Feshbach Resonances and the Control of the Group Velocity of Atoms Propagating Through a Bose Condensate

The Matter Wave equivalent of *slow light.*

Eite Tiesinga

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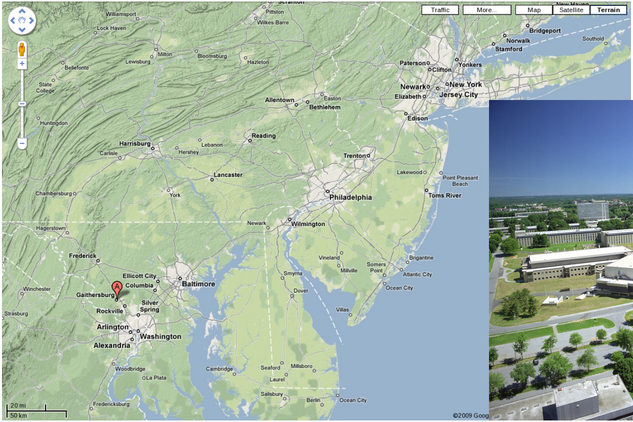
National Institute of Standards and Technology (NIST)
and University of Maryland, College Park, USA

April, 2014

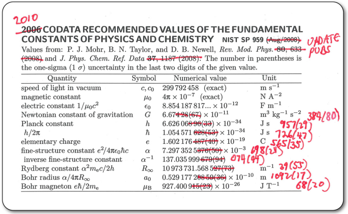
About NIST/JQI

- NIST's mission is to promote innovation and industrial competitiveness by advancing measurement science, standards, and technology to improve our quality of life.



Standards I

- Calibrate measuring tapes of all types in a 60-meter long tape tunnel.
- U.S. National Prototype kilogram. It contains of 90% platinum and 10% iridium and was made in 1889.
- Update the fundamental constants.



2010

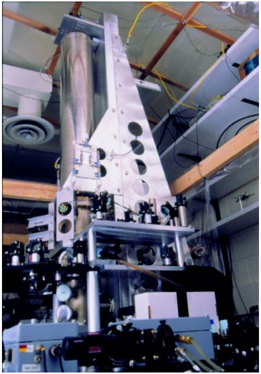
~~2006~~ CODATA RECOMMENDED VALUES OF THE FUNDAMENTAL CONSTANTS OF PHYSICS AND CHEMISTRY NIST SP 959 ~~(Aug/2008)~~ UPDATE PUBS

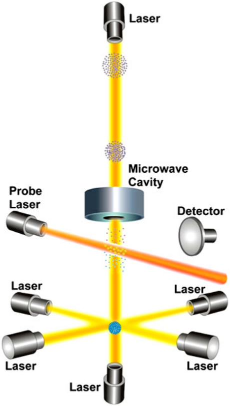
Values from: P. J. Mohr, B. N. Taylor, and D. B. Newell, *Rev. Mod. Phys.* ~~80, 633 (2008)~~ and *J. Phys. Chem. Ref. Data* ~~37, 1187 (2008)~~. The number in parentheses is the one-sigma ($1\ \sigma$) uncertainty in the last two digits of the given value.

Quantity	Symbol	Numerical value	Unit	
speed of light in vacuum	c, c_0	299 792 458 (exact)	$\mathrm{m\ s^{-1}}$	
magnetic constant	μ_0	$4\pi \times 10^{-7}$ (exact)	$\mathrm{N\ A^{-2}}$	
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	$\mathrm{F\ m^{-1}}$	
Newtonian constant of gravitation	G	$6.674\cancel{28(67)} \times 10^{-11}$	$\mathrm{m^3\ kg^{-1}\ s^{-2}}$	384(80)
Planck constant	h	$6.626\,068\cancel{96(33)} \times 10^{-34}$	J s	957(29)
$h/2\pi$	$\hbar$	$1.054\,571\,\cancel{628(53)} \times 10^{-34}$	J s	726(47)
elementary charge	e	$1.602\,176\,\cancel{487(40)} \times 10^{-19}$	C	565(35)
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5\cancel{376(50)} \times 10^{-3}$		698(24)
inverse fine-structure constant	α^{-1}	$137.035\,999\,\cancel{679(94)}$		074(44)
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	$10\,973\,731.568\,5\cancel{27(73)}$	$\mathrm{m^{-1}}$	39(55)
Bohr radius $\alpha/4\pi R_\infty$	a_0	$0.529\,177\,2\cancel{08\,59(36)} \times 10^{-10}$	m	1092(17)
Bohr magneton $e\hbar/2m_e$	μ_B	$927.400\,9\cancel{15(23)} \times 10^{-26}$	$\mathrm{J\ T^{-1}}$	68(20)

