Measuring Scale Invariance and Viscosity in Fermi Gases

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Outline

Introduction

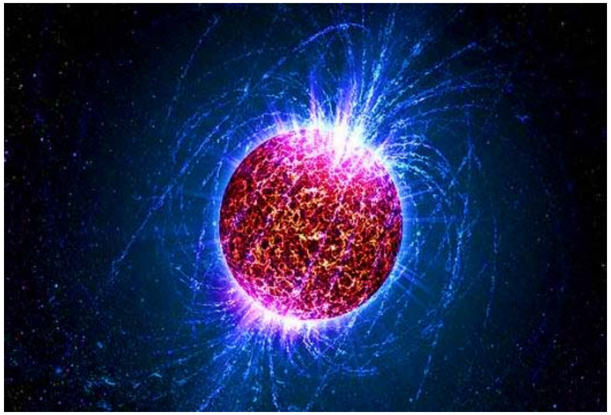
- *Optically trapped Fermi gases:*
  - Creating strong interactions
- *Universal Regime:*
  - Thermodynamics, Quantum viscosity, String theory conjecture

Topics

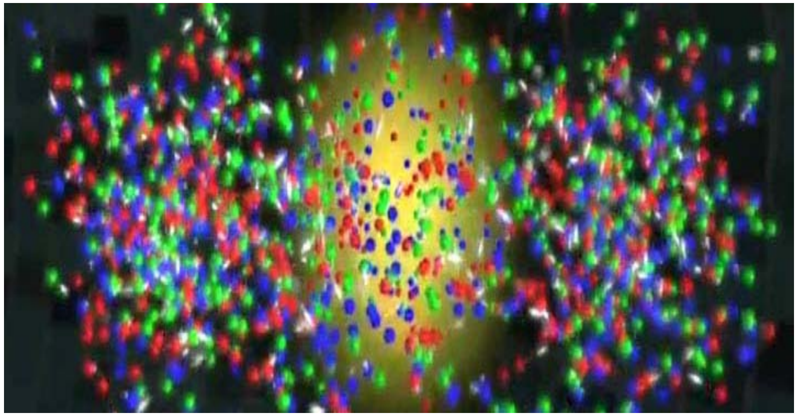
- *Scale Invariance in Expanding Fermi gases:*
  - Scale invariance: *Ballistic* expansion of a *hydrodynamic* gas
- *Viscosity measurement on/off resonance*
  - Minimum on the BEC side of resonance
  - Measurements at constant S

Why Study Strongly Interacting Fermi Gases?

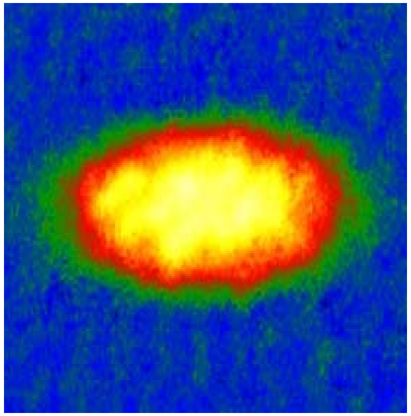
Strongly Interacting Fermionic Systems



Neutron Star



Quark Gluon Plasma

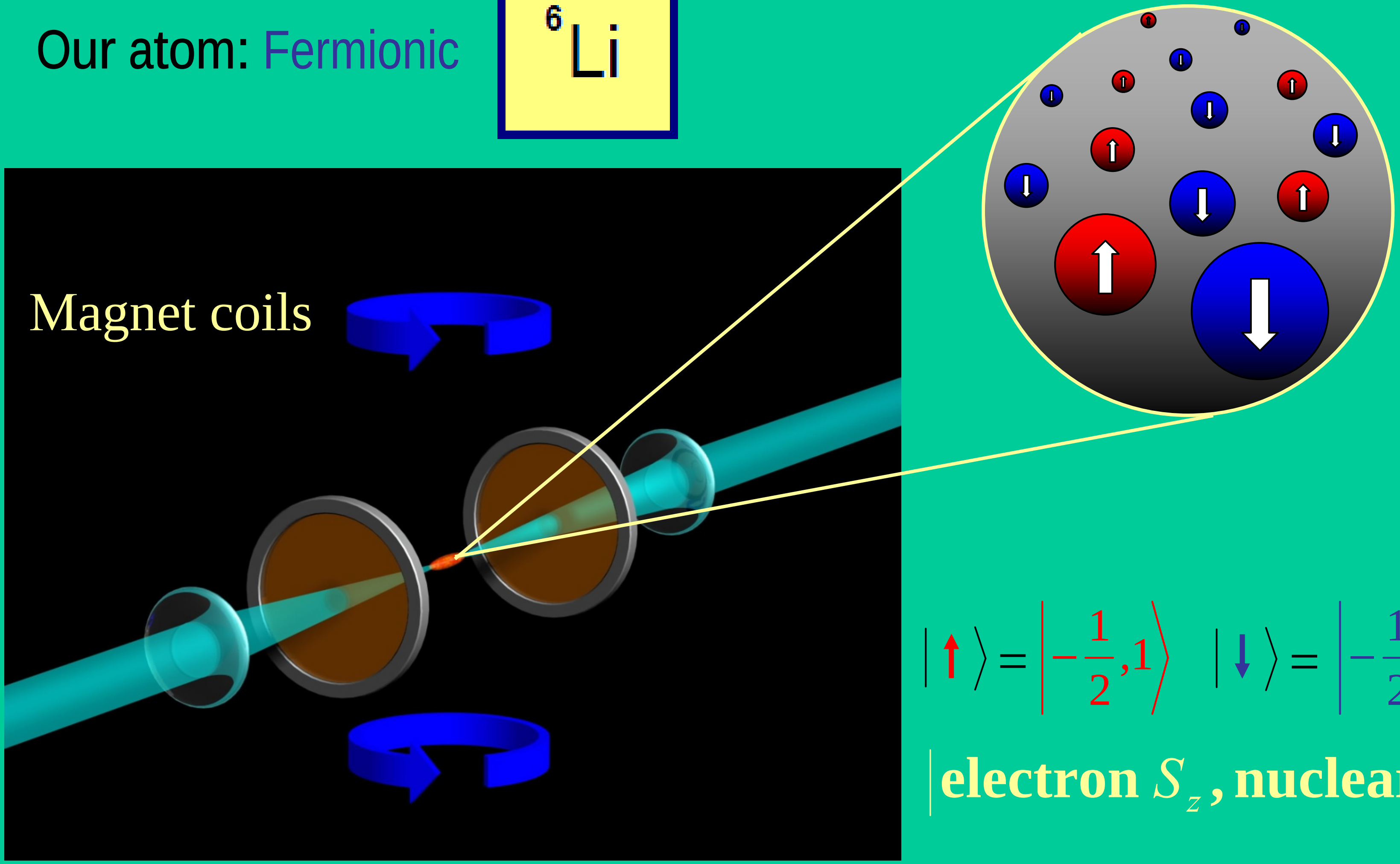


Ultra-Cold Fermi Gas

High Temperature Superconductors

Optically Trapped Fermi Gas

Our atom: Fermionic ^{6}Li



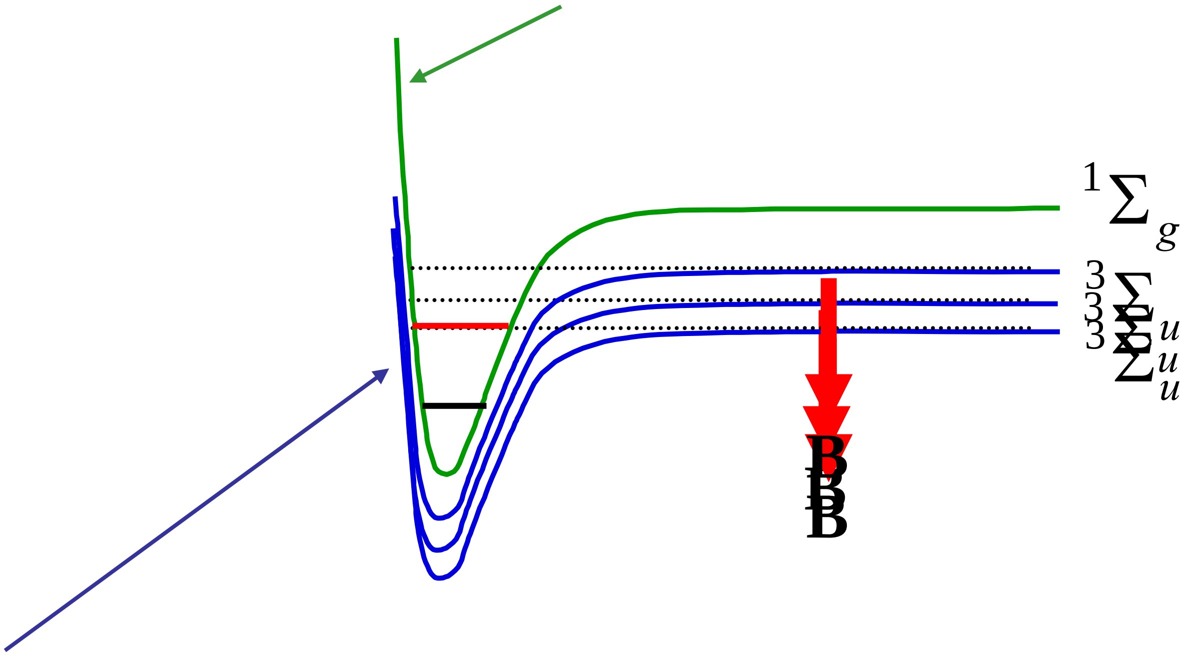
$$|\uparrow\rangle = \left|-\frac{1}{2}, 1\right\rangle \quad |\downarrow\rangle = \left|-\frac{1}{2}, 0\right\rangle$$

$$\left|\textbf{electron } S_z, \textbf{nuclear } I_z\right\rangle$$

Feshbach Resonance

Resonant Coupling between Colliding Atom Pair – Bound Molecular State

Singlet Diatomic Potential: Electron Spins Anti-Parallel

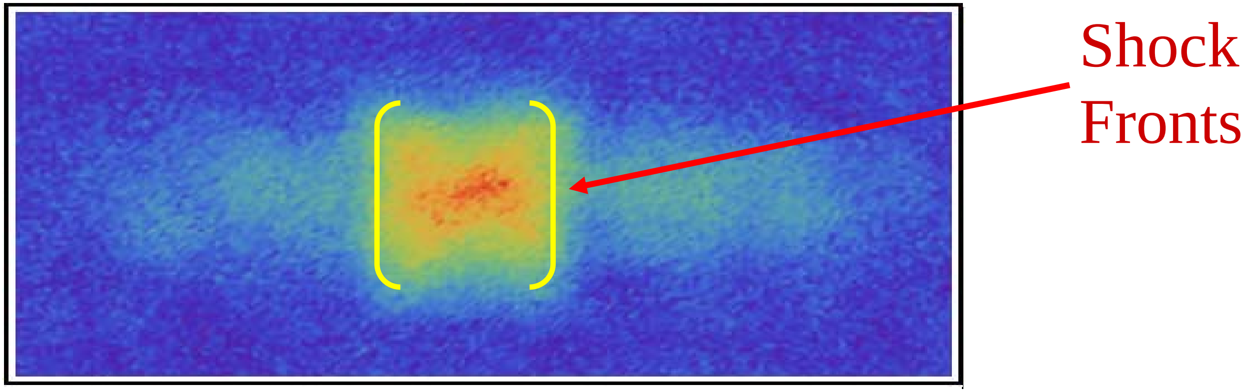


Triplet Diatomic Potential: Electron Spins *Parallel*

832 G-Resonant Scattering

Strong *Interactions:*
Shock waves in Fermi gases

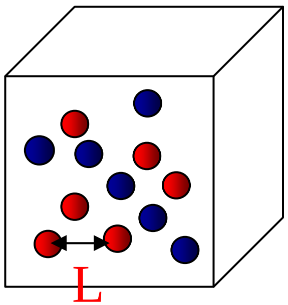
- Trapped gas is divided into two clouds with a repulsive optical potential.
- The repulsive potential is extinguished, the two clouds accelerate towards each other and collide.



