Few-Body Problems in Spin-Orbit Coupled Cold Atom Systems

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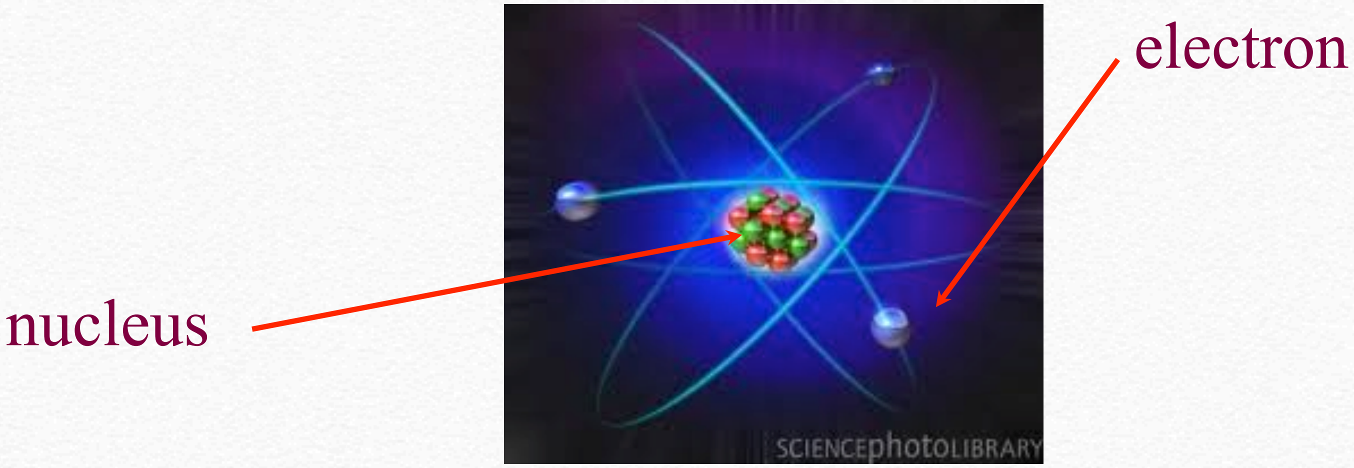
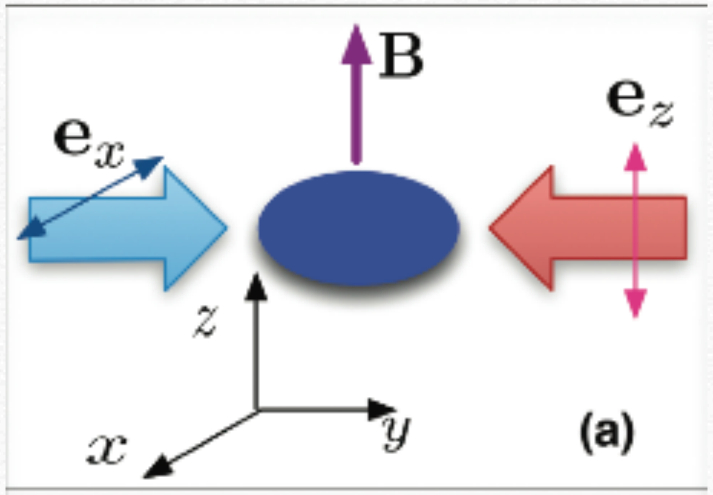
Workshop “ Few-body Universality in Atomic and Nuclear Physics: Recent Experimental and Theoretical Advances”
Institute for Nuclear Theory
University of Washington
May 2014

Spin-Orbit Coupling

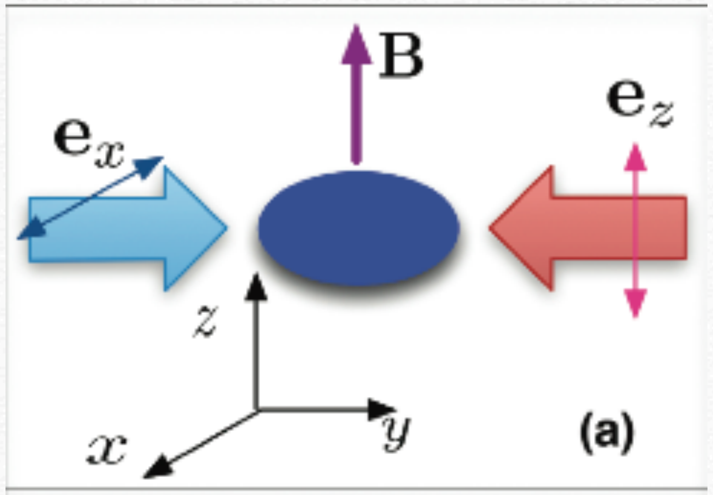


$$\vec{L} \cdot \vec{S}$$

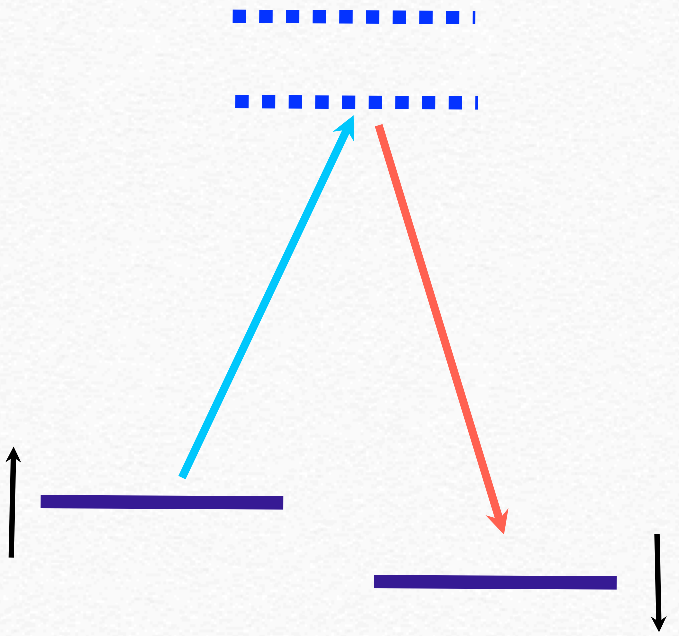


Two-photon Raman process

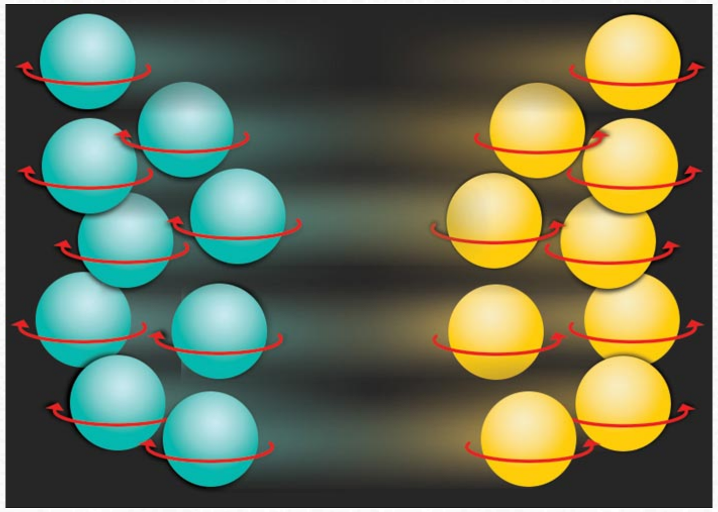
Raman-Coupling induced Spin-Orbit Coupling

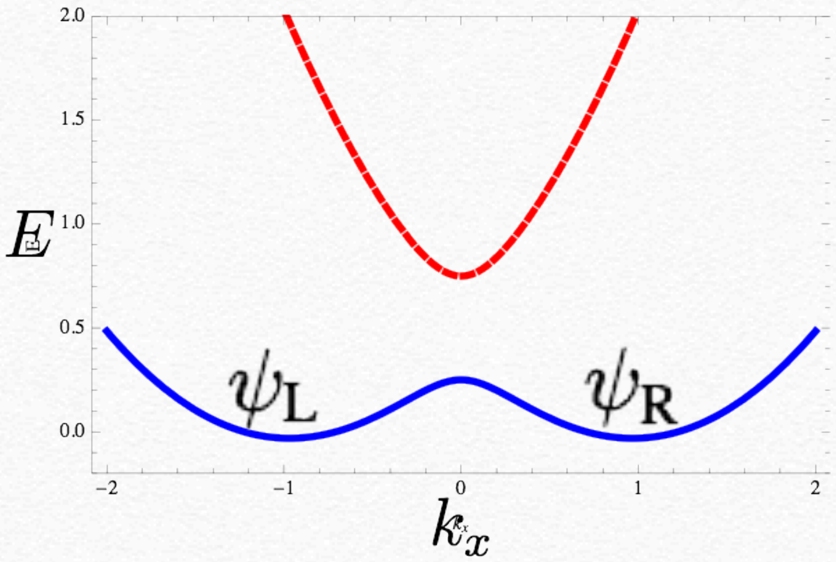


Two-photon Raman process



$$\hat{H}_0 = \frac{k_x^2}{2m} + \vec{h}_k \cdot \vec{\sigma} \qquad \vec{h}_k = \left(\frac{\Omega}{2}, 0, -\frac{k_x k_0}{m} + \frac{\delta}{2}\right)$$



