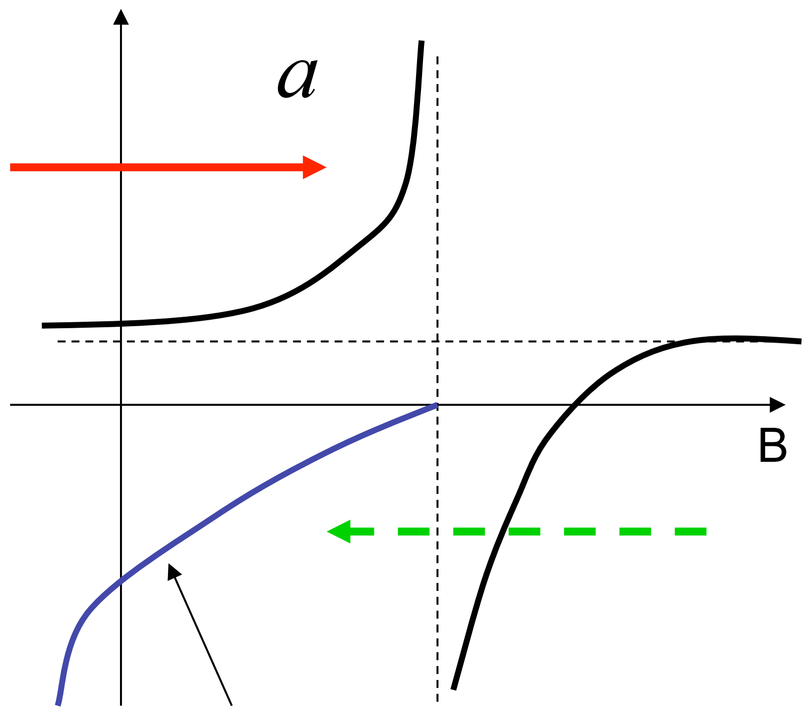
Quantum Bose Gas near Feshbach Resonance (Upper Branch)

Dilute limit



Binding energy

…. Papp + Pino et. al, (Wieman, Jin, Cornell, 2009);
Pollack+ Dries et al., (Hulet, 2009); Navon+Piatecki et al.,
(Chevy and Salomon, 2011); Wild +Makotyn et al., (Cornell, Jin, 2012);
Ha+Hung et al., (Chin, 2012); Makotyn et al., (Cornell, Jin, 2013)…

Dilute Bose Gases

Lee-Yang-Huang (56; 57-58) and Beliaev (58)

And is valid for small scattering lengths.

$$\sqrt{na^3} \propto \frac{a}{\xi} << 1$$

There have been efforts to improve LYH-Beliaev theory by taking into the higher order contributions. (Wu, Sawada, 59; ….. Braatten et al, 02…) At infinity a, each term diverges.

$$E = \frac{2\pi\hbar^2 n^2 a}{m}(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na^3}$$

$$+ 8(4\pi - 3\sqrt{3})/3 \times [\ln(na^3) + 4.72 + 2B]na^3 + ...)$$

Approaches to unitary Bose gases

1) Variational approaches (numerical):

Cowell et al. (Pethick group), 2002; Song & Zhou, 2009…

Diederix et al. (Stoof group), 2011; Yin & Radzihovsky, 2013. Sykes et al. (JILA +Greene), 2014 ….

2D: Pilati et al., (Giorgini group), 2005.

2) Effective potential via Loop summation ("single shot renormalization method")

Borzov, Mashayekhi, Song, Bernier, Jiang, Liu, Semenoff, Maki, FZ (2011---now)

………

Q: Can one have an analogue of BEC-BCS theory but for upper branch Bose gases?

Outline

1) Ideas/Cartoons: the running of coupling or scale dependent interactions;

2) Implementation: Theory frame work for unitary Bose gases (an analogue of BEC-BCS CO theory for bosons beyond the dilute limit.);

3) Results

Ref:

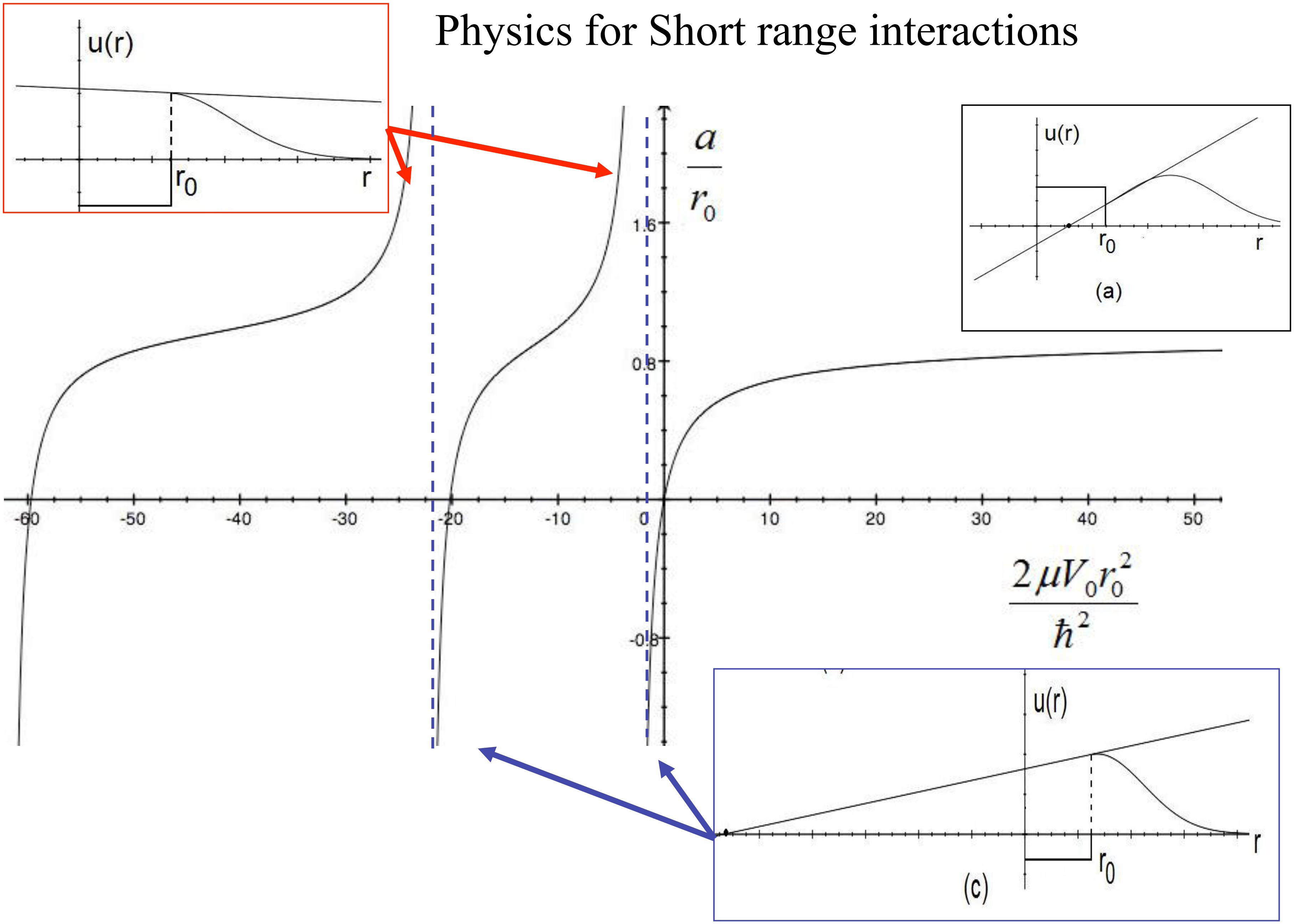
a) Borzov, Mashayekhi, Zhang, Song and FZ, PRA 85, 023620 (2012).

b) FZ and Mashayekhi, 2012, Ann. Phys. 328, 83 (2013).

c) Mashayekhi, Bernier, Borzov, Zhou, PRL 110, 145301 (2013).