Really strong interactions!

Universal Regime: Natural Units

Universal Regime: *For resonant* scattering,
the scattering cross section is the square of the de Broglie wavelength, which is independent of the details of the collisional interactions!



Atom spacing L
becomes the *only* length scale.

Heisenberg Uncertainty Principle: $\Delta x \Delta p \approx \hbar$ $\Delta p \approx p \approx \frac{\hbar}{L}$

Physical Properties, like Energy and Temperature have *Natural Units* determined by L.

Fermi Energy: $\frac{\hbar^2}{2mL^2}$

Universal Thermodynamics

When the interparticle spacing sets the scale of energy and temperature, the pressure p is a function only of density n and temperature T:

$$p(n, T)$$

Using elementary thermodynamics, one then can show that

$$p = \frac{2}{3}\varepsilon$$

$\varepsilon =$ energy density

(Ho, 2004)

This elementary result has several amazing consequences.

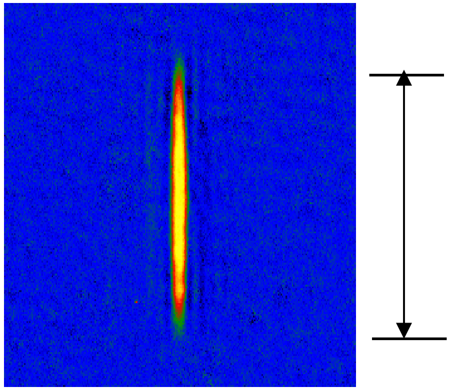
Global energy E measurement

Universal **Gas obeys the Virial Theorem**

Thomas (2005)
Castin (2004)
Werner and Castin (2006)
Son (2007)

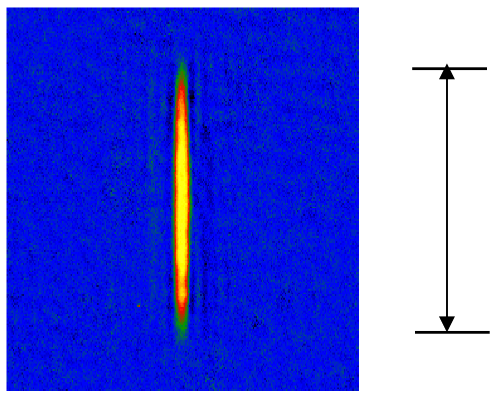
In a HO potential: $E = 2\langle U \rangle$

Energy per particle $\mathrm{E} = 3m\omega_z^2 \langle z^2 \rangle$



For a *universal* quantum gas, the energy E is determined by the *cloud size*

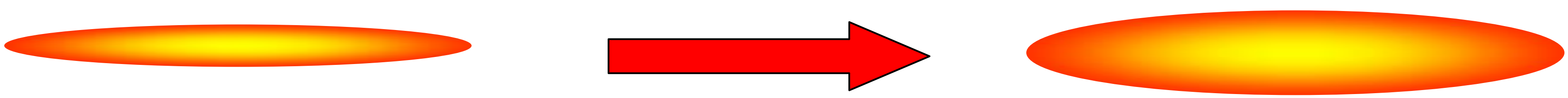
Measuring the Energy E and Entropy S



For a *universal* quantum gas,
the energy E is determined
by the *cloud size*

For a *weakly interacting* quantum gas
the entropy S can always be determined
from the *cloud size* (textbook problem)

<u>Experiment</u>

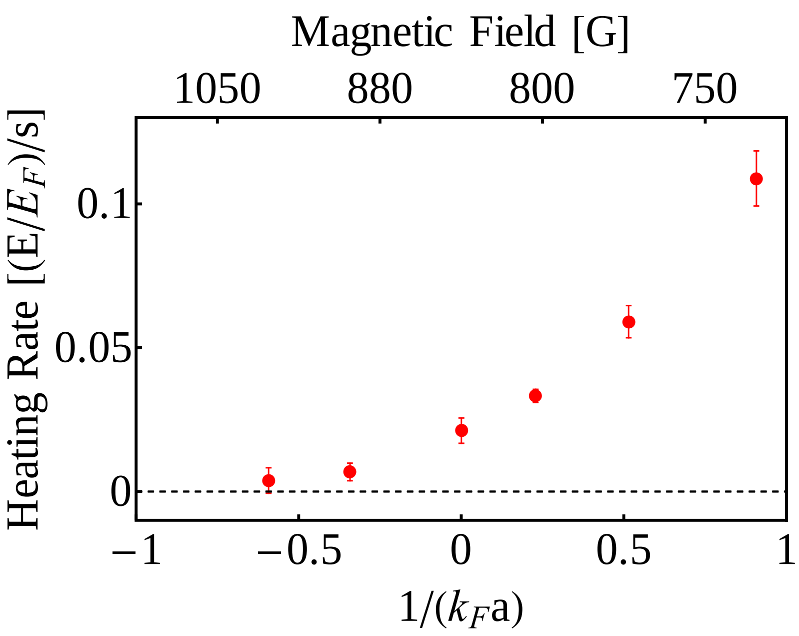


Start
Universal
Strongly interacting

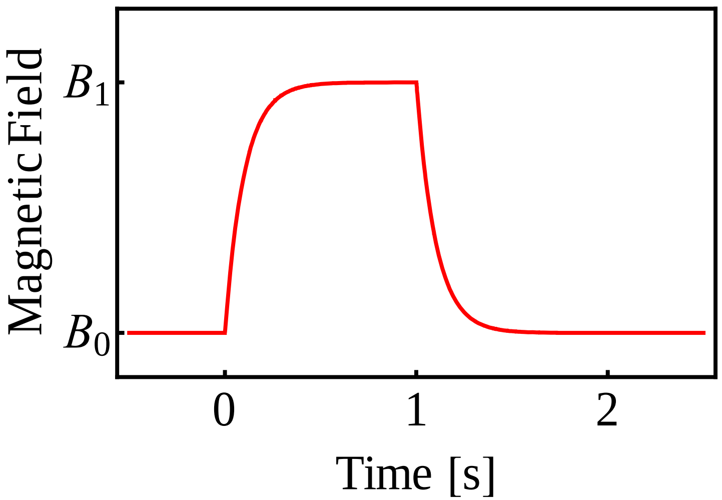
Sweep magnetic field

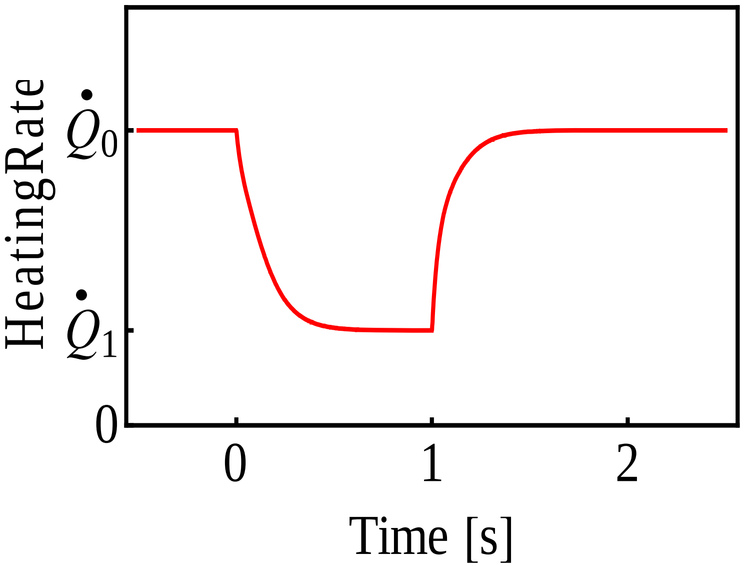
End
Weakly interacting

Is the B-Field Sweep Adiabatic?

