Combining ab initio calculations and EFT for loosely bounded systems: $^{7}\mathrm{Li}(n,\gamma)^{8}\mathrm{Li}$ and $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$

Xilin Zhang (Ohio University)

INT "Universality in few-body systems" Program, University of Washington, Seattle, WA, April 22, 2014

X. Z, K. M. Nollett and D. R. Phillips, arXiv:1311.6822 (PRC.89.024613) ; 1401.4482 [PRC(R)]

Outline

- Motivations

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- A toy model: spinless nucleon and core

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- Li7 capture: spins, core excitation, leading order (LO) results
- Be7 capture: nonperturbative Coulomb, LO results

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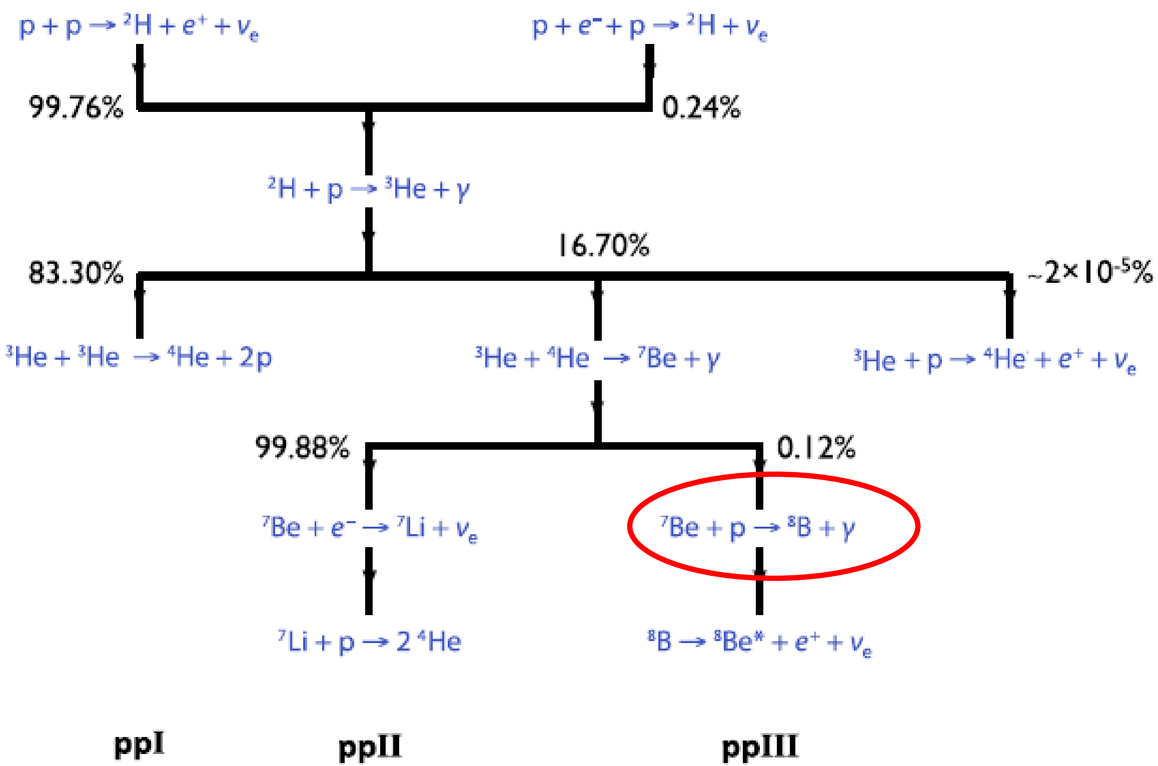
- A toy model: spinless nucleon and core
- Li7 capture: spins, core excitation, leading order (LO) results
- Be7 capture: nonperturbative Coulomb, LO results
- Outlook: Next-to-LO

Motivations

- Astrophysics: solar neutrino flux; solar model;…
- Neutrino mixing parameters

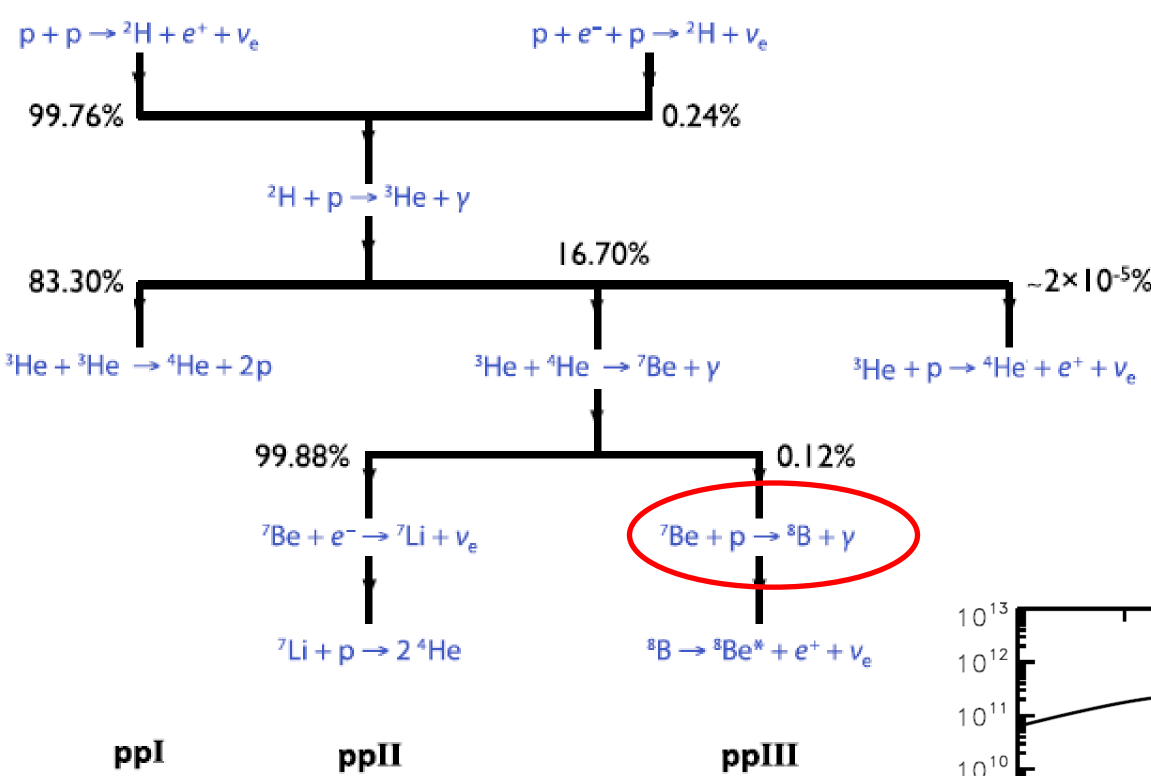
Solar neutrino generation



Solar neutrino generation

W. C. Haxton et.al., arXiv:1208.5723

Solar neutrino generation



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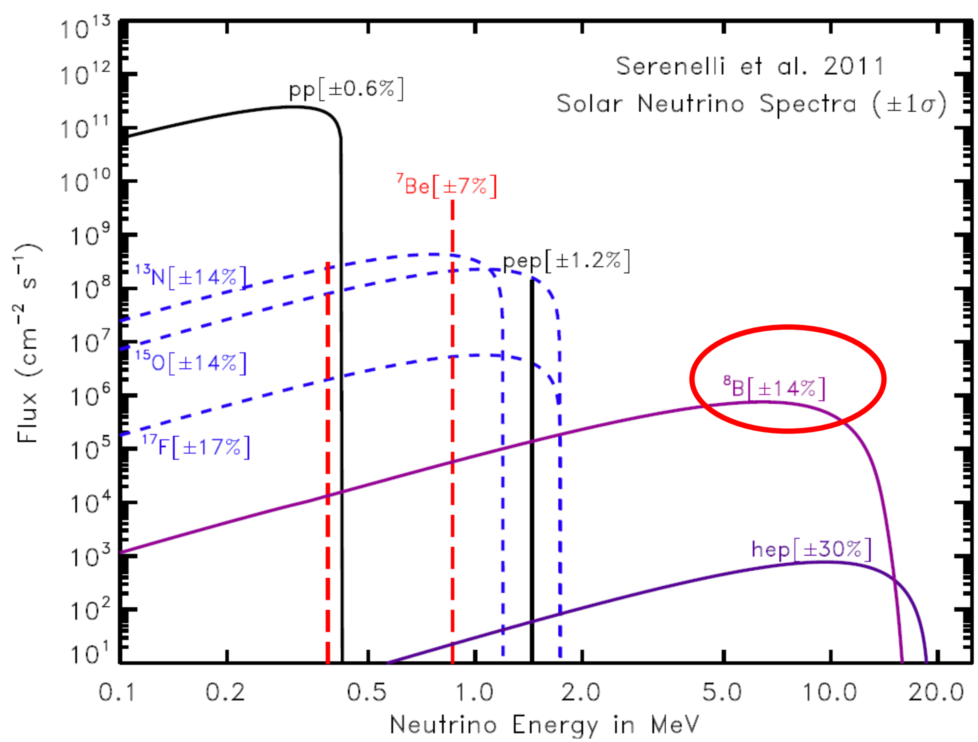

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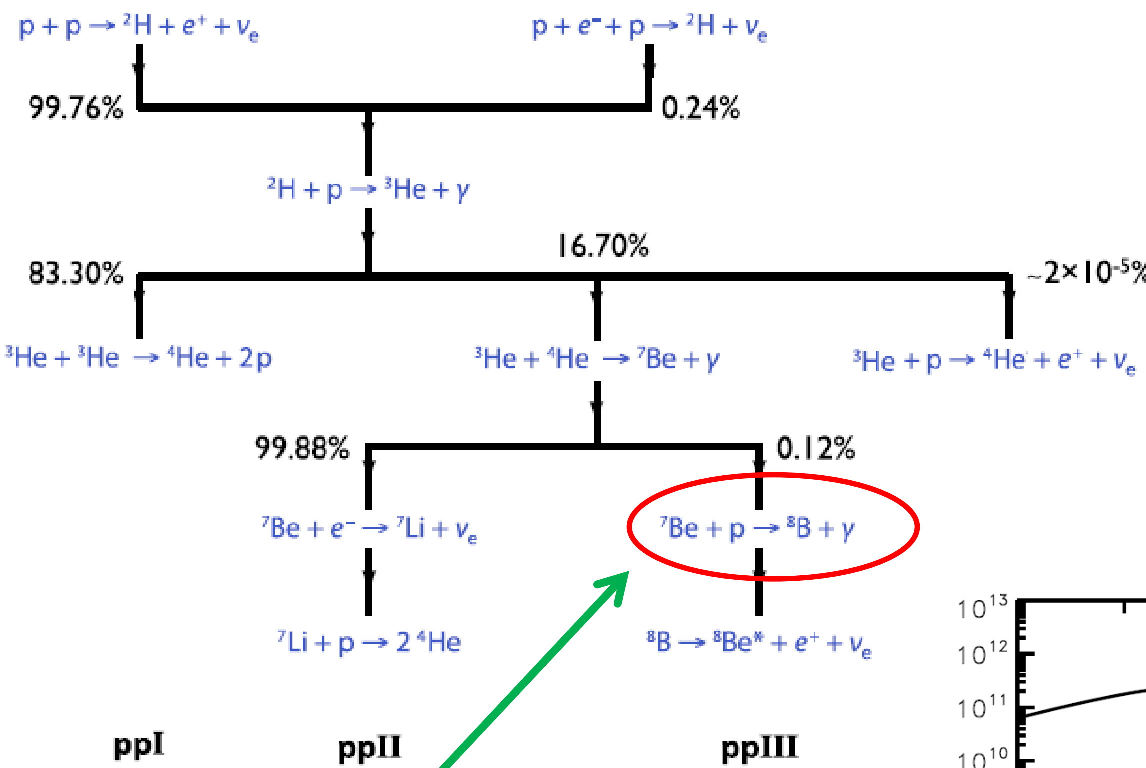
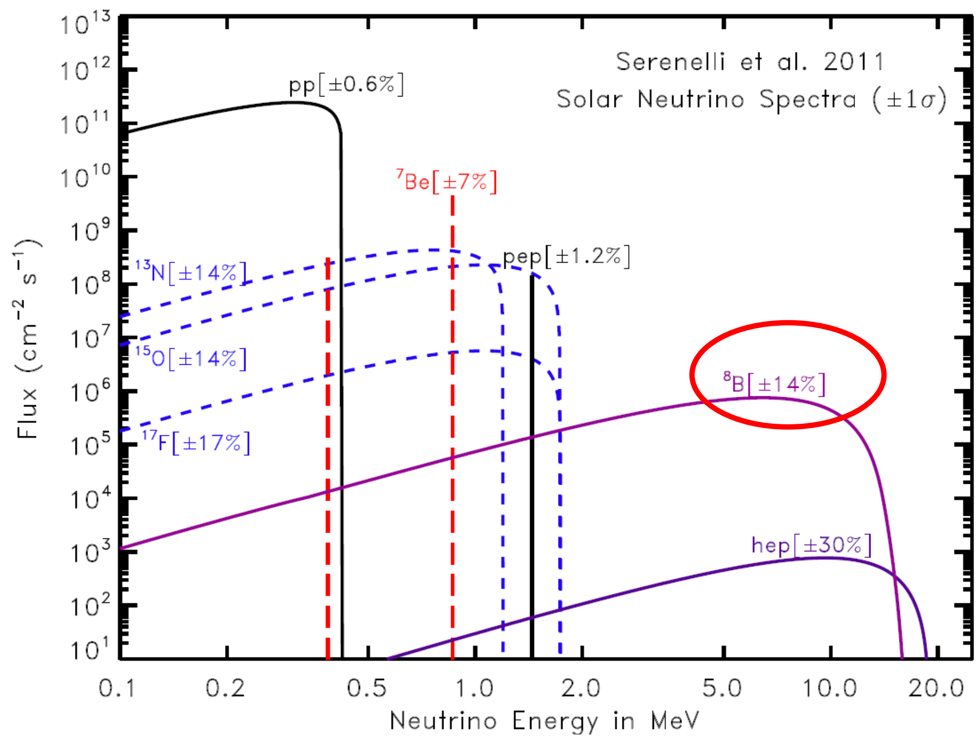


Not experimentally accessible

W. C. Haxton et.al., arXiv:1208.5723



SENSITIVITY OF *r*-PROCESS NUCLEOSYNTHESIS TO LIGHT-ELEMENT NUCLEAR REACTIONS

TAKAHIRO SASAQUI[1] AND T. KAJINO
National Astronomical Observatory, 2-21-1 Osawa, Mitaka, Tokyo 181-8588; and Department of Astronomy, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan; sasaqui@th.nao.ac.jp

G. J. MATHEWS AND K. OTSUKI[2]
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AND

MOST IMPORTANT 18 LIGHT-MASS NUCLEAR REACTIONS, ADOPTED "STANDARD" THERMONUCLEAR REACTION RATES $\lambda_i(0)$, AND UNCERTAINTIES

No.	Reaction	$N_{\rm Av}\langle\sigma v\rangle$	1 σ[a]	Reference[b]
(1)........................	$\alpha(\alpha n,\gamma)^9$Be	$N_{\rm Av}^2\langle\alpha\alpha n\rangle = 2.43\times10^9 T_9^{-2/3}\exp[-13.490T_9^{-1/3}-(T_9/0.15)^2](1+74.5T_9)$ $+6.09\times10^5 T_9^{-3/2}\exp(-1.054/T_9)(1-58.80T_9-1.794\times10^4T_9^2$ $+2.969\times10^6T_9^3-1.535\times10^8T_9^4+2.610\times10^9T_9^5)$	±35%	1
(2)........................	$\alpha(t,\gamma)^7$Li[c]	$3.032\times10^5T_9^{-2/3}\exp(-8.09/T_9^{1/3})(1.0+0.0516T_9^{1/3}+0.0229T_9^{2/3}$ $+8.28\times10^{-3}T_9-3.28\times10^{-04}T_9^{4/3}-3.01\times10^{-04}T_9^{5/3})$ $+5.109\times10^5T_{9*}^{5/6}T_9^{-3/2}\exp(-8.068/T_{9*}^{1/3})$	±30%	2
(3)........................	^{7}Li$(n,\gamma)^8$Li	$4.90\times10^3+9.96\times10^3T_9^{-3/2}\exp(-2.62/T_9)$	±35%	3

Li7 capture is used to constrain models of Be7 capture.

Motivations

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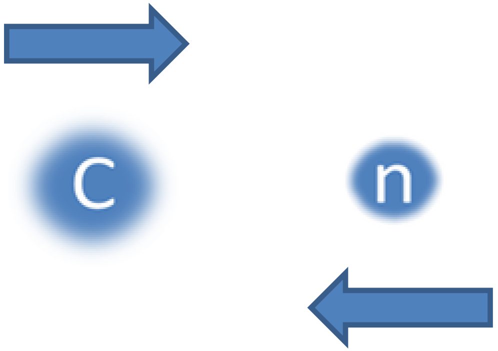
- Astrophysics: solar neutrino flux; solar model;…
- Neutrino mixing parameters
- EFT: a simple picture; systematic expansion (Lagrangian); uncertainty estimate
- Parameters: ab initio bound state information
- Ab initio reaction calculation

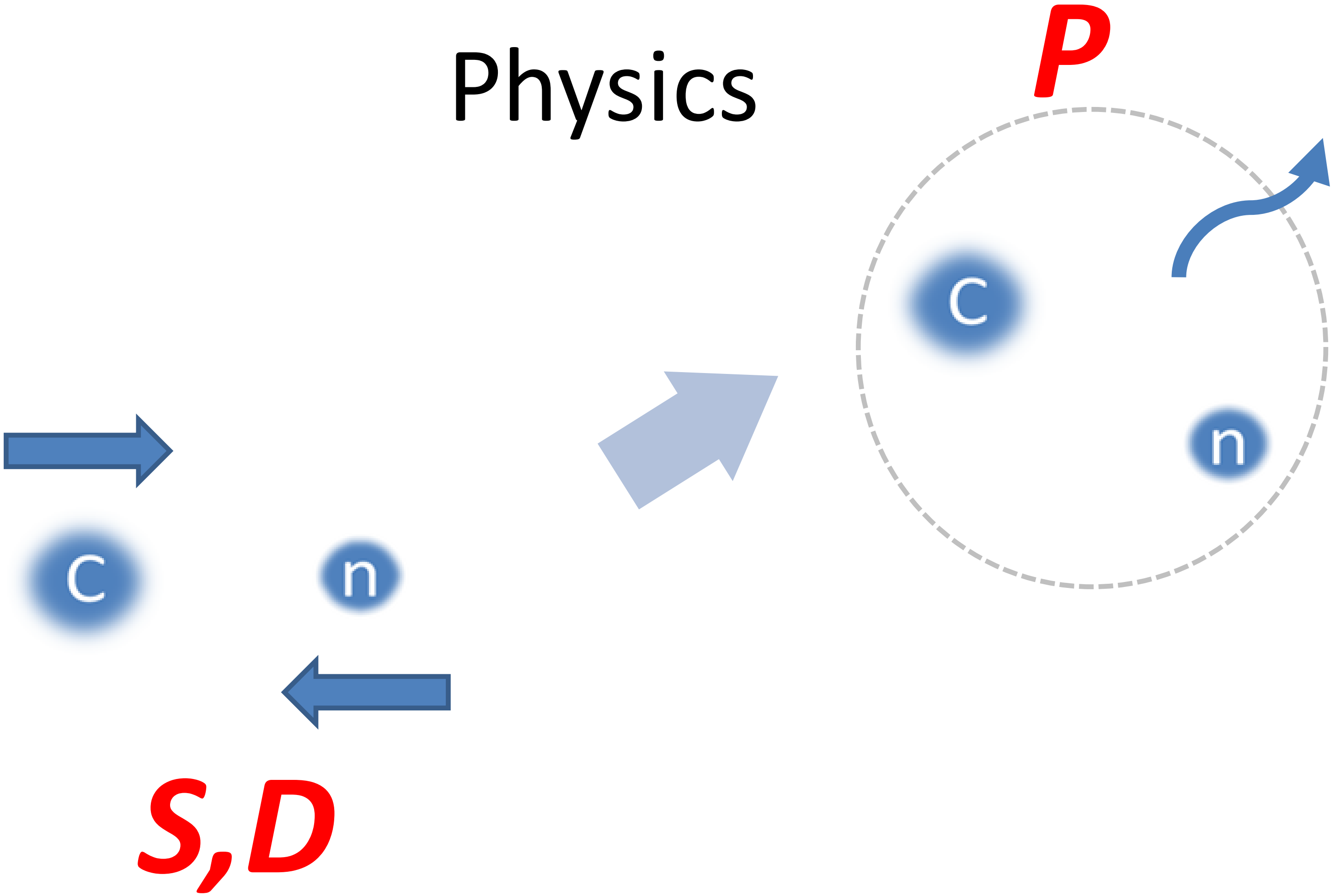
A toy model

Physics



S,D

Physics



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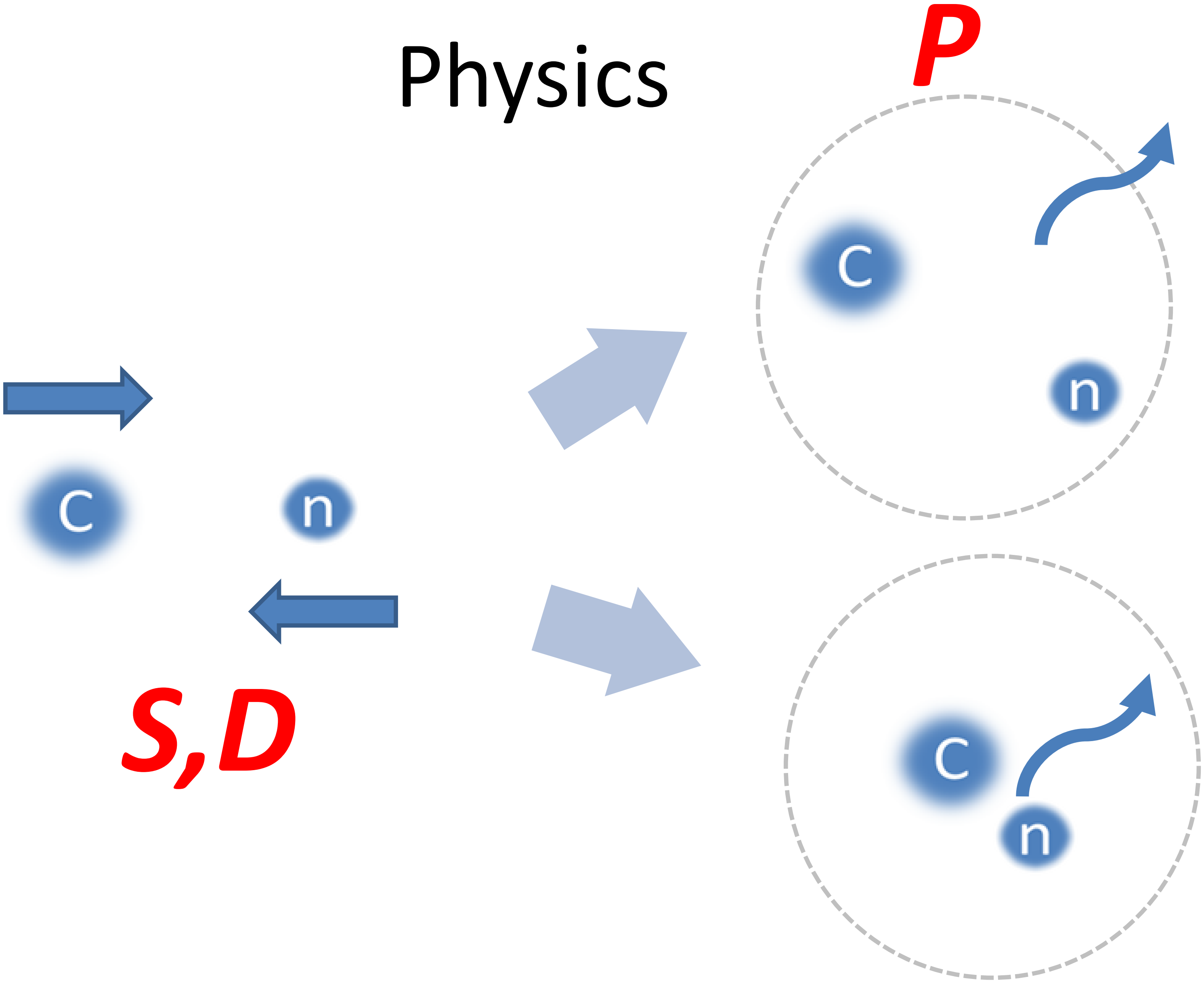