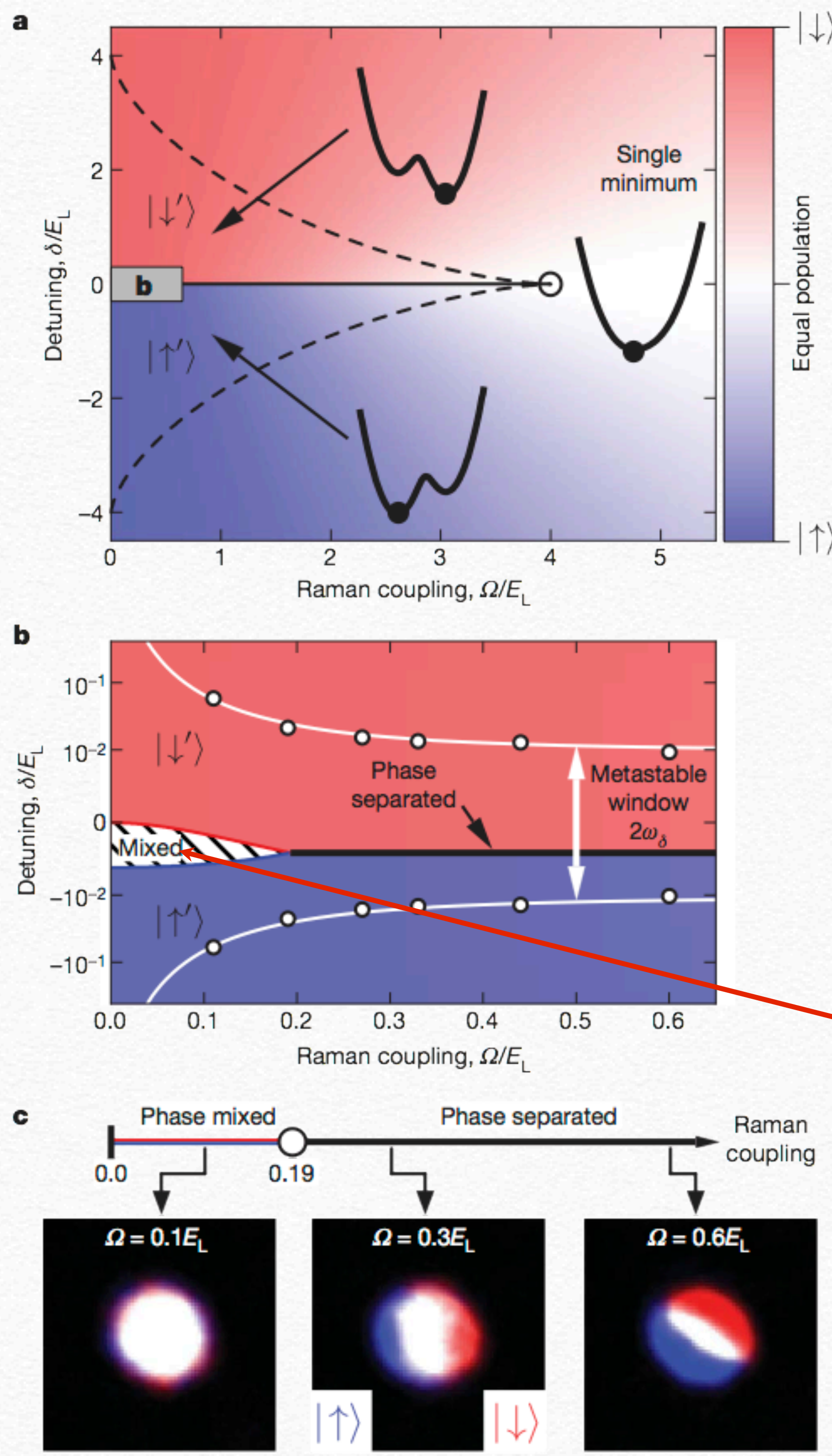


Spielman's group in NIST, 2010

Shanxi, MIT, USTC, Washington State, Purdue

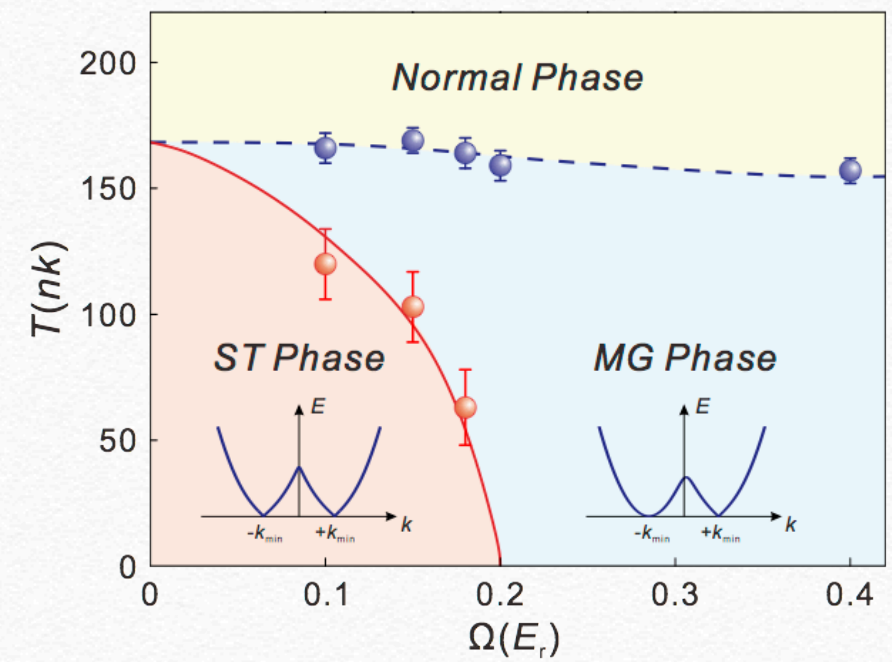
Experimental Progress --- Bosons

Zero Temperature Phase Diagram:

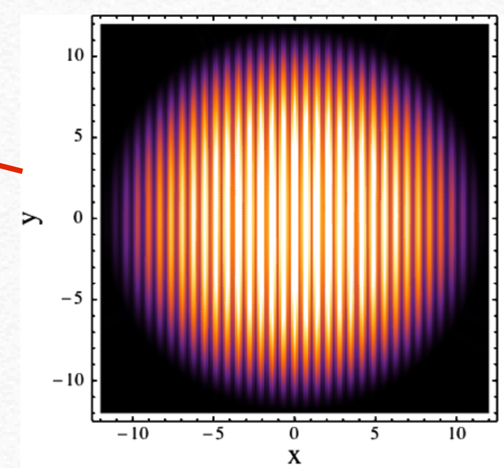


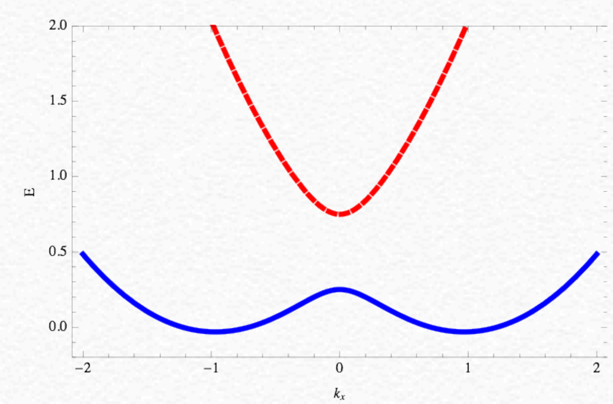
Spielman Nature, 2011

Finite Temperature Phase Diagram:



Si-Cong Ji, Jin-Yi Zhang, Long Zhang, Zhi-Dong Du, Wei Zheng,You-Jin Deng, HZ, Shuai Chen and Jian-Wei Pan, Nature Physics, 10, 314 (2014)





Superfluid with density stripe order

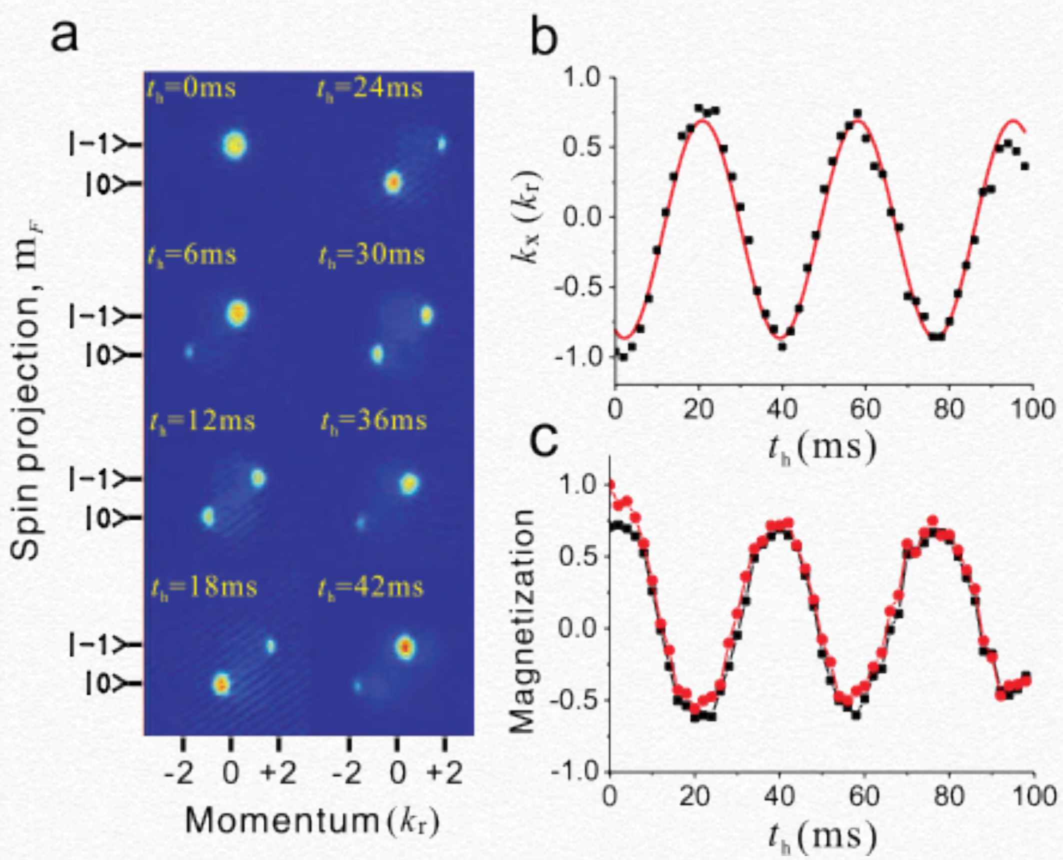
Theory:

C. J. Wang, C. Gao, C. M. Jian and HZ, PRL, (2010)

T. L. Ho and S. Zhang, PRL, (2011)

Experimental Progress --- Dynamics

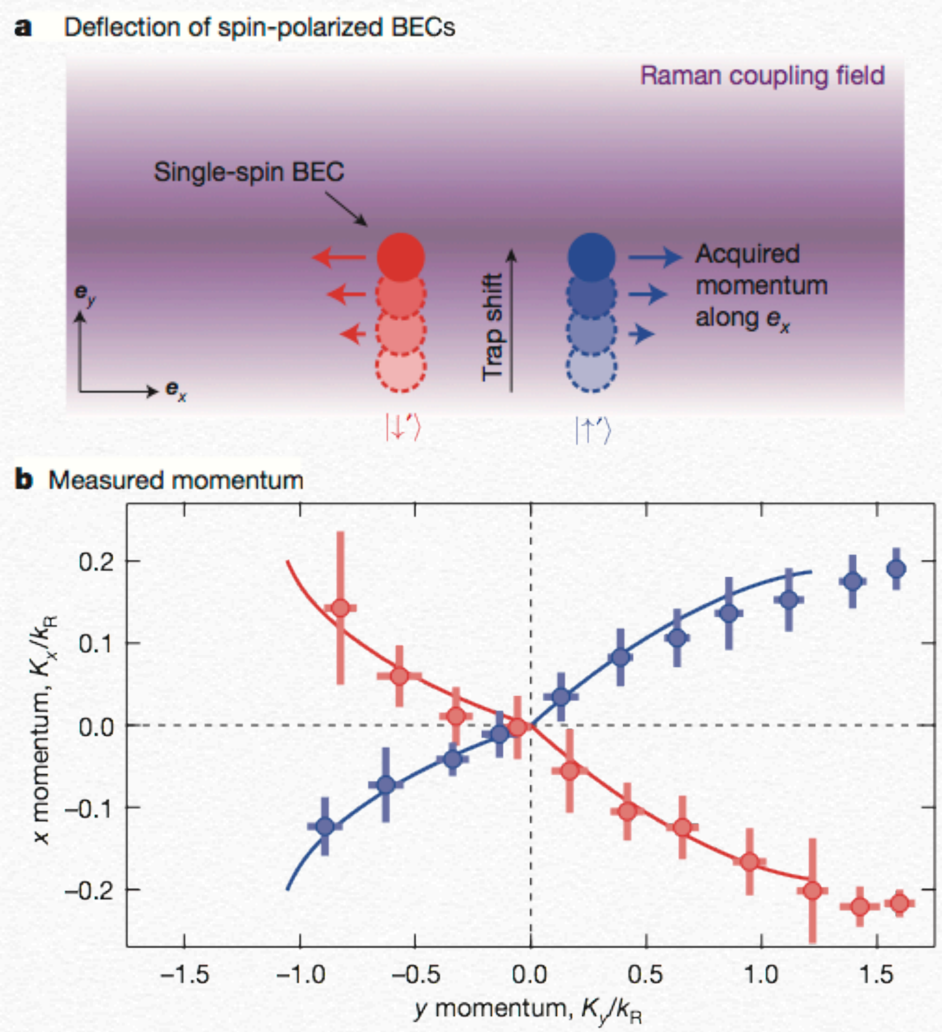
Collective Modes



Jin-Yi Zhang, Si-Cong ji, Zhu Chen, Long Zhang, Zhi-Dong Du, Bo Yan, Ge-Sheng Pan, Bo Zhao, You-Jin Deng, HZ, Shuai Chen, Jian-Wei Pan, Phys. Rev. Lett. 109, 115301 (2012)