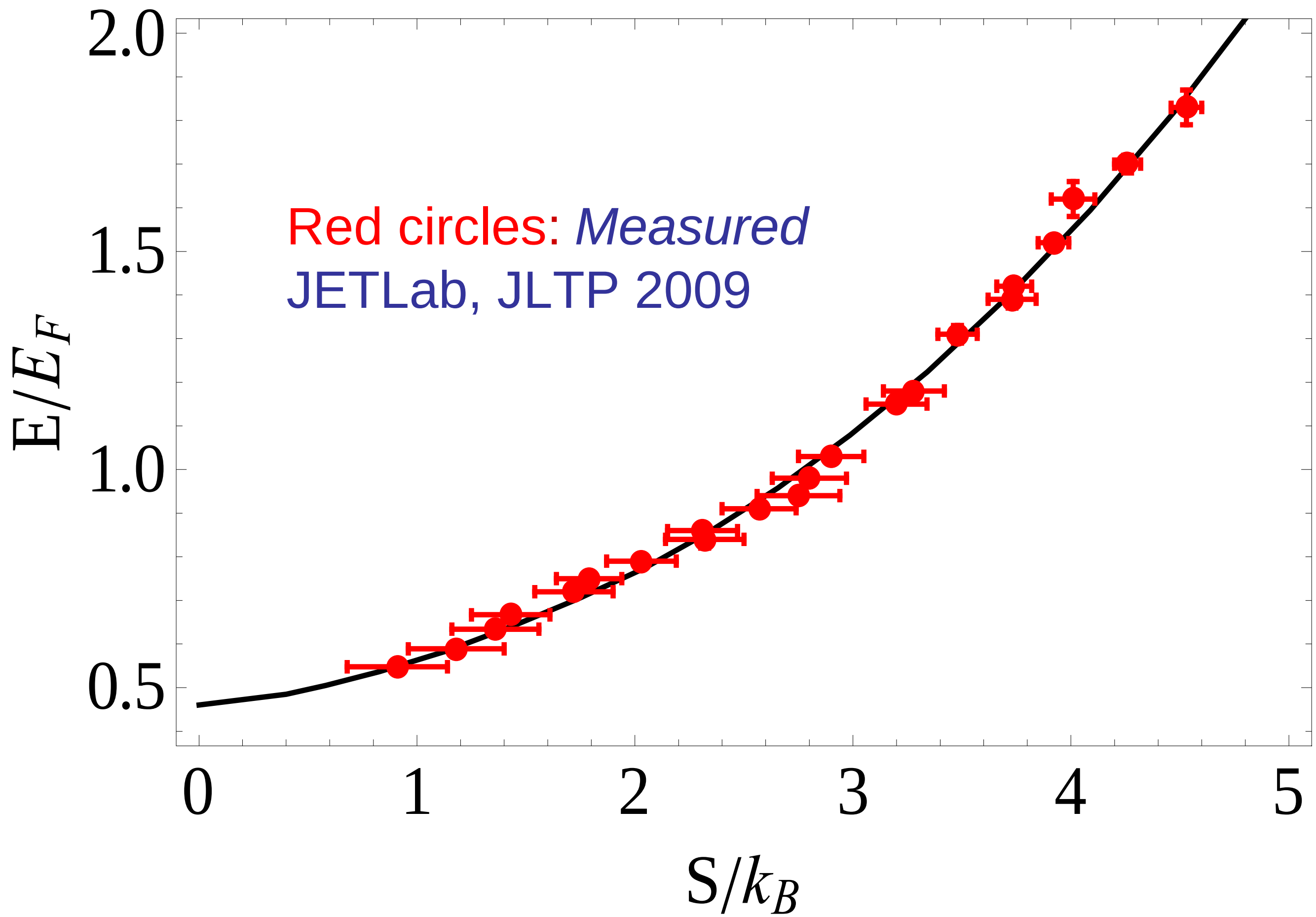


B_0	B_1	ΔE measured	$\int \dot{Q}\, dt$
832 G	770 G	0.081(8)	.077
832 G	800 G	0.053(6)	.054
832 G	900 G	0.028(3)	.029





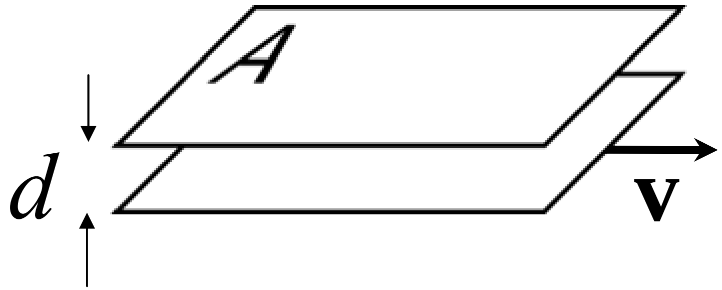
Energy per particle versus Entropy per Particle



Solid line—from measured equation of state: Ku et al., *Science*, 2012

Universal Regime: Viscosity Scale

Shear forces:



$$\mathbf{F} = \eta \frac{A}{d} \mathbf{v}$$

Viscosity scale:

$$\eta \approx \frac{p}{A} \qquad p \approx \frac{\hbar}{L} \qquad A \approx L^2$$

$$\eta \approx \frac{\hbar}{L^3} = \hbar n$$

n = density

Quantum scale—requires Planck's constant!

Quantum Viscosity

Viscosity: $\eta = \alpha \hbar n$ n = density (particles/cc)

dimensionless parameter

Water:	n = 3.3 x 10^{22}	$\eta = 300\ \hbar n$
Air:	n = 2.7 x 10^{19}	$\eta = 6000\ \hbar n$
Fermi gas:	n = 3.0 x 10^{13}	$\eta = 0.4\ \hbar n$
Quark Gluon Plasma:	n = 3.0 x 10^{38}	$\eta = ?\ \hbar n$

Minimum Viscosity Conjecture
Experimentalist's Approach!

Viscosity $\hbar n$ —Hydrodynamics

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

Kovtun et al., PRL 2005

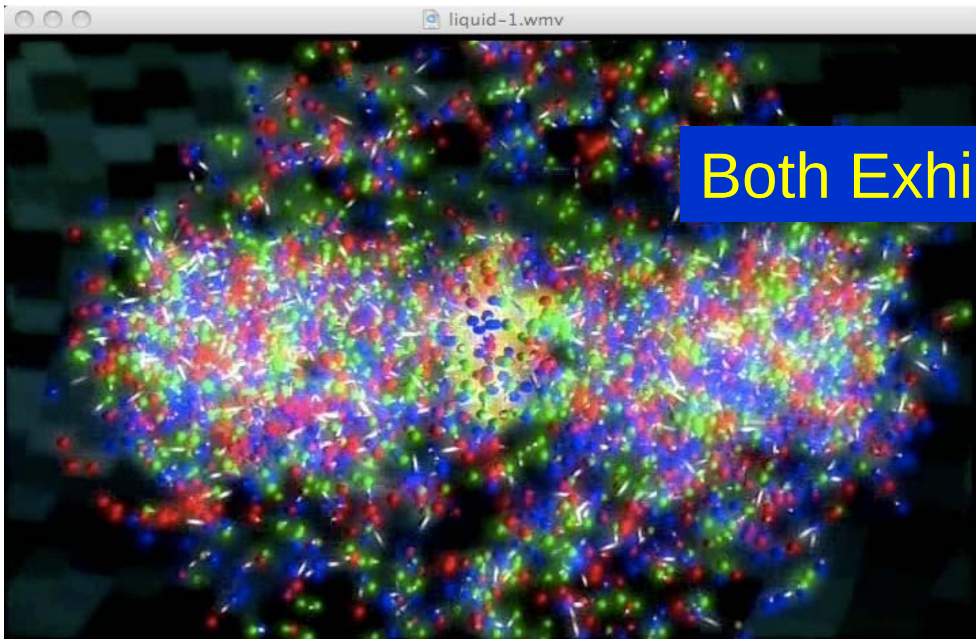
Entropy density $k_B n$ —Thermodynamics

Minimum defines a *Perfect* normal fluid

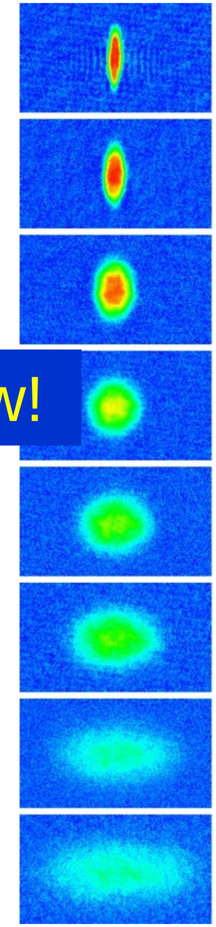
In a ^{6}Li gas we can *measure* η and s.

Perfect Fluidity—Viscosity

Quark-Gluon Plasma: $T = 10^{12}$ K



Both Exhibit Elliptic Flow!



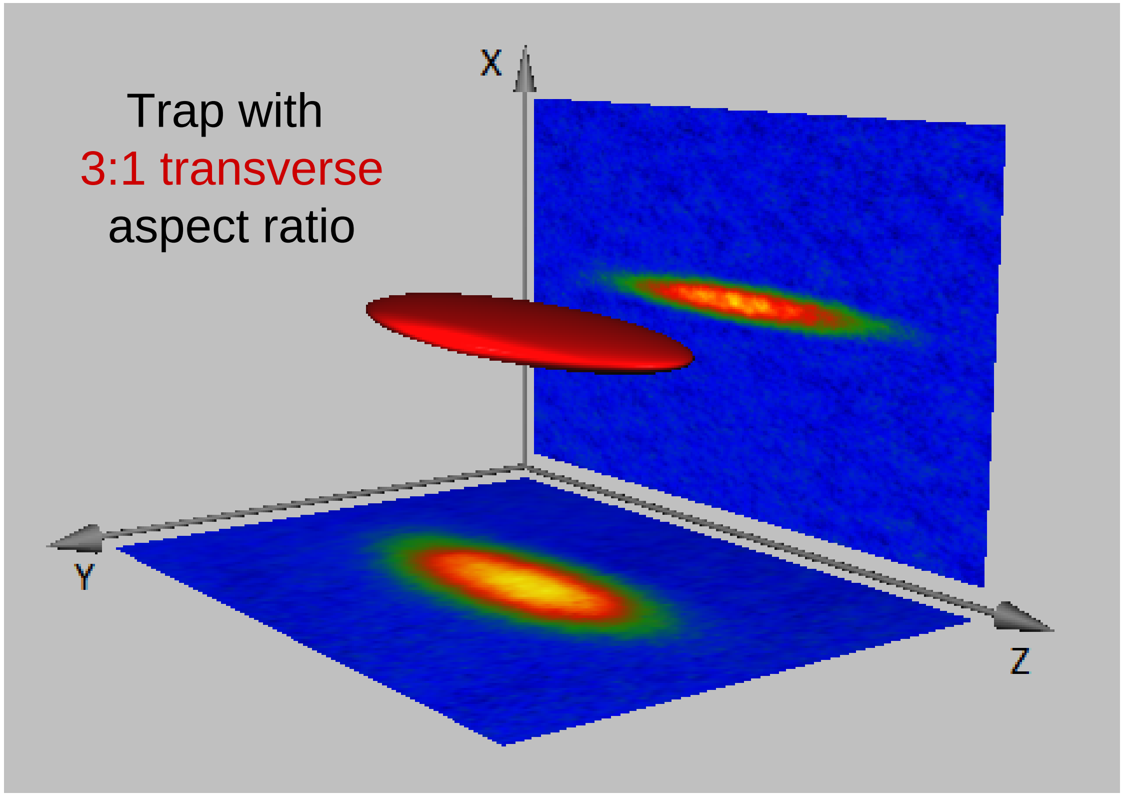
JETLab
2002

Computer simulation of RHIC collision

BIG BANG

Ultra-cold Atomic
Fermi gas: $T = 10^{-7}$ K

Measuring Viscosity in 3D



- Measure *all three* cloud radii using two cameras.