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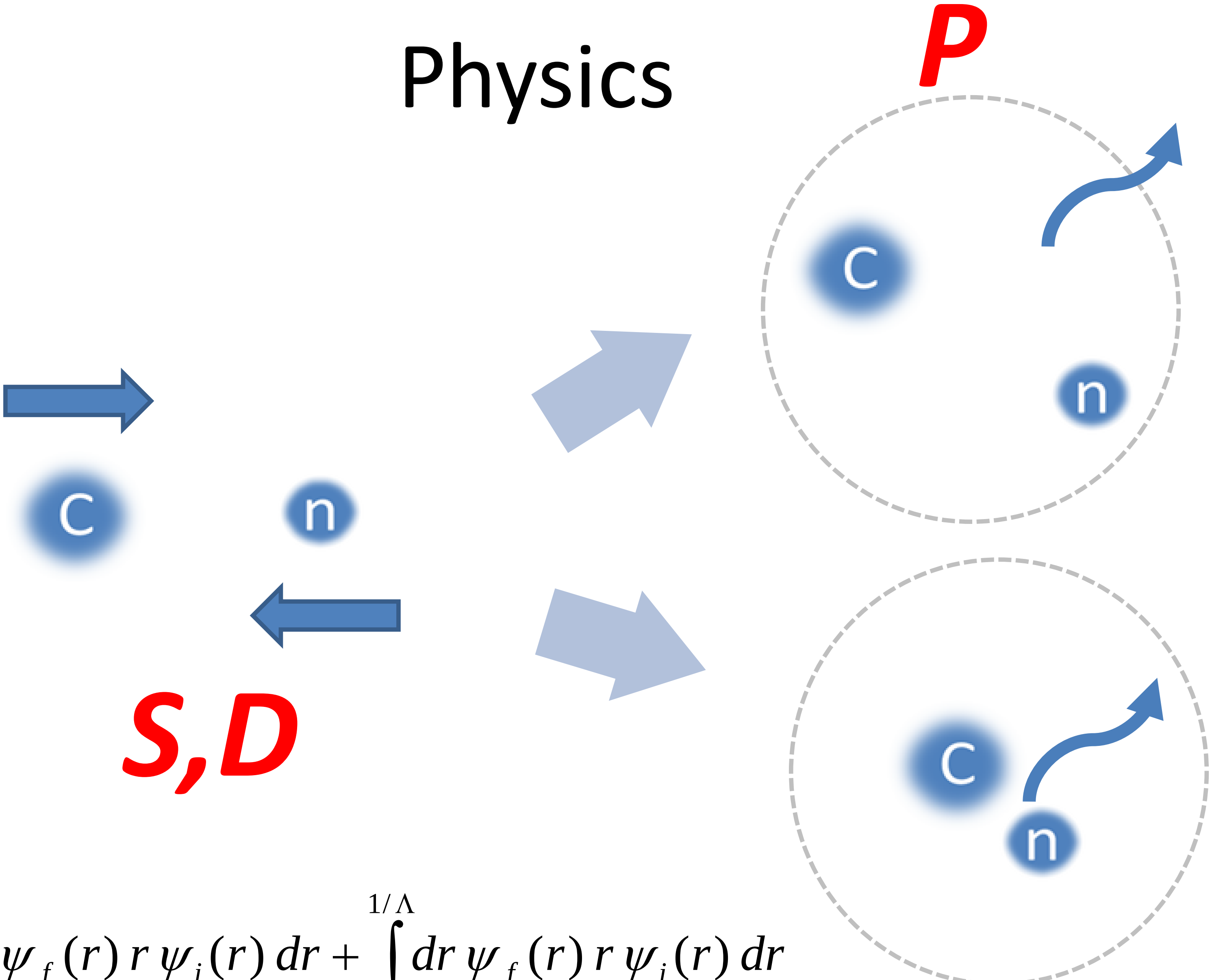
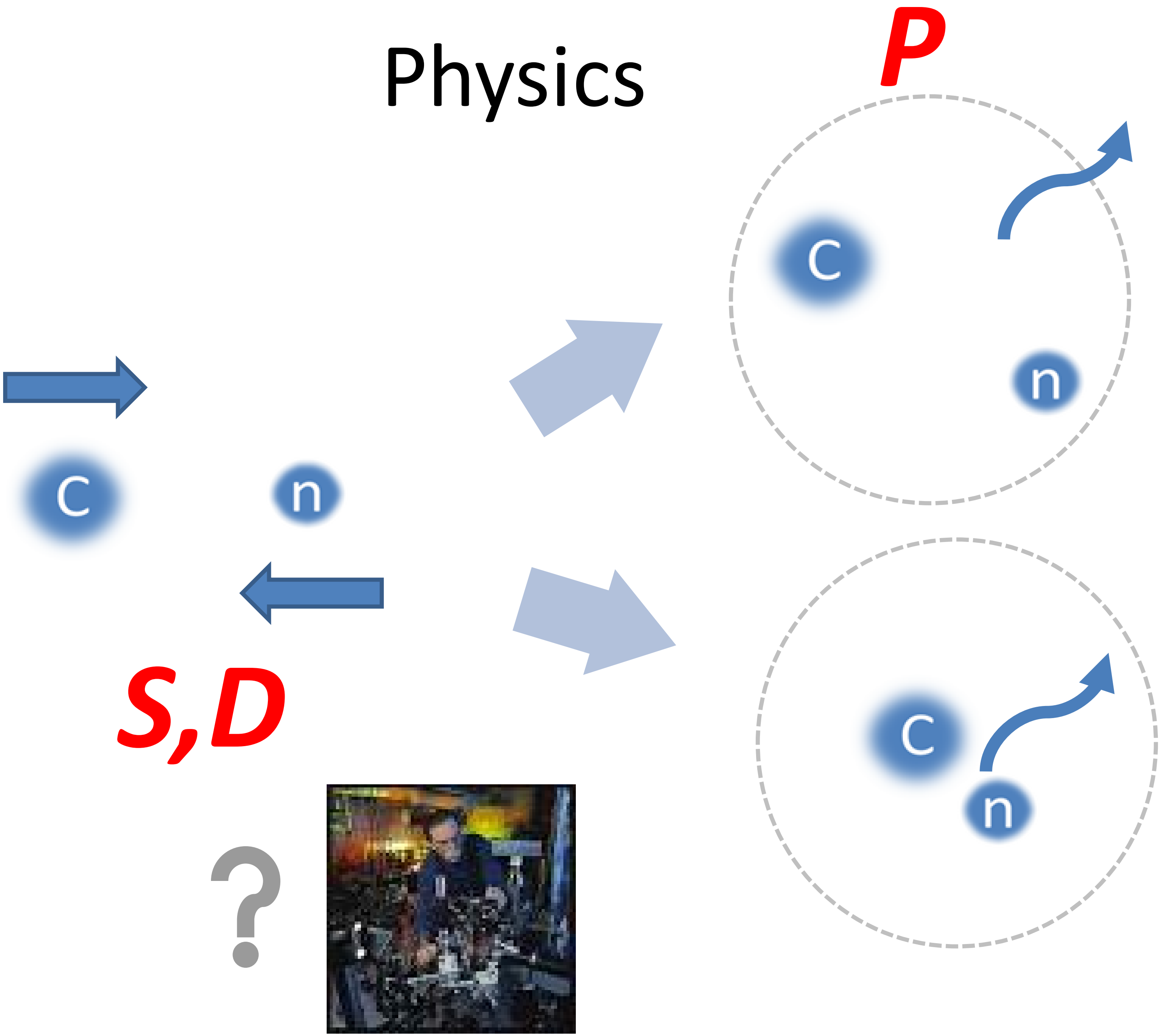
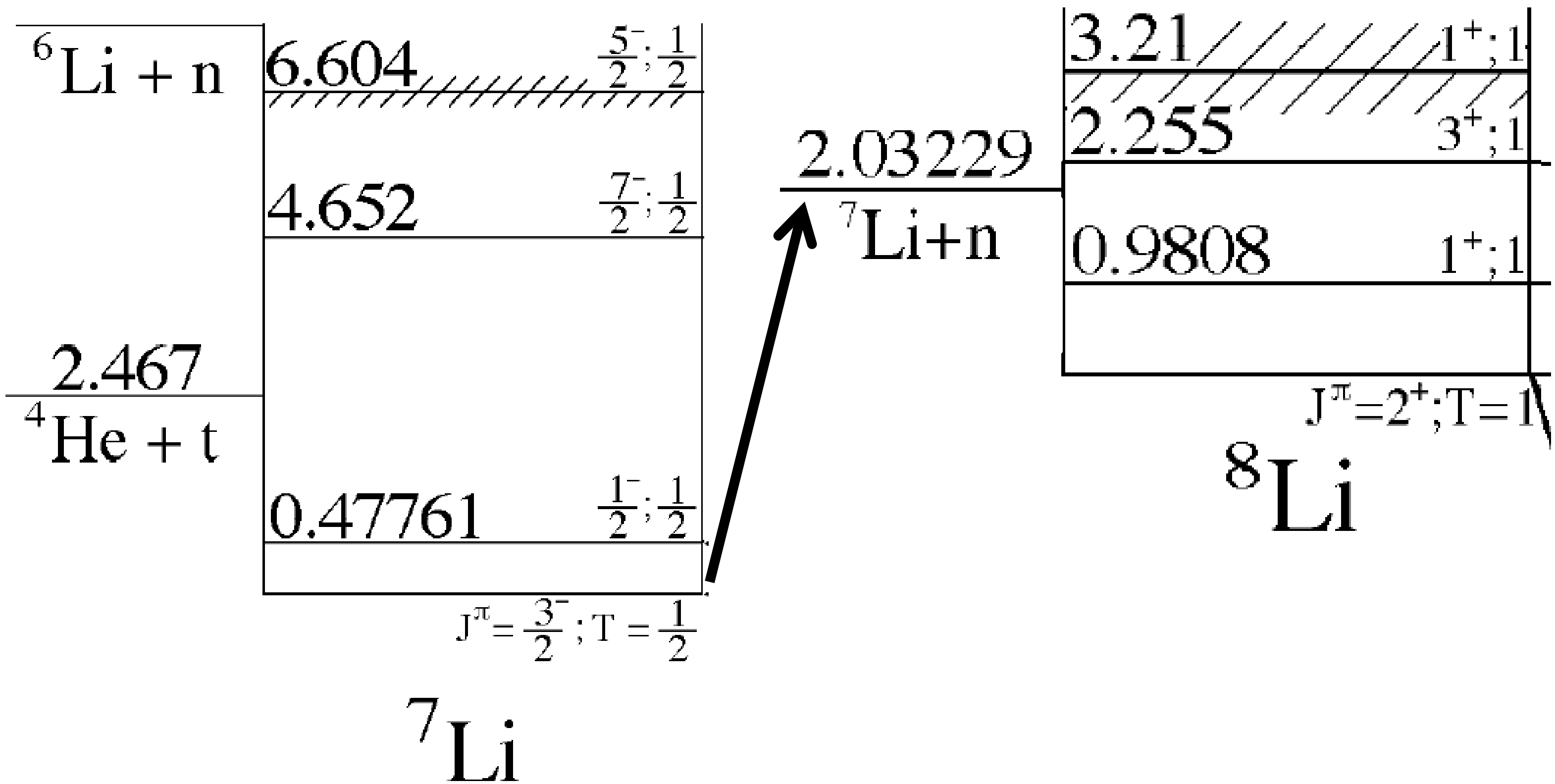


$$M \sim \int_{1/\Lambda}^{+\infty} dr\, \psi_f(r)\, r\, \psi_i(r)\, dr + \int_{0}^{1/\Lambda} dr\, \psi_f(r)\, r\, \psi_i(r)\, dr$$

Physics



Gross features: p-wave



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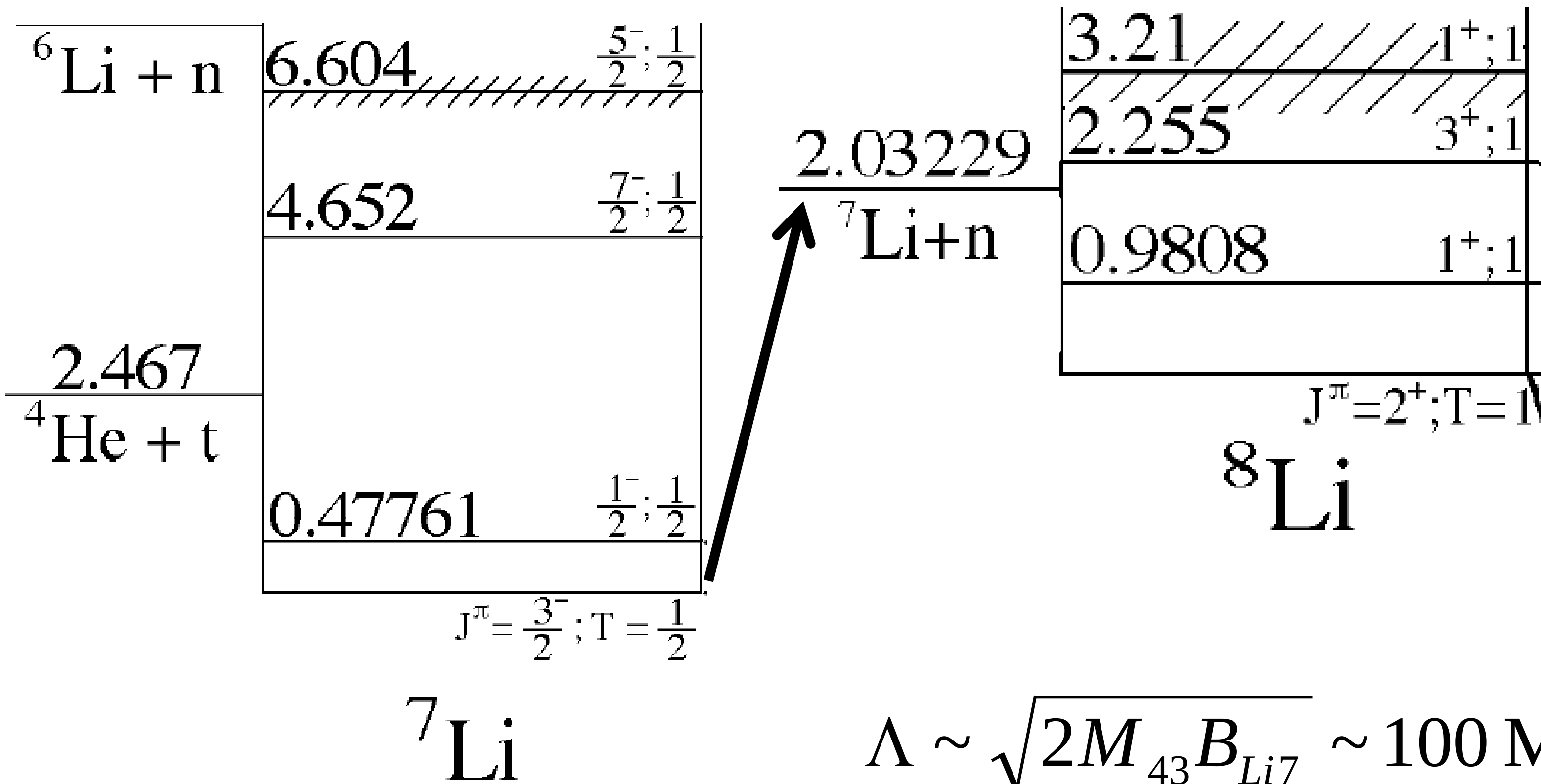

$$\Lambda \sim \sqrt{2M_{43}B_{Li7}} \sim 100\,\mathrm{MeV}$$

$$\gamma \sim \sqrt{2M_{71}B_{Li8}} = 57.8\,\mathrm{MeV}$$

$$\frac{\gamma}{\Lambda} \sim 0.5$$

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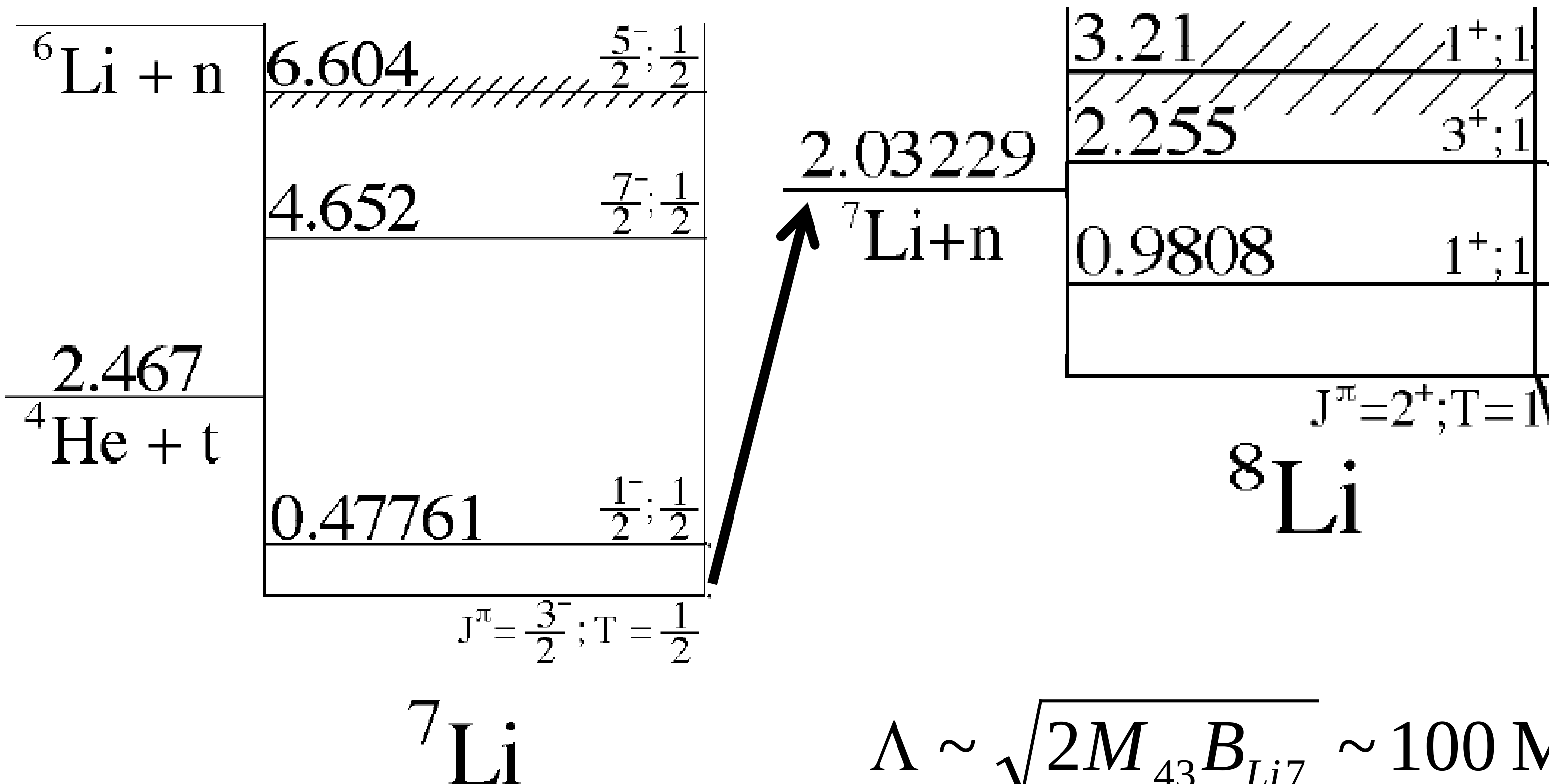

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Shallow p-wave bound state

Gross features: s-wave

Parameter	Channel	Value	Assigned scaling
$a_{(^5S_2)}$	S-wave, $S=2$	$-3.63(5)$ fm	$1/\gamma$
$a_{(^3S_1)}$	S-wave, $S=1$	$0.87(7)$ fm	$1/\Lambda$

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L. Koester, K. Knopf, and W. Waschkowski, Z. Phys. A 312, 81 (1983)

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Large s-wave scattering length

L. Koester, K. Knopf, and W. Waschkowski, Z. Phys. A 312, 81 (1983)

S-wave in EFT

$$\mathcal{L}_0 = n^\dagger \left(i\partial_t + \frac{\nabla^2}{2M_{\rm n}} \right) n + c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}} \right) c$$

$$\mathcal{L}_S = g c^\dagger n^\dagger c n \qquad\qquad g(\mu) = \frac{2\pi}{M_{\rm R}\left(2\mu - \frac{1}{a_0}\right)}$$

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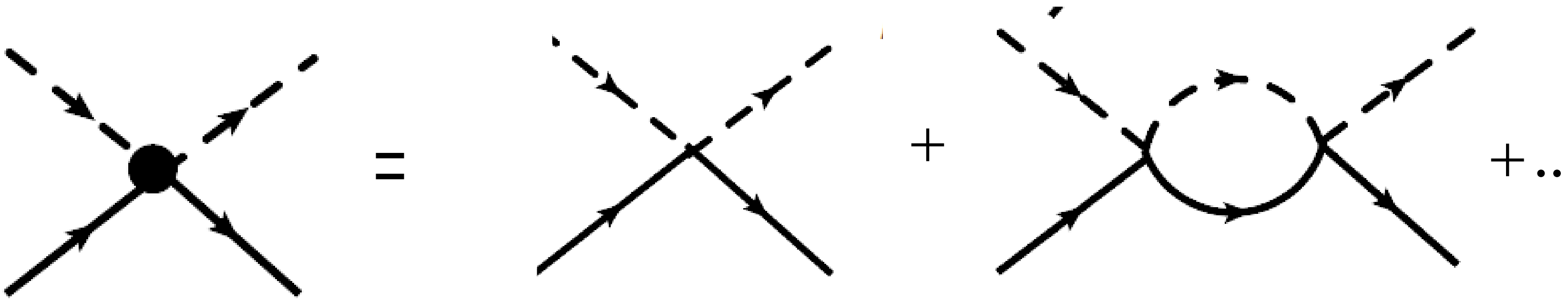

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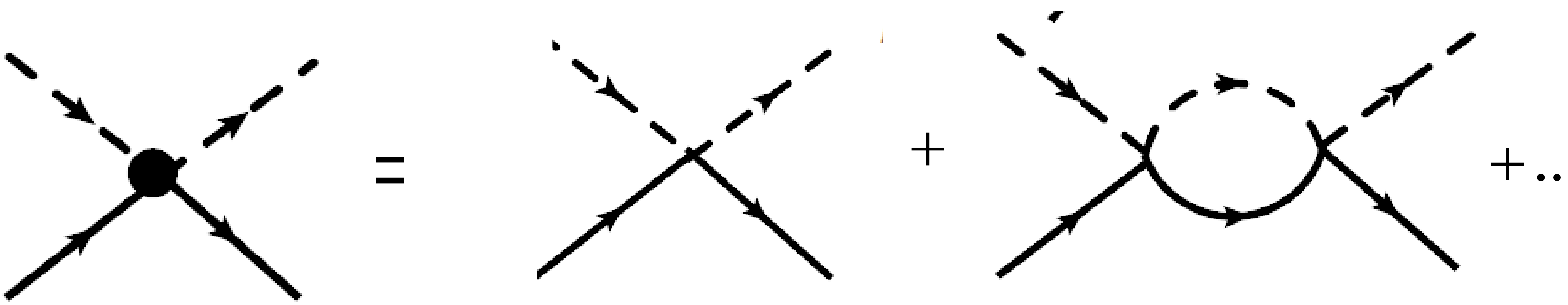
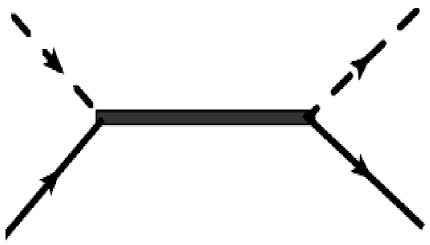


$$T = \frac{2\pi}{M_\mathrm{R}} \frac{1}{a_0^{-1} + ik}$$

One parameter: g (or a0)

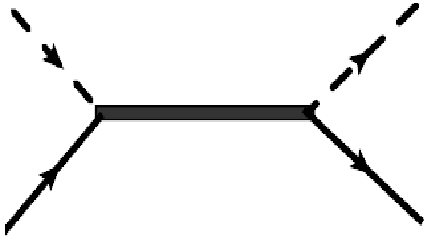
P-wave in EFT



$$\langle \boldsymbol{p}'|T(E)|\boldsymbol{p}\rangle = \frac{6\pi}{M_{\mathrm{R}}} \frac{\boldsymbol{p}' \cdot \boldsymbol{p}}{a_1^{-1} - \frac{1}{2} r_1 k^2 + i k^3}$$

$$k^3 \cot \delta_1 = -\frac{1}{a_1} + \frac{1}{2} r_1 k^2 + \dots$$

P-wave in EFT



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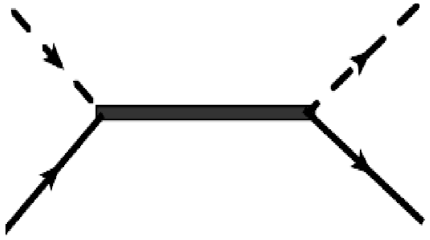
Shallow p-wave bound state: $$\frac{1}{a_1} + \frac{1}{2} r_1 \gamma^2 + \gamma^3 = 0$$

