Note: The following contains
-a somewhat dirty version of what I wrote on the blackboard
-some of what I said
-the plots that I showed at the end

INT, Avril 2014

Three-body loss rate in the non-degenerate Bose gas at infinite or negative scattering length within the universal zero-range theory

with D. Petrov , F. Chevy, C. Salomon et al. [PRL 110, 163202 (2013)]

I Experiment (ENS)

Bose gas , ("Li 7") $a = \infty$

"non-degenerate" $n\lambda^3 \ll 1$ $\qquad \lambda = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$ $\quad \hbar = 1,\ m = 1$

(ie $T \gg T_c^{ideal}$)

"universal regime" $\lambda \gg$ range

"3-body losses" 3 atoms $\longrightarrow$ $\underbrace{\text{1 deep dimer + 1 atom}}_{E_{kin} > \text{trap depth}}$

$$\dot{n} = -L_3(T)\cdot n^3$$

exp. data : $L_3(T) \propto \frac{1}{T^2}$

II Universal zero-range theory

- 3-body parameter R_0 "length"
- inelasticity parameter η_* "dimensionless"

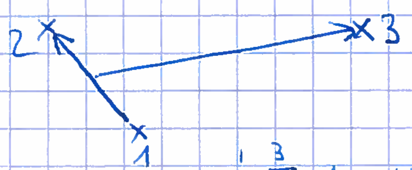
$$L_3(T) = \frac{1}{(k_B T)^2}\cdot\frac{\hbar^5}{m^3}\cdot fct\left(\frac{\lambda}{R_t}, \eta_*\right)$$

↳ log-periodic

"$T \in$ [$\simeq 2$ to $10\,\mu K$] $\Rightarrow \sim \frac{1}{4}$ period"

"Calculation" "convenient" coordinates :

$$\vec{R} \equiv \left(\vec{r}_1 - \vec{r}_2 \ ;\ \left(\vec{r}_3 - \frac{\vec{r}_1+\vec{r}_2}{2}\right)\cdot\frac{2}{\sqrt{3}}\right)$$



in CM frame : $\psi(\vec{R})$; $\sum_{i=1}^{3} \Delta_{\vec{r}_i}\psi = \Delta_{\vec{R}}\psi$

$R \equiv \|\vec{R}\| = \sqrt{\sum_{i<j} r_{ij}^2} \times$ cst "hyperradius" $\qquad r_{ij} = \|\vec{r}_i - \vec{r}_j\|$

$\vec{\Omega} \equiv \vec{R}/R$ "hyperangles"

3-body scattering state

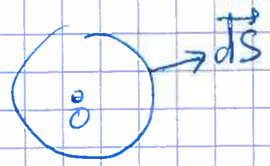
"Eigenstate of the 3-body pb. with the boundary cond. :"

$$\Psi_{\vec k}(\vec R) \underset{R\to\infty}{\simeq} \hat S\, e^{i\vec k\cdot\vec R} + f_{\vec k}(\vec\Omega)\cdot\frac{e^{ikR}}{R^{5/2}}$$

$L_3 \leftsquigarrow \Psi_{\vec k}$:

$$\vec j = \frac{\hbar}{m}\sqrt{2}\ \mathrm{Im}\left(\Psi^*\vec\nabla_{\vec R}\Psi\right)$$

$$\varphi_{out} = -\oiint \vec j\cdot d^5\vec S$$



$$\varphi_{out} \xrightarrow[\text{avg over }\vec k]{} L_3(k) \xrightarrow[\text{Thermal avg}]{} L_3(T)$$

Zero-range model

$$\begin{cases} (1)\ -\frac{\hbar^2}{m}\Delta_{\vec R}\Psi = E\Psi, & \text{if all } r_{ij}>0 \\ (2)\ \cdot\ \text{2-body contact condition :} & \Psi \underset{r_{ij}\to 0}{\propto} \frac{1}{r_{ij}} - \frac{1}{a} + o(1) \\ (3)\ \cdot\ \text{3-body co co :} & \Psi \underset{R\to 0}{\propto} \frac{1}{R^2}\left[\left(\frac{R}{R_0}\right)^{-is_0} - e^{-2\eta_*}\left(\frac{R}{R_0}\right)^{+is_0} + o(1)\right] \end{cases}$$