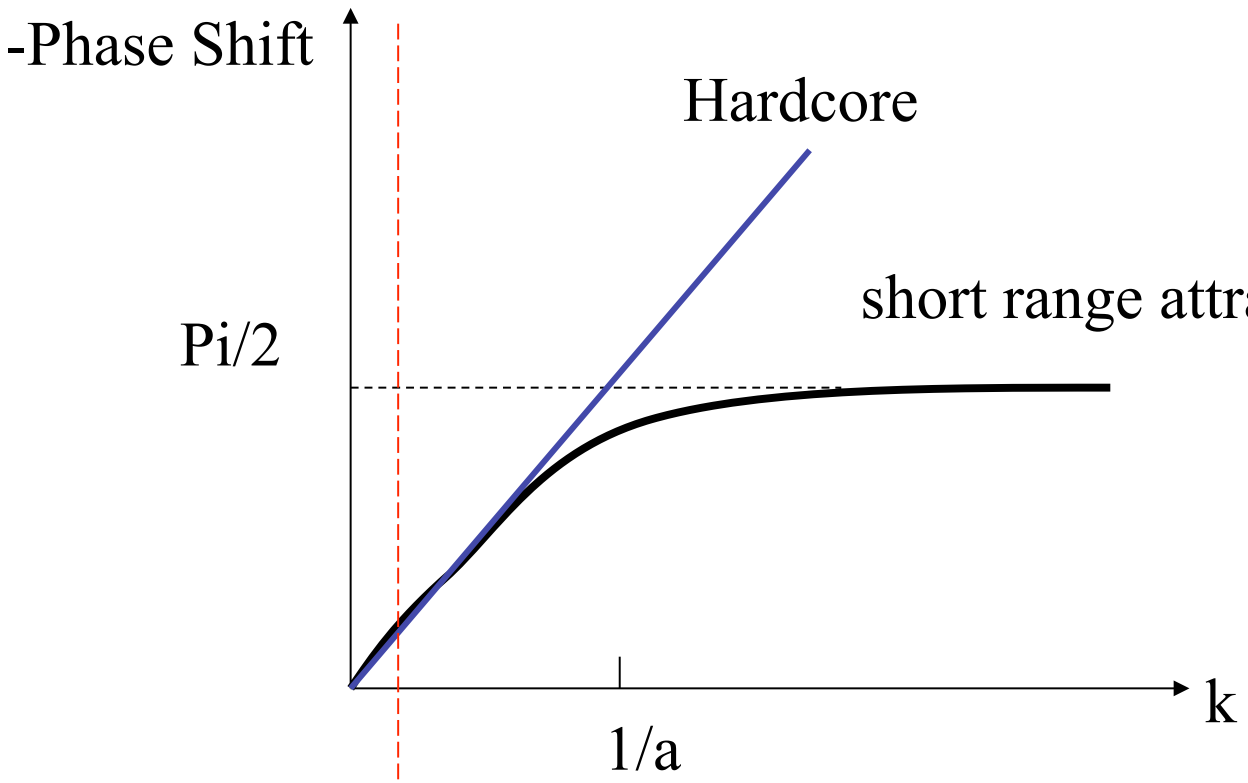
d) Jiang, Liu, Semenoff and Zhou, Rigorous solution to strong coupling fixed pt near 4 spatial dimension. ArXiv: 1307.4263 (2013), PRA, 2014.

Physics for Short range interactions



Positive scattering lengths = Effective repulsive interactions ?

$$V_{pseudo}(r) = \frac{4\pi a}{m}\delta(r)\frac{\partial}{\partial r}(r\ldots) \sim \frac{4\pi a}{m}\delta(r)$$



Low energy scattering physics of a short range 2-body attractive potential is equivalent to a repulsive interaction.

Scale dependence and deviation from “4\pi a”:

Energy for two atoms in a box of size L (kinetic energy ~ 1/L^2.)

$$E_{2-b}(L) \sim \frac{a}{mL^3}\left(1 + C\frac{a}{2L} + \ldots\right), a << L;$$

$$\Rightarrow g_2(\Lambda \sim 0) = 4\pi a(1 + C\Lambda a + \ldots.)$$

Scale dependence II:
A cartoon in real Space at short distance L<<a

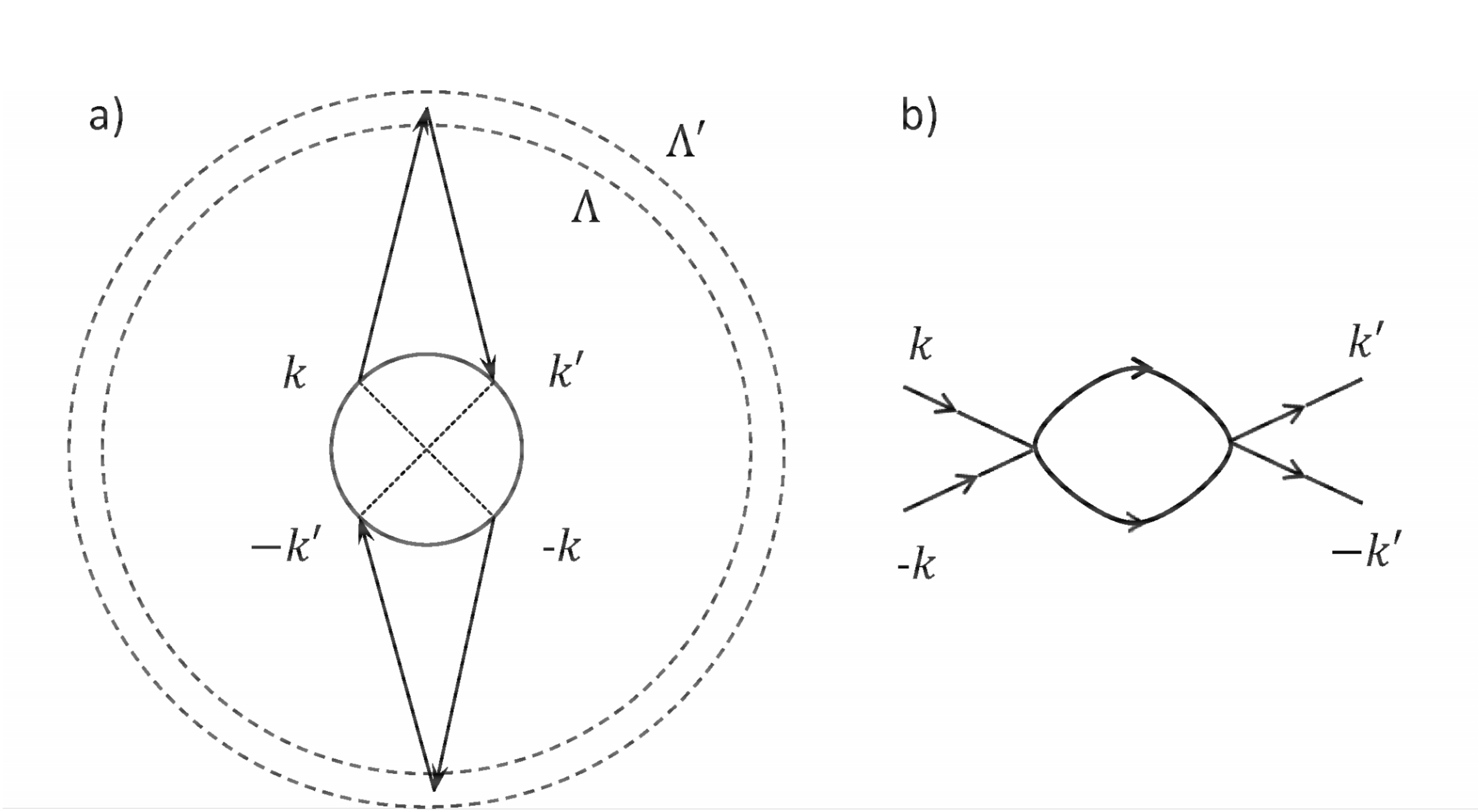
$$r_0 \to \lambda r_0, V_0 \to V_0 \frac{1}{\lambda^2},$$