P-wave in EFT



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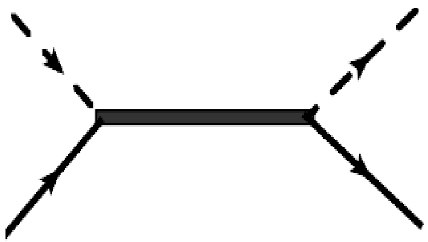
$$a_1 \sim \frac{1}{\Lambda\gamma^2} \text{ and } r_1 \sim \Lambda$$

Unnatural *Natural*

P-wave in EFT



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Unnatural *Natural*

P. F. Bedaque, H.-W. Hammer and U. van Kolck, PLB 569, 159 (2003)

P-wave in EFT

$$\mathcal{L}_P = \pi^{\dagger i}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \Delta\right)\pi_i + h\pi^{\dagger i} n i\left(\boldsymbol{V}_n - \boldsymbol{V}_c\right)_i c + \text{C.C.}\ .$$

P-wave in EFT

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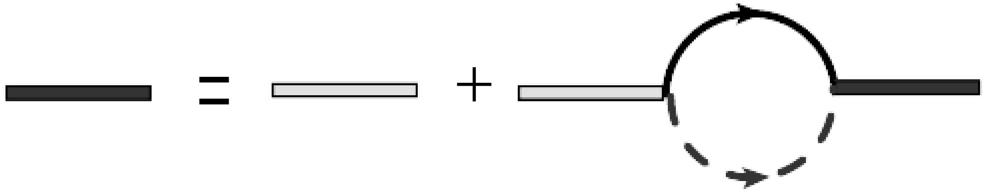

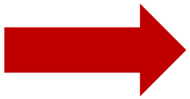

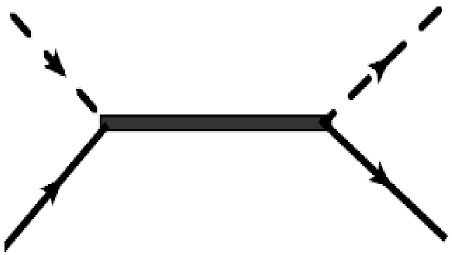

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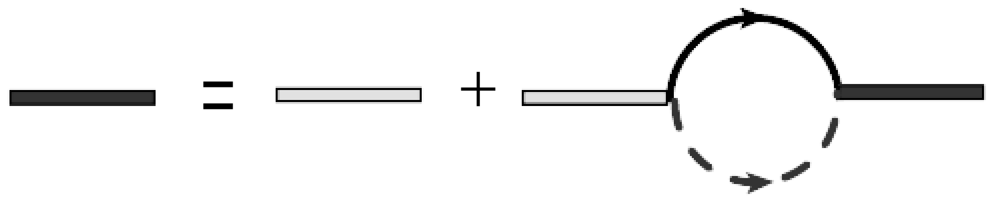

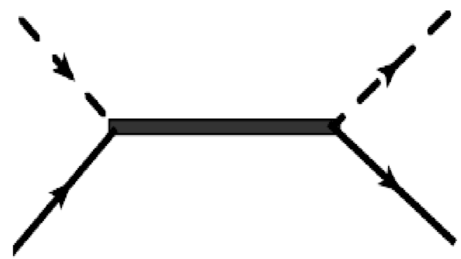

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Two parameters: Delta and h (or a1 and r1)

P-wave in EFT

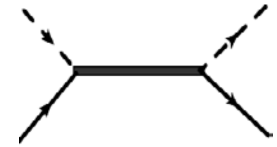
$$\langle \boldsymbol{r}^{'} | \frac{1}{E-H} | \boldsymbol{r} \rangle = \langle \boldsymbol{r}^{'} | \frac{1}{E-H_0} + \frac{1}{E-H_0} T \frac{1}{E-H_0} | \boldsymbol{r} \rangle$$

Asymptotic normalization coefficient (ANC)

$$\xrightarrow{E \to -B} C^2 \times \sum_j \frac{\phi_j(\boldsymbol{r}^{'})\phi_j^*(\boldsymbol{r})}{E+B} \ .$$

$$\phi_j(\mathrm{r}) = \left(1 + \frac{1}{\gamma r}\right) Y_{1j}(\hat{r}) \frac{e^{-\gamma r}}{r}$$

P-wave in EFT



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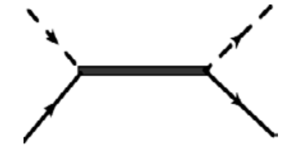

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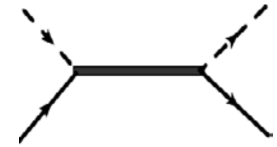
$$C = \sqrt{\frac{-2\gamma^2}{r_1 + 3\gamma}}$$

$$\frac{1}{a_1} + \frac{1}{2} r_1 \gamma^2 + \gamma^3 = 0$$

a_1 and r_1 (or h and Δ)

P-wave in EFT



$$\langle r'|\frac{1}{E-H}|r\rangle = \langle r'|\frac{1}{E-H_0} + \frac{1}{E-H_0}T\frac{1}{E-H_0}|r\rangle$$

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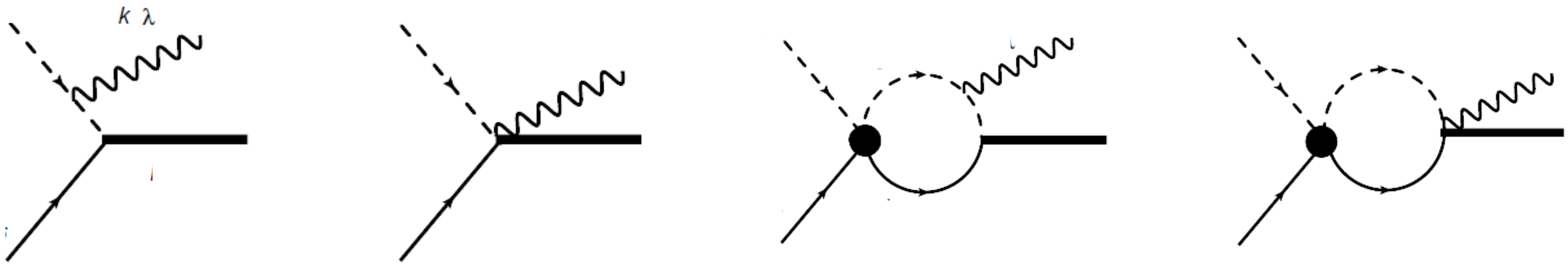
a_1 and r_1 (or h and Δ)

K. M. Nollett and R. B. Wiringa, PRC 83, 041001 (2011)

Radiative capture: LO



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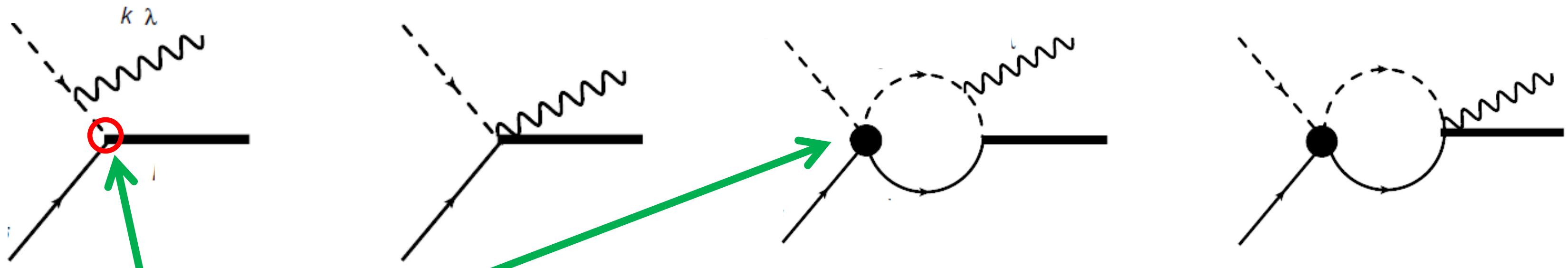
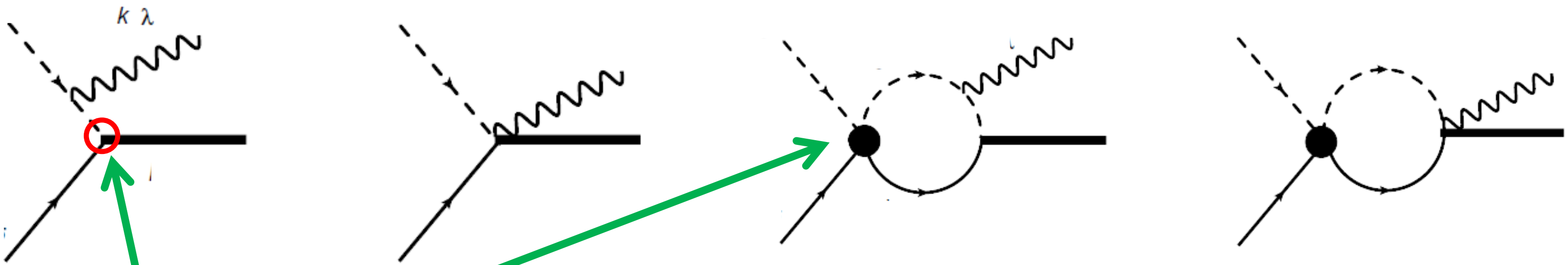


Halo-EFT parameters

Radiative capture: LO

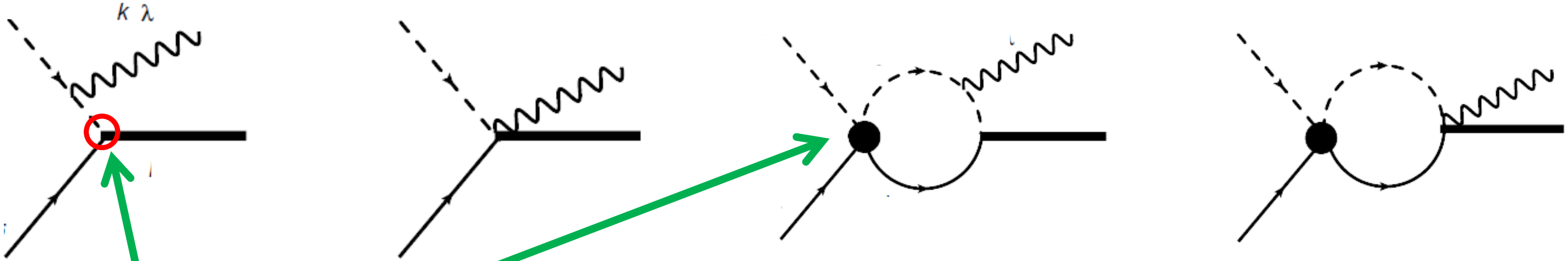


Halo-EFT parameters

$$\mathcal{M} \sim i e_c h \sqrt{Z} \left[\frac{\boldsymbol{\epsilon}^*(\lambda) \cdot \boldsymbol{V}_c}{p_c^0 - \omega - \frac{(\boldsymbol{p}_c - \boldsymbol{k})^2}{2M_c} + i\epsilon} \left(\frac{\boldsymbol{p}_c}{M_{\rm R}} - \frac{\boldsymbol{k}}{M_c} \right)_j + (1 + X(p_c;\gamma,a_0)) \frac{\boldsymbol{\epsilon}^*(\lambda)_j}{M_c} \right]$$

$$X(p_c;\gamma,a) \equiv \frac{(-)i}{a^{-1} + i p_c} \left[p_c - \frac{2}{3} i \frac{\gamma^3 - i p_c^3}{\gamma^2 + p_c^2} \right]$$

Radiative capture: LO



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C

$$X(p_c;\gamma,a) \equiv \frac{(-)i}{a^{-1}+ip_c}\left[p_c-\frac{2}{3}i\frac{\gamma^3-ip_c^3}{\gamma^2+p_c^2}\right]$$

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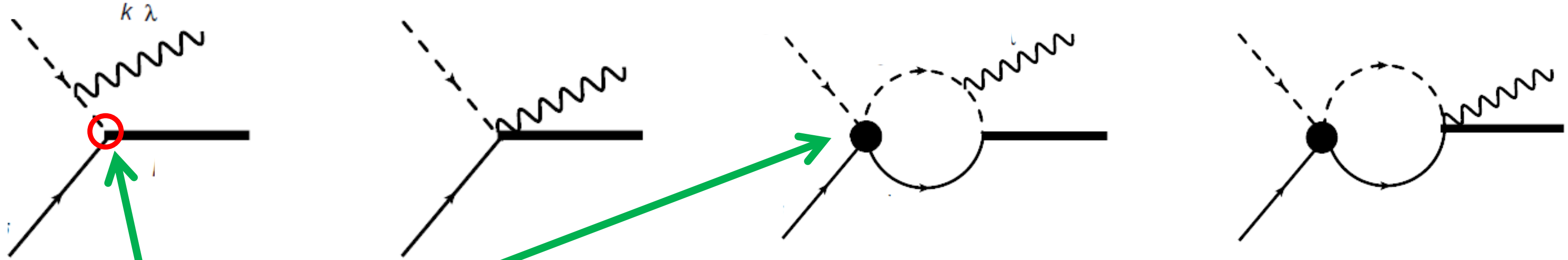


Halo-EFT parameters

$$\mathcal{M} \sim i e_c h \sqrt{Z} \left[\frac{\boldsymbol{\epsilon}^*(\lambda) \cdot \boldsymbol{V}_c}{p_c^0 - \omega - \frac{(\boldsymbol{p}_c - \boldsymbol{k})^2}{2M_c} + i\epsilon} \left(\frac{\boldsymbol{p}_c}{M_{\mathrm{R}}} - \frac{\boldsymbol{k}}{M_c} \right)_j + \left(1 + X(p_c;\gamma,a_0)\right) \frac{\boldsymbol{\epsilon}^*(\lambda)_j}{M_c} \right]$$

C

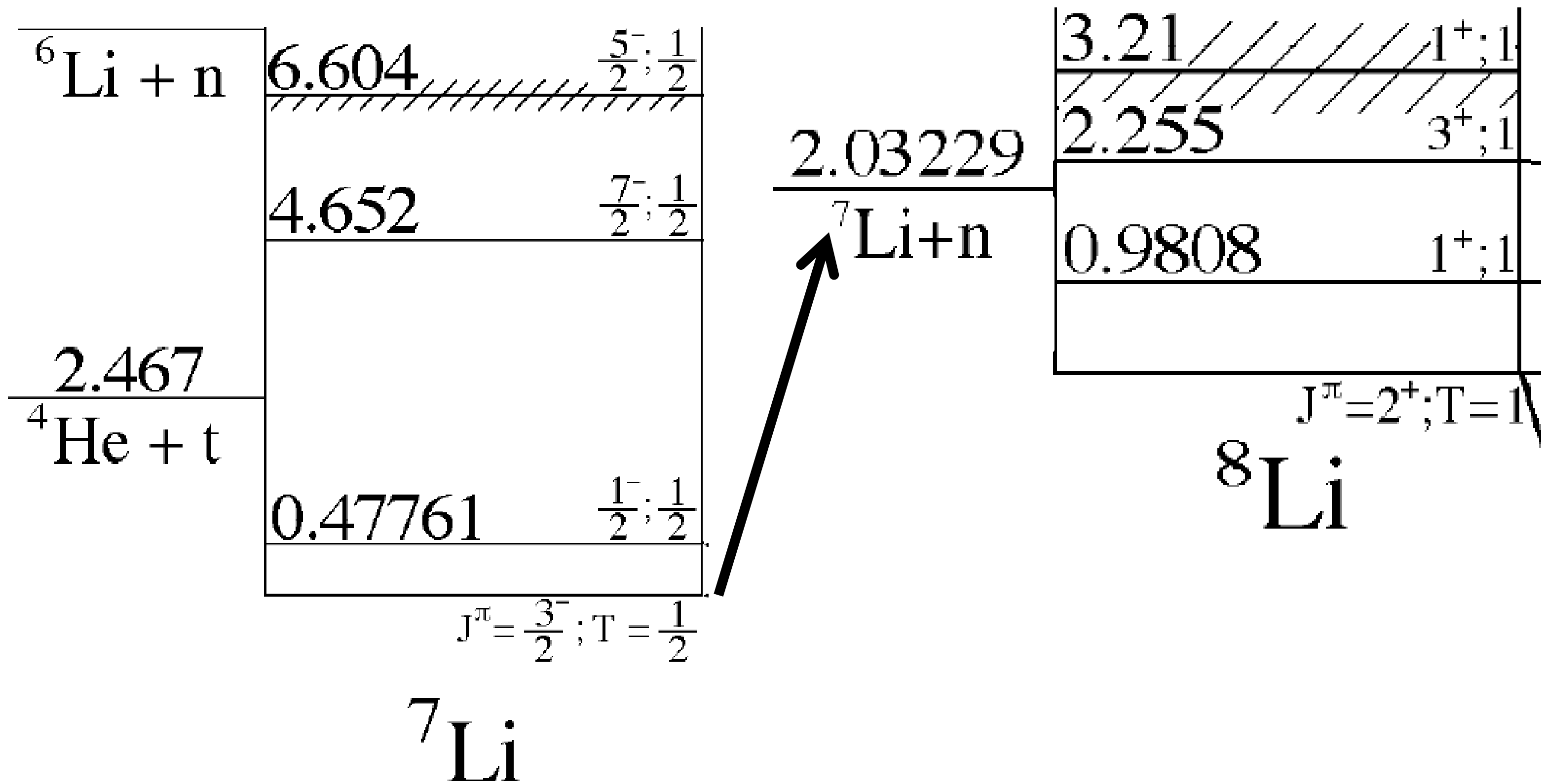
$$X(p_c;\gamma,a) \equiv \frac{(-)i}{a^{-1} + i p_c} \left[p_c - \frac{2}{3} i \frac{\gamma^3 - i p_c^3}{\gamma^2 + p_c^2} \right]$$

$$a \sim \frac{1}{\gamma} \Rightarrow X \sim 1, \quad a \sim \frac{1}{\Lambda} \Rightarrow X \sim \frac{\gamma}{\Lambda}$$

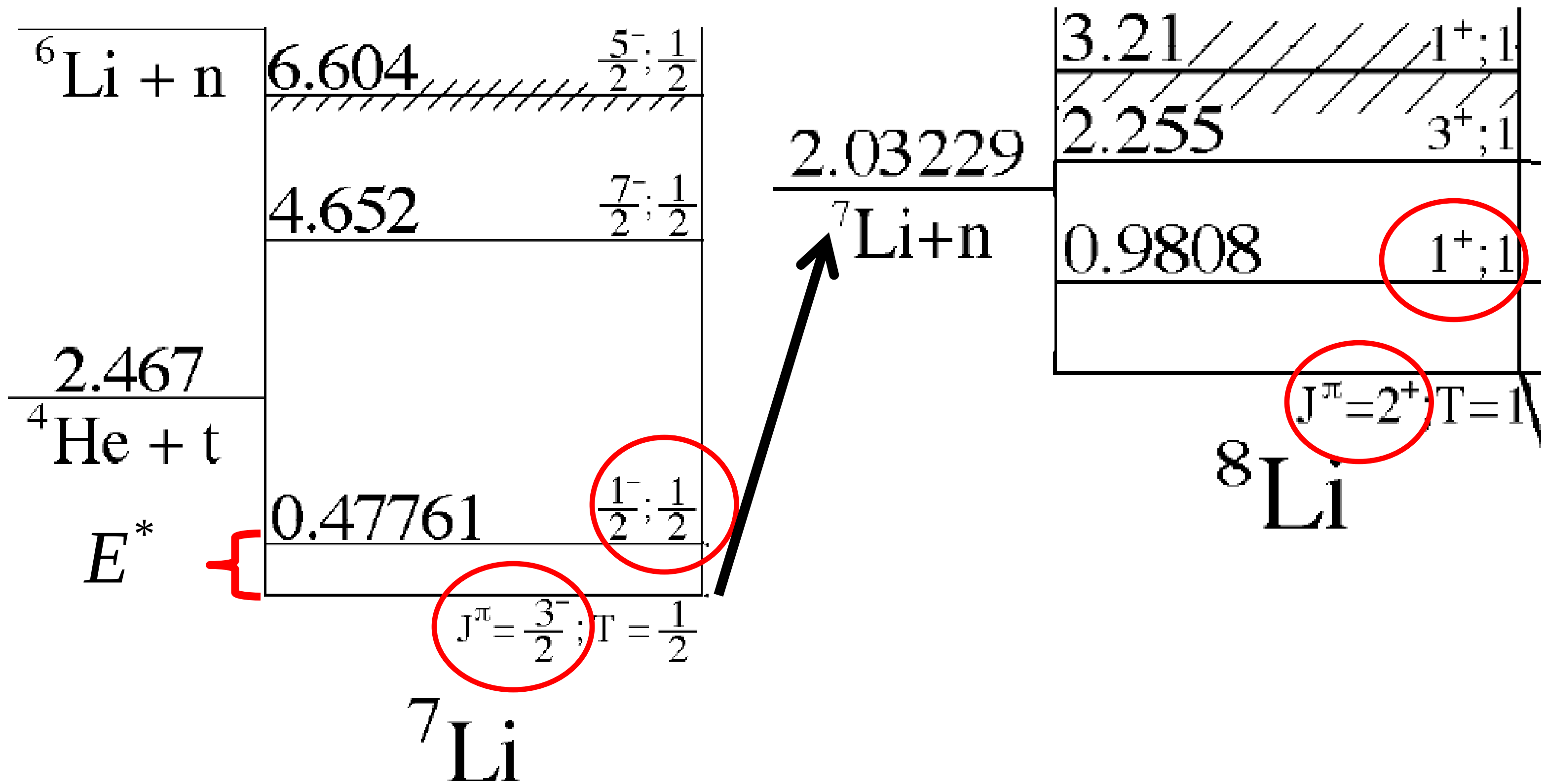
$$^{7}\mathbf{Li}\left(\mathbf{n},\gamma\right)^{8}\mathbf{Li}$$