See also Spielman Nature Physics, 2011

Classical Spin Hall Effect



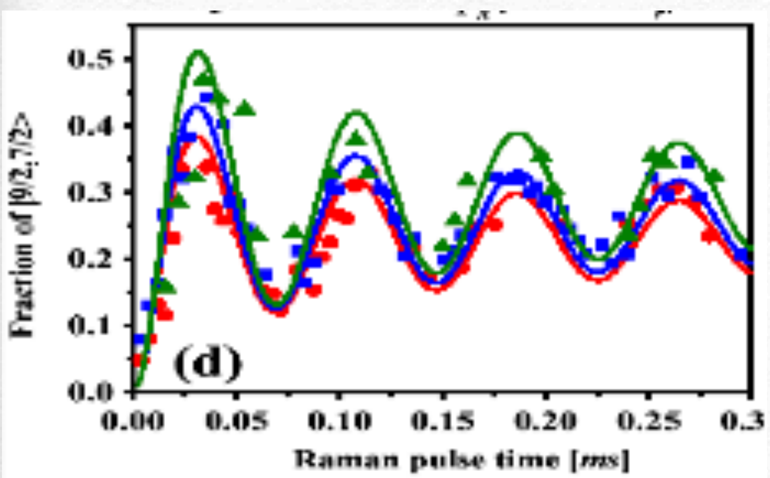
Spielman Nature 2013

Zittwebebewegung: Spielman, NJP, 2013; Engels, PRA, 2013

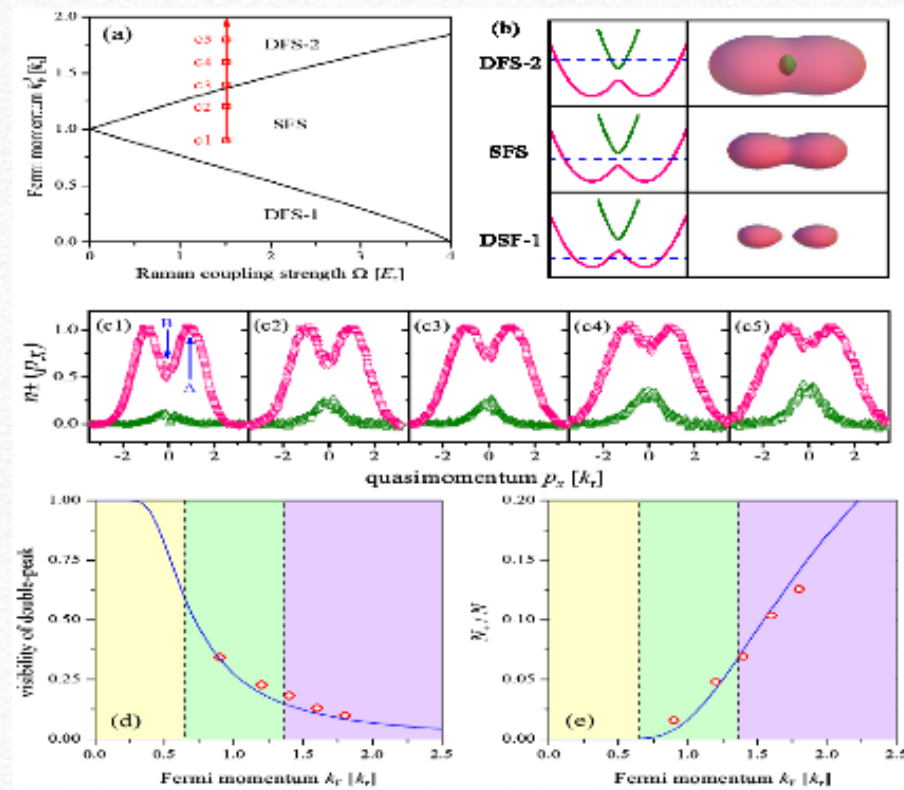
Landau Zener Tunneling: Y. Chen, Purdue, 2013

Experimental Progress --- Fermions

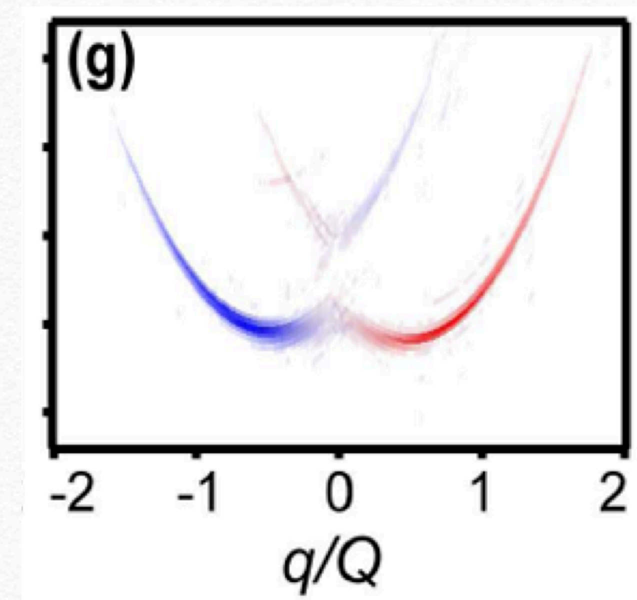
Spin Diffusion



Topological Changes of Fermi Surfaces



Spin-Resolved Imagine of Dispersion

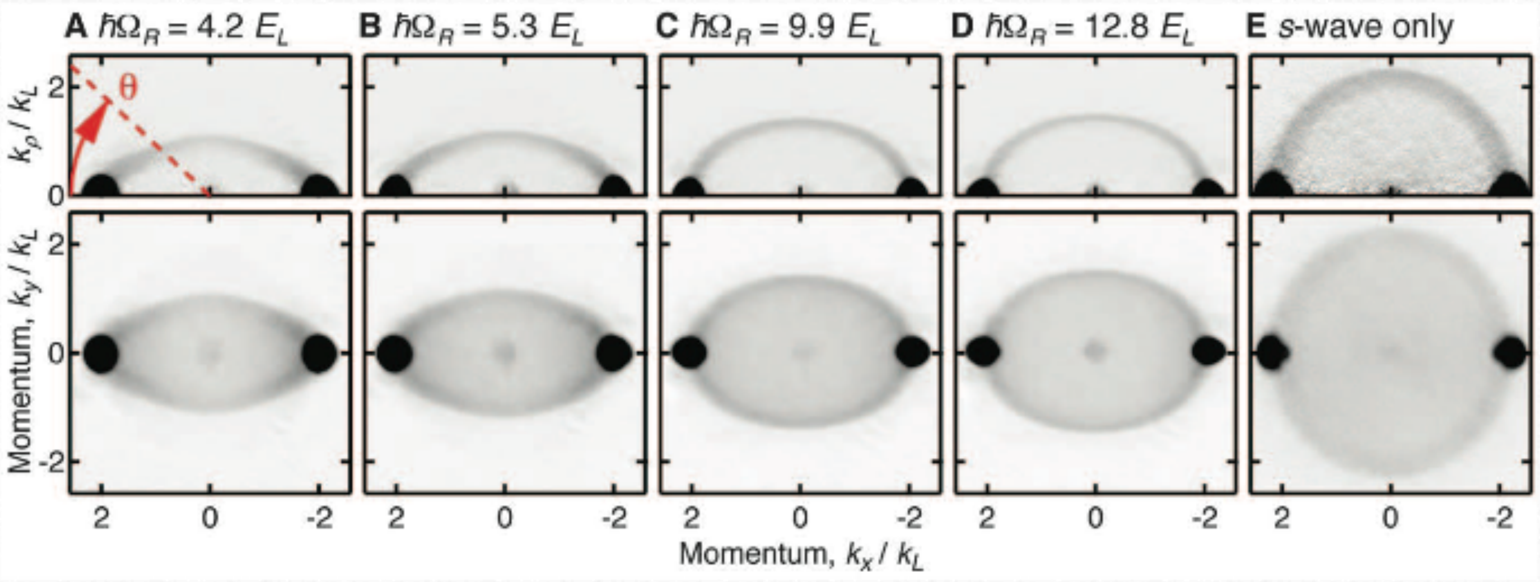


Pengjun Wang, Zeng-Qiang Yu, Zhengkun Fu, Jiao Miao, Lianghui Huang, Shijie Chai, HZ and Jing Zhang, Phys. Rev. Lett. 109, 095301 (2012)

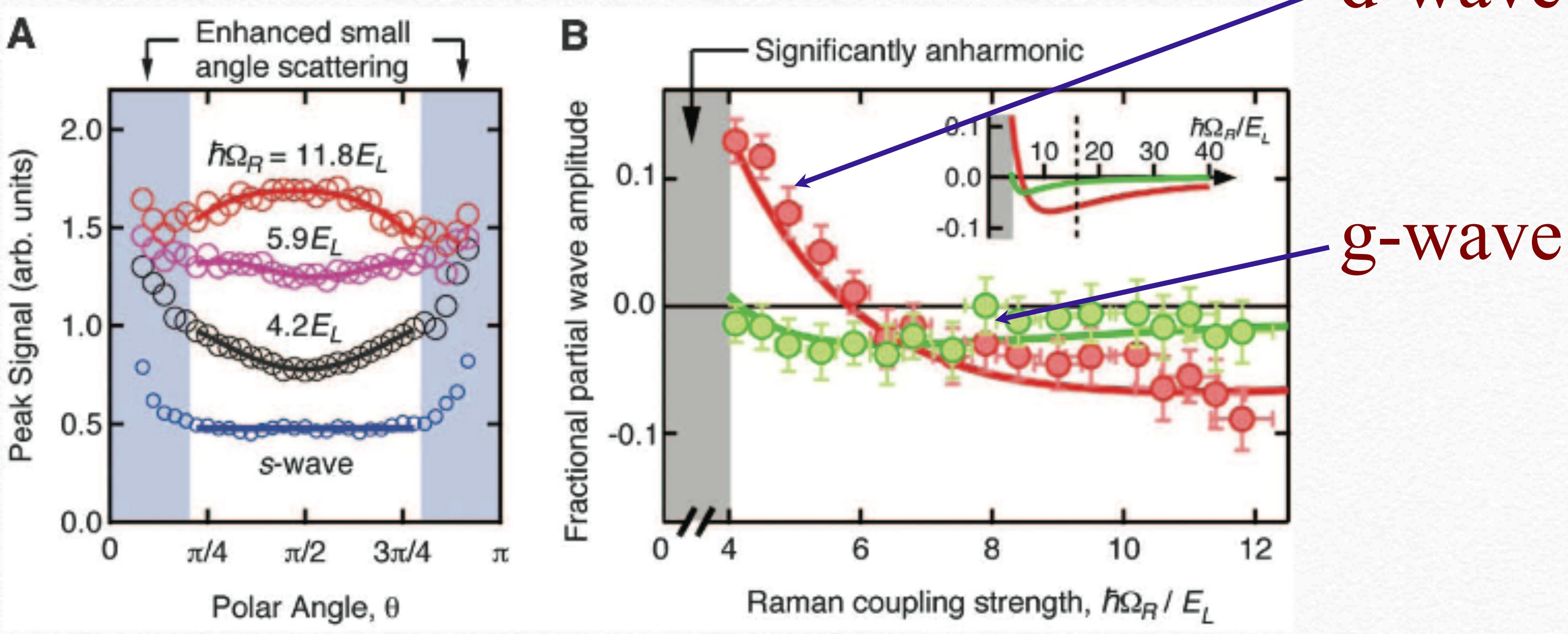
Lawrence W. Cheuk, Ariel T. Sommer, Zoran Hadzibabic, Tarik Yefsah, Waseem S. Bakr, and Martin W. Zwierlein, Phys. Rev. Lett. 109, 095302 (2012)

