Tsinghua University

Ultracold collisions in the presence of spin orbit coupling

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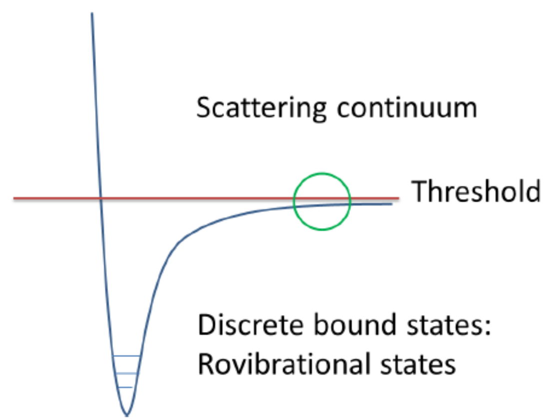
Physical Review A **87**, 052708 (2013)

fewbody@INT, May 12-16th, 2014

Fermi-contact interaction

$$\frac{4\pi\hbar^2 a_{sc}}{M}\delta(\vec{r})$$

$$U(r) = g\,\delta(\mathbf{r})\,\frac{\partial}{\partial r}(r\,\cdot\,)$$



Spinor atomic quantum gases

$$V\left(\vec{r}\right) \rightarrow \left[c_0 \left(\overrightarrow{F_1} \bullet \overrightarrow{F_2} \right)^0 + c_2 \left(\overrightarrow{F_1} \bullet \overrightarrow{F_2} \right)^1 + \bullet\bullet\bullet \right] \delta\left(\vec{r}\right)$$

1-atom spin orbit coupling

$$\oplus H_{SOC}^{(j)} = \frac{\hbar k_{so}}{M} \overrightarrow{F}_j \cdot \vec{p}_j$$

Effective 1D (1-atom interaction)

ME 81, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUS

Atomic Scattering in the Presence of an External Confinement and a Gas of Impenetrable Bosons

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Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
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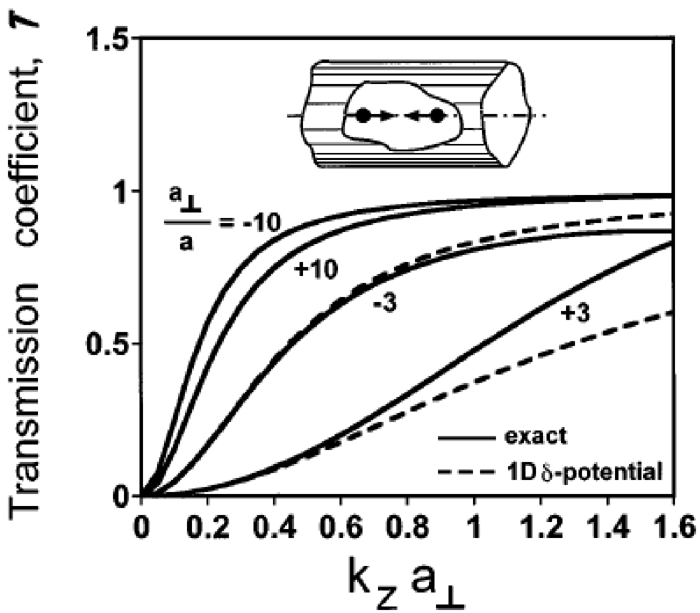
(Received 22 January 1998)



$$U_{1D}(z) = g_{1D}\,\delta(z)$$

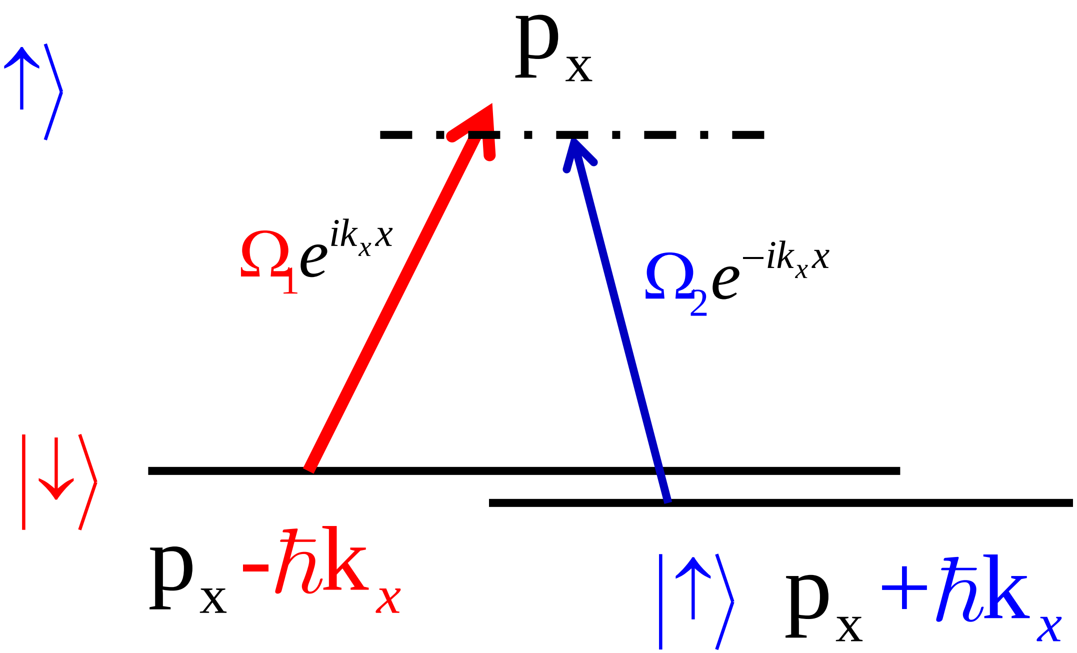
$$g_{1D} = -\frac{\hbar^2}{\mu a_{1D}} = g|\phi_{0,0}(0)|^2\left(1 - C\,\frac{a}{a_\perp}\right)^{-1}$$

Spin orbit coupling (SOC)

A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, PRL **61**, 826 (1988).

Dark state & VSCPT

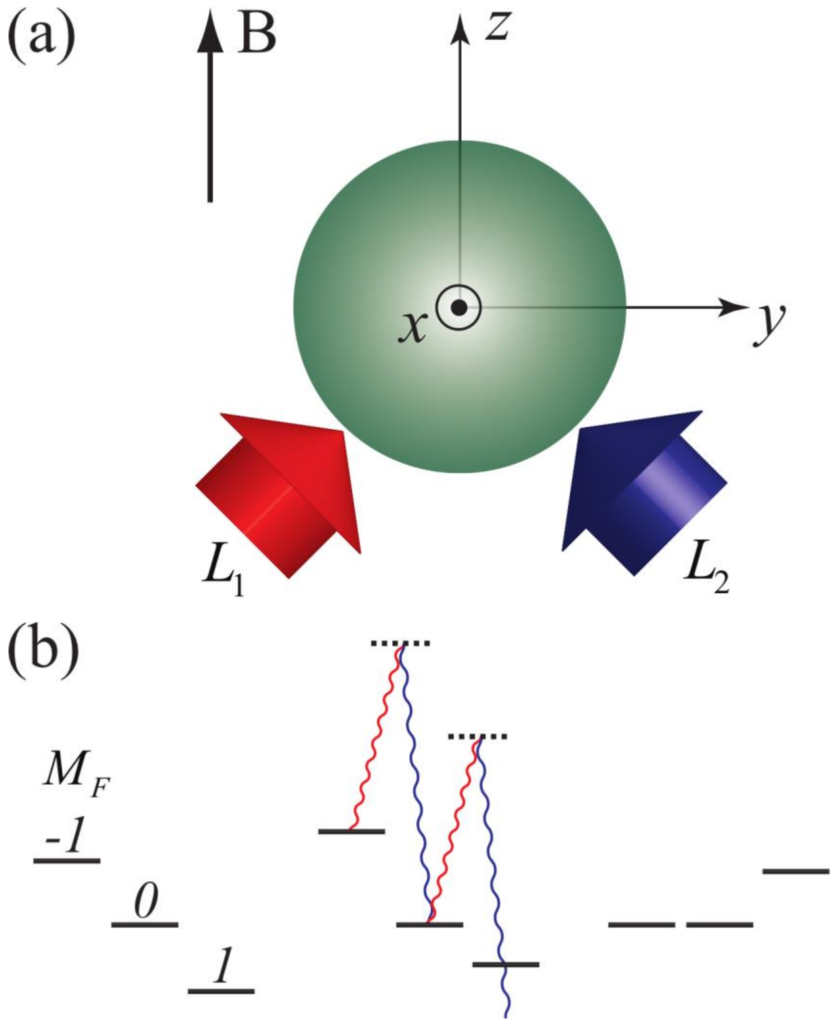
$$|\text{Dark}\rangle \sim \Omega_2 e^{-ik_x x}|\downarrow\rangle - \Omega_1 e^{ik_x x}|\uparrow\rangle$$



$$e^{-ik_{so}x\sigma_x} p_x e^{ik_{so}x\sigma_x} = p_x + k_{so}\sigma_x$$

$$k_{so} = 2k_x$$

The JQI protocol



Weak

More symmetric forms

Abelian/nonabelian

Lattice/inhomogeneous

2D/3D

…

- Rashba SOC

$$\propto \hat{k}_y\sigma_z - \hat{k}_z\sigma_y$$

- Dresselhaus SOC

$$\propto \hat{k}_y\sigma_z + \hat{k}_z\sigma_y$$

- JQI protocol

$$\hat{k}_y\sigma_z = 1(\hat{k}_y\sigma_z - \hat{k}_z\sigma_y) + 1(\hat{k}_y\sigma_z + \hat{k}_z\sigma_y)$$

Y.-J. Lin, K. Jimenez-Garcia, and I. B. Spielman, Nature (London) **473**, 83 (2011).