G. Rupak and R. Higa, *Phys. Rev. Lett.* 106, 222501 (2011)

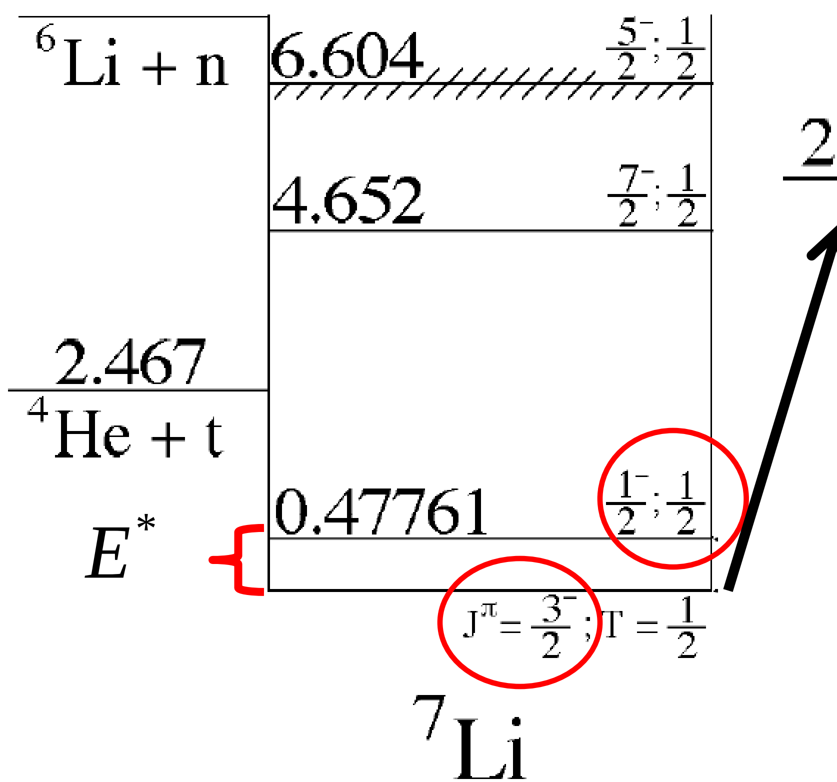
Scales, spins, core excitations

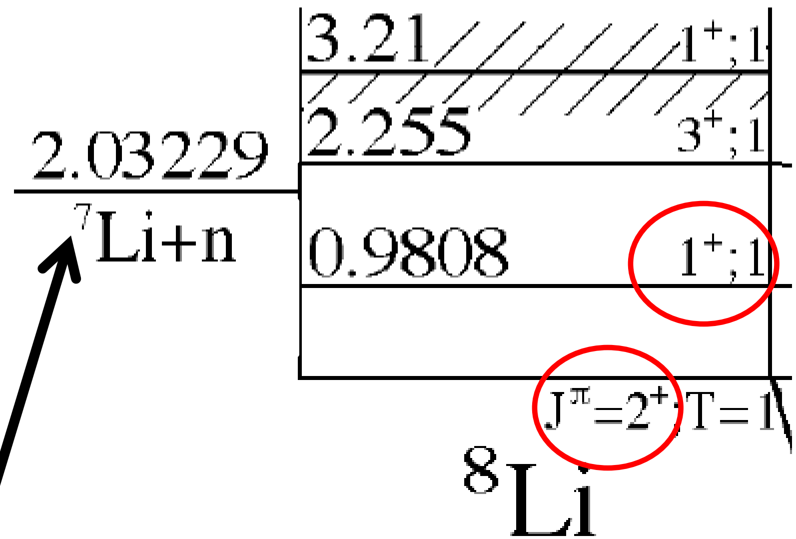


Scales, spins, core excitations



Scales, spins, core excitations

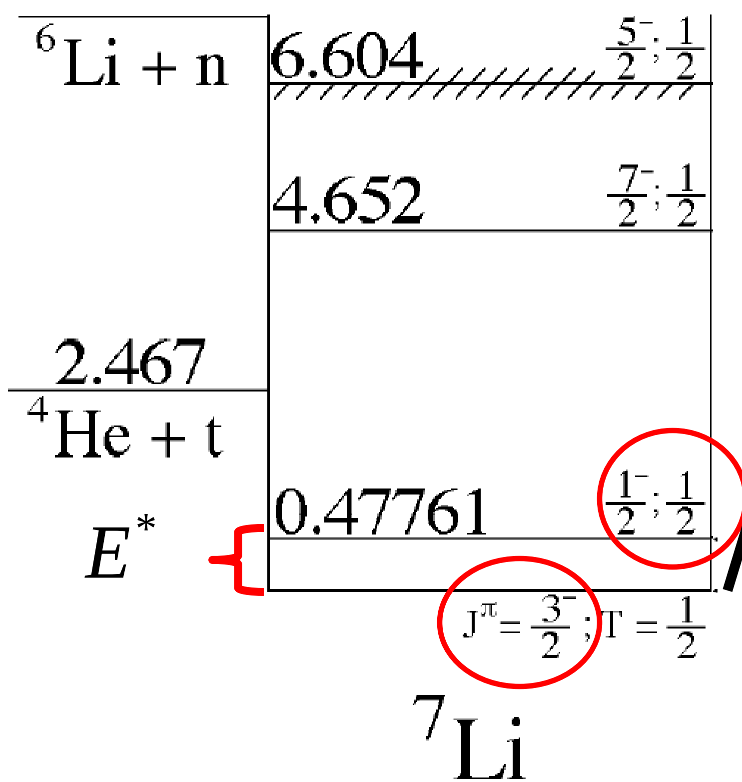


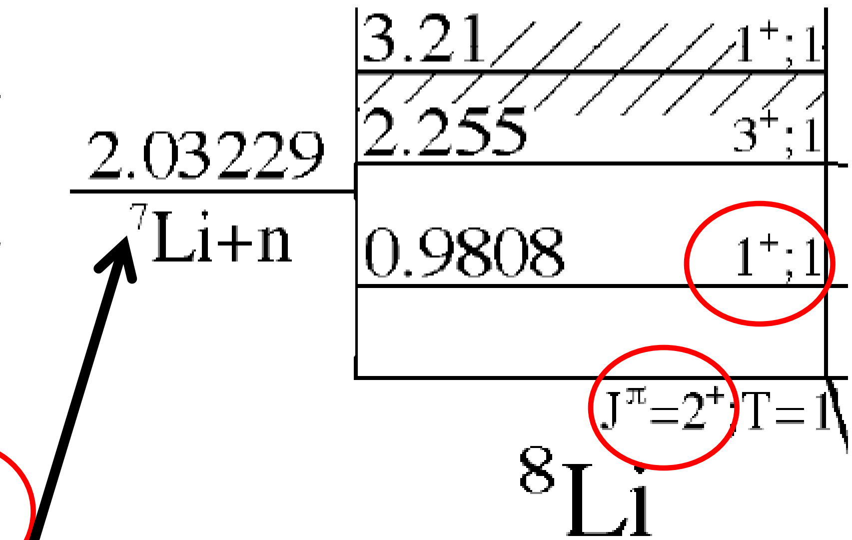


$$\text{IS Li7} + \text{n}: \ {}^3S_1,\ {}^5S_2, D$$

$$\text{IS Li7}^* + \text{n}: \ {}^1S^*{}_0,\ {}^3S^*{}_1$$

Scales, spins, core excitations

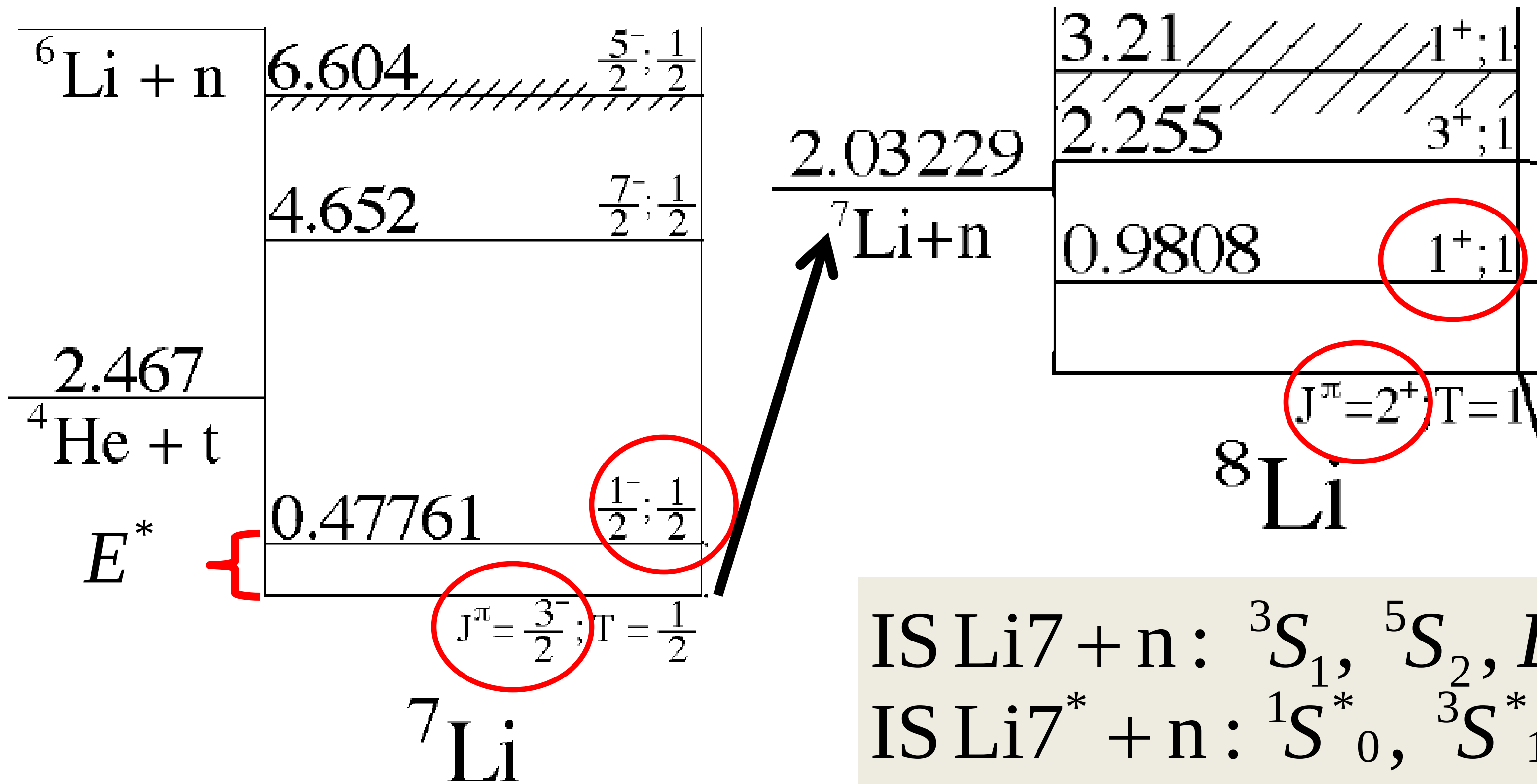




$$\mathrm{IS\ Li7 + n:}\ \ {}^3S_1,\ \ {}^5S_2,\ D$$
$$\mathrm{IS\ Li7^* + n:}\ {}^1S^*{}_0,\ \ {}^3S^*{}_1$$

$$\mathrm{FS(2^+)\ Li7 + n:}\ \ {}^3P_2,\ \ {}^5P_2$$
$$\mathrm{FS(2^+)\ Li7^* + n:}\ \ {}^3P^*{}_2$$

Scales, spins, core excitations



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$$\text{FS}(1^+)\,\text{Li7}^* + \text{n}: \; {}^1P^*{}_1, \; {}^3P^*{}_1$$

Scales, spins, core excitations

$$\Lambda \approx 100 - 300\ \mathrm{MeV}$$

Momentum scale	Definition	Value
γ	$\sqrt{2M_R B_{^8\mathrm{Li}}}$	57.8 MeV
γ^*	$\sqrt{2M_R(B_{^8\mathrm{Li}} + E^*)}$	65.1 MeV
γ_Δ	$\sqrt{2M_R E^*}$	30.0 MeV
$\tilde{\gamma}$	$\sqrt{2M_R B_{^8\mathrm{Li}^*}}$	41.6 MeV
$\tilde{\gamma}^*$	$\sqrt{2M_R(B_{^8\mathrm{Li}^*} + E^*)}$	51.3 MeV

Parameter	Channel	Value	Assigned scaling
$a_{(^5S_2)}$	S-wave, $S = 2$	$-3.63(5)$ fm	$1/\gamma$
$a_{(^3S_1)}$	S-wave, $S = 1$	$0.87(7)$ fm	$1/\Lambda$
r	P-wave, $J = 2$	$-1.43(2)\ \mathrm{fm}^{-1}$	Λ
$\tilde{r}$	P-wave, $J = 1$	$-1.86(6)\ \mathrm{fm}^{-1}$	Λ

L. Koester, K. Knopf, and W. Waschkowski, Z. Phys. A 312, 81 (1983)

EFT

$$\mathcal{L}_0 = n^{\dagger\sigma}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm n}}\right)n_\sigma + c^{\dagger a}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}}\right)c_a$$
$$+d^{\dagger\delta}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}}\right)d_\delta + \pi^{\dagger\alpha}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \Delta\right)\pi_\alpha$$
$$+\tilde{\pi}^{\dagger i}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \tilde{\Delta}\right)\tilde{\pi}_i \ ,$$

EFT

$$\mathcal{L}_0 = n^{\dagger\sigma}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm n}}\right)n_\sigma + c^{\dagger a}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}}\right)c_a$$
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$$+\tilde{\pi}^{\dagger i}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \tilde{\Delta}\right)\tilde{\pi}_i \ ,$$

$$\mathcal{L}_S = g_{(^3S_1)} c^{\dagger a'} n^{\dagger\sigma'} T^i_{a'\sigma'} T_i^{a\sigma} c_a n_\sigma$$
$$+g_{(^5S_2)} c^{\dagger a'} n^{\dagger\sigma'} T^\alpha_{a'\sigma'} T_\alpha^{a\sigma} c_a n_\sigma$$
$$+g_{(^3S_1^*)} d^{\dagger\delta} n^{\dagger\sigma'} T^i_{\delta\sigma'} T_i^{a\sigma} c_a n_\sigma + \text{C.C.} \ ,$$

EFT

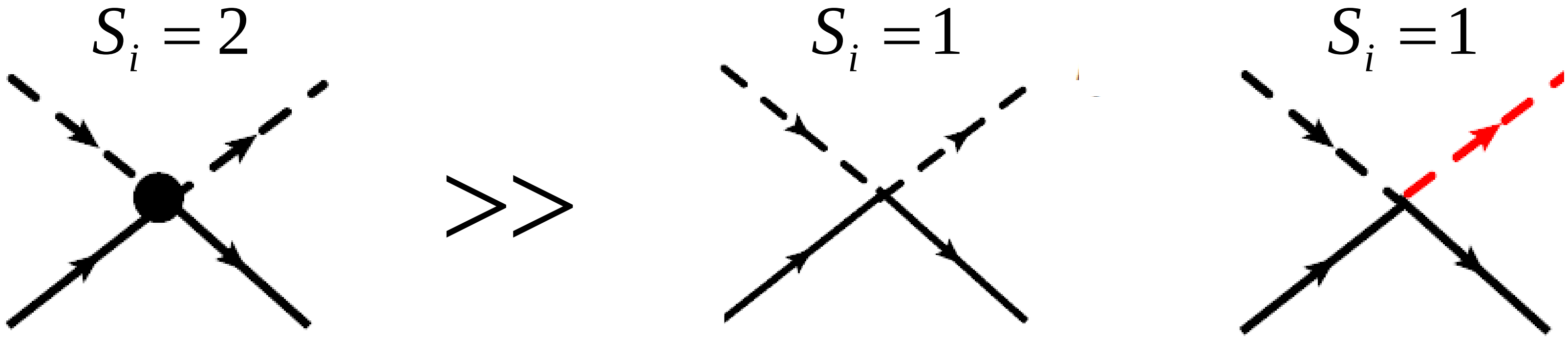
$$\mathcal{L}_0 = n^{\dagger\sigma}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm n}}\right)n_\sigma + c^{\dagger a}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}}\right)c_a$$
$$+d^{\dagger\delta}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm c}}\right)d_\delta + \pi^{\dagger\alpha}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \Delta\right)\pi_\alpha$$
$$+\tilde{\pi}^{\dagger i}\left(i\partial_t + \frac{\nabla^2}{2M_{\rm nc}} + \tilde{\Delta}\right)\tilde{\pi}_i\ ,$$

$$\mathcal{L}_S = g_{(^3S_1)} c^{\dagger a'} n^{\dagger\sigma'} T^{i}_{a'\sigma'} T^{a\sigma}_{i} c_a n_\sigma$$
$$+g_{(^5S_2)} c^{\dagger a'} n^{\dagger\sigma'} T^{\alpha}_{a'\sigma'} T^{a\sigma}_{\alpha} c_a n_\sigma$$
$$+g_{(^3S_1^*)} d^{\dagger\delta} n^{\dagger\sigma'} T^{i}_{\delta\sigma'} T^{a\sigma}_{i} c_a n_\sigma + \text{C.C.}$$

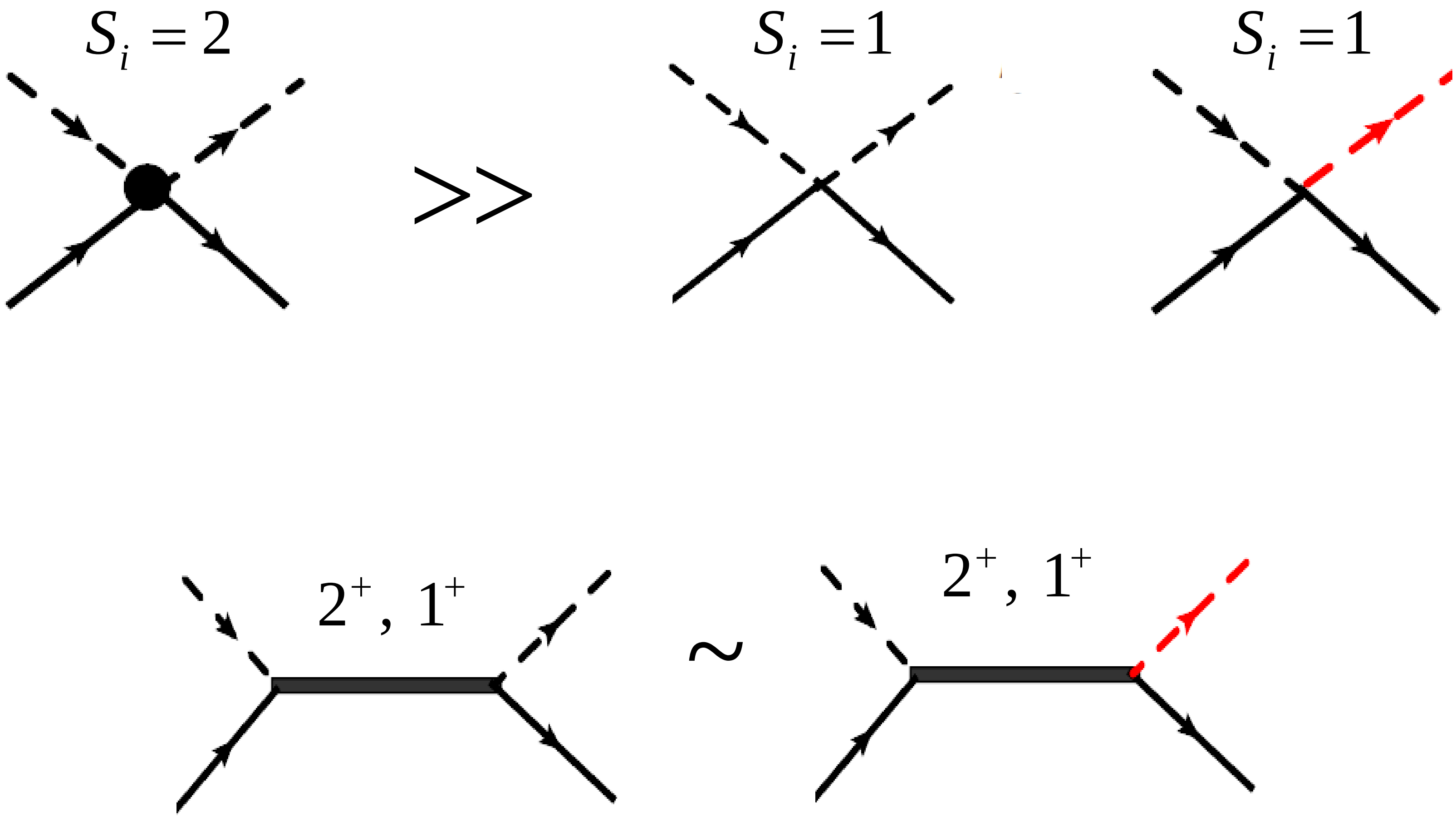
$$\mathcal{L}_{P,gs} = h_{(^3P_2)} \pi^{\dagger\alpha} T^{ij}_{\alpha} T^{\sigma a}_{i} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_c\right)_j c_a$$
$$+h_{(^5P_2)} \pi^{\dagger\alpha} T^{\beta j}_{\alpha} T^{\sigma a}_{\beta} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_c\right)_j c_a$$
$$+h_{(^3P_2^*)} \pi^{\dagger\alpha} T^{jk}_{\alpha} T^{\delta\sigma}_{k} n_\sigma i(\boldsymbol{V}_n - \boldsymbol{V}_{c^*})_j d_\delta + \text{C.C.}\ ,$$

$$\mathcal{L}_{P,es} = \tilde{h}_{(^3P_1)} \tilde{\pi}^{\dagger k} T^{ij}_{k} T^{\sigma a}_{i} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_c\right)_j c_a$$
$$+\tilde{h}_{(^5P_1)} \tilde{\pi}^{\dagger k} T^{\beta j}_{k} T^{\sigma a}_{\beta} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_c\right)_j c_a$$
$$+\tilde{h}_{(^1P_1^*)} \tilde{\pi}^{\dagger k} T^{0j}_{k} T^{\sigma\delta}_{0} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_{c^*}\right)_j d_\delta$$
$$+\tilde{h}_{(^3P_1^*)} \tilde{\pi}^{\dagger k} T^{ij}_{k} T^{\sigma\delta}_{i} n_\sigma i\left(\boldsymbol{V}_n - \boldsymbol{V}_{c^*}\right)_j d_\delta + \text{C.C.}\ .$$

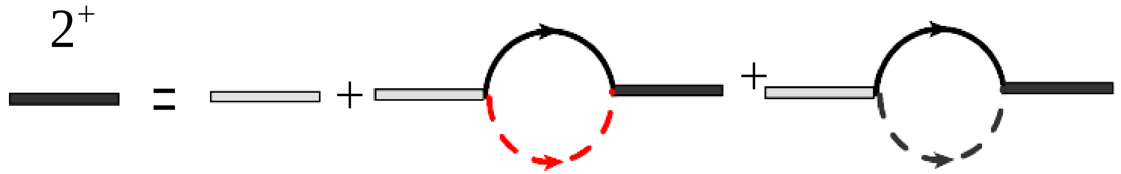
EFT



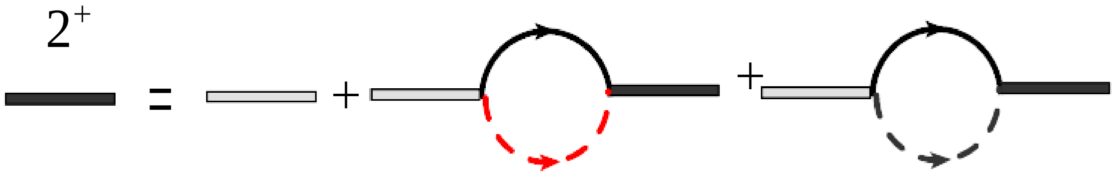
EFT



P-wave

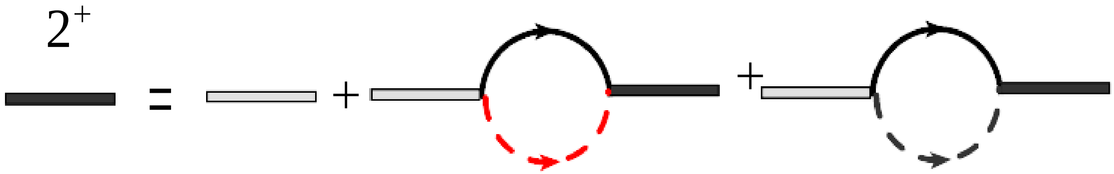


P-wave



$$\frac{C^2_{(^3P_2)}}{h^2_{(^3P_2)}\gamma^2} = \frac{C^2_{(^5P_2)}}{h^2_{(^5P_2)}\gamma^2} = \frac{C^2_{(^3P_2^*)}}{h^2_{(^3P_2^*)}\gamma^{*2}} = \frac{Z}{3\pi}$$

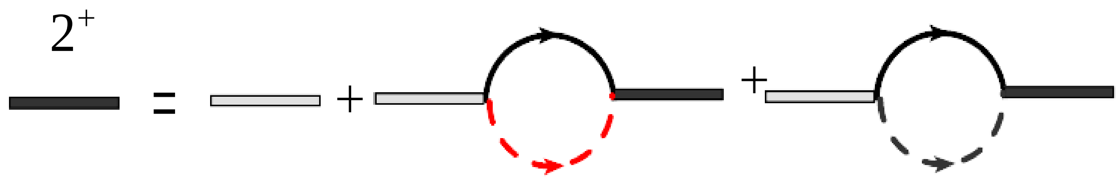
P-wave



$$\frac{C^2_{(^3P_2)}}{h^2_{(^3P_2)}\gamma^2} = \frac{C^2_{(^5P_2)}}{h^2_{(^5P_2)}\gamma^2} = \frac{C^2_{(^3P_2^*)}}{h^2_{(^3P_2^*)}\gamma^{*2}} = \frac{Z}{3\pi}$$

A function of parameters

P-wave



$$\frac{C^2_{(^3P_2)}}{h^2_{(^3P_2)}\gamma^2} = \frac{C^2_{(^5P_2)}}{h^2_{(^5P_2)}\gamma^2} = \frac{C^2_{(^3P_2^*)}}{h^2_{(^3P_2^*)}\gamma^{*2}} = \frac{Z}{3\pi}$$

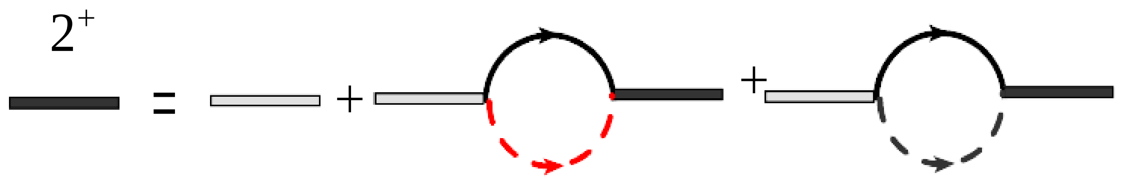
4 parameters: 3 h + 1 Delta,
or 3 C + gamma

	$C_{(^3P_2)}$	$C_{(^5P_2)}$	$C_{(^3P_2^*)}$
Nollett	$-0.283(12)$	$-0.591(12)$	$-0.384(6)$
	$-0.284(23)$	$-0.593(23)$	

