Exotic pairing states in 2D Fermi gases with Rashba spin-orbit coupling

An interest in few-body physics from a many-body perspective

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- ￥￥￥

NSFC

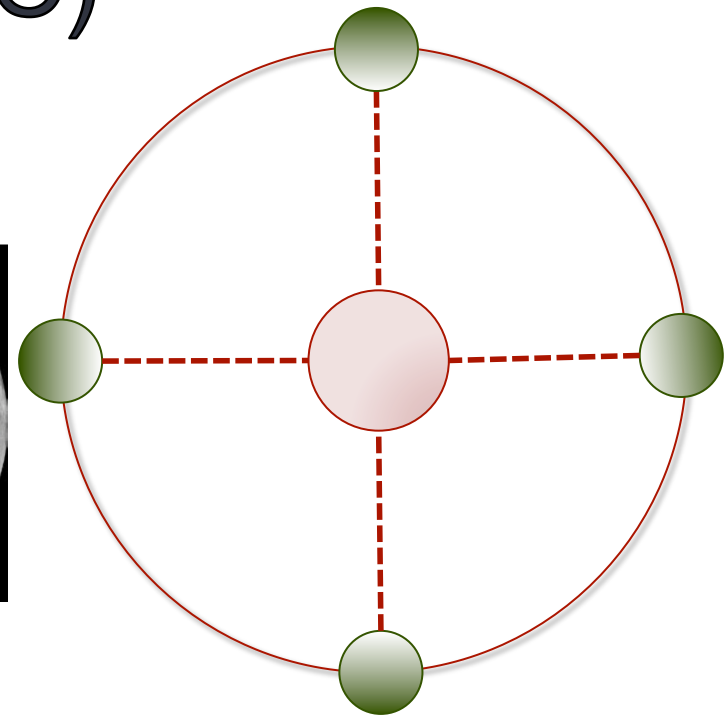
教育部
Ministry of Education

Outline

- Background
  - Spin-orbit coupling
  - Realization of spin-orbit coupling (1D, ERD)
  - Single particle property
- 2D Fermi gas with Rashba SOC & B-fields
  - Topological superfluid
  - FFLO pairing within a single branch
  - Topological FFLO state
- Few-body physics from a many-body perspective
  - Quasi-2D versus 2D: When and how the 3rd dimension matters?
  - Validity of a contact interaction

Spin-orbit coupling (SOC)

- SOC is well observed in nature



- SOC in continuous system

$$H_{\mathrm{SO}} = \alpha_x \sigma_x k_x + \alpha_y \sigma_y k_y + \alpha_z \sigma_z k_z$$

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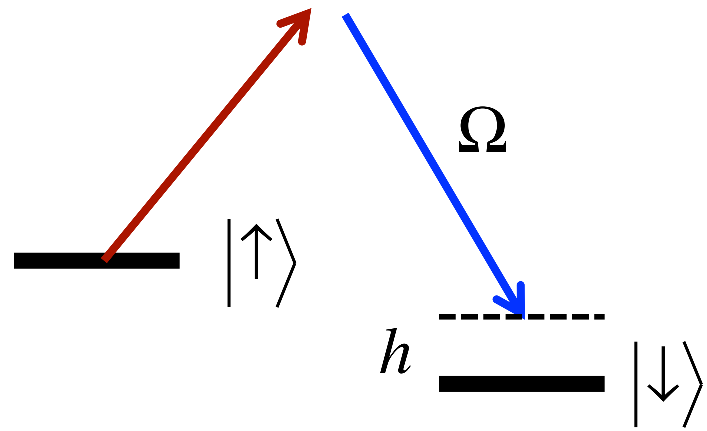
$\alpha_x = \alpha_y = \alpha_z$	**3D isotropic SOC**	$\alpha \vec{\sigma} \cdot \vec{k}$
$\alpha_x = \alpha_y$ $\alpha_z = 0$	**Rashba SOC**	$\alpha\left(\sigma_x k_x + \sigma_y k_y\right)$
$\alpha_x = -\alpha_y$ $\alpha_z = 0$	**Dresselhaus SOC**	$\alpha\left(\sigma_x k_x - \sigma_y k_y\right)$
$\alpha_y = \alpha_z = 0$	**1D SOC** **Equal R-D SOC**	$\alpha \sigma_x k_x$

Realization of ERD SOC: NIST-scheme

Lin et. al., Nature 471, 83 (2011)



$$\begin{array}{cc} |\uparrow\rangle & |\downarrow\rangle \end{array}$$

$$H = \begin{pmatrix} \frac{k_x^2}{2m} + \frac{h}{2} & \frac{\Omega}{2} e^{i2k_0 x} \\ \frac{\Omega}{2} e^{-i2k_0 x} & \frac{k_x^2}{2m} - \frac{h}{2} \end{pmatrix} \begin{array}{c} |\uparrow\rangle \\ |\downarrow\rangle \end{array} \qquad U = \begin{pmatrix} e^{-ik_0 x} & 0 \\ 0 & e^{ik_0 x} \end{pmatrix}$$

Realization of ERD SOC: NIST-scheme

$$H_{\mathrm{SO}} = UHU^\dagger = \begin{pmatrix} \frac{(k_x+k_0)^2}{2m} + \frac{h}{2} & \frac{\Omega}{2} \\ \frac{\Omega}{2} & \frac{(k_x-k_0)^2}{2m} - \frac{h}{2} \end{pmatrix}$$

$$= \frac{1}{2m}(k_x + k_0\sigma_z)^2 + \frac{\Omega}{2}\sigma_x + \frac{h}{2}\sigma_z.$$

- Spin rotation, x to –z, z to x

$$H_{\mathrm{SO}} = \frac{1}{2m}\left(k_x^2 + k_0^2 + 2\sigma_x k_x\right) - \frac{\Omega}{2}\sigma_z + \frac{h}{2}\sigma_x$$

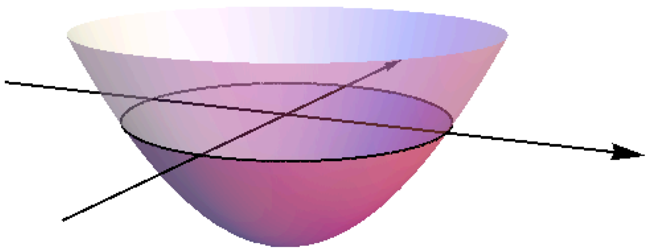
1D (ERD) SOC

x-field
2-photon
detuning

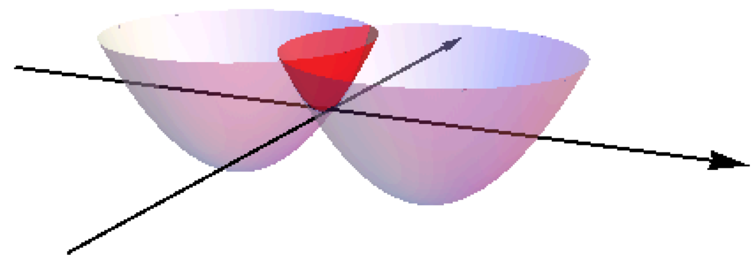
z-field
Rabi Frequency

Single-particle dispersion

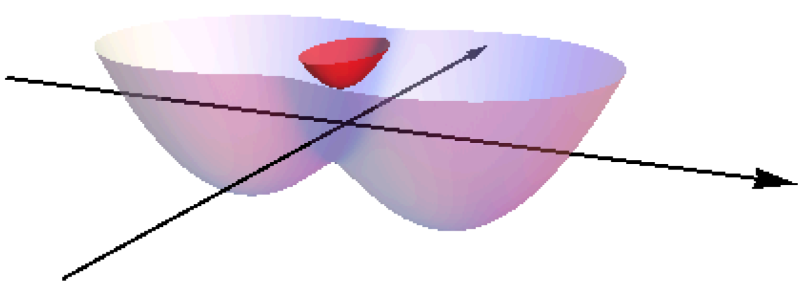
w/o SOC



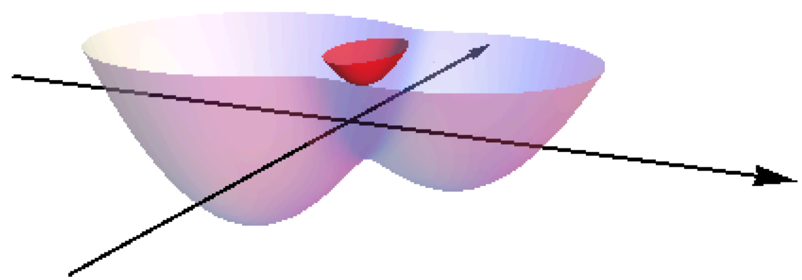
w/ SOC



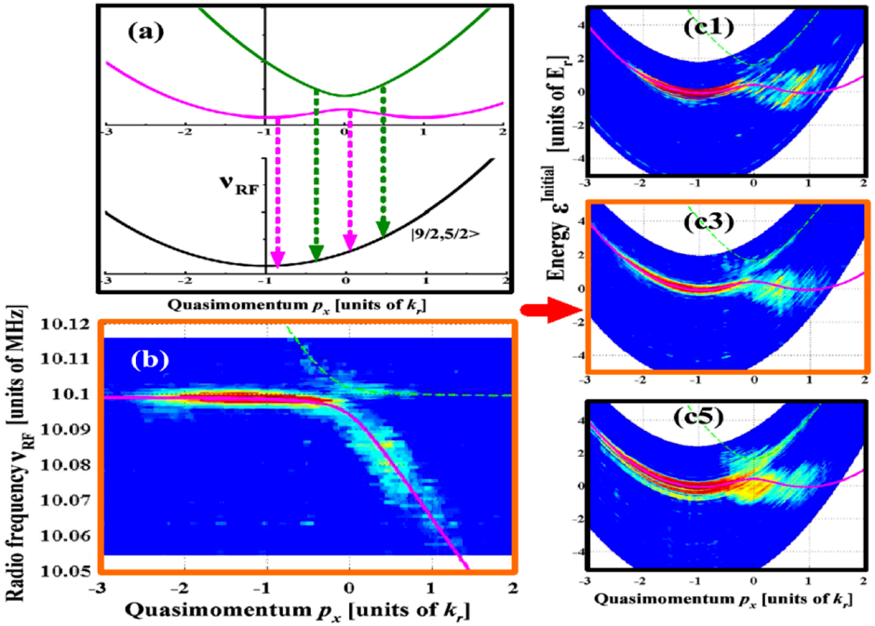
w/ SOC + z-field

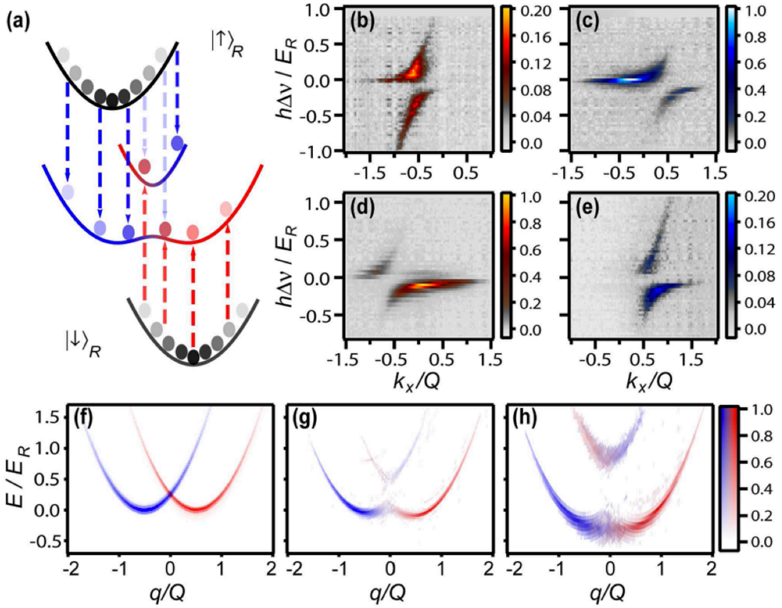


w/ SOC + z-field + x-field



Implementation in Fermi gases





Shanxi Univerisity, Taiyuan, China

Wang *et al.*, PRL 109, 095301 (2012)

