# Fully Perturbative Calculation of *nd* Scattering to Next-to-next-to-leading-order

Jared Vanasse

Duke University

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# Ingredients of Pionless Effective Field Theory

- For momenta $p < m_\pi$ pions can be integrated out as degrees of freedom and only nucleons and external currents are left.
- For any effective (field) theory one writes down all terms with degrees of freedom that respect symmetries.
- Develop a power counting to organize terms by their relative importance.
- Calculate respective observables up to a given order in the power counting.

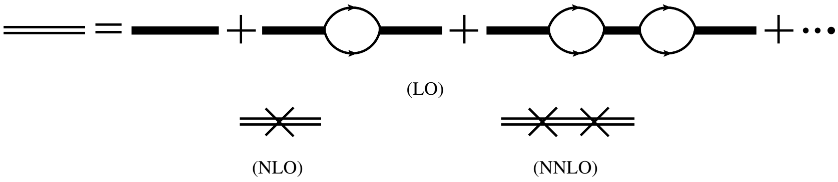
# Lagrangian

The Lagrangian in $\mathrm{EFT}_{\not\pi}$ is

$$\mathcal{L} = \hat{N}^\dagger\left(i\partial_0+\frac{\vec{\nabla}^2}{2M_N}\right)\hat{N} - \hat{t}_i^\dagger\left(i\partial_0+\frac{\vec{\nabla}^2}{4M_N}-\Delta_{(-1)}^{(^3S_1)}-\Delta_{(0)}^{(^3S_1)}\right)\hat{t}_i - \hat{s}_a^\dagger\left(i\partial_0+\frac{\vec{\nabla}^2}{4M_N}-\Delta_{(-1)}^{(^1S_0)}-\Delta_{(0)}^{(^1S_0)}\right)\hat{s}_a + y_t\left[\hat{t}_i^\dagger\hat{N}^TP_i\hat{N}+H.c.\right] + y_s\left[\hat{s}_a^\dagger\hat{N}^T\bar{P}_a\hat{N}+H.c.\right].$$

The projector $P_i = \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2$ ($\bar{P}_a = \frac{1}{\sqrt{8}}\tau_a\tau_2\sigma_2$) projects out the spin-triplet iso-singlet (spin-singlet iso-triplet) combination of nucleons.

The LO dressed deuteron propagator is given by a bubble sum




($Z$-parametrization) At LO coefficients are fit to reproduce the deuteron pole and at NLO to reproduce the residue about the deuteron pole.

$$\frac{1}{y_t^2} = \frac{M_N^2}{8\pi\gamma_t}\frac{Z_t-1}{1+(Z_t-1)}, \quad \Delta^{(^3S_1)}_{(-1)} = \frac{2y_t^2}{M_N}\frac{\gamma_t-\mu}{Z_t-1}, \quad \Delta^{(^3S_1)}_{(0)} = \frac{\gamma_t^2}{M_N}$$

The spin-triplet ("deuteron") and spin-singlet dibaryon propagator to NNLO are given by

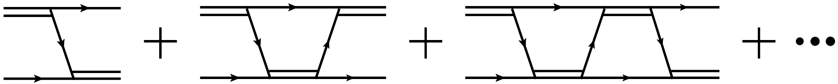
$$iD_{t,s}^{NNLO}(p_0,\vec{\mathbf{p}}) = \frac{4\pi i}{M_N y_{t,s}^2}\frac{1}{\gamma_{t,s}-\sqrt{\frac{\vec{\mathbf{p}}^2}{4}-M_N p_0 - i\epsilon}}\times$$

$$\times\left[\underbrace{1}_{\text{LO}}+\underbrace{\frac{Z_{t,s}-1}{2\gamma_{t,s}}\left(\gamma_{t,s}+\sqrt{\frac{\vec{\mathbf{p}}^2}{4}-M_N p_0 - i\epsilon}\right)}_{\text{NLO}}\right.$$

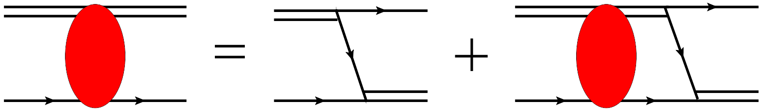
$$\left.+\underbrace{\left(\frac{Z_{t,s}-1}{2\gamma_{t,s}}\right)^2\left(\frac{\vec{\mathbf{p}}^2}{4}-M_N p_0-\gamma_t^2\right)}_{\text{NNLO}}+\cdots\right].$$

# Quartet Channel (*nd* Scattering)

At LO in the Quartet channel, *nd* scattering is given by an infinite sum of diagrams.



This infinite sum of diagrams can be represented by an integral equation.



The integral equation gives

$$
\begin{aligned}
\left(it^{ji}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{p}},h) &= \frac{y_t^2}{2}(\sigma^i\sigma^j)^{\beta}_{\alpha}\delta^b_a \frac{i}{-\frac{\vec{\mathbf{k}}^2}{4M_N}-\frac{\gamma_t^2}{M_N}+h-\frac{(\vec{\mathbf{k}}+\vec{\mathbf{p}})^2}{2M_N}+i\epsilon}+ \\
&+\frac{y_t^2}{2}(\sigma^i\sigma^k)^{\beta}_{\gamma}\delta^b_c\int\frac{d^4q}{(2\pi)^4}(it^{jk})^{\gamma c}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{q}},h+q_0)\times \\
&\times iD_t^{(0)}\left(\frac{\vec{\mathbf{k}}^2}{4M_N}-\frac{\gamma_t^2}{M_N}+h+q_0,\vec{\mathbf{q}}\right)\frac{i}{\frac{\vec{\mathbf{k}}^2}{2M_N}-h-q_0-\frac{\vec{\mathbf{q}}^2}{2M_N}+i\epsilon}\times \\
&\times\frac{i}{-\frac{\vec{\mathbf{k}}^2}{4M_N}-\frac{\gamma_t^2}{M_N}+2h+q_0-\frac{(\vec{\mathbf{q}}+\vec{\mathbf{p}})^2}{2M_N}+i\epsilon}.
\end{aligned}
$$

Projecting spin and isospin in the Quartet channel and projecting out in angular momentum gives

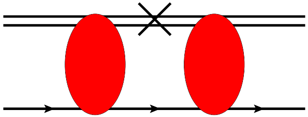
$$t_q^l(k,p) = -\frac{y_t^2 M_N}{pk} Q_l\left(\frac{p^2+k^2-M_N E-i\epsilon}{pk}\right) - \\ + \frac{2}{\pi}\int_0^\Lambda dq q^2 t_q^l(k,q)\frac{1}{\gamma_t - \sqrt{\frac{3\vec{\mathbf{q}}^2}{4} - M_N E - i\epsilon}}\frac{1}{qp}\times \\ Q_l\left(\frac{p^2+q^2-M_N E-i\epsilon}{pq}\right),$$

where

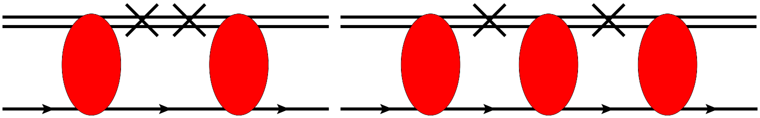
$$Q_l(a) = \frac{1}{2}\int_{-1}^{1} dx \frac{P_l(x)}{x+a}.$$

# Higher Orders

NLO correction is



NNLO corrections are



Note the second diagram contains full off-shell scattering amplitude.

