Universal Aspects of Dipolar Scattering

Christopher Ticknor

- May 15 at INT

Outline of Talk

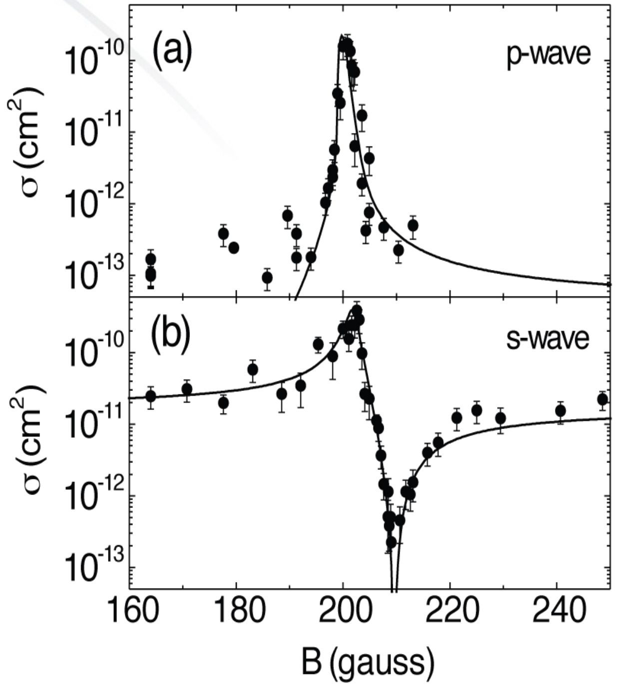
- Universal Dipolar Scattering
  - Theory of a long range scattering potential
- 2D Universal and tilted dipolar scattering
  - Interplay of interaction and geometry
- Universal 3-body dipolar recombination (not included)

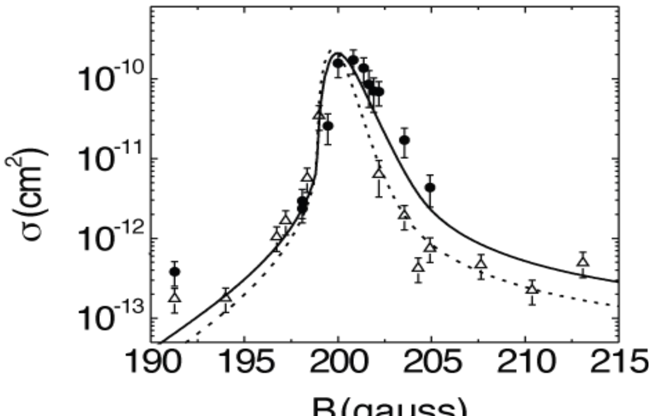
$$L_3 \propto D^4 \, , a^2 D^2$$

Ultracold Scattering

- Feshbach resonances
- s-wave
- p-wave

$$\sigma = \frac{4\pi}{k^2}\sin^2(\delta)$$

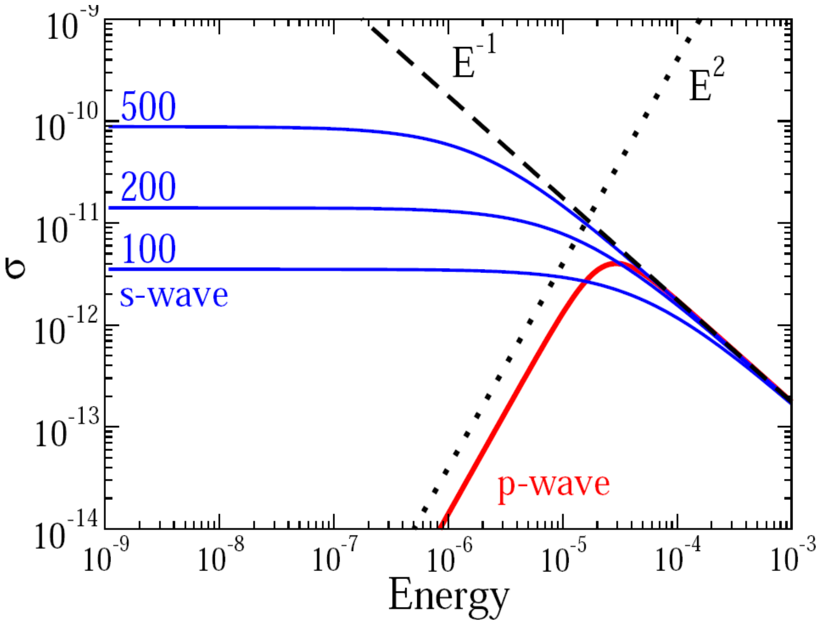




Regal et al. PRL **90** 053201 (2003)

Ultracold Scattering

- Threshold laws for partial waves
- s-wave, energy independent
- p-wave, suppressed



$$\delta \propto \langle \psi | 1/r^s | \psi \rangle$$

$$\delta \sim \alpha k^{2l+1} + \beta k^{s-2}$$

$$\sigma_l \propto E^p$$

$$p = min(2l, s-3)$$

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Ultracold Scattering

- But dipoles are different

$$s = 3$$

$$\sigma_l \propto E^0$$

- Long range
- Anisotropic
- Strong

$$V_{dd} = \frac{\vec{d} \cdot \vec{d} - 3(\hat{R} \cdot \vec{d})^2}{R^3}$$

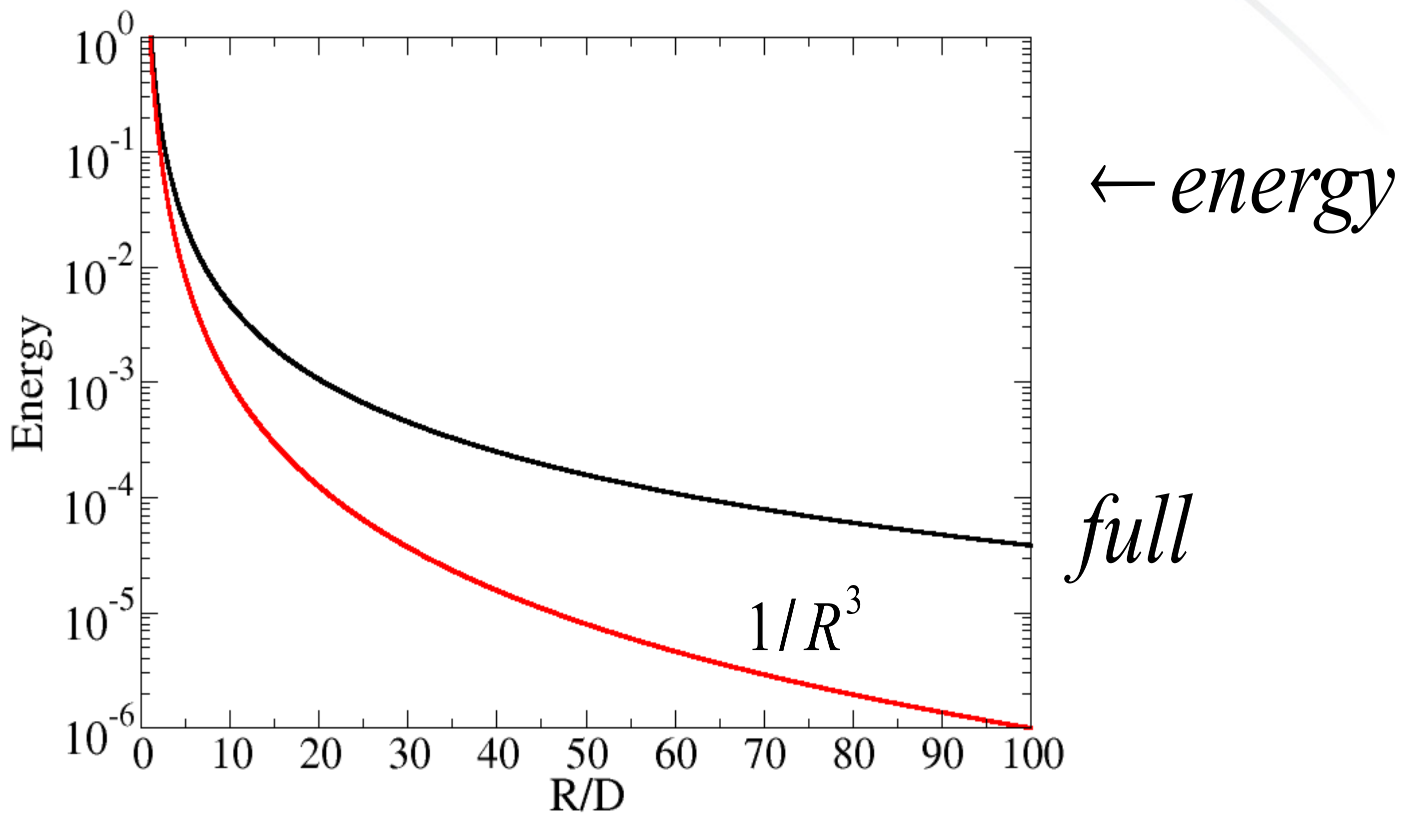
$$V_{dd} \sim \langle d \rangle^2 \frac{1 - 3\cos^2(\theta)}{R^3}$$

Dipolar Scattering

- The system can be magnetic or electric.
- Dipolar length scale $D = \langle d \rangle^2 m_r / \hbar^2$
- Energy scale $E_D = \langle d \rangle^2 / D^3 = \hbar^6 / \langle d \rangle^4 m_r^3$
- Strong interactions, many exotic theories.