MIT, Boston, USA

Cheuk *et al.*, PRL 109, 095302 (2012)

Rashba SOC

Campbell, Juzeliunas, and Spielman, PRA 84, 025602 (2011)
Liu, Law, and Ng, PRL 112, 086401 (2014)

$\alpha_x = \alpha_y = \alpha_z$	**3D isotropic SOC**	$\alpha \vec{\sigma} \cdot \vec{k}$
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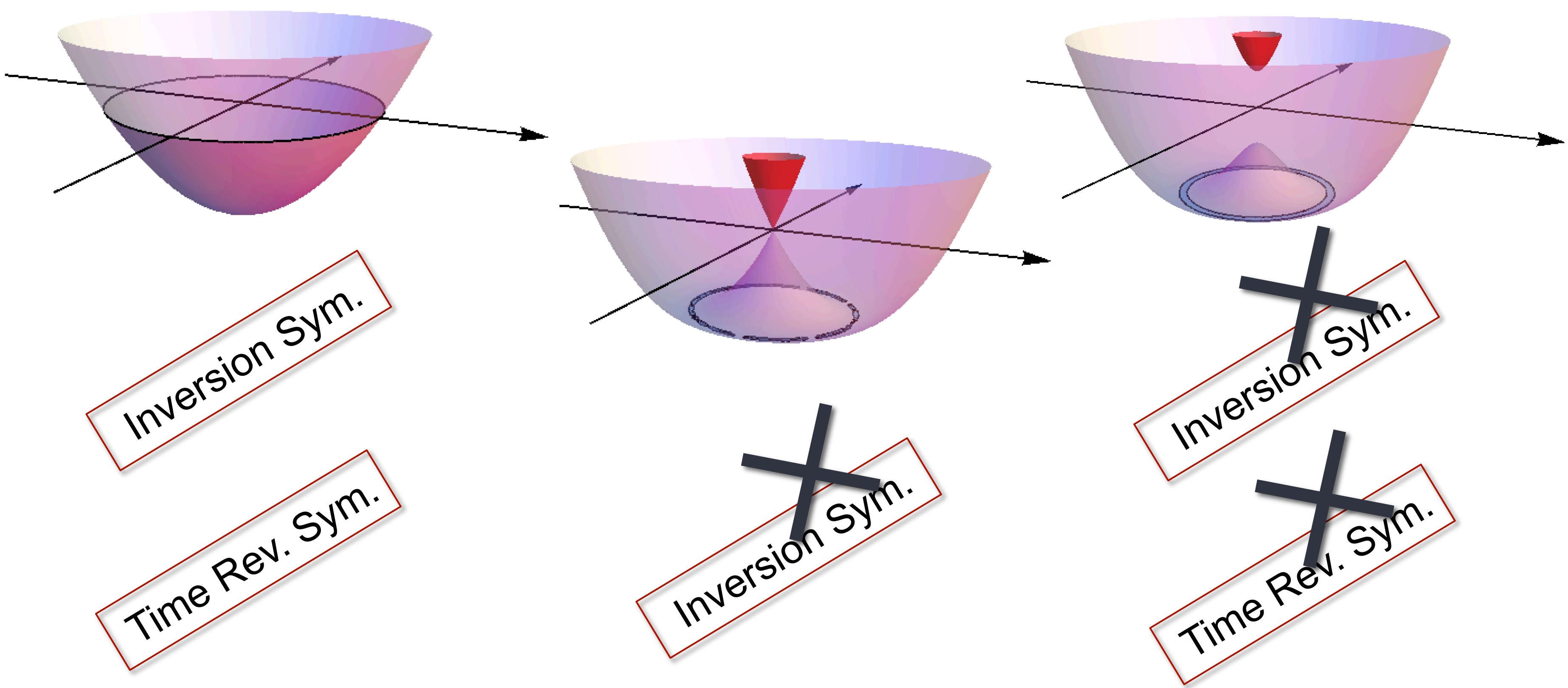
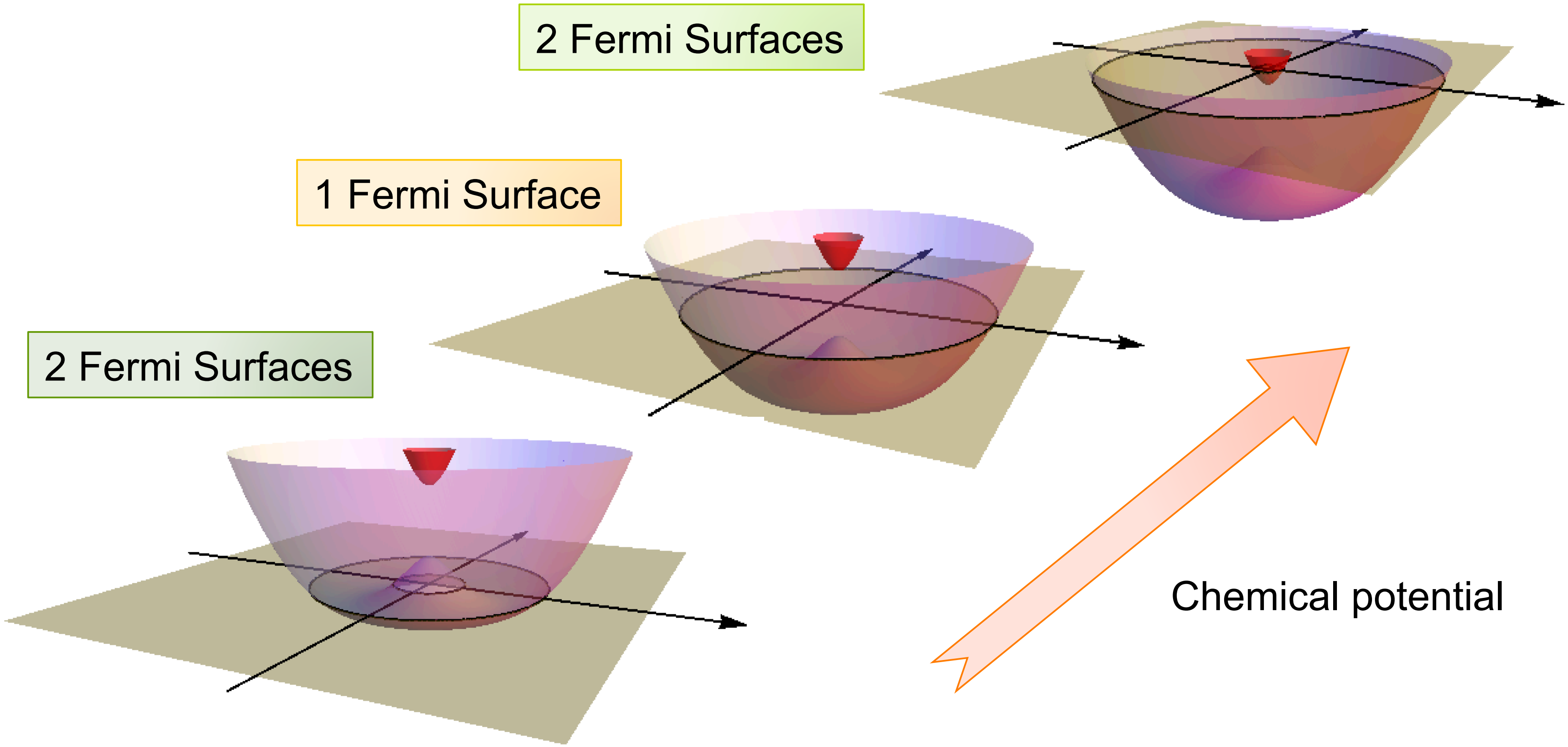
Single-particle dispersion

w/o SOC w/o z-field	w/ SOC w/o z-field	w/ SOC w/ z-field



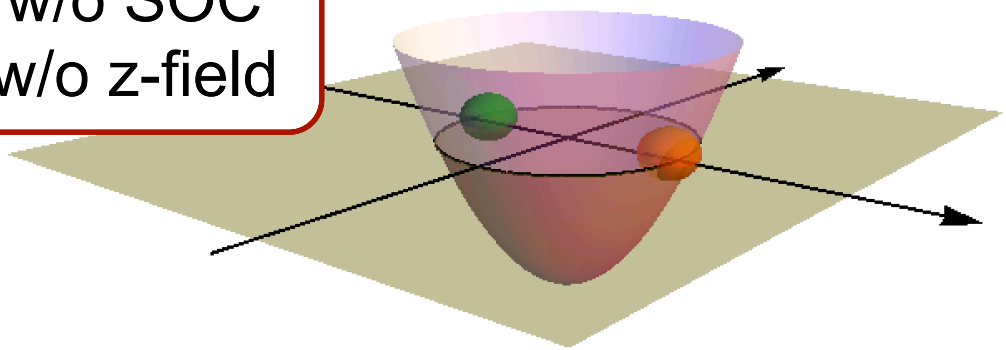
Lifshitz transition

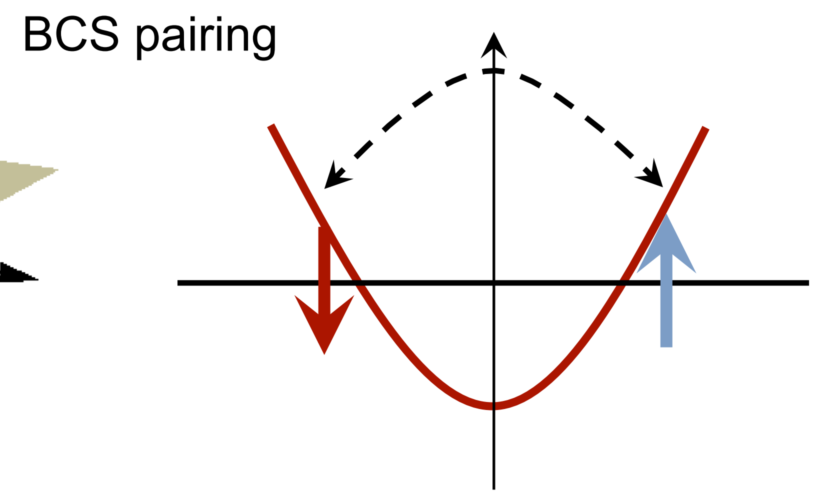
- 2D Fermi gas with Rashba SOC + z-field