Hydrodynamic Expansion

From the Navier-Stokes and continuity equations, it is easy to show that a single component fluid obeys:

Pressure Trap potential

$$\frac{d^2}{dt^2}\frac{m\left\langle x_i^2\right\rangle}{2} = m\left\langle \mathrm{v}_i^2\right\rangle + \frac{1}{N}\int d^3\mathbf{r}\; p - \left\langle x_i \partial_i U\right\rangle - \hbar\left\langle \alpha_S \sigma_{ii} + \alpha_B \sigma'\right\rangle$$

Stream KE Shear and Bulk Viscosity

Equilibrium:

$$\frac{3}{N}\int d^3\mathbf{r}\; p_0 = \left\langle \mathbf{r}\cdot\nabla U\right\rangle_0 \equiv \tilde{E}$$

Measured from the cloud profile and trap parameters

Need to find the volume integral of the pressure:

Energy Conservation

For a temporally constant potential energy U,
the internal energy change during expansion is: $dE_{\text{int}} = dQ - p\,dV$

$$\Rightarrow \quad \dot{E}_{\text{int}} = \dot{Q} - p\dot{V}$$

The local volume dilates at a rate: $\dot{V} = d^3\mathbf{r}\,\nabla\cdot\mathbf{v}$

$E_{\text{int}} = \int d^3\mathbf{r}\,\varepsilon$

energy density (ε)

$$\frac{d}{dt}\int d^3\mathbf{r}\,\varepsilon = \dot{Q} - \int d^3\mathbf{r}\,(\nabla\cdot\mathbf{v})\,p$$

$$p = \tfrac{2}{3}\varepsilon + \Delta p$$

$\Delta p = 0$ for resonantly interacting gas

Easy to solve in scaling approximation: $\nabla\cdot\mathbf{v} = \dot{\Gamma}/\Gamma$

$\Gamma(t) =$ volume scale factor

Scaling Approximation

$$n(x,y,x,t)=\frac{n_0(x/b_x,y/b_y,z/b_z)}{\Gamma}$$

$$\Gamma=b_xb_yb_z$$

Volume scale factor

$$\mathrm{v}_i=x_i\dot{b}_i/b_i$$

Velocity field is linear in the spatial coordinates

$$\langle x_i^2\rangle=\langle x_i^2\rangle_0 b_i^2(t)$$

$$\langle \mathrm{v}_i^2\rangle=\langle x_i^2\rangle_0 \dot{b}_i^2(t)$$

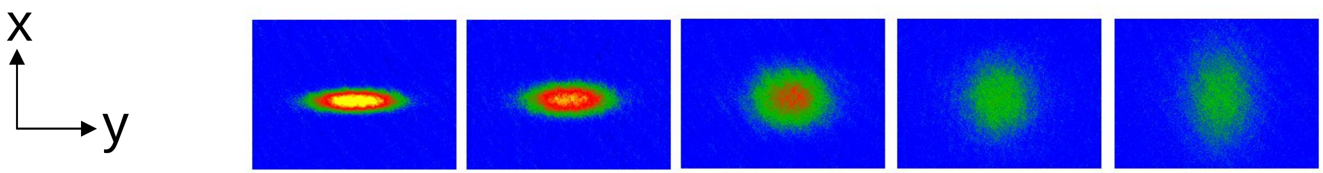
$$\overline{\omega_i^2}\equiv\frac{\langle \mathbf{r}\cdot\nabla U\rangle_0}{3m\langle x_i^2\rangle_0}$$

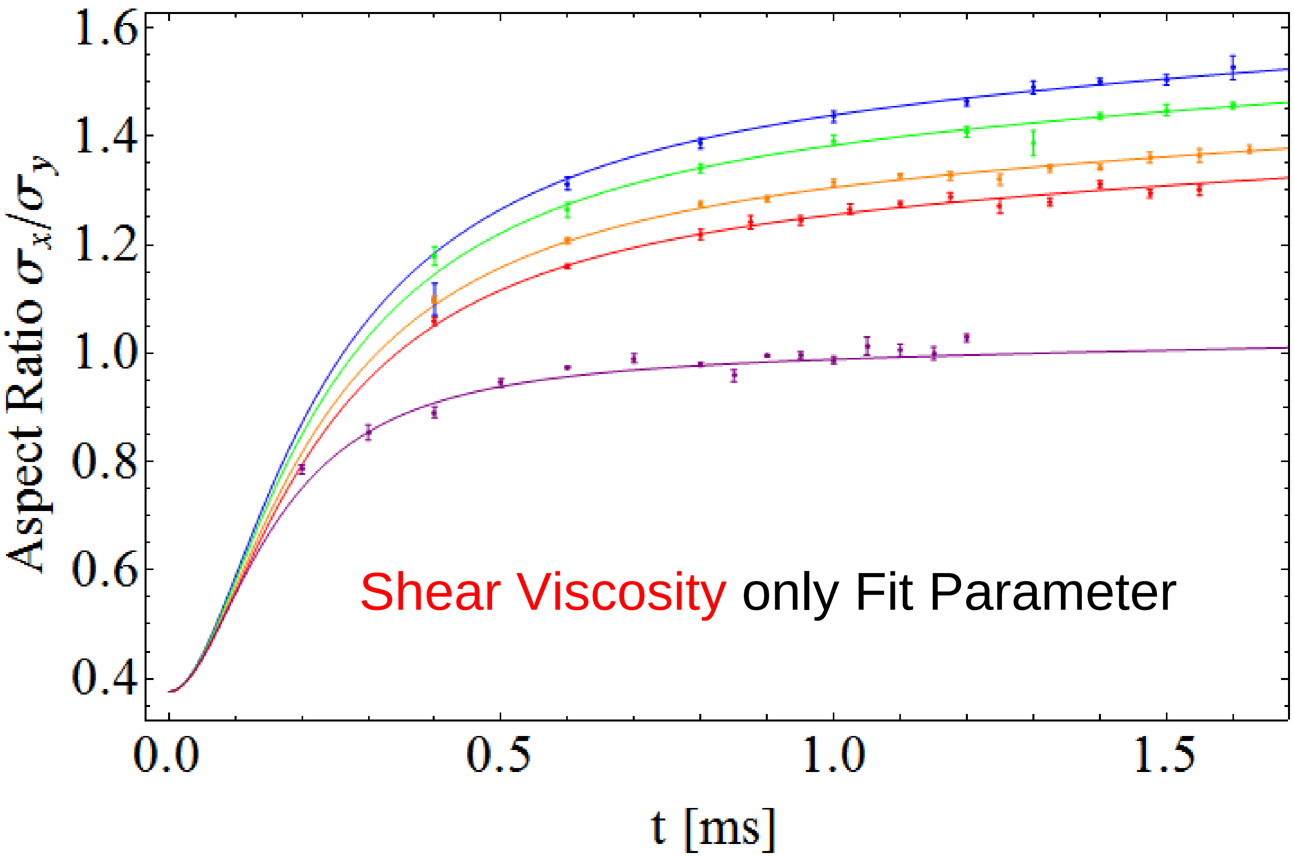
$$\sigma_{ii}=2\frac{\dot{b}_i}{b_i}-\frac{2}{3}\frac{\dot{\Gamma}}{\Gamma}$$

Cloud-averaged shear viscosity coefficient

$$\ddot{b}_i=\frac{\overline{\omega_i^2}}{\Gamma^{2/3}b_i}\left[1+C_Q(t)+C_{\Delta p}(t)\right]-\frac{\hbar\langle\alpha_S\rangle\sigma_{ii}}{m\langle x_i^2\rangle_0 b_i}-\omega_{imag}^2 b_i$$