Ultracold Scattering

- Long Range



Dipolar Scattering

- Equation of Motion

$$\left(-\frac{\nabla^2}{2\mathrm{m}_r}+\langle\mu\rangle^2\frac{1-3\cos^2(\theta)}{R^3}\right)\psi=E\psi$$

$$y=R/D$$
$$R\psi=\sum_l Y_{lm}F_l(R)$$

Dipolar Scattering

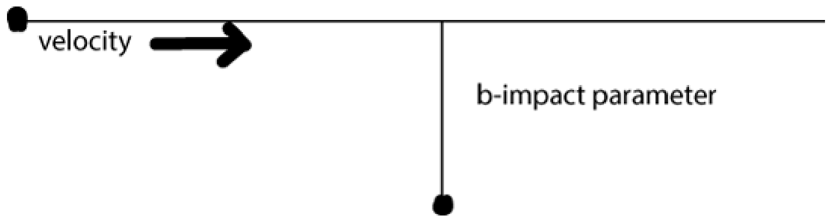
- Equation of Motion → Universal!
  - Except for short range $y_0 = R_0 / D \ll 1$

$$\left(-\frac{d^2}{2dy^2} + \frac{l(l+1)}{2y^2}\right) F_l + \sum_{l'} \frac{C_{ll'}}{y^3} F_{l'} = \frac{E}{E_D} F_l$$

$$C_{ll'} = \langle lm | 1 - 3\cos^2(\theta) | l'm \rangle$$

Dipolar Scattering

- Semi-classical solution from Eikonal approximation



$$\sigma \leftarrow \int_{line} V_{dd}(z, \vec{b}, \vec{\mu})\, dz$$

$$\frac{E}{E_D} \gg 1$$

$$\sigma = \frac{8}{\pi}\frac{D}{k}$$

Bohn, Cavagnero, and Ticknor NJP **11** 055039 (2009)

Ticknor PRL **100,** 133202 (2008)

Dipolar Scattering

- Threshold behavior $$\frac{E}{E_D} \ll 1$$
- Born Approximation: **all** dipolar coupled partial waves have an energy independent cross section!

$$\sigma_{l\,l'} \propto \left| \langle F_l | 1/R^3 | F_{l'} \rangle \langle lm | 1 - 3\cos^2(\theta) | l'm \rangle \right|^2$$

$$\sigma_e = 1.117\, D^2$$
$$\sigma_o = 3.351\, D^2$$

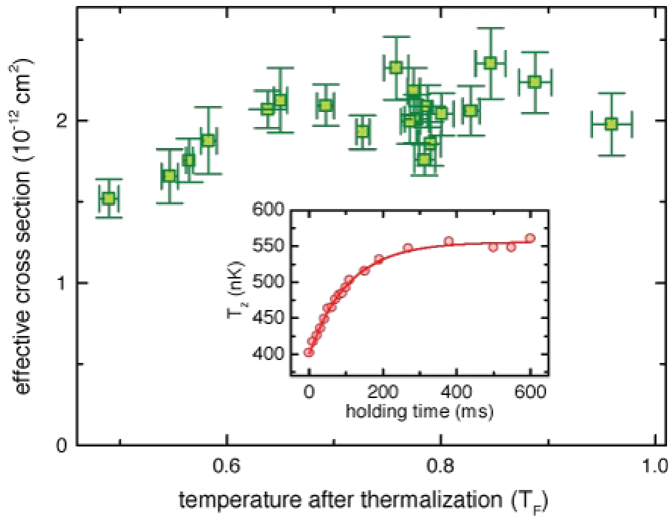
Bohn, Cavagnero, and Ticknor NJP **11** 055039 (2009)

Dipolar Scattering

- Experimental observation from Innsbruck.
  - Threshold universality

$$D = 100\, a_0$$

$$E_D = 200\, \mu K$$



$\leftarrow \sigma_{universal}$

From K. Aikawa et al. PRL **112** 010404 (2013)

2D dipolar scattering with a tilt

Christopher Ticknor

- May 15 at INT

2D dipolar scattering

- Long range
- Anisotropic
- Strong

$$V_{dd} = \frac{\vec{d}\cdot\vec{d} - 3(\hat{R}\cdot\vec{d})^2}{R^3}$$

$$R \rightarrow \rho$$

$$\hat{d} \rightarrow \cos(\alpha)\hat{z} + \sin(\alpha)\hat{x}$$

$$V_{dd} = \langle d \rangle^2 \frac{1 - 3\cos(\theta)^2 \sin^2(\alpha)}{\rho^3}$$

2D Universal Dipolar Scattering

$$\hat{d}\cdot\hat{\rho}=0 \qquad \tilde{\rho}=\rho/D$$

$$\left(-\frac{d^2}{2\mathrm{d}\tilde{\rho}^2}+\frac{m^2-1/4}{2\tilde{\rho}^2}+\frac{1}{\tilde{\rho}^3}\right)\phi_m=\frac{E}{E_D}\phi_m$$

- Universal
  - Repulsive diagonal potential for m=0
  - Diagonal in partial wave
  - Short range not really important

2D Universal Dipolar Scattering

Semi-classical regime $\frac{E}{E_D} \gg 1$

$$\sigma_{sc} = \frac{4}{k}\sqrt{\pi D k}$$

Ticknor, PRA **80** 052702 (2009)

2D Universal Dipolar Scattering

Threshold regime $$\frac{E}{E_D} \ll 1$$

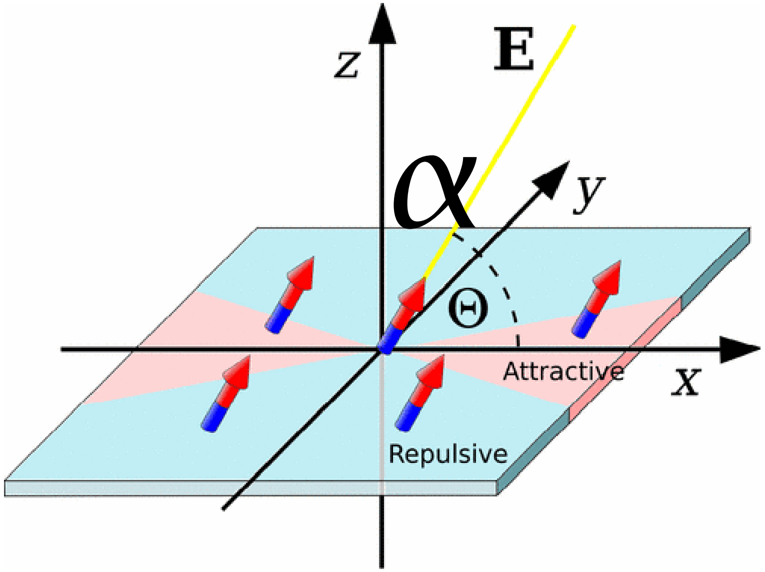
$m=0$ $$a/D = e^{2\gamma + \ln 2} \approx 6.344$$

$m^2 > 0$ $$\sigma_m = \frac{4}{k}\frac{(Dk)^2}{(m^2 - 1/4)^2}$$

Ticknor, PRA **80** 052702 (2009) $$\sigma_m \propto k^{4m-1}$$

Interplay of geometry and interaction

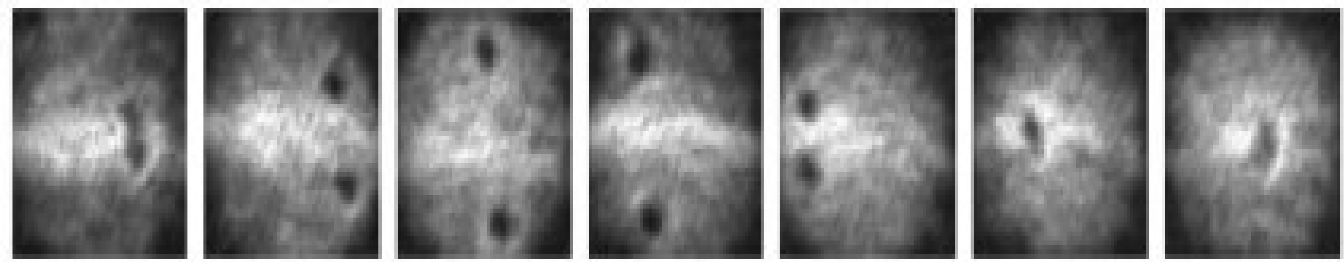
$$\hat{d}\cdot\hat{\rho}\neq 0$$



$$V_{dd}=\langle d\rangle^2\frac{1-3\cos(\theta)^2\sin^2(\alpha)}{\rho^3}$$

Breakdown of superfluidity in a BEC

An amazing BEC experiment.