Helicity: 1-atom with SOC

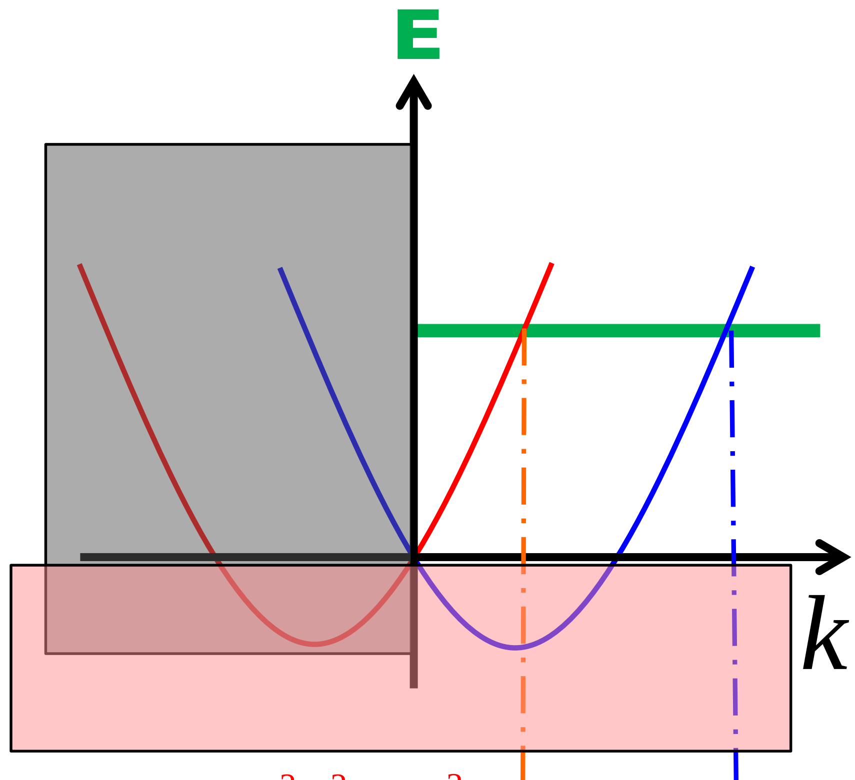
$$H_i = H_i^{(0)} + V_{trap}\left(\vec{r}\right)$$

$$H_i^{(0)} = \frac{\vec{p}_i^{\,2}}{2m_i} + \frac{\hbar k_{so}}{m_i}\vec{\sigma}_i \cdot \vec{p}_i$$

$$\left|+,\hat{k}\right\rangle e^{i\vec{k}\cdot\vec{r}}$$

$$\left|-,\hat{k}\right\rangle e^{i\vec{k}\cdot\vec{r}}$$

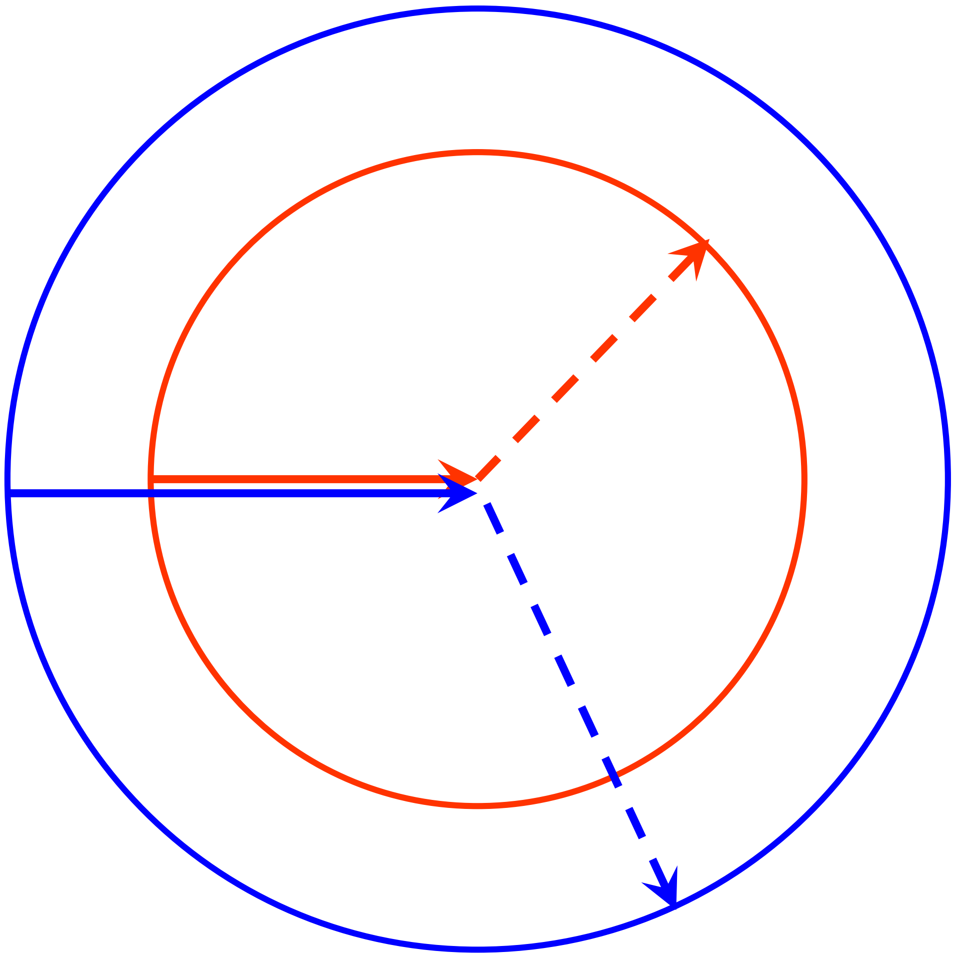
$$\lambda_i = \frac{\vec{\sigma}_i \cdot \vec{p}_i}{p_i} = \pm 1$$

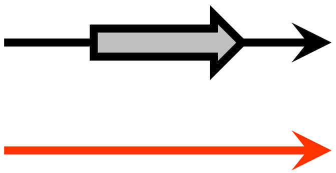


Slow: $E = \frac{\hbar^2 k_+^2}{2m} + \frac{\hbar^2 k_{so} k_+}{m}$

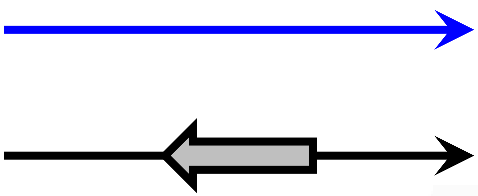
Fast: $E = \frac{\hbar^2 k_-^2}{2m} - \frac{\hbar^2 k_{so} k_-}{m}$

Locking spin to direction of motion

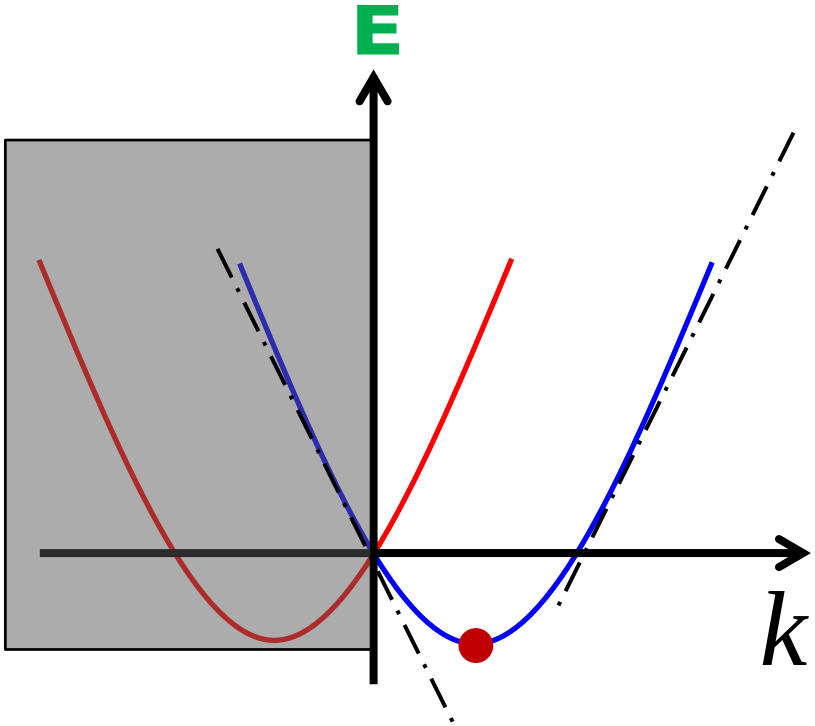




$$\left[\frac{\vec{\sigma}_i \cdot \vec{p}_i}{p_i}, V_{trap}\left(\vec{r}_i\right)\right] \neq 0$$



Canonical/kinetic momentum



Dirac cone

particle/hole symmetry

...

Gauge potentials

condensation at nonzero canonical momenta

but zero kinetic momentum

2-atom collision with SOC

$$\mathrm{H} = \frac{\overrightarrow{P_1}^2}{2m_1} + \frac{\overrightarrow{P_2}^2}{2m_2} + \frac{\hbar k_{so}}{m_{i=1,2}}\left(\overrightarrow{\sigma_1} \bullet \overrightarrow{P_1} + \overrightarrow{\sigma_2} \bullet \overrightarrow{P_2}\right) + V\left(\vec{r}\right)$$

$$= \frac{\overrightarrow{P}^2}{2M} + \frac{\hbar k_{so}}{2M}\left(\overrightarrow{\sigma_1} + \overrightarrow{\sigma_2}\right) \bullet \vec{P} \qquad \text{(center of mass)}$$

$$+ \frac{\overrightarrow{p}^2}{2\mu} + \frac{\hbar k_{so}}{2\mu}\left(\overrightarrow{\sigma_1} - \overrightarrow{\sigma_2}\right) \bullet \vec{p} + V\left(\vec{r}\right) \qquad \text{(relative)}$$

X. Cui, Phys. Rev. A **85**, 022705 (2012);
P. Zhang, L. Zhang, and W. Zhang, Phys. Rev. A **86**, 042707 (2012);
Yuxiao Wu and Zhenghua Yu, Phys. Rev. A **87**, 032703 (2013).

Separation of timescale

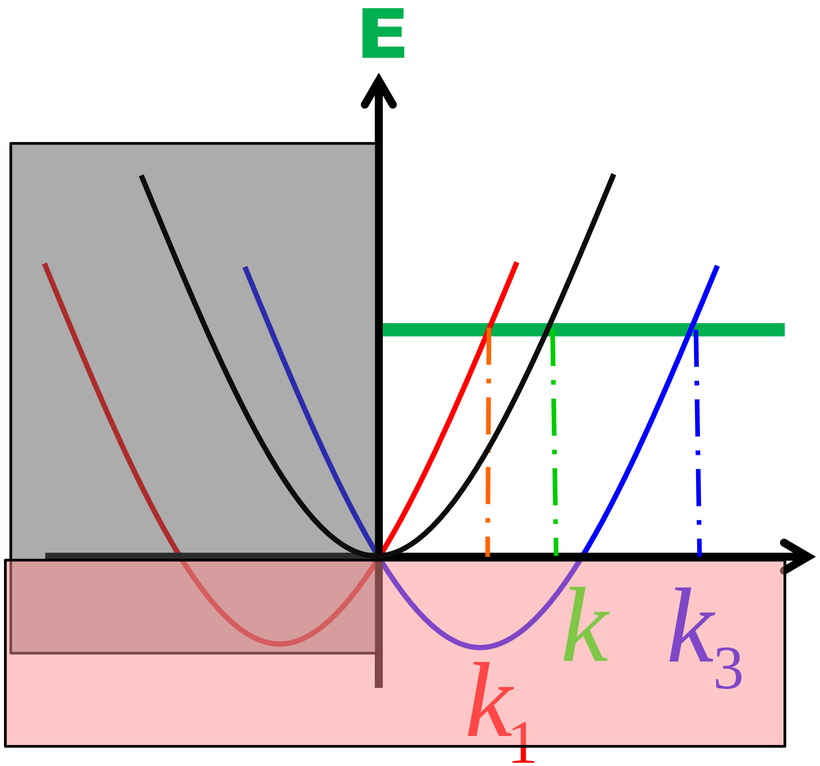
$$
\begin{aligned}
[H_{\rm com}, H_{\rm rel}] &= [\frac{\hbar}{M} C_{\rm so}(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot \vec{P}, \frac{\hbar}{m} C_{\rm so}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot \vec{p}] \\
&= \frac{\hbar^2}{2m^2} k_{\rm so}^2 \left([\sigma_1 \cdot \vec{P}, \vec{\sigma}_1 \cdot \vec{p}] - [\vec{\sigma}_2 \cdot \vec{P}, \vec{\sigma}_2 \cdot \vec{p}]\right) \\
&= \frac{i\hbar^2 k_{\rm so}^2}{m^2} (\vec{\sigma}_1 - \vec{\sigma}_2)\cdot \left(\vec{P} \times \vec{p}\right).
\end{aligned}
$$

center of mass time: $\dfrac{2\pi}{\omega_{trap}}$ $\qquad \dfrac{\beta_6}{v/\omega_{trap}} \ll 1$

collision time: $\dfrac{\beta_6}{v} \ll$ center of mass timescale

The center of mass frame

$$H=\frac{\vec{p}^{\,2}}{2\mu}+\frac{\hbar k_{so}}{2\mu}\left(\vec{\sigma}_1-\vec{\sigma}_2\right)\bullet\vec{p}\,\theta\left(r-r_0\right)+V\left(\vec{r}\right)$$