The NNLO scattering amplitude is

$$t_{0,q}^{l}(k,p)+t_{1,q}^{l}(k,p)+t_{2,q}^{l}(k,p)=B_0^l(k,p)+B_1^l(k,p)+B_2^l(k,p)+$$
$$+\left(K_0^l(q,p,E)+K_1^l(q,p,E)+K_2^l(q,p,E)\right)\otimes\left(t_{0,q}^{l}(q,k)+t_{1,q}^{l}(q,k)+t_{2,q}^{l}(q,k)\right),$$

where

$$A(q)\otimes B(q)=\frac{2}{\pi}\int_0^\Lambda dq q^2 A(q)B(q).$$

The inhomogeneous and homogeneous terms are

$$B_0^l(k,p)=-\frac{y_t^2M_N}{pk}Q_l\left(\frac{p^2+k^2-M_NE-i\epsilon}{pk}\right),B_1^l(k,p)=B_2^l(k,p)=0$$

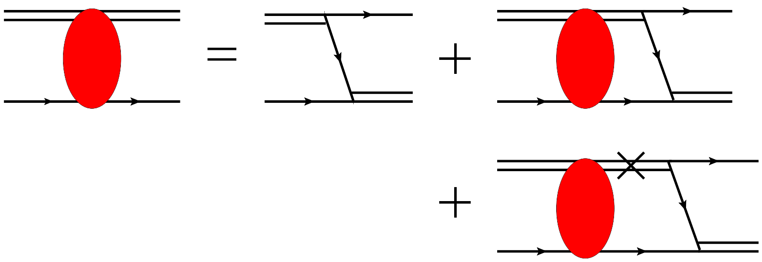
$$K_n^l(q,p,E)=-\frac{M_Ny_t^2}{4\pi}D_t^{(n)}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\frac{1}{qp}Q_l\left(\frac{q^2+p^2-M_NE-i\epsilon}{pq}\right)$$

# Partial Resummation Technique

Denoting $t^{l}_{NLO} = t^{l}_{0,q} + t^{l}_{1,q}$, for the partial resummation technique one finds

$$t^{l}_{NLO}(k,p) = B^{l}_{0}(k,p) + B^{l}_{1}(k,p) + (K^{l}_{0}(q,p,E) + K^{l}_{1}(q,p,E)) \otimes t^{l}_{NLO}(k,q),$$

with the diagrammatic representation



# New Full Perturbative technique

Picking out only LO pieces gives

$$t^{l}_{0,q}(k,p) = B^{l}_{0}(k,p) + K^{l}_{0}(q,p,E) \otimes t^{l}_{0,q}(k,q),$$

only NLO pieces

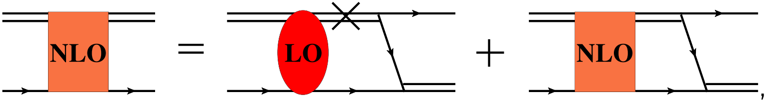
$$t^{l}_{1,q}(k,p) = B^{l}_{1}(k,p) + K^{l}_{1}(q,p,E) \otimes t^{l}_{0,q}(k,q) + K^{l}_{0}(q,p,E) \otimes t^{l}_{1,q}(k,q),$$

and only NNLO pieces

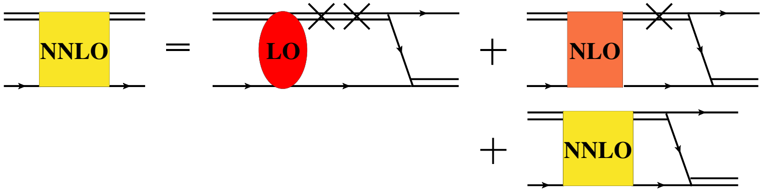
$$t^{l}_{2,q}(k,p) = B^{l}_{2}(k,p) + K^{l}_{2} \otimes t^{l}_{0,q}(k,q) + K^{l}_{1}(q,p,E) \otimes t^{l}_{1,q}(k,q) + K^{l}_{0}(q,p,E) \otimes t^{l}_{2,q}(k,q).$$

Terms are reshuffled to inhomogeneous term. Kernel at each order is the same

Diagrammatically NLO correction is now given by



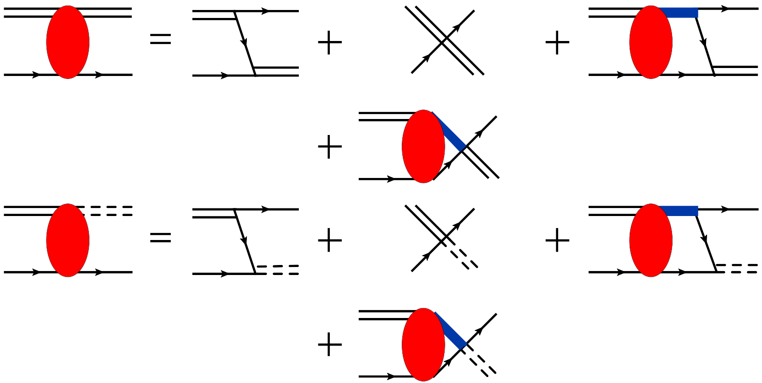

and NNLO correction by




Note all corrections are half off-shell.

# Doublet Channel *nd* scattering

At LO in the Doublet channel, *nd* scattering is given by a coupled set of integral equations



Using the perturbative technique, the Doublet *nd* scattering amplitude integral equation at LO is

$$\mathbf{t}_{0,d}^{l}(k,p)=\mathbf{B}_{0}^{l}(k,p)+\mathbf{K}_{0}^{l}(q,p,E)\otimes\mathbf{t}_{0,d}^{l}(k,q),$$

at NLO

$$\mathbf{t}_{1,d}^{l}(k,p)=\mathbf{B}_{1}^{l}(k,p)+\mathbf{K}_{1}^{l}(q,p,E)\otimes\mathbf{t}_{0,d}^{l}(k,q)+\mathbf{K}_{0}^{l}(q,p,E)\otimes\mathbf{t}_{1,d}^{l}(k,q),$$

and at NNLO

$$\begin{aligned}\mathbf{t}_{2,d}^{l}(k,p)=\mathbf{B}_{2}^{l}(k,p)+\mathbf{K}_{2}^{l}(q,p,E)\otimes\mathbf{t}_{0,d}^{l}(k,q)+&\\ +\mathbf{K}_{1}^{l}(q,p,E)\otimes\mathbf{t}_{1,d}^{l}(k,q)+\mathbf{K}_{0}^{l}(q,p,E)\otimes\mathbf{t}_{2,d}^{l}(k,q).&\end{aligned}$$

The Equations are the same as in Quartet case but are now matrix equations in cluster configuration space.