$$or \quad g_2 \sim V_0 r_0^3 \to \lambda V_0 r_0^3$$

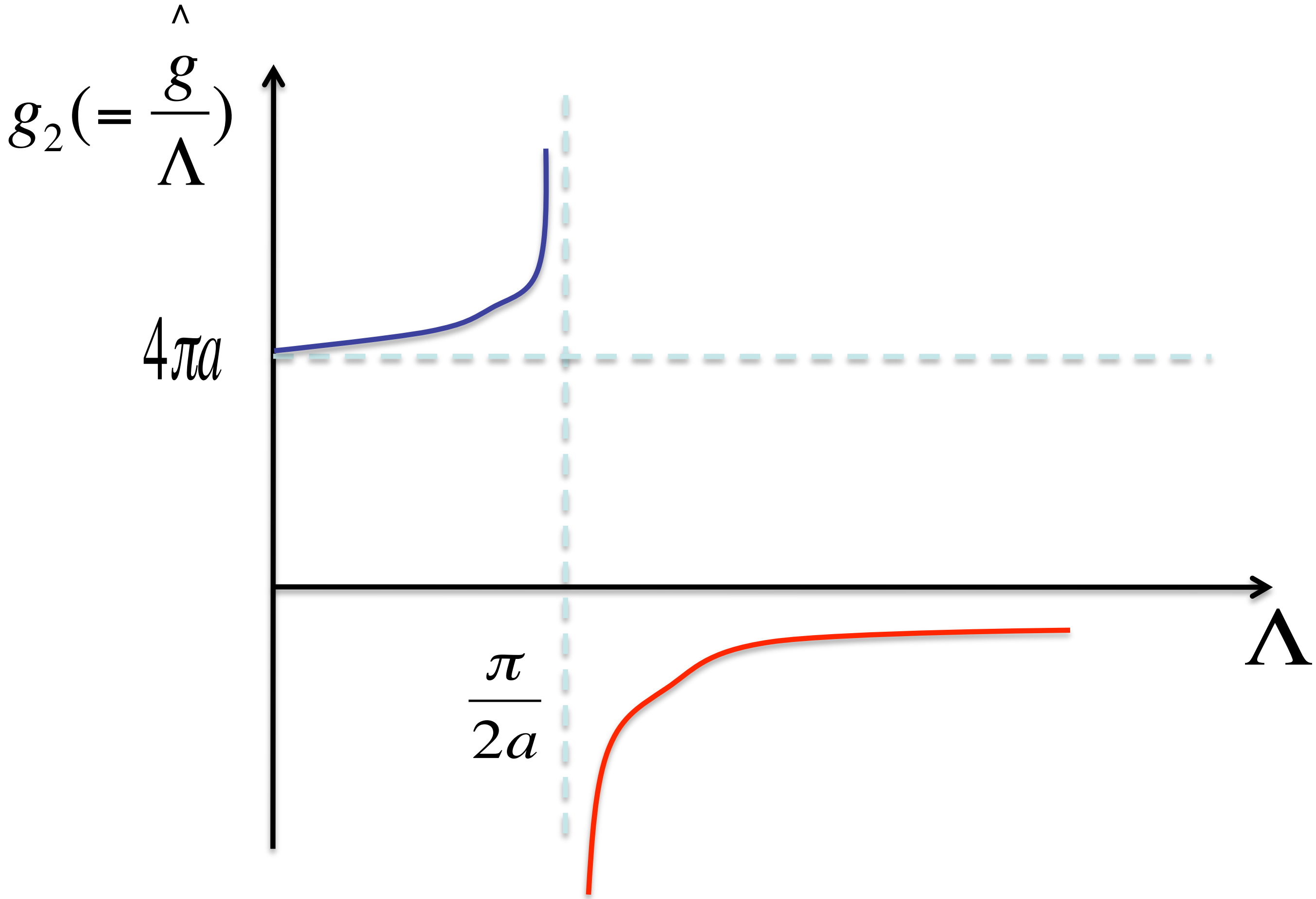
$$\Lambda = \frac{1}{r_0} \to \frac{\Lambda}{\lambda},\ g_2(\Lambda) \to g(\frac{\Lambda}{\lambda}) = \lambda g_2(\Lambda)$$