Aspect Ratio versus Expansion Time





- $E/E_F=0.52$
- $E/E_F=0.75$
- $E/E_F=1.22$
- $E/E_F=1.69$

832 G

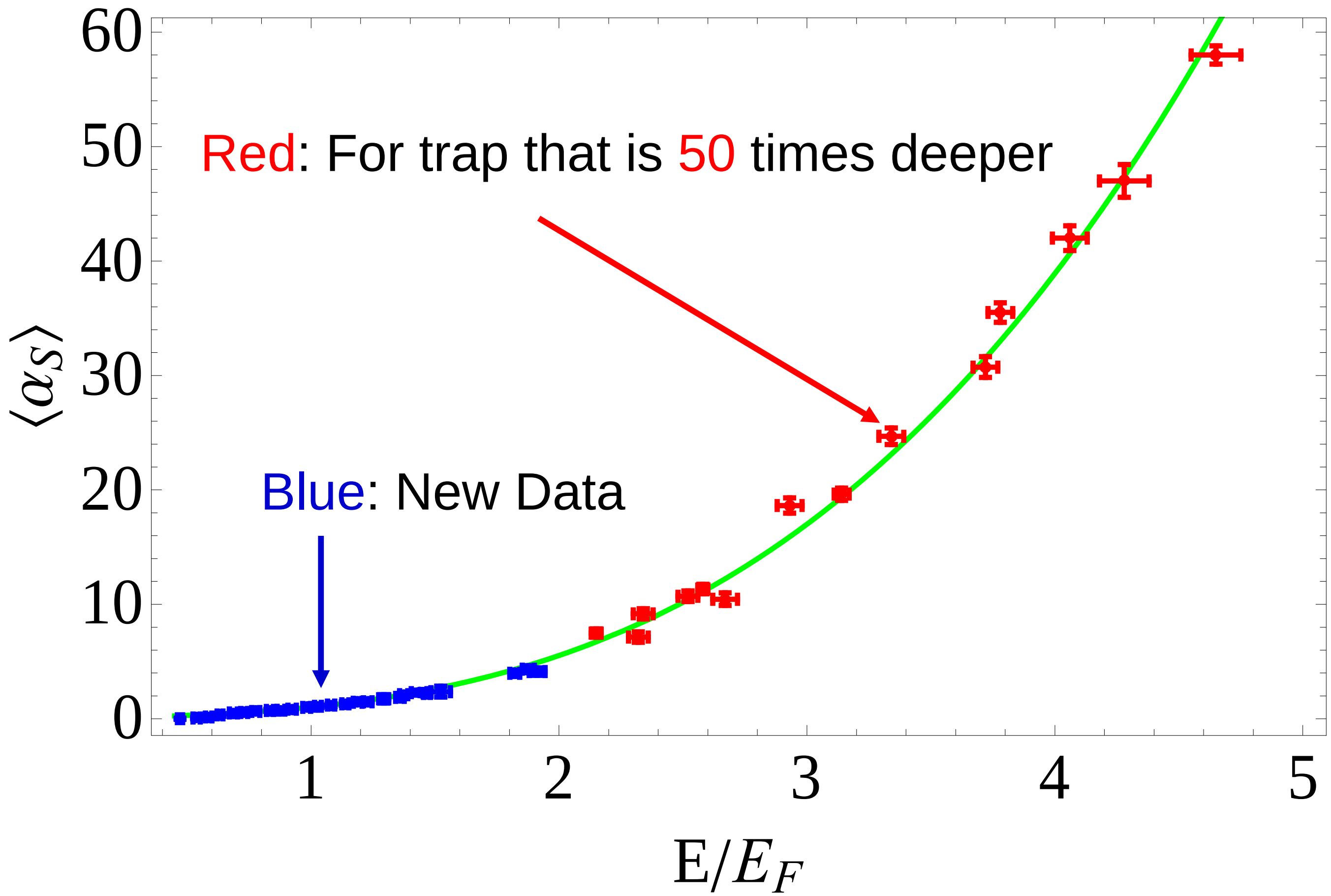
- *Ballistic*

527.5 G

$$\frac{\sigma_x}{\sigma_y} = \frac{\omega_y}{\omega_x} \frac{b_x(t)}{b_y(t)}$$

Shear Viscosity: Universal Scaling

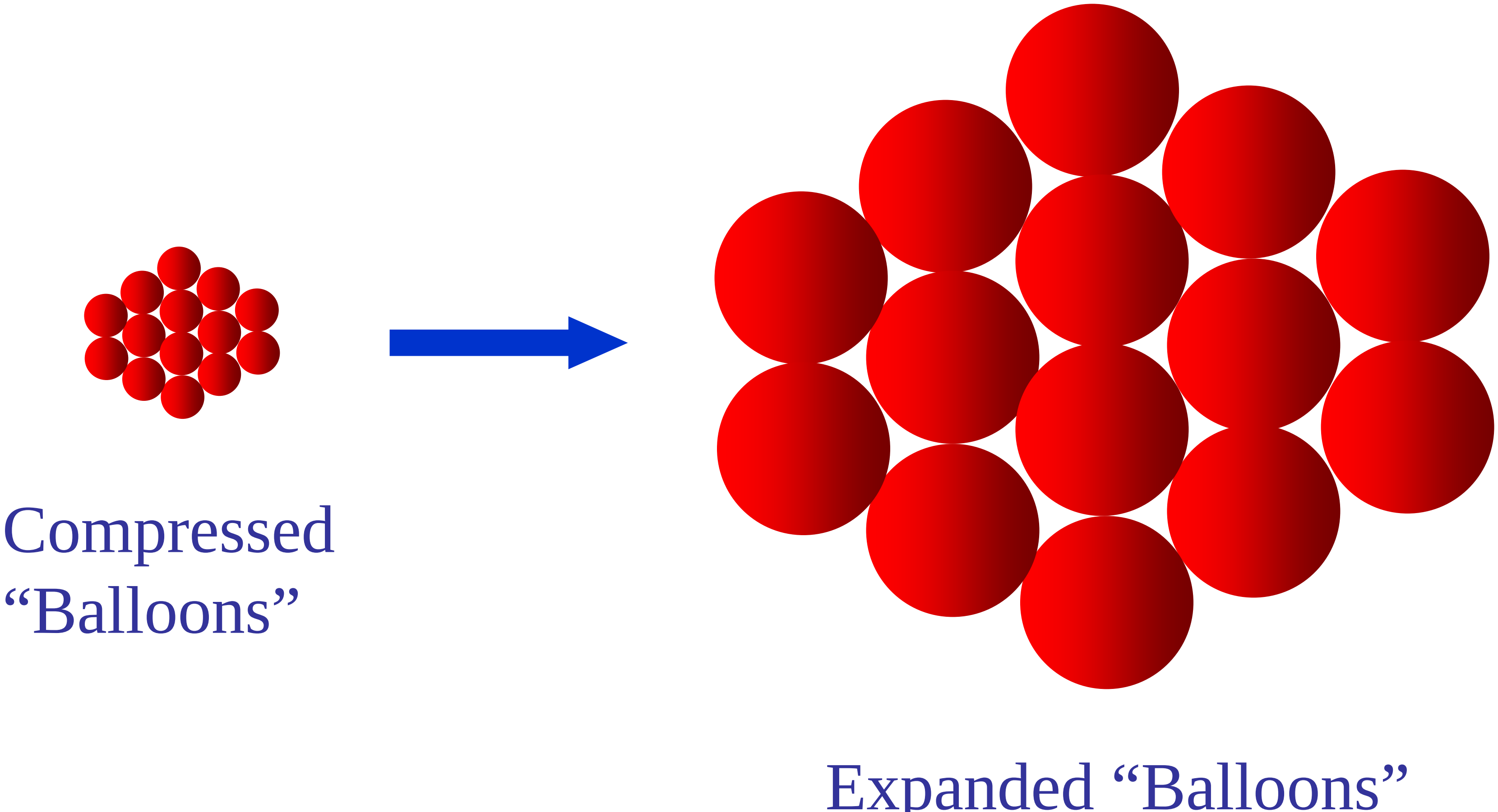
$$\eta = \alpha_S \hbar n$$



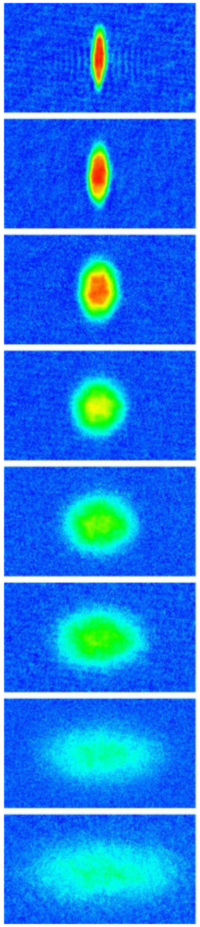
JETLab, *Science* **331**, 58 (2011)

Scale-Invariance

in Expanding *Universal* Fermi Gases



Scale-invariance: Connecting *Strongly* to *Weakly* Interacting



- Anti-de Sitter-Conformal Field Theory Correspondence: Connects strongly interacting fields in 4-dimensions to weakly interacting gravity in 5-dimensions.

- Can we connect elliptic flow in 2D to the ballistic flow of an *ideal* gas in 3D?

For both, the pressure is 2/3 of the energy density:

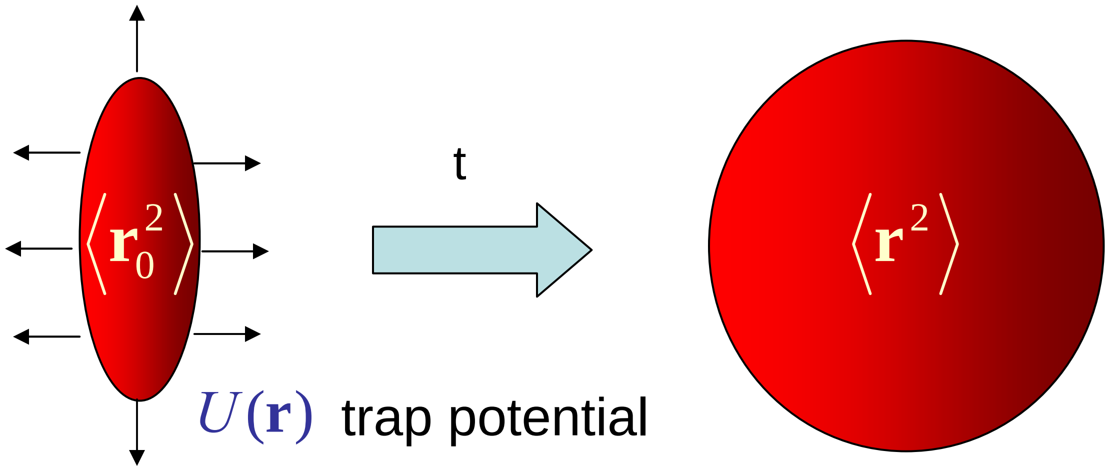
$$\Delta p \equiv p - \frac{2}{3}\varepsilon = 0$$

Scale Invariant?

Elliptic Flow: Observe 2 dimensions + time

Scale Invariance: Ideal Gas

Ideal gas: $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ Ballistic flow



$$\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + t^2 \langle \mathbf{v}^2 \rangle_0$$

Cloud average

How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

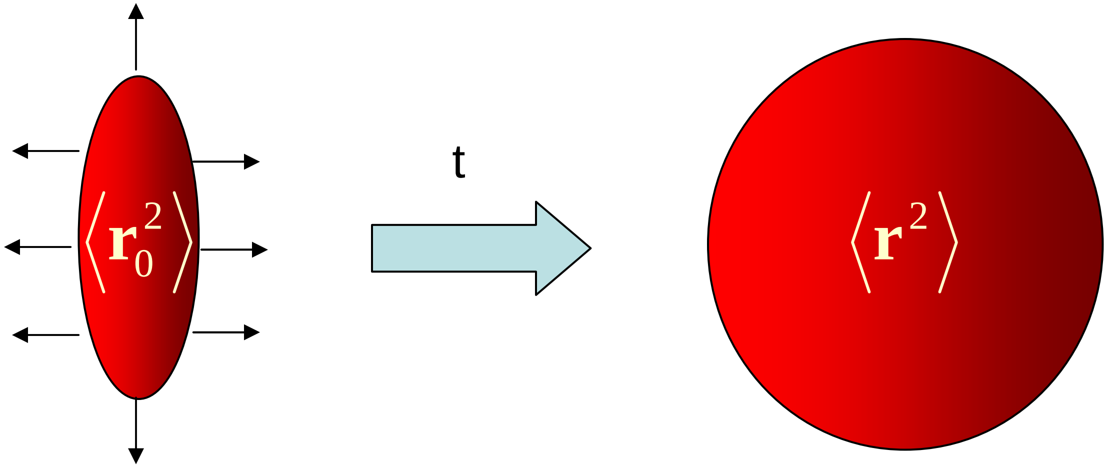
Virial Theorem:

$$m\langle \mathbf{v}^2 \rangle = \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0$$

$$\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + \frac{t^2}{m} \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0$$

Ballistic Flow

Scale Invariance: Ideal Gas



$$\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$$

Cloud average

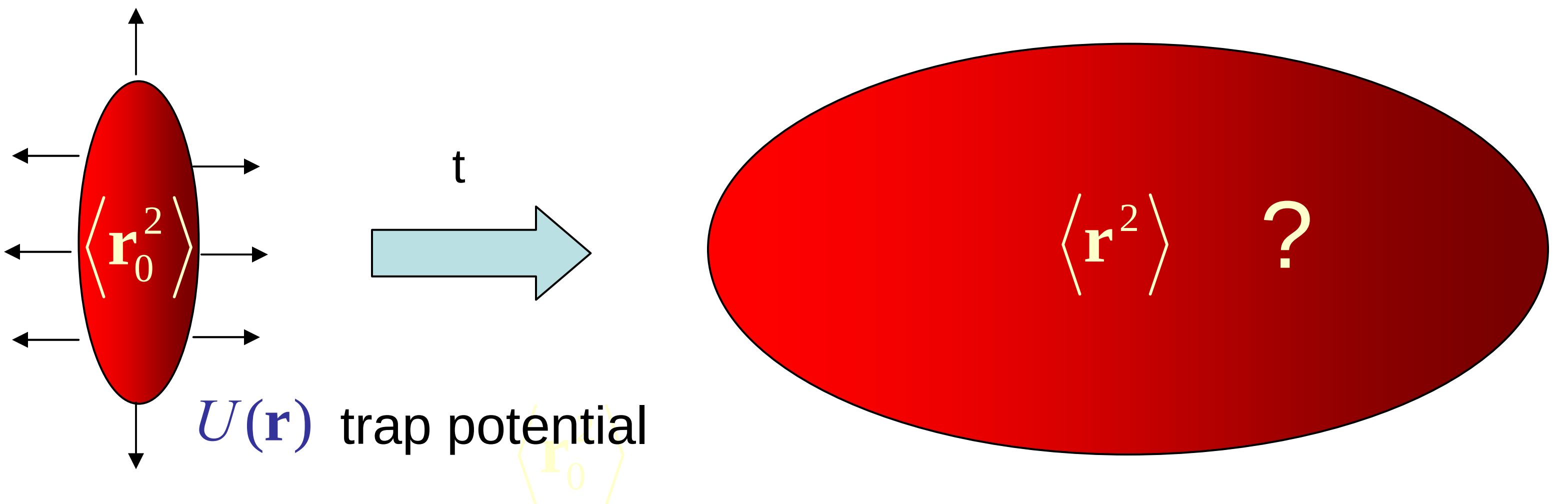
$$\frac{m\left(\langle \mathbf{r}^2 \rangle - \langle \mathbf{r}^2 \rangle_0\right)}{\langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0} = t^2$$