The vector $\vec{\mathbf{t}}^{l}_{n,d}(k,q)$ is

$$\mathbf{t}^{l}_{n,d}(k,q) = \begin{pmatrix} t^{l}_{n,Nt\to Nt}(k,q) \\ t^{l}_{n,Nt\to Ns}(k,q) \end{pmatrix}.$$

The inhomogeneous term is

$$\mathbf{B}^{l}_{0}(k,p) = \begin{pmatrix} \frac{y_t^2 M_N}{pk} Q_l\left(\frac{p^2+k^2-M_N E-i\epsilon}{pk}\right) + \mathcal{H}_0(E,\Lambda)\delta_{l0} \\ -\frac{3y_t y_s M_N}{pk} Q_l\left(\frac{p^2+k^2-M_N E-i\epsilon}{pk}\right) - \mathcal{H}_0(E,\Lambda)\delta_{l0} \end{pmatrix}$$

$$\mathbf{B}^{l}_{1}(k,p) = \begin{pmatrix} \mathcal{H}_1(E,\Lambda)\delta_{l0} \\ -\mathcal{H}_1(E,\Lambda)\delta_{l0} \end{pmatrix}, \mathbf{B}^{l}_{2}(k,p) = \begin{pmatrix} \mathcal{H}_2(E,\Lambda)\delta_{l0} \\ -\mathcal{H}_2(E,\Lambda)\delta_{l0} \end{pmatrix}.$$

The homogeneous term is

$$\mathbf{K}_n^l(q,p,E) = \mathbf{D}^{(n)}(E,\vec{\mathbf{q}})\frac{1}{qp}Q_l\left(\frac{q^2+p^2-M_NE-i\epsilon}{qp}\right)\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} + \delta_{l0}\sum_{j=0}^{n}\mathbf{D}^{(j)}(E,\vec{\mathbf{q}})\mathcal{H}_{n-j}(E,\Lambda)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},$$

where

$$\mathbf{D}^n(E,\vec{\mathbf{q}}) = \begin{pmatrix} D_t^{(n)}(E,\vec{\mathbf{q}}) & 0 \\ 0 & D_s^{(n)}(E,\vec{\mathbf{q}}) \end{pmatrix},$$

and the three-body force terms defined by

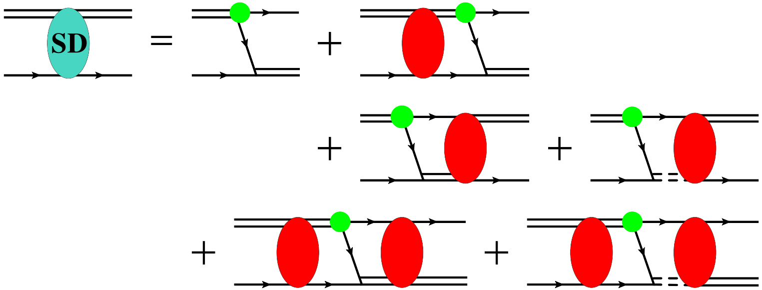
$$\mathcal{H}(E,\Lambda) = \frac{2H_0^{LO}(\Lambda)}{\Lambda^2} + \frac{2H_0^{NLO}(\Lambda)}{\Lambda^2} + \frac{2H_0^{N^2LO}(\Lambda)}{\Lambda^2} + \frac{2H_2^{N^2LO}(\Lambda)}{\Lambda^4}(M_NE+\gamma_d^2).$$

# SD-mixing Term

The SD-mixing Lagrangian is

$$\mathcal{L}_{Nd}^{SD} = y_{SD}\hat{d}_i^{\dagger}\left[\hat{N}^T\left((\overrightarrow{\partial}-\overleftarrow{\partial})^i(\overrightarrow{\partial}-\overleftarrow{\partial})^j-\frac{1}{3}\delta^{ij}(\overrightarrow{\partial}-\overleftarrow{\partial})^2\right)P_j\hat{N}\right]+H.c.$$

The SD-mixing amplitude is given by the sum of diagrams



The sum of all diagrams gives the amplitude

$$
\begin{aligned}
(t_{SD}^{xw})_{\alpha a}^{\beta b}(\vec{\mathbf{k}},\vec{\mathbf{p}}) &= \frac{4M_N}{\sqrt{8}}\mathbf{v}_p^T\,(\mathcal{K}^{xw})_{\alpha a}^{\beta b}(\vec{\mathbf{k}},\vec{\mathbf{p}})\mathbf{v}_p \\
&- \frac{4M_N}{\sqrt{8}}\int\frac{d^3q}{(2\pi)^3}\mathbf{v}_p^T(\mathcal{K}^{xy})_{\gamma c}^{\beta b}(\vec{\mathbf{q}},\vec{\mathbf{p}})\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\left((\mathbf{t}^{yw})_{\alpha a}^{\gamma c}(\vec{\mathbf{k}},\vec{\mathbf{q}})\right) \\
&- \frac{4M_N}{\sqrt{8}}\int\frac{d^3q}{(2\pi)^3}\left((\mathbf{t}^{xy})_{\gamma c}^{\beta b}(\vec{\mathbf{q}},\vec{\mathbf{p}})\right)^T\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)(\mathcal{K}^{yw})_{\alpha a}^{\gamma c}(\vec{\mathbf{k}},\vec{\mathbf{q}})\mathbf{v}_p \\
&+ \frac{4M_N}{\sqrt{8}}\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3\ell}{(2\pi)^3}\left((\mathbf{t}^{xz})_{\delta d}^{\beta b}(\vec{\ell},\vec{\mathbf{p}},)\right)^T\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right) \\
&\qquad (\mathcal{K}^{zy})_{\gamma c}^{\delta d}(\vec{\mathbf{q}},\vec{\ell})\mathbf{D}\left(E-\frac{\vec{\ell}^2}{2M_N},\vec{\ell}\right)\left((\mathbf{t}^{yw})_{\alpha a}^{\gamma c}(\vec{\mathbf{k}},\vec{\mathbf{q}})\right),
\end{aligned}
$$

where

$$\mathbf{v}_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

All angular dependence is contained in

$$(\mathcal{K}^{xw})^{\beta b}_{\alpha a}(\vec{\mathbf{q}},\vec{\ell}) = \frac{1}{\vec{\mathbf{q}}^2+\vec{\mathbf{q}}\cdot\vec{\ell}+\vec{\ell}^2-M_N E-i\epsilon}\times$$
$$\times\begin{pmatrix} \left(\mathcal{K}^{11\ xw}_{SD}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{q}},\vec{\ell}) & \left(\mathcal{K}^{12\ xw}_{SD}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{q}},\vec{\ell}) \\ \left(\mathcal{K}^{21\ xw}_{SD}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{q}},\vec{\ell}) & \left(\mathcal{K}^{22\ xw}_{SD}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{q}},\vec{\ell}) \end{pmatrix},$$

where

$$\left(\mathcal{K}^{11\ xw}_{PV}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{p}}) = y_t y_{SD}(\sigma^y\sigma^x)^\beta_\alpha\delta^b_a\left[(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^w(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^y-\frac{1}{3}\delta_{yw}(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^2\right]$$

$$\left(\mathcal{K}^{12\ xA}_{PV}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{p}}) = y_s y_{SD}(\sigma^y)^\beta_\alpha(\tau^A)^b_a\left[(2\vec{\mathbf{k}}+\vec{\mathbf{p}})^x(2\vec{\mathbf{k}}+\vec{\mathbf{p}})^y-\frac{1}{3}\delta_{yx}(2\vec{\mathbf{k}}+\vec{\mathbf{p}})^2\right]$$

$$\left(\mathcal{K}^{21\ Bw}_{PV}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{p}}) = y_t y_{SD}(\sigma^y)^\beta_\alpha(\tau)^b_a\left[(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^w(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^y-\frac{1}{3}\delta_{yw}(2\vec{\mathbf{p}}+\vec{\mathbf{k}})^2\right]$$

$$\left(\mathcal{K}^{22\ BA}_{PV}\right)^{\beta b}_{\alpha a}(\vec{\mathbf{k}},\vec{\mathbf{p}}) = 0.$$