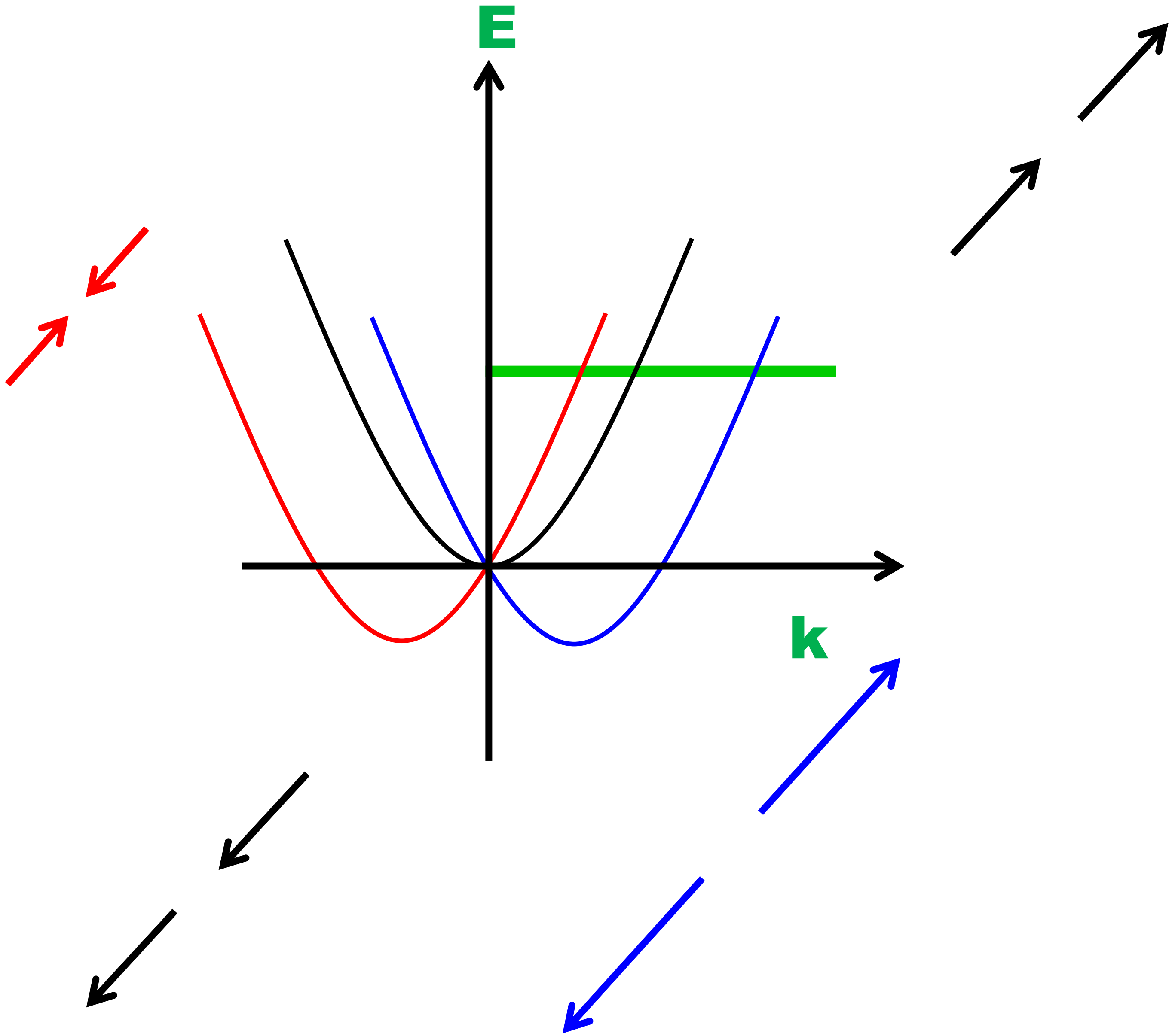
$$\left(\vec{\sigma}_1-\vec{\sigma}_2\right)\bullet\vec{p}=\left(2,0,-2\right)\hbar k$$

$$|1\rangle_{\vec{k}}:\ |+\rangle_{\vec{k}}|+\rangle_{-\vec{k}}$$

$$|2\rangle_{\vec{k}}:\ |+\rangle_{\vec{k}}|-\rangle_{-\vec{k}}\sim|-\rangle_{\vec{k}}|+\rangle_{-\vec{k}}$$

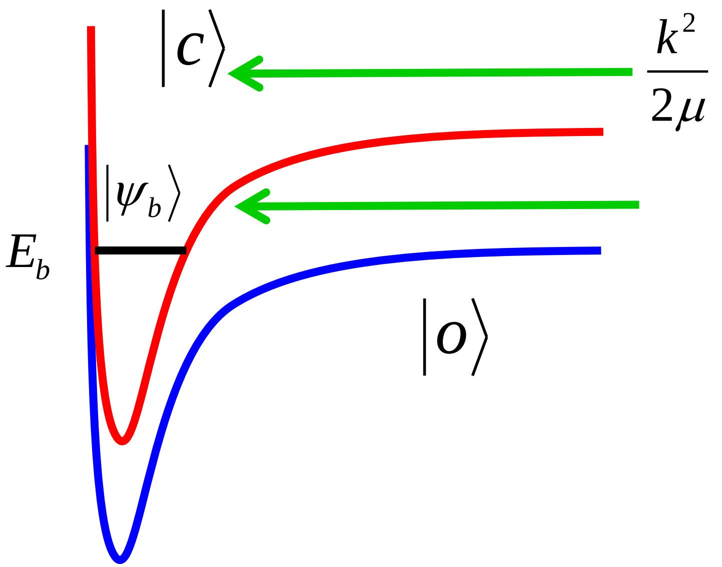
$$|3\rangle_{\vec{k}}:\ |-\rangle_{\vec{k}}|-\rangle_{-\vec{k}}$$

4 scattering channels: bosons



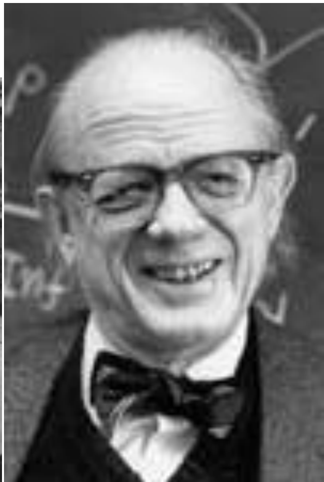
On-shell collision without SOC

$$e^{i\vec{k}\bullet\vec{r}} + f(\vec{k},\vec{k}'=k\hat{r})\frac{e^{ikr}}{r}$$



Free particle dispersion

Multiple threshold
Fano-Feshbach resonance

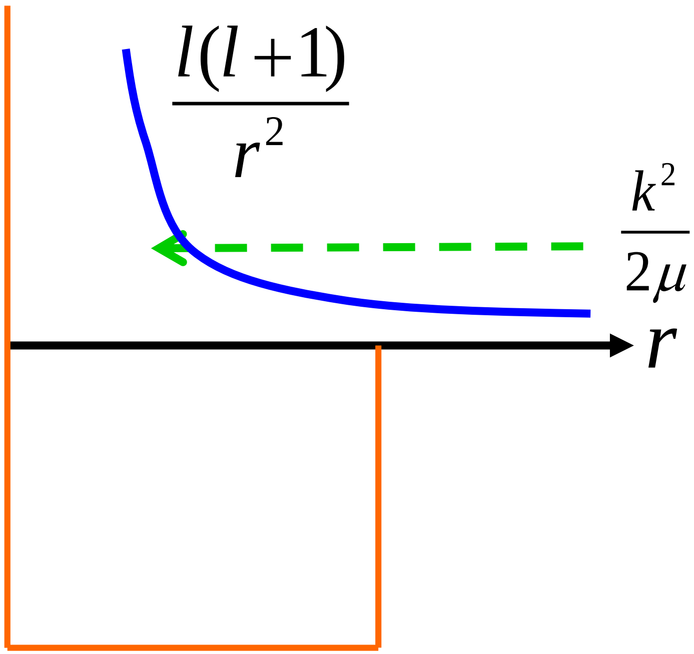


E. Tiesinga, B. J. Verhaar, and H. T. C. Stoof ,
Phys. Rev. A **47**, 4114 (1993)

Wigner threshold law

Short-range central potential

$$f(\vec{k},\vec{k}')\Big|_{k=k'} \sim \sum_l (2l+1)\frac{\delta_l(k)}{k} P_l(\hat{k}\bullet\hat{k}') \propto \sum_l (2l+1) k^{2l} P_l(\hat{k}\bullet\hat{k}')$$



$$\delta_l(k) \propto k^{2l+1},$$

$$\psi_0(r) \sim \frac{1}{r} - \frac{1}{a_{sc}}$$

Effective-range expansion

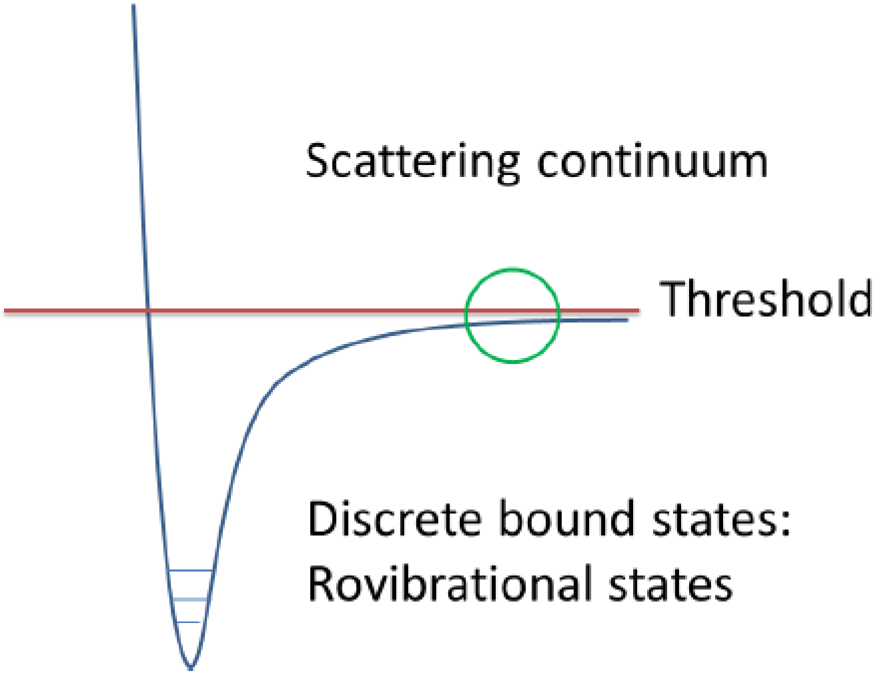
$$k^{2l+1}\cot\delta_l = -\frac{1}{a_l} + \frac{1}{2} r_{eff}^{(l)} k^2 + \ldots$$