Defines Scale Invariant Flow!

t = expansion time

Scale Invariance: Resonant Gas

Hyrdodynamic gas: Elliptic flow



How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

Global Energy Conservation

Just after the optical trap is abruptly extinguished*: $t = 0^+$:

Stream KE (t)

Internal Energy ($t = 0^+$)

$$\frac{1}{N}\int d^3\mathbf{r}\,\mathcal{E} + \frac{m}{2}\left\langle \mathbf{v}^2 \right\rangle + \left\langle U \right\rangle = \frac{1}{N}\int d^3\mathbf{r}\,\mathcal{E}_0 + \left\langle U \right\rangle_0$$

Internal Energy (t)

Potential Energy (t)

Potential Energy ($t = 0^+$)

*Note: for $t > 0^+$, $U = U_{mag}$ arises from curvature in the bias magnetic field

Scale Invariant Expansion

Using the hydrodynamic equations for $\langle x_i^2 \rangle$ and global energy conservation it is easy to obtain the exact result:

$$\frac{d^2}{dt^2}\frac{m\langle \mathbf{r}^2 \rangle}{2} = \langle \mathbf{r}\cdot\nabla\mathbf{U}\rangle_0 + \frac{3}{N}\int d^3\mathbf{r}\left(\Delta\mathbf{p}-\Delta\mathbf{p}_0\right) - \frac{3}{N}\int d^3\mathbf{r}\,\zeta_B \nabla\cdot\mathbf{v}$$

Initial trap potential

$$= \tilde{E}$$

Conformal symmetry breaking Δp

$$\Delta p \equiv p - \tfrac{2}{3}\varepsilon$$

Bulk viscosity

Scale Invariance!

Resonant gas $\Delta p \equiv p - \frac{2}{3}\varepsilon = 0$

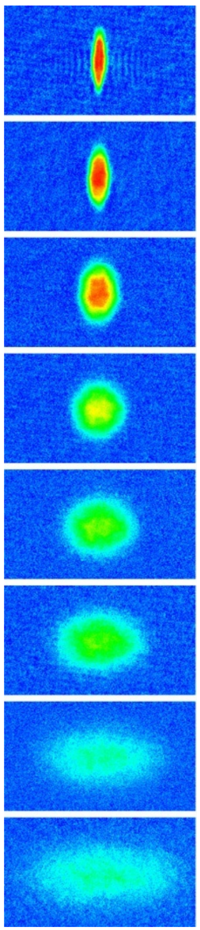
The bulk viscosity also vanishes so

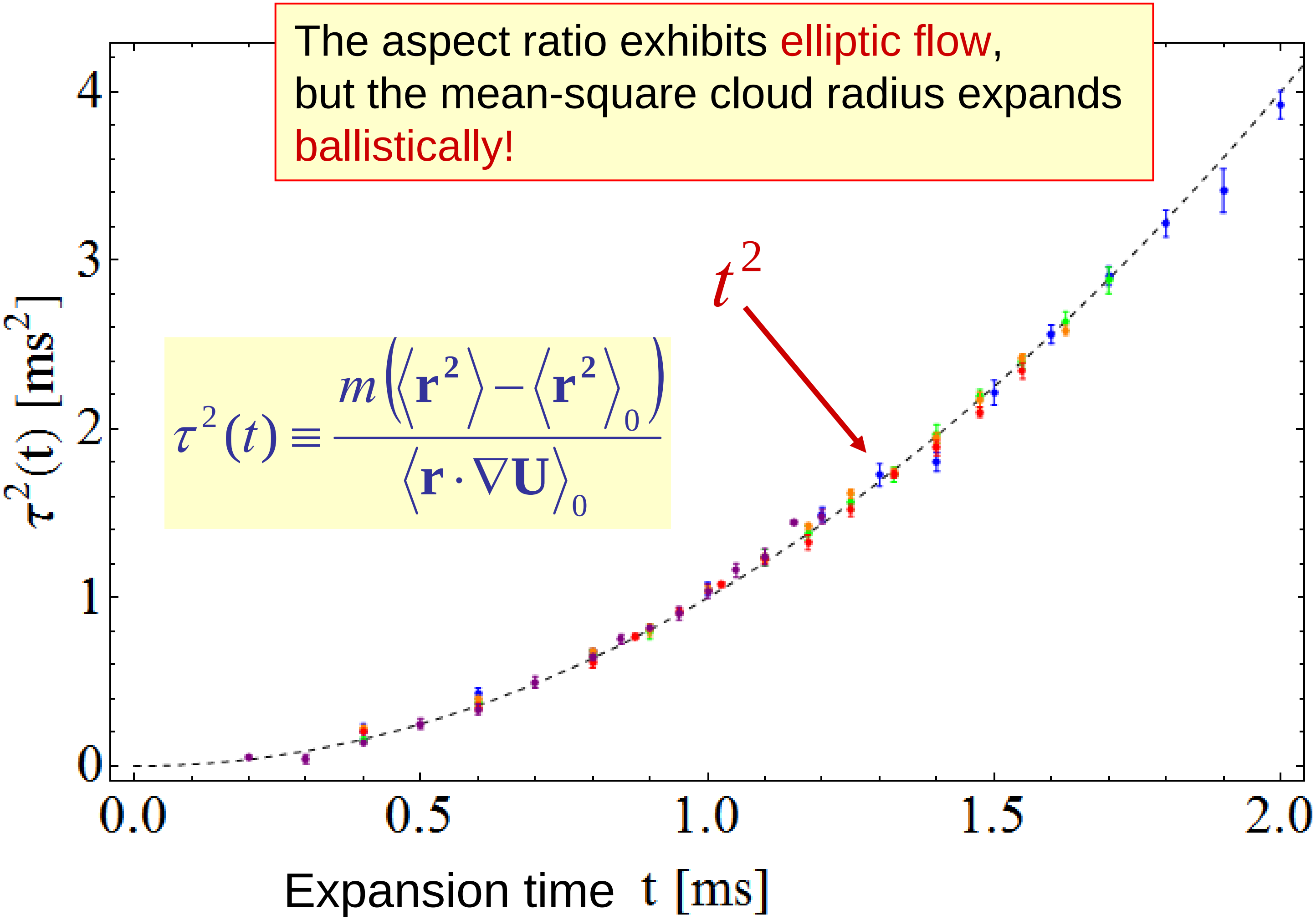
$$\left\langle \mathbf{r}^2 \right\rangle = \left\langle \mathbf{r}^2 \right\rangle_0 + \frac{t^2}{m}\left\langle \mathbf{r} \cdot \nabla \mathbf{U} \right\rangle_0$$

Ballistic Flow!

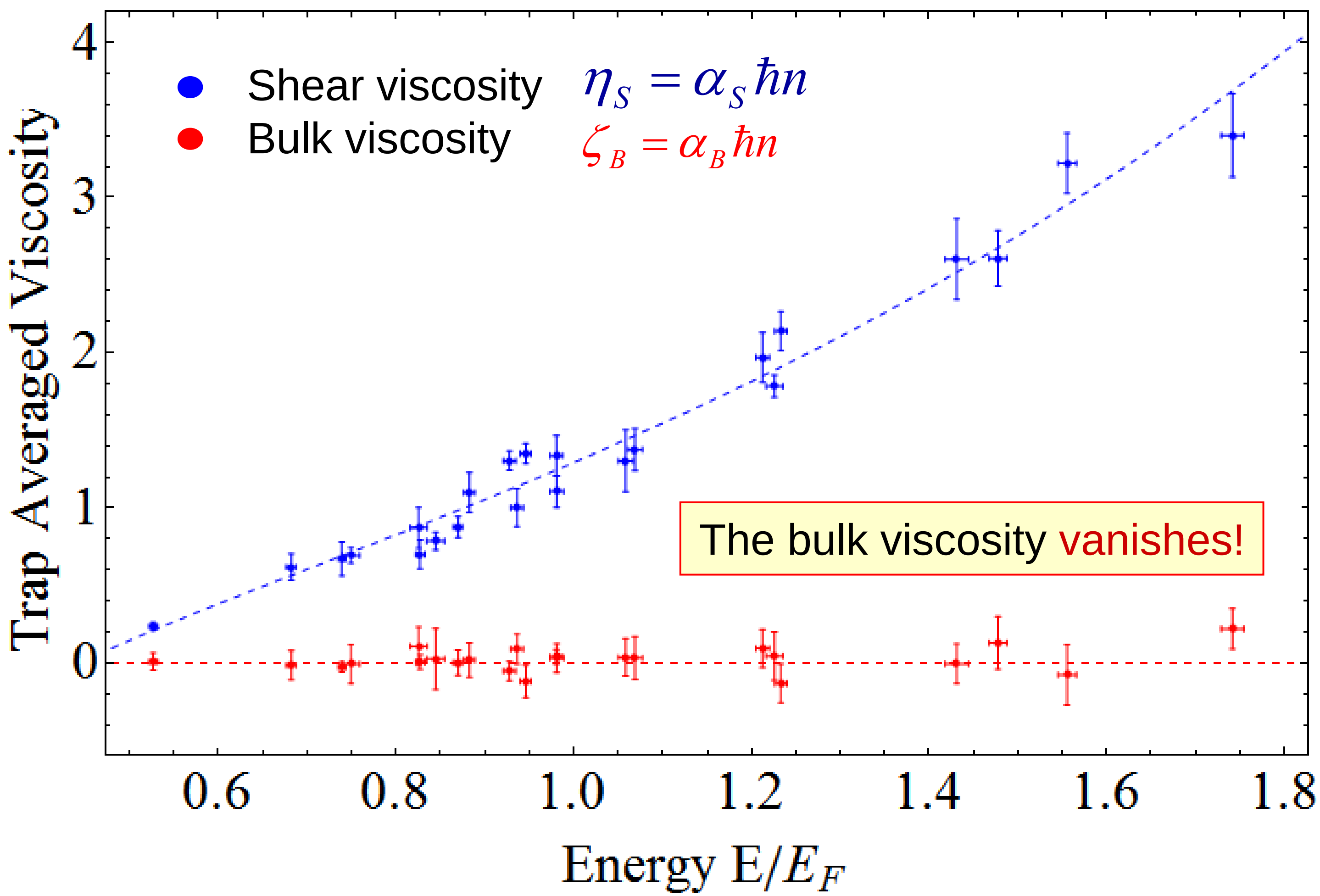
Can we observe *ballistic* flow of an *elliptically* expanding gas?