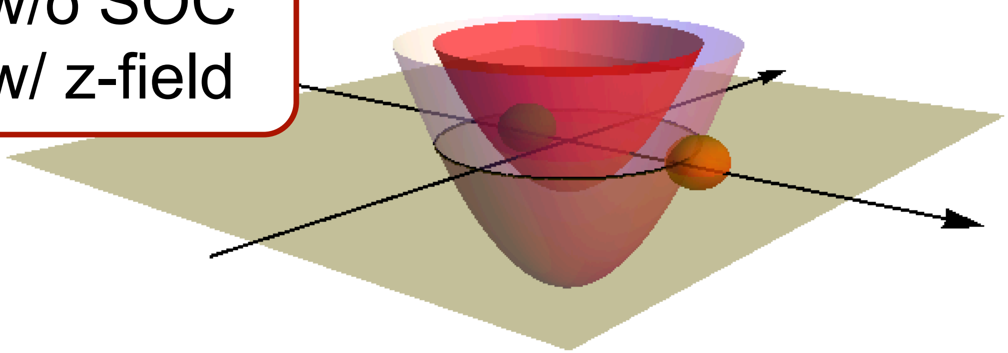
Now we add in the interaction…

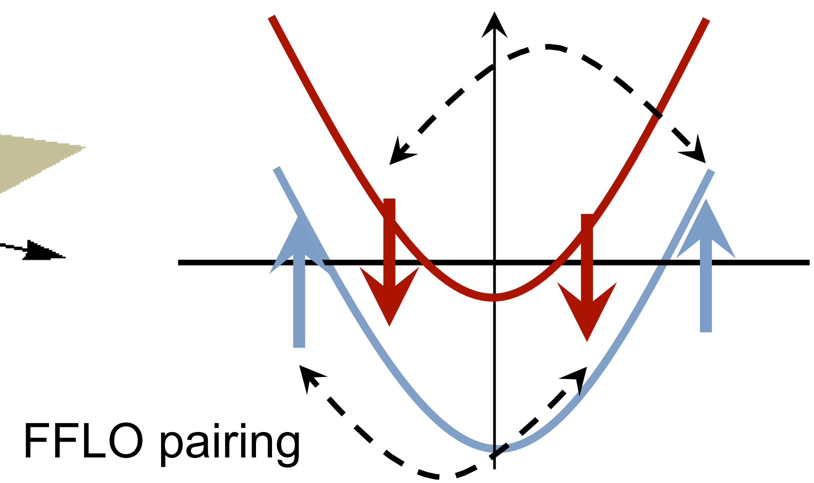
w/o SOC
w/o z-field





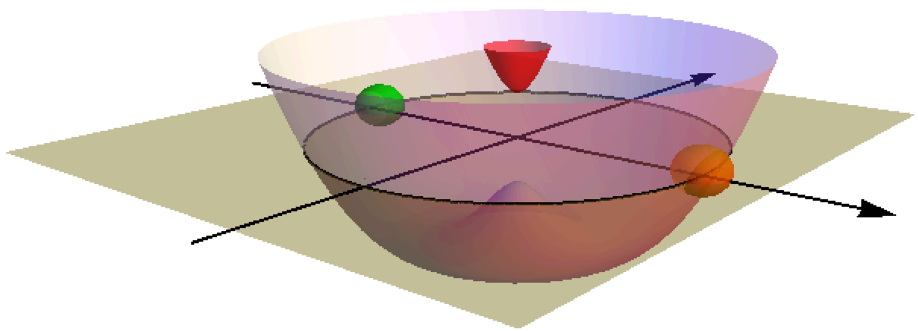
w/o SOC
w/ z-field

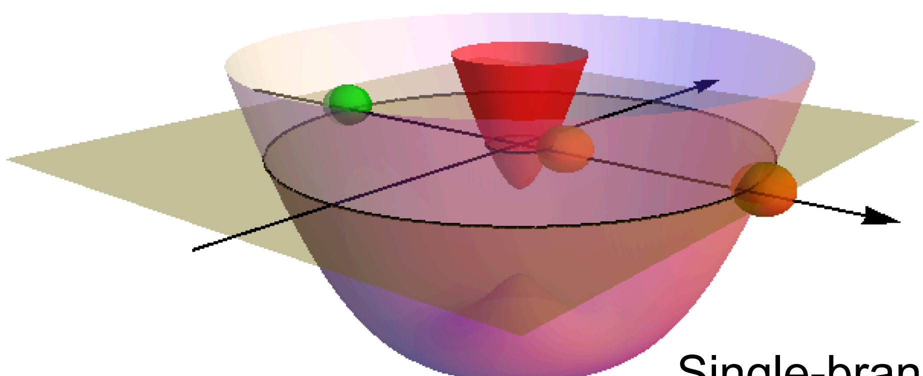




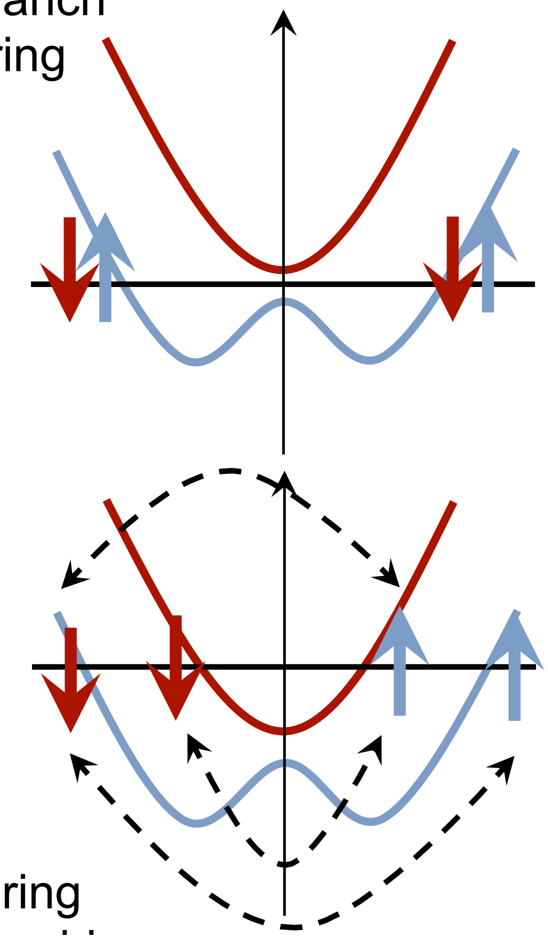
Now we add in the interaction…

w/ SOC +
z-field





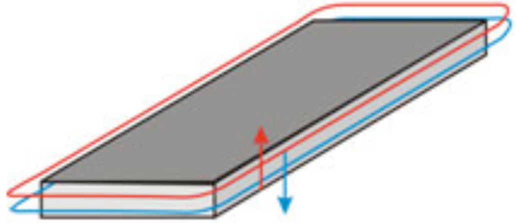
Single-branch
BCS pairing



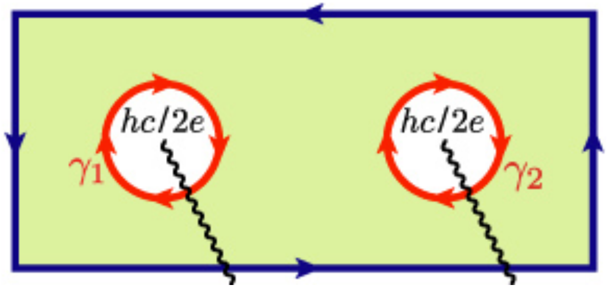
Single-branch BCS pairing
v.s. inter-branch FFLO pairing

Topological Superfluid (TSF)

- Fully gapped in the bulk
- Topologically non-trivial properties
  - Not characterized by local order parameters
  - Topologically protected gapless edge modes



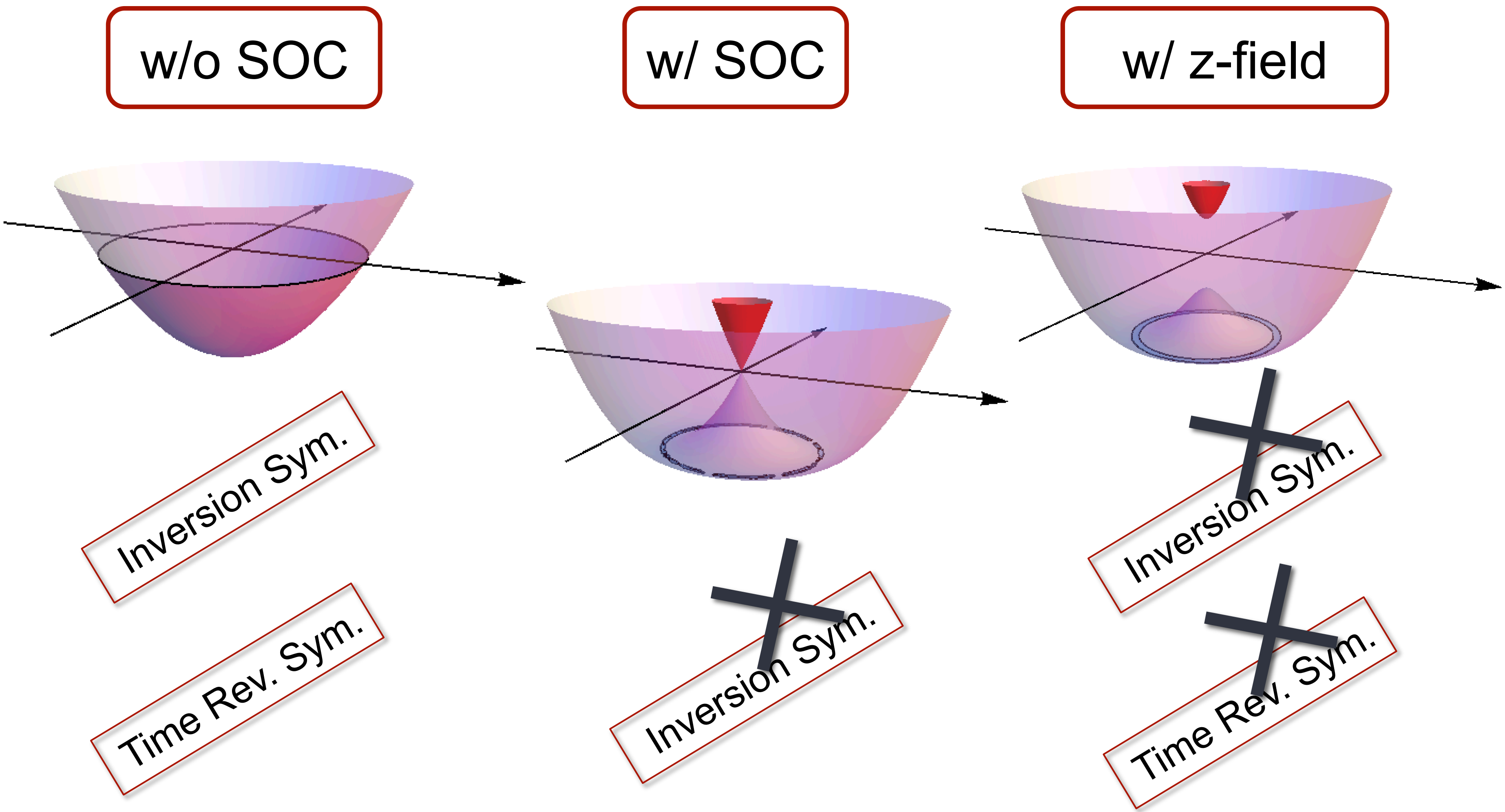
- Majorana fermions at vortex cores
  - Fault-tolerant quantum computing



- 2D Fermi system which breaks inversion and time-reversal symmetries

Sau et. al., PRL 104, 040502 (2010)