K. M. Nollett and R. B. Wiringa, PRC 83, 041001 (2011)

L. Trache,et.al., Phys. Rev. C 67, 062801(R) (2003)

P-wave



$$\frac{C^2_{(^3P_2)}}{h^2_{(^3P_2)}\gamma^2} = \frac{C^2_{(^5P_2)}}{h^2_{(^5P_2)}\gamma^2} = \frac{C^2_{(^3P_2^*)}}{h^2_{(^3P_2^*)}\gamma^{*2}} = \frac{Z}{3\pi}$$

4 parameters: 3 h + 1 Delta,

or 3 C + gamma

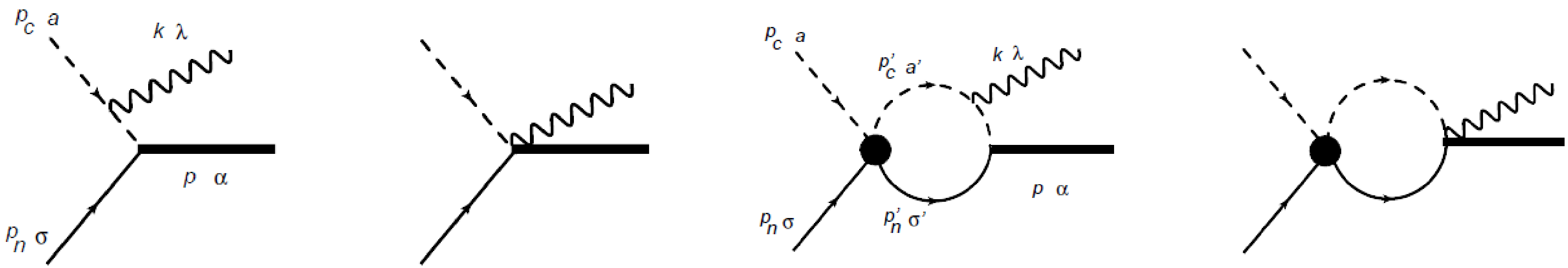
5 parameters

	$C_{(^3P_2)}$	$C_{(^5P_2)}$	$C_{(^3P_2^*)}$	$\tilde{C}_{(^3P_1)}$	$\tilde{C}_{(^5P_1)}$	$\tilde{C}_{(^1P_1^*)}$	$\tilde{C}_{(^3P_1^*)}$
Nollett	$-0.283(12)$	$-0.591(12)$	$-0.384(6)$	$0.220(6)$	$0.197(5)$	$-0.195(3)$	$-0.214(3)$
	$-0.284(23)$	$-0.593(23)$		$0.187(16)$	$0.217(13)$		

K. M. Nollett and R. B. Wiringa, PRC 83, 041001 (2011)

L. Trache, et.al., Phys. Rev. C 67, 062801(R) (2003)

Radiative captures: LO



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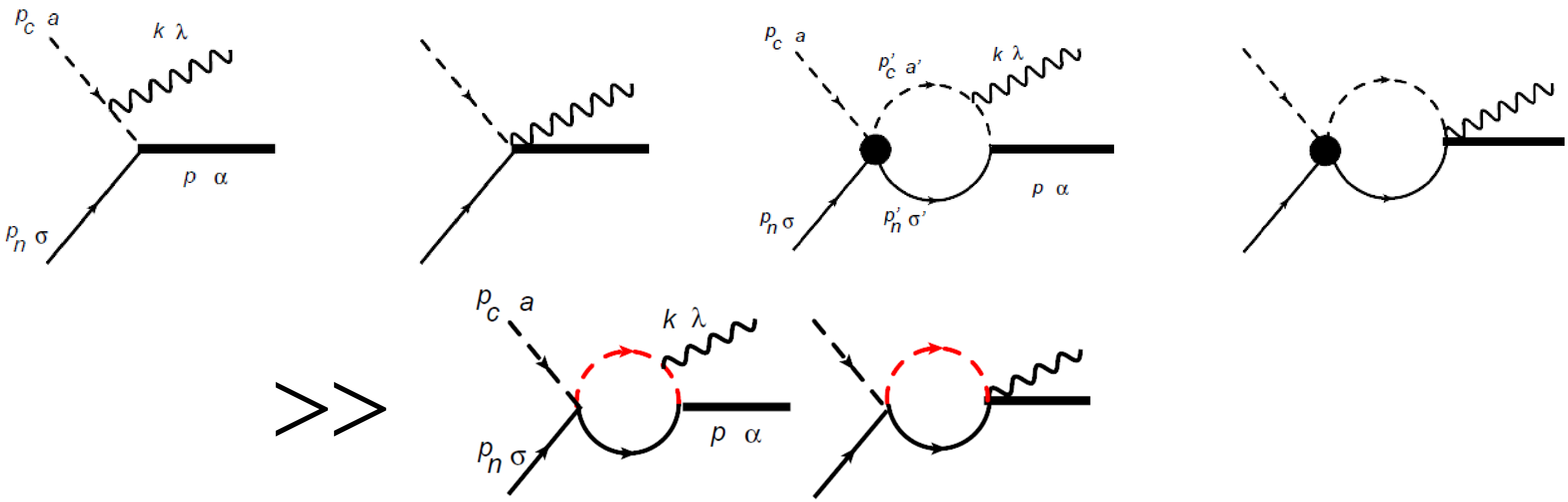
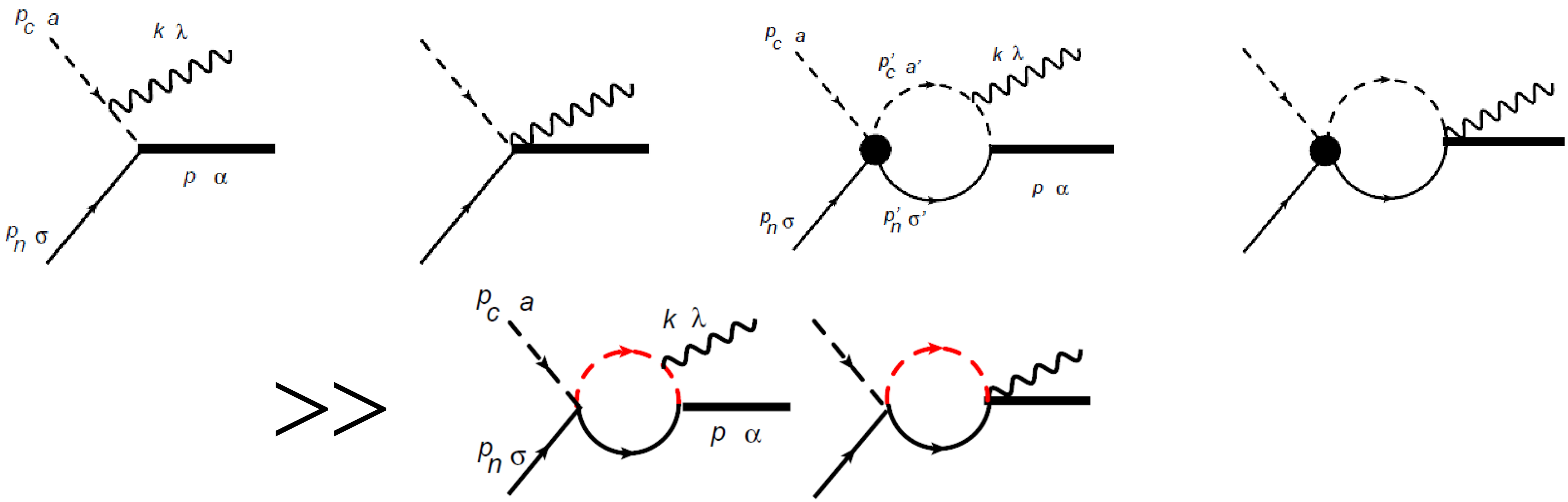


Radiative captures: LO



Initial total spin Si=2

$$\mathcal{M} = i e_c h_{(^5P_2)} \sqrt{8 Z^{\mathrm{LO}} M_{\mathrm{n}} M_{\mathrm{c}} M_{\mathrm{nc}}}\, T_{\beta}^{\sigma a} T_{\alpha}^{\beta j} \left[\frac{\boldsymbol{\epsilon}^*(\lambda) \cdot \boldsymbol{V}_c}{p_c^0 - \omega - \frac{(\boldsymbol{p}_c - \boldsymbol{k})^2}{2M_c} + i\epsilon} \left(\frac{\boldsymbol{p}_c}{M_{\mathrm{R}}} - \frac{\boldsymbol{k}}{M_{\mathrm{c}}} \right)_j + \left(1 + X(p_c; \gamma, a_{(^5S_2)})\right) \frac{\boldsymbol{\epsilon}^*(\lambda)_j}{M_{\mathrm{c}}} \right]$$

$$X(p_c; \gamma, a) \equiv \frac{(-)i}{a^{-1} + i p_c} \left[p_c - \frac{2}{3} i \frac{\gamma^3 - i p_c^3}{\gamma^2 + p_c^2} \right] \qquad a \sim \frac{1}{\gamma} \Rightarrow X \sim 1, \quad a \sim \frac{1}{\Lambda} \Rightarrow X \sim \frac{\gamma}{\Lambda}$$

$$\sum_{\sigma,a}^{\alpha,\lambda} |\mathcal{M}|^2 = \frac{5}{3} 64\pi\alpha Z_c^2 \frac{3\pi}{\gamma^2} \frac{M_{\rm n}^2}{M_{\rm R}} \left(C^{\rm LO}_{(^5P_2)}\right)^2 \left[|1 + X(p_c;\gamma, a_{(^5S_2)})|^2 - \frac{2\boldsymbol{p}_c^2 \sin^2\theta}{\boldsymbol{p}_c^2 + \gamma^2} \left(\frac{\gamma^2}{\boldsymbol{p}_c^2+\gamma^2} + \mathrm{Re}\left\{ X(p_c;\gamma, a_{(^5S_2)}) \right\} \right) \right]$$

$$+ \quad \frac{5}{3} 64\pi\alpha Z_c^2 \frac{3\pi}{\gamma^2} \frac{M_{\rm n}^2}{M_{\rm R}} \left(C^{\rm LO}_{(^3P_2)}\right)^2 \left[1 - \frac{\boldsymbol{p}_c^2 \sin^2\theta}{\boldsymbol{p}_c^2+\gamma^2} \frac{2\gamma^2}{\boldsymbol{p}_c^2+\gamma^2} \right]$$

$$X(p_c;\gamma,a) \equiv \frac{(-)i}{a^{-1} + ip_c} \left[p_c - \frac{2}{3} i \frac{\gamma^3 - ip_c^3}{\gamma^2 + p_c^2} \right]$$

$$\sum_{\sigma,a}^{\alpha,\lambda}|\mathcal{M}|^2=\frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\mathrm{n}}^2}{M_{\mathrm{R}}}\left(C^{\mathrm{LO}}_{({}^5P_2)}\right)^2\left[|1+X(p_c;\gamma,a_{({}^5S_2)})|^2-\frac{2\boldsymbol{p}_c^2\sin^2\theta}{\boldsymbol{p}_c^2+\gamma^2}\left(\frac{\gamma^2}{\boldsymbol{p}_c^2+\gamma^2}+\mathrm{Re}\left\{X(p_c;\gamma,a_{({}^5S_2)})\right\}\right)\right]$$

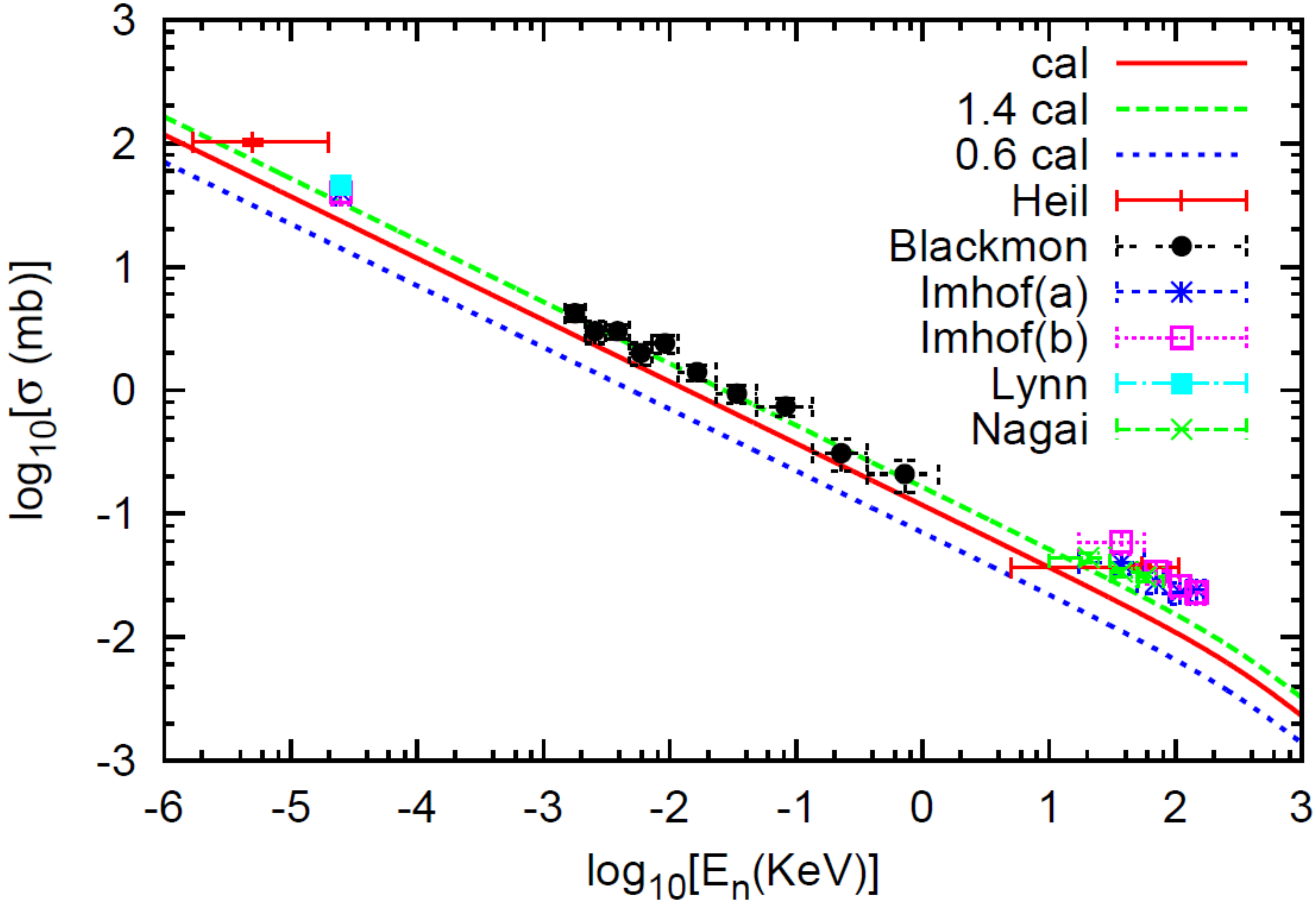
$$+\ \frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\mathrm{n}}^2}{M_{\mathrm{R}}}\left(C^{\mathrm{LO}}_{({}^3P_2)}\right)^2\left[1-\frac{\boldsymbol{p}_c^2\sin^2\theta}{\boldsymbol{p}_c^2+\gamma^2}\frac{2\gamma^2}{\boldsymbol{p}_c^2+\gamma^2}\right]$$

$$X(p_c;\gamma,a)\equiv\frac{(-)i}{a^{-1}+ip_c}\left[p_c-\frac{2}{3}i\frac{\gamma^3-ip_c^3}{\gamma^2+p_c^2}\right]$$

$$\sum_{i,f}|\mathcal{M}|^2\ =\ 64\pi\alpha Z_c^2\frac{3\pi}{\tilde{\gamma}^2}\frac{M_{\mathrm{n}}^2}{M_{\mathrm{R}}}\left\{\left(\tilde{C}^{\mathrm{LO}}_{({}^3P_1)}\right)^2\left[1-\frac{\boldsymbol{p}_c^2\sin^2\theta}{\boldsymbol{p}_c^2+\tilde{\gamma}^2}\left(\frac{2\tilde{\gamma}^2}{\boldsymbol{p}_c^2+\tilde{\gamma}^2}\right)\right]\right.$$

$$\left.+\left(\tilde{C}^{\mathrm{LO}}_{({}^5P_1)}\right)^2\left[|1+X(p_c;\tilde{\gamma},a_{({}^5S_2)})|^2-\frac{2\boldsymbol{p}_c^2\sin^2\theta}{\boldsymbol{p}_c^2+\tilde{\gamma}^2}\left(\frac{\tilde{\gamma}^2}{\boldsymbol{p}_c^2+\tilde{\gamma}^2}+\mathrm{Re}\left\{X(p_c;\tilde{\gamma},a_{({}^5S_2)})\right\}\right)\right]\right\}$$

LO results on Li7(n,gamma)Li8(Li8*)



N. K. Timofeyuk *et.al., PRL* 91, 232501 (2003); D. Howell *et.al., PRC* 88, 025804 (2013);
D. Gul'ko *et.al., SJNP* 6, 477 (1968); E. Lynn *et.al., PRC* 44, 764 (1991);
Y. Nagai *et. al., PRC* 71, 055803 (2005); J. C. Blackmon *et. al., PRC* 54, 383 (1996); J. E. Lynn *et. al., PRC* 44, 764 (1991); M. Heil *et.al., Astro. J.* 507, 997 (1998); W. L. Imhof *et.al., PR* 114, 1037 (1959).

LO results on Li7(n,gamma)Li8(Li8*)

$$\frac{\sigma[(S_i=1)\to 2^+]}{\sigma[(S_i=2)\to 2^+]} = \frac{\left(C^{\mathrm{LO}}_{(^3P_2)}\right)^2}{\left(C^{\mathrm{LO}}_{(^5P_2)}\right)^2 (1-\frac{2}{3}\gamma a_{(^5S_2)})^2}$$

$$\Rightarrow \quad \frac{\sigma[(S_i=2)\to 2^+]}{\sigma(\to 2^+)} = 0.93(2) \; [> 0.86]$$

A. D. Gul'ko, S. S. Trostin, and A. Hudoklin, *Sov. J. Nucl. Phys.* 6, 477 (1968);
J. E. Lynn, E. T. Jurney, and S. Raman, *Phys. Rev. C* 44, 764 (1991);
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$$\frac{\sigma[(S_i=2)\to 1^+]}{\sigma(\to 1^+)} = 0.65(6) \quad \mathrm{or} \quad 0.75(7).$$

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$$\frac{\sigma[(S_i=1)\rightarrow 2^+]}{\sigma[(S_i=2)\rightarrow 2^+]} = \frac{\left(C^{\rm LO}_{(^3P_2)}\right)^2}{\left(C^{\rm LO}_{(^5P_2)}\right)^2 (1-\frac{2}{3}\gamma a_{(^5S_2)})^2}$$

$$\frac{\sigma[(S_i=2)\rightarrow 2^+]}{\sigma(\rightarrow 2^+)} = 0.93(2)\ [>0.86]$$

$$\frac{\sigma[(S_i=2)\rightarrow 1^+]}{\sigma(\rightarrow 1^+)} = 0.65(6) \ \text{ or } \ 0.75(7).$$

$$\frac{\sigma(\rightarrow 1^+)}{\sigma(\rightarrow 2^+)} = \frac{3}{5}\frac{\left(\tilde{C}^{\rm LO}_{(^3P_1)}\right)^2 + \left(\tilde{C}^{\rm LO}_{(^5P_1)}\right)^2 |1-\frac{2}{3}a_{(^5S_2)}\tilde{\gamma}|^2}{\left(C^{\rm LO}_{(^3P_2)}\right)^2 + \left(C^{\rm LO}_{(^5P_2)}\right)^2 |1-\frac{2}{3}a_{(^5S_2)}\gamma|^2}$$

$$\frac{\sigma(\rightarrow 2^+)}{\sigma} = 0.88(4) \qquad [0.89(1)]$$

A. D. Gul'ko, S. S. Trostin, and A. Hudoklin, *Sov. J. Nucl. Phys.* 6, 477 (1968);
J. E. Lynn, E. T. Jurney, and S. Raman, *Phys. Rev. C* 44, 764 (1991);
Y. Nagai et. al., *Phys. Rev. C* 71, 055803 (2005).

$^{7}\mathbf{Be}\left(\mathbf{p},\boldsymbol{\gamma}\right)^{8}\mathbf{B}$

- It is considered as isospin mirror of Li7 capture on the nucleon level
- From EFT/core+proton picture, they are quite different due to strong Coulomb effect

$^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$

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- From EFT/core+proton picture, they are quite different due to strong Coulomb effect

E. Ryberg, C. Forssén, H.-W. Hammer and L. Platter, PRC 89, 014325 (2014)

Nonperturbative Coulomb effect

$k_C \equiv Q_c Q_n \alpha_{EM} M_{\mathrm{R}}$ $\quad$ $\eta \equiv \dfrac{k_C}{k} \sim 1$ *Sommerfeld para.*

Nonperturbative Coulomb effect

$$k_C \equiv Q_c Q_n \alpha_{EM} M_{\mathrm{R}} \qquad \eta \equiv \frac{k_C}{k} \sim 1$$ *Sommerfeld para.*

$$\frac{1}{E - H_0 - V_c - V_s} = \frac{1}{E - H_0 - V_c} + \frac{1}{E - H_0 - V_c} V_s \frac{1}{E - H_0 - V_c} + \cdots$$

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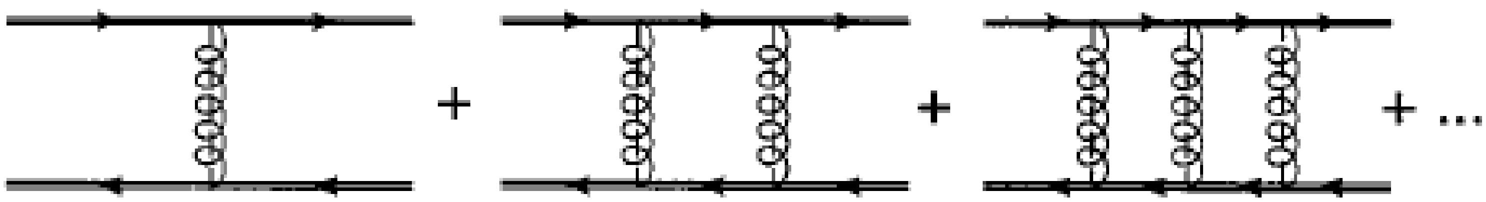

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$$\chi_{\boldsymbol{k}}^{(\pm)}(\boldsymbol{r}) = e^{-\frac{\pi}{2}\eta} e^{i\boldsymbol{kr}} \Gamma(1 \pm i\eta) M(\mp i\eta, 1; \pm ikr - i\boldsymbol{kr})$$

Kummer function

$$\chi_{\boldsymbol{k}}^{(\pm)*}(r=0) \chi_{\boldsymbol{k}}^{(\pm)}(r=0) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} = C_{\eta,0}^2$$

Coulomb barrier, and phase

$$\chi_{\boldsymbol{k}}^{(\mp)*}(r=0) \chi_{\boldsymbol{k}}^{(\pm)}(r=0) = C_{\eta,0}^2 e^{\pm 2i\sigma_0}$$

$$C_{\eta,l} = \frac{2^l e^{-\frac{\pi}{2}\eta} |\Gamma(l+1+i\eta)|}{\Gamma(2l+2)} \qquad e^{2i\sigma_l} \equiv \frac{\Gamma(l+1+i\eta)}{\Gamma(l+1-i\eta)}$$

Effective range expansion in EFT

$$\langle \chi_{\boldsymbol{p}'}^{(-)} | T_{cs}(E) | \chi_{\boldsymbol{p}}^{(+)} \rangle = (-)\frac{2\pi}{M_{\rm R}} \frac{\chi_{\boldsymbol{p}'}^{(-)*}(0)\chi_{\boldsymbol{p}}^{(+)}(0)}{-a_0^{-1} - 2k_C H(\eta)}$$

$$\rightarrow (-)\frac{2\pi}{M_{\rm R}} \frac{C_{\eta,0}^2 e^{2i\sigma_0}}{-a_0^{-1} - 2k_C H(\eta)}$$

$$C_{\eta,0}^2 k(\cot\delta_0 - i) = -\frac{1}{a_0} + \cdots - 2k_C H(\eta) \qquad H(\eta) = \psi(i\eta) + 1/(2i\eta) - \ln(i\eta)$$

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X. Kong and F. Ravndal, NPA 665, 137 (2000).
R. Higa, H. -W.Hammer and U. van Kolck, NPA 809, 171 (2008).