Experimental Progress --- Two-Body Physics

(1) Spin-Orbit Coupling Mixes Different Partial Waves



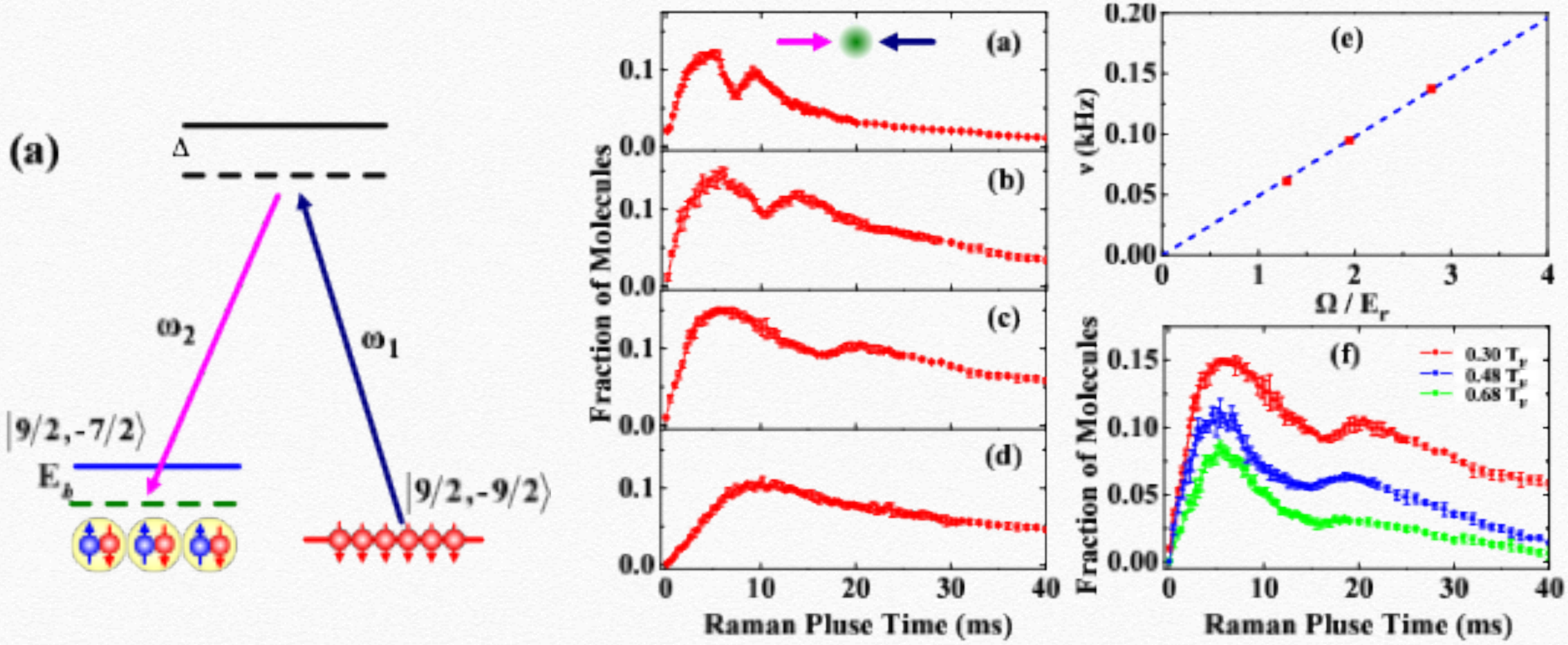
Scattering halos of two Spin-Orbit coupled BECs



Spielman Science, 2012

Experimental Progress --- Two-Body Physics

(2) Spin-Orbit Coupling Mixes Singlet with Triplet

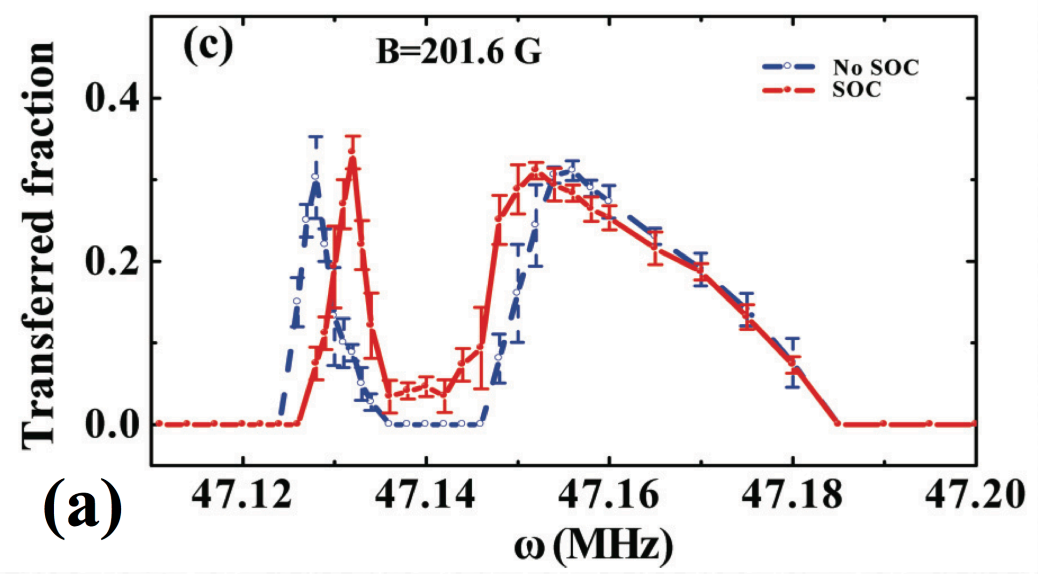


Zhengkun Fu, Lianghui Huang, Zengming Meng, Pengjun Wang, Long Zhang, Shizhong Zhang, HZ, Peng Zhang and Jing Zhang, Nature Physics, 10, 110 (2014)

Experimental Progress --- Two-Body Physics

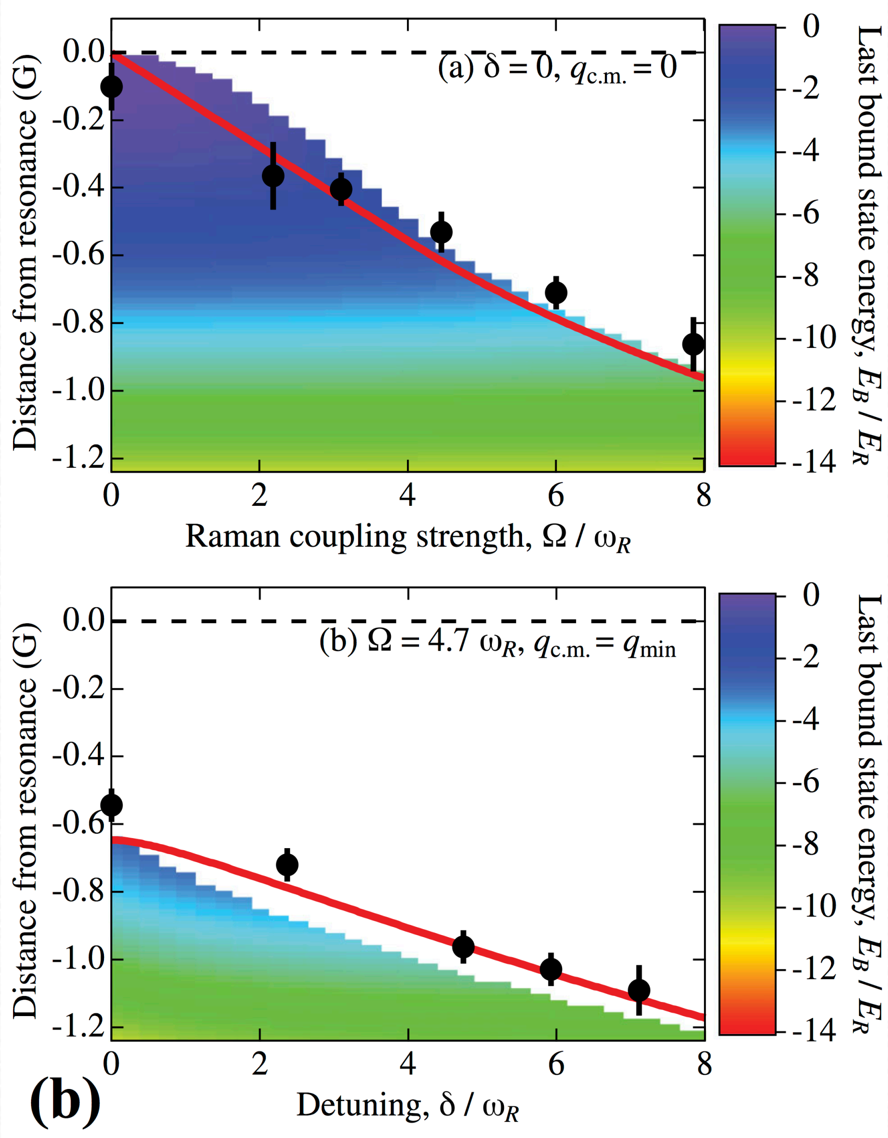
(3) Spin-Orbit Coupling Shifts Molecule Eenergy

Molecule binding energy changes
with spin-orbit coupling



Jing Zhang's group, PRA 2013

Resonance position changes



Spielman's group, PRL, 2013

For a review of the progresses in last 3-4 years, see

arXiv: 1403.8021

Degenerate Quantum Gases with Spin-Orbit Coupling

Hui Zhai*
Institute for Advanced Study, Tsinghua University, Beijing, 100084, China
(Dated: April 1, 2014)

This review focuses on recent developments on studying synthetic spin-orbit (SO) coupling in ultracold atomic gases. Two types of SO coupling are discussed. One is Raman process induced coupling between spin and motion along one of the spatial directions, and the other is Rashba SO coupling. We emphasize their common features in both single-particle and two-body physics and their consequences in many-body physics. For instance, single particle ground state degeneracy leads to novel features of superfluidity and richer phase diagram; increased low-energy density-of-state enhances interaction effects; the absence of Galilean invariance and spin-momentum locking give rise to intriguing behaviors of superfluid critical velocity and novel quantum dynamics; and mixing of two-body singlet and triplet states yields novel fermion pairing structure and topological superfluids. With these examples, we show that investigating SO coupling in cold atom systems can enrich our understanding of basic phenomena such as superfluidity, provide a good platform for simulating condensed matter states such as topological superfluids, and more importantly, result in novel quantum systems such as SO coupled unitary Fermi gas or high spin quantum gases. Finally we also point out major challenges and possible future directions.

Contents

Something not discussed......

Three-body Problem with Spin-Orbit Coupling

Some early works on nuclear physics system:

For instance:

A. N. Mitra, Phys. Rev. 150, 839 (1966).Y.
Matsui, Phys. Rev. C 22, 2591 (1980).
R. B. Wiringa, Phys. Rev. C. 43, 1585 (1991).

Models and Focuses are quite different

Something not discussed......