The amplitude can be projected in partial waves of $\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}}$

$$t_{SD}{}^{JM}_{L'S',LS}(k,p) = \frac{1}{4\pi}\int d\Omega_k \int d\Omega_p \left(\mathscr{Y}^M_{J,L'S'}(\hat{\mathbf{p}})\right)^* t_{SD}(\vec{\mathbf{k}},\vec{\mathbf{p}})\mathscr{Y}^M_{J,LS}(\hat{\mathbf{k}}).$$

The projected amplitude is

$$\begin{aligned}
t_{SD}{}^{JM}_{L'S',LS}(k,p) = {} & \frac{M_N}{\sqrt{8\pi}}\mathbf{v}_p^T\mathcal{K}(k,p)^J_{L'S',LS}\mathbf{v}_p + \\
& -\frac{M_N}{2\sqrt{8}\pi^3}\int_0^\infty dq q^2 \mathbf{v}_p^T \mathcal{K}(q,p)^J_{L'S',LS}\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\left(\mathbf{t}^{JM}_{LS,LS}(k,q)\right) \\
& -\frac{M_N}{2\sqrt{8}\pi^3}\int_0^\infty dq q^2 \left(\mathbf{t}^{JM}_{L'S',L'S'}(q,p)\right)^T\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\mathcal{K}(k,q)^J_{L'S',LS}\mathbf{v}_p \\
& +\frac{M_N}{4\sqrt{8}\pi^5}\int_0^\infty dq q^2 \int_0^\infty d\ell \ell^2 \left(\mathbf{t}^{JM}_{L'S',L'S'}(p,\ell)\right)^T\mathbf{D}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\times \\
& \quad \times\mathcal{K}(q,\ell)^{JM}_{L'S',LS}\mathbf{D}\left(E-\frac{\vec{\ell}^2}{2M_N},\vec{\ell}\right)\left(\mathbf{t}^{JM}_{LS,LS}(k,q)\right).
\end{aligned}$$

$$\left[\mathcal{K}(k,p)^{J}_{L'S',LS}\right]_{11} = 4\pi\sqrt{\bar{S}\bar{S}'\bar{L}}\sqrt{\frac{10}{3}}\left(\delta_{S',1/2}+2\delta_{S',3/2}\right)C^{0,0,0}_{L,2,L'}(-1)^{2S'+S+L-J}\times$$

$$\times\left\{\begin{array}{ccc} 2 & 1 & 1 \\ 1/2 & S & S' \end{array}\right\}\left\{\begin{array}{ccc} S' & 2 & S \\ L & J & L' \end{array}\right\}]\frac{1}{kp}(k^2Q_{L'}(a)+4p^2Q_L(a))$$

$$+8\pi\sqrt{\bar{S}\bar{S}'\bar{L}}\left(\delta_{S',1/2}+2\delta_{S',3/2}\right)\sum_{L''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{L'',1,L'}\times$$

$$\times(-1)^{3/2-S'-L-L''}\sqrt{\bar{L''}}\left\{\begin{array}{ccc} 1/2 & 1 & S \\ 1 & L'' & L \\ S' & L' & J \end{array}\right\}Q_{L''}(a)$$

$$+8\pi\sqrt{\bar{S}\bar{S}'\bar{L}}\left(\delta_{S',1/2}+2\delta_{S',3/2}\right)\sum_{L''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{L'',1,L'}(-1)^{1/2+S'+L+L''}\times$$

$$\times\sqrt{\bar{L''}}\left\{\begin{array}{ccc} 1/2 & 1 & S' \\ L' & J & L'' \end{array}\right\}\left\{\begin{array}{ccc} L & 1 & L'' \\ 1/2 & J & S \end{array}\right\}Q_{L''}(a)$$

$$-\frac{16\pi}{3}\frac{1}{\sqrt{\bar{L}}}\left(\delta_{S',1/2}+2\delta_{S',3/2}\right)(-1)^{1/2-S'}\delta_{L,L'}\delta_{S,S'}\sum_{L''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{1,L'',L}\sqrt{\bar{L}''}Q_{L''}(a)$$

$$+(S\longleftrightarrow S')(L\longleftrightarrow L')(k\longleftrightarrow p)$$

$$\left[\mathcal{K}(k,p)^{J}_{L'S',LS}\right]_{21}=8\pi\sqrt{5}\sqrt{\bar{S}\bar{L}}\delta_{S'1/2}C^{0,0,0}_{L,2,L'}(-1)^{1+S+L-J}$$

$$\left\{\begin{array}{ccc}2 & 1 & 1\\ 1/2 & S & S'\end{array}\right\}\left\{\begin{array}{ccc}S' & 2 & S\\ L & J & L'\end{array}\right\}\frac{1}{kp}(k^2Q_{L'}(a)+4p^2Q_L(a))$$

$$+8\pi\sqrt{6}\sqrt{\bar{L}\bar{S}}\delta_{S'1/2}\sum_{L''}\sqrt{\bar{L}''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{L'',1,L'}\left\{\begin{array}{ccc}1/2 & 1 & S\\ 1 & L'' & L\\ S' & L' & J\end{array}\right\}Q_{L''}(a)$$

$$+8\pi\sqrt{6}\sqrt{\bar{L}\bar{S}}\delta_{S'1/2}\sum_{L''}(-1)^{1+L+L''}\sqrt{\bar{L}''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{L'',1,L'}\times$$

$$\times\left\{\begin{array}{ccc}L' & 1 & L''\\ 1/2 & J & S'\end{array}\right\}\left\{\begin{array}{ccc}L & 1 & L''\\ 1/2 & J & S\end{array}\right\}Q_{L''}(a)$$

$$+\frac{16\pi}{\sqrt{3}}(-1)^{L''-L}\sqrt{\frac{1}{\bar{L}'}}\sum_{L''}\sqrt{\bar{L}''}C^{0,0,0}_{L,1,L''}C^{0,0,0}_{L'',1,L'}\delta_{S'1/2}\delta_{S'S}\delta_{L'L}Q_{L''}(a)$$