Standards II

- The best clocks rely on the measuring a hyperfine transition of laser-cooled cesium atoms. (NIST-F1)





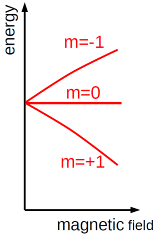
Collaborators and other recent work

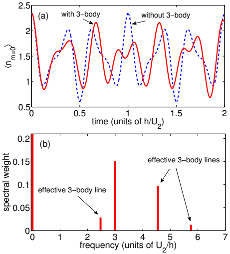
- The results are by student Ranchu Mathew and published in Phys. Rev. A **87**, 053608 (2013).
- With Prof. Phil Johnson at American University and Prof. Doerte Blume at Washington State University studied the few-body problem with field theoretical tools. In particular, we derived effective three- and four-body interactions for atoms in individual lattice sites. Ergo

$$\frac{1}{2}U_2 a^\dagger a^\dagger a a + \frac{1}{6}U_3 a^\dagger a^\dagger a^\dagger a a a$$

Phys. Rev. A **87**, 013423 (2013) and New. J. Phys. **14**, 053037 (2012).

- With postdoc Khan Mahmud we studied quantum dynamics of spin-1 bosons in optical lattices and looked at spin mixing and effective three-body interactions. Phys. Rev. A **87**, 053608 (2013).



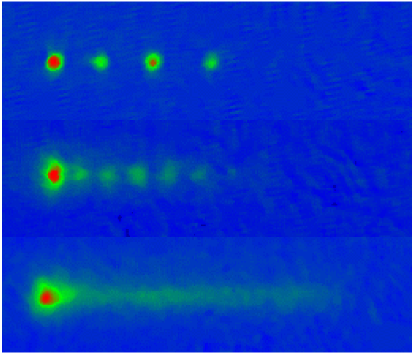


Collaborators

- With student Saurabh Paul I studied the collisional stability of atoms in excited bands of optical lattices.
- Post-doc Lei Jiang set up a time-dependent BdG code for interacting spin-orbit-coupled Fermions in a trap. We investigate the properties of topological states.
- Studied soliton dynamics with spin-1 Bose condensates with Prof. Indu I. Satija (George Mason University).
  Phys. Rev. A **87**, 033608 (2013).
- Experiment on spinor oscillations in *thermal* Bose gases with Dr. Paul Lett (NIST). Phys. Rev. Lett. **111**, 025301 (2013)

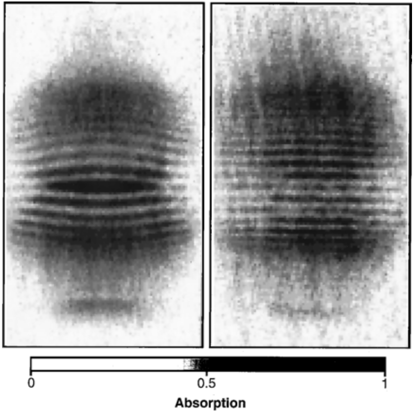
Motivation: Matter-wave Equivalences, Atom optics

- atom lasers



Motivation: Matter-wave Equivalences II

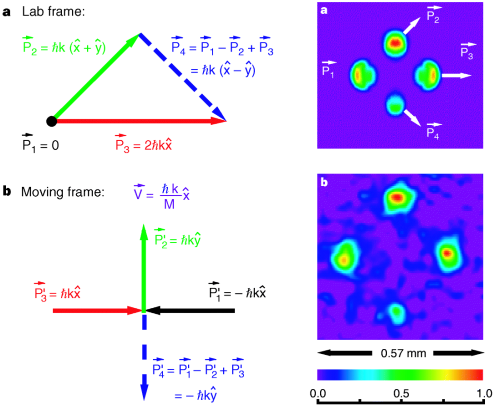
- Interference between two BECs. Showing the phase of a condensate.