S. Tan, Ann. Phys. (N.Y.) 323, 2952/ 2971/2987 (2008)

Real atoms: power law potentials

$$V(r \to \infty) \propto -\frac{1}{r^n},$$

$$\delta_l(k) \propto \begin{cases} k^{2l+1}, & l < \frac{1}{2}(n-3) \\ k^{n-2}, & l > \frac{1}{2}(n-3) \end{cases}$$



van der Waals potential

$$V(r) \propto -\frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{C_{10}}{r^{10}}$$

Length scale separation

TABLE I. Sample scale parameters for $A+A$ type of systems where A is an alkali-metal atom. The $\beta_6=(2\mu C_6/\hbar^2)^{1/4}$ is the length scale. The $s_E=(\hbar^2/2\mu)(1/\beta_6)^2$ is the corresponding energy scale. It is given both in units of μK and in units of MHz. $s_T=\hbar/s_E$ is the corresponding time scale. All are determined by the C_6 coefficient and atomic masses.

Atom	C_6 (a.u.)	β_6 (a.u.)	s_E/k_B (μK)	s_E/h (MHz)	s_T (ns)
^{6}Li	1393.39[a]	62.52	7368	153.5	1.037
^{23}Na	1556[b]	89.86	933.1	19.44	8.186
^{40}K	3897[b]	129.8	257.3	5.360	29.69
$^{85\partial}$Rb	4707[c]	164.3	75.58	1.575	101.1
^{133}Cs	6860[d]	201.9	31.97	0.6662	238.9

β_6,

$\frac{1}{k}$

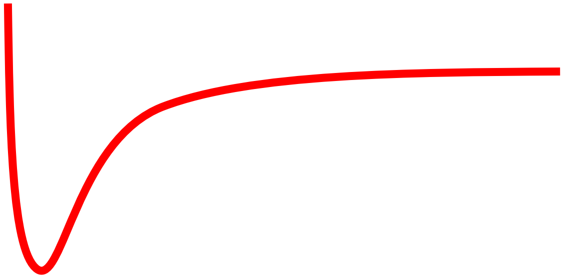
$$\frac{C_6}{r^6}\sim\frac{10^4}{(100)^6}\doteq 10^{-8}$$

Methodology: analytic results

all partial waves

$$U(r) = g\,\delta(\mathbf{r})\,\frac{\partial}{\partial r}(r\,\cdot\,)$$

all partial waves



MQDT
Frame transformation

Multichannel QDT for ion-atom interactions
arXiv:1402.3704//Ming Li, Li You, Bo Gao

Alternative basis set

$$\vec{F}_{t} = \vec{R}\times\vec{P} + \vec{r}\times\vec{p} + \frac{\hbar}{2}(\vec{\sigma}_1+\vec{\sigma}_2) = \vec{r}\times\vec{p} + \frac{\hbar}{2}(\vec{\sigma}_1+\vec{\sigma}_2) = \vec{l} + \vec{F}$$

$$\left|\lambda_1,\lambda_2;\mathrm{F}_t,\mathrm{M}_t\right\rangle$$

$$\left|\lambda_1,\lambda_2;\vec{p}=\hbar\vec{k}\right\rangle$$

$$f\left(|i\rangle_{k_i} \rightarrow |j\rangle_{\hat{r}}\right) \sim \frac{1}{k_j} T_{j,i}(k)\, \mathrm{D}^{(\mathrm{F}_t)}_{M_t,M_t'}\left(k_i\right) \mathrm{D}^{(\mathrm{F}_t\,*)}_{M_t,M_t''}\left(\hat{r}\right)$$

Channel structures (spin ½ fermion)

F+L=even

$$\mathrm{F}_t, \quad \{F=1, l=F_t\}$$

$$\mathrm{F}_t=2, \quad \{F=1, l=2\}$$

$$\text{odd } \mathrm{F}_t=1, \quad \{F=1, l=1\}$$

$$\mathrm{F}_t, \quad \{F=0, l=F_t\},$$

$$\{F=1, l=F_t-1\}, \{F=1, l=F_t+1\}$$

$$\text{even } \mathrm{F}_t=0, \quad \{F=0, l=0\}, \{F=1, l=1\}$$

Channel structures (spin ½ boson)

F+L=odd

even $\mathrm{F}_t \neq 0, \quad \{F=1, l=F_t\}$

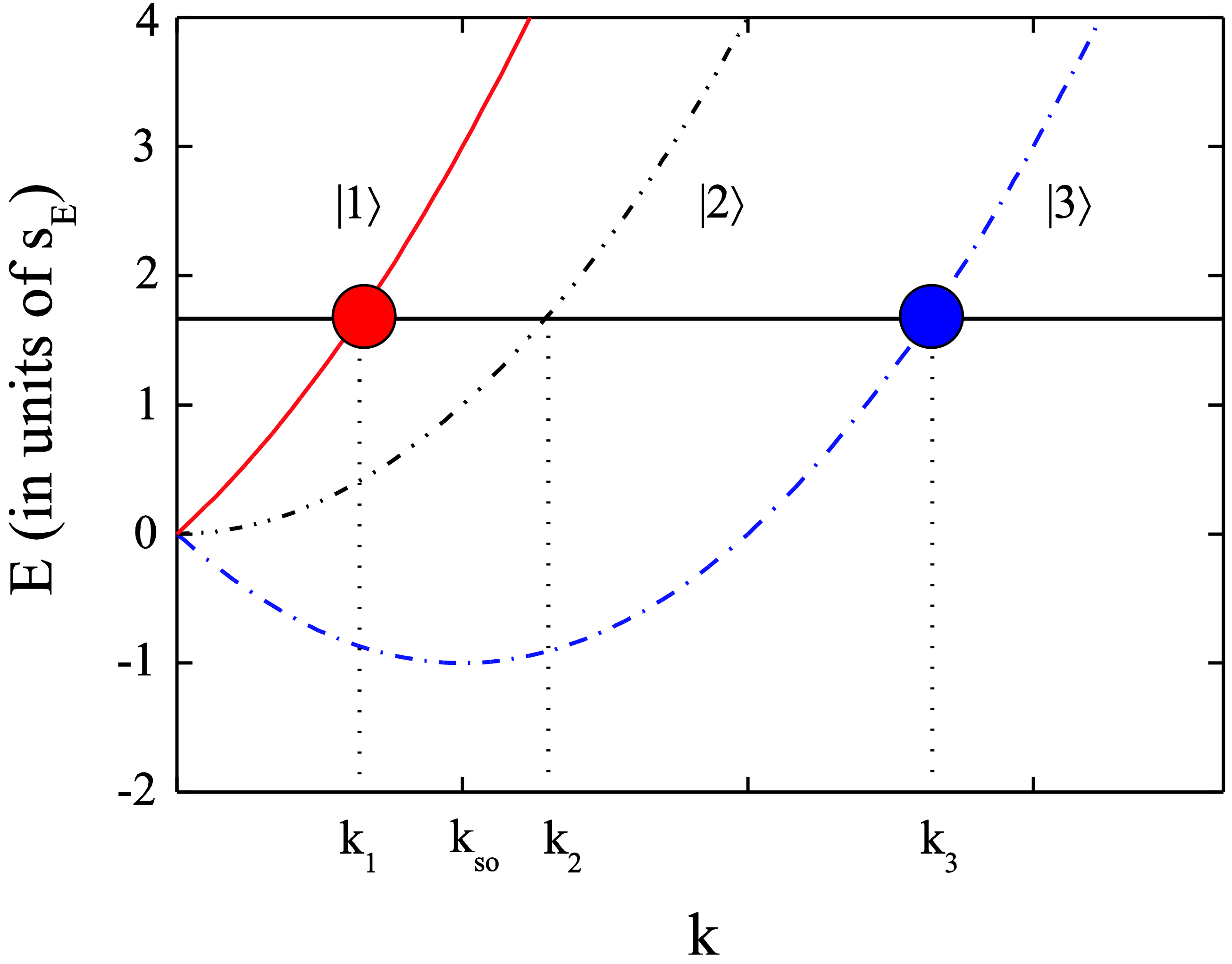
$\mathrm{F}_t, \quad \{F=0, l=F_t\},$

$\{F=1, l=F_t-1\}, \{F=1, l=F_t+1\}$

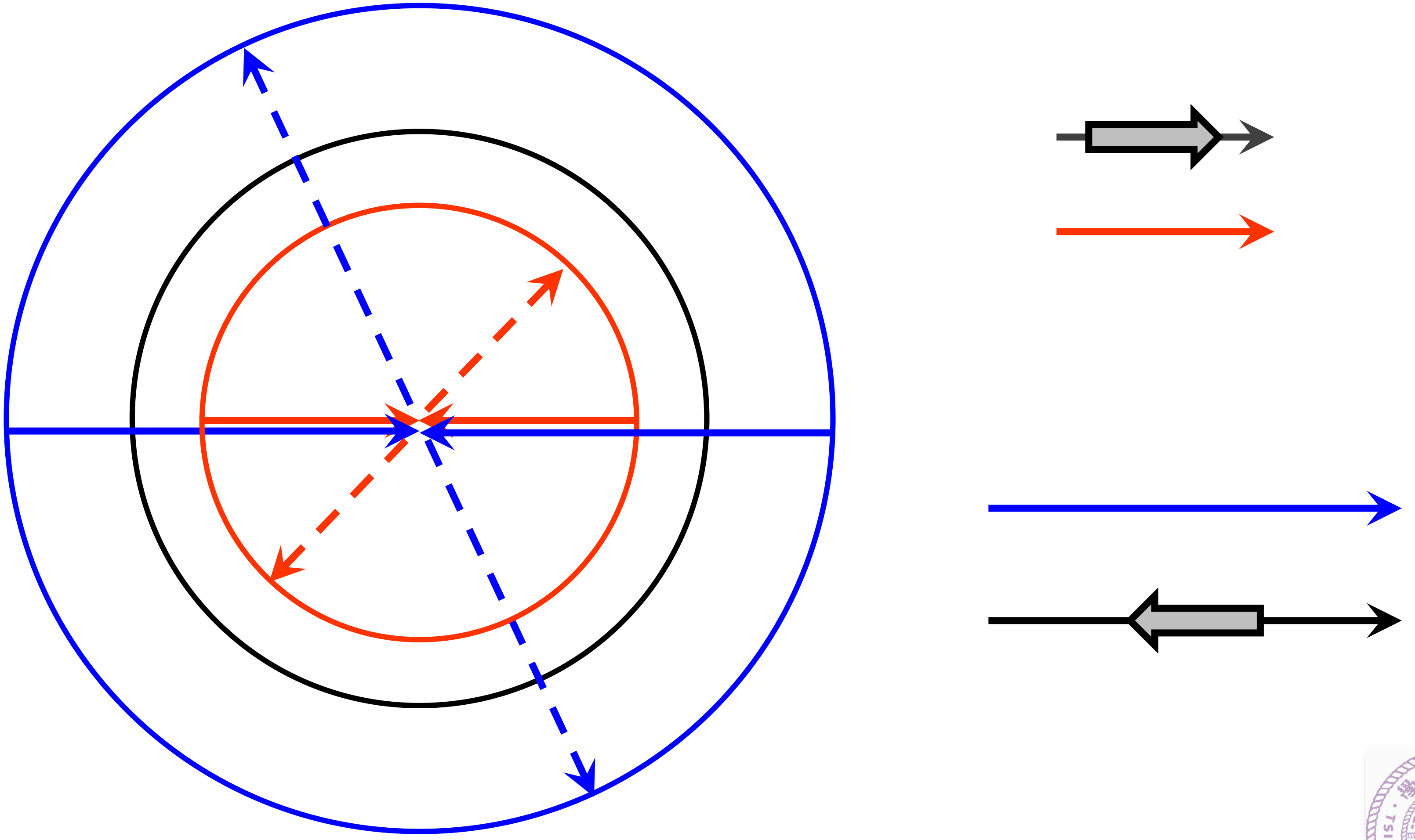
odd $\mathrm{F}_t=1, \quad \{F=0, l=1\}, \{F=1, l=0\}, \{F=1, l=2\}$