"We'll come back to ~~this~~ meaning of this later.
For now, just note that this depends on R_0 and η_*"

$a=\infty$

Separability $(R,\vec\Omega)$

$$\Psi(\vec R) = \sum_s \frac{F_s(R)}{R^2}\,\phi_s(\vec\Omega)$$

$$\Delta_{\vec R}\Psi = \frac{1}{R^2}\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \frac{1}{R^2}T_{\vec\Omega}\right)R^2\Psi$$

$$\begin{cases} (1) \longrightarrow \begin{cases} \boxed{T_{\vec\Omega}\phi_s(\vec\Omega) = -s^2\phi_s(\vec\Omega)} \\ -F''(R) - \frac{1}{R}F'(R) + \frac{s^2}{R^2}F(R) = \frac{m}{\hbar^2}E\cdot F(R) \end{cases} \\ (2) \longrightarrow \boxed{\text{~~boundary condition~~ c.c. on } \phi_s(\vec\Omega)} \\ (3) \longrightarrow \text{c. on } F(R) \end{cases}$$

spectrum for s : $\cdot\ s = is_0$, $s_0 = 1{,}00624\ldots$

$\cdot$ other $s \in \mathbb{R}$

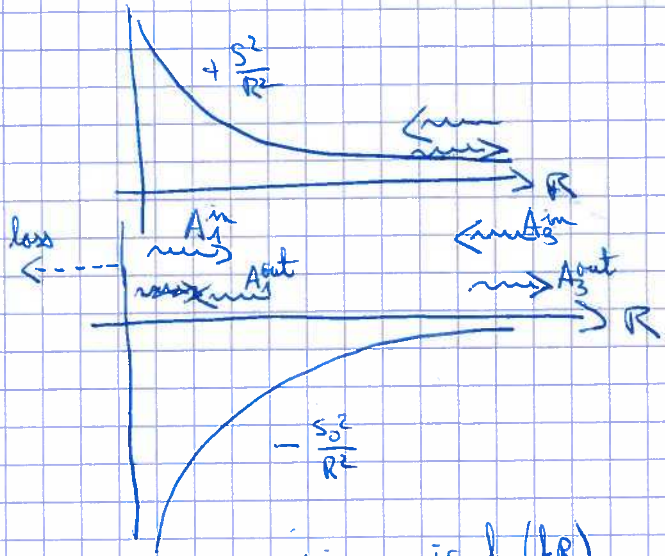
For $(f|g) \equiv \int d^5\Omega\ f(\vec\Omega)^*\, g(\vec\Omega)$, $(\phi_s)_s$ ONBasis

$$(\phi_s|\phi_{s'}) = \delta_{s,s'}$$

$$\sum_s |\phi_s)(\phi_s| = \mathbb{1}$$

"This allows us to write $\Psi = \sum_s \ldots$"

- $s \in \mathbb{R}$:



"not contribute to φ_{out}"

- $s = i s_0$:

$$F_{is_0}(R) \underset{R\to 0}{\simeq} \frac{A_1^{in}\, e^{is_0 \ln(kR)} + A_1^{out}\, e^{-is_0 \ln(kR)}}{\sqrt{2s_0}}$$

3-body c.c. : $\frac{A_1^{in}}{A_1^{out}} = -(kR_0)^{-2is_0} e^{-2\eta_*} =: \mathcal{A}$

$$F_{is_0}(R) \underset{R\to \infty}{\simeq} \frac{A_3^{in}\, e^{-ikR} + A_3^{out}\, e^{+ikR}}{\sqrt{2kR}}$$

$$\begin{pmatrix} A_1^{out} \\ A_3^{out} \end{pmatrix} = \underbrace{\begin{pmatrix} S_{11} & S_{13} \\ S_{31} & S_{33} \end{pmatrix}}_{\text{computed using } F_{is_0}(R) = \sum_{\pm} c_\pm J_{\pm i s_0}(kR)} \begin{pmatrix} A_1^{in} \\ A_3^{in} \end{pmatrix} \qquad (*)$$

$$S_{11} = -e^{-\pi s_0} \exp\left\{ 2i\left[s_0 \ln 2 + \arg \Gamma(1+is_0) \right] \right\}$$