Three-body Problem with Spin-Orbit Coupling

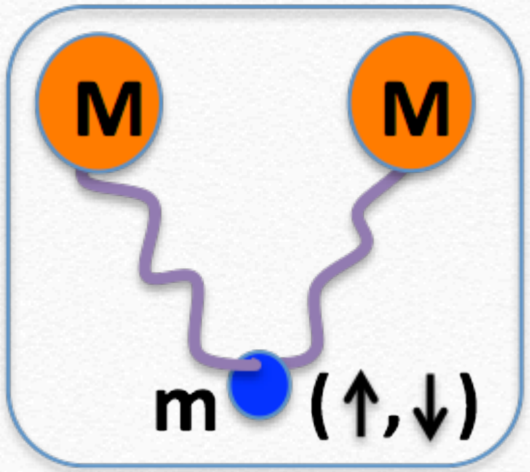
This three-body problem is important for cold atom physics:

1. Basic properties of this system:
how atom-dimer scattering length changes

2. Universal phenomenon:
how spin-orbit coupling changes universal bound states

3. Correlations:
help to understand many-body correlation in presence of spin-orbit coupling

The Three-Body System: Model



Two heavy fermions plus one light spin-1/2 fermion

$$\hat{H}_0 = \frac{\mathbf{p_1}^2}{2M} + \frac{\mathbf{p_2}^2}{2M} + \frac{(\mathbf{p_3} - \lambda\hat{\sigma})^2}{2m}$$

$$\hat{U} = [g\delta(\mathbf{r_1} - \mathbf{r_3}) + g\delta(\mathbf{r_2} - \mathbf{r_3})]\mathbf{I},$$

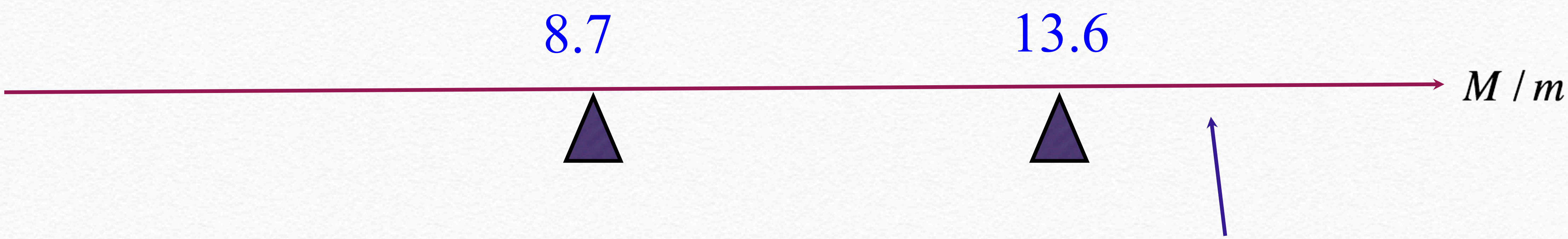
Isotropic SO coupling for light particle

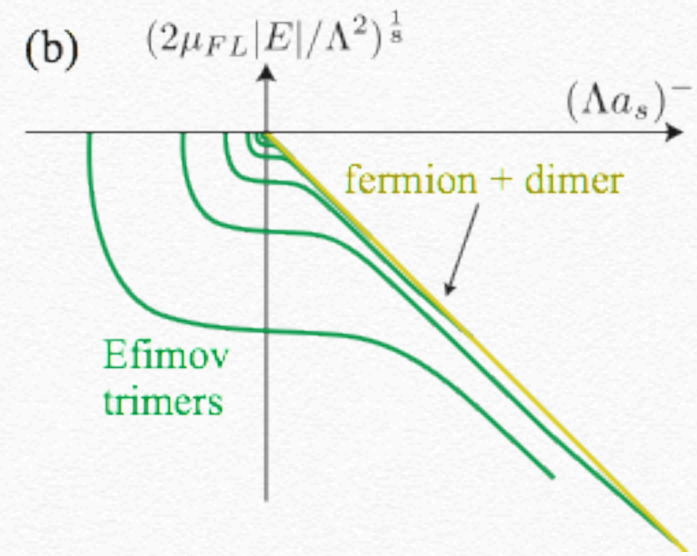
$$(\lambda / m) p \cdot \sigma$$

Ref: Anderson, Spielman, Juzeliunas, PRL, 2013
Anderson, Juzeliunas, Spielman, Galitski, PRL, 2012

Resonant interaction between heavy and light fermions, independent of spin-degree of freedom

Three-body physics without SO coupling





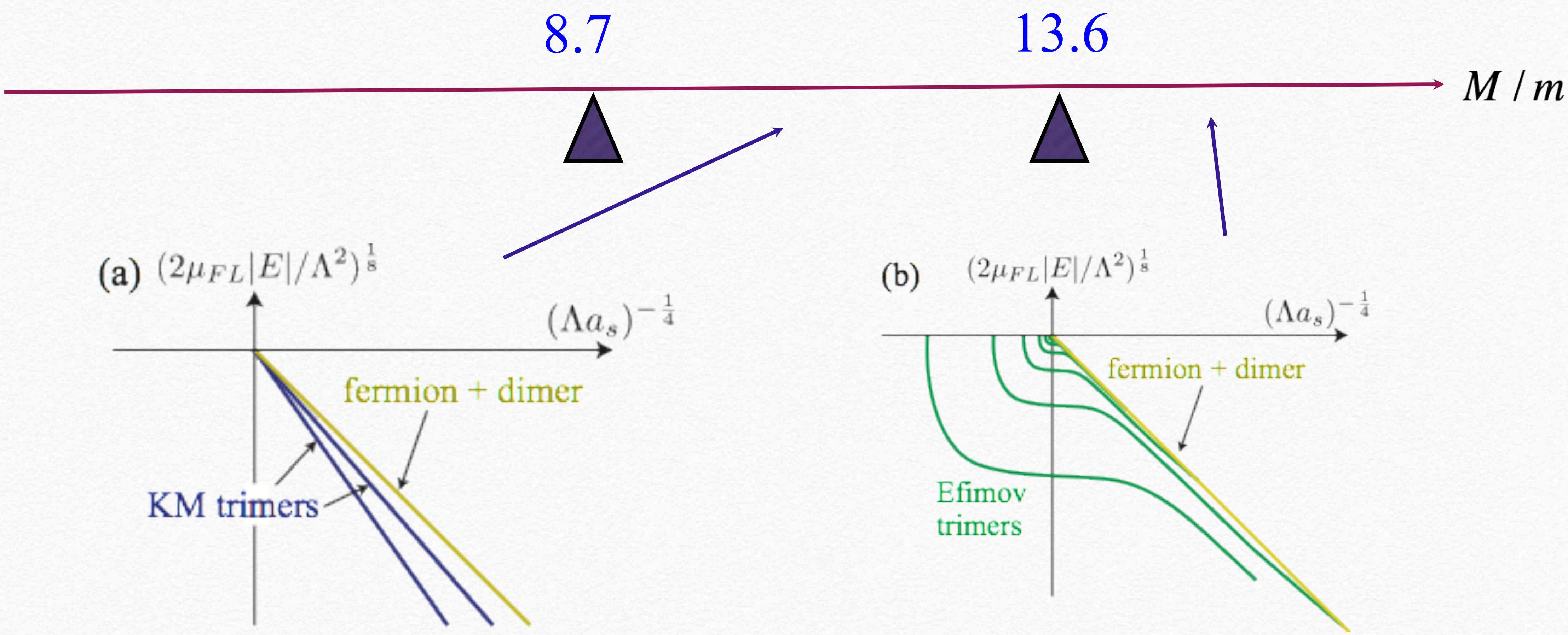
$M/m > 13.6$

Efimov trimer
Universal Scaling

Two-types of Universal Behaviors

S. Endo, P. Naidon & M. Ueda, PRA 86, 062703 (2012)

Three-body physics without SO coupling



$8.17 < M/m < 13.6$

Kartavtsev-Malykh trimer
Universal energy
(independent of high-energy cutoff)

Kartavtsev and Malykh, 2007

$M/m > 13.6$

Efimov trimer
Universal Scaling

Two-types of Universal Behaviors

S. Endo, P. Naidon & M. Ueda, PRA 86, 062703 (2012)

Outline of how to solve the problem

Wave-function assumption:

$$|\Psi\rangle = \sum_{\mathbf{p},\mathbf{q},\sigma} \Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q}) \hat{\alpha}^\dagger_{\mathbf{q}} \hat{\alpha}^\dagger_{\mathbf{K}_0-\mathbf{p}} \hat{\beta}^\dagger_{\sigma,\mathbf{p}-\mathbf{q}} |0\rangle,$$

Auxiliary function:

$$f_\sigma(\mathbf{p}) = g \sum_{\mathbf{q}} \Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q})$$

Three-body Schrodinger equation can be reduced to a self-consistent equation:

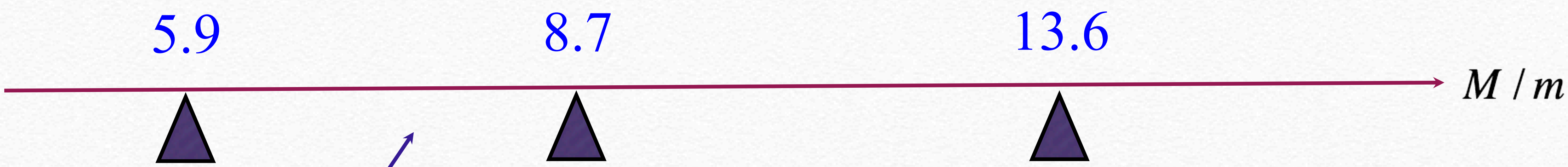
$$f_\sigma(\mathbf{k}) = g \sum_{\mathbf{p},\sigma'} G_{\sigma\sigma'}(\mathbf{p}, \mathbf{K}_0 - \mathbf{k}, \mathbf{k} - \mathbf{p})[f_{\sigma'}(\mathbf{k}) - f_{\sigma'}(\mathbf{K}_0 - \mathbf{p})].$$

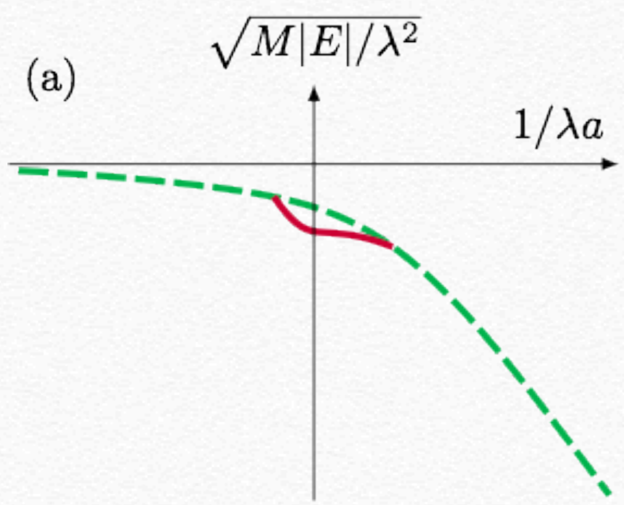
Key: use symmetry to simplify this equation

$$\left.\begin{matrix}\mathbf{L}\\ \mathbf{S}\end{matrix}\right\} \longrightarrow \mathbf{J} = \mathbf{L} + \mathbf{S}$$

$$Z(k)\begin{pmatrix} f_0(k) \\ f_1(k) \end{pmatrix} = \int_0^\infty dp K_j(k,p) \begin{pmatrix} f_0(p) \\ f_1(p) \end{pmatrix}$$