Selected for a Viewpoint in *Physics*

PRL **104**, 160401 (2010) PHYSICAL REVIEW LETTERS week ending 23 APRIL 2010

Observation of Vortex Dipoles in an Oblate Bose-Einstein Condensate

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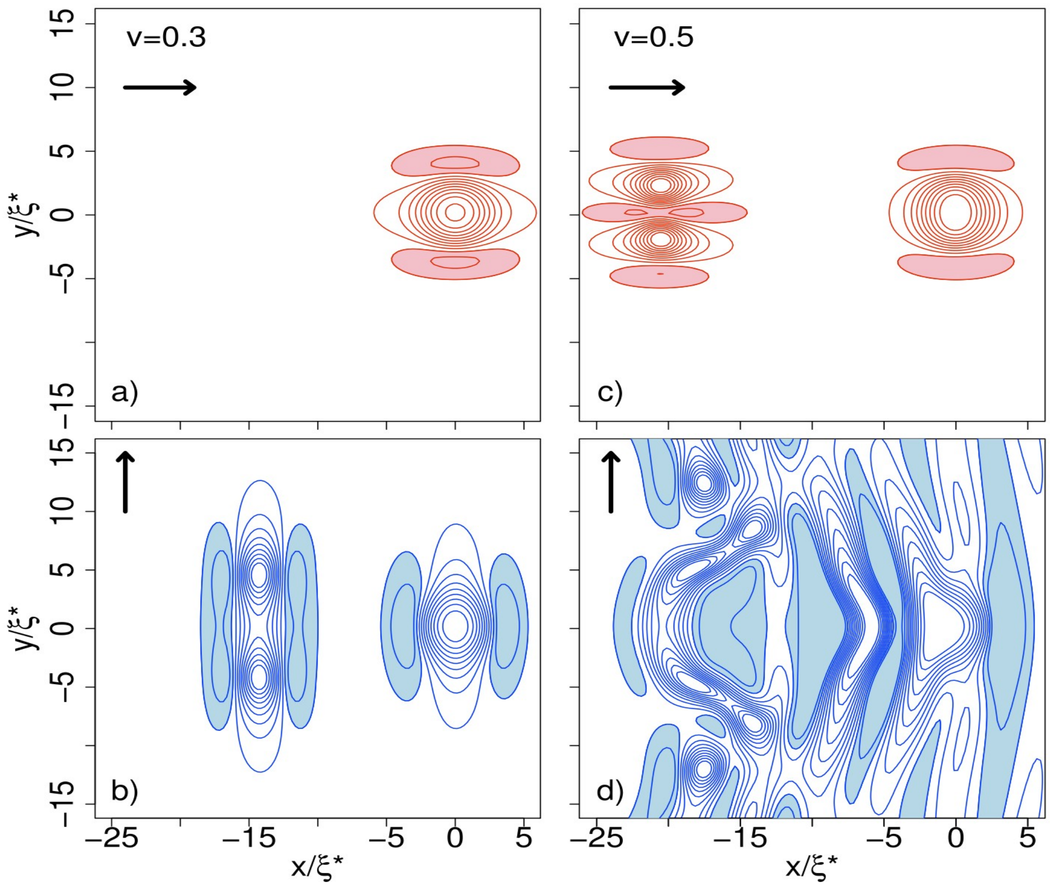
[4]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

(Received 17 December 2009; published 19 April 2010)

Interplay of geometry and interaction

- 2D dipolar gas
  with tilted polarization
- Anisotropic superfluid character!

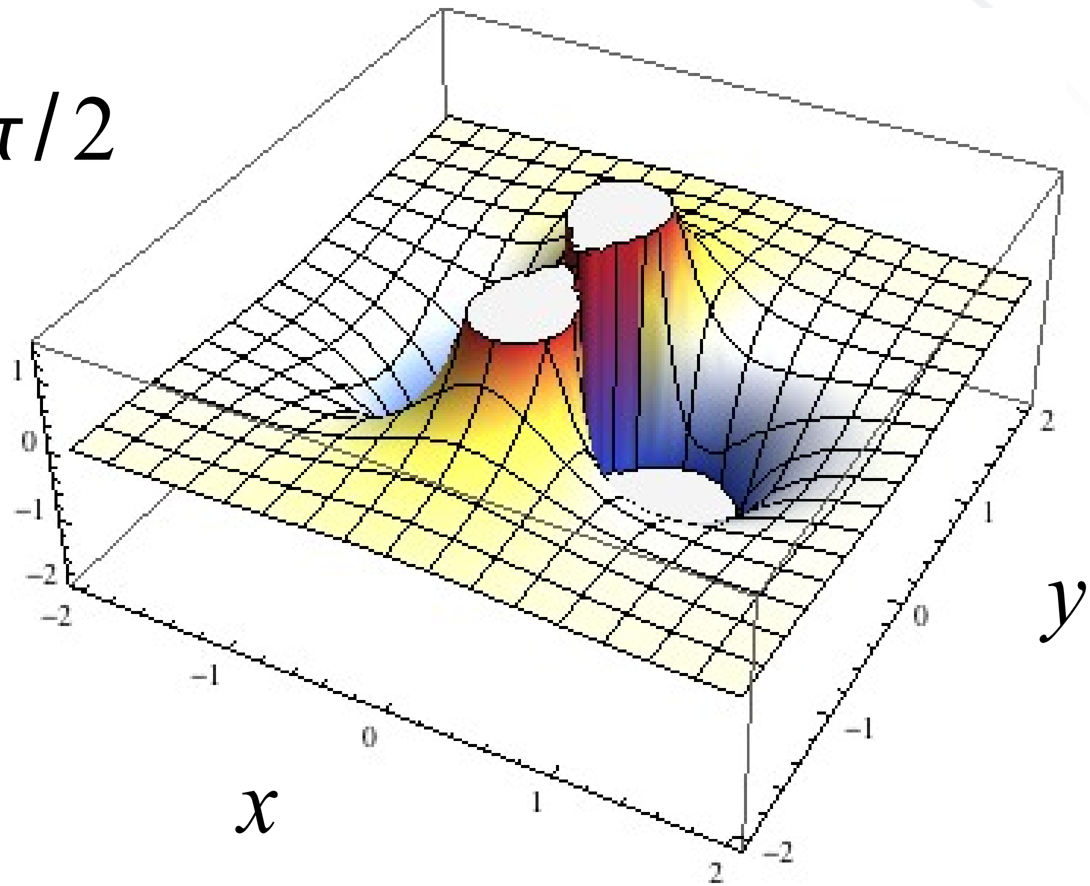


Ticknor et al. PRL **106** 065301

Interplay of geometry and interaction

$$V_{dd} = \frac{1 - 3\cos^2(\theta)\sin^2(\alpha)}{\rho^3} \rightarrow \frac{1 - 3\cos^2(\theta)}{\rho^3}$$

$$\alpha = \pi / 2$$



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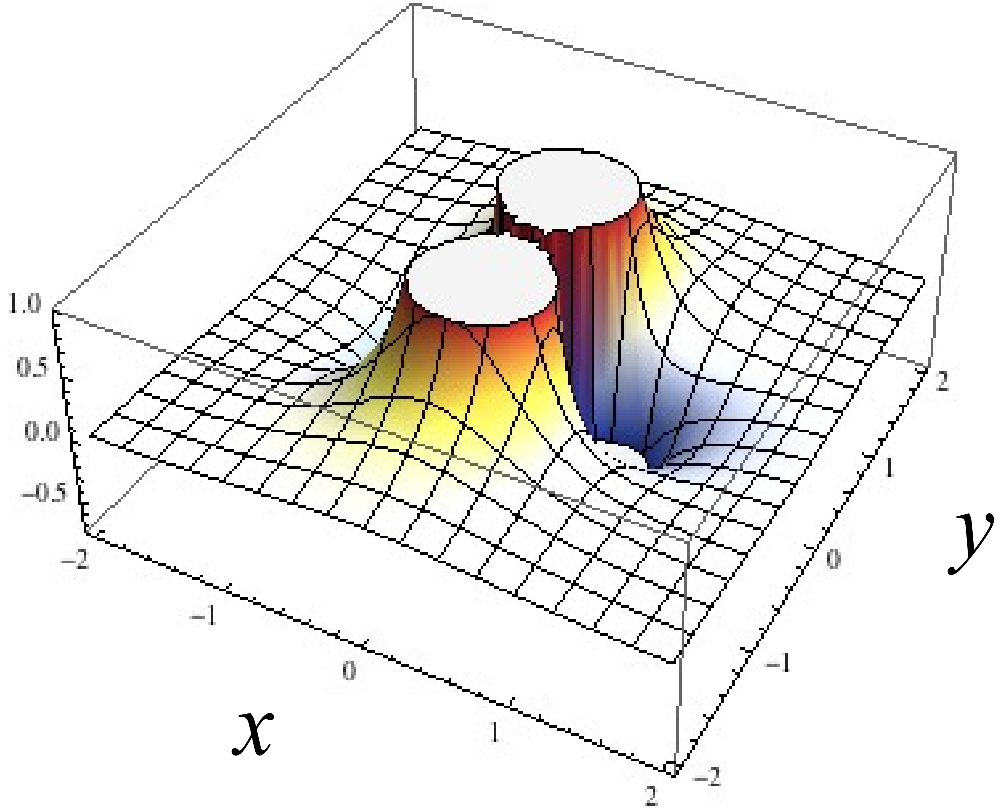
Interplay of geometry and interaction