M. R. Andrews et al, Science **275**, *637 (1997).*

Motivation: Matter-wave Four-wave mixing

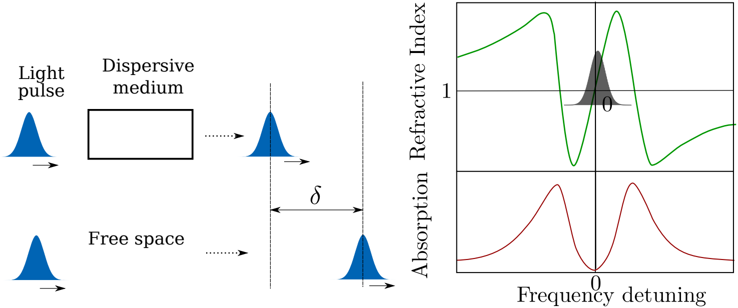
- Four-wave mixing with BECs. Atomic interactions lead to non-linear behavior.



L. Deng et al., Nature **398***, 218 (1999). Experiment conducted at NIST.*

Motivation: Slow light

- Sharp variation of refractive index with frequency of light.



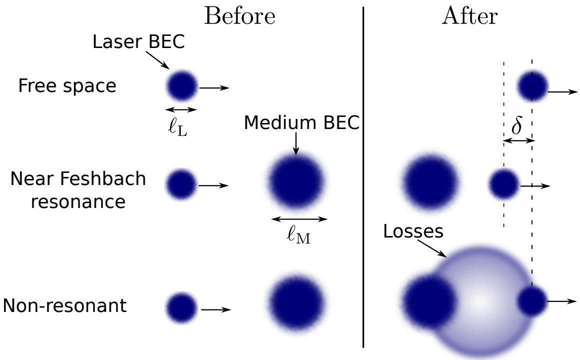
- Delay is determined by the group velocity in the medium

$$v_g = \frac{\partial \omega}{\partial k}$$

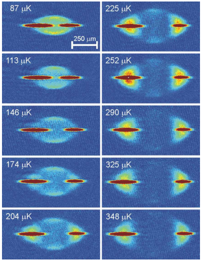
- Resonant Scattering of photons with the material in the dispersive medium.
- We expect analogous behavior in collisions of Bose condensates near a collisional resonance.

Problem statement

- A small moving BEC (*laser*) propagates through a larger stationary BEC (*medium*) and gets, hopefully, delayed.

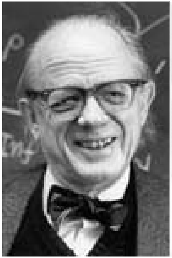


This halo was already observed



Thomas et al., Phys. Rev. Lett. **93** *173201 (2004); Buggle et al., Phys. Rev. Lett.* **93** *173202 (2004).*

Feshbach or Fano resonance



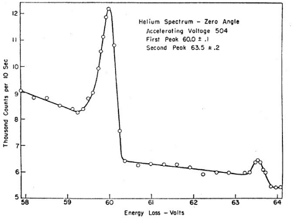
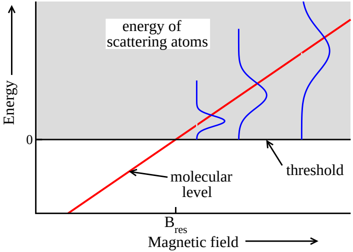


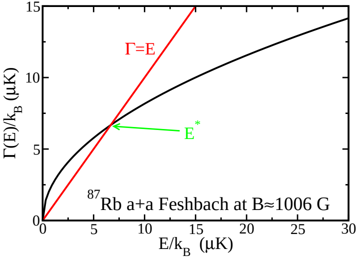
FIG. 2. Experimental results by Silverman and Lassettre[7] on the inelastic scattering of electrons by He.

- Image from Fano's 1961 paper explaining the energy loss of forward electron scattering from Helium. We temporarily form a negative ion. (from 2s2p level)
- Light scattering from an atom when photon energy is resonant with atomic transition. In other words, laser cooling. We temporarily form an atom-photon complex (excited state).
- A resonance in an ultra-cold collision of two atoms. We temporarily form a di-atomic molecule.

So what is special about our resonances?

- In ultra-cold experiments it is hard to change the collision energy (or temperature of the gas) relative to large natural energy scales in the collision. Another talk on van der Waals potentials.
- Rather we change an external magnetic field to move the position of the resonance.
- More crucial is that we are at the "edge" of a threshold.