The S matrix can be decomposed into irreducible representations of $\vec{\mathbf{J}}$ and parity. For $J = 1/2$ it is a $2\times 2$ unitary matrix

$$\mathbf{S}^{\frac{1}{2}+} = \begin{pmatrix} S^{\frac{1}{2}}_{2\frac{3}{2},2\frac{3}{2}} & S^{\frac{1}{2}}_{2\frac{3}{2},0\frac{1}{2}} \\ S^{\frac{1}{2}}_{0\frac{1}{2},2\frac{3}{2}} & S^{\frac{1}{2}}_{0\frac{1}{2},0\frac{1}{2}} \end{pmatrix}, \mathbf{S}^{\frac{1}{2}-} = \begin{pmatrix} S^{\frac{1}{2}}_{1\frac{1}{2},1\frac{1}{2}} & S^{\frac{1}{2}}_{1\frac{1}{2},1\frac{3}{2}} \\ S^{\frac{1}{2}}_{1\frac{3}{2},1\frac{1}{2}} & S^{\frac{1}{2}}_{1\frac{3}{2},1\frac{3}{2}} \end{pmatrix},$$

which is decomposed by

$$\mathbf{S}^{J\pi} = \left(\mathbf{u}^{J\pi}\right)^T e^{2i\boldsymbol{\delta}^{J\pi}}\mathbf{u}^{J\pi}$$

$$\boldsymbol{\delta}^{\frac{1}{2}+} = \begin{pmatrix} \delta^{\frac{1}{2}}_{2\frac{3}{2}} & 0 \\ 0 & \delta^{\frac{1}{2}}_{0\frac{1}{2}} \end{pmatrix}, \boldsymbol{\delta}^{\frac{1}{2}-} = \begin{pmatrix} \delta^{\frac{1}{2}}_{1\frac{1}{2}} & 0 \\ 0 & \delta^{\frac{1}{2}}_{1\frac{3}{2}} \end{pmatrix},$$

$$\mathbf{u}^{\frac{1}{2}+} = \begin{pmatrix} \cos\eta^{\frac{1}{2}+} & \sin\eta^{\frac{1}{2}+} \\ -\sin\eta^{\frac{1}{2}+} & \cos\eta^{\frac{1}{2}+} \end{pmatrix}, \mathbf{u}^{\frac{1}{2}-} = \begin{pmatrix} \cos\epsilon^{\frac{1}{2}-} & \sin\epsilon^{\frac{1}{2}-} \\ -\sin\epsilon^{\frac{1}{2}-} & \cos\epsilon^{\frac{1}{2}-} \end{pmatrix}$$

For $J \geq 3/2$ $\mathbf{S}$ is a unitary $3\times3$ matrix

$$\mathbf{S}^{J\pi} = \begin{pmatrix} S^J_{J\mp\frac{3}{2}\frac{3}{2},J\mp\frac{3}{2}\frac{3}{2}} & S^J_{J\mp\frac{3}{2}\frac{3}{2},J\pm\frac{1}{2}\frac{1}{2}} & S^J_{J\mp\frac{3}{2}\frac{3}{2},J\pm\frac{1}{2}\frac{3}{2}} \\ S^J_{J\pm\frac{1}{2}\frac{1}{2},J\mp\frac{3}{2}\frac{3}{2}} & S^J_{J\pm\frac{1}{2}\frac{1}{2},J\pm\frac{1}{2}\frac{1}{2}} & S^J_{J\pm\frac{1}{2}\frac{1}{2},J\pm\frac{1}{2}\frac{3}{2}} \\ S^J_{J\pm\frac{1}{2}\frac{3}{2},J\mp\frac{3}{2}\frac{3}{2}} & S^J_{J\pm\frac{1}{2}\frac{3}{2},J\pm\frac{1}{2}\frac{1}{2}} & S^J_{J\pm\frac{1}{2}\frac{3}{2},J\pm\frac{1}{2}\frac{3}{2}} \end{pmatrix}$$

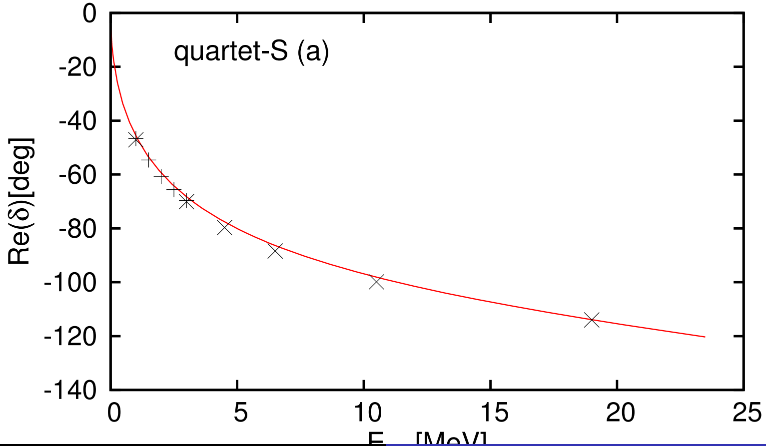
which is decomposed by

$$\boldsymbol{\delta}^{J\pi} = \begin{pmatrix} \delta^J_{J\mp\frac{3}{2}\frac{3}{2}} & 0 & 0 \\ 0 & \delta^J_{J\pm\frac{1}{2}\frac{1}{2}} & 0 \\ 0 & 0 & \delta^J_{J\pm\frac{1}{2}\frac{3}{2}} \end{pmatrix} \quad , \quad \mathbf{u}^{J\pi} = \mathbf{v}^{J\pi}\mathbf{w}^{J\pi}\mathbf{x}^{J\pi}$$

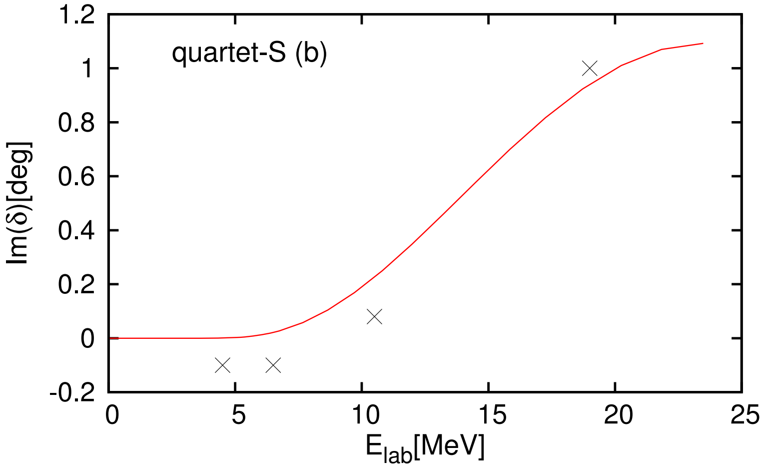
$$\mathbf{v}^{J\pi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon^{J\pi} & \sin\epsilon^{J\pi} \\ 0 & -\sin\epsilon^{J\pi} & \cos\epsilon^{J\pi} \end{pmatrix}, \mathbf{w}^{J\pi} = \begin{pmatrix} \cos\zeta^{J\pi} & 0 & \sin\zeta^{J\pi} \\ 0 & 1 & 0 \\ -\sin\zeta^{J\pi} & 0 & \cos\zeta^{J\pi} \end{pmatrix},$$

$$\mathbf{x}^{J\pi} = \begin{pmatrix} \cos\eta^{J\pi} & \sin\eta^{J\pi} & 0 \\ -\sin\eta^{J\pi} & \cos\eta^{J\pi} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

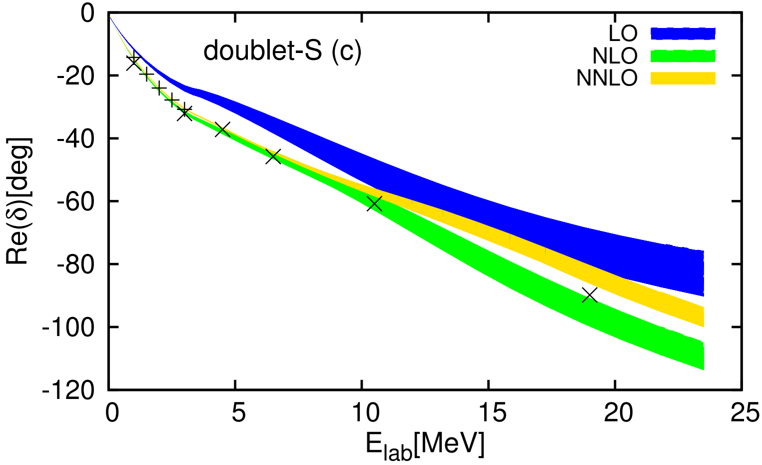
quartet S-wave phase shift at NNLO. Crosses are AV-18+UIX with hyperspherical harmonics method, and x's Bonn-B with Faddeev equations.