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$$\langle\chi_{\boldsymbol{p}'}^{(-)}|T_{cs}(E)|\chi_{\boldsymbol{p}}^{(+)}\rangle = (-)\frac{6\pi}{M_{\mathrm{R}}}\frac{\partial\chi_{\boldsymbol{p}'}^{(-)*}(0)\partial\chi_{\boldsymbol{p}}^{(+)}(0)}{-\frac{1}{a_1}+\frac{r_1}{2}k^2-k^2(1+\eta^2)2k_C H(\eta)}$$
$$\to (-)\frac{6\pi}{M_{\mathrm{R}}}\frac{k^2 C_{\eta,1}^2 e^{2i\sigma_1}}{-\frac{1}{a_1}+\frac{r_1}{2}k^2-k^2(1+\eta^2)2k_C H(\eta)}$$

$$C_{\eta,1}^2 k^3(\cot\delta_1 - i) = -\frac{1}{a_1}+\frac{r_1}{2}k^2+\cdots-k^2(1+\eta^2)2k_C H(\eta)$$

Effective range expansion in EFT

$$\langle\chi_{\boldsymbol{p}'}^{(-)}|T_{cs}(E)|\chi_{\boldsymbol{p}}^{(+)}\rangle = (-)\frac{2\pi}{M_{\mathrm{R}}}\frac{\chi_{\boldsymbol{p}'}^{(-)*}(0)\chi_{\boldsymbol{p}}^{(+)}(0)}{-a_0^{-1}-2k_C H(\eta)}$$
$$\rightarrow (-)\frac{2\pi}{M_{\mathrm{R}}}\frac{C_{\eta,0}^2 e^{2i\sigma_0}}{-a_0^{-1}-2k_C H(\eta)}$$

One parameter: g (or a0)

$$C_{\eta,0}^2 k(\cot\delta_0 - i) = -\frac{1}{a_0}+\cdots-2k_C H(\eta) \qquad H(\eta)=\psi(i\eta)+1/(2i\eta)-\ln(i\eta)$$

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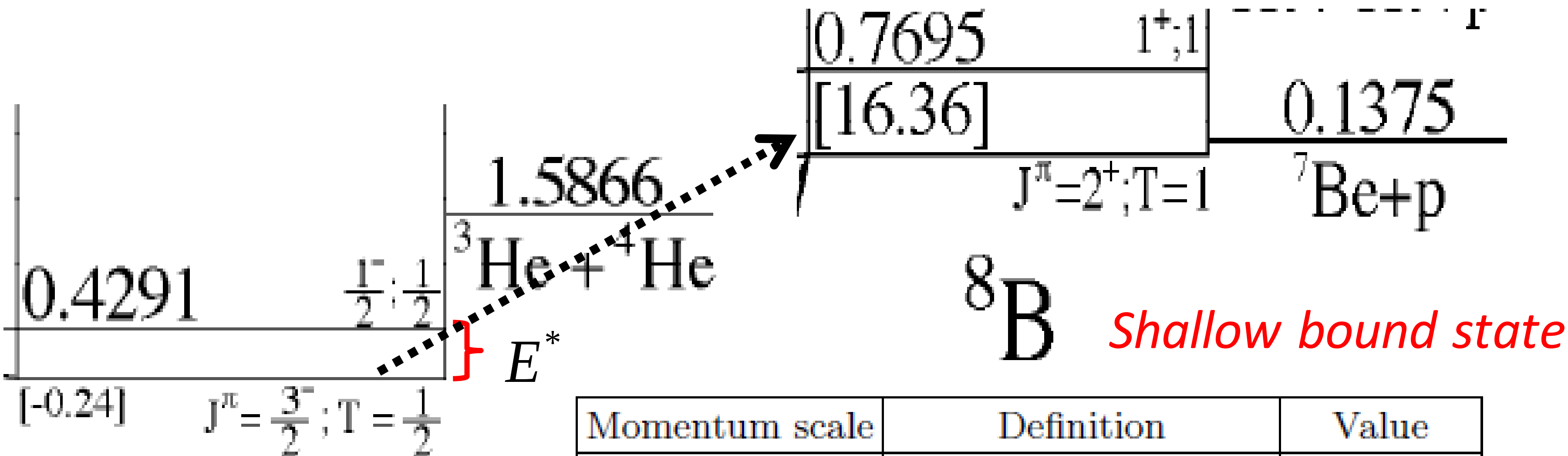
$$C_{\eta,1}^2 k^3(\cot\delta_1 - i) = -\frac{1}{a_1}+\frac{r_1}{2}k^2+\cdots-k^2(1+\eta^2)2k_C H(\eta)$$

Two parameters: Delta and h (or a1 and r1)

Scales, spins, core excitations



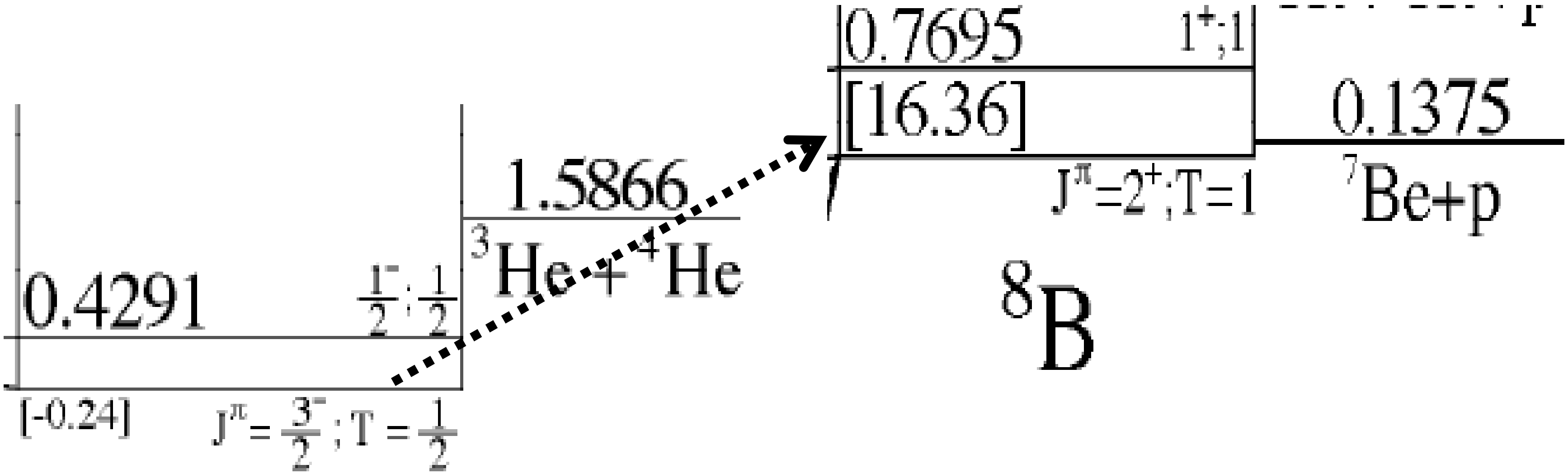
^{7}Be

$$\frac{p_c}{\Lambda},\ \frac{\gamma}{\Lambda},\ \frac{\tilde{\gamma}}{\Lambda} \sim 0.2$$

$$\eta = \frac{k_C}{k} \sim 1$$

Momentum scale	Definition	Value
$k_C \sim \gamma$	$Q_c Q_n \alpha_{EM} M_R$	24.02 MeV
γ	$\sqrt{2M_R B_{^8\mathrm{B}}}$	15.04 MeV
Λ	$\sqrt{2M'_R B_{^7\mathrm{Be}}}$	70 MeV
$\gamma^* \sim \gamma$	$\sqrt{2M_R(B_{^8\mathrm{B}} + E^*)}$	30.53 MeV
$\gamma_\Delta \sim \gamma$	$\sqrt{2M_R E^*}$	26.57 MeV
$a_{^3S_1}, a_{^5S_2} \sim 1/\gamma$	scattering lengths	Varies
$r_0 \sim 1/\Lambda$	$l = 0$ effective ranges	Varies
$a_1 \sim \gamma^{-2}\Lambda^{-1}$	scattering volume	1054.1 fm^3
$r_1 \sim \Lambda$	$l = 1$ effective "range"	-0.34 fm^{-1}

Repeat

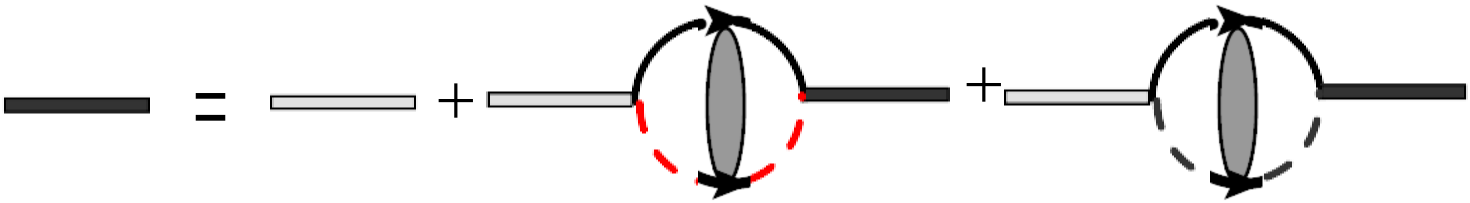


^{7}Be

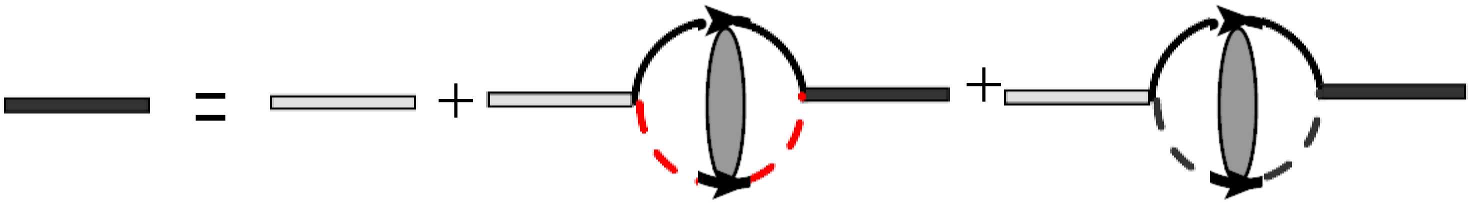
$$\mathrm{IS\ Be7+p:}\ \ {}^3S_1,\ {}^5S_2,\ D$$
$$\mathrm{IS\ Be7^*+p:}\ {}^1S^*{}_0,\ {}^3S^*{}_1$$

$$\mathrm{FS}(2^+)\ \mathrm{Be7+p:}\ \ {}^3P_2,\ {}^5P_2$$
$$\mathrm{FS}(2^+)\ \mathrm{Be7^*+p:}\ \ {}^3P^*{}_2$$

P-wave



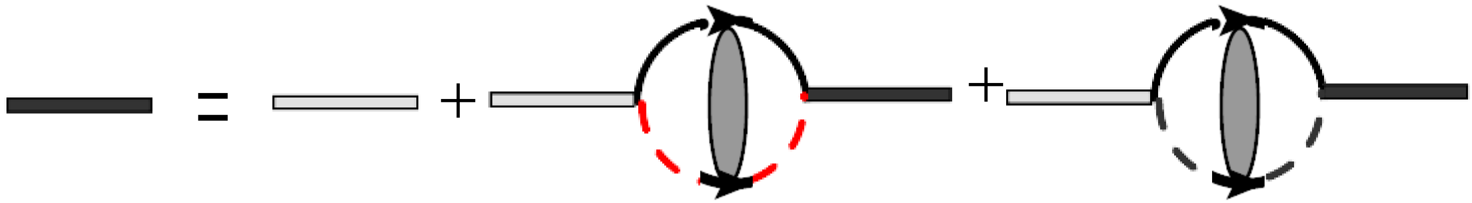
P-wave



$$\frac{C_Y^2}{h_Y^2\gamma^2\Gamma^2(2+k_C/\gamma)} = \frac{C^2_{({}^3P_2^*)}}{h^2_{({}^3P_2^*)}\gamma^{*2}\Gamma^2(2+k_C/\gamma^*)} = \frac{Z}{3\pi}$$

$Y = {}^3P_2$ and ${}^5P_2$

P-wave

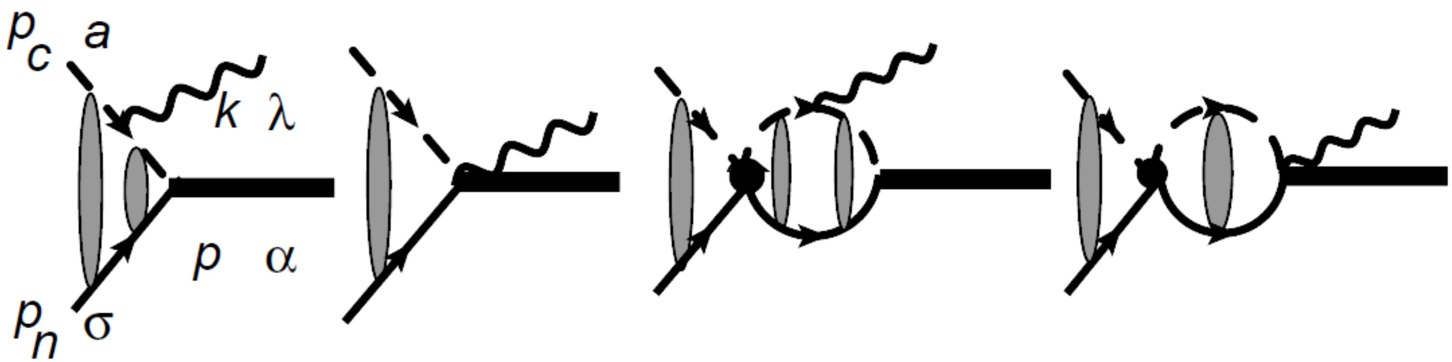


$$\frac{C_Y^2}{h_Y^2\gamma^2\Gamma^2(2+k_C/\gamma)} = \frac{C^2_{(^3P_2^*)}}{h^2_{(^3P_2^*)}\gamma^{*2}\Gamma^2(2+k_C/\gamma^*)} = \frac{Z}{3\pi}$$

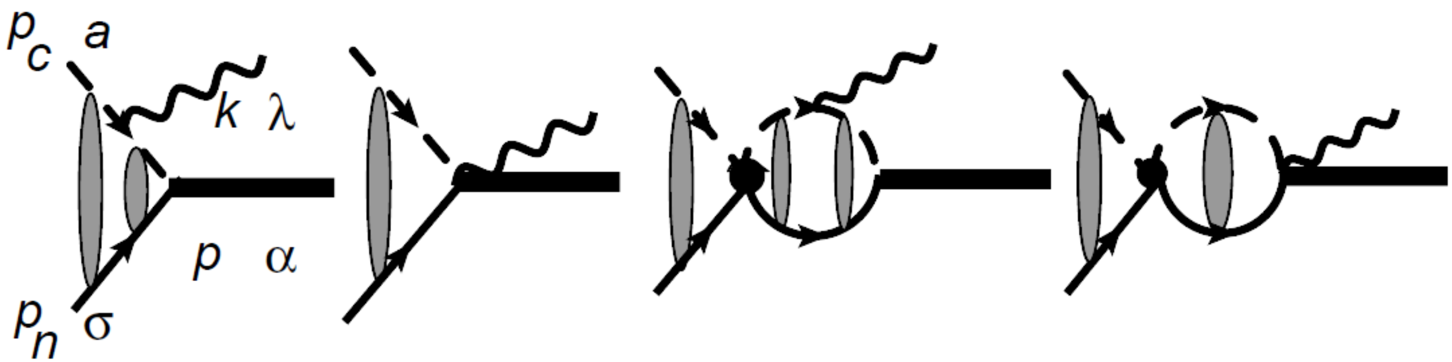
$$Y = {}^3P_2 \text{ and } {}^5P_2$$

4 parameters: 3 h + 1 Delta, or 3 C + gamma

Radiative captures: LO



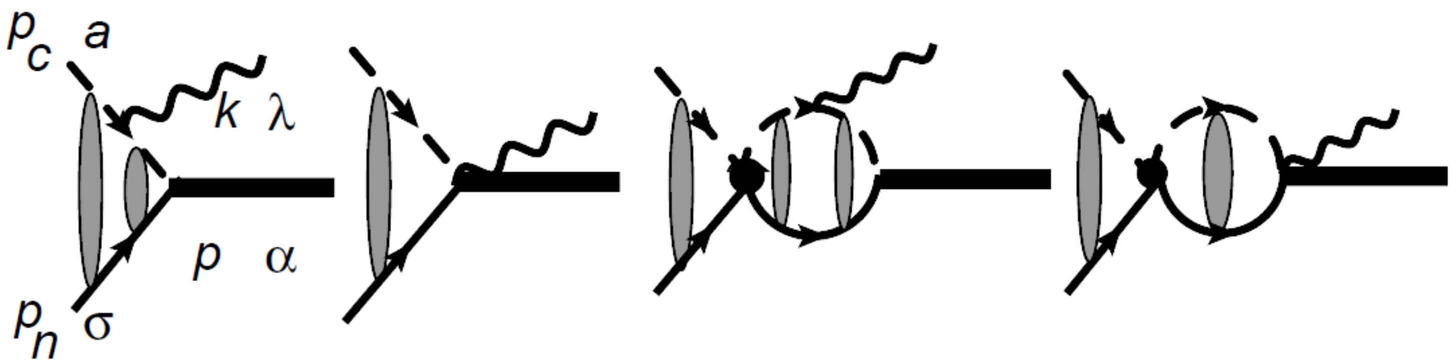
Radiative captures: LO



$$S(E) = \frac{e^{2\pi\eta}}{e^{2\pi\eta}-1}\frac{Z_{eff}^2}{M_{\mathrm{R}}^2}\frac{\pi}{24}\omega k_C\left(\gamma^2+k^2\right)^2 \ \frac{5}{3} \ \times$$

$$\left[{C_{(^3P_2)}^{\mathrm{LO}}}^2\left(|\ \mathcal{S}(^3S_1)\ |^2+2\ |\ \mathcal{D}\ |^2\right)+{C_{(^5P_2)}^{\mathrm{LO}}}^2\left(|\ \mathcal{S}(^5S_2)\ |^2+2\ |\ \mathcal{D}\ |^2\right)\right]$$

Radiative captures: LO



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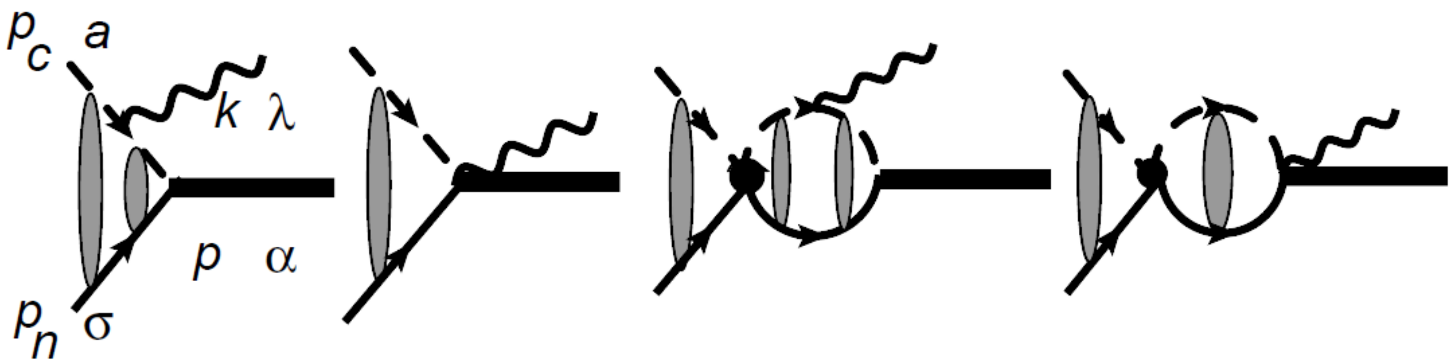
$$\left[{C^{\rm LO}_{(^3P_2)}}^2\left(|\;\mathcal{S}(^3S_1)\;|^2+2\;|\;\mathcal{D}\;|^2\right)+{C^{\rm LO}_{(^5P_2)}}^2\left(|\;\mathcal{S}(^5S_2)\;|^2+2\;|\;\mathcal{D}\;|^2\right)\right]$$

$$\mathcal{S}(X) \equiv \int_0^{+\infty} dr W_{-\eta_B,\frac{3}{2}}(2\gamma r) r \left[\frac{C_{\eta,0}G_0(k,r)}{-a_{(X)}^{-1}-2k_C H(\eta)} + \frac{F_0(k,r)}{C_{\eta,0}k}\frac{-a_{(X)}^{-1}-2k_C {\rm Re}\left[H(\eta)\right]}{-a_{(X)}^{-1}-2k_C H(\eta)}\right]$$

$$\mathcal{D} \equiv \int_0^{+\infty} dr W_{-\eta_B,\frac{3}{2}}(2\gamma r) r \frac{F_2(k,r)}{C_{\eta,0}k}$$

Radiative captures: LO



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Coulomb wavefunc.

$F \rightarrow j$
$G \rightarrow n$
$W \rightarrow h$

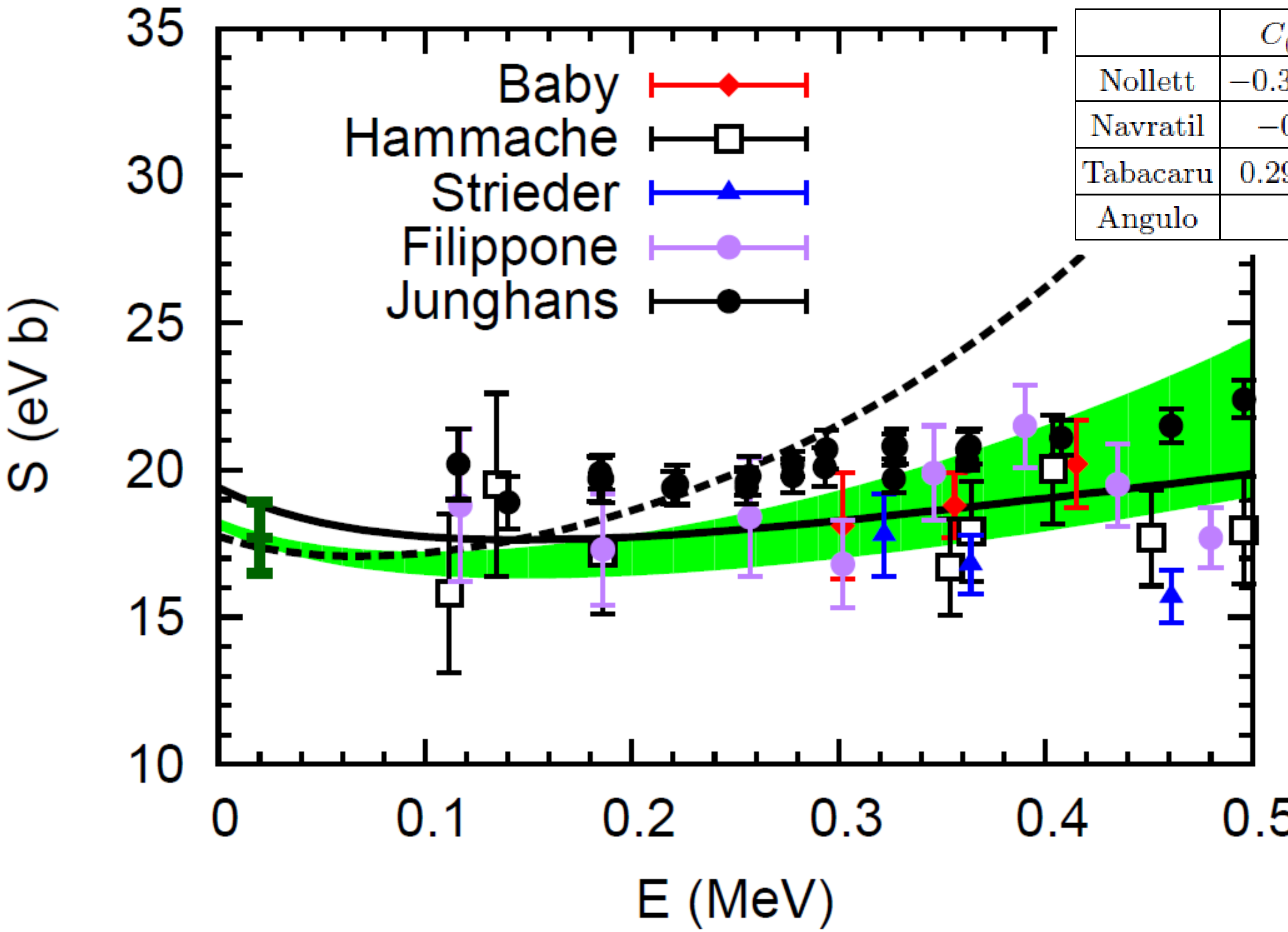
LO results on Be7(p,gamma)B8

	$C_{(^3P_2)}$	$C_{(^5P_2)}$	$a_{(^3S_1)}$	$a_{(^5S_2)}$
Nollett	$-0.315(19)$	$-0.662(19)$		
Navratil	-0.294	-0.650	-5.2	-15.3
Tabacaru	$0.294(45)$	$0.615(45)$		
Angulo			$25(9)$	$-7(3)$

$$C_{(^3P_2^*)} = -0.3485(51)$$

P. Navratil, R. Roth and S. Quaglioni, *Phys. Lett. B* 704, 379 (2011);
C. Angulo *et. al., Nucl. Phys. A* 716, 211 (2003);
G. Tabacaru, *et. al., Phys. Rev. C 73,* 025808 (2006)