$$V_{dd} \to \frac{1-1.5\cos^2(\theta)}{\rho^3}$$

$\alpha = \pi/4$

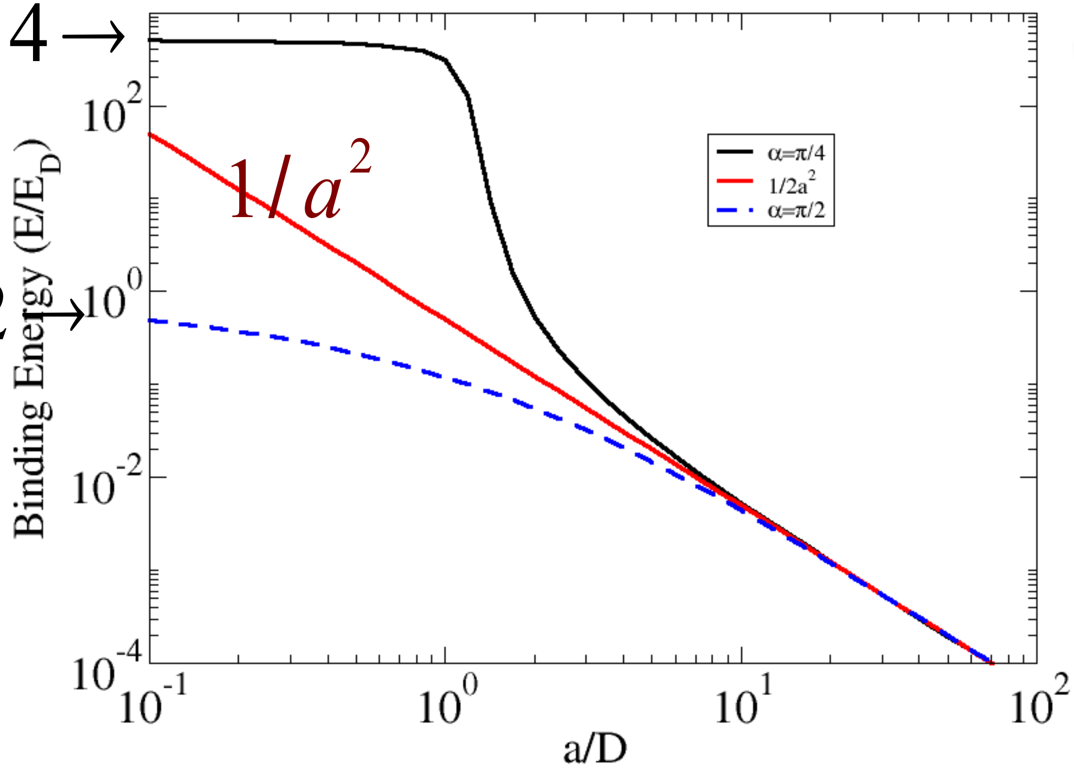


Interplay of geometry and interaction

Binding Energies as function of a/D and α

$\alpha = \pi/4 \rightarrow$

$\alpha = \pi/2 \rightarrow$



Ticknor PRA **84** 032702

Interplay of geometry and interaction

$$f(\theta_i,\theta_f)=\frac{e^{i\pi/4}}{\sqrt{2\pi k}}\sum_{mm'} e^{-im\theta_i} T_{mm'} e^{im'\theta_f}$$

$$\sigma(\theta_i)=\int d\theta_f \left|f(\theta_i,\theta_f)\right|^2$$

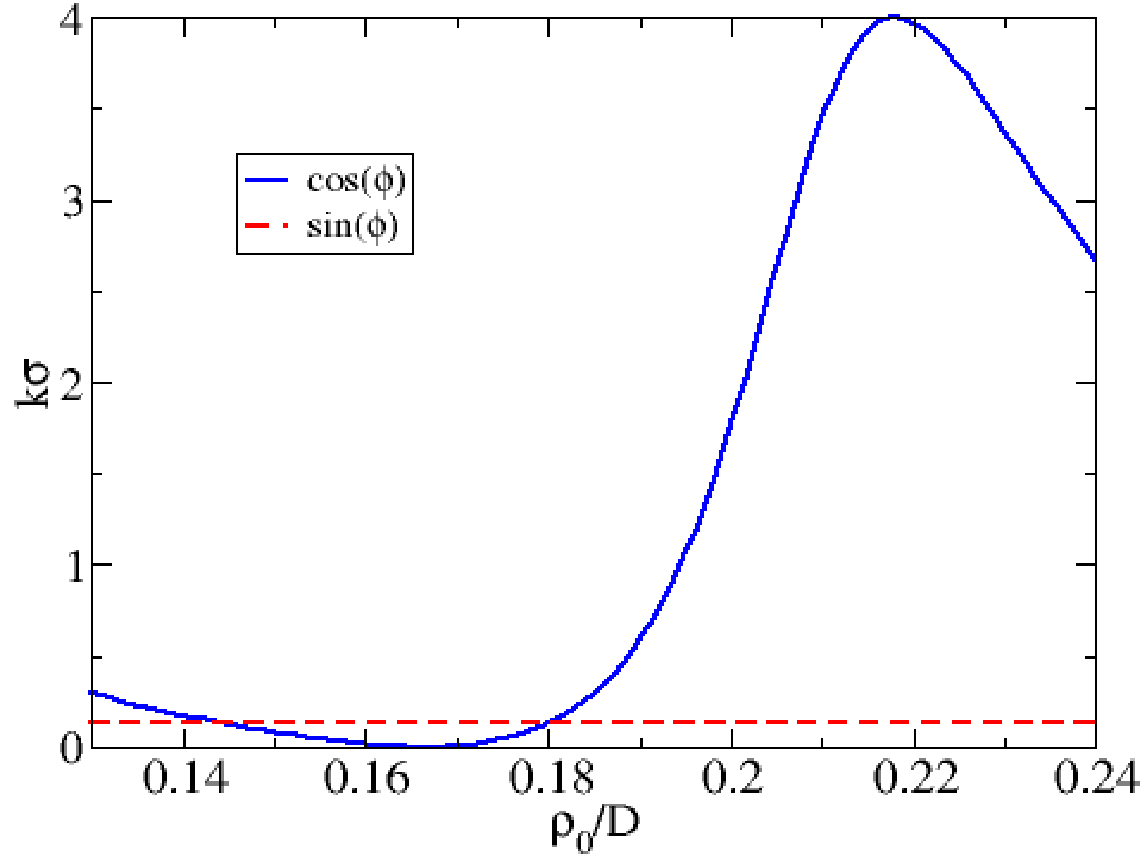
$$\sigma=\frac{1}{k}\sum_{mm'}\left|T_{mm'}\right|^2$$

Interplay of geometry and interaction

Fermions with tilted polarization

$\alpha/\pi = 0.315$

$$\cos(m\theta)$$
$$\sin(m\theta)$$



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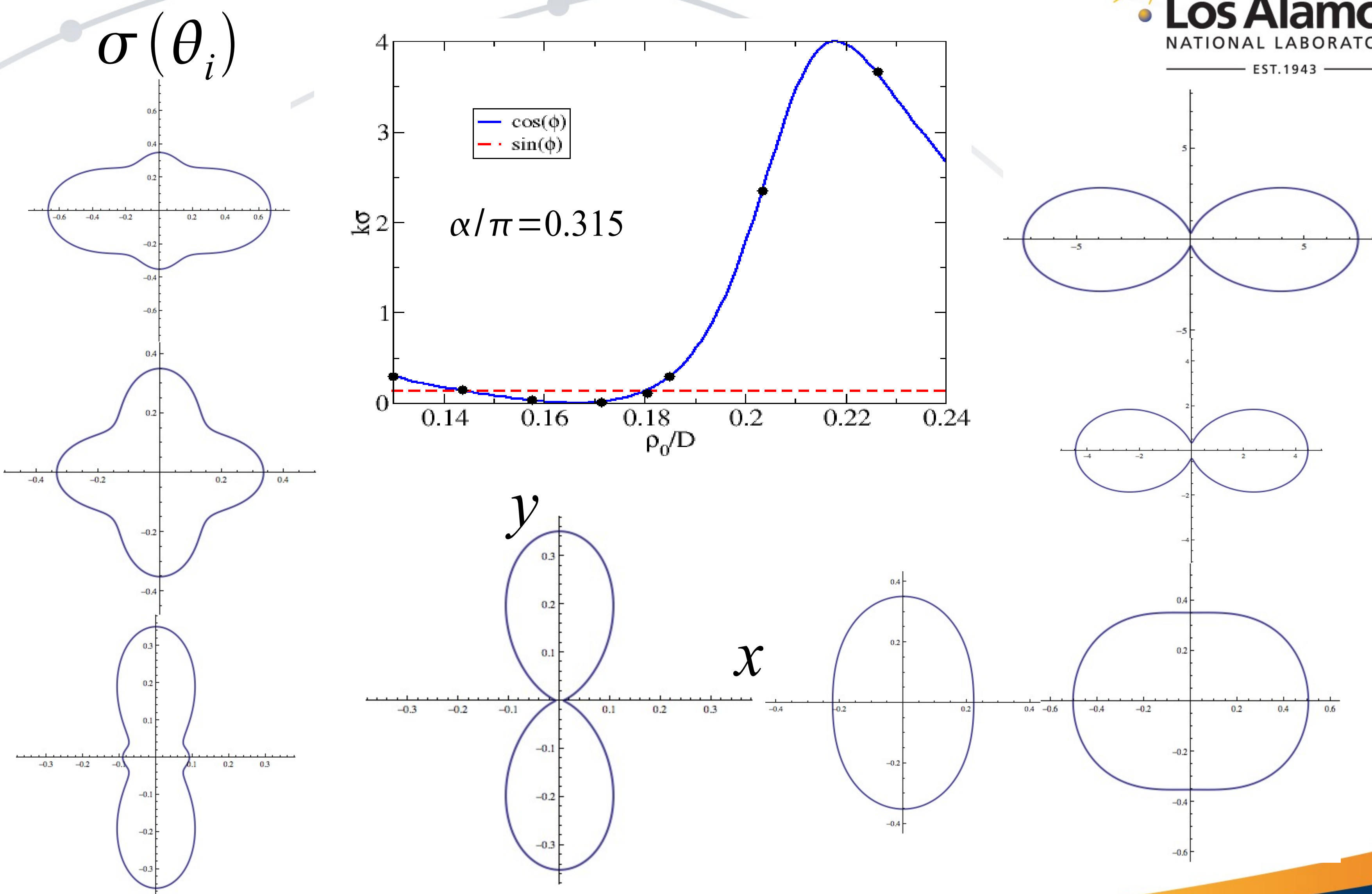
Interplay of geometry and interaction