$$\Rightarrow g_2(\Lambda) \sim -\frac{1}{\Lambda} + o(\frac{1}{a\Lambda})$$

Wilsonian renormalization of interaction constant

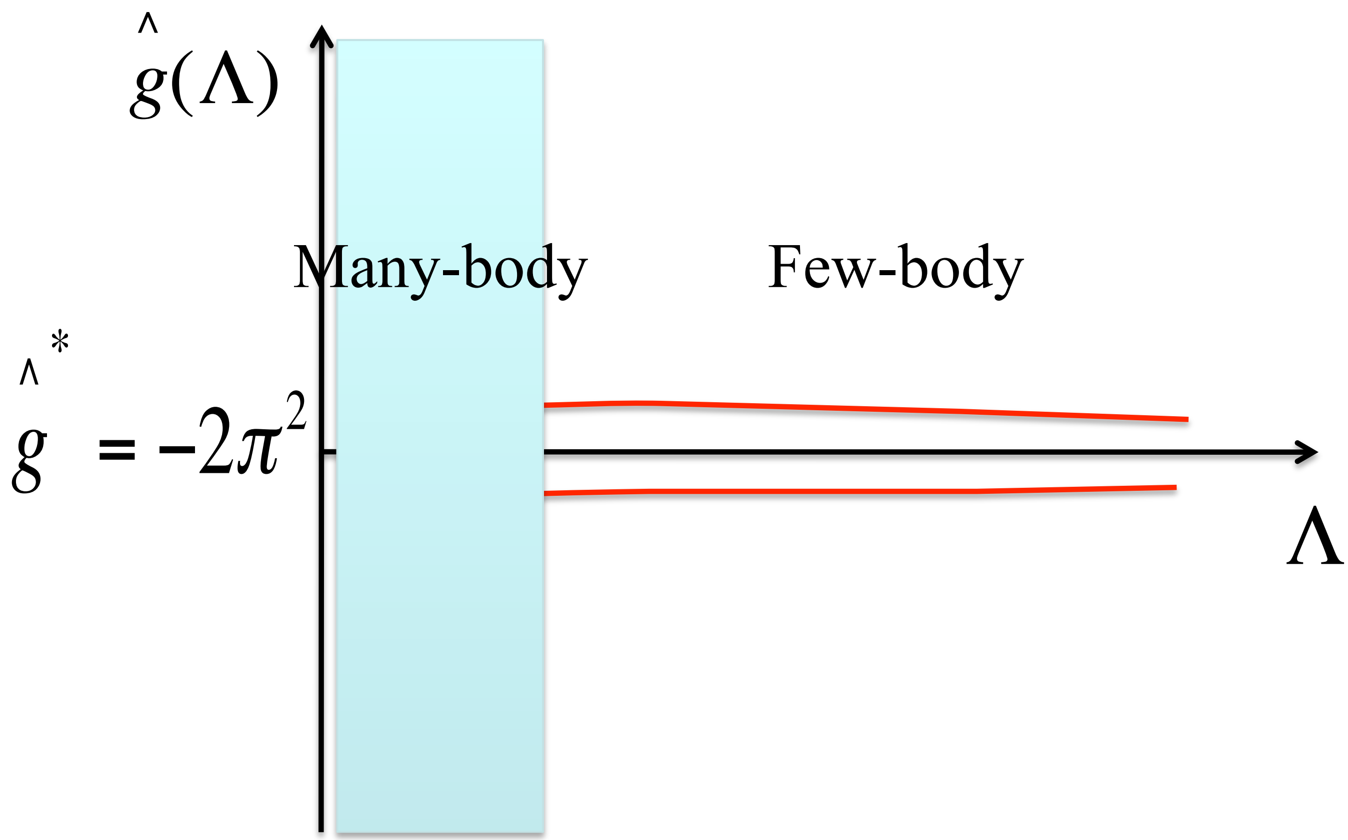


The running of 2-body Coupling Constant:

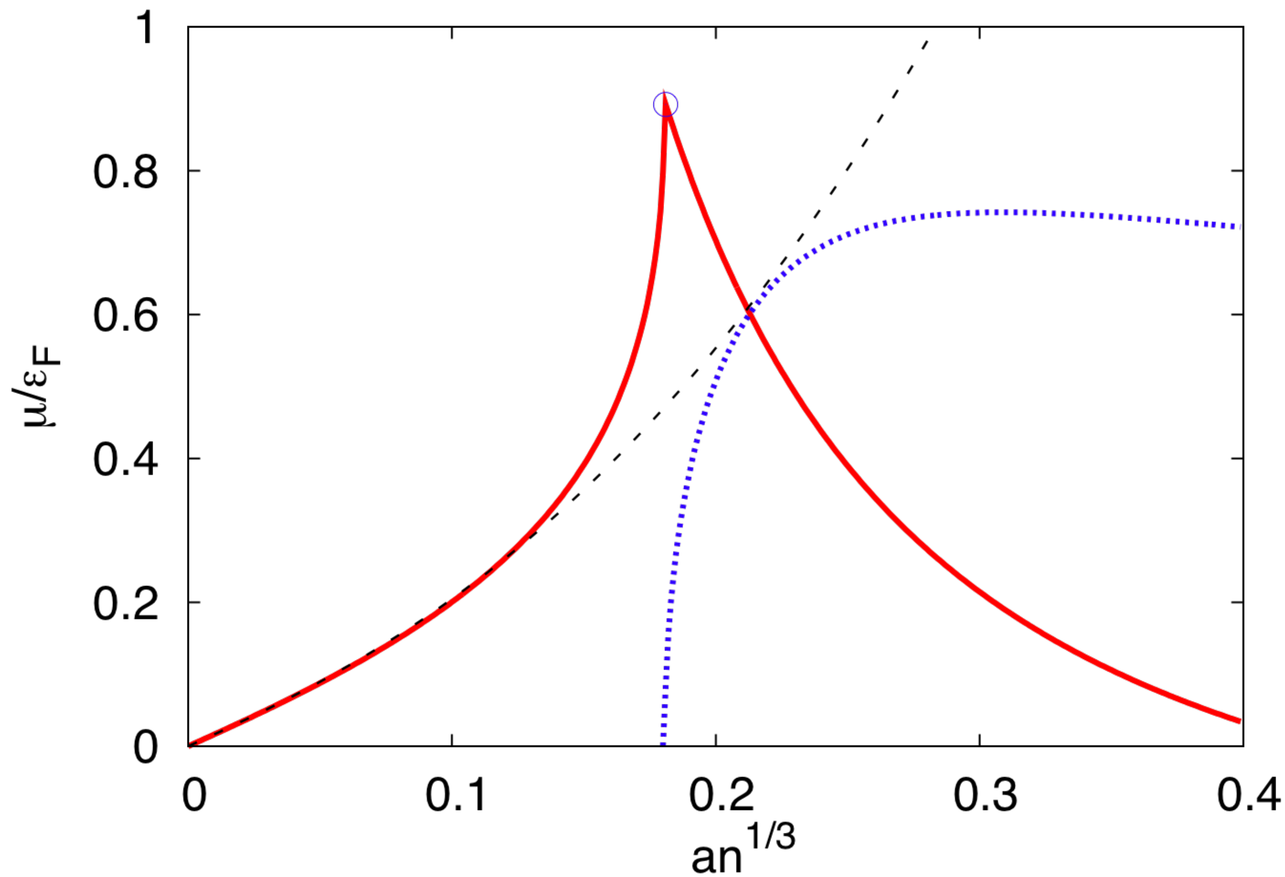


$$g_2(\Lambda \sim 0) = g_2(0)(1 + C\Lambda a \ldots), \qquad \Lambda a \Rightarrow \sqrt{2\mu} a \sim \sqrt{na^3}$$

Patching: Flow and boundary condition



$$\mu \sim g_2(\Lambda \sim \sqrt{\mu})n \qquad [MF : \mu = g_2(\Lambda = 0)n]$$



Dashed line---LHY dilute gas theory.
Dotted line --- imaginary part of chemical potential

1) Nearly fermionized near resonance (analogue of 1D TG gases);
2) Chemical potential \mu reaches a maximum;
3) Maximum accompanied by an instability.

\mu calculated using the Running coupling at \mu.