Spin ½ fermions

$\vec{P} = 0$



$F_t = 0$



Cross sections

$$k \to 0, \quad f\left(|i\rangle_{\vec{k}} \to |j\rangle_{k\hat{r}}\right) \sim \frac{-a_{sc}}{1 + ik_{so}a_{sc}}$$

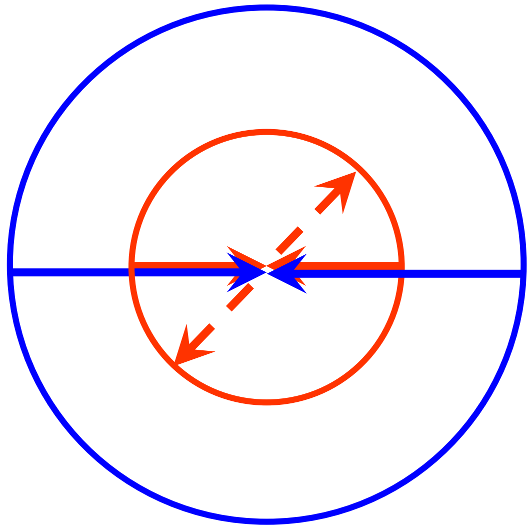
$$\sigma^{F_t=0}(|1\rangle_{\text{in}} \to |1\rangle_{\text{out}}) = \sigma^{F_t=0}(|3\rangle_{\text{in}} \to |1\rangle_{\text{out}})$$

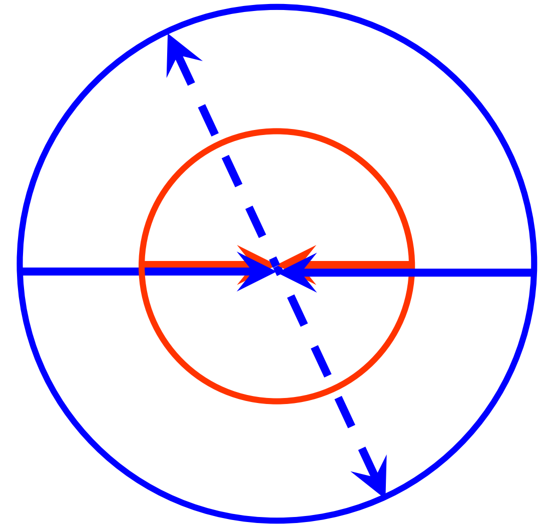
$$= \frac{8\pi a_{F=0,l=0}^2 k_1^2}{(k_1+k_3)^2 + a_{F=0,l=0}^2 (k_1^2 + k_3^2)^2},$$

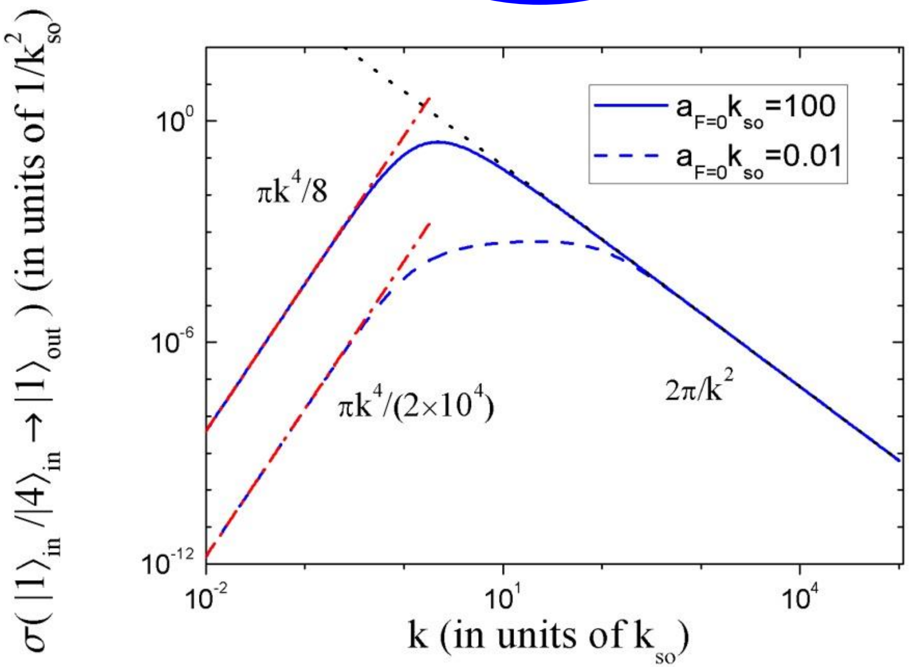
$$\sigma^{F_t=0}(|1\rangle_{\text{in}} \to |3\rangle_{\text{out}}) = \sigma^{F_t=0}(|3\rangle_{\text{in}} \to |3\rangle_{\text{out}})$$

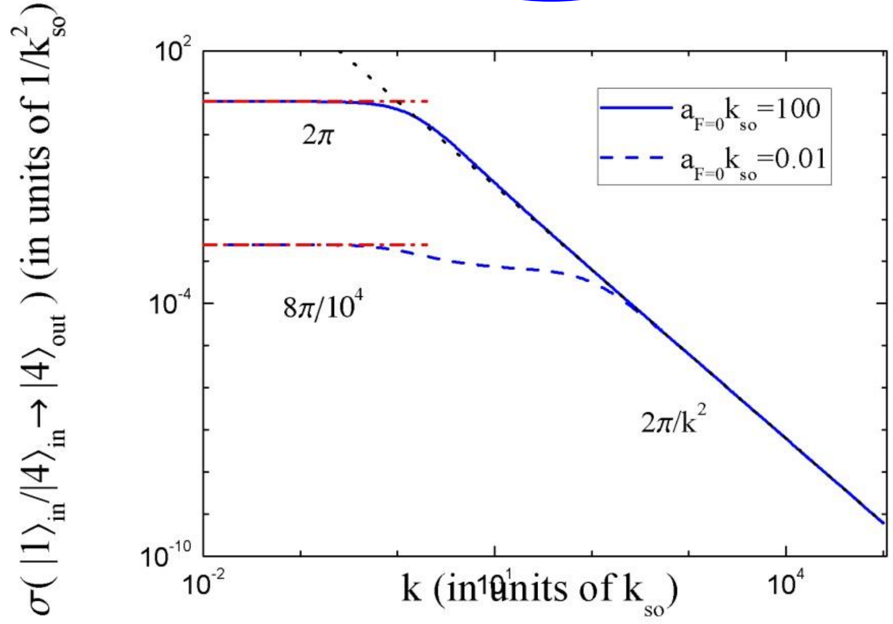
$$= \frac{8\pi a_{F=0,l=0}^2 k_3^2}{(k_1+k_3)^2 + a_{F=0,l=0}^2 (k_1^2 + k_3^2)^2}.$$

Elastic & inelastic (final state)