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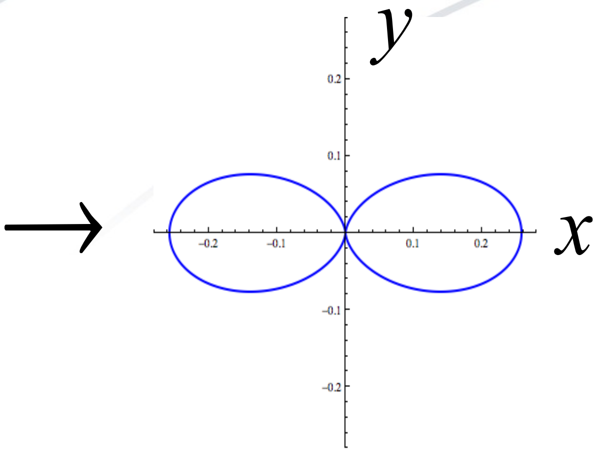
Conclusions

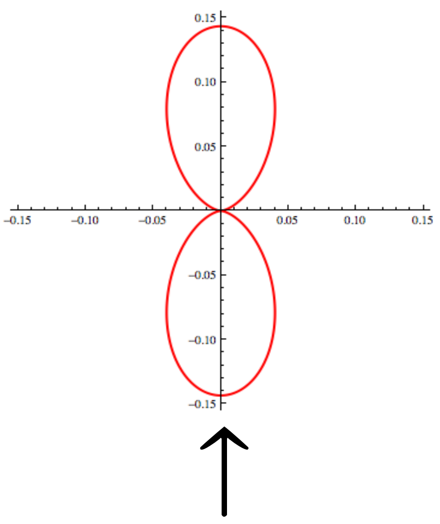
- Universal dipolar scattering in 3D.
  - Many partial waves always contribute.
- Dipolar scattering in 2D is universal.
- If polarization is tilted then:
  - Tune anisotropy of system.

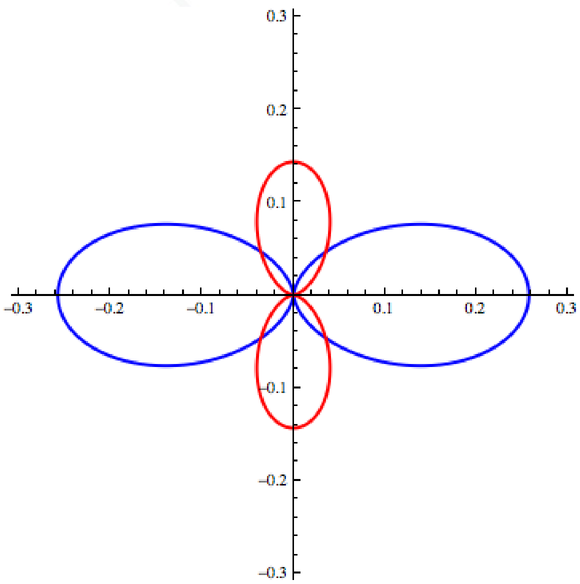
- Money from LANL and LDRD.

Interplay of geometry and interaction



$$\left|f\left(\theta_i,\theta_f\right)\right|^2$$





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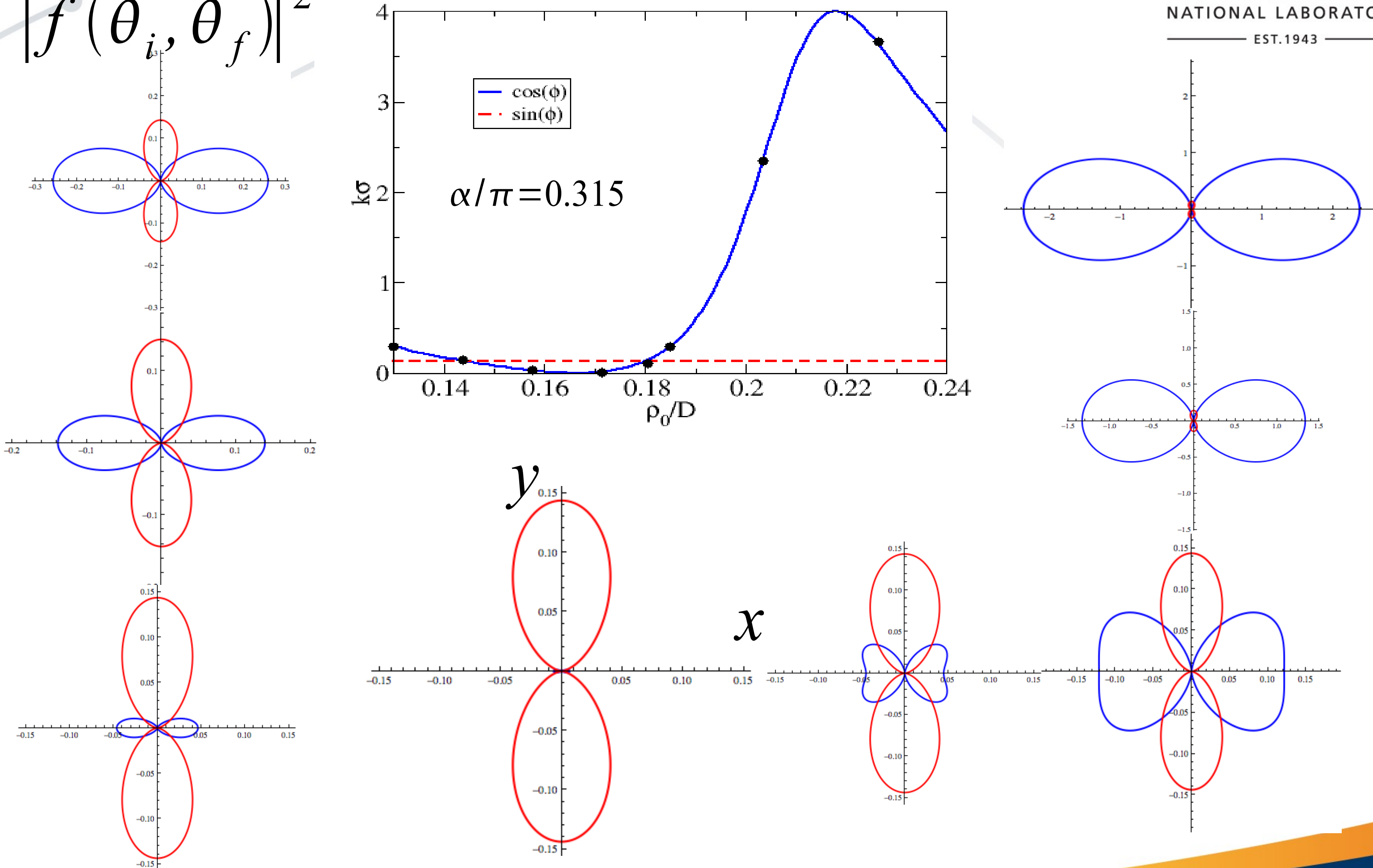
Interplay of geometry and interaction