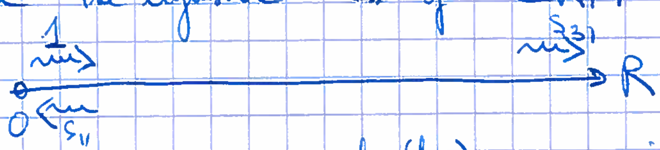
A_3^{in} given from $\left(\phi_{is_0} \middle| e^{i\vec{k}\cdot\vec{R}} \right)$

"4 unknowns, 4 eqs. → done"

"before discussing the result, treat finite a case, and discuss both cases at the end"

(X) $\boxed{\text{Finite negative } a}$ $\qquad \frac{1}{a} \leq 0$

"Consider the eigenstate of ZRM



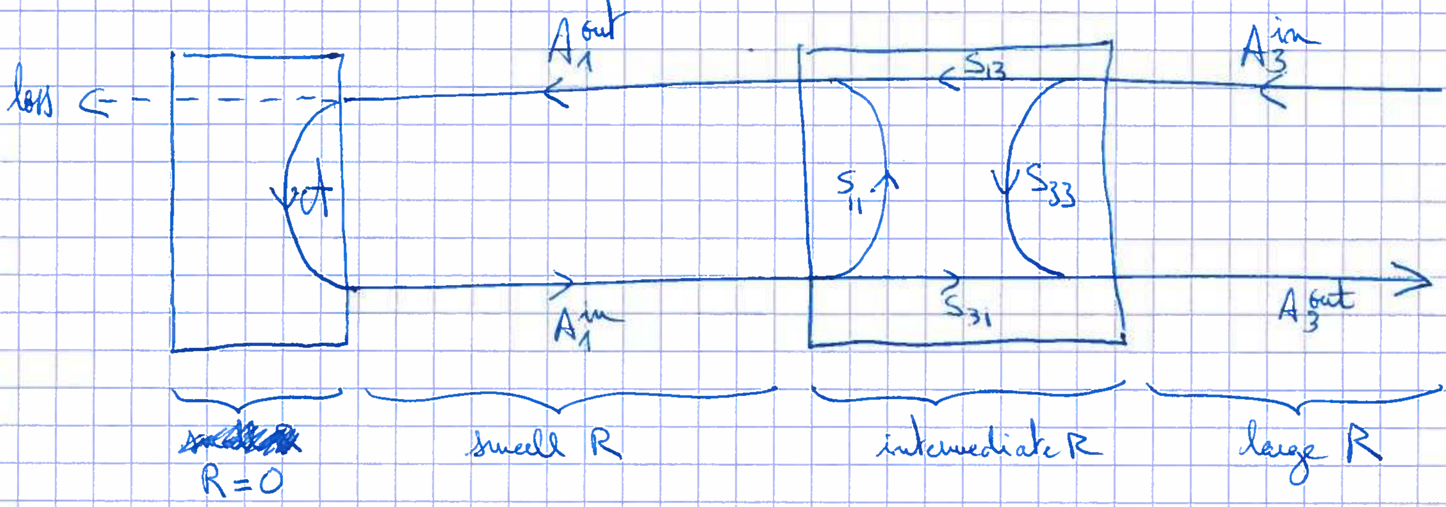
$$\begin{cases} \Psi(\vec{R}) \underset{R\to 0}{\simeq} \dfrac{e^{is_0 \ln(kR)} + S_{11}\, e^{-is_0 \ln(kR)}}{R^2 \sqrt{2s_0}} \cdot \phi_{is_0}(\vec{\Omega}) \\ \Psi \underset{R\to\infty}{\simeq} S_{31} \dfrac{e^{ikR}}{\sqrt{2kR}\, R^2} \cdot \Phi_3(\vec{\Omega}) \end{cases} \quad \text{"defines } \Phi_3\text{"}$$

$\left(\Phi_n(\vec\Omega)\right)_{n\geq 3}$ arbitrary ~~ONB~~ s.t.