Single-particle dispersion



2D Fermi gas with Rashba SOC & z-field

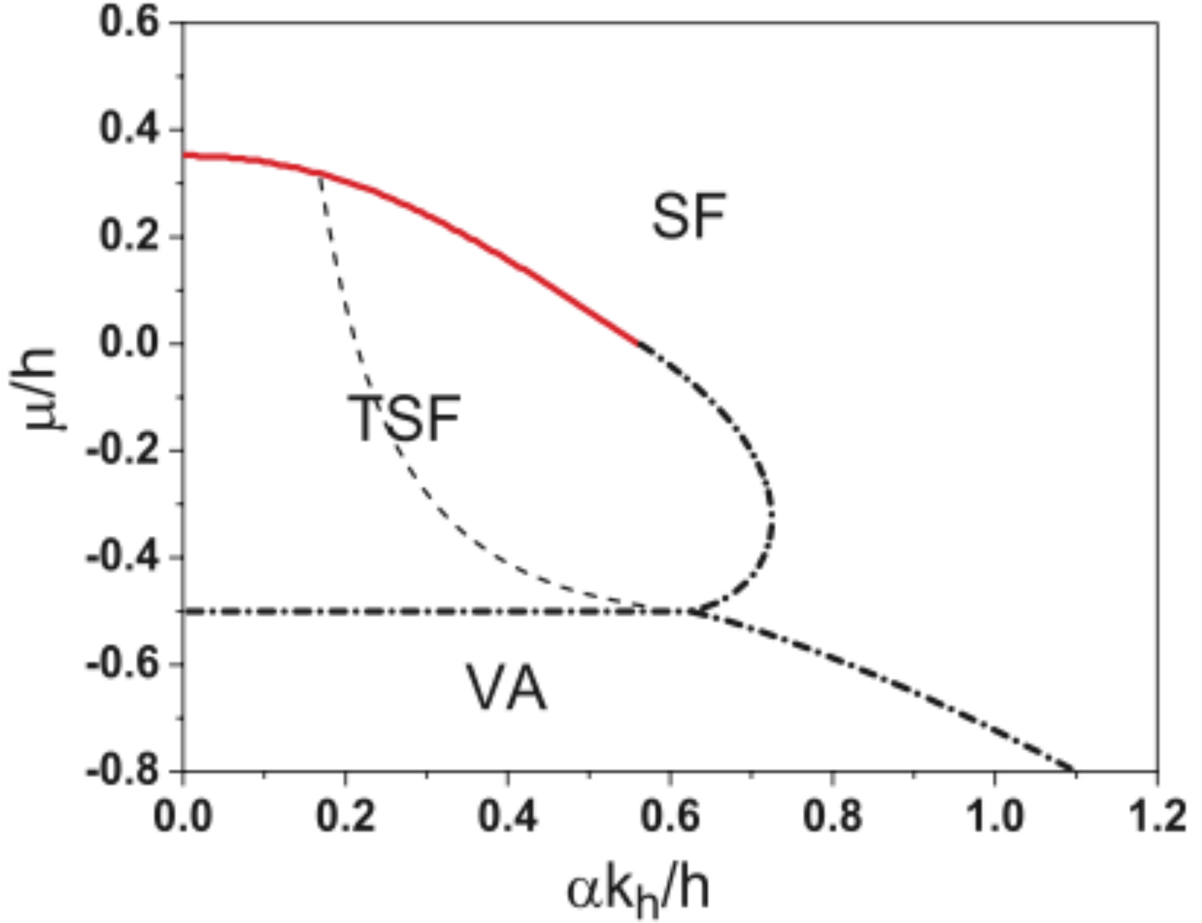
- Hamiltonian

$$
\begin{aligned}
H - \sum_\sigma \mu_\sigma N_\sigma &= H_0 + H_{\text{soc}} + H_{\text{int}} \\
&= \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_\sigma) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} \alpha k \left(e^{-i\varphi_{\mathbf{k}}} a^\dagger_{\mathbf{k},\uparrow} a_{\mathbf{k},\downarrow} + \text{H.C.} \right) \\
&+ \frac{U}{\mathcal{V}} \sum_{\mathbf{k},\mathbf{k}'} a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} a_{-\mathbf{k}',\downarrow} a_{\mathbf{k}',\uparrow}, \qquad (1)
\end{aligned}
$$

- Thermodynamic potential

$$
\begin{aligned}
\Omega &= -\frac{1}{\beta} \ln \operatorname{tr} \left[e^{-\beta (H_m - \sum_\sigma \mu_\sigma N_\sigma)} \right] \Big|_{T\to 0} \\
&= \frac{1}{2} \sum_{\mathbf{k},\lambda=\pm} (\xi_\lambda - E_{\mathbf{k},\lambda}) - \mathcal{V} \frac{|\Delta|^2}{U}.
\end{aligned}
$$

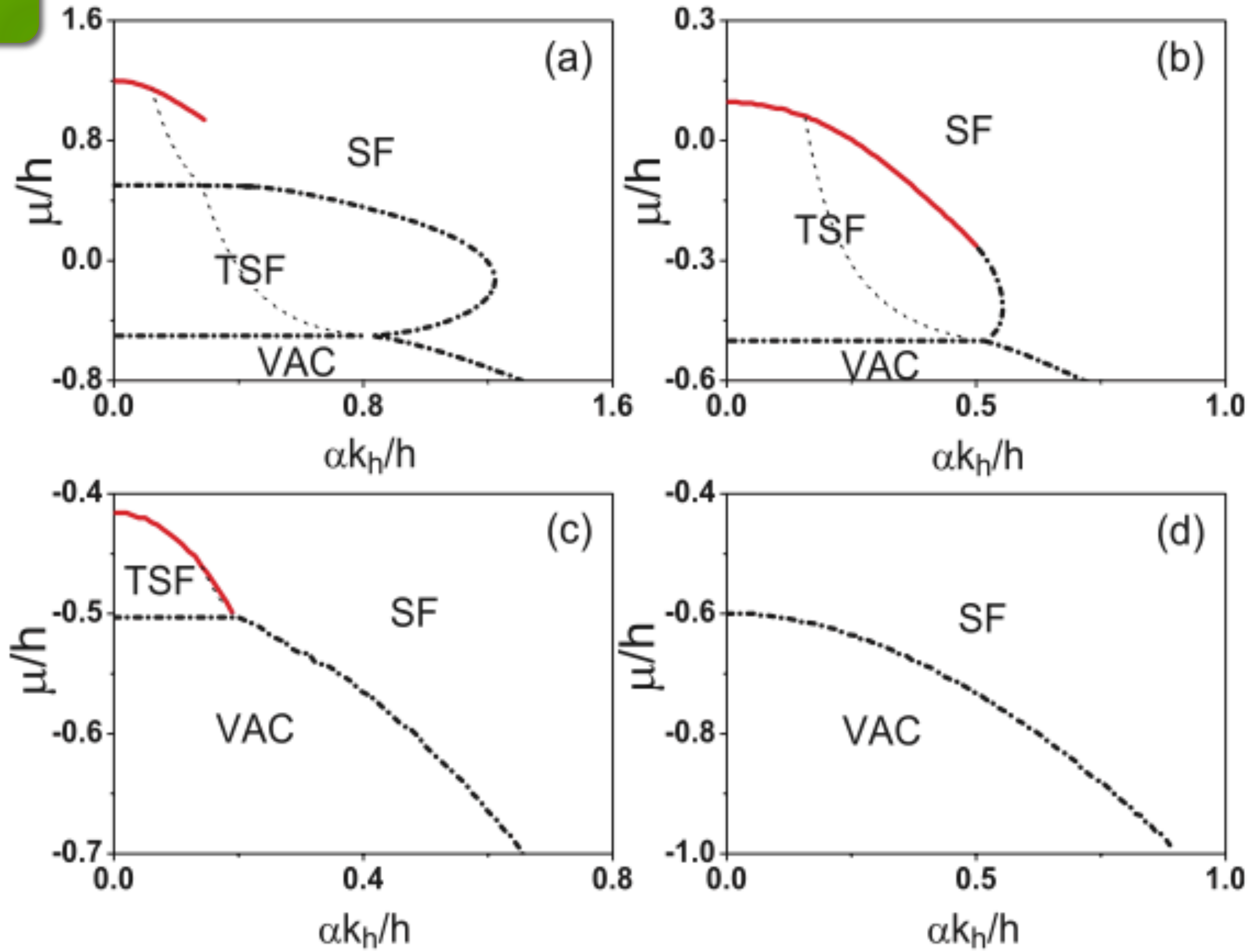
Zero-T phase diagram



Zhou, **WZ**, and Yi, PRA 84 063603 (2011)

BCS-BEC crossover

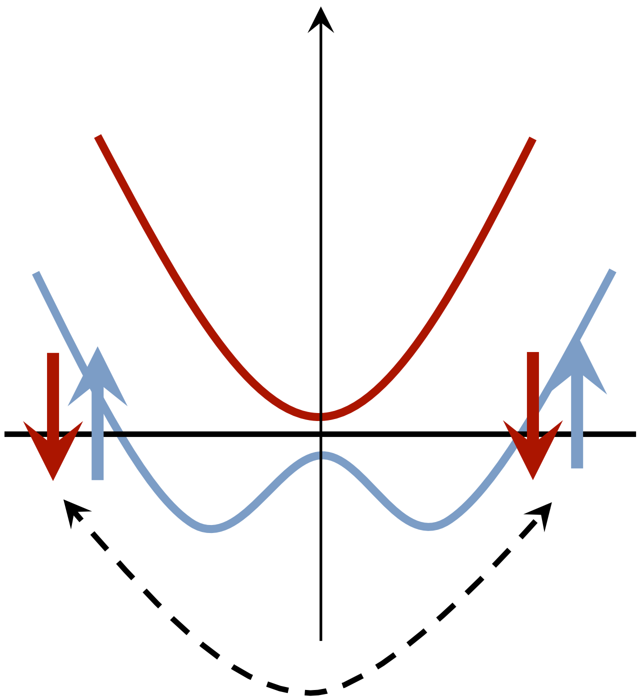
BCS

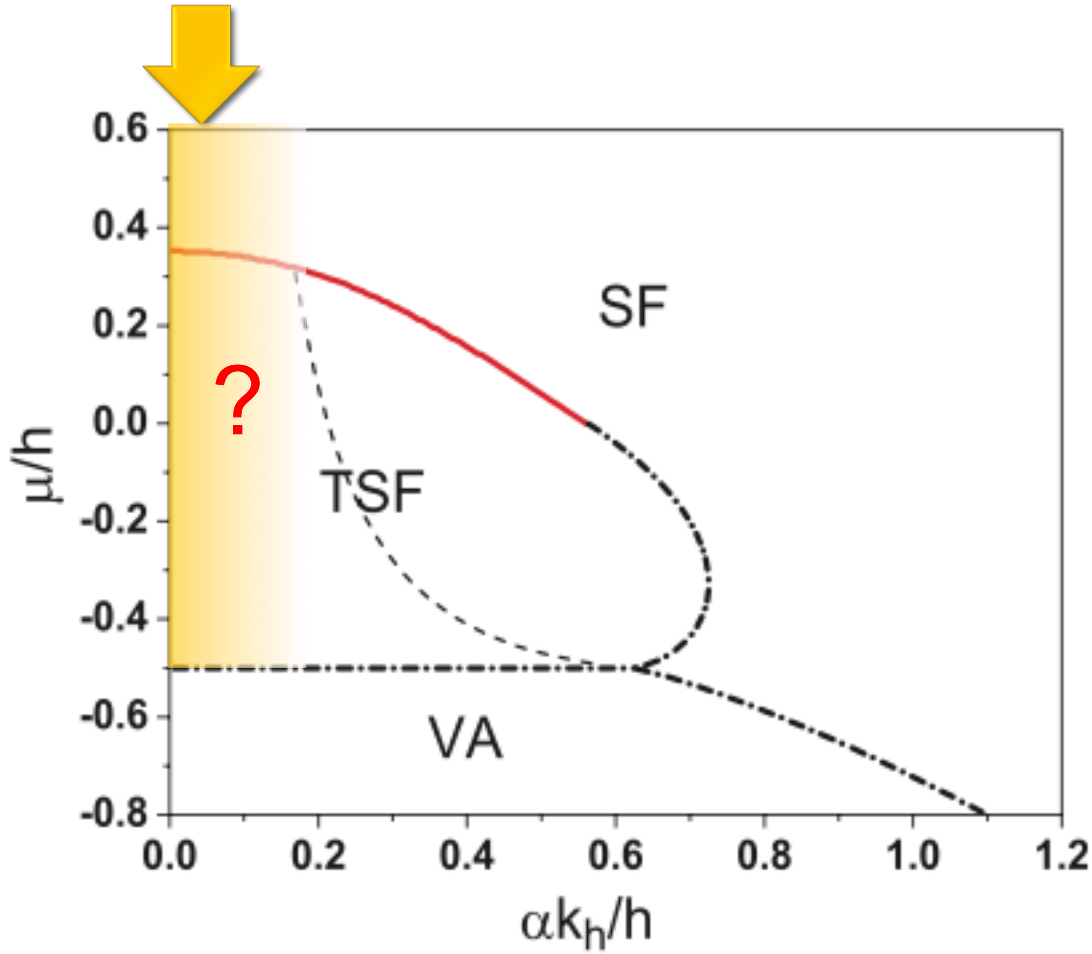


BEC

Zhou, **WZ**, and Yi, PRA 84 063603 (2011)