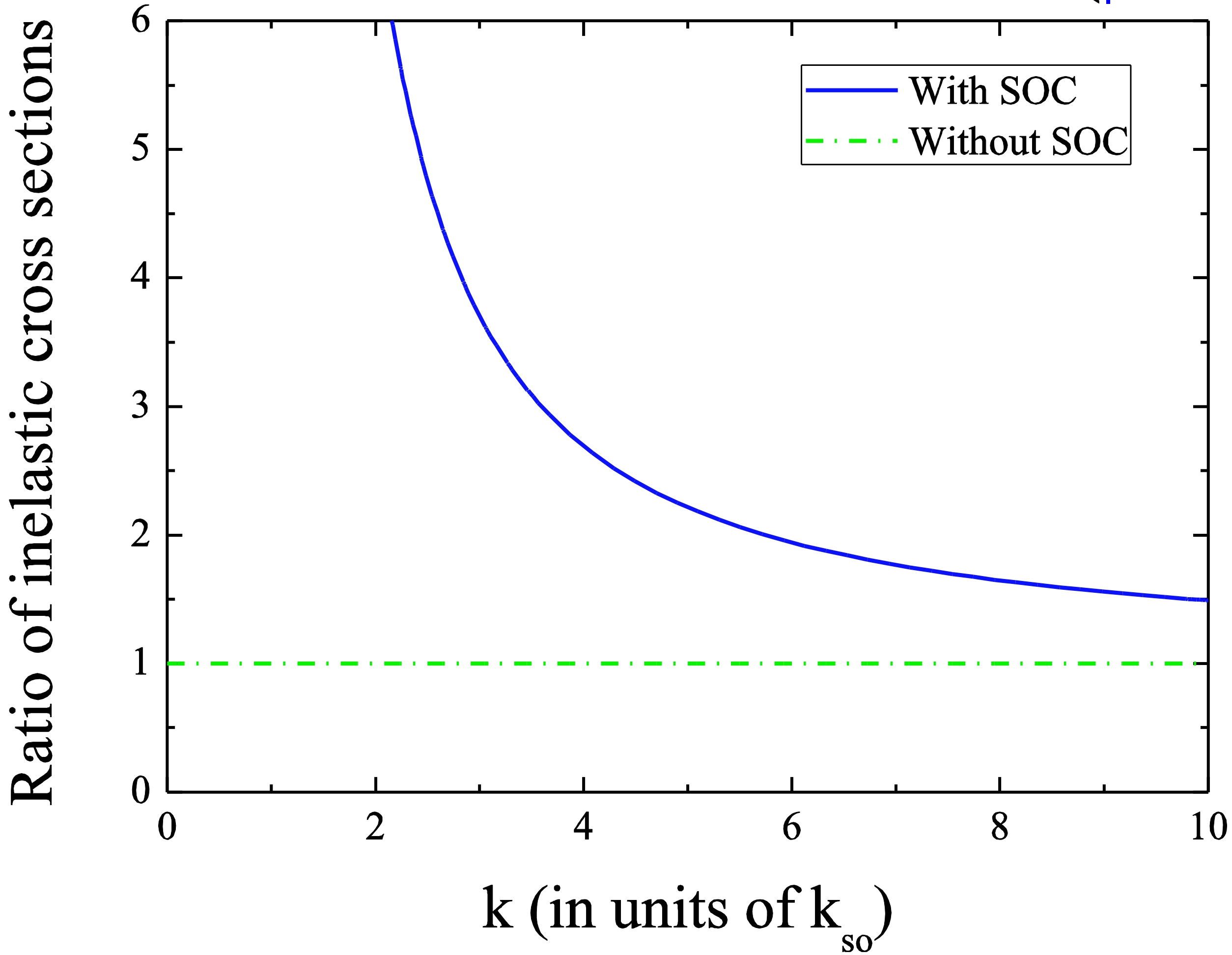




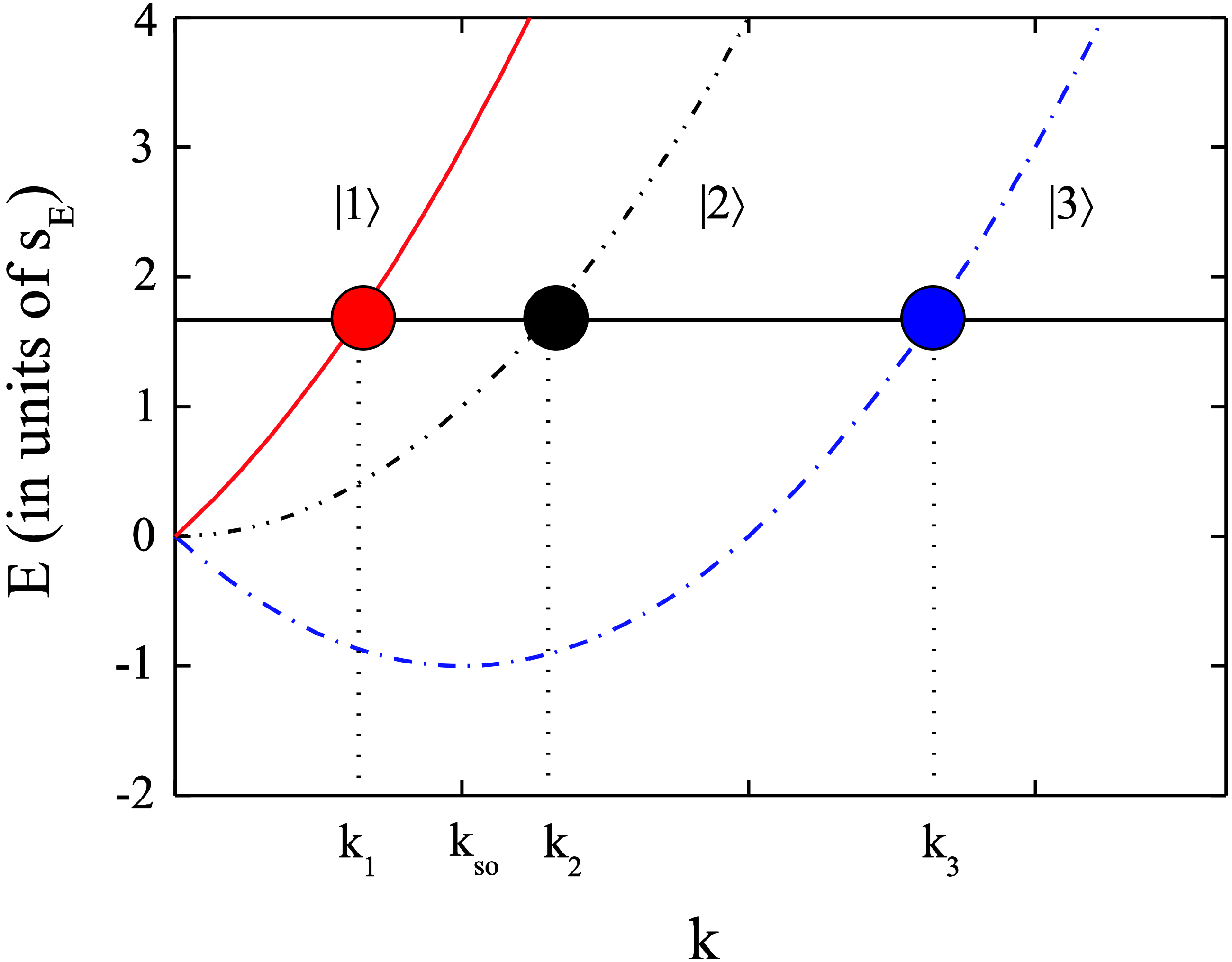


Detailed balance: time reversal

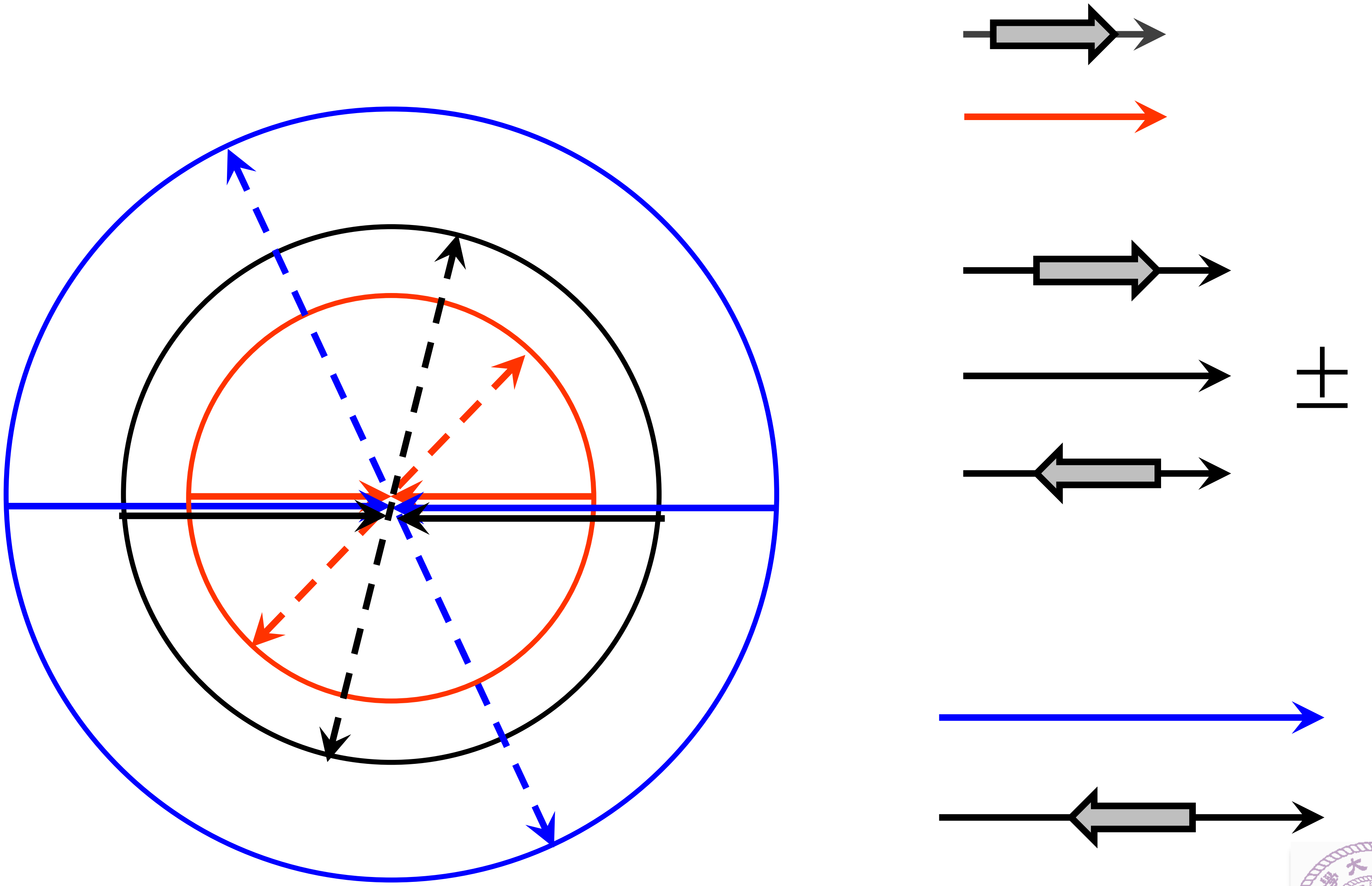
$$\frac{\sigma^{F_t=0}\left(|+,+\rangle_{in} \rightarrow |-,-\rangle_{out}\right)}{\sigma^{F_t=0}\left(|-,-\rangle_{in} \rightarrow |+,+\rangle_{out}\right)} = \frac{k_3^2}{k_1^2}$$