Theory Frame Work

Condensate

Non-condensed Chem. potential

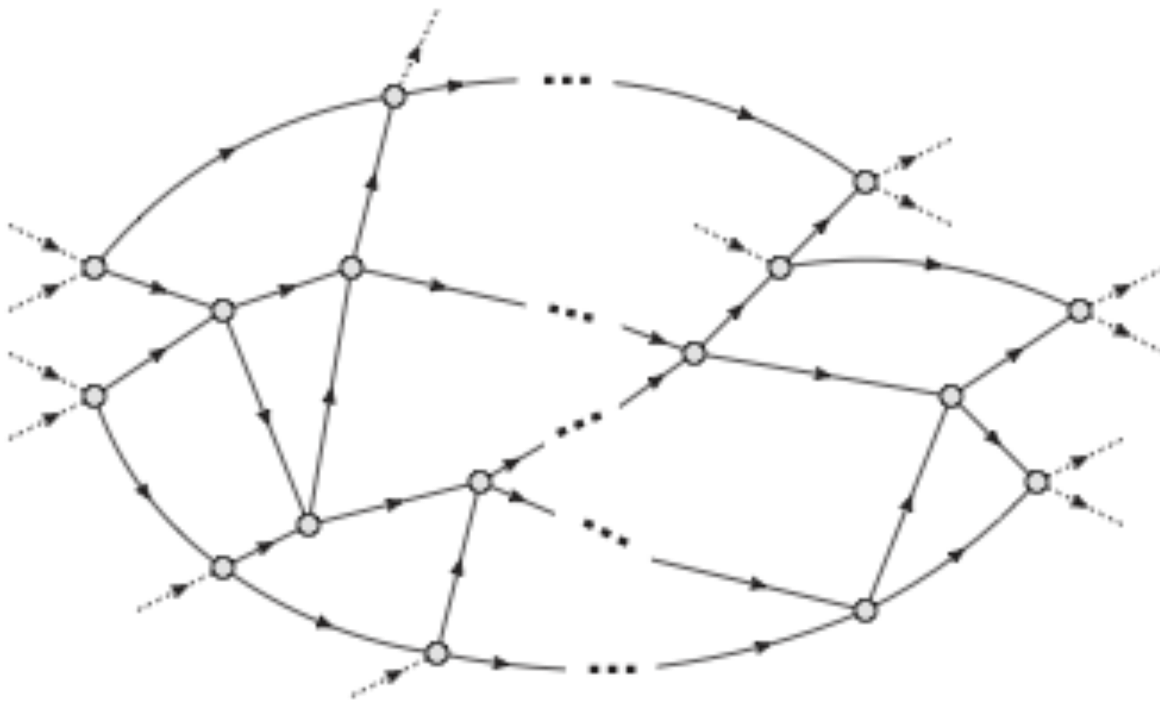
$$\mu_c(n_0,\mu) = \frac{\partial E(n_0,\mu)}{\partial n_0}, n = n_0 - \frac{\partial E(n_0,\mu)}{\partial \mu},$$
$$\mu = \mu_c(n_0,\mu),$$

Self-consistent Equilibrium Cond.

E is the total interaction energy of condensate at fixed n_0
Is also the effective potential for the quantum bosonic field
(Coleman-Weinberg type but calculated at a finite \mu).

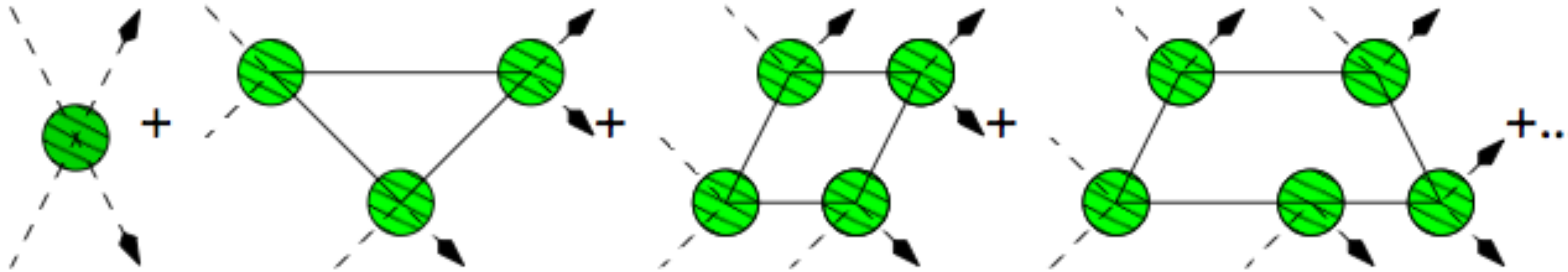
A typical L-loop diagram for E

Diag(L,N)

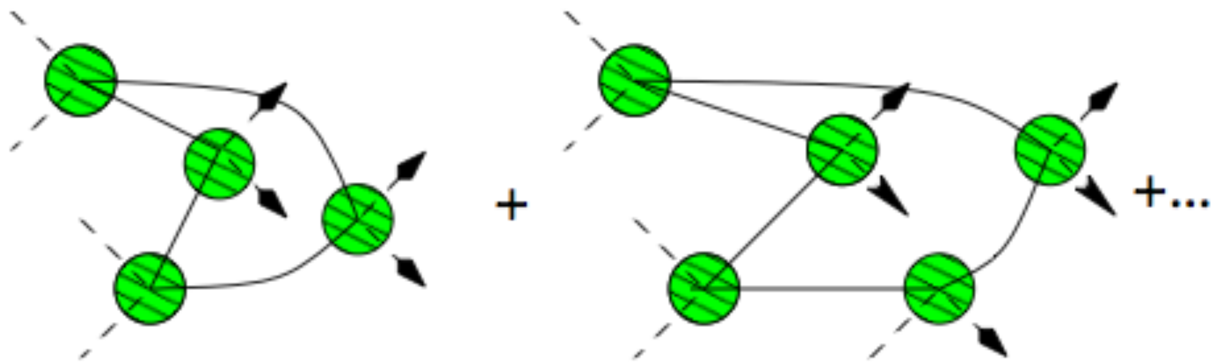


2D and 3D: arbitrary L, N but with up to 3-body irreducible diags;

4D: epsilon expansion---systematic.