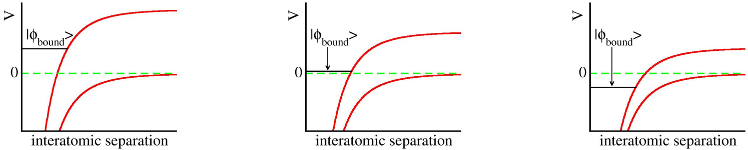




- The width of the resonance, $\Gamma(E)$, changes with collision energy. It is larger than the collision energy for small coll. energies.
- It is zero below threshold.

Molecular picture

- The collision is described by mixing between two (or more) potentials as a function of the separation between the atoms.

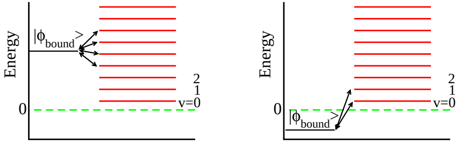


A decaying resonance

On resonance

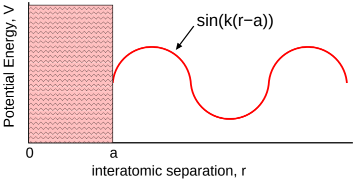
A molecular bound state

- Or consider the two atoms held in a harmonic trap

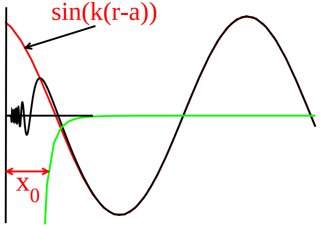


Scattering length

- The "effective size" of an atom at zero collision energy
- Best introduced with scattering from a hard wall potential at separation $a > 0$.



- In reality, scattering is more complex

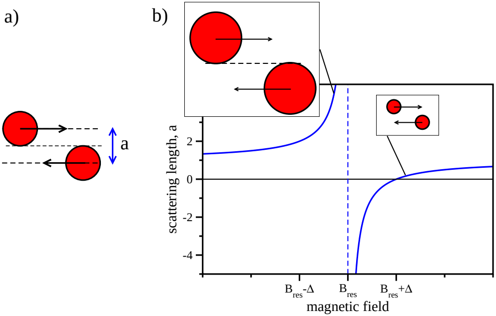


- In fact, $a < 0$ is allowed by quantum mechanics. This is captured by a delta-function potential

$$V(r) = 4\pi\frac{\hbar^2}{2\mu}\, a\, \delta(\vec{r})\frac{\partial}{\partial r}r$$

Scattering in a magnetic field

- For scattering near a magnetic resonance the scattering length depends on B



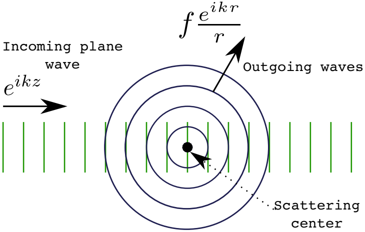
Boudewijn Verhaar

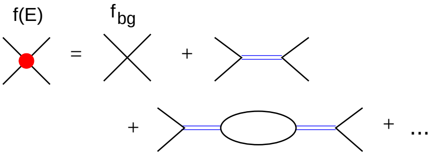
Henk Stoof

- The scattering length is infinite at $B = B_{\text{res}}$. It approaches a backbround value away from resonance. It is zero at $B = B_{\text{res}} + \Delta$.
- It is well described by $a = a_{\text{bg}} \left(1 - \dfrac{\Delta}{B - B_{\text{res}}} \right)$

Feshbach resonance at non-zero collision energy

- Feshbach and Fano, assuming an isolated resonance, derived a non-perturbative expression for scattering amplitude $f(E)$





- Scattering Amplitude, $f(E) = -a_{\rm bg} - \frac{1}{k}\frac{\hbar\Gamma(E)/2}{E - \underbrace{E_{\rm res}}_{\text{Resonance position}} + i\hbar\underbrace{\Gamma(E)}_{\text{Resonance width}}/2}$