Scale-invariant "Ballistic" Expansion of a Resonant Fermi gas

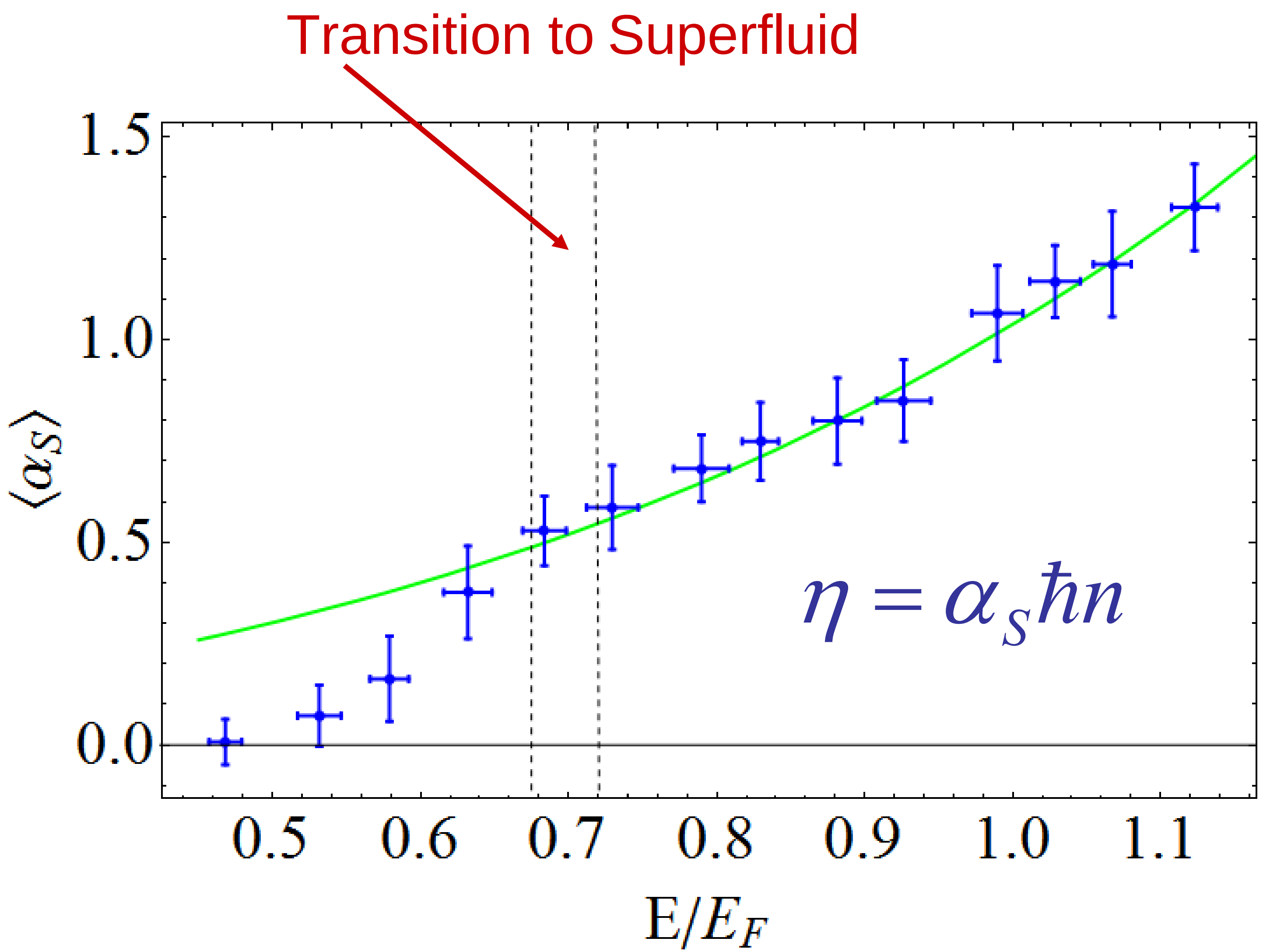




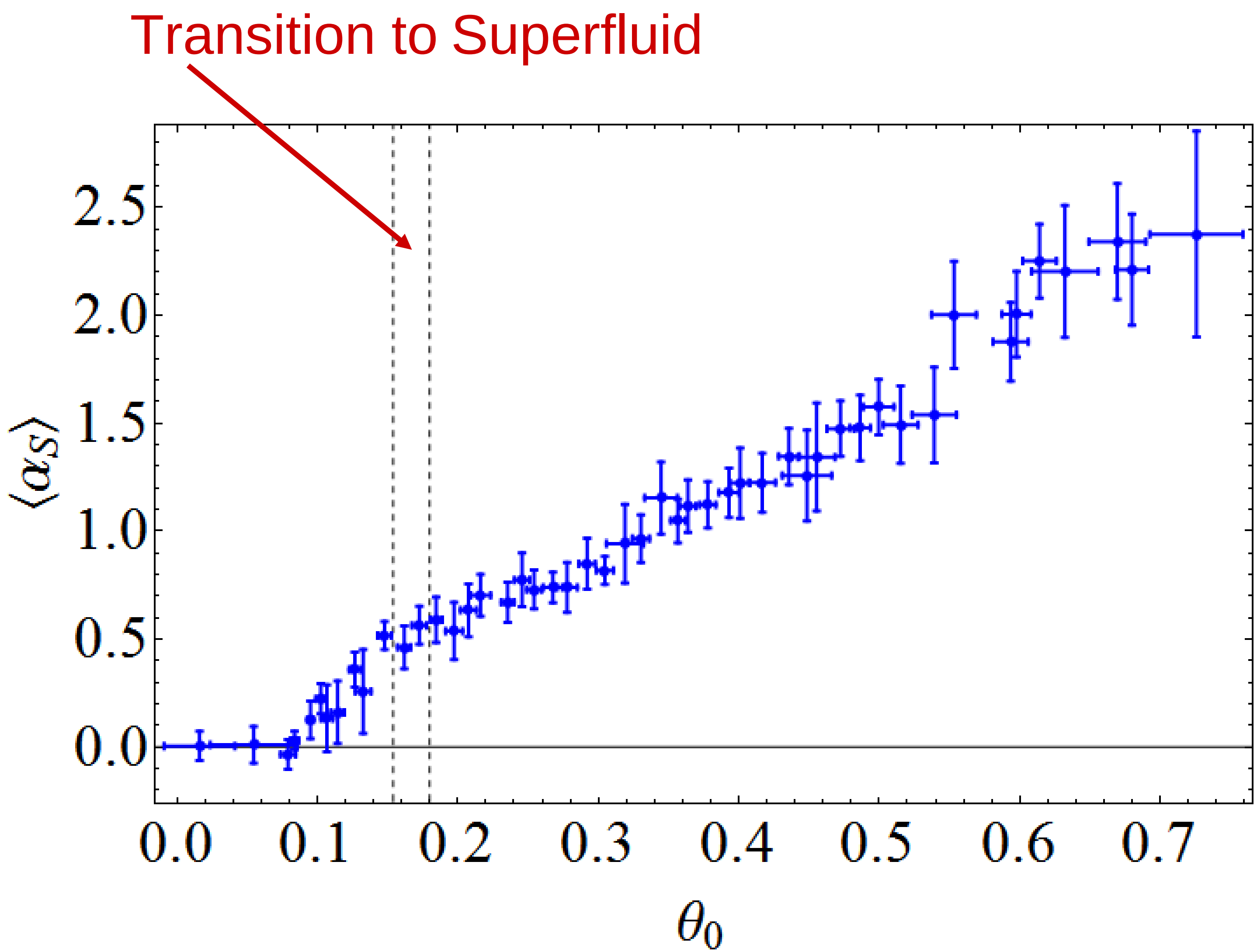
Vanishing Bulk Viscosity: Unitary Fermi Gas



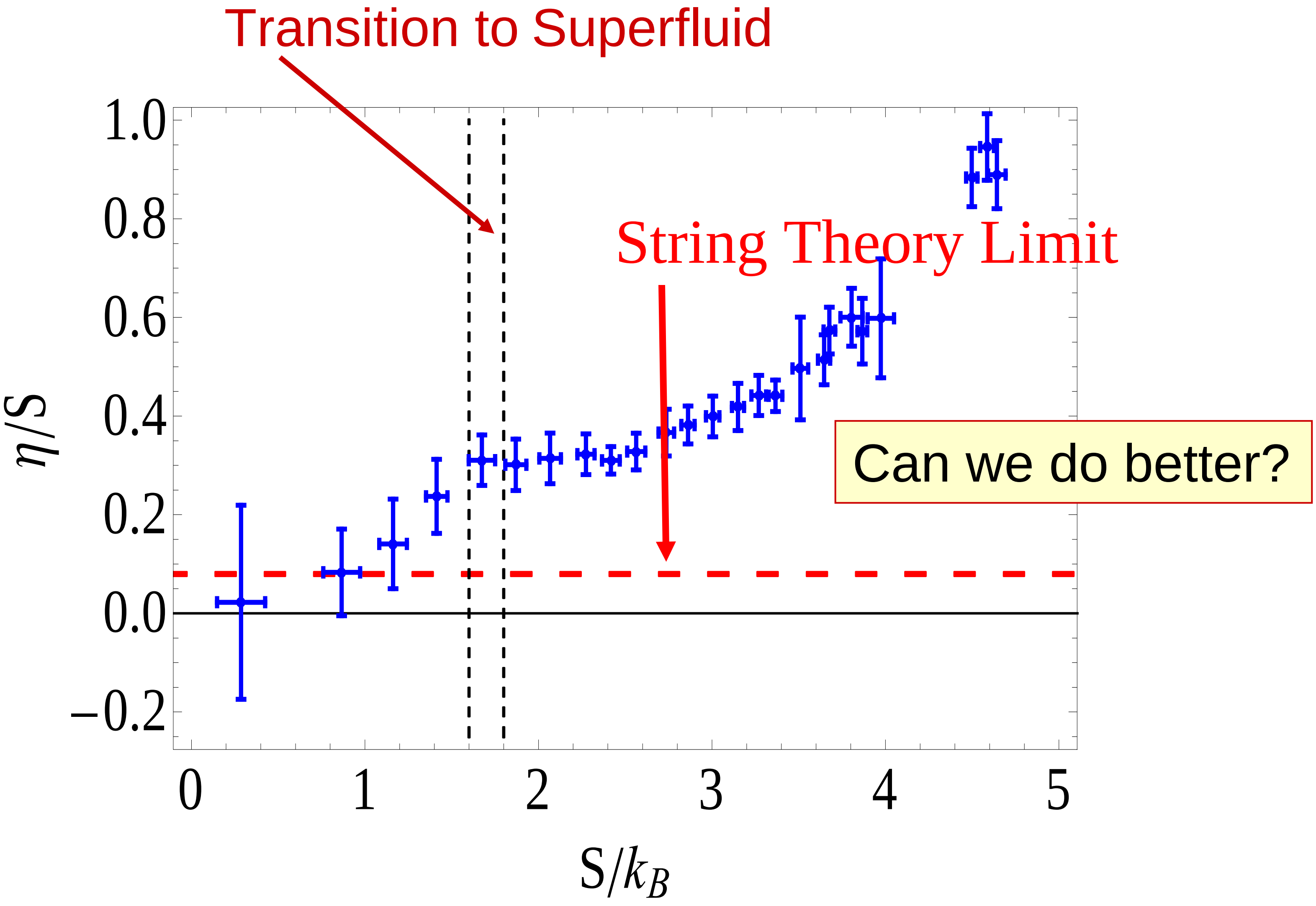
Shear Viscosity at Resonance versus Energy