$(\Phi_n)_{n\geq3}$ ONB

$$\begin{cases} \Psi(\vec R) \underset{R\to 0}{\simeq} \dfrac{A_1^{in}\, e^{is_0\ln(kR)} + A_1^{out}\, e^{-is_0 \ln(kR)}}{R^2\sqrt{2 s_0}}\, \phi_{is_0}(\vec\Omega) \\ \Psi(\vec R) \underset{R\to\infty}{\simeq} \displaystyle\sum_{n\geq3} \dfrac{A_n^{in}\, e^{-ikR}\,\Phi_n(\vec\Omega)^* + A_n^{out}\, e^{ikR}\,\Phi_n(\vec\Omega)}{\sqrt{2kR}\; R^2}\end{cases}$$

(*) holds.



$A_3^{in} \rightsquigarrow \left(\Phi_3^* \,\middle|\, e^{i\vec k\cdot\vec R}\right)$

"loss rate indep of Φ_3, due to hyper-triangular avg on $\hat k$"

$$L_3(k) = \underbrace{\frac{\hbar}{m}\cdot\frac{72\sqrt3\,\pi^2}{k^4}}_{L_3^{max}(k)} \cdot \left(1-e^{-4\eta_*}\right)\cdot \left|\underbrace{\frac{1}{1-s_{11}\mathcal{A}}}_{1+s_{11}\mathcal{A}+(s_{11}\mathcal{A})^2+\dots}\right|^2 \cdot \underbrace{\left(1-|s_{11}|^2\right)}_{|s_{13}|^2}$$

"L_3 if $A_3^{out}=0$" "multiple reflections"

$a=\infty$: $|s_{11}| = e^{-\pi s_0} \simeq \dots \ll 1 \Rightarrow$ "multiple reflections" $\simeq 1$

"no multiple reflection interf., no dependence on 3-body param via $\mathcal{A}$"

$s_{11}(ka)$ computed numerically by D. Petrov

If ~~For~~ ~~Eq~~ $\begin{cases} k(a) \to 0 : & |S_{11}| \to 1 \\ \eta_* \to 0 : & |A| \to 1 \end{cases}$

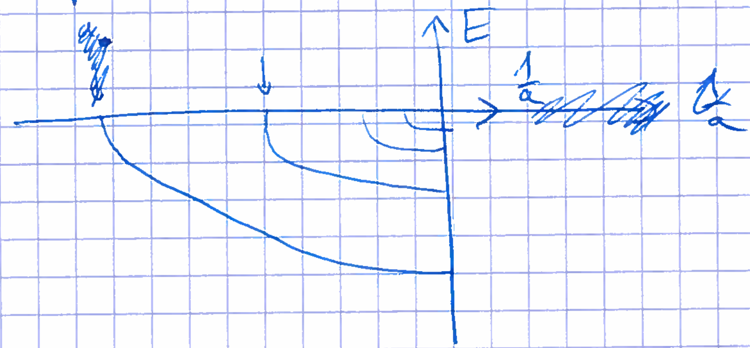
"This mirror becomes almost completely reflecting".