LO results on Be7(p,gamma)B8



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One standard deviation in S-wave scattering lengths

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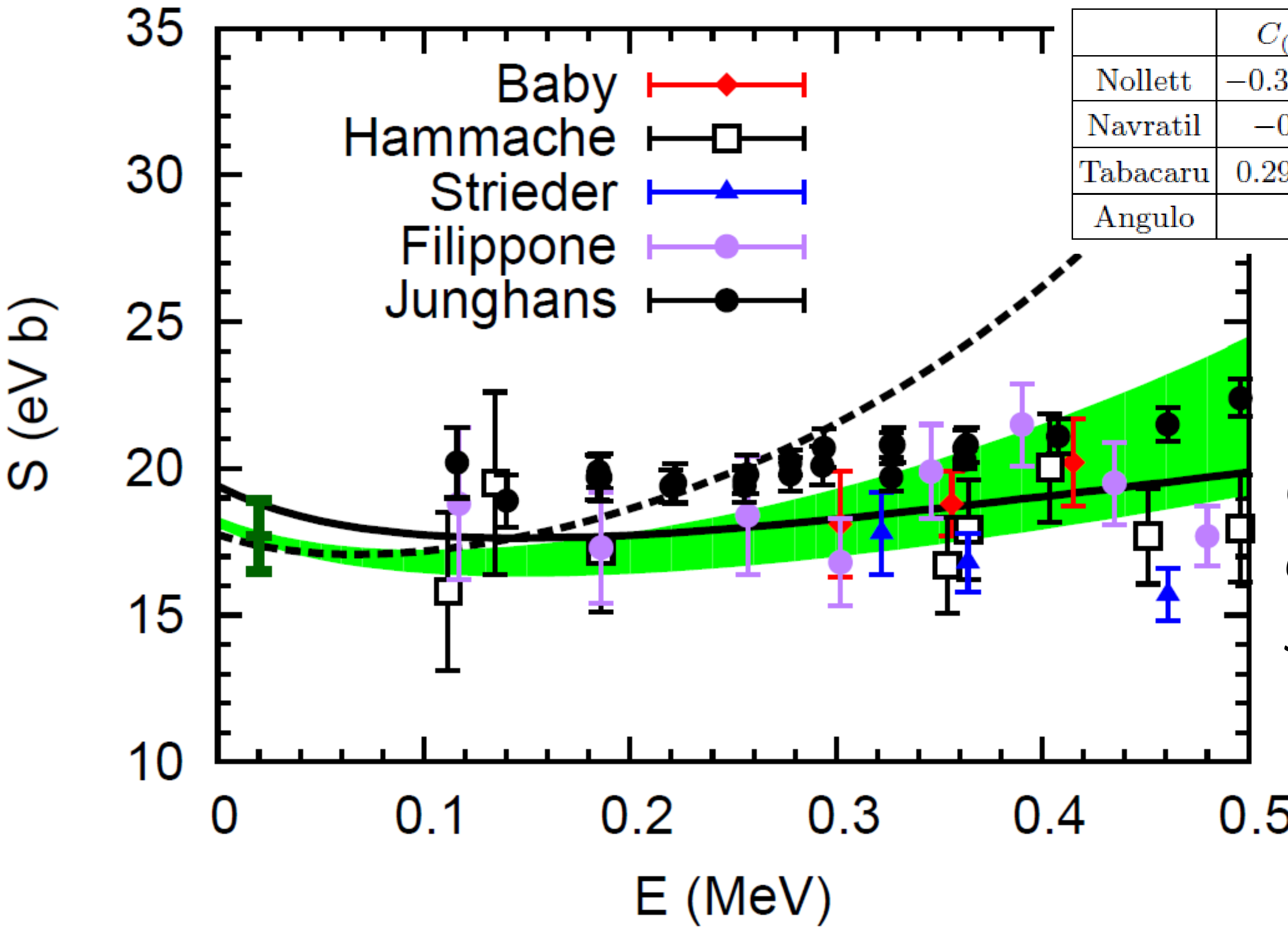

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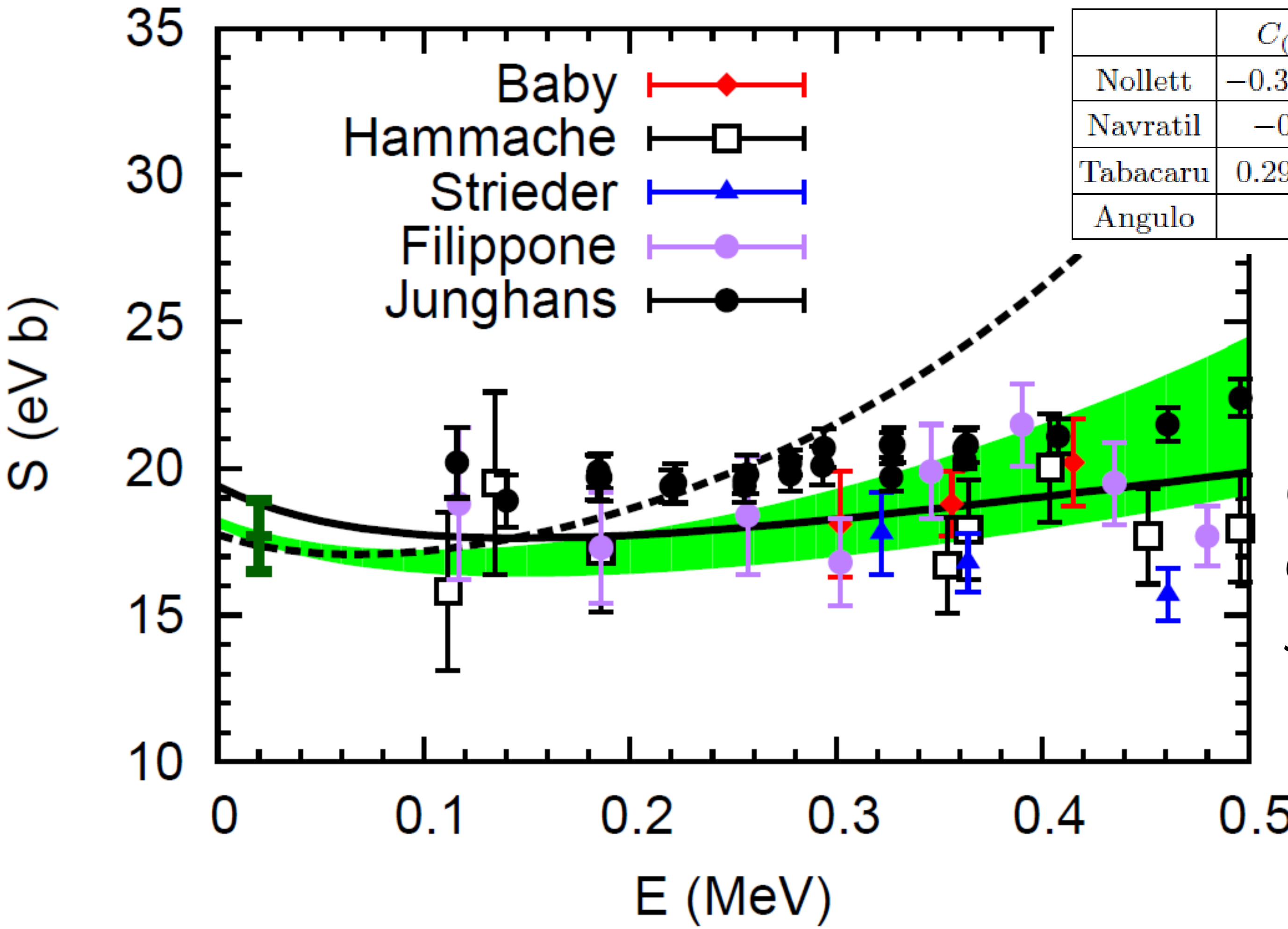
Need better measurement of S-wave scattering lengths and/or effective ranges to extrapolate data to zero energy

P. Navratil, R. Roth and S. Quaglioni, *Phys. Lett. B* 704, 379 (2011)

LO results on Be7(p,gamma)B8



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One standard deviation in S-wave scattering lengths

TRIUMF: Ecm=120 keV

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LO results on Be7(p,gamma)B8

$$S(E) = S(0)(1 + d_1 E + d_2 E^2)$$ *Fit to 0<E<50 keV*

L. T. Baby, *et. al.,* [ISOLDE Collaboration], *Phys. Rev.*Lett. 90, 022501 (2003);
F. Hammache, *et. al., Phys. Rev. Lett*. 86, 3985 (2001);
F. Strieder, *et. al., Nucl. Phys. A* 696, 219 (2001);
B. W. Filippone, *et. al., Phys. Rev. C* 28, 2222 (1983);
A. R. Junghans, *et. al., Phys. Rev. C* 68, 065803 (2003);
A. R. Junghans, *et. al., Phys. Rev. C* 81, 012801 (2010).

LO results on Be7(p,gamma)B8

$$S(E) = S(0)(1 + d_1 E + d_2 E^2)$$ *Fit to 0<E<50 keV*

	$S(0)$ (eV b)	$S_{(^3S_1)}(0)$	$d_1\,(\mathrm{MeV}^{-1})$	$d_2\ (\mathrm{MeV}^{-2})$
No+A	18.2 ± 1.2	3.1 ± 0.4	-1.62	10.3
Na	17.8	3.0	-1.26	10.8
T+A	15.7 ± 2.7	2.7 ± 0.8	-1.62	10.3
	20.8 ± 1.6		-1.5 ± 0.1	6.5 ± 2.0

E. G. Adelberger, *et al., Rev. Mod. Phys.* 83, 195 (2011)

L. T. Baby, *et. al.,* [ISOLDE Collaboration], *Phys. Rev.*Lett. 90, 022501 (2003);
F. Hammache, *et. al., Phys. Rev. Lett*. 86, 3985 (2001);
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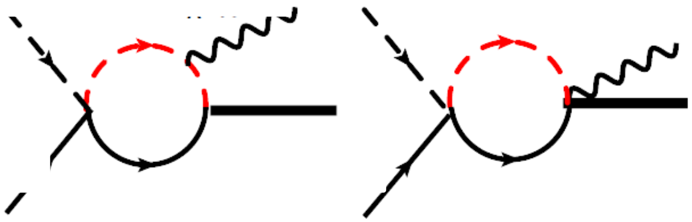
A. R. Junghans, *et. al., Phys. Rev. C* 68, 065803 (2003);
A. R. Junghans, *et. al., Phys. Rev. C* 81, 012801 (2010).

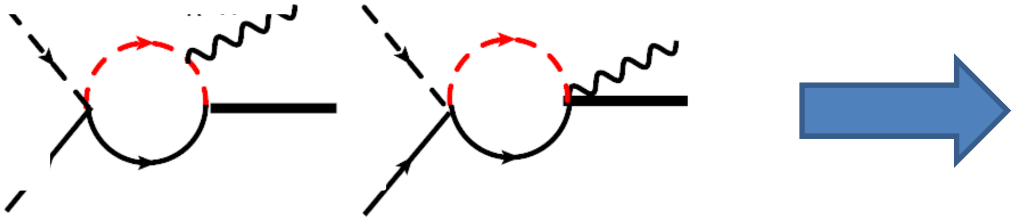
Summary

- EFT + ab initio works as expected at LO
- LO needs s-wave scattering length, p-wave ANCs, and binding momentum
- The p-wave is a coupled-channel problem
- For Be7 capture, improving s-wave measurement is important for extrapolating data to stellar energies.

Outlook: NLO

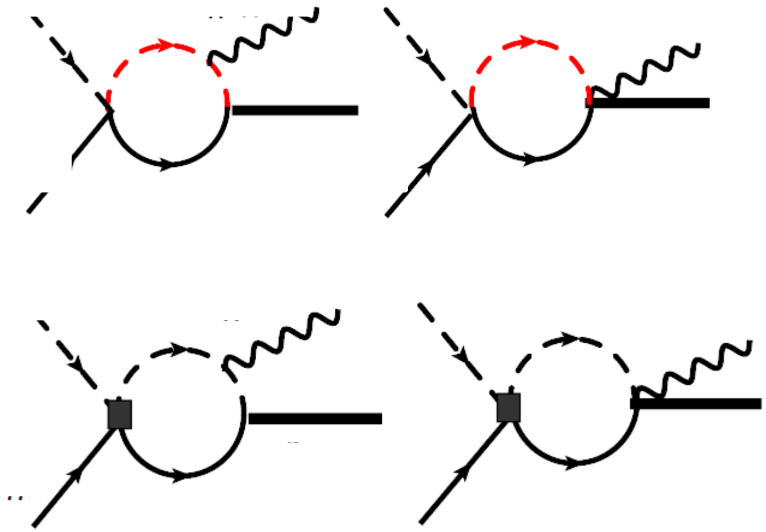


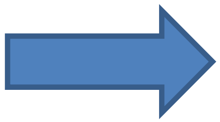
Outlook: NLO



Existence of threshold

Outlook: NLO





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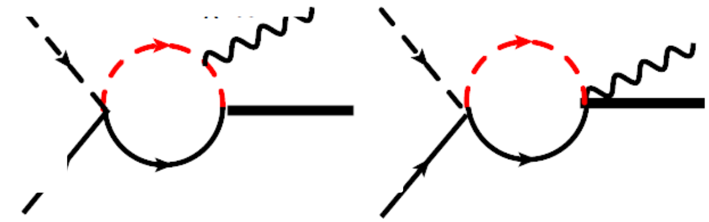
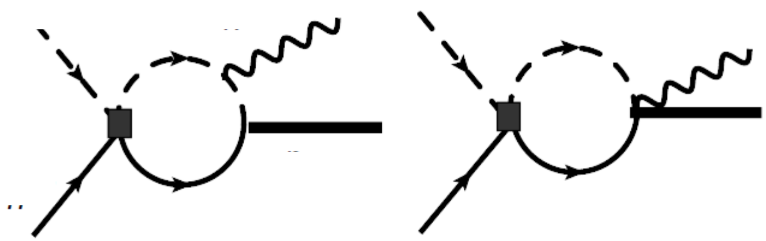
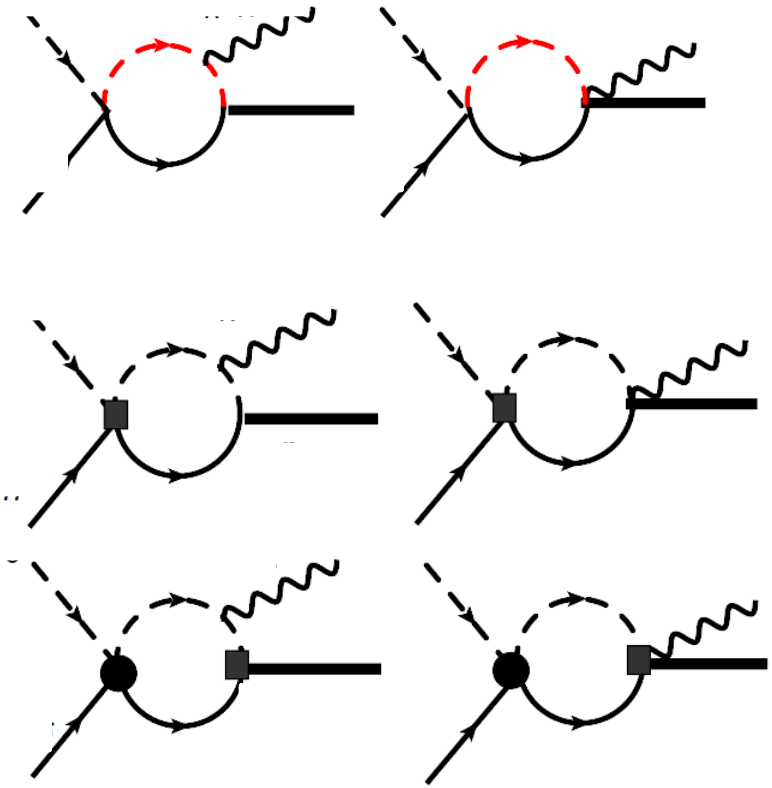


Existence of threshold



Improve the initial state multiple scattering

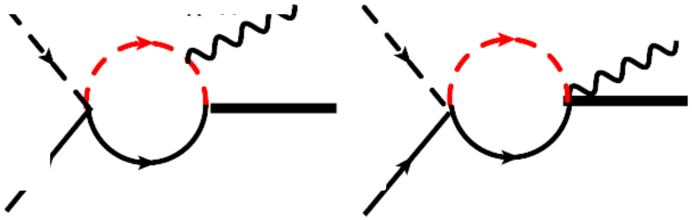
Outlook: NLO



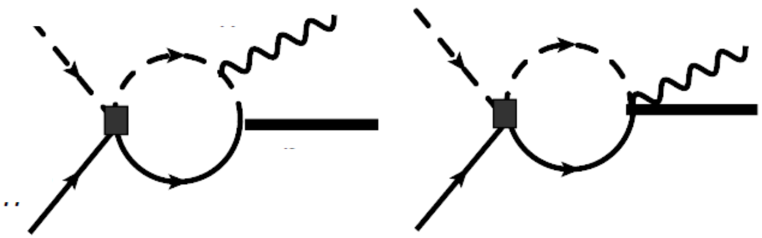
Existence of threshold

Improve the initial state multiple scattering

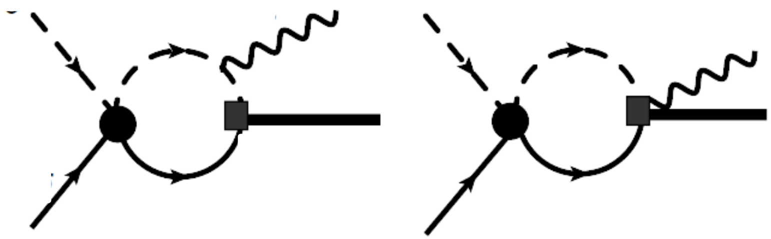
Outlook: NLO



Existence of threshold

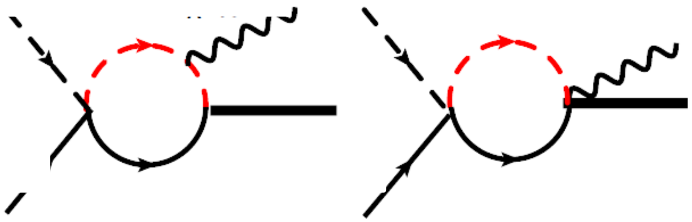


Improve the initial state multiple scattering

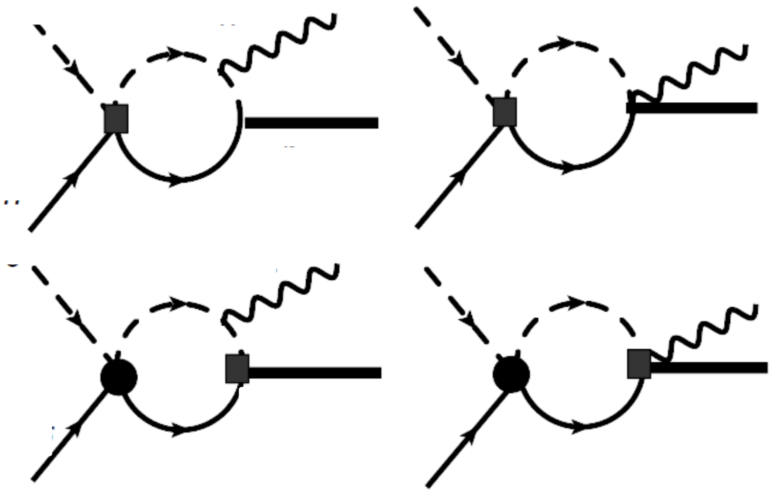


Improve the final state interaction

Outlook: NLO

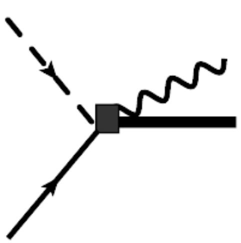


Existence of threshold

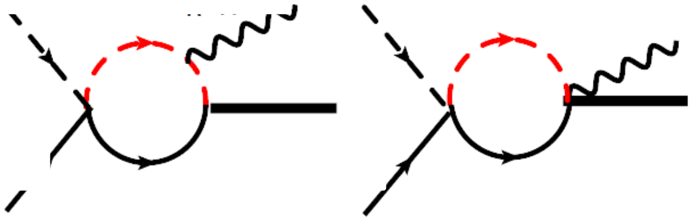


Improve the initial state multiple scattering

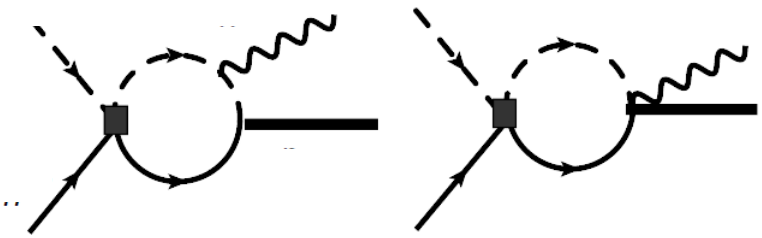
Improve the final state interaction



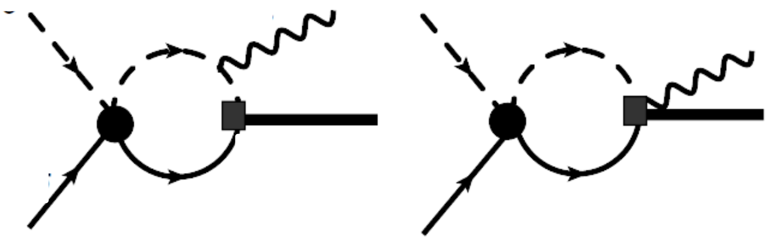
Outlook: NLO



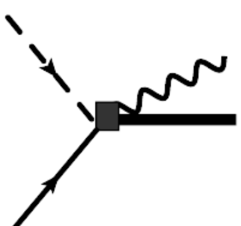
Existence of threshold



Improve the initial state multiple scattering

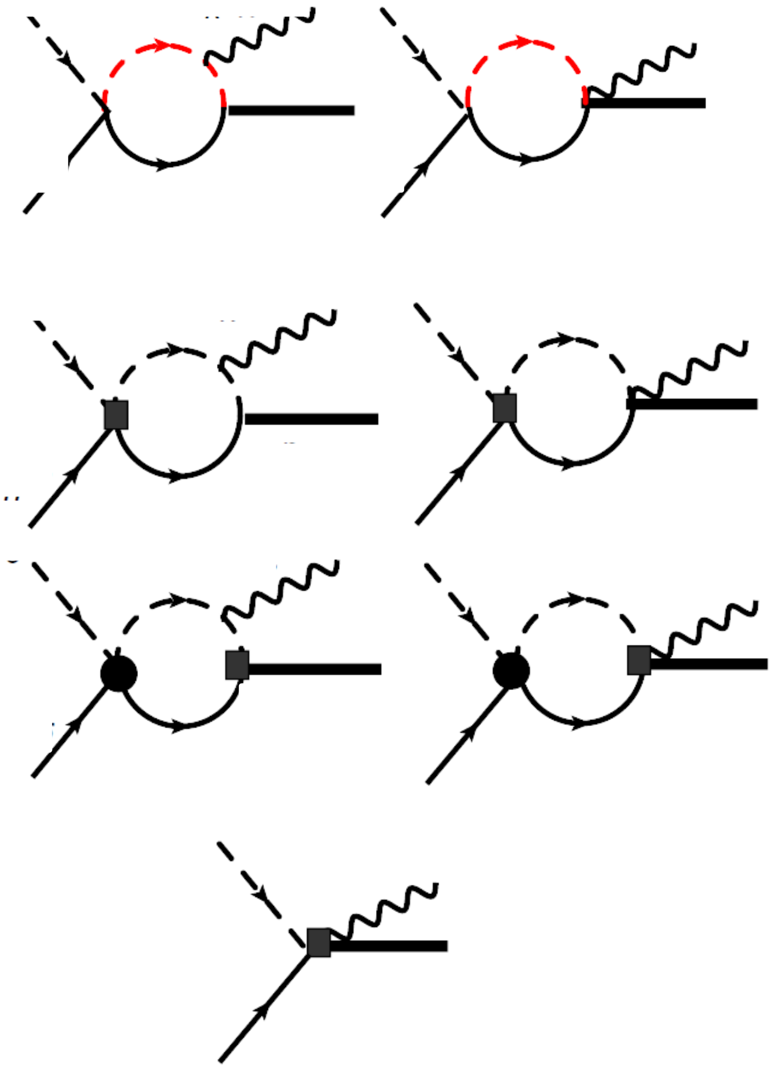


Improve the final state interaction



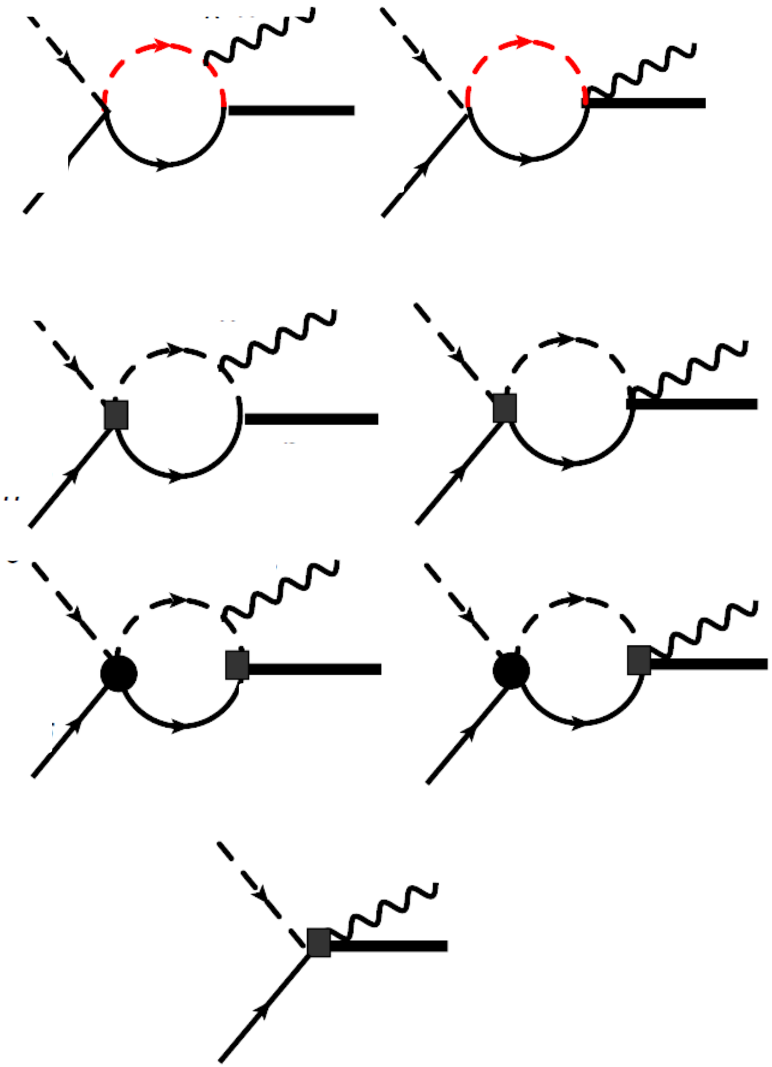
Improve the short distance contribution

Outlook: NLO



- Need to fix higher order couplings, i.e., need more "observables".