Spin ½ bosons



$F_t = 1$

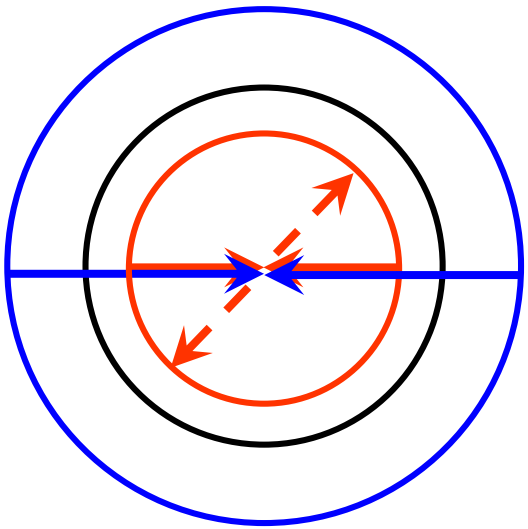


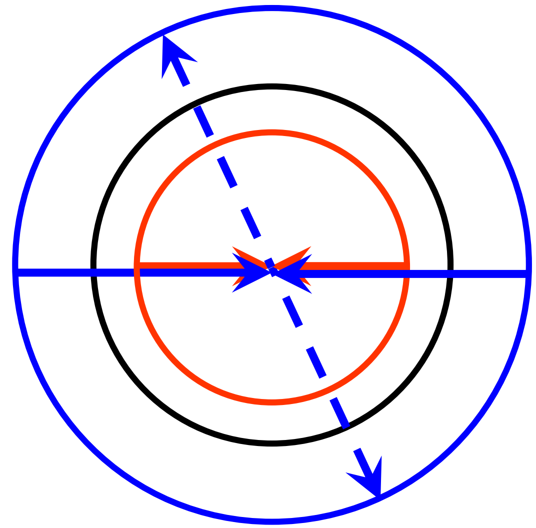
Scattering amplitudes

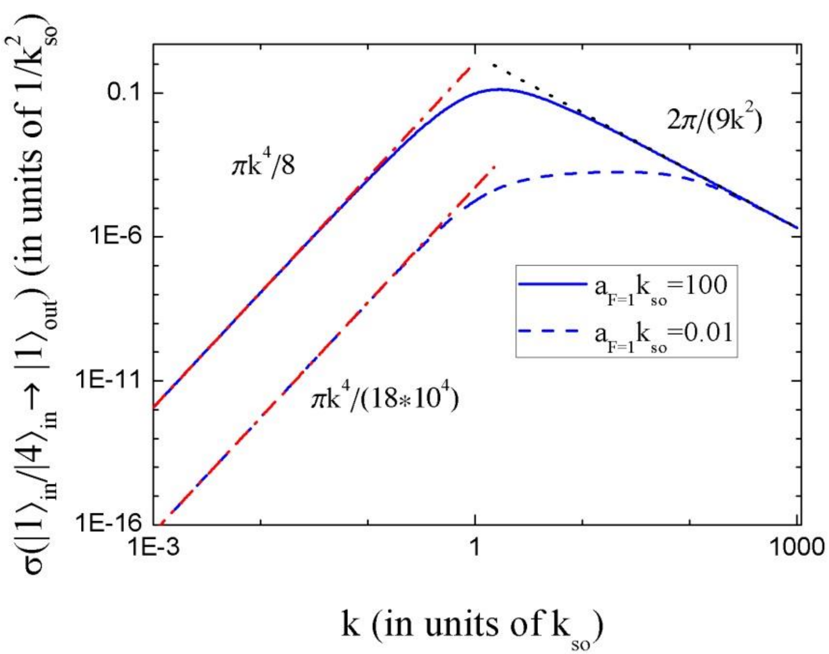
$$k \to 0, \quad f_0 \sim \frac{-a_{sc}}{1 + 2ik_{so}a_{sc}/3}$$

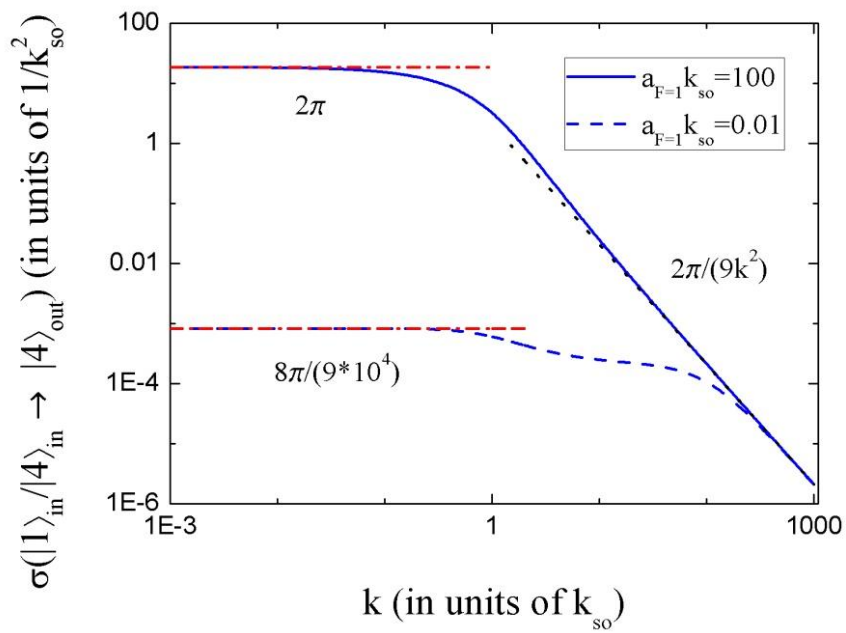
$$\begin{aligned}
f^{(1)}(|1\rangle_{\hat{z}}, |3\rangle_{\hat{z}} \to |1\rangle_{\hat{r}}) &= \frac{1}{2\sqrt{2}} f_0 \frac{k^2}{k_{\rm so}^2} \sqrt{\frac{4\pi}{3}} Y_{10}(\hat{r}), \\
f^{(1)}(|2\rangle_{\hat{z}} \to |1\rangle_{\hat{r}}) &= \frac{1}{2} f_0 \frac{k^2}{k_{\rm so}^2} \sqrt{\frac{4\pi}{3}} Y_{11}(\hat{r}), \\
f^{(1)}(|1\rangle_{\hat{z}}, |3\rangle_{\hat{z}} \to |2\rangle_{\hat{r}}) &= f_0 \sqrt{\frac{4\pi}{3}} Y_{1-1}(\hat{r}), \\
f^{(1)}(|2\rangle_{\hat{z}} \to |2\rangle_{\hat{r}}) &= f_0 \sqrt{\frac{2\pi}{3}} \left[\sqrt{3}\, Y_{00}(\hat{r}) + Y_{10}(\hat{r})\right], \\
f^{(1)}(|1\rangle_{\hat{z}}, |3\rangle_{\hat{z}} \to |3\rangle_{\hat{r}}) &= \sqrt{2} f_0 \sqrt{\frac{4\pi}{3}} Y_{10}(\hat{r}), \\
f^{(1)}(|2\rangle_{\hat{z}} \to |3\rangle_{\hat{r}}) &= 2 f_0 \sqrt{\frac{4\pi}{3}} Y_{11}(\hat{r}), \qquad (2)
\end{aligned}$$

Cross-sections: boson









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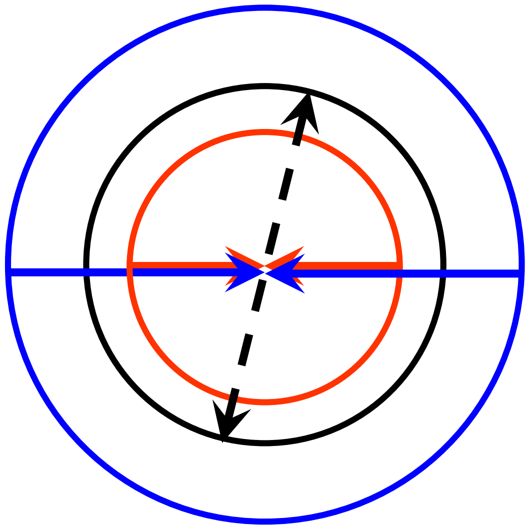
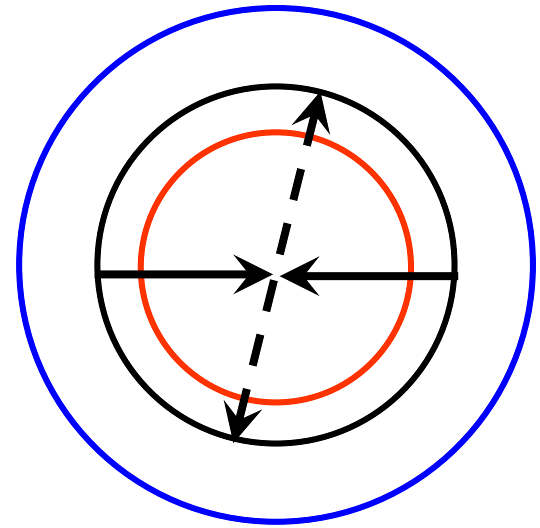
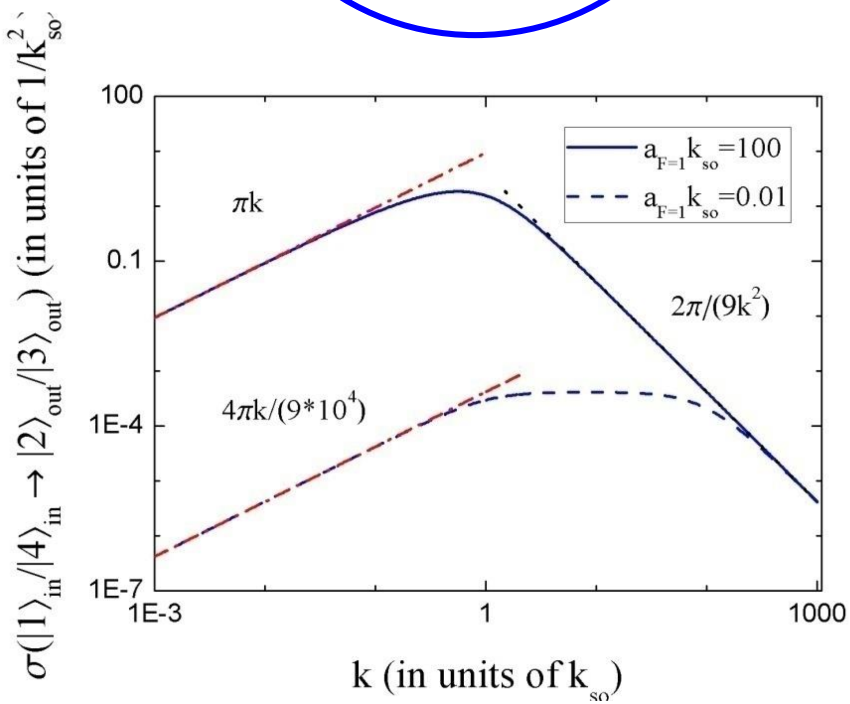
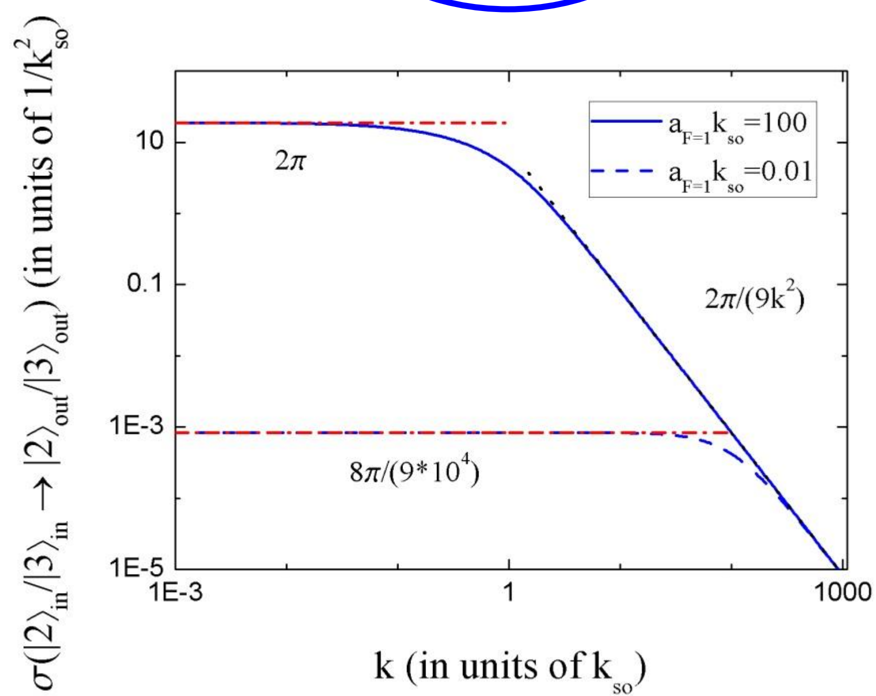
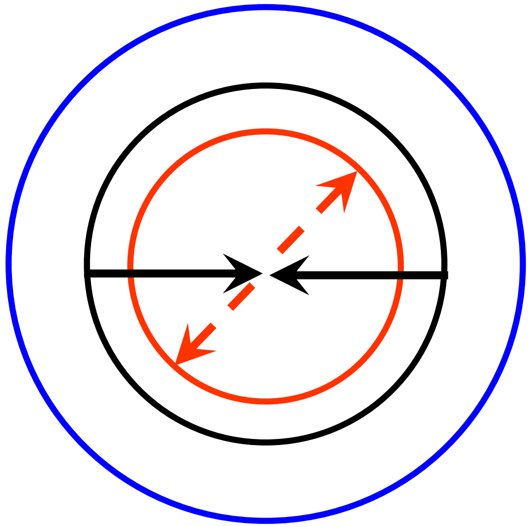