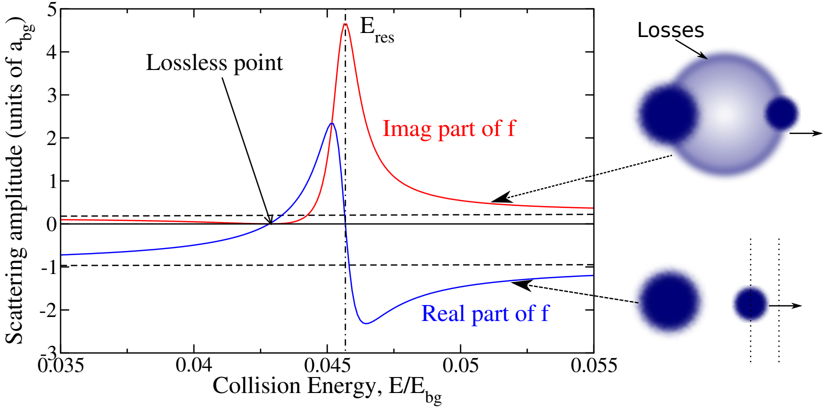
- The resonance width is

$$\hbar\Gamma(E) = 2\pi|\langle\phi_{\rm bound}|V|\phi_{\rm bg}, E\rangle|^2 \equiv 2\cdot\underbrace{\Gamma_0}_{\text{Reduced width}}\cdot(a_{\rm bg}k) \quad \propto \sqrt{E},$$

- In the limit of small energies $f(E) \to -a$.

Feshbach resonance

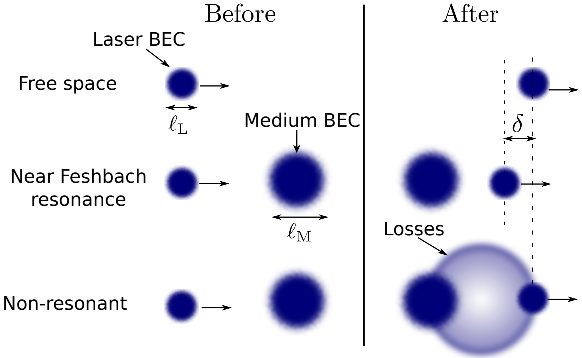
- Real and imaginary parts of scattering amplitude, $f(E)$, near a Feshbach resonance.



- The variation of Re f with energy, leads to the change in group velocity of *laser* BEC.

Remember our Problem statement

- A small moving BEC (*laser*) propagates through a larger stationary BEC (*medium*) with mean "collision energy" $E_{\rm coll}$, and corresponding mean wavevector k_0 and mean velocity v_0.



Description of a Bose condensate

- All atoms are in the same spatial wavefunction $\psi(\vec{x},t)$
- There exists a non-zero order parameter $\langle\hat{\Psi}(\vec{x},t)\rangle \equiv \psi(\vec{x},t)$.
- It satisfies the Gross-Pitaevskii (GP) or non-linear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\vec{x})\right)\psi(\vec{x},t) + \underbrace{g|\psi(\vec{x},t)|^2\psi(\vec{x},t)}_{\rm Interaction\ \ term}$$

  where $g = 4\pi\frac{\hbar^2}{2\mu}\,a$ and $V_{\rm ext}(\vec{x})$ is a trapping potential.
- The GP can not describe a collision near a Feshbach resonance.

Description of BEC near a Feshbach resonance

- We use a cumulant expansion[†]. In essence, we assume that variances, skewnesses, etc. become smaller and smaller

$$\psi(\vec{x},t) = \langle \hat{\Psi}(\vec{x},t) \rangle$$

$$\sigma(\vec{x},t) = \langle (\, \hat{\Psi}(\vec{x},t) - \langle \hat{\Psi}(\vec{x},t) \rangle \,)^2 \rangle$$
$$\kappa(\vec{x},t) = \langle (\, \hat{\Psi}(\vec{x},t) - \langle \hat{\Psi}(\vec{x},t) \rangle \,)^3 \rangle$$

Cumulants are moments only upto third order and I do not show averages that do not look like moments.

- We also assume that the momenta contained in the laser and medium condensate do not overlap. The condensate wave function can then be split in terms of *laser* and *medium* parts,

$$\psi(\vec{x},t) = \psi_L(\vec{x},t) + \psi_M(\vec{x},t)$$

† *T. Köhler, K. Burnett, Microscopic quantum dynamics approach to the dilute condensed Bose gas. Phys. Rev. A* **65**, *033601 (2002).*