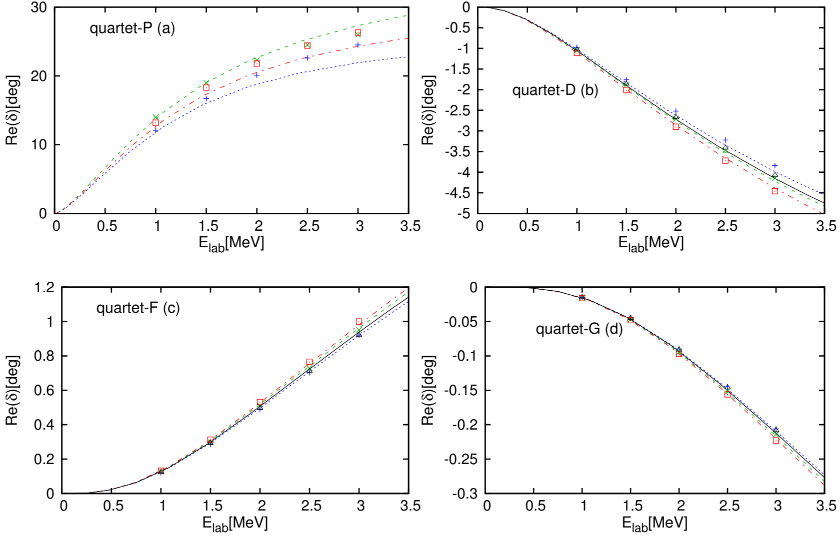
imaginary part of quartet S-wave phase shift at NNLO. Note here imaginary part is positive unlike in partial resummation technique.

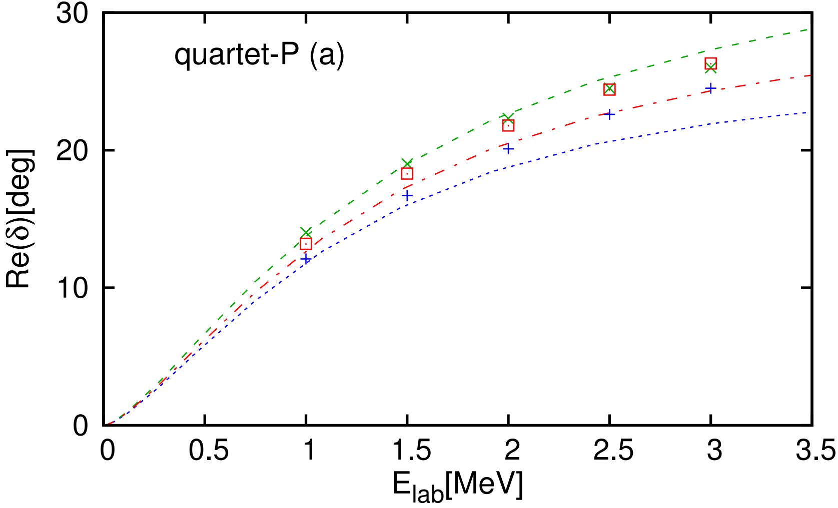


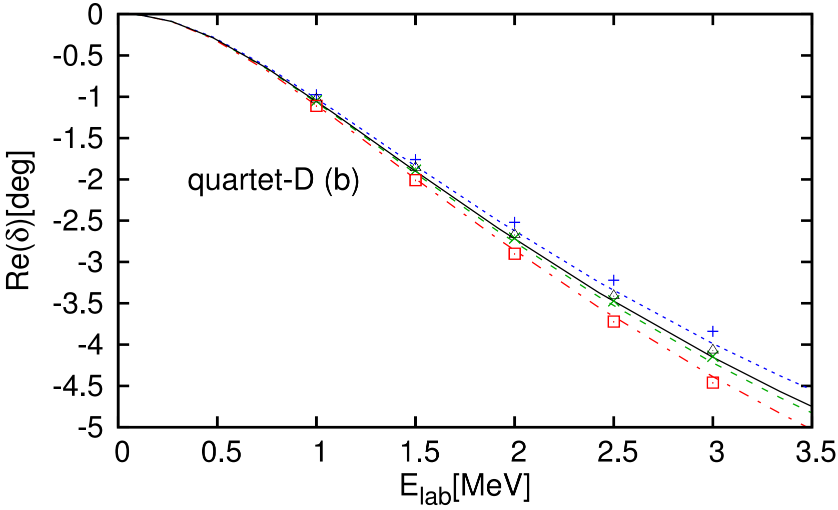
The bands are due to cutoff variation from $\Lambda = 200 - 1600$ MeV