Three-body physics with SO coupling

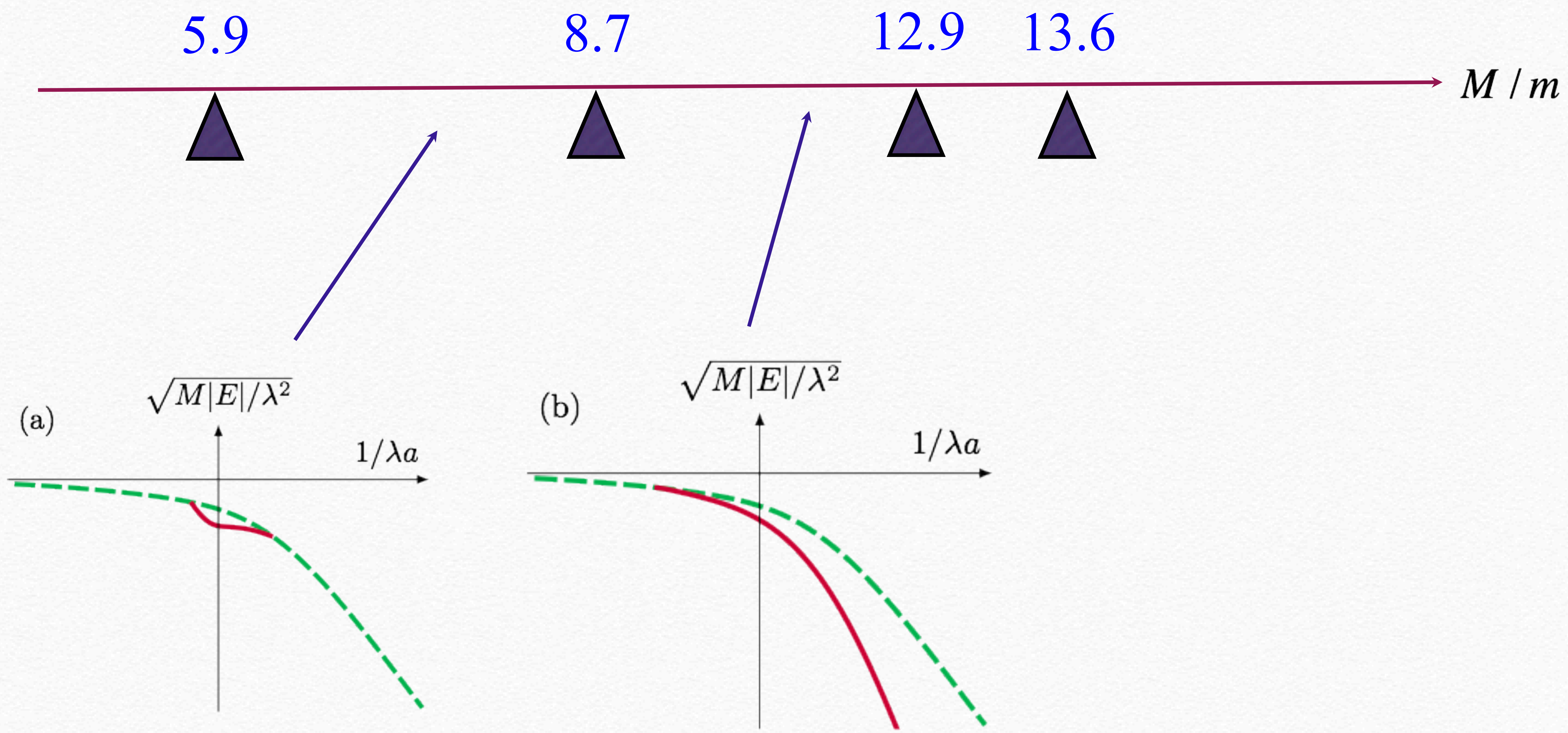




One Universal KM trimer
around unitary regime

Lower bound 5.9 can be satisfied by Li-K mixture

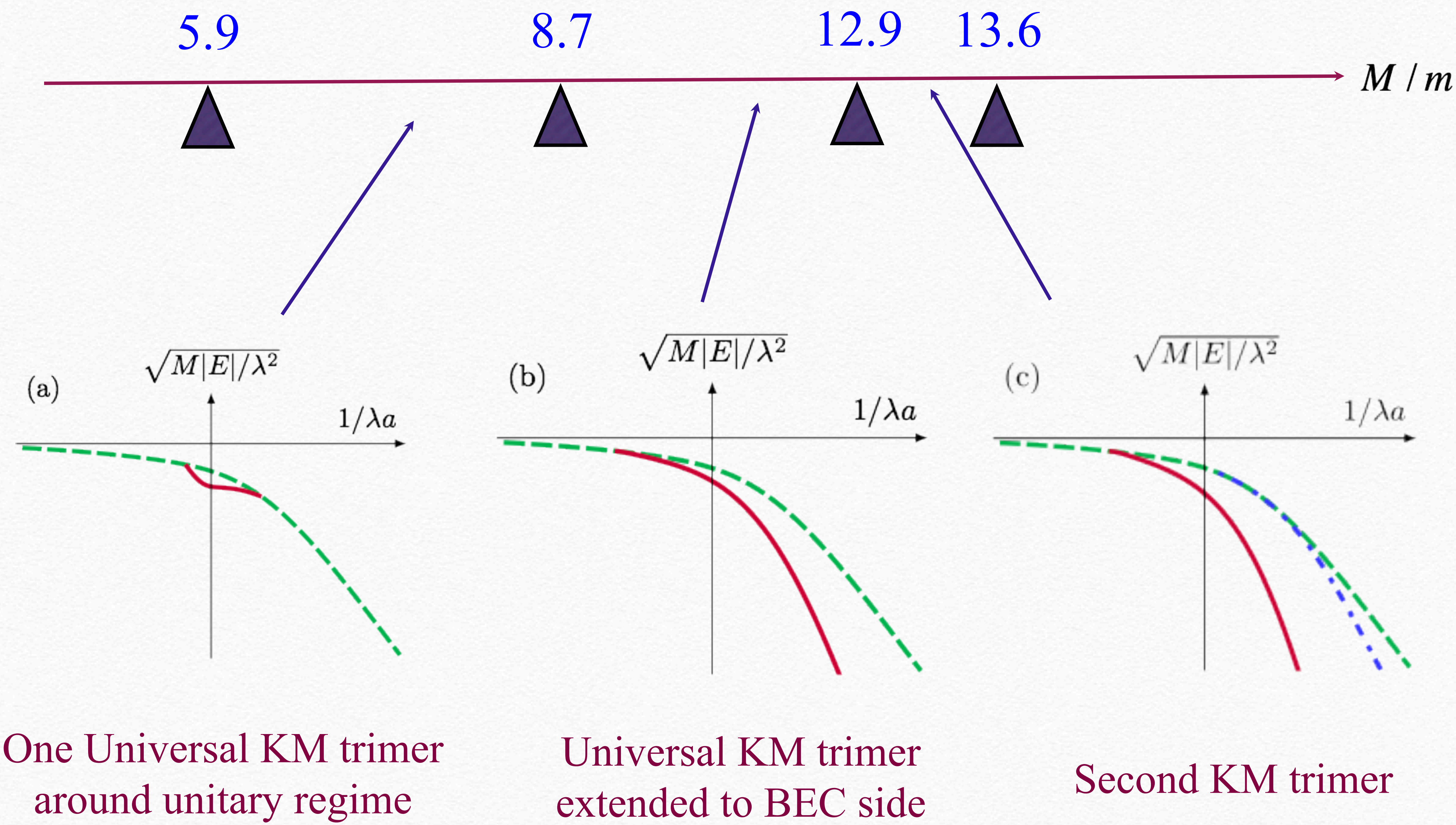
Three-body physics with SO coupling



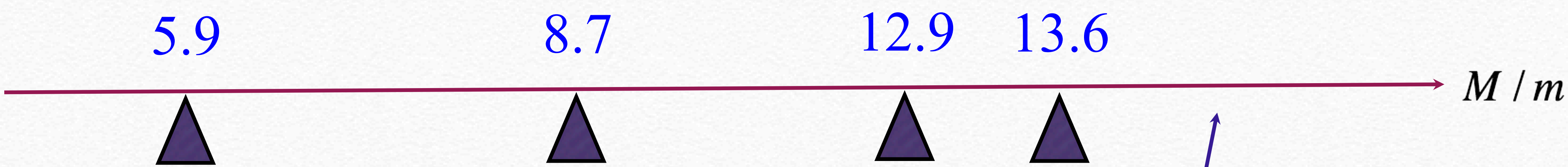
One Universal KM trimer
around unitary regime

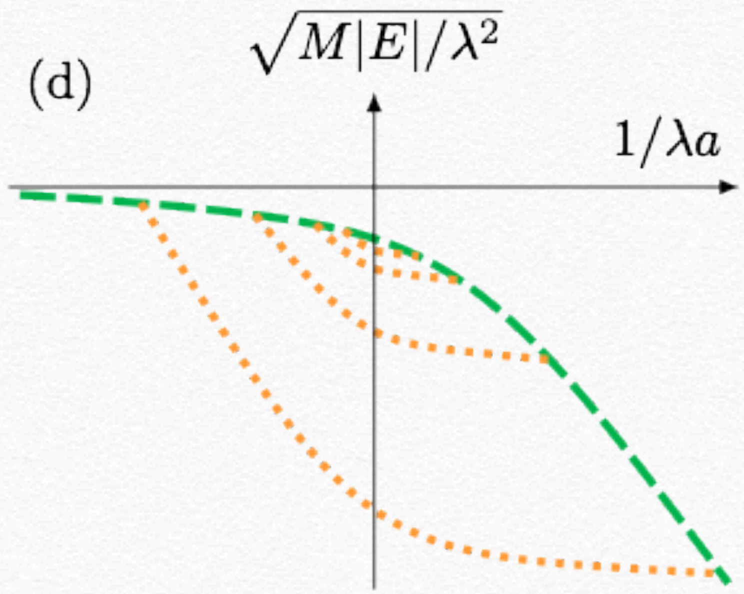
Universal KM trimer
extended to BEC side

Three-body physics with SO coupling



Three-body physics with SO coupling

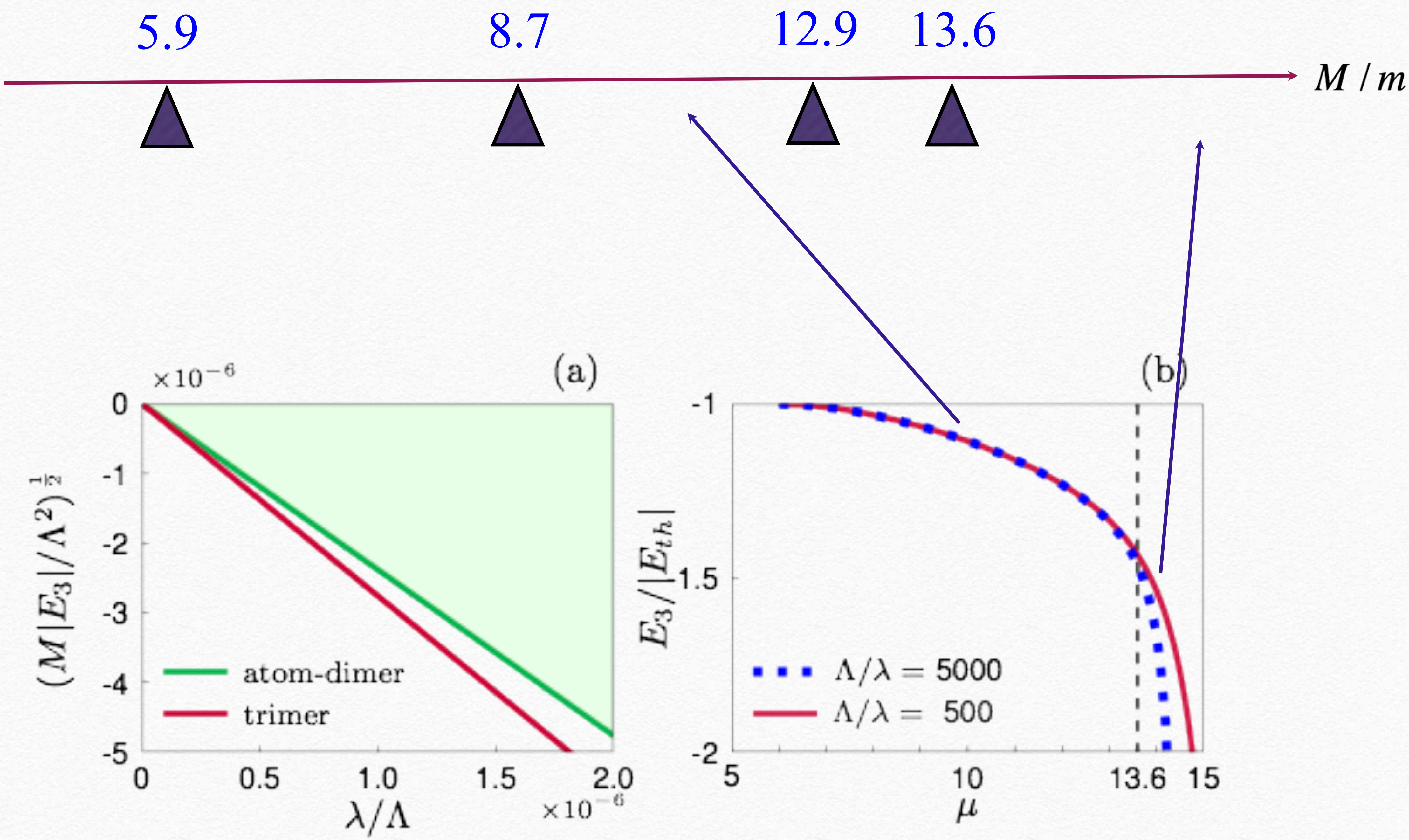




“ Efimov trimer ”

No Universal Scaling

Three-body physics with SO coupling



Trimer are indeed universal for mass ratio < 13.6

What do we learn from this calculation ?