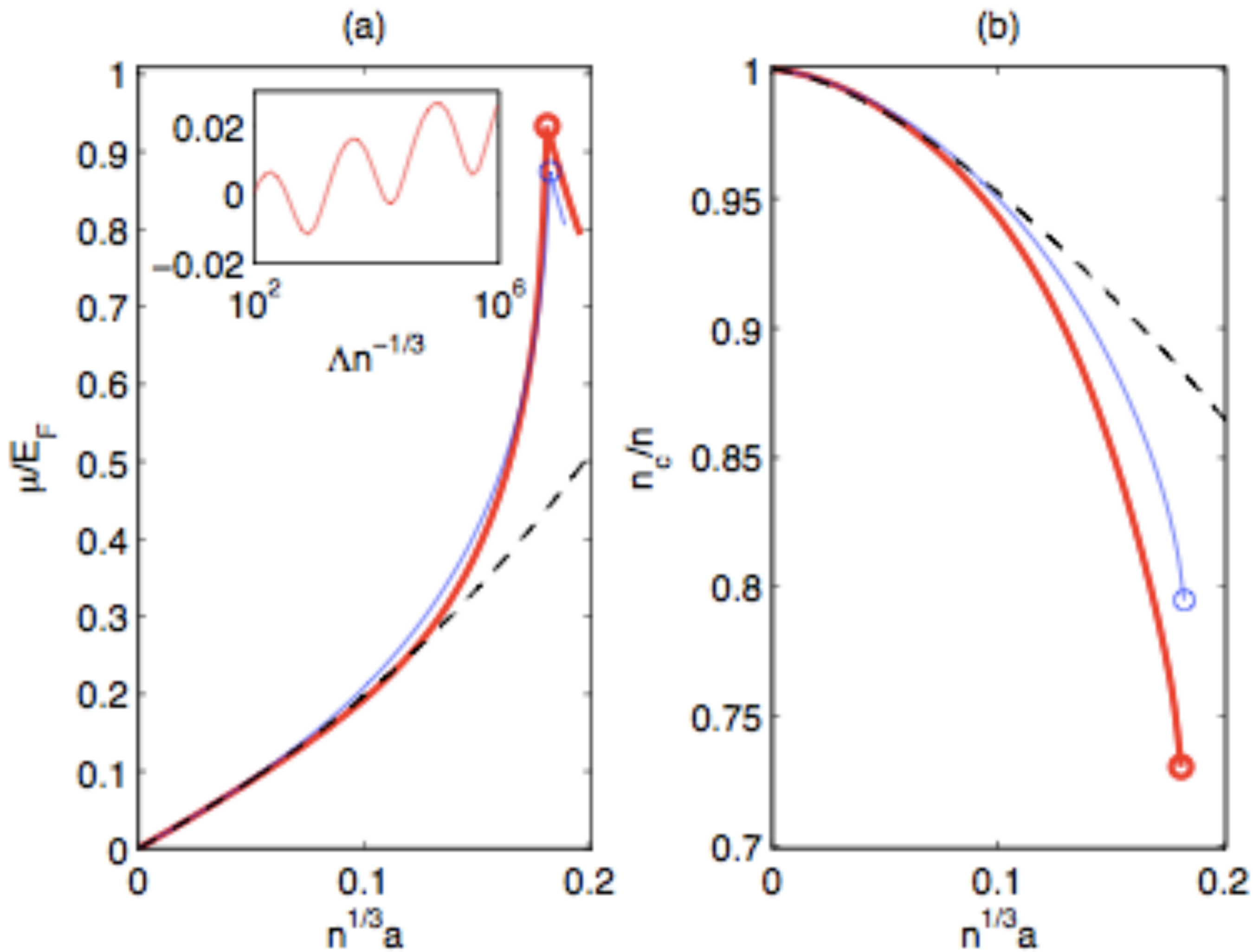


c)



d)

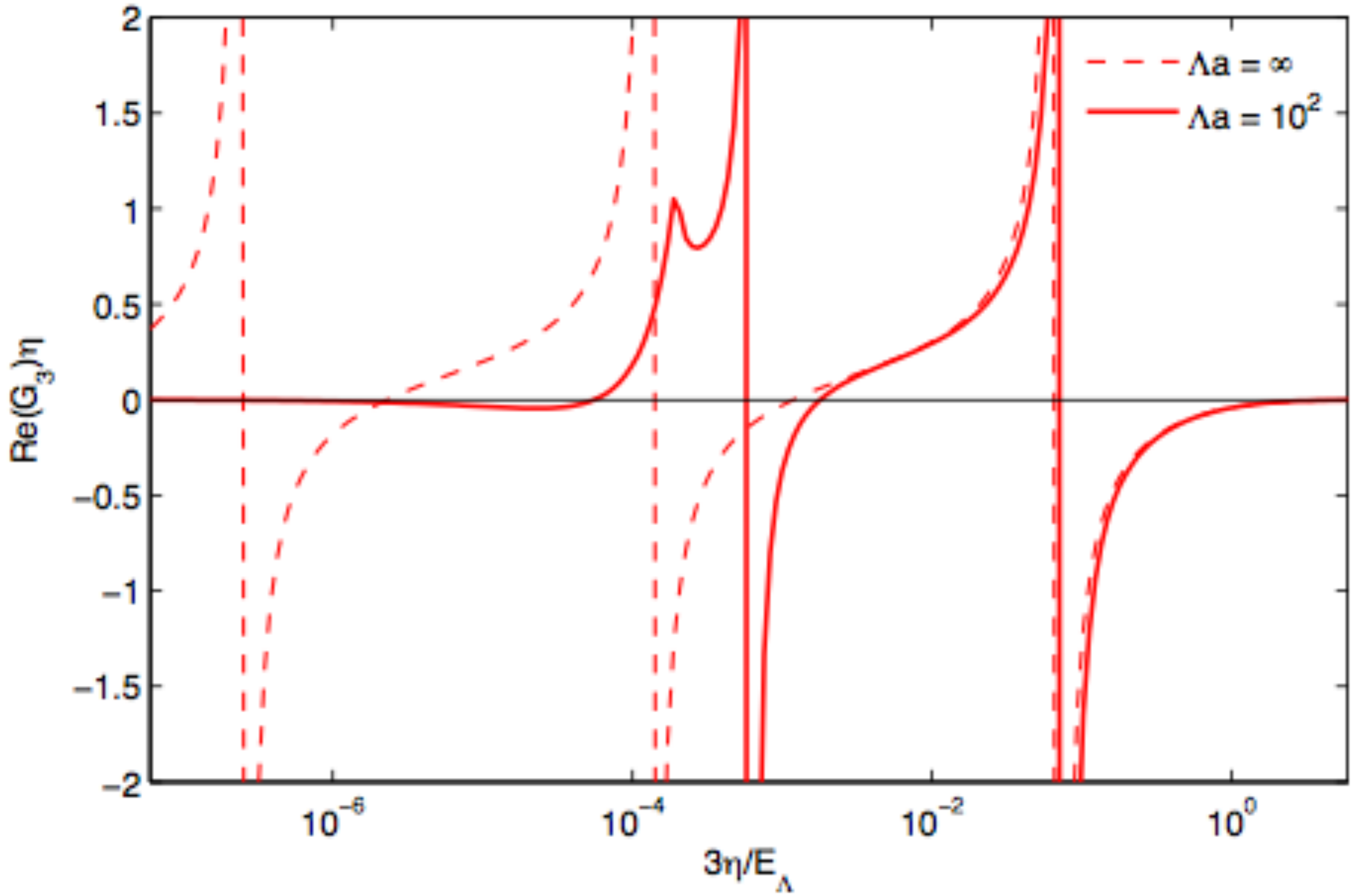
$$\frac{g_2(\Lambda_\mu) - g_2(0)}{LHY} = \frac{(c)}{(c)+(d)} = \frac{9\pi\sqrt{2}}{40} = 99.96\%$$



Blue: resummation predicts a critical point.