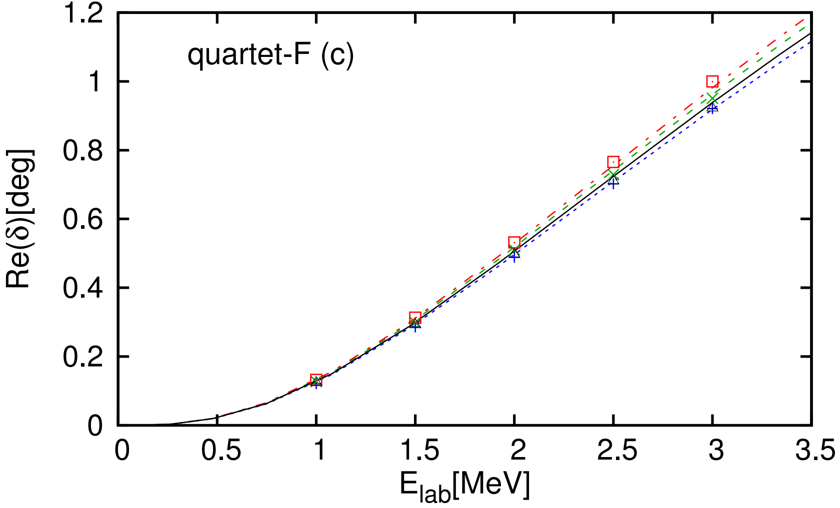


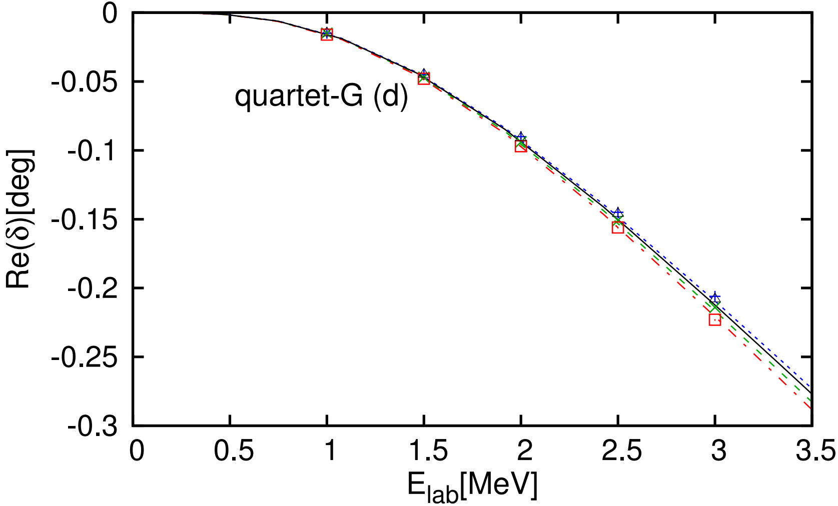
for $l=1$: $j = l-\frac{1}{2},\ l+\frac{1}{2},\ l+\frac{3}{2}$ :: for $l>1$: $j = l-\frac{3}{2},\ l-\frac{1}{2},\ l+\frac{1}{2},\ l+\frac{3}{2}$

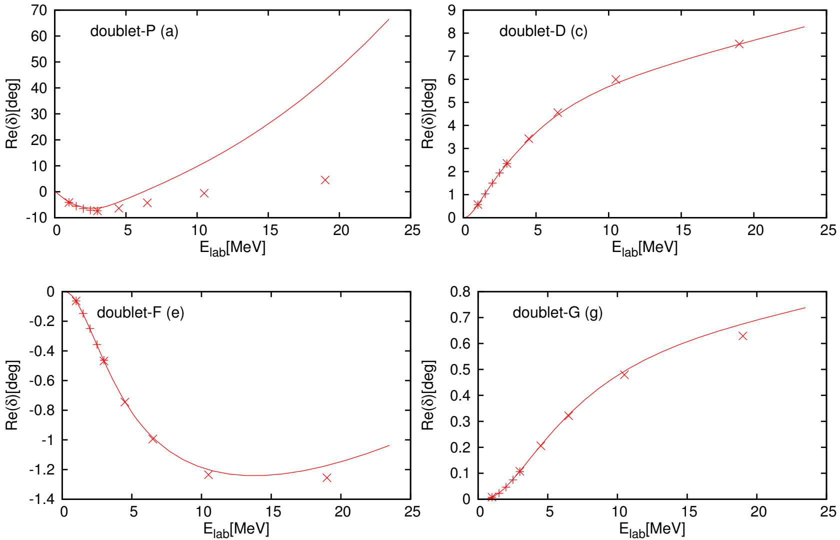


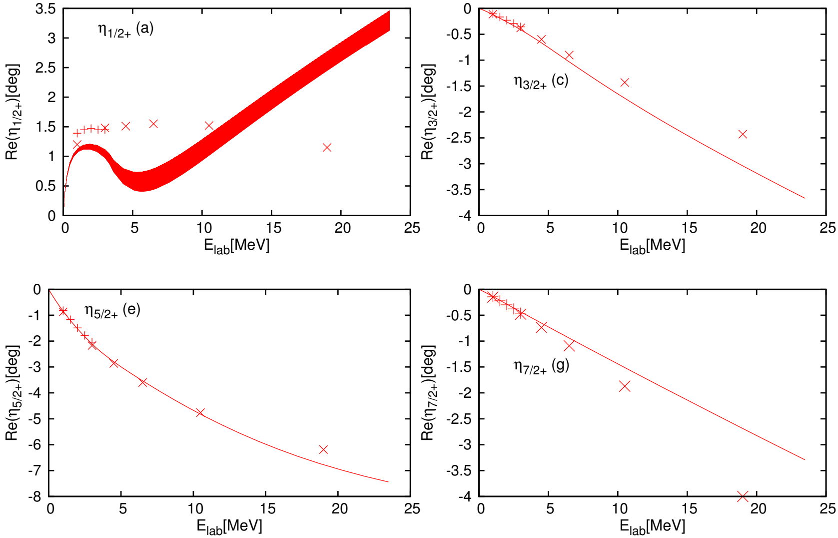

quartet-P (a)
Re(δ)[deg]
E_lab[MeV]
0
10
20
30
0
0.5
1
1.5
2
2.5
3
3.5

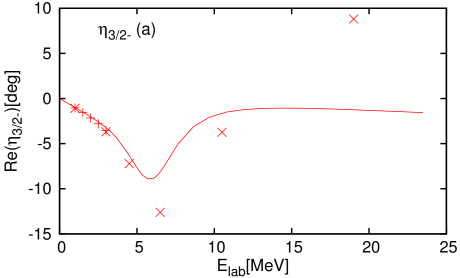

quartet-D (b)
Re(δ)[deg]
E_lab[MeV]
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
0
0.5
1
1.5
2
2.5
3
3.5

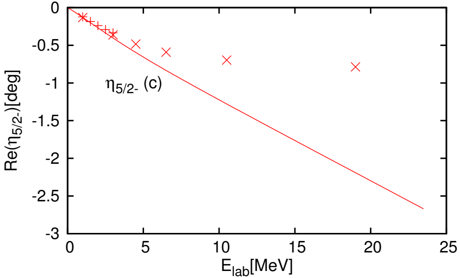

quartet-F (c)
Re(δ)[deg]
E_lab[MeV]
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5

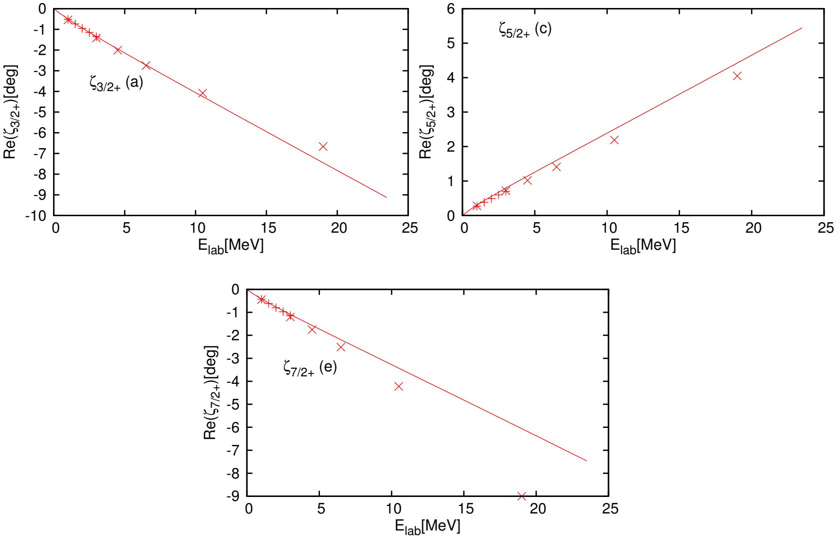

0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
Re(δ)[deg]
quartet-G (d)
0
0.5
1
1.5
2
2.5
3
3.5
E_lab[MeV]