How general are our results ?

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❒ SO coupling introduces an extra length scale to the problem, which destroys Efimov universal scaling.

Efimov trimer with Universal Scaling → **with SO coupling** → Broken

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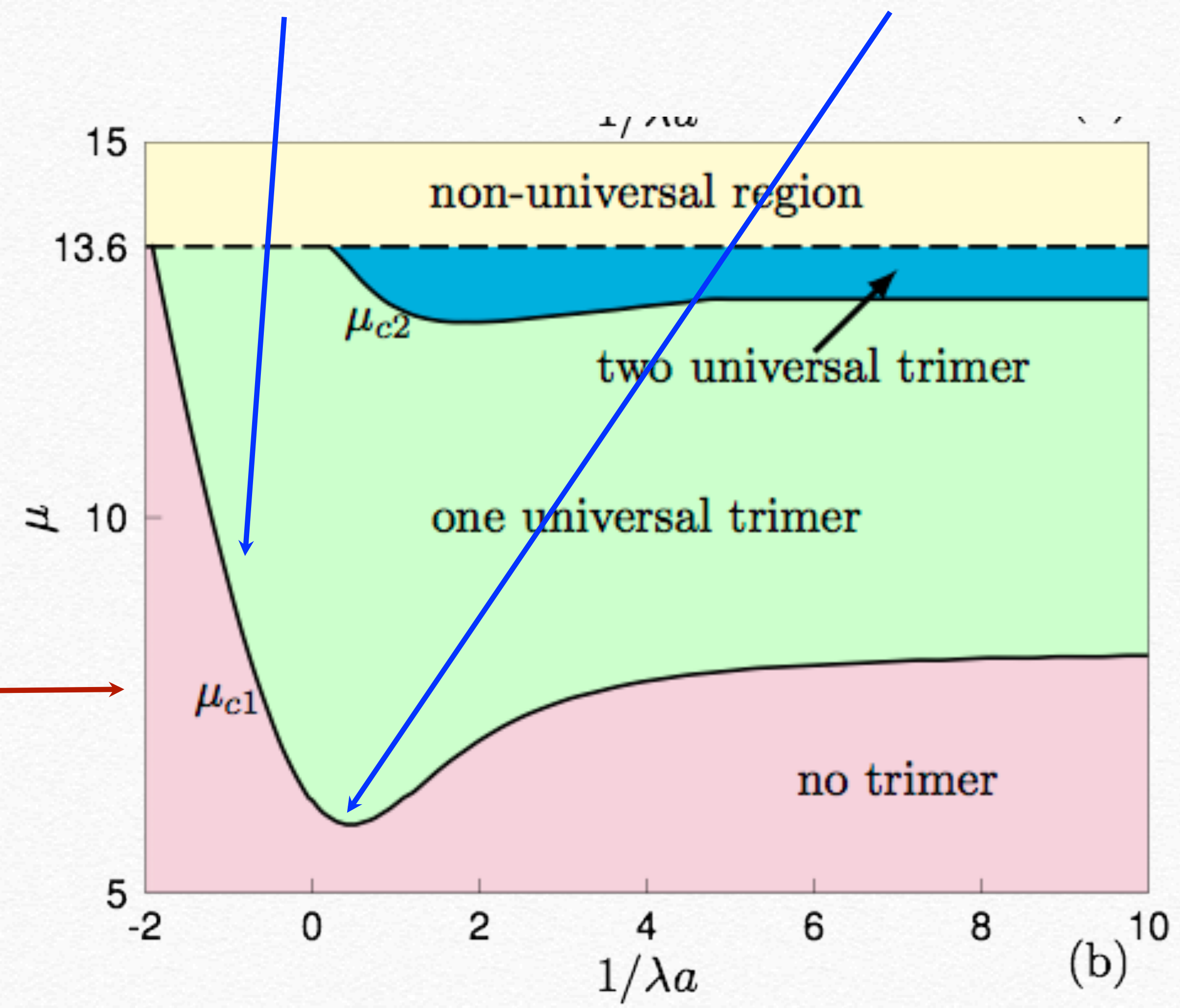
- ❒ SO coupling enhances KM trimer

KM trimer with Universal Energy → **with SO coupling** → Enhanced

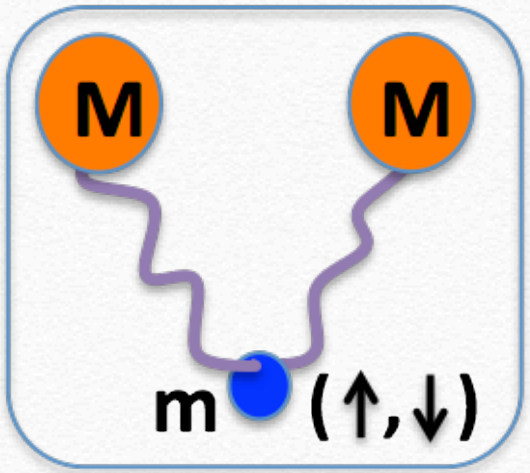
“ Phase diagram ” for three-body problem

Universal trimer is induced by SO coupling
(i) at BCS side also; (ii) for smaller mass ratio

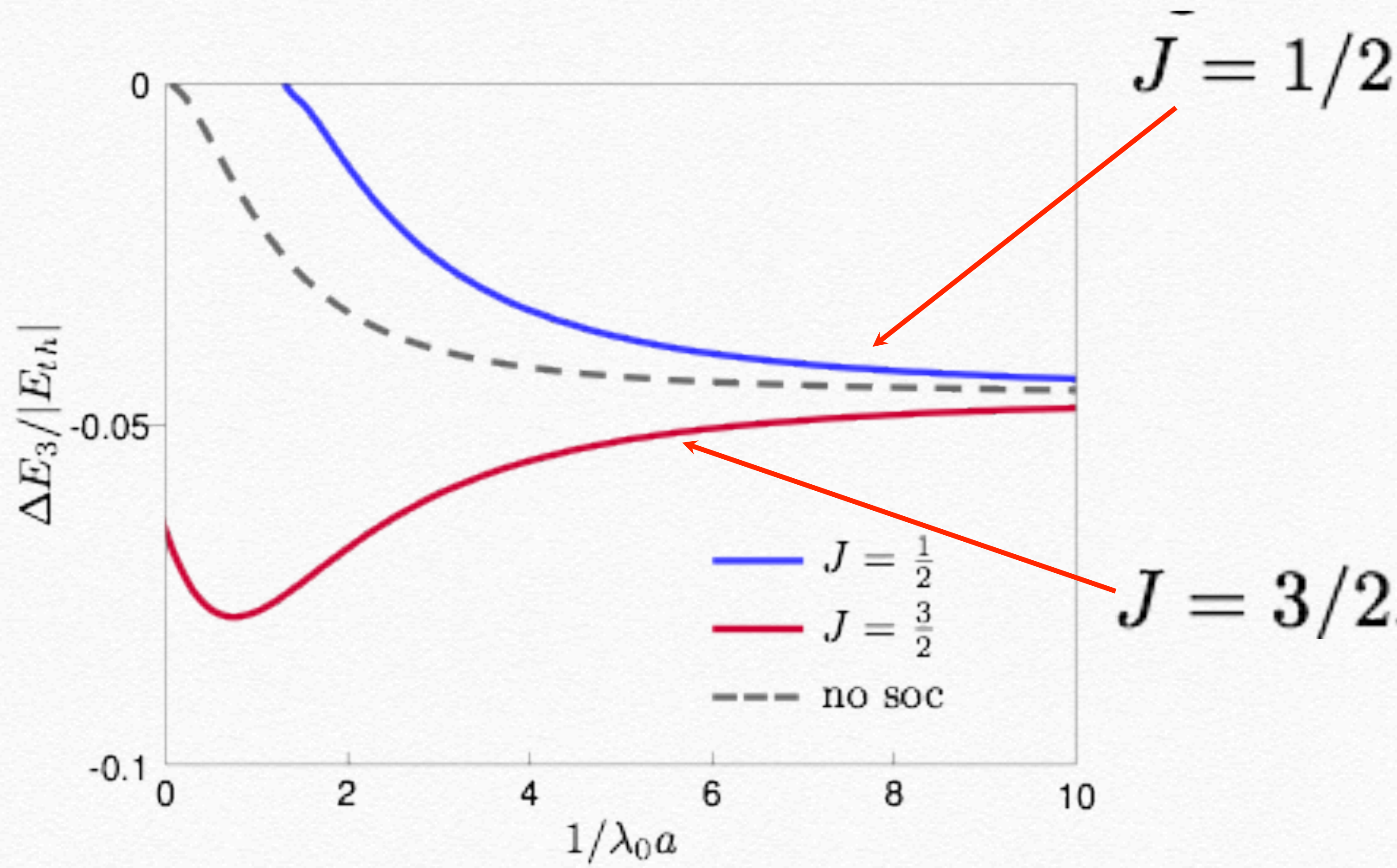
8.7



Physical Understanding



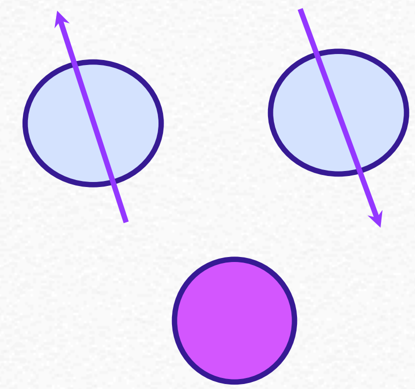
$$L = 1$$
$$S = 1/2$$



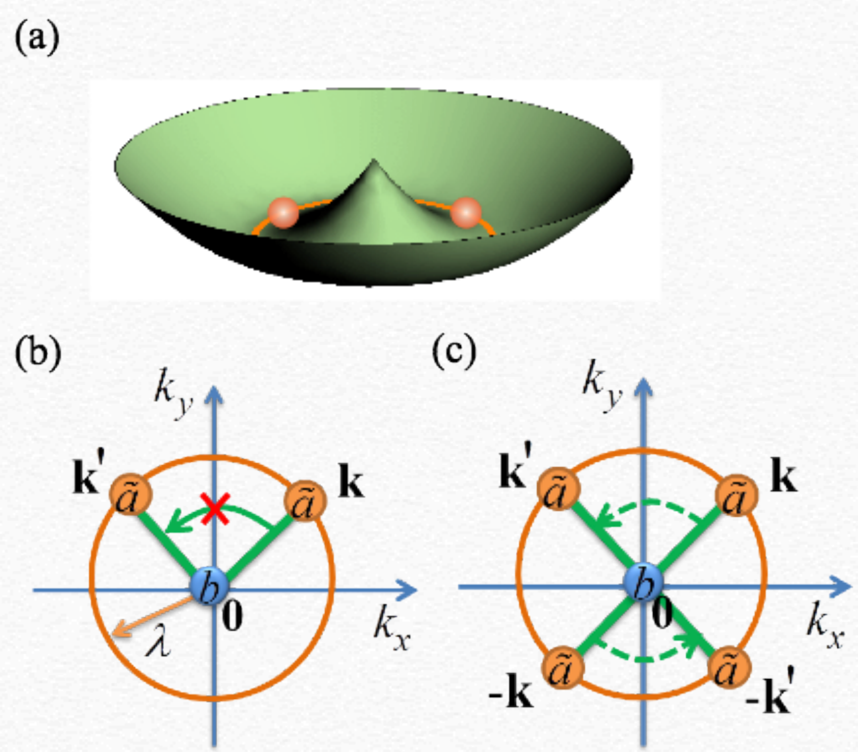
Spin-orbit coupling couples L and S,
lower energies of certain states