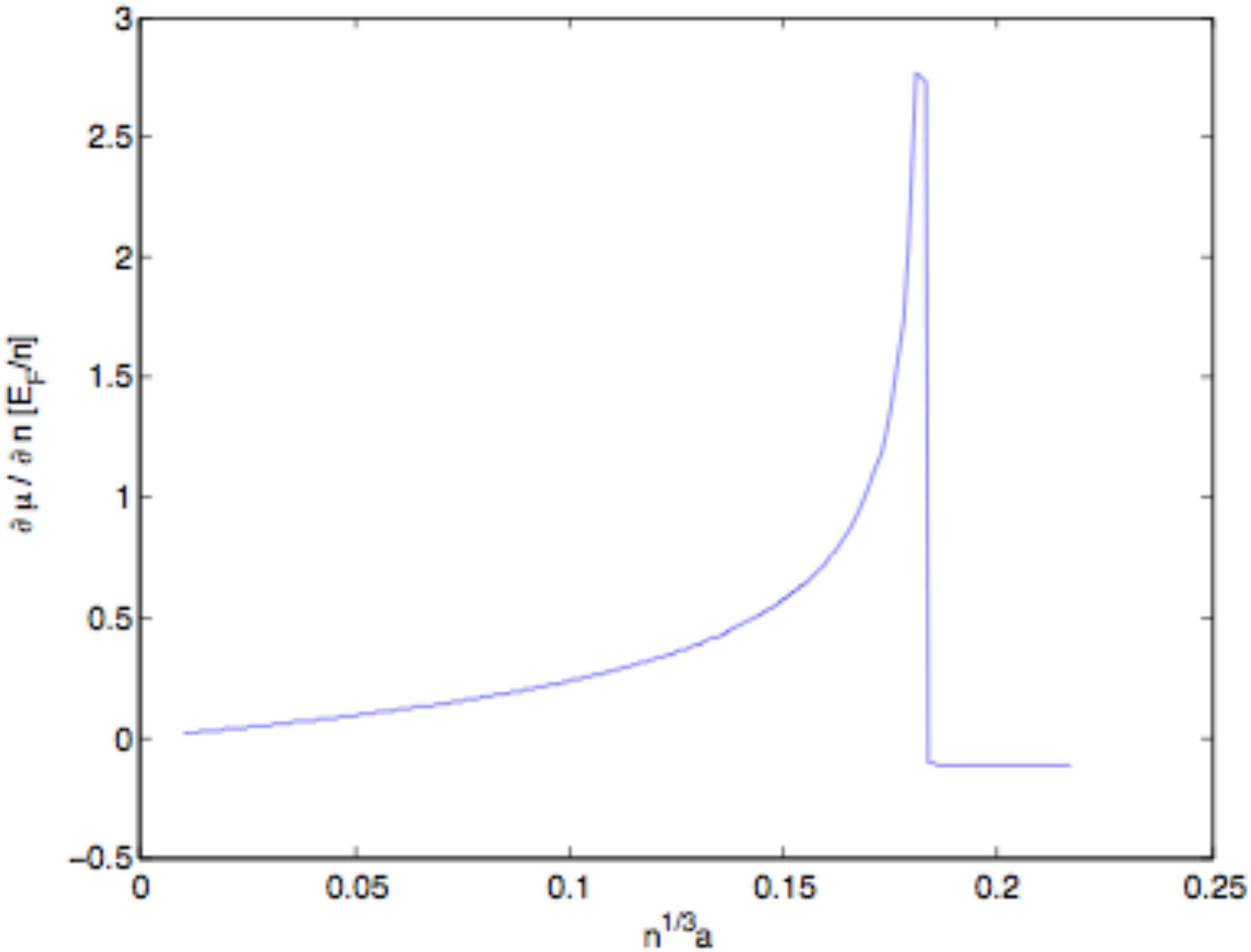
Red: with Efimov physics; Dashed line: Lee-Yang-Huang

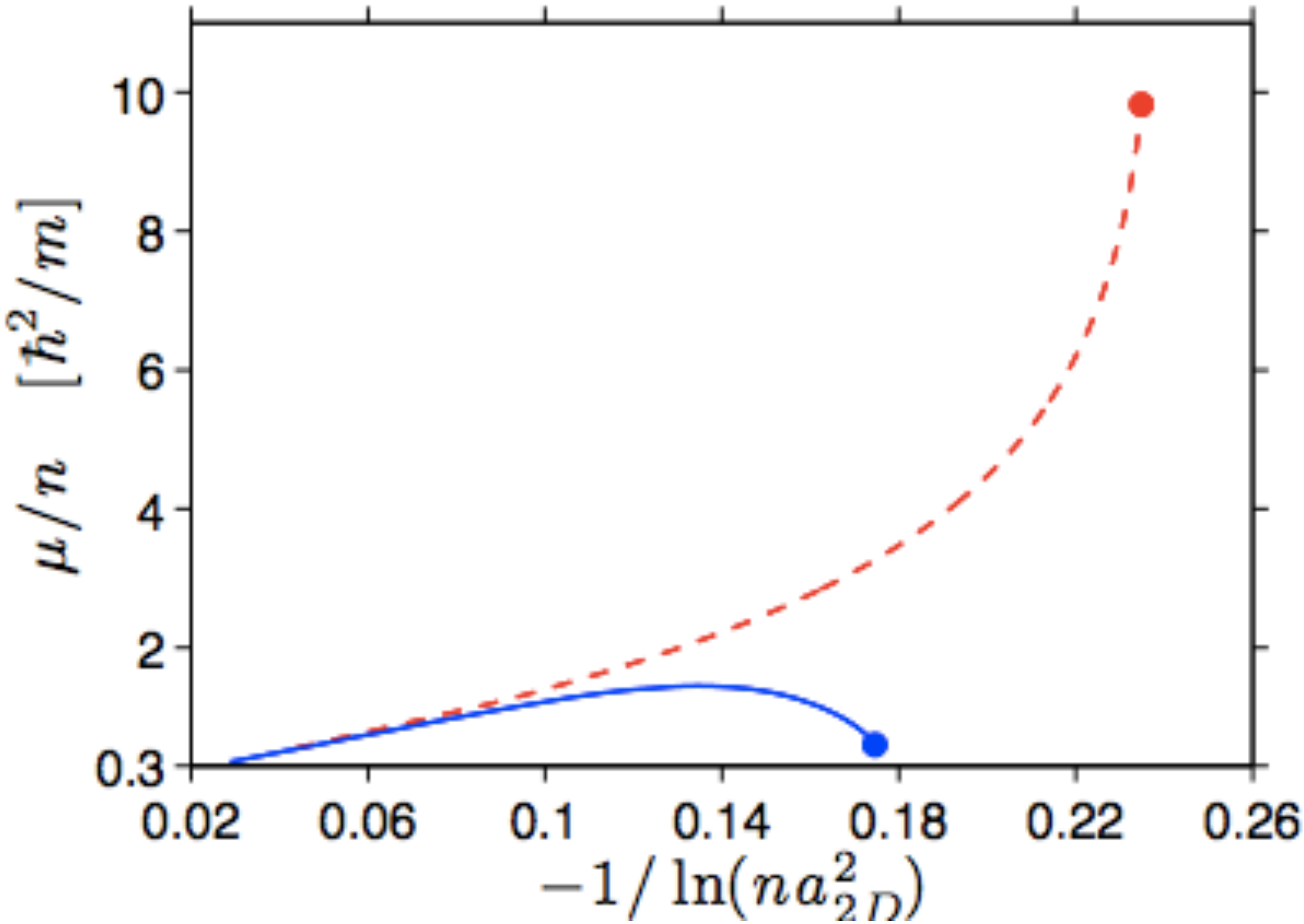
three-body physics in BECs at varying \mu