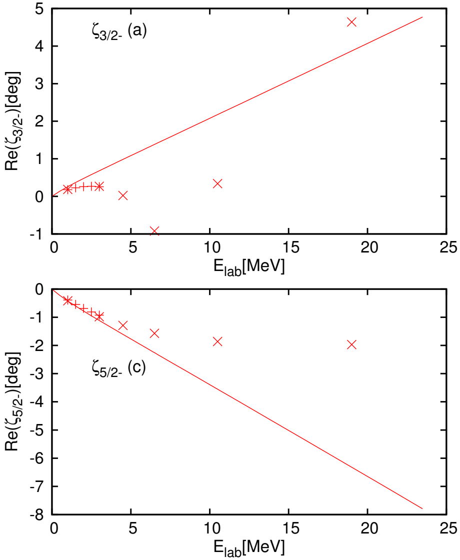

doublet-P (a)
doublet-D (c)
doublet-F (e)
doublet-G (g)
Re(δ)[deg]
E_lab[MeV]

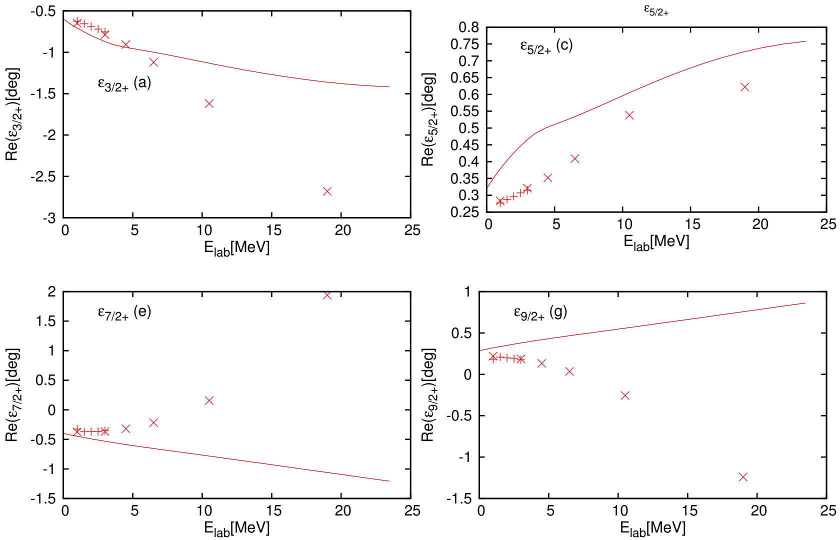

η1/2+ (a)
Re(η1/2+)[deg]
Elab[MeV]
η3/2+ (c)
Re(η3/2+)[deg]
Elab[MeV]
η5/2+ (e)
Re(η5/2+)[deg]
Elab[MeV]
η7/2+ (g)
Re(η7/2+)[deg]
Elab[MeV]

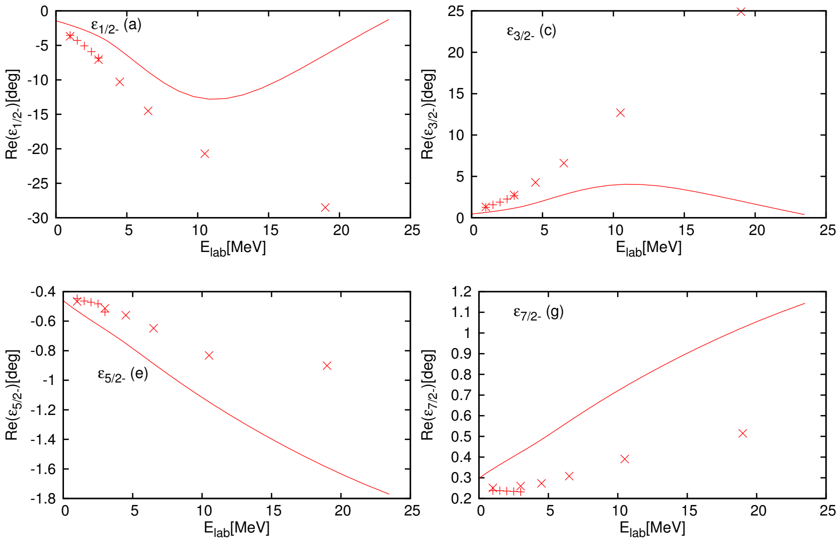

$\eta_{3/2-}$ (a)
$\mathrm{Re}(\eta_{3/2-})$[deg]
$E_{lab}$[MeV]
10
5
0
-5
-10
-15
0
5
10
15
20
25


$\eta_{5/2-}$ (c)
$\mathrm{Re}(\eta_{5/2-})$[deg]
$E_{lab}$[MeV]
0
-0.5
-1
-1.5
-2
-2.5
-3
0
5
10
15
20
25


Re(ζ3/2+)[deg]
ζ3/2+ (a)
Elab[MeV]
Re(ζ5/2+)[deg]
ζ5/2+ (c)
Elab[MeV]
Re(ζ7/2+)[deg]
ζ7/2+ (e)
Elab[MeV]


5
4
3
2
1
0
-1
$\zeta_{3/2-}$ (a)
Re($\zeta_{3/2-}$)[deg]
0
5
10
15
20
25
$E_{lab}$[MeV]
0
-1
-2
-3
-4
-5
-6
-7
-8
$\zeta_{5/2-}$ (c)
Re($\zeta_{5/2-}$)[deg]
0
5
10
15
20
25
$E_{lab}$[MeV]


$\varepsilon_{3/2+}$ (a)
Re($\varepsilon_{3/2+}$)[deg]
$E_{lab}$[MeV]
$\varepsilon_{5/2+}$
$\varepsilon_{5/2+}$ (c)
Re($\varepsilon_{5/2+}$)[deg]
$E_{lab}$[MeV]
$\varepsilon_{7/2+}$ (e)
Re($\varepsilon_{7/2+}$)[deg]
$E_{lab}$[MeV]
$\varepsilon_{9/2+}$ (g)
Re($\varepsilon_{9/2+}$)[deg]
$E_{lab}$[MeV]


$\varepsilon_{1/2-}$ (a)
$\text{Re}(\varepsilon_{1/2-})$[deg]
$E_{lab}$[MeV]
$\varepsilon_{3/2-}$ (c)
$\text{Re}(\varepsilon_{3/2-})$[deg]
$E_{lab}$[MeV]
$\varepsilon_{5/2-}$ (e)
$\text{Re}(\varepsilon_{5/2-})$[deg]
$E_{lab}$[MeV]
$\varepsilon_{7/2-}$ (g)
$\text{Re}(\varepsilon_{7/2-})$[deg]
$E_{lab}$[MeV]

# Conclusions and Future Directions

- Most phase shifts and mixing angles are described well by NNLO $\mathrm{EFT}_{\not{\pi}}$ at low energies.
- $\mathrm{N}^3$LO contributions will be important as they contain two-body P wave contact interactions, which may help fix certain mixing-angles and phase shifts.
- New technique makes higher order perturbative calculations much easier, which is important in order to calculate polarizaiton observables such as $A_y$.
- The new technique can be used to calculate any diagram with full off-shell scattering amplitudes. This makes calculations involving external currents much simpler to calculate (e.g. $^3H$ and $^3He$ Compton scattering and photodisintegration both parity conserving and violating).

# Numerical Techniques

$$t_i = B_i + \sum_j K_{ij} w_j t_j$$

$$\sum_j \left(\delta_{ij} - K_{ij} w_j\right) t_j = B_i$$

Integral equation is discretized and solved along contour in complex plane (Hetherington-Schick Method)