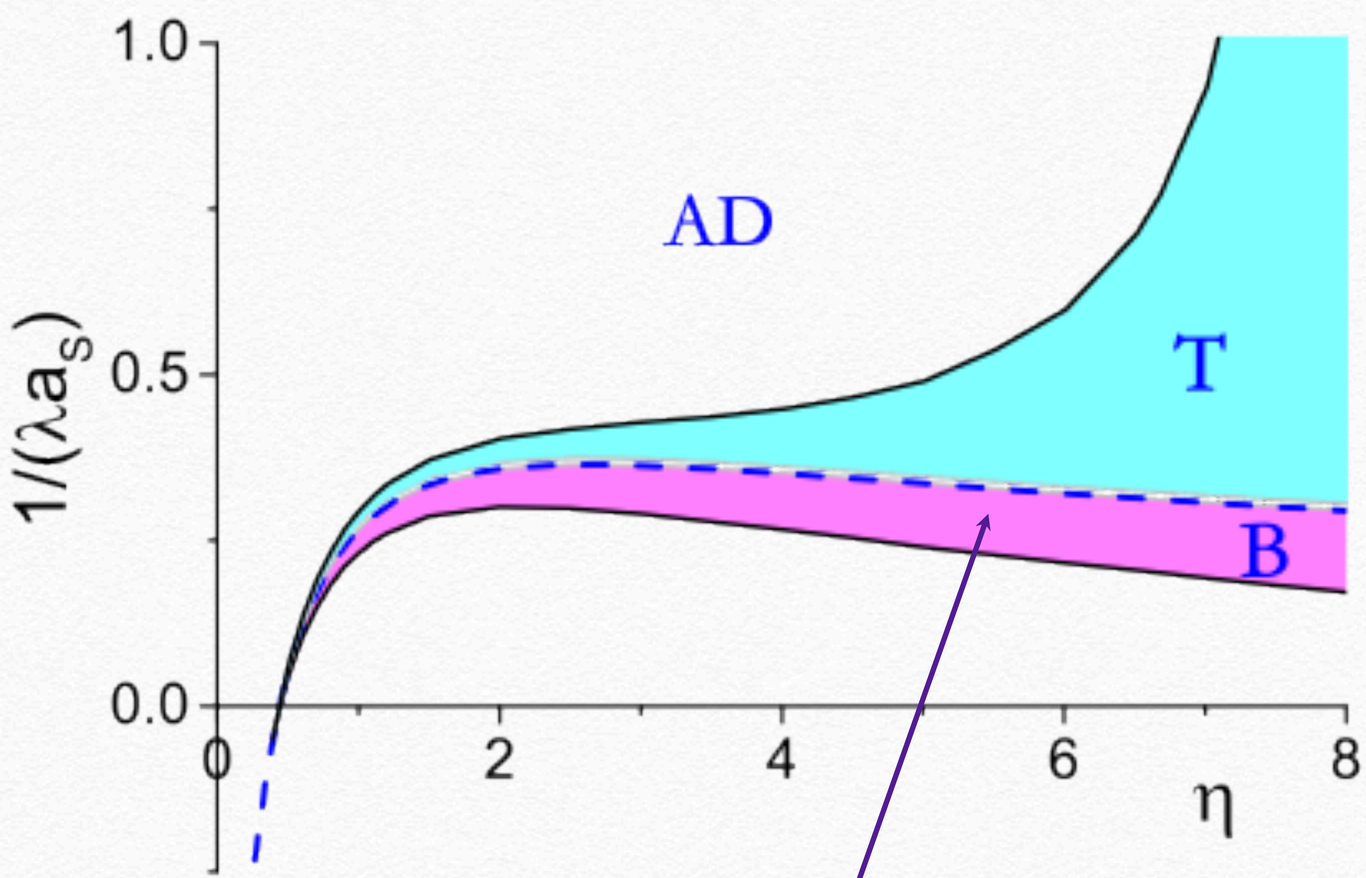
Expert: our results hold for general SO coupling

A later related work



Rashba SO coupling





Universal Borromean binding

Xiaoling Cui and Wei Yi, arXiv: 1403.0649

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Efimov trimer with Universal Scaling → with SO coupling → Broken

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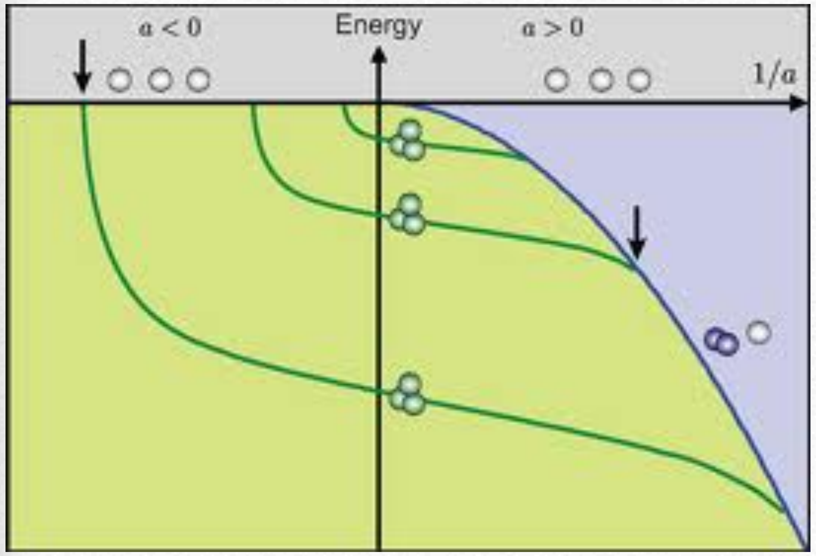
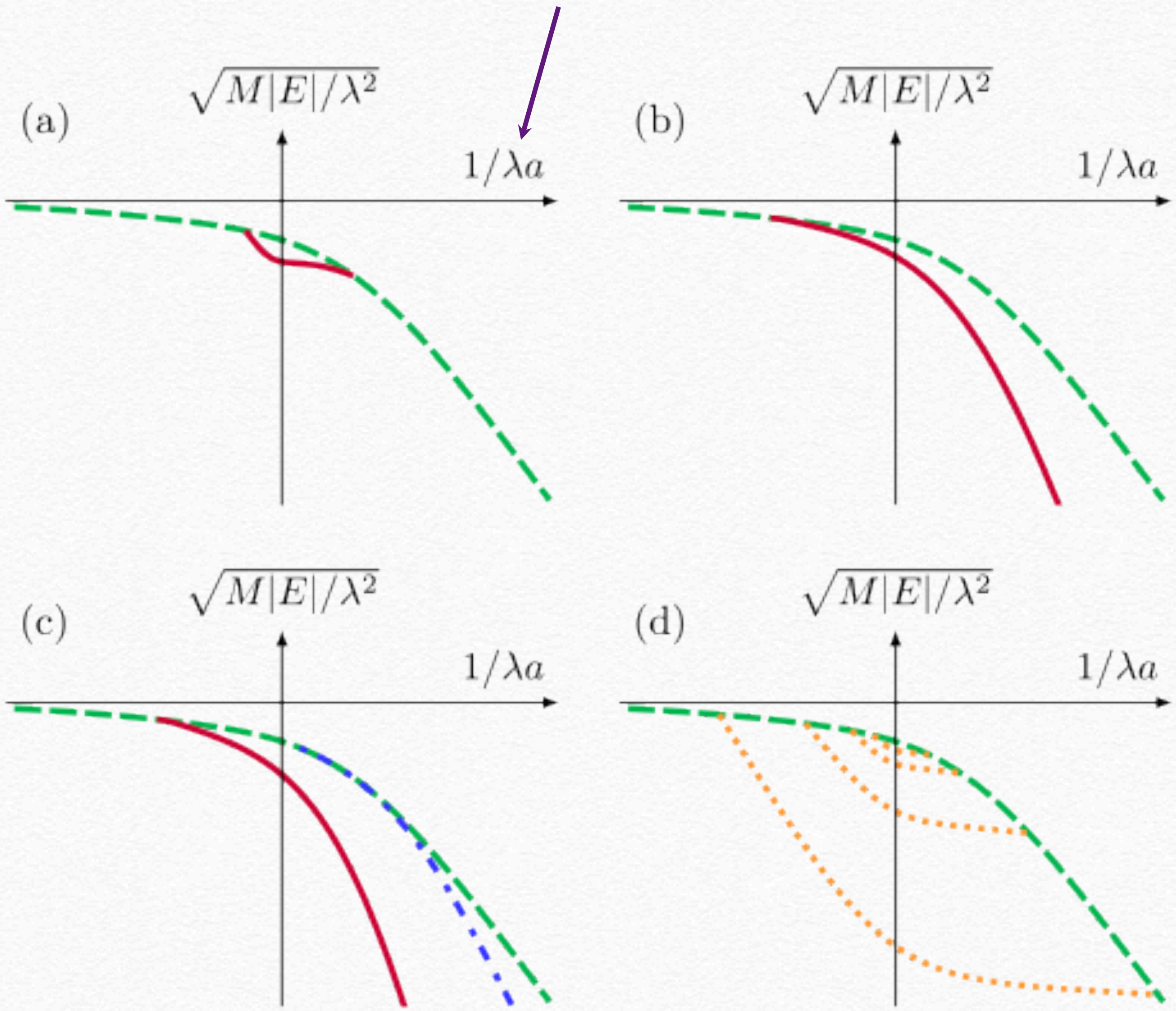
KM trimer with Universal Energy → with SO coupling → Enhanced

❒ SO coupling allows one to control three-body problem and atom-dimer scattering length.

Control atom-dimer scattering



Spin-orbit coupling strength



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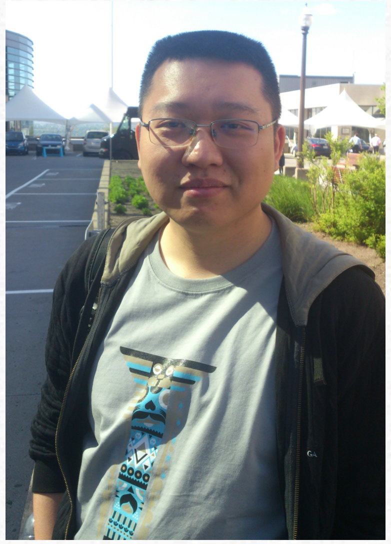
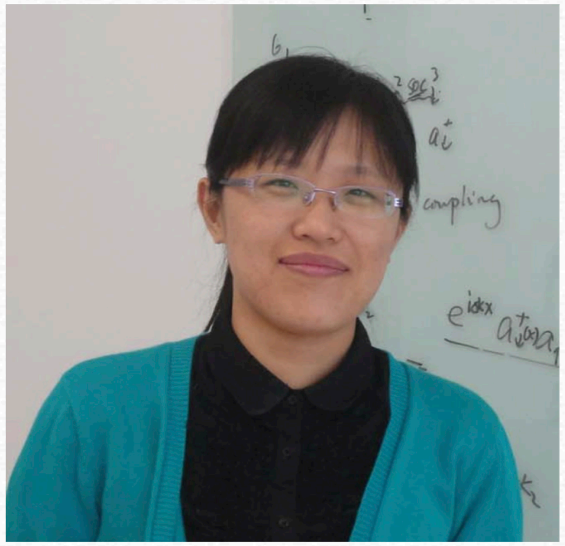
Thanks to my collaborators



Zheyu Shi



Xiaoling Cui

Ref:

Zhe-Yu Shi, Xiaoling Cui and HZ, Phys. Rev. Lett. 112, 013201 (2014)
Zhe-Yu Shi, Xiaoling Cui and HZ, A long paper to appear

Thank you for your attention !