Outlook: NLO



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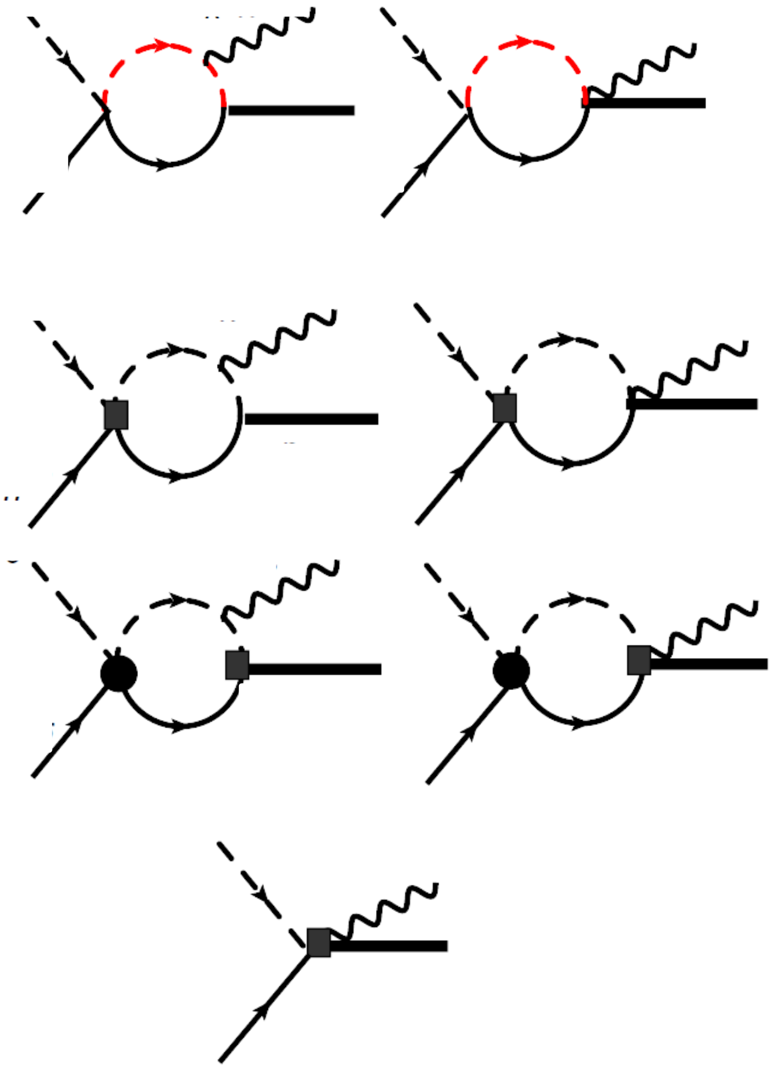

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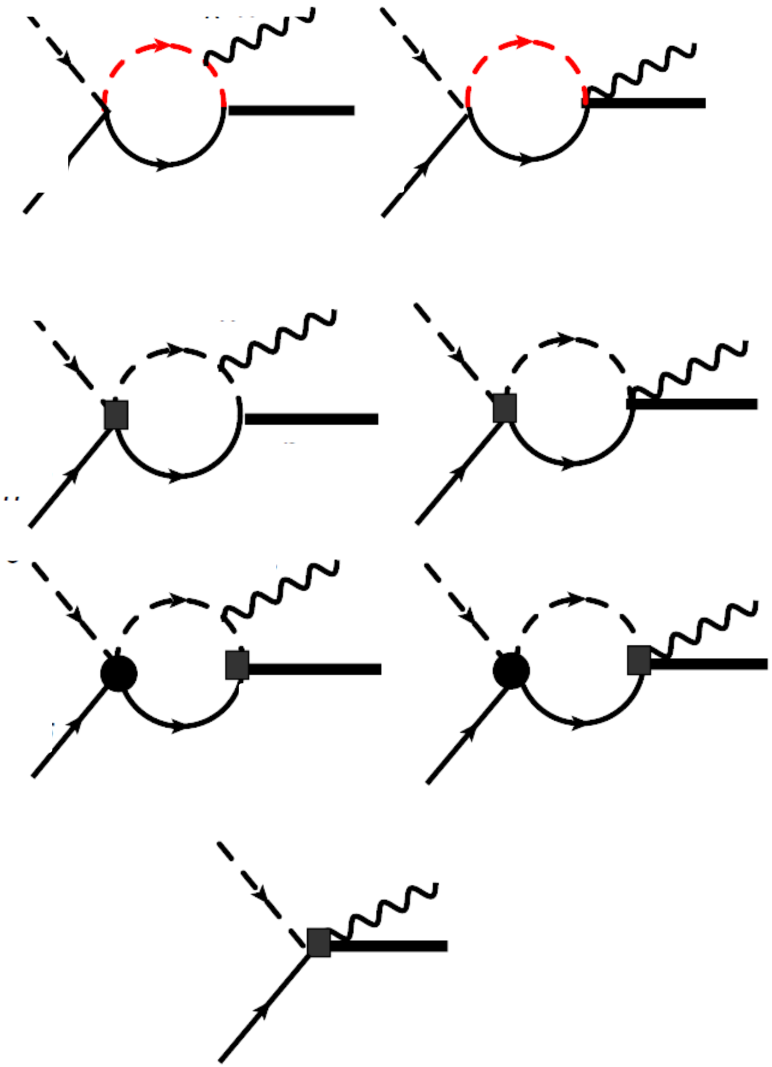


- Need to fix higher order couplings, i.e., need more "observables".
- Extract from *direct* ab initio calculations (short distance)?
- Change the boundary conditions and the background fields?
- Use data directly?

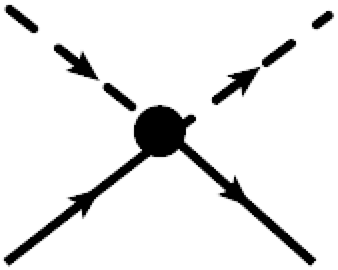
backup

- Capture cross section

$$\sigma(E) = \frac{S(E)}{E} \exp[-2\pi\eta(E)],$$

- 20 keV ~ fb
- 1MeV ~mb

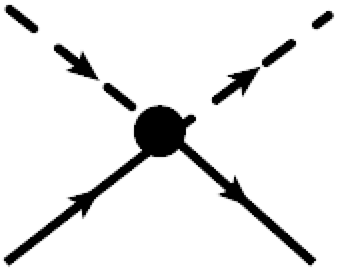
S-wave in EFT



$$T = \frac{2\pi}{M_{\mathrm{R}}} \frac{1}{-k\cot\delta_0 + ik}$$

Effective range expansion: $$-k\cot\delta_0 = \frac{1}{a_0} - \frac{1}{2} r_0 k^2 + \ldots$$

S-wave in EFT



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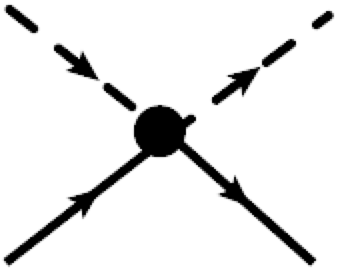
$$T = \frac{2\pi}{M_{\rm R}} a_0 \left[1 - i a_0 k + \left(\frac{a_0 r_0}{2} - a_0^2\right) k^2 + \ldots\right]$$

Natural

$a_0 \sim \frac{1}{\Lambda}$ and $r_0 \sim \frac{1}{\Lambda}$

S-wave in EFT



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Natural

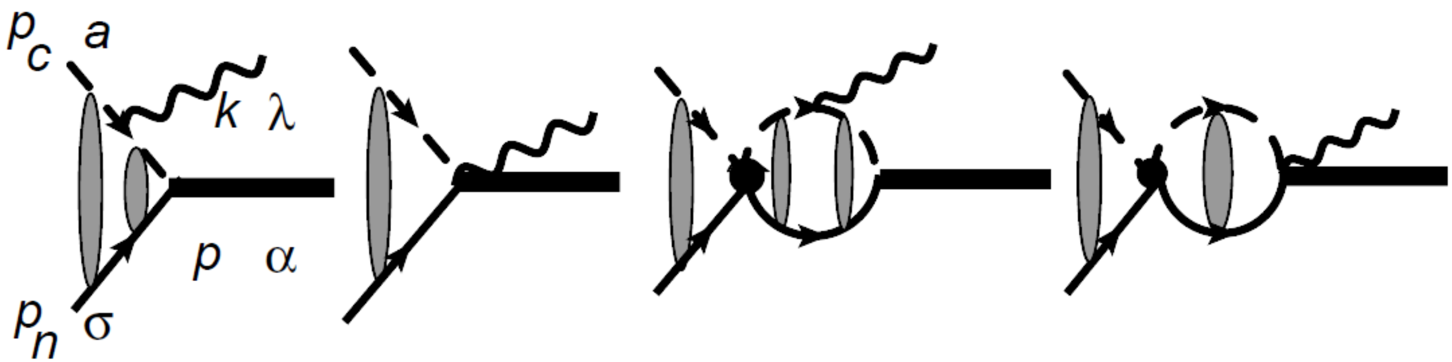
$$a_0 \sim \frac{1}{\Lambda} \text{ and } r_0 \sim \frac{1}{\Lambda}$$

$$T = \frac{2\pi}{M_{\rm R}} \frac{1}{a_0^{-1} + ik} \left(1 + \frac{r_0 k^2}{2} \frac{1}{a_0^{-1} + ik} + \ldots\right)$$

Unnatural

$$a_0 \sim \frac{1}{\gamma} \text{ but } r_0 \sim \frac{1}{\Lambda}$$

Radiative captures: LO

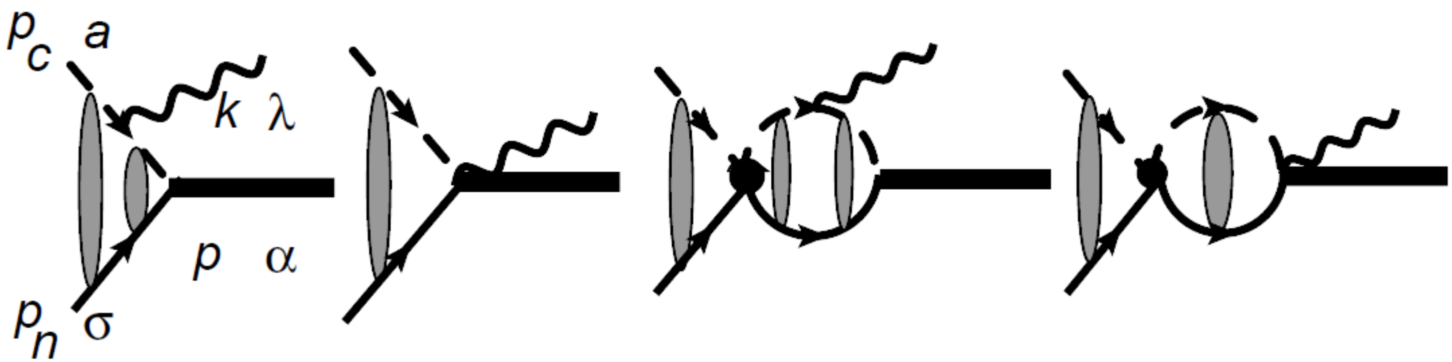


$$\langle \pi^\alpha | L_{EM} | \chi_{\boldsymbol{p}}^{(+)}, \delta, a \rangle \equiv T_i^{\delta a} T_\alpha^{ij} \mathcal{M}_j$$

Initial total spin Si=1

$$\mathcal{M}_j = (-i) C_{\eta,0} C_{(^3P_2)}^{\mathrm{LO}} \frac{Z_{eff}}{M_{\mathrm{R}}} \frac{2\pi}{\sqrt{3}} \left(\gamma^2 + k^2\right) \left[e^{i\sigma_0} \epsilon_j^* Y_{00}(\hat{p}) \mathcal{S}(^3S_1) + e^{i\sigma_2} \epsilon_k^* \sqrt{2} T_j^{ka} Y_{2a}(\hat{p}) \mathcal{D} \right]$$

Radiative captures: LO



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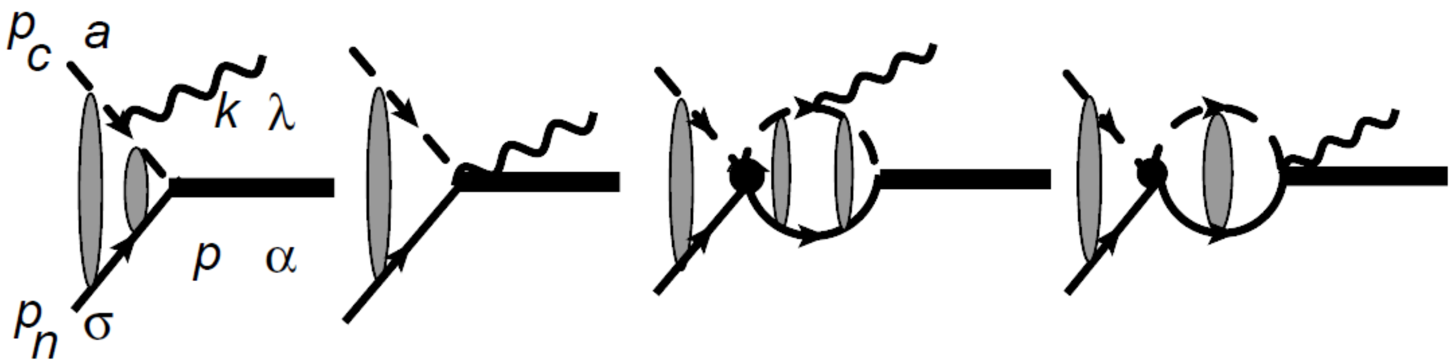
$$\mathcal{S}(X) \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \left[\frac{C_{\eta,0} G_0(k,r)}{-a_{(X)}^{-1} - 2k_C H(\eta)} + \frac{F_0(k,r)}{C_{\eta,0} k} \frac{-a_{(X)}^{-1} - 2k_C \mathrm{Re}[H(\eta)]}{-a_{(X)}^{-1} - 2k_C H(\eta)} \right]$$

$$\mathcal{D} \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \frac{F_2(k,r)}{C_{\eta,0} k}$$

Radiative captures: LO



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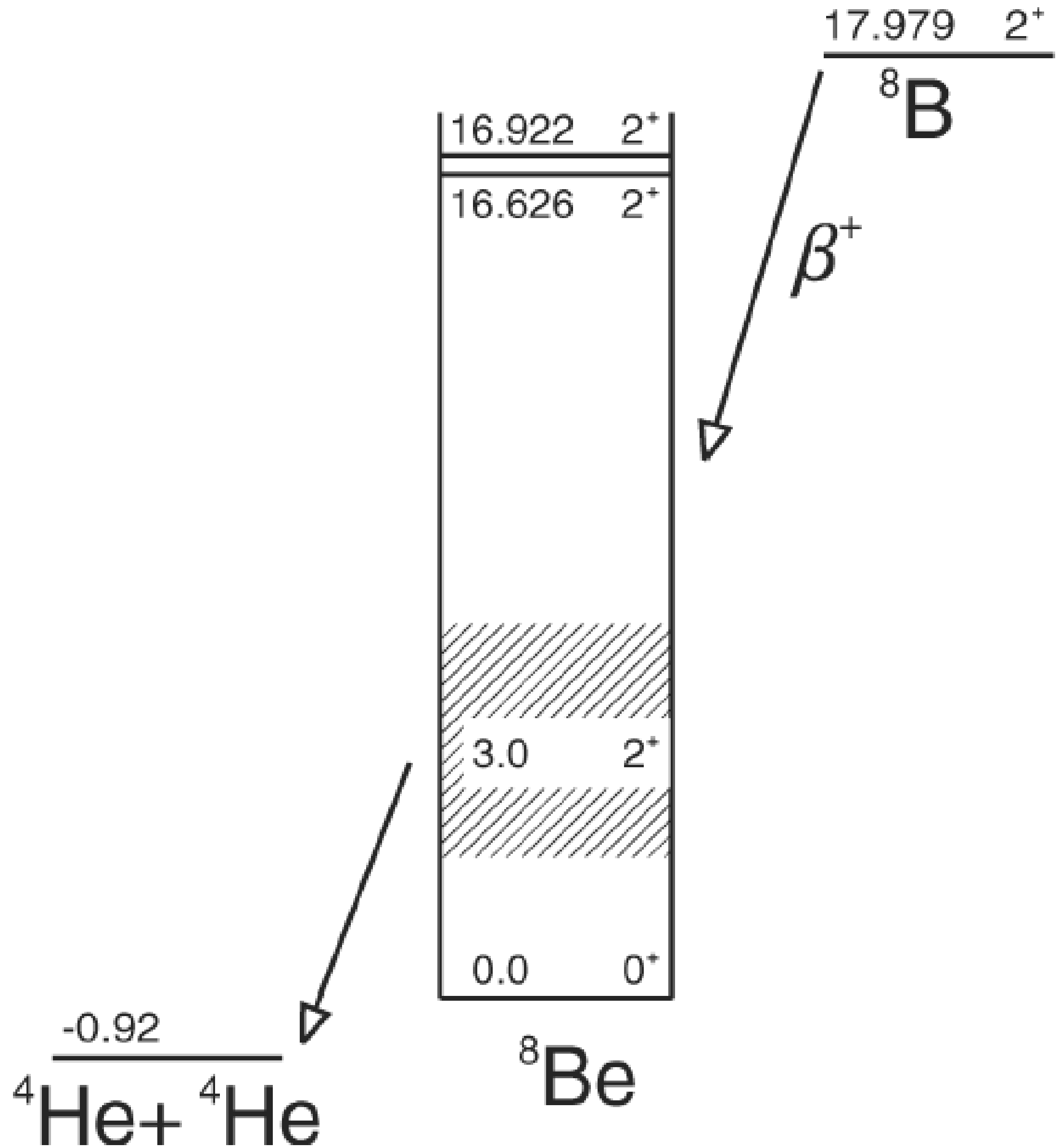
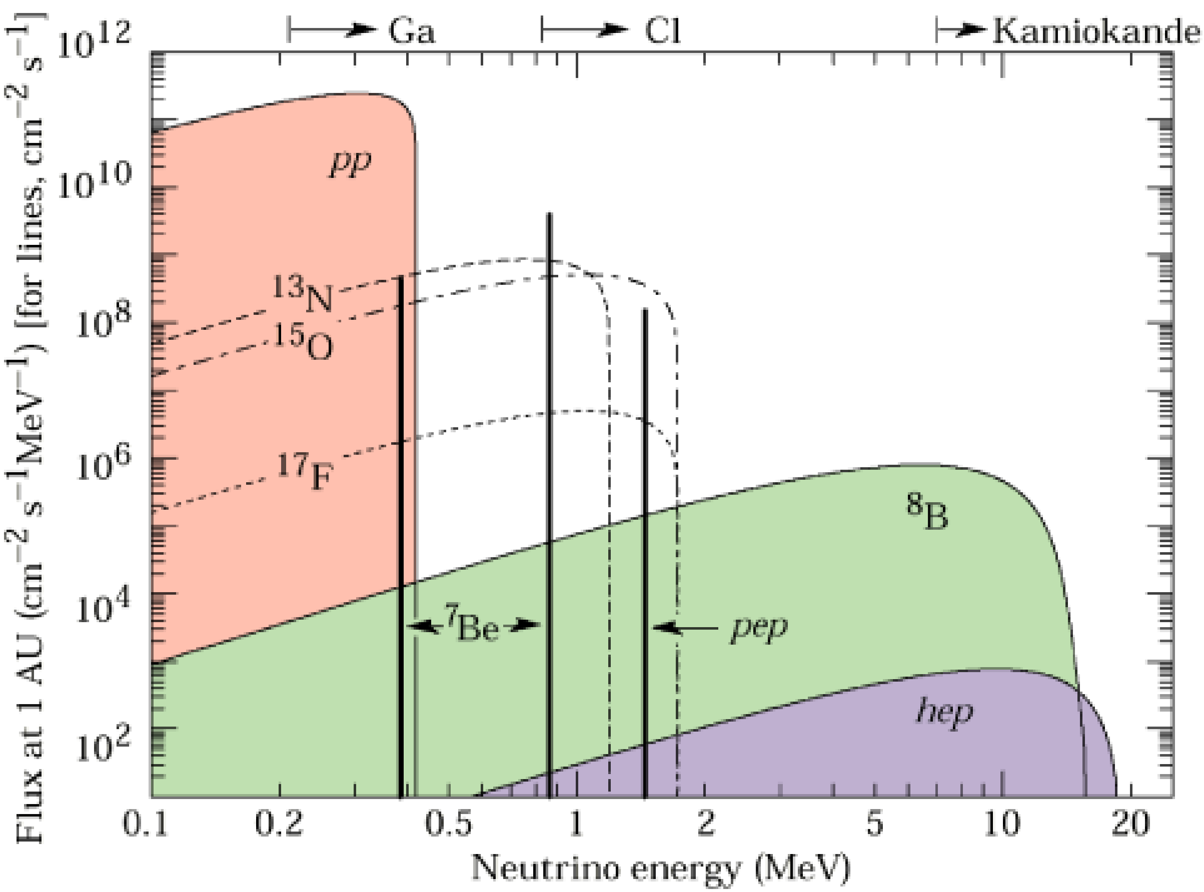
Coulomb wavefunc.

$F \to j$
$G \to n$
$W \to h$


17.979 2^+
^8B
β^+
16.922 2^+
16.626 2^+
3.0 2^+
0.0 0^+
^8Be
-0.92
$^4He + ^4He$


Ga
Cl
Kamiokande
pp
^{13}N
^{15}O
^{17}F
^{8}B
^{7}Be
pep
hep
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
0.1
0.2
0.5
1
2
5
10
20
Neutrino energy (MeV)
Flux at 1 AU (cm^{-2} s^{-1}MeV^{-1}) [for lines, cm^{-2} s^{-1}]