Zero-T phase diagram

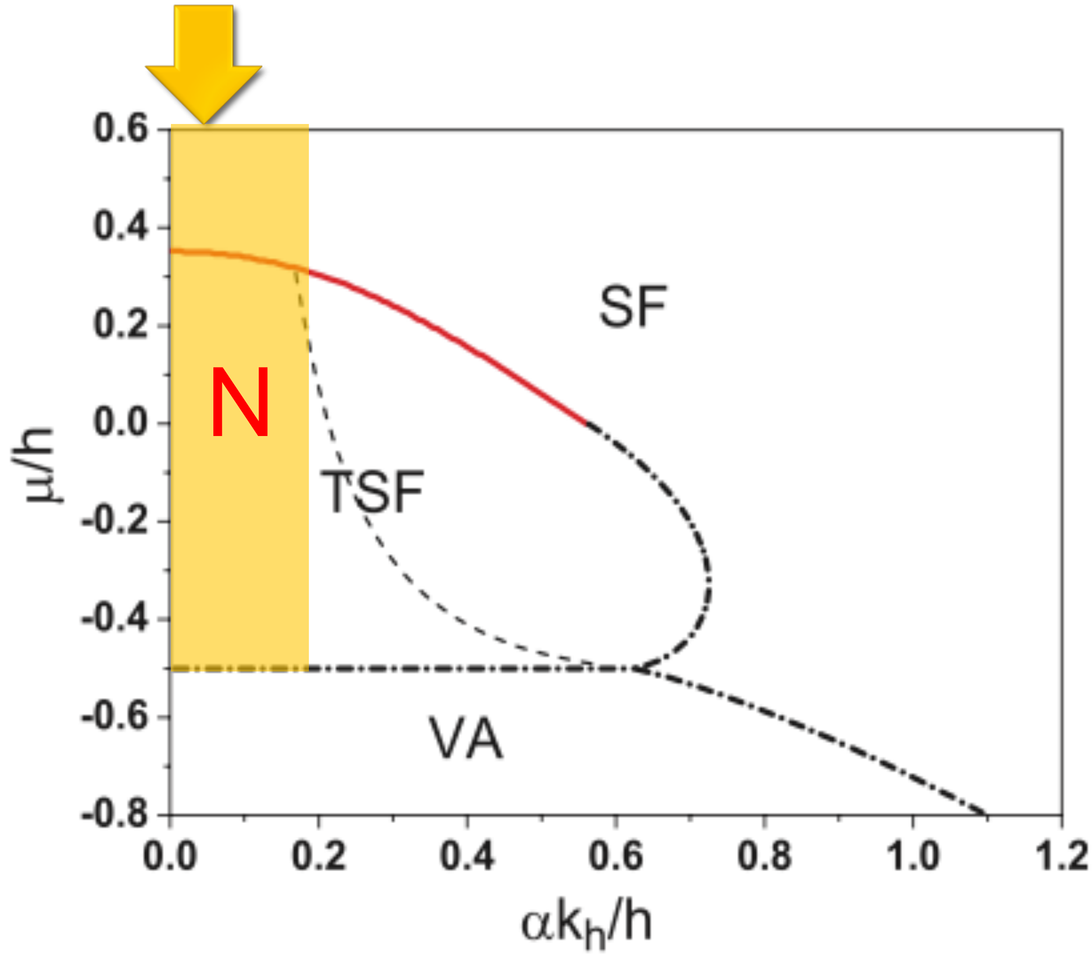




Zhou, **WZ**, and Yi, PRA 84 063603 (2011)

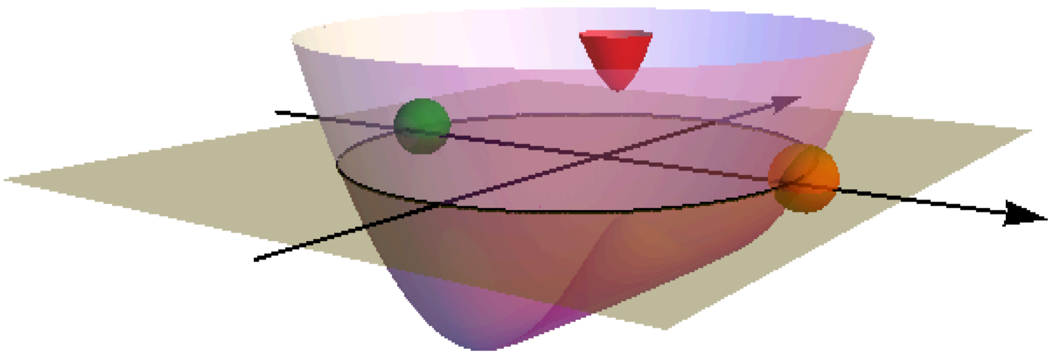
Zero-T phase diagram

- Pairing physics at large magnetic field limit
- Polaron-molecule transition
- Normal gas in the large h limit

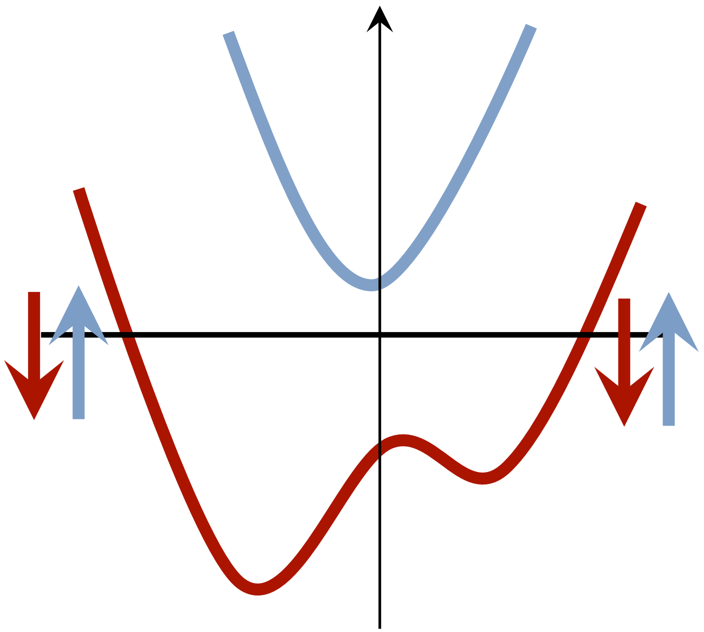


Yi and **WZ**, PRL 109, 140402 (2012)

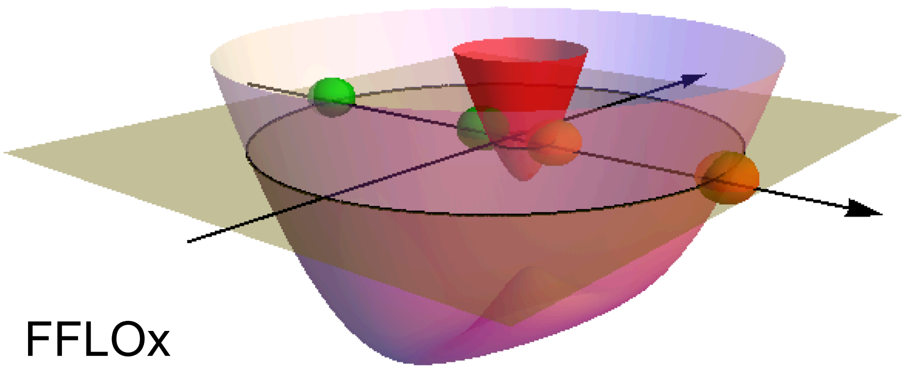
FFLO pairing



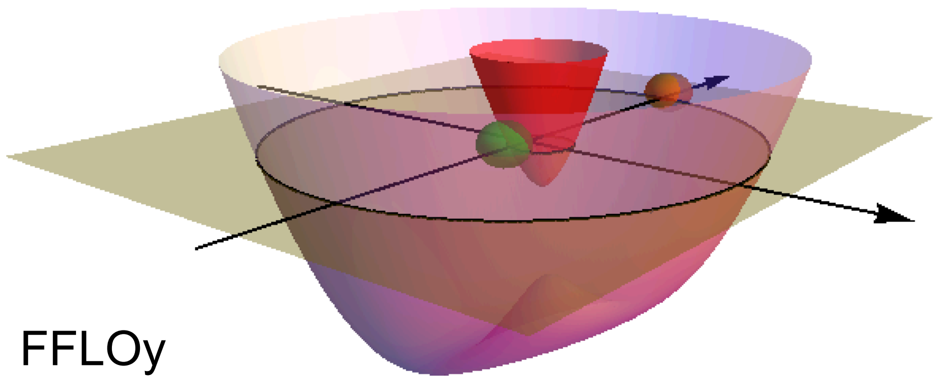
- BCS pairing becomes unstable
- SOC-induced spin mixing
- In-plane field induced Fermi surface asymmetry