$$|f(\theta_i, \theta_f)|^2$$



$$\theta_i = 0, \pi/2$$

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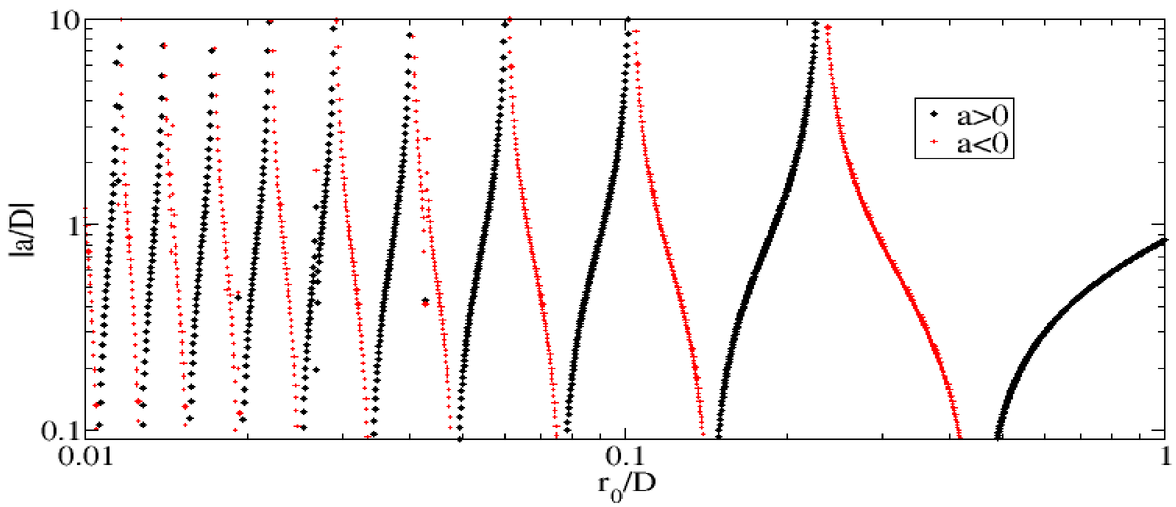
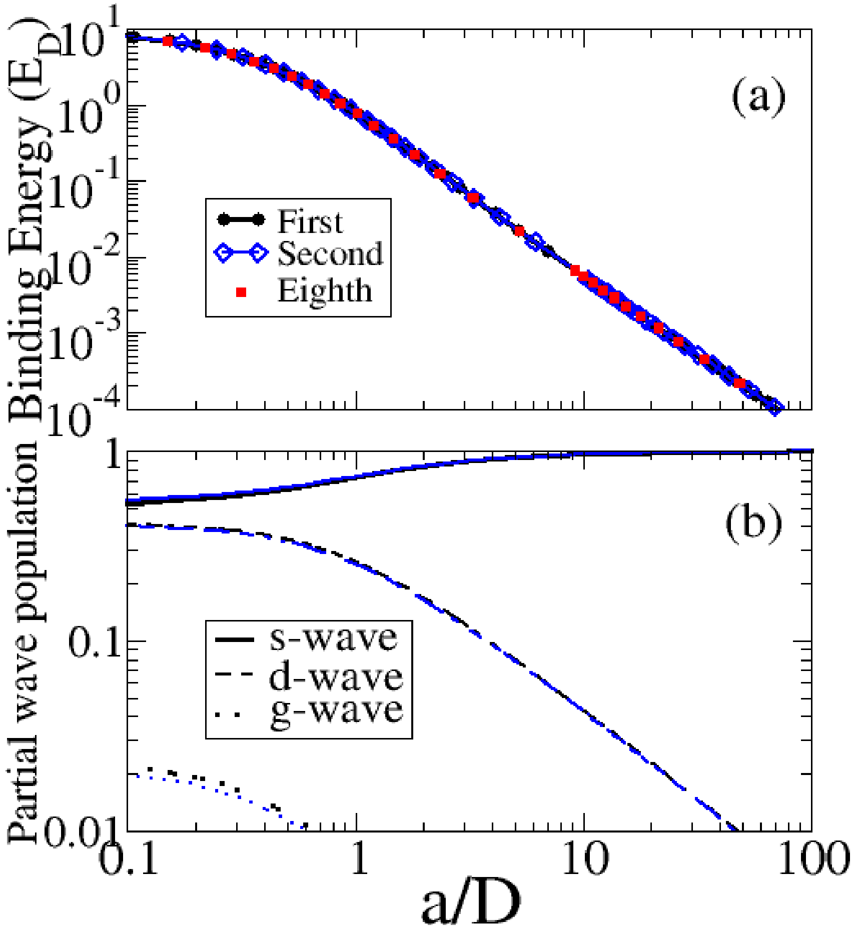
3-body dipolar recombination

- Studied formation of 2 body molecule from 3 body dipolar system.



Ticknor and Rittenhouse, PRL **105** 013201 (2010)

3-body dipolar recombination

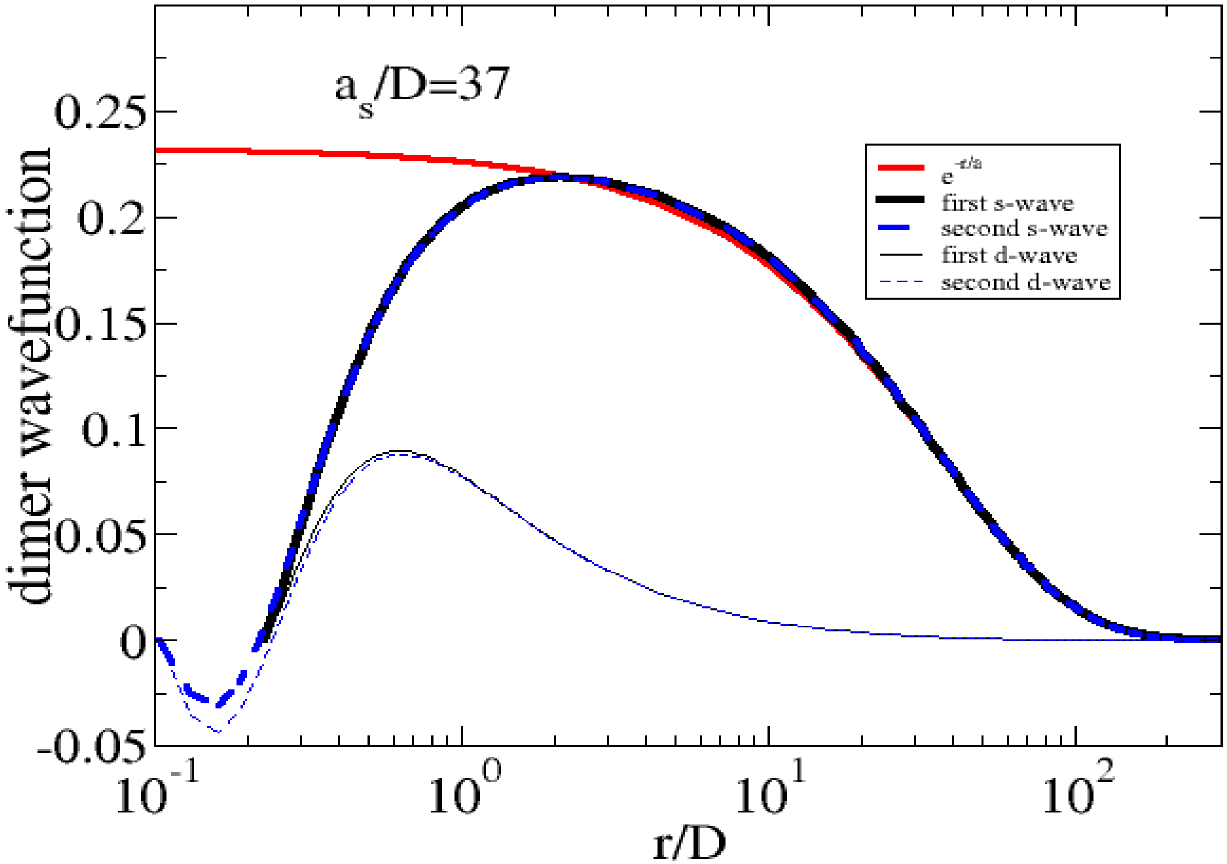


- a/D
- bound states
- Shape

Also see: Wang and Greene PRA **85** 022704 (2012)

3-body dipolar recombination

- Example wave functions.