Generalized interaction

- There are many terms to generalize. For example

self interaction of laser condensate,	$\lvert\Psi_L(\vec{x},t)\rvert^2\Psi_L(\vec{x},t)$
self interaction of medium condensate,	$\lvert\psi_M(\vec{x},t)\rvert^2\psi_M(\vec{x},t)$
interaction of laser condensate by medium,	$\lvert\psi_M(\vec{x},t)\rvert^2\psi_L(\vec{x},t)$

- The most-relevant GP interaction term $g|\psi_M(\vec{x},t)|^2\psi_L(\vec{x},t)$ is replaced by

$$-2\times 4\pi\frac{\hbar^2}{2\mu}\Psi^*_{\mathrm{M}}(\vec{x},t)\overbrace{f\left(i\hbar\partial/\partial t+\hbar^2\nabla^2/2M\right)}^{\text{Operator}\rightarrow}\Psi_{\mathrm{M}}(\vec{x},t)\Psi_{\mathrm{L}}(\vec{x},t)\,.$$

- The generalized interaction term is both non-local in time and space.
- For a resonance width that is small compared to energy of the laser condensate, a linear approximation of $f(E)$ about energy E_{coll} gives

$$\approx f(E_{\mathrm{coll}})+\left.\frac{\partial f}{\partial E}\right|_{E=E_{\mathrm{coll}}}\left(i\hbar\partial/\partial t+\hbar^2\nabla^2/2M-E_{\mathrm{coll}}\right).$$

Final Evolution equation

- Now we need two more approximations. That is that the medium condensate is much larger than the laser condensate and that, moreover, we can ignore the medium evolution.
- Local equation for the laser BEC becomes

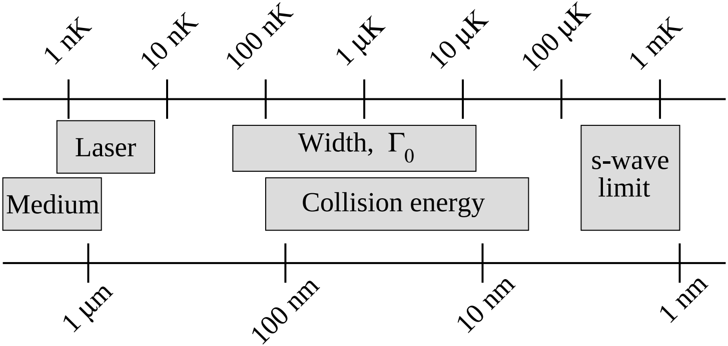
$$i\hbar\frac{\partial}{\partial t}\Psi_{\rm L}(\vec{x},t) = \left[-\frac{\hbar^2}{2m^*(\vec{x})}\nabla^2 + V_{\rm mf}(\vec{x}) + V_{\rm deriv}(\vec{x})\right]\Psi_{\rm L}(\vec{x},t).$$

- A spatially dependent mass $m^*(\vec{x})$ that is a function of

$$\left.\frac{\partial f}{\partial E}\right|_{E=E_{\rm coll}} \times |\Psi_M(\vec{x})|^2$$

- A mean-field GP-like potential $V_{\rm mf}(\vec{x}) \propto f(E_{\rm coll})|\Psi_M(\vec{x})|^2$
- A "derivative" potential $V_{\rm deriv}(\vec{x}) \propto \left.\frac{\partial f}{\partial E}\right|_{E=E_{\rm coll}} \times |\Psi_M(\vec{x})|^2$.
- The potentials are complex as $f(E)$ is complex. There are losses.

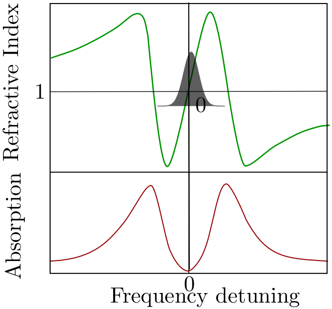
Energy and length scales in the problem

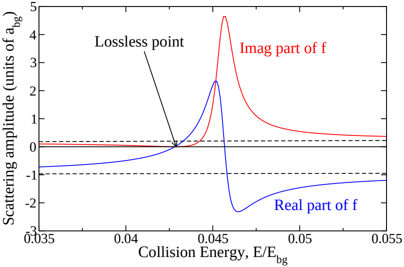


- The numbers are based on widths of "narrow" Feshbach resonances and typical condensate sizes.