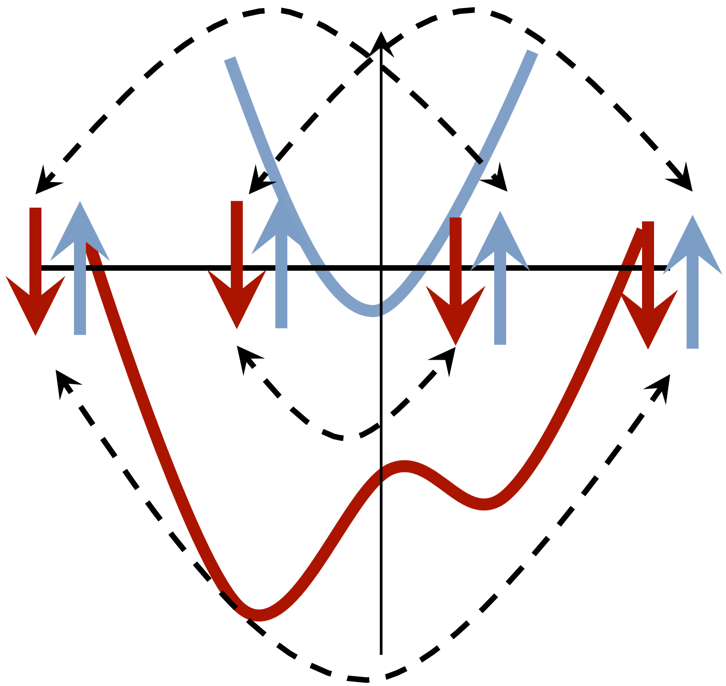
Competition of FFLO states



FFLOx

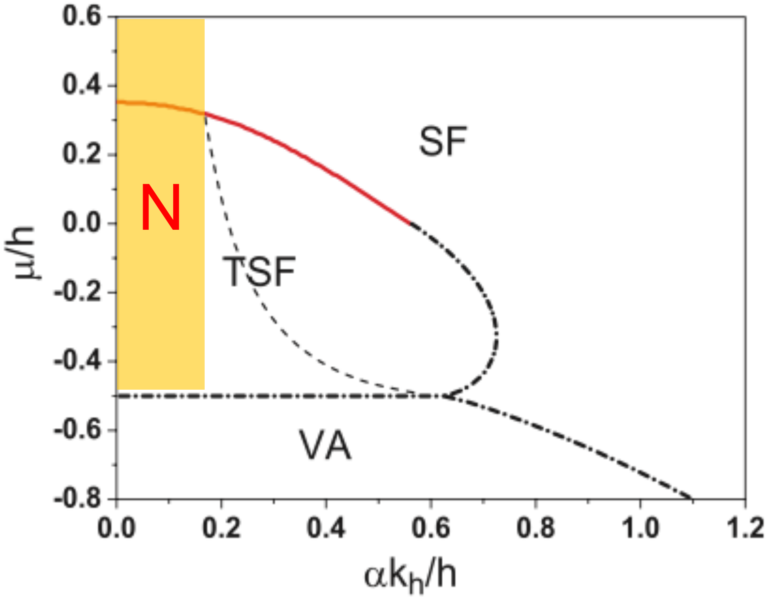


FFLOy

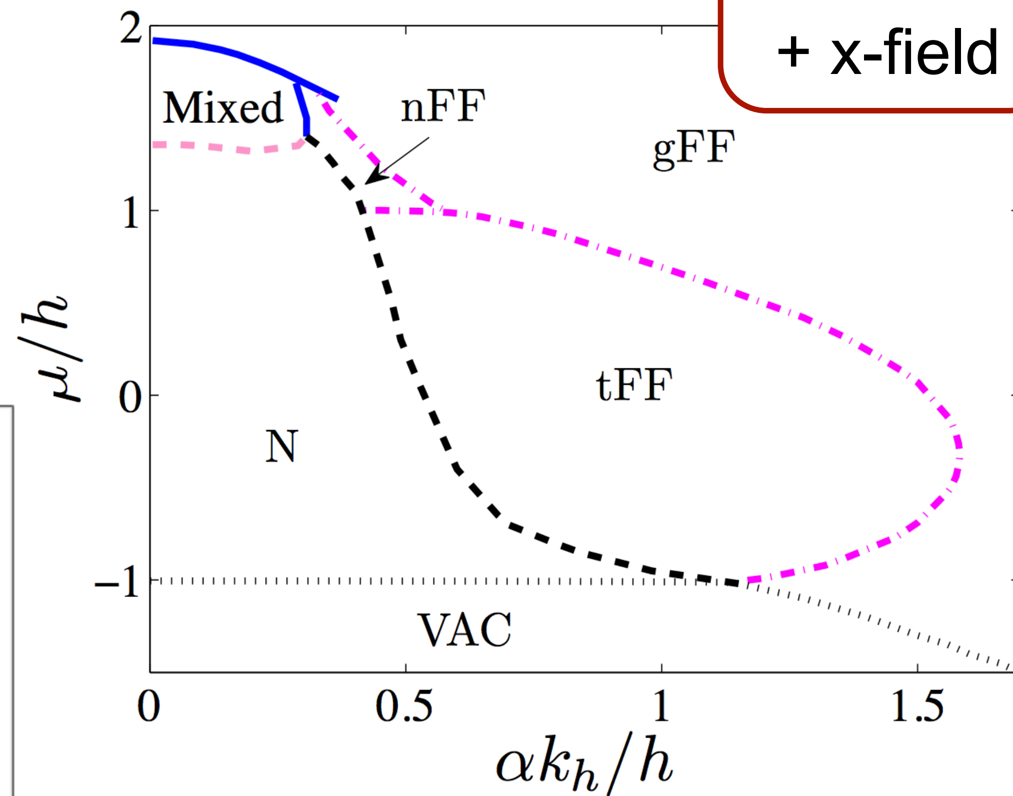


Topological FF state

w/ SOC + z-field



w/ SOC
+ z-field
+ x-field



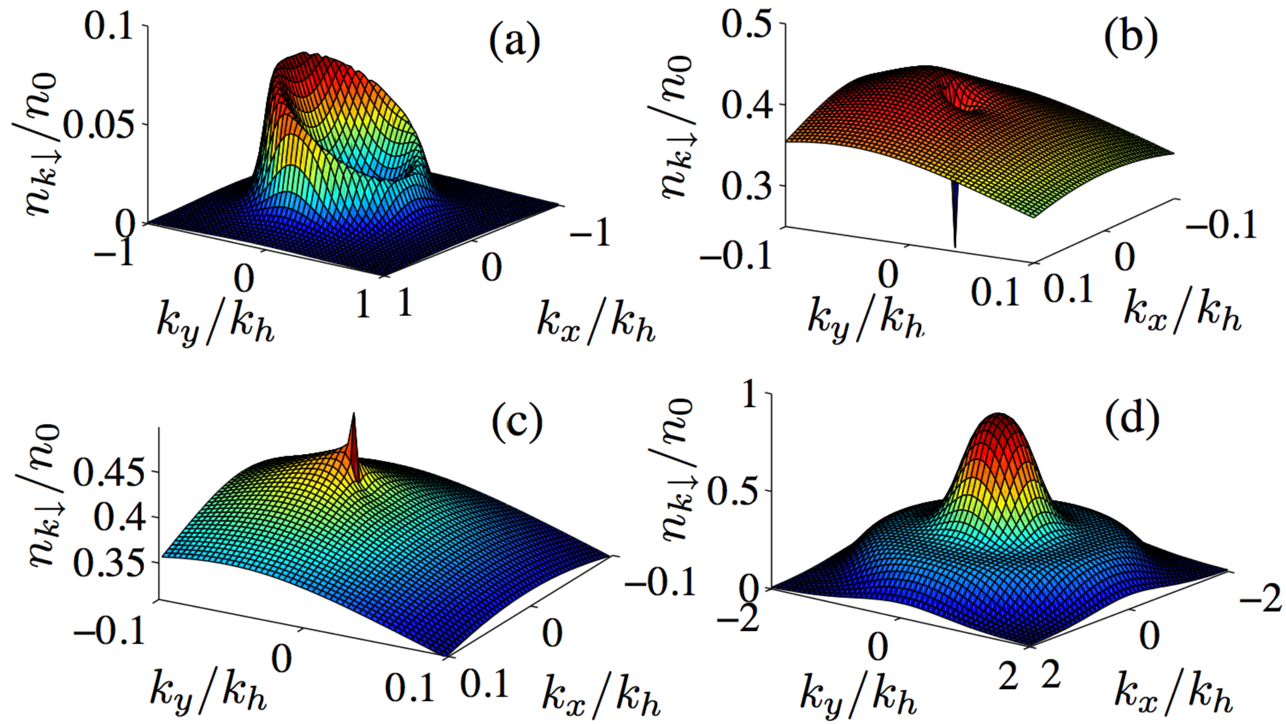
WZ and Yi, Nat. Comm. 4, 2711 (2013)

Detection of tFF

- Momentum distribution of minority fermions

Topologically nontrivial

Topologically trivial

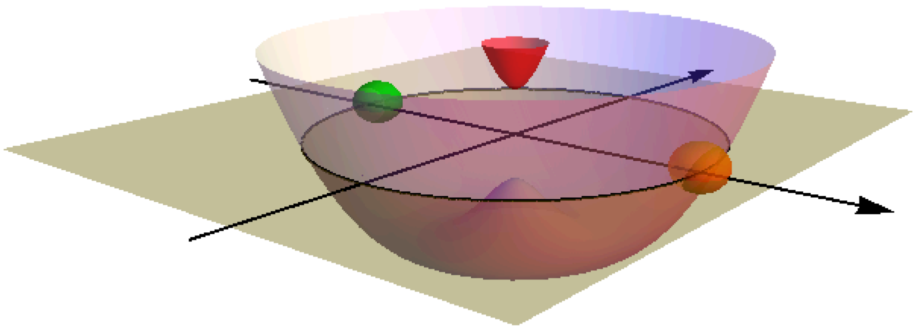


WZ and Yi, Nat. Comm. 4, 2711, (2013)

Summary

- 2D Fermi gas with Rashba SOC

+ z-field

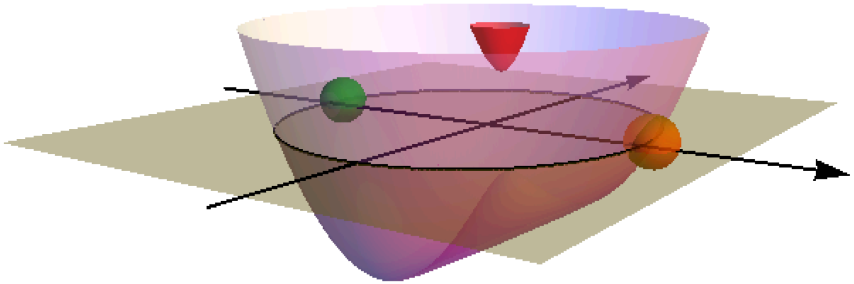


- Topological superfluid
- Pairing instability at large z-field limit

Zhou, **WZ**, Yi, PRA 84 063603 (2011)

Yi and **WZ**, PRL 109, 140402 (2012)

+ z-field + x-field



- Breakdown of BCS pairing
- Topological FFLO state

WZ and Yi, Nat. Comm. 4, 2711, (2013)

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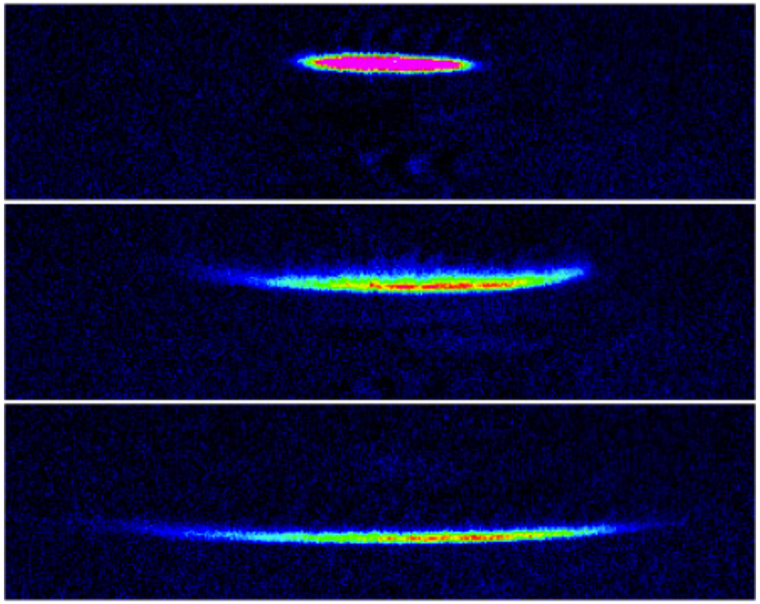
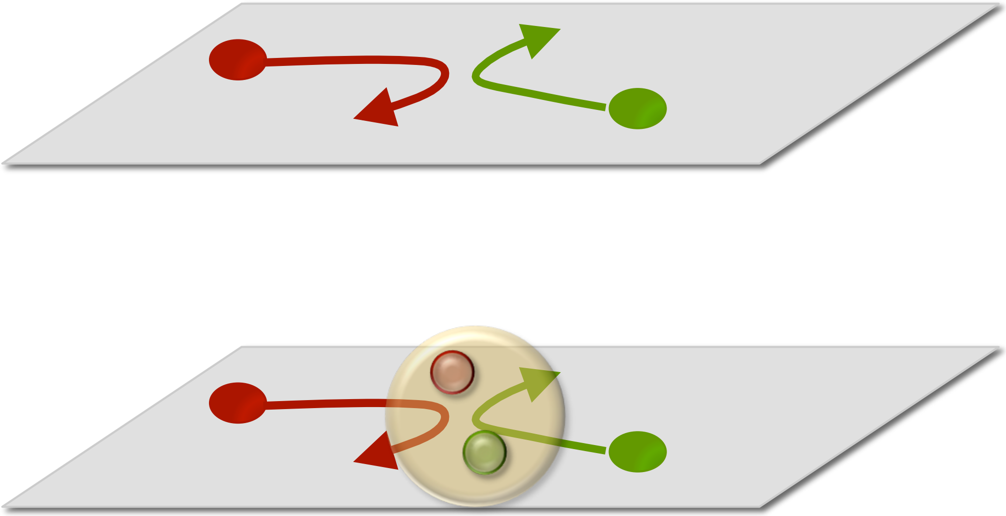
A typical 2D system in cold atoms