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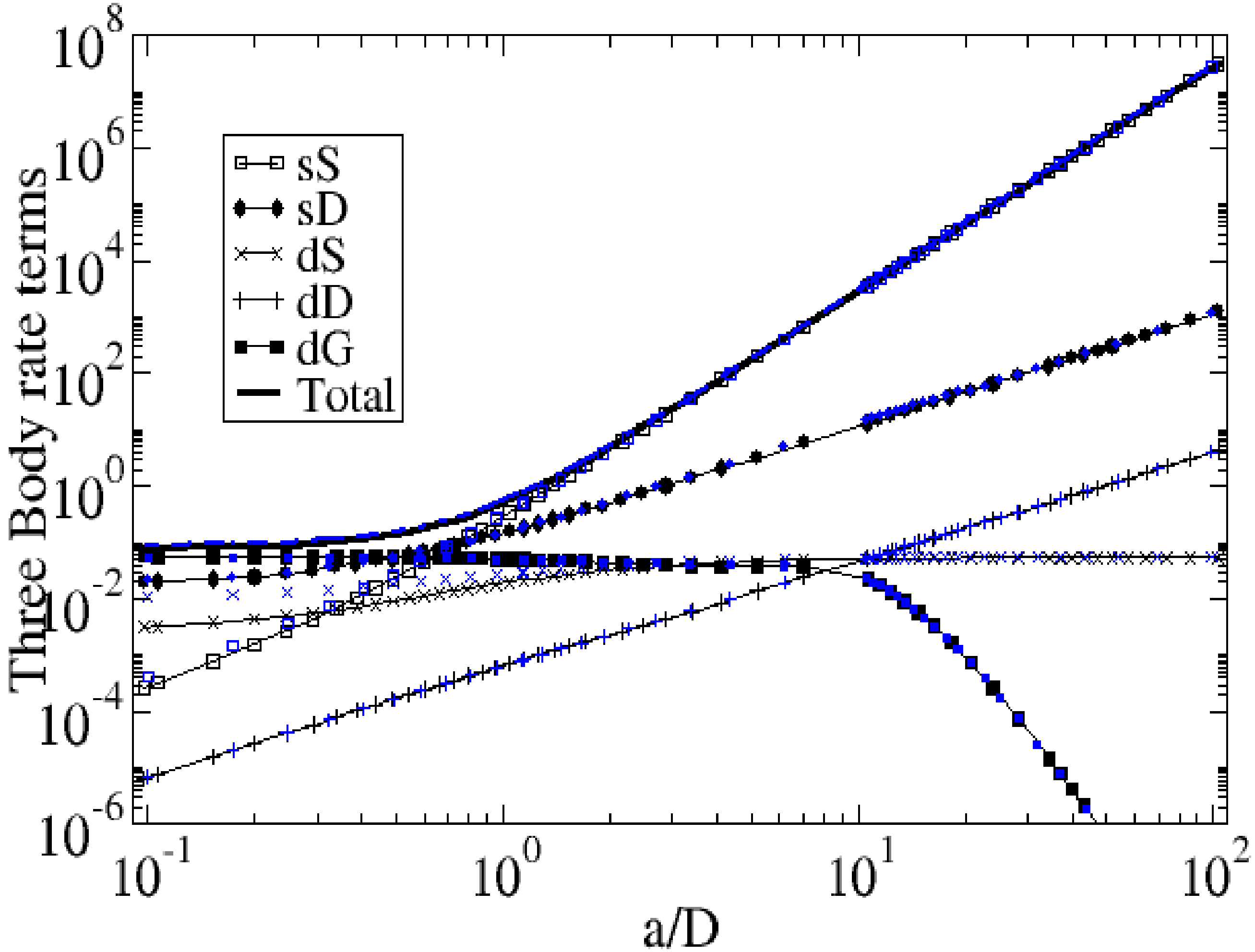
3-body dipolar recombination

- Now use Fermi Golden Rule (FGR)

$$\langle 3-body\ plane\,wave | V_{dd} | molecule + third \rangle$$

$$V_{dd} \propto \sum_{k\,m_1 m_2} \frac{R_<^k}{R_>^{3+k}} \left[Y_{k,m_1} \otimes Y_{k+2,m_2} \right]_{20}$$