Loss-less Regime

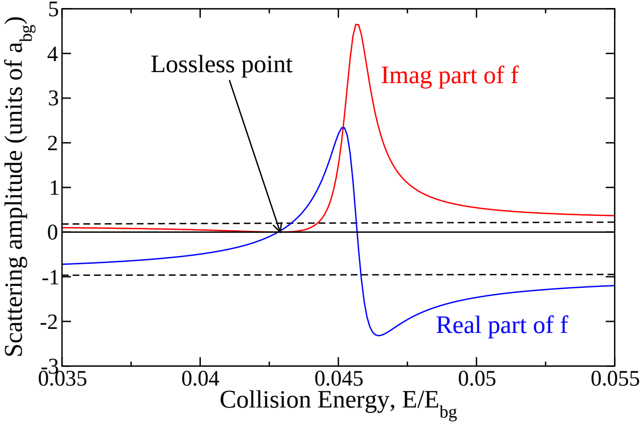
- In a slow-light experiment we use pulses with photon energies where the index of refraction changes fastest with frequency. The losses are minimal.
- For atoms we might think the same will hold. However, now atom loss is large.
- The compromise is to set the magnetic field or collision energy at the loss-less point: The imag. part of the scattering ampl. is zero and the Schrödinger equation is real.





At the loss-less point

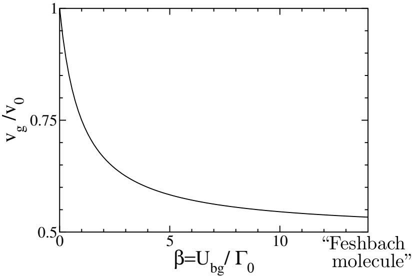
- Curiously $f(E_{\text{coll}}) = 0$ as follows from the optical theorem.
- The slope satisfies
$$\left.\frac{\partial f}{\partial E}\right|_{E=E_{\text{coll}}} = \frac{a_{\text{bg}}}{\Gamma_0}$$
which suggests using narrow resonances with small Γ_0.



Homogeneous medium at the loss-less point

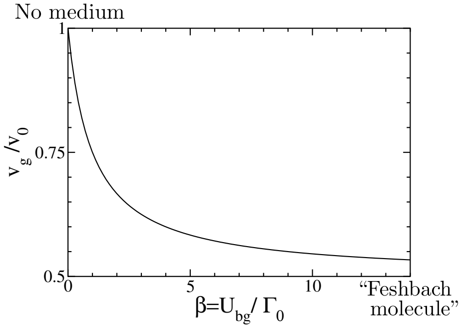
- The laser BEC satisfies $i\hbar\frac{\partial}{\partial t}\Psi_{\rm L}(\vec{x},t) = -\frac{\hbar^2}{2m^*}\nabla^2\Psi_{\rm L}(\vec{x},t)$ with an effective mass.

- The group velocity v_g in units of v_0, the velocity in free space.



- Along the x-axis $\beta = c|\Psi_{\rm M}|^2\,\frac{\partial f}{\partial E}$ is a convenient dimension-less parameter. $\beta = U_{\rm bg}/\Gamma_0$ with $U_{\rm bg} = 4\pi(\hbar^2/2\mu)a_{\rm bg}|\Psi_{\rm M}|^2$.
- For increasing β either the medium density increases or the resonance gets narrower. $(\Gamma_0$ decreases.$)$

Limit on the group velocity



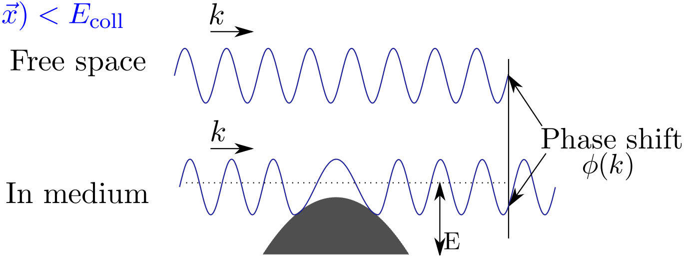
- We always have $v_0/2 < v_g < v_0$.
- The smallest value is half of the free space velocity. It is as if the Feshbach molecule, with twice the atomic mass, is propagating through the medium. Momentum conservation then suggests this lowest velocity.
- We have $\beta > 0$ for Feshach resonances. We can not make “fast” atoms.