LPL, Paris

- Trapping potential along z ~ $10^4 – 10^5$ Hz
- Trap size ~ 10^{-7} m
- Particle separation in the radial plane ~ 10^{-6} m
- Interatomic potential range ~ 10^{-9} m

Two-body bound state (Q2D)

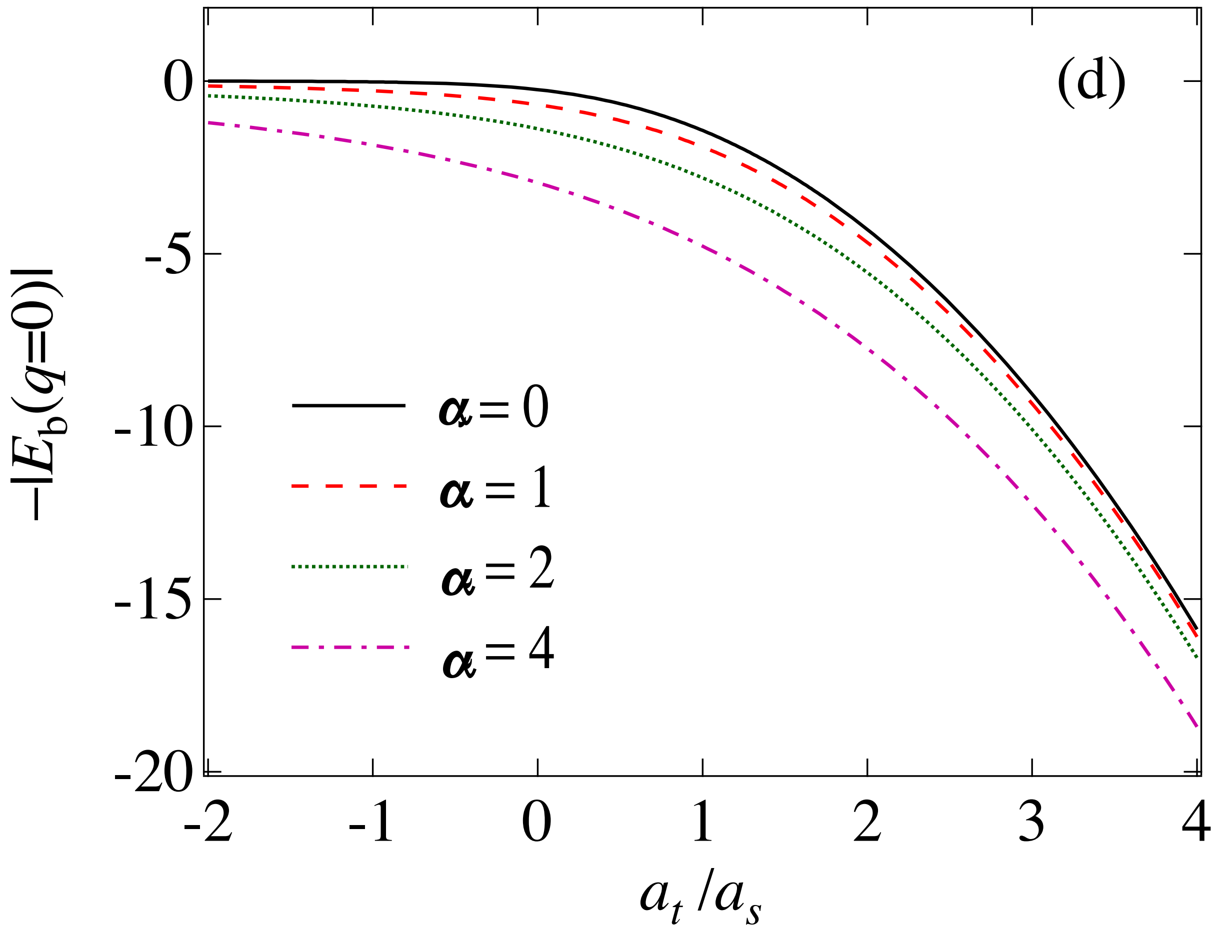
- Two-channel model

$$H = H_0 + H_{\mathrm{soc}} + H_{\mathrm{bf}} + H_{\mathrm{int}}.$$

$$|\Psi\rangle_{\ell,\boldsymbol{q}} = \Big(\beta_{\ell,\boldsymbol{q}} b^\dagger_{\ell,\boldsymbol{q}} + \sum_{m,n,\boldsymbol{k}}{}' \sum_{\sigma,\sigma'} \eta^{\sigma\sigma'}_{m,n,\boldsymbol{k},\boldsymbol{q}} c^\dagger_{m,\boldsymbol{k}+\boldsymbol{q}/2,\sigma} c^\dagger_{n,-\boldsymbol{k}+\boldsymbol{q}/2,\sigma'}\Big)|0\rangle$$

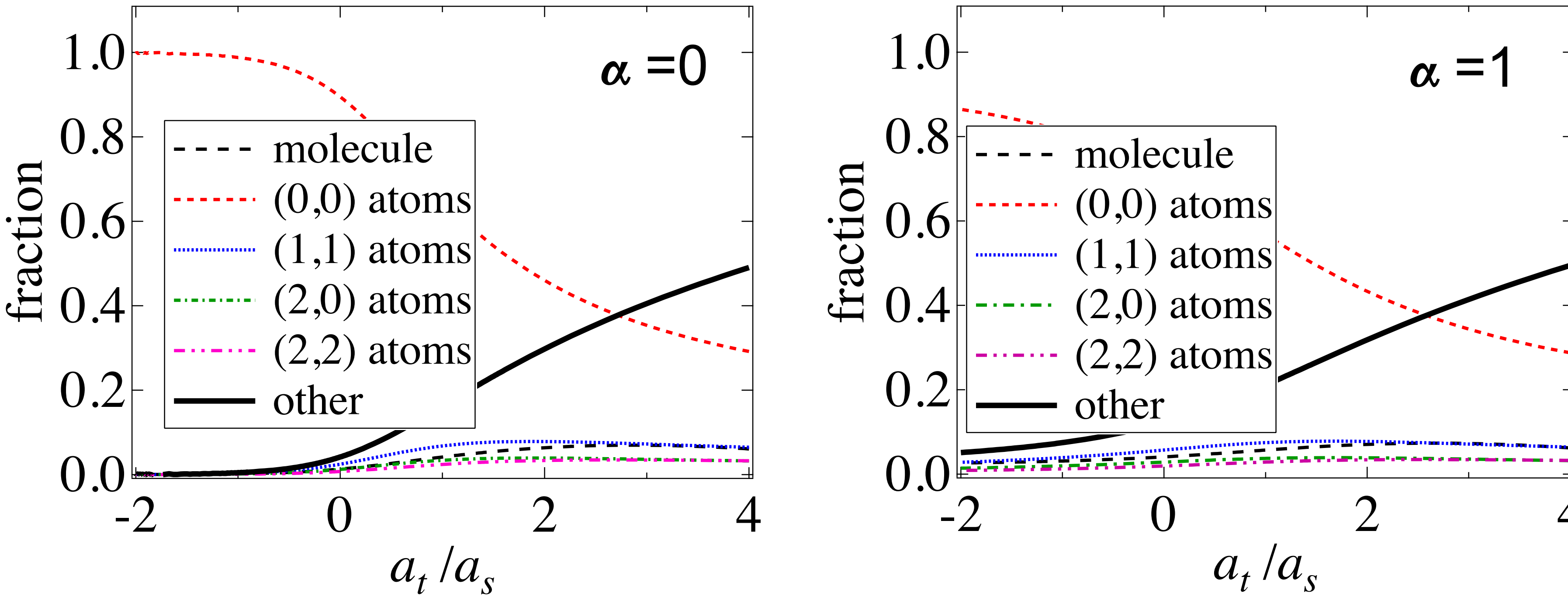
Two-body bound state (Q2D)

- q=0



Zhang, Wu, Tang, Guo, Yi, **WZ**, PRA 87, 033629 (2013)

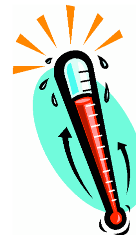
Excitations along the z-direction

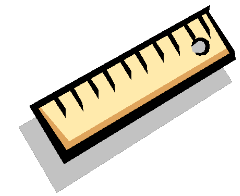


$$|\Psi\rangle_{\ell,\boldsymbol{q}} = \Big(\beta_{\ell,\boldsymbol{q}} b^\dagger_{\ell,\boldsymbol{q}} + \sum_{m,n,\boldsymbol{k}}{}' \sum_{\sigma,\sigma'} \eta^{\sigma\sigma'}_{m,n,\boldsymbol{k},\boldsymbol{q}} c^\dagger_{m,\boldsymbol{k}+\boldsymbol{q}/2,\sigma} c^\dagger_{n,-\boldsymbol{k}+\boldsymbol{q}/2,\sigma'}\Big)|0\rangle$$

- When a Q2D Fermi gas can be looked as 2D?
  - Only on the BCS side.
- What if we introduce Rashba SOC?
  - It makes things worse.
  - SOC tends to increase the two-body binding energy.
- Can we still simulate 2D physics with Q2D system?
  - Yes, but need another way.