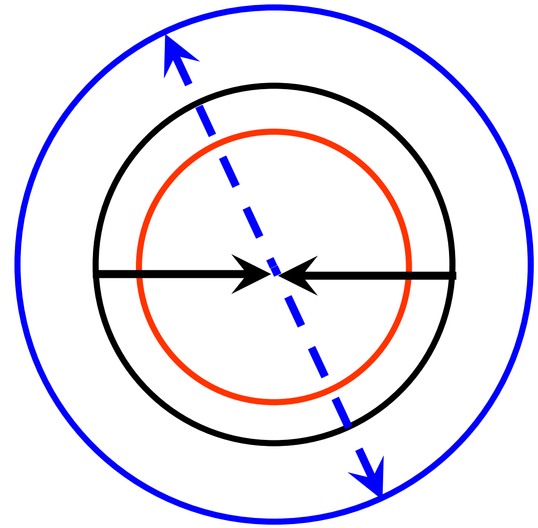


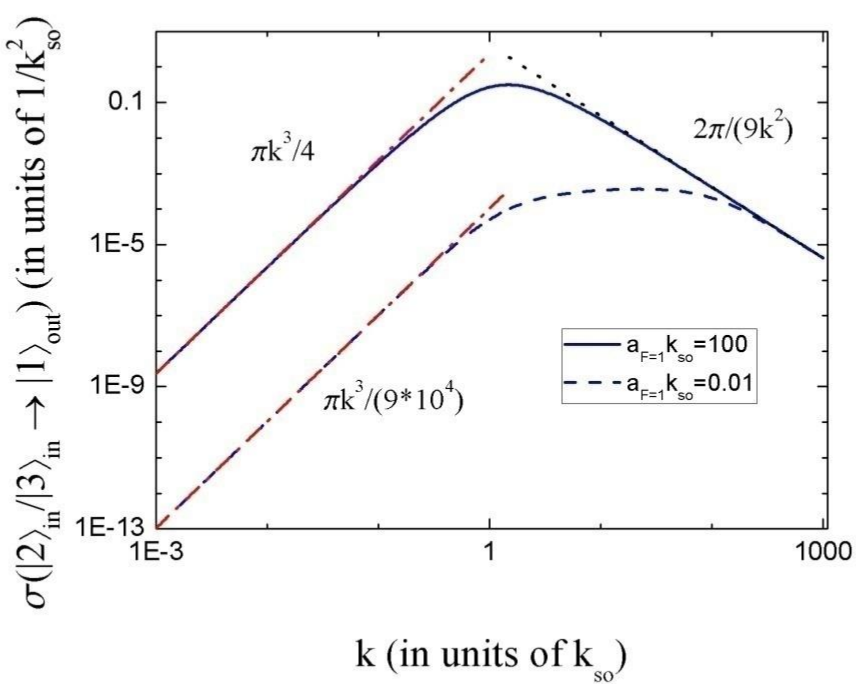

$\sigma(|1\rangle_{in}/|4\rangle_{in} \to |2\rangle_{out}/|3\rangle_{out})$ (in units of $1/k_{so}^2$)
100
0.1
1E-4
1E-7
1E-3
1
1000
k (in units of k_{so})
$a_{F=1}k_{so}=100$
$a_{F=1}k_{so}=0.01$
πk
$2\pi/(9k^2)$
$4\pi k/(9*10^4)$


$\sigma(|2\rangle_{in}/|3\rangle_{in} \to |2\rangle_{out}/|3\rangle_{out})$ (in units of $1/k_{so}^2$)
10
0.1
1E-3
1E-5
1E-3
1
1000
k (in units of k_{so})
$a_{F=1}k_{so}=100$
$a_{F=1}k_{so}=0.01$
2π
$2\pi/(9k^2)$
$8\pi/(9*10^4)$

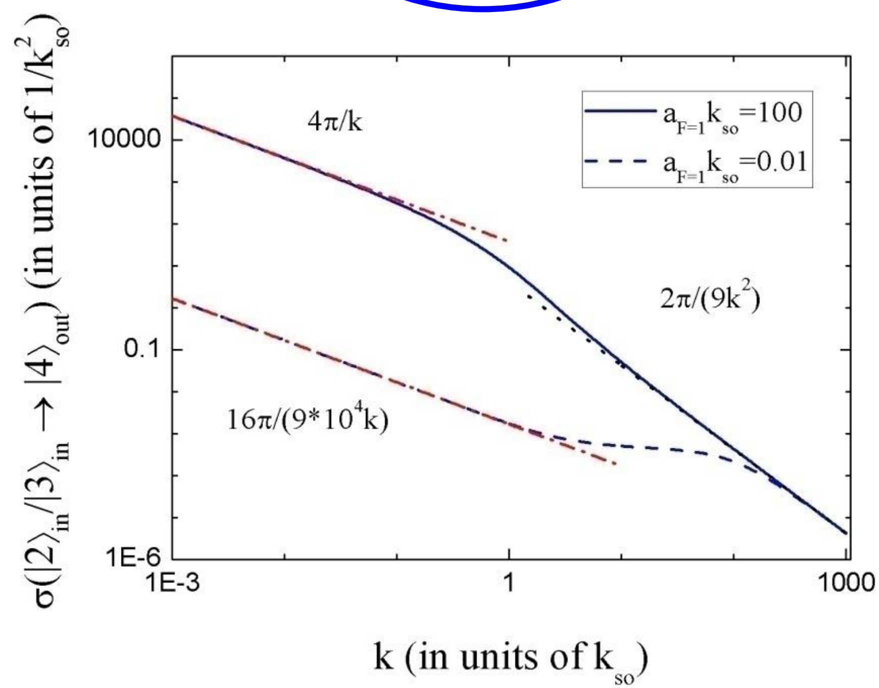
continued





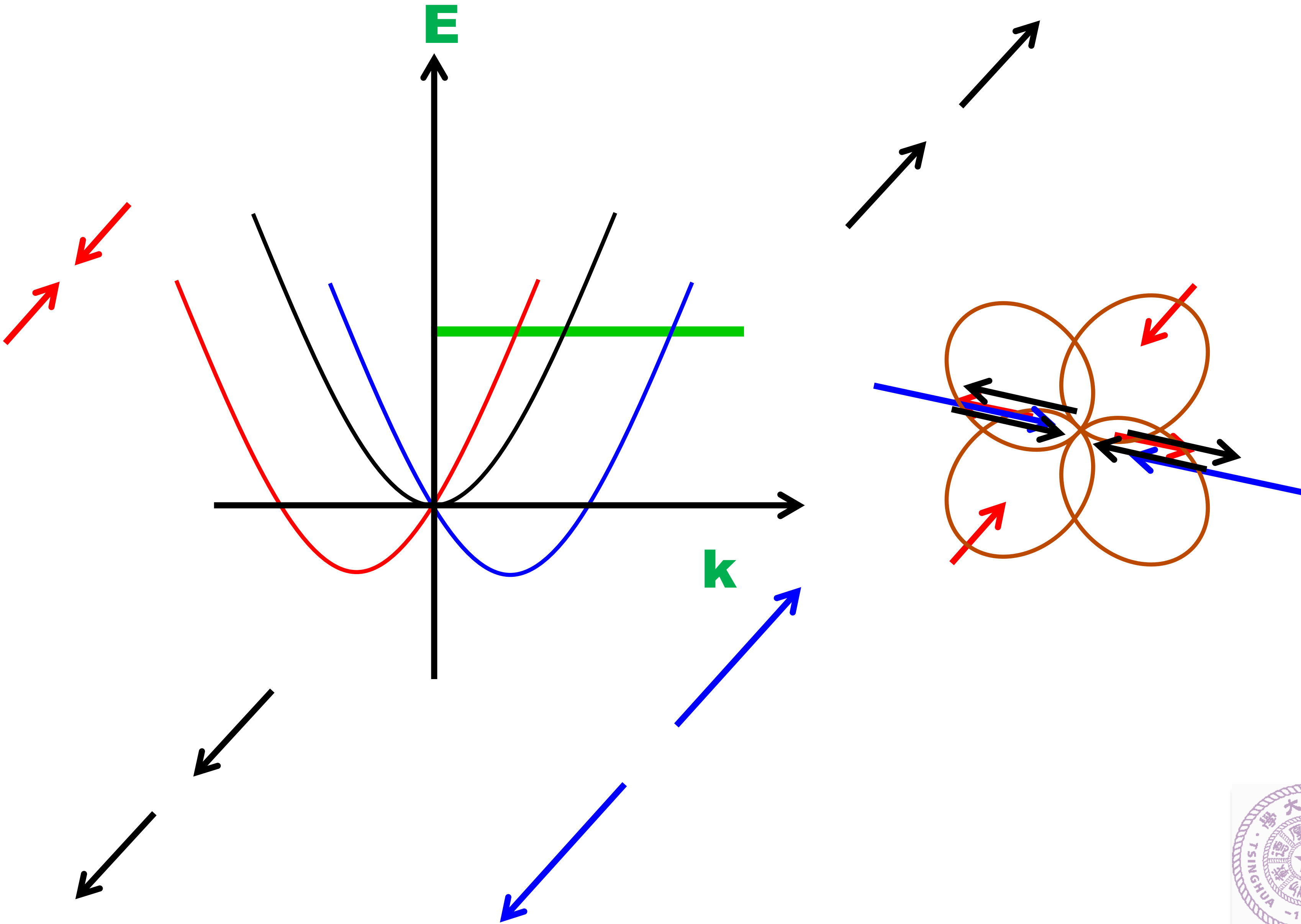


σ(|2⟩in/|3⟩in → |1⟩out) (in units of 1/k²so)
0.1
1E-5
1E-9
1E-13
1E-3
1
1000
πk³/4
2π/(9k²)
πk³/(9*10⁴)
aF=1kso=100
aF=1kso=0.01
k (in units of kso)



σ(|2⟩in/|3⟩in → |4⟩out) (in units of 1/k²so)
10000
0.1
1E-6
1E-3
1
1000
4π/k
2π/(9k²)
16π/(9*10⁴k)
aF=1kso=100
aF=1kso=0.01
k (in units of kso)

4 scattering channels: bosons



Summary

- **Different dispersion/threshold**

- **Revised Wigner threshold**

  higher than s-wave survives

  low momentum cutoff at universality limit

  isotropic for fermions (novel pairing symmetry)

  anisotropic for bosons/distinguishable particles

- **Development of handedness**

  preferential scattering into lower helicity branches

- **More surprises for E<0**

- **1D, 2D, anisotropic SOC?**