3-body dipolar recombination

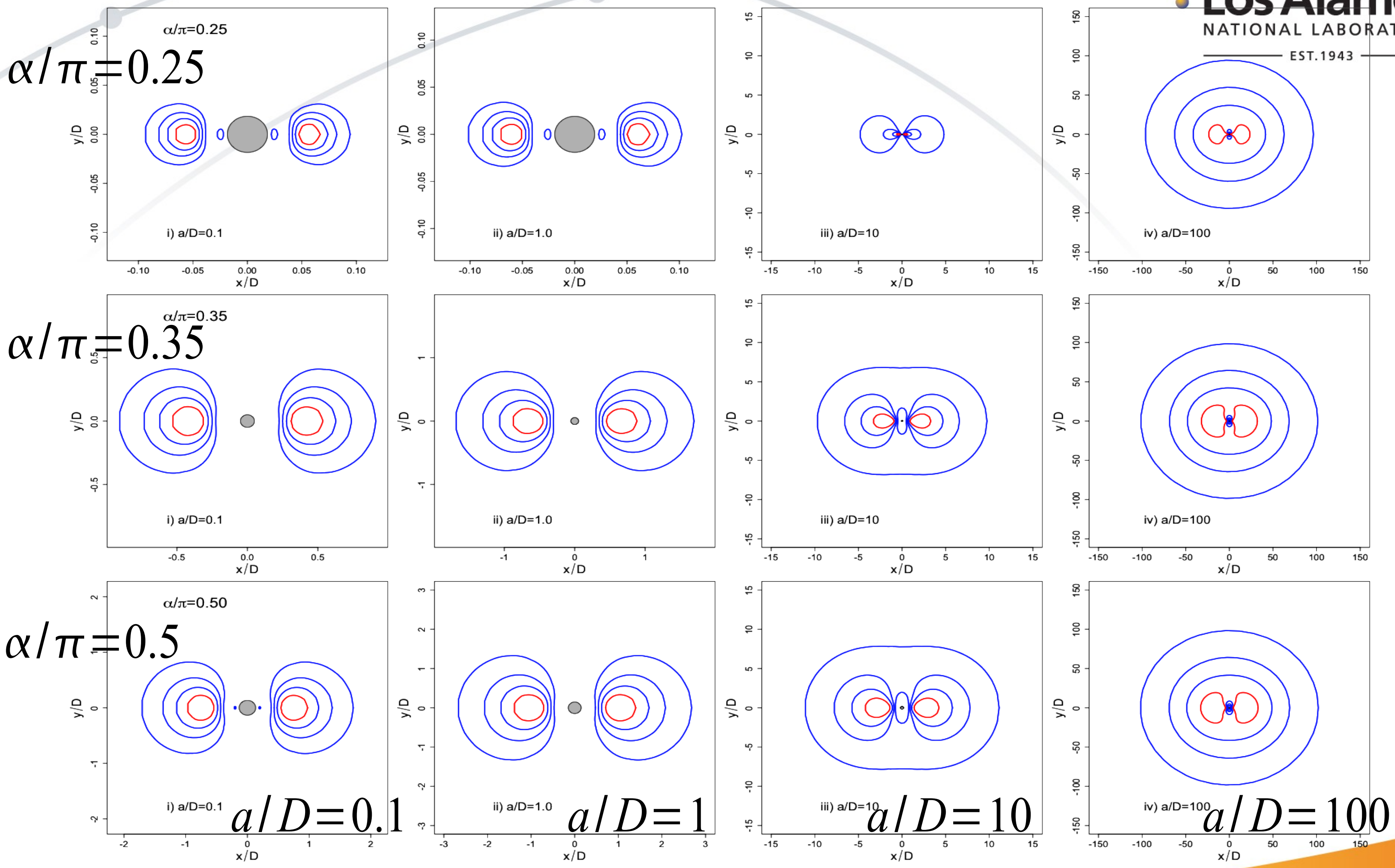


a^4

$a^2 D^2$

D^4

Interplay of geometry and interaction

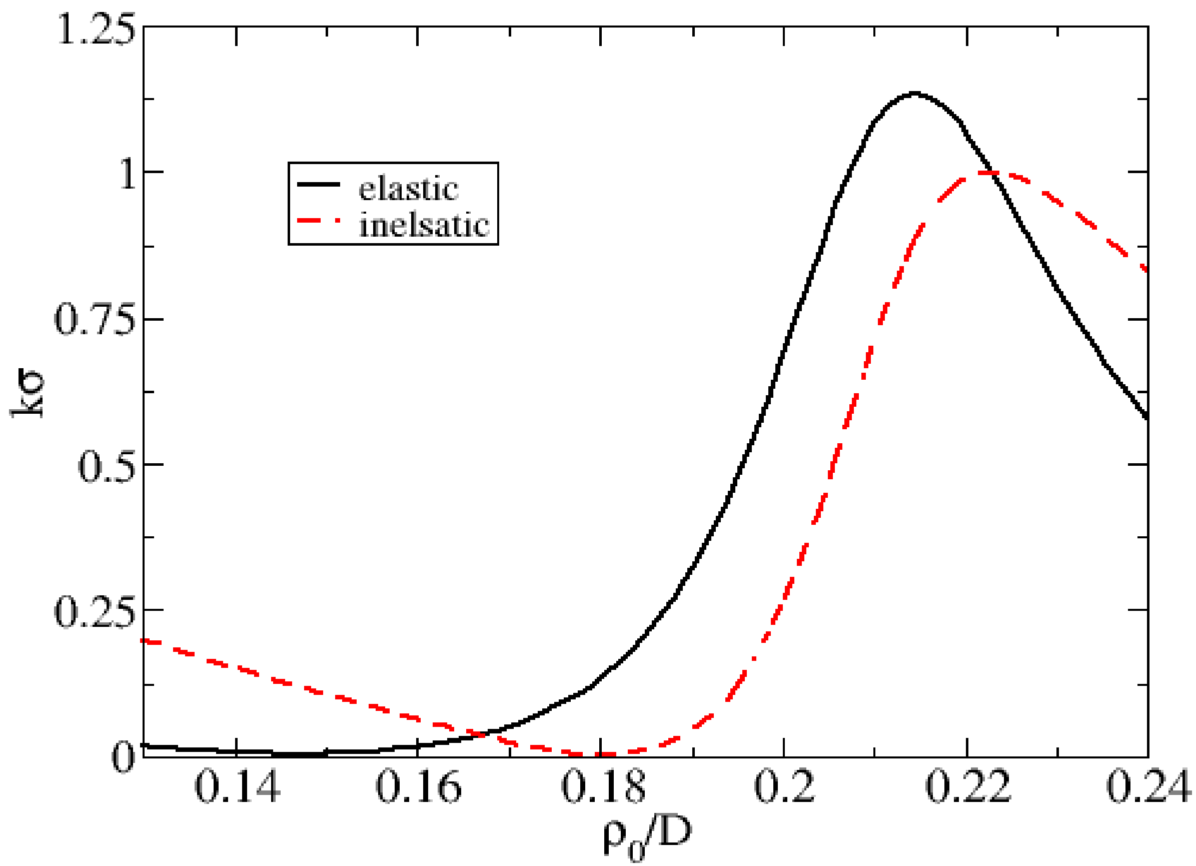


Interplay of geometry and interaction

Fermions in 2D with tilted polarization

$\alpha/\pi = 0.315$

$$e^{\pm i m\theta}$$



Interplay of geometry and interaction

$$\hat{d}\cdot\hat{\rho}\neq 0$$

$$\langle m|1-3(\hat{d}\cdot\hat{\rho})^2|m'\rangle$$

$$=\left(1-\frac{3}{2}\sin^2(\alpha)\right)\delta_{mm'}-\frac{3}{4}\sin^2(\alpha)\,\delta_{mm'\pm 2}$$

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Interplay of geometry and interaction

$$\langle m | 1 - 3(\hat{d}\cdot\hat{\rho})^2 | m' \rangle$$

$$\cos(m\phi)$$

$$= \left(1 - \frac{9}{4}\sin^2(\alpha)\right)\delta_{mm'}$$

$$- \frac{3}{4}\sin^2(\alpha)\,\delta_{mm'\pm 2}$$

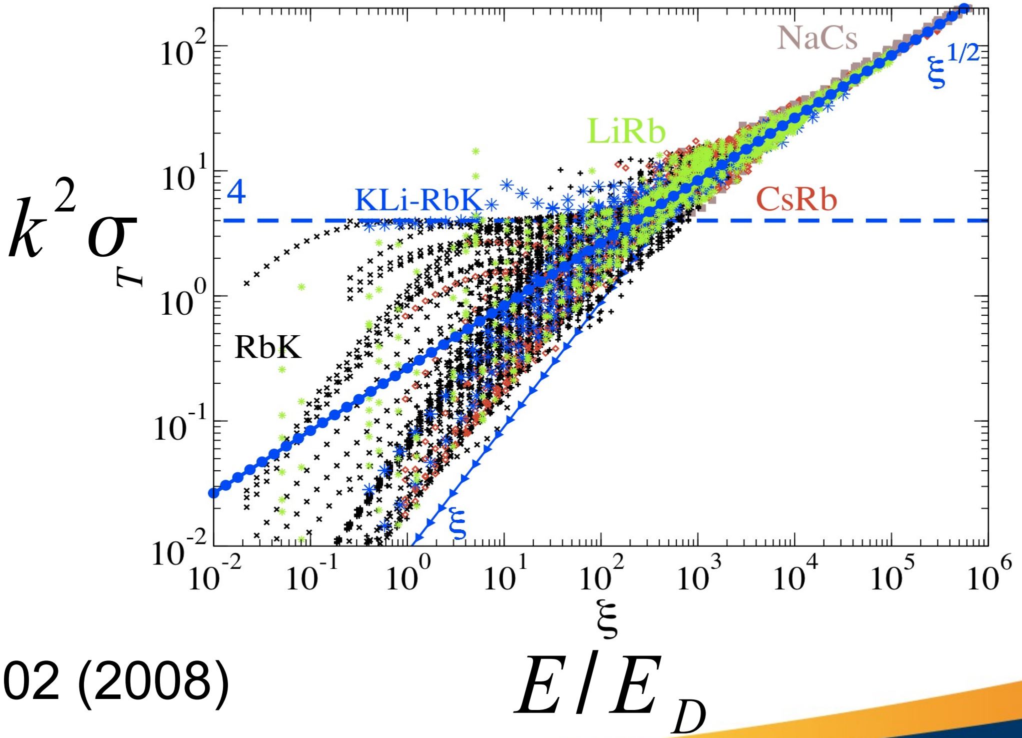
$$\sin(m\phi)$$

$$= \left(1 - \frac{3}{4}\sin^2(\alpha)\right)\delta_{mm'}$$

$$- \frac{3}{4}\sin^2(\alpha)\,\delta_{mm'\pm 2}$$

Dipolar Scattering

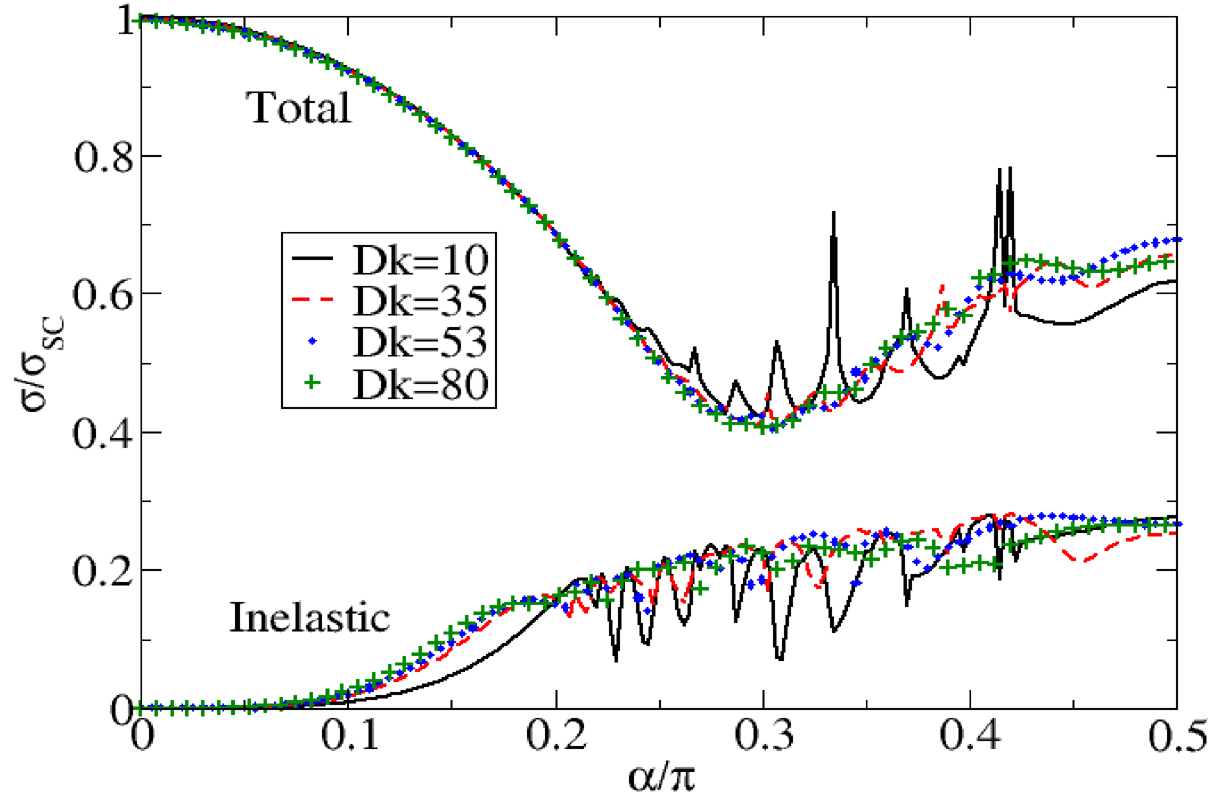
- Semiclassical Universality first predicted numerically
- True of Fermions and Bosons and distinguishable particles.



Ticknor PRL **100,** 133202 (2008)

Interplay of geometry and interaction

$$k\sigma_{sc} = 4\sqrt{\pi Dk}$$



Interplay of geometry and interaction

$$\hat{d}\cdot\hat{\rho}\neq 0$$

$$k\sigma_{m\to m'}=\frac{4(Dk)^2}{(m^2-1/4)^2}\left(1-\frac{3}{4}\sin(\alpha)^2\right)^2$$

$$k\sigma_{m'\to m+2}=\frac{4(Dk)^2}{(m-1/2)(m+3/2)}\left(\frac{3}{4}\sin(\alpha)^2\right)^2$$

$$k\sigma_{\mp 1\to\pm 1}=\frac{4(Dk)^2}{(m^2-1/4)^2}\left(\frac{3}{4}\sin(\alpha)^2\right)^2$$