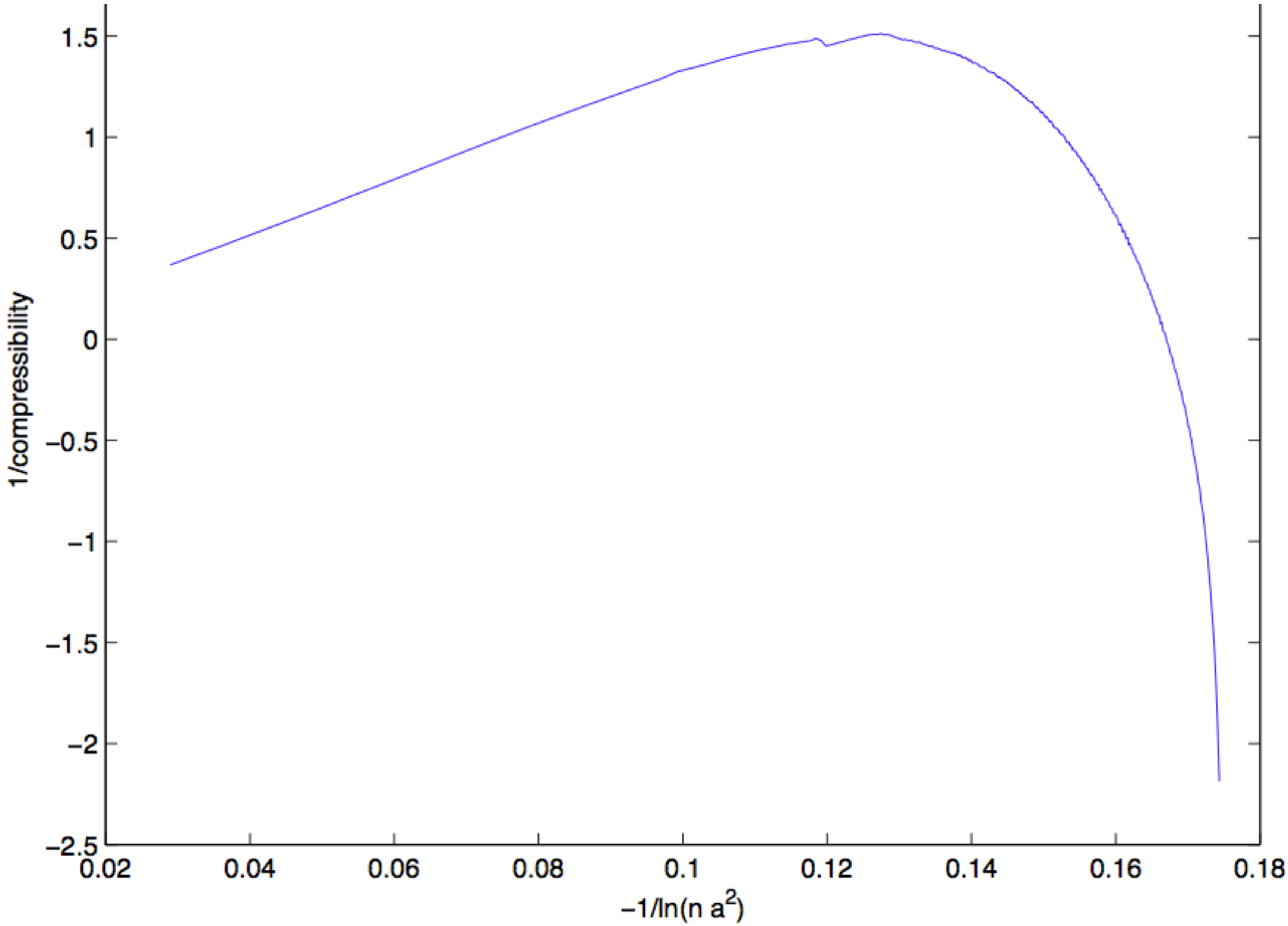
Consistent with RGE in Bedaque, Hammer, van Klock, 1999.

Anomalous Compressibility





2D, Mashayekhi+ Bernier et al, 2013. Consistent with the variational QMC simulations by Giorgini et al., 2005.



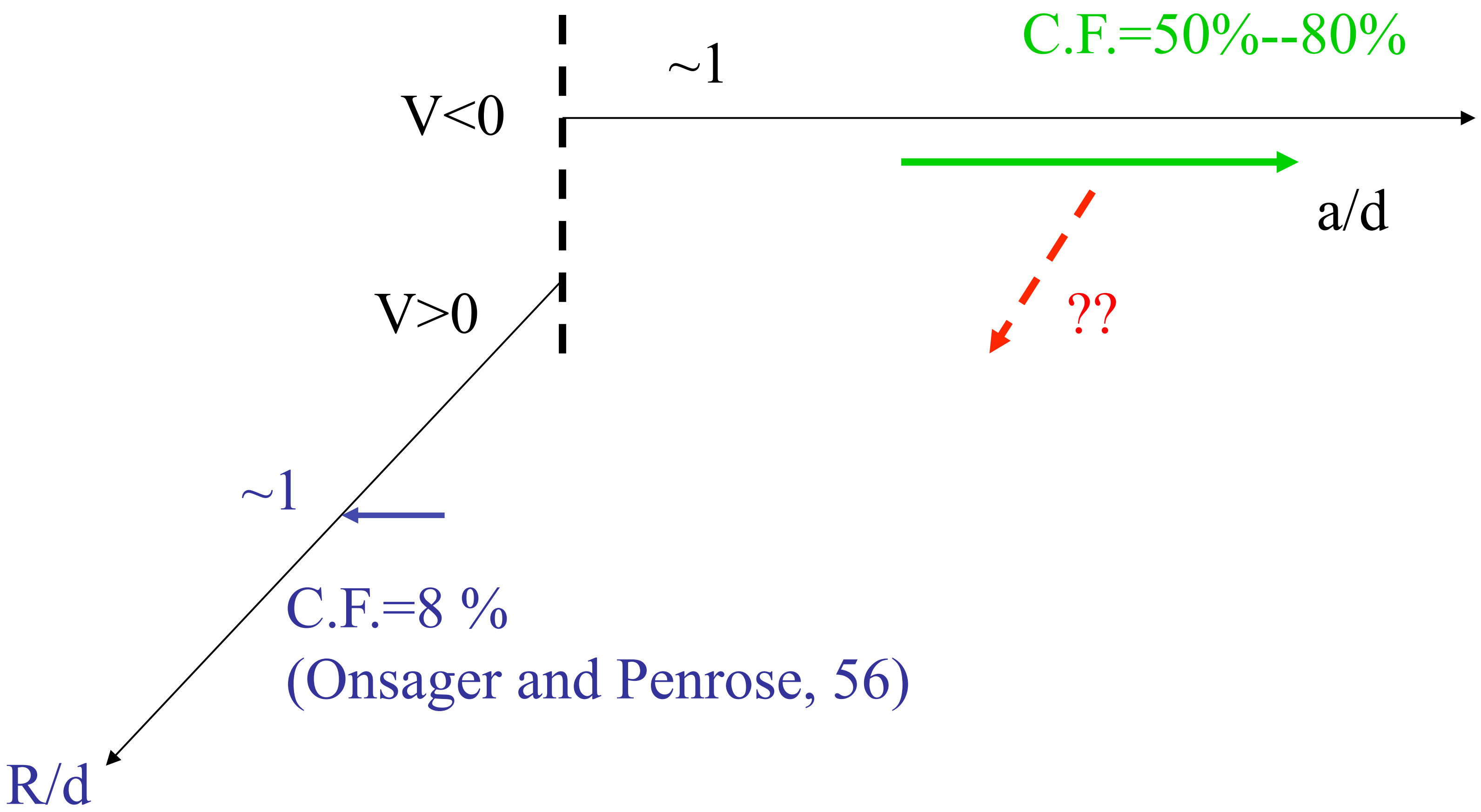
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
1/compressibility
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
−1/ln(n a2)

Implications of epsilon expansion

$$\mu = \epsilon^{\frac{2}{4-\epsilon}} \epsilon_F \sqrt{\frac{2}{3}} (1 + 0.474\epsilon - i1.217\epsilon + \cdots),$$

$$n_0 = \frac{2}{3} n (1 + 0.0877\epsilon + \cdots).$$

Relation to the estimate of liquid Helium



Conclusions

1) 3D Bose gases near resonance (Beyond Lee-Huang-Yang limit)
   a) are nearly fermionized;
   b) chemical potential reaches a maximum;
   c) an onset of instability near resonance---
   unexpected in the dilute theory;
   d) Efimov effects play a role though not significant near instability.
2) However 3-body effects significant in 2D gases.
3) Generally, the rigorous solution suggests that near 4D are a collection of independent scattering pairs.
   Moving away from 4D, 3, 4-body etc effects get stronger and stronger, while the life time gets shorter. Consistent with 1) And 2).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