Separation of energy scales





- DOF in Q2D model:

fermions in ground state n=0	2D Fermi energy	Interatomic distance
fermions in excited states n=1,2,3…	Trapping potential	Trap size
Feshbach molecules	atomic interaction	Interaction range

- Effective 2D Hamiltonian (2-channel model)
  - 2D Fermions
  - dressed molecules (structureless bosons)

Effective 2D Hamiltonian

$$H_{\text{eff}} = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \delta_b d^\dagger_0 d_0 + \frac{\alpha_b}{L} \sum_{\mathbf{k}} \left(d^\dagger_0 a_{\mathbf{k},\uparrow} a_{-\mathbf{k},\downarrow} + \text{H.C.} \right)$$
$$+ \frac{V_b}{L^2} \sum_{\mathbf{k},\mathbf{k}'} a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} a_{-\mathbf{k}',\downarrow} a_{\mathbf{k}',\uparrow} + \gamma' \sum_{\mathbf{k}} \left[(k_x - i k_y) a^\dagger_{\mathbf{k},\uparrow} a_{\mathbf{k},\downarrow} + (k_x + i k_y) a^\dagger_{\mathbf{k},\downarrow} a_{\mathbf{k},\uparrow} \right]$$

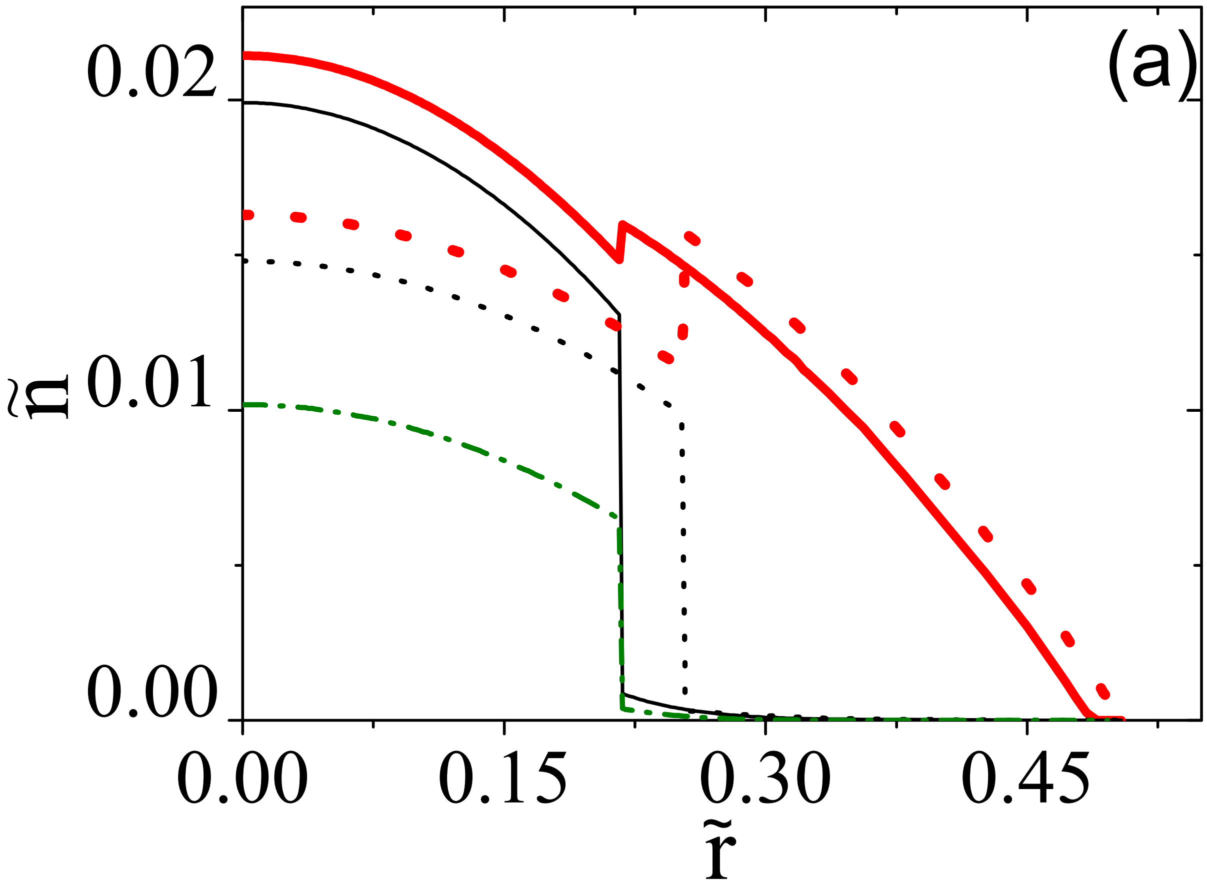
- Single-particle physics
  - Single particle dispersion
- Two-particle physics
  - background scattering
  - two-body binding energy
  - # of fermions in ground state

singular point of T(x)

first derivative of 1/T(x)
at singular point

$$\Delta \varepsilon = O\left(\frac{\mu - E_b / 2}{\hbar \omega_z} \right)^2$$

Q2D Fermi gas with Rashba SOC



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Two-body scattering state (2D)

Short-range

$$H^{(2\mathrm{D})} = H_0^{(2\mathrm{D})} + V_{2\mathrm{D}}(\rho),$$

$$_{\perp}\langle\boldsymbol{\rho}|\psi_c^{(0)}\rangle = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{\rho}}}{2^{3/2}\pi}|\boldsymbol{\alpha}(\boldsymbol{q},\boldsymbol{k})\rangle_S - \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{\rho}}}{2^{3/2}\pi}|\bar{\boldsymbol{\alpha}}(\boldsymbol{q},-\boldsymbol{k})\rangle_S.$$

$$_{\perp}\langle\boldsymbol{\rho}|\psi_c^{(+)}\rangle \approx {}_{\perp}\langle\boldsymbol{\rho}|\psi_c^{(0)}\rangle + A(c)\,{}_{\perp}\langle\boldsymbol{\rho}|g(\varepsilon_c)|\mathbf{0}\rangle_{\perp}|0,0\rangle_S,$$

$$f^{(2\mathrm{D})}(c' \leftarrow c) = -2\pi^2\langle\psi_{c'}^{(0)}|\mathbf{0}\rangle_{\perp}|00\rangle_S A(c).$$

Two-body scattering state (2D)

$$A(c) = \frac{(2\pi)_S\langle 00|_\perp \langle \mathbf{0}|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln\left(d\sqrt{\varepsilon_c}/2\right) - (2\pi)\lambda\left(\varepsilon_c, \boldsymbol{q}\right)}.$$

- Scattering amplitude is ***q***-dependent
- Qualitative change of behavior at low-energy limit

w/o SOC

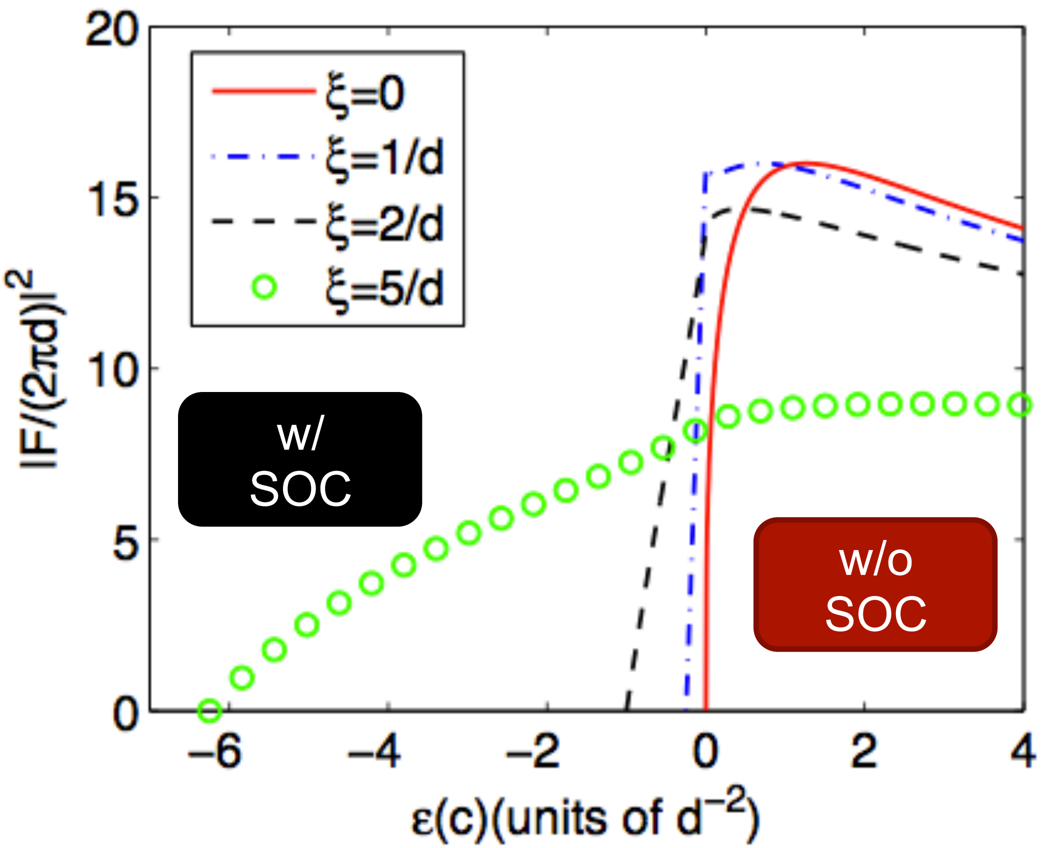
$$\lim_{\varepsilon\to 0} f_0^{(2\mathrm{D})} \propto \frac{1}{\ln \varepsilon_c}.$$

w/ SOC

$$\lim_{\varepsilon_c\to\varepsilon_{\mathrm{thre}}(q)} f^{(2\mathrm{D})} \propto \sqrt{\varepsilon_c - \varepsilon_{\mathrm{thre}}(q)}.$$

Two-body scattering state (2D)

$$F \equiv \frac{f^{(2\mathrm{D})}(c' \leftarrow c)}{\langle \psi_{c'}^{(0)} | \mathbf{0} \rangle_\perp |00\rangle_S \langle 00|_\perp \langle \mathbf{0} | \psi_c^{(+)} \rangle}$$



Zhang, Zhang, **WZ**,
PRA 86 042707 (2012)

Two-body scattering state (Q2D)

$$H = H_0^{(2\mathrm{D})} + H_z + V_{3\mathrm{D}}(r).$$

Short-range

$$H_z = -\frac{\partial^2}{\partial z^2} + \frac{\omega^2 z^2}{4} - \frac{\omega}{2}$$

$$A_{\mathrm{eff}}(c) = \frac{(2\pi)_S\langle 00|_\perp\langle \mathbf{0}|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln\left\{d_{\mathrm{eff}}(\varepsilon_c, \boldsymbol{q})\sqrt{\varepsilon_c}/2\right\} - (2\pi)\lambda(\varepsilon_c, \boldsymbol{q})}.$$

2D

$$A(c) = \frac{(2\pi)_S\langle 00|_\perp\langle \mathbf{0}|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln\left(d\sqrt{\varepsilon_c}/2\right) - (2\pi)\lambda(\varepsilon_c, \boldsymbol{q})}.$$

Effective Hamiltonian

- Around threshold

$$\hat{V}_o = \frac{1}{\mathcal{S}} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''}{}' g(\boldsymbol{k}+\boldsymbol{k}') a^\dagger_{\boldsymbol{k},\uparrow} a^\dagger_{\boldsymbol{k}',\downarrow} a_{\boldsymbol{k}'+\boldsymbol{k}'',\downarrow} a_{\boldsymbol{k}'-\boldsymbol{k}'',\uparrow}.$$

$$\ln k_c - \frac{1}{2\pi g(\boldsymbol{q})} = -C - \ln\left[\frac{d_{\text{eff}}\left(-\frac{\xi^2}{4}, \boldsymbol{q}\right)}{2}\right]$$

***q*-dependent**

Effective Hamiltonian

- Around 2-body binding energy

$$\hat{V}_b = \hat{V}_o + \sum_{\boldsymbol{q}} \left(\frac{q^2}{4} + v(\boldsymbol{q}) \right) b_{\boldsymbol{q}}^\dagger b_{\boldsymbol{q}} + \frac{1}{\sqrt{S}} \sum_{\boldsymbol{k},\boldsymbol{k}'} {}' u(\boldsymbol{k}+\boldsymbol{k}') a_{\boldsymbol{k},\uparrow}^\dagger a_{\boldsymbol{k}',\downarrow}^\dagger b_{\boldsymbol{k}+\boldsymbol{k}'} + h.c.$$

***q*-dependent**

A contact interaction is valid provided that…

- The interaction is short-range
- The SOC intensity is weak
- The effective contact interaction strength is ***q***-dependent