Inhomogeneous medium at loss-less point

- Local equation for the laser BEC becomes

$$i\hbar\frac{\partial}{\partial t}\Psi_{\mathrm{L}}(\vec{x},t) = \left[-\frac{\hbar^2}{2m^*(\vec{x})}\nabla^2 + V_{\mathrm{deriv}}(\vec{x})\right]\Psi_{\mathrm{L}}(\vec{x},t).$$

- We are left with a 1D scattering problem where it turns out that $V_{\mathrm{deriv}}(\vec{x}) < E_{\mathrm{coll}}$



- The spatial shift/delay is given by $\delta = -\dfrac{\partial\phi}{\partial k}$.

WKB analysis

- We assume a stationary Thomas-Fermi profile for the *medium* BEC

$$|\Psi_{\mathrm{M}}(\mathbf{x})|^2 = n_{\mathrm{M}}(1 - x^2/\ell_{\mathrm{M}}^2).$$

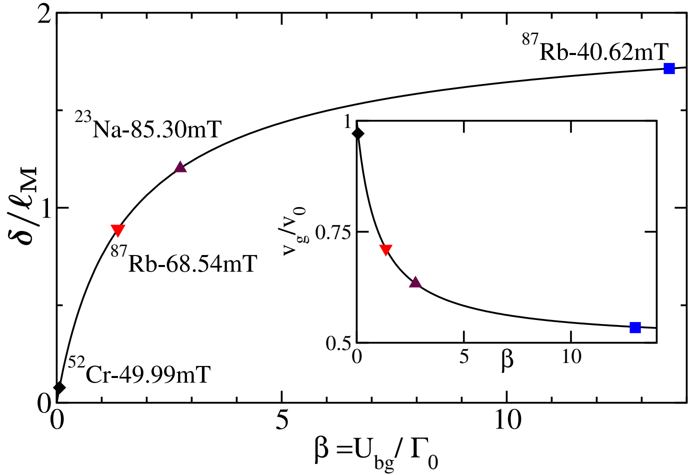
  with radius ℓ_M and peak density n_M.
- The WKB approximation

$$\delta/\ell_{\mathrm{M}} = 2\left(1 - \frac{\operatorname{arctanh}\left(\sqrt{\beta/(1+\beta)}\right)}{\sqrt{\beta(1+\beta)}}\right).$$

- In the limit $\beta \to \infty$ we find $\delta \to 2\ell_{\mathrm{M}}$.

Shift δ for selected resonances

- Shift δ (in units of Thomas-Fermi radius ℓ_{M}) for $n_{\mathrm{M}} = 10^{15}\mathrm{cm}^{-3}$.



- Markers correspond to four actual resonances of chromium, sodium, and rubidium.

Is the shift measurable?

- We assumed that the size of the medium condensate, ℓ_M, is large compared to that of the laser BEC, ℓ_L.
- We need shifts that are sizeable compared to size of the laser condensate. Discuss expansion of clouds during experiment.
- For $\ell_{\rm M} = 10\ell_{\rm L}$, the shift δ can be around $0.1\text{-}20\,\ell_{\rm L}$, which should be experimentally observable.
- Table of widths Δ, Γ_0, and shift δ for selected Feshbach resonances and $n_{\rm M} = 10^{15}{\rm cm}^{-3}$.

Atom	B_0 (mT)	Δ (mT)	$\hbar\Gamma_0/k_{\rm B}$ (μm)	$\delta/\ell_{\rm M}$
^{23}Na	119.5	-0.14	14	0.15
″	85.3	2.5×10^{-4}	0.64	1.20
^{87}Rb	91.17	1.3×10^{-4}	0.24	1.25
″	68.54	6×10^{-4}	0.54	0.89
″	40.62	4×10^{-5}	0.054	1.7
″	0.913	1.5×10^{-3}	2.0	0.38
^{52}Cr	49.99	0.008	22	0.078

Summary and extensions

- Analogous phenomenon of slowing of light occurs in colliding BECs near a magnetic Feshbach resonance.
- Using modified GP-equation, we estimated the shift δ of *laser* BEC.
- Details: R. Mathew and E. Tiesinga, *Controlling the group velocity of colliding atomic Bose-Einstein condensates using Feshbach resonances.* Phys. Rev. A **87**, 053608 (2013).
- What happens with optical Feshbach resonances?
  - Use a resonant laser to excite a molecular ro-vibrational level.
  - These levels decay by spontaneous emission. (Not true for magnetic Feshbach resonances.)
  - Losses never go to zero.
  - Is there a balance?