"This mirror also becomes of good quality

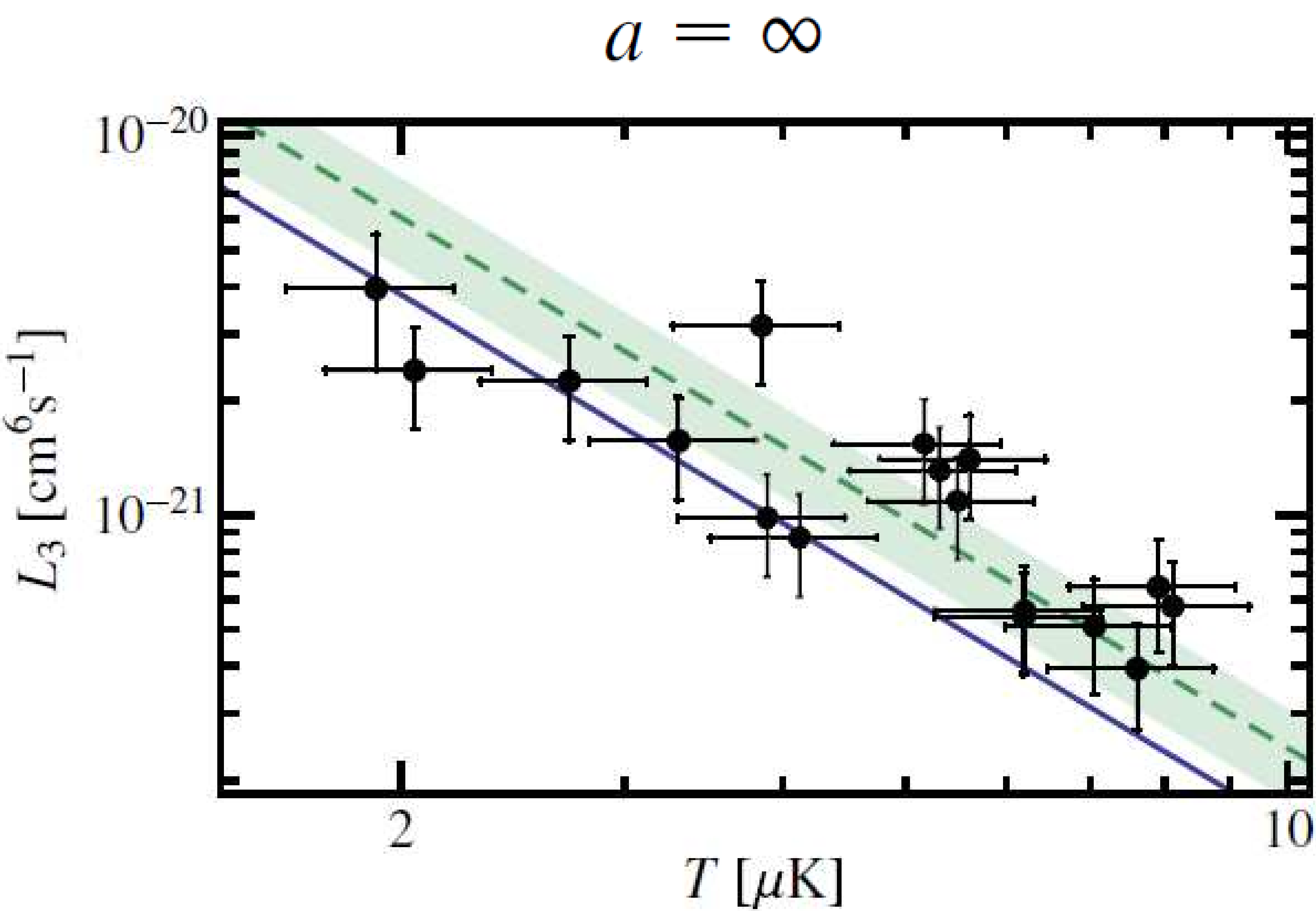
→ we get a high-finesse cavity

⇒ sharp resonances in L_3 ξ for a s.t. $E_{\text{Trimer}} = 0$

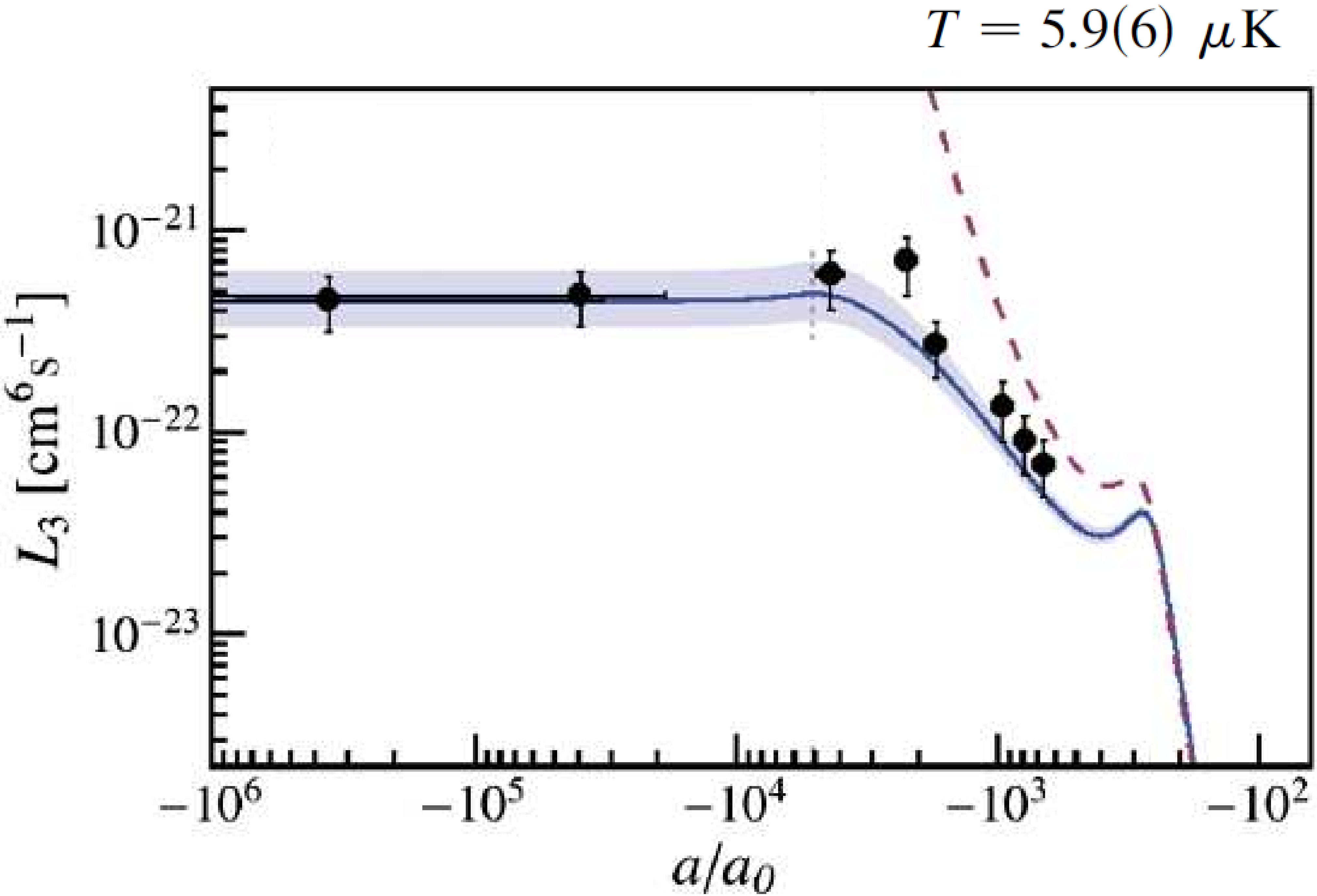
$$a = a_-(R_0) \times \Big(\underbrace{e^{\frac{\pi}{s_0}}}_{22.7}\Big)^n$$



Expts Plots



$a = \infty$
10^{-20}
10^{-21}
L_3 [cm^{6}s^{-1}]
2
10
T [μK]



$T = 5.9(6)\ \mu\mathrm{K}$
10^{-21}
10^{-22}
10^{-23}
$L_3\ [\mathrm{cm}^6\mathrm{s}^{-1}]$
-10^6
-10^5
-10^4
-10^3
-10^2
a/a_0

Huang, Sidorenkov, Grimm, Hutson, arXiv 2014