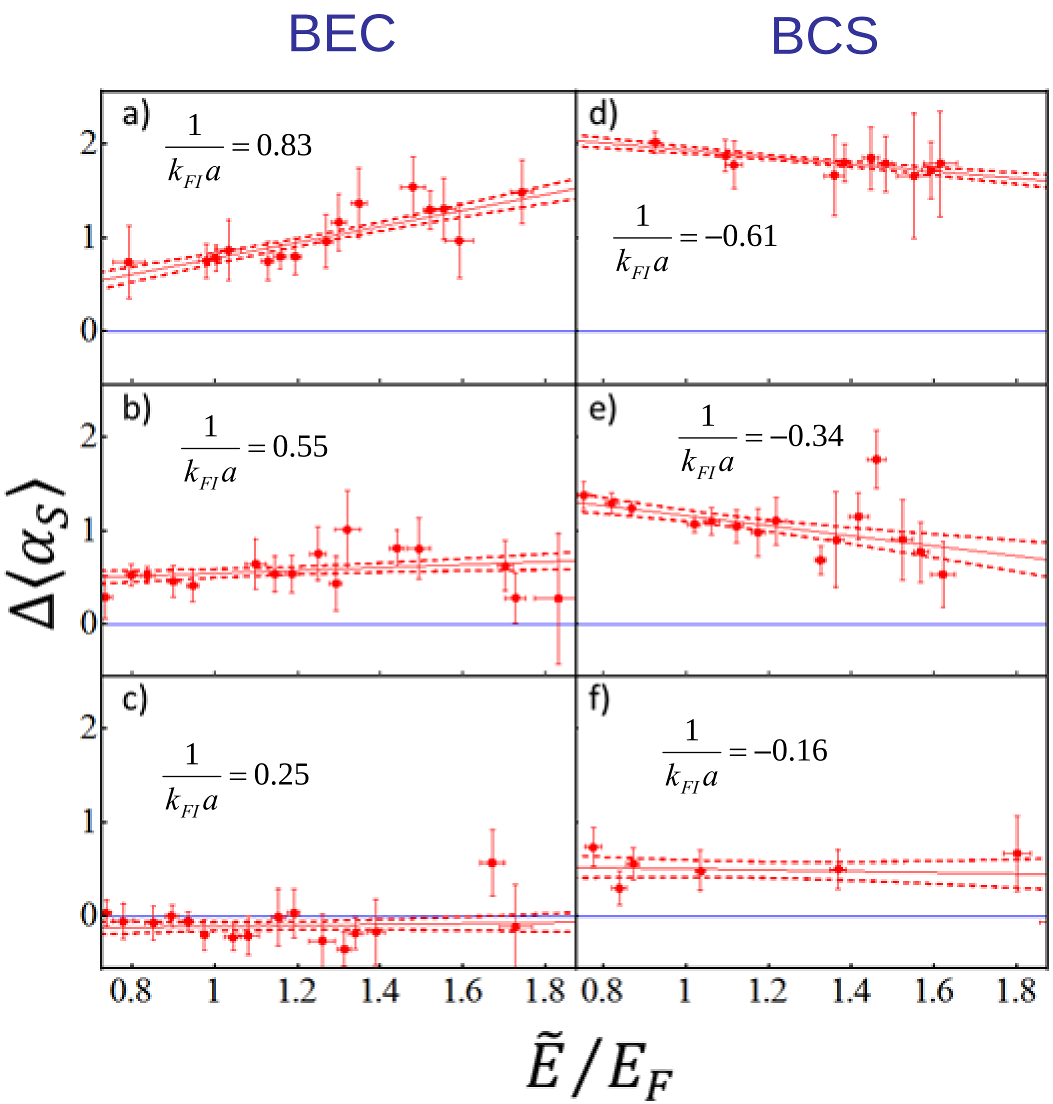
Shear Viscosity at Resonance versus Reduced Temperature



Ratio of the Shear Viscosity to the Entropy Density: Resonance



Change in Shear Viscosity at fixed Interaction Strength vs Energy



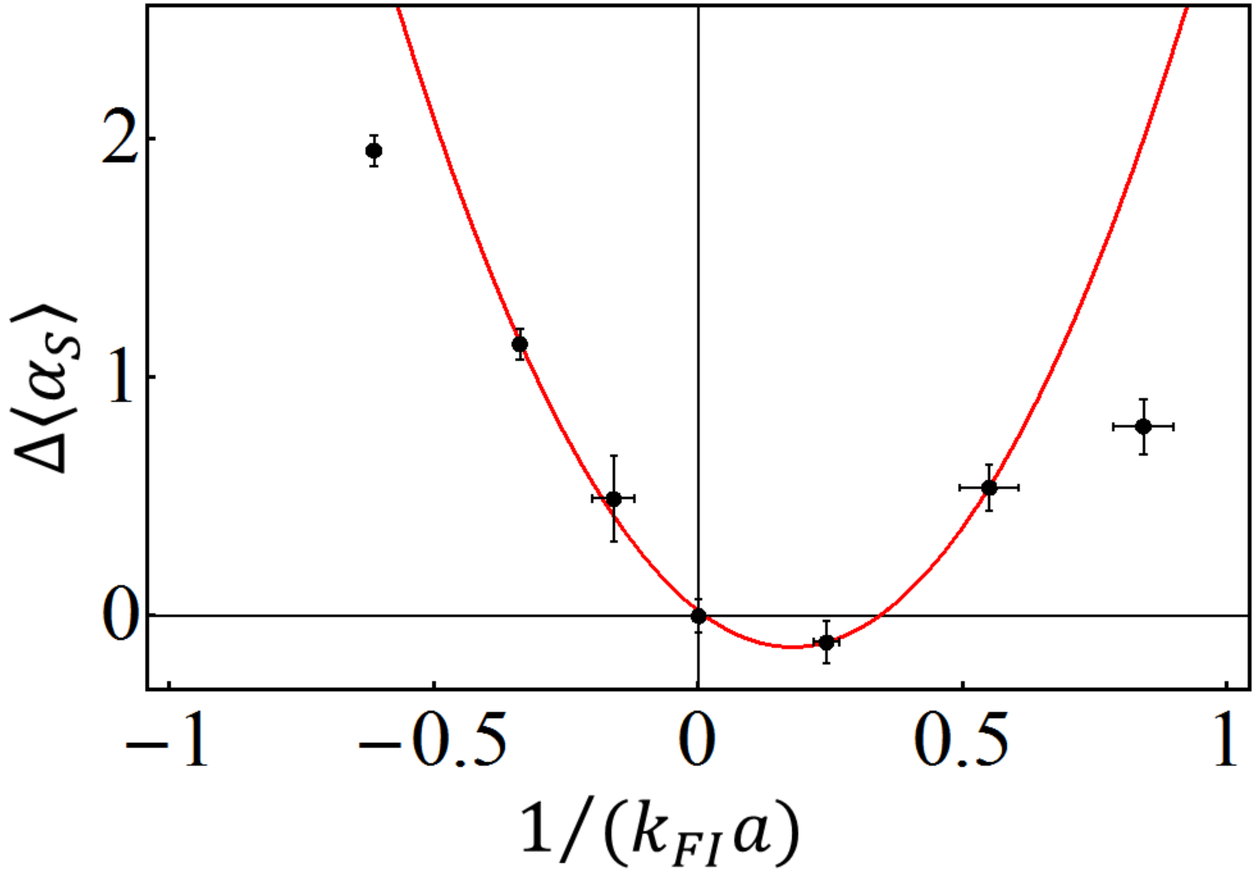
Resonance

Resonance

Resonance

Change in Shear Viscosity versus Interaction Strength

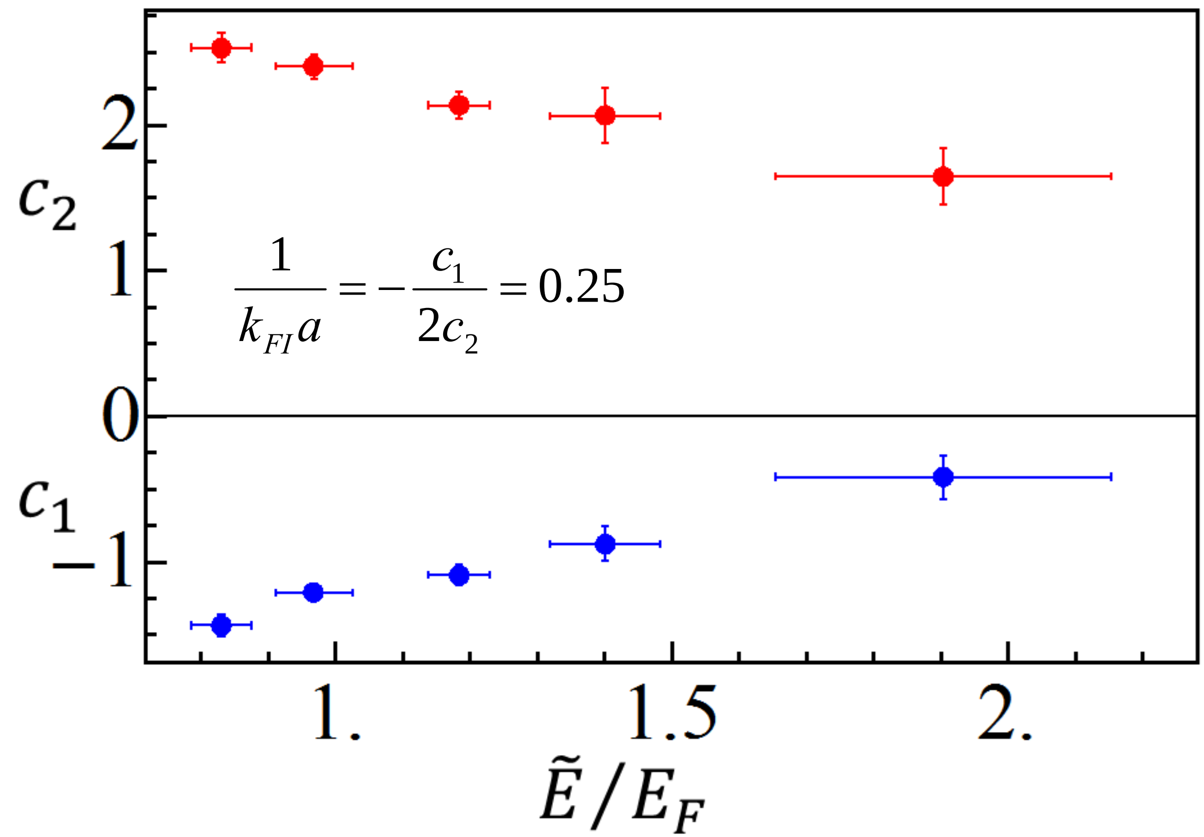
$\tilde{E} / E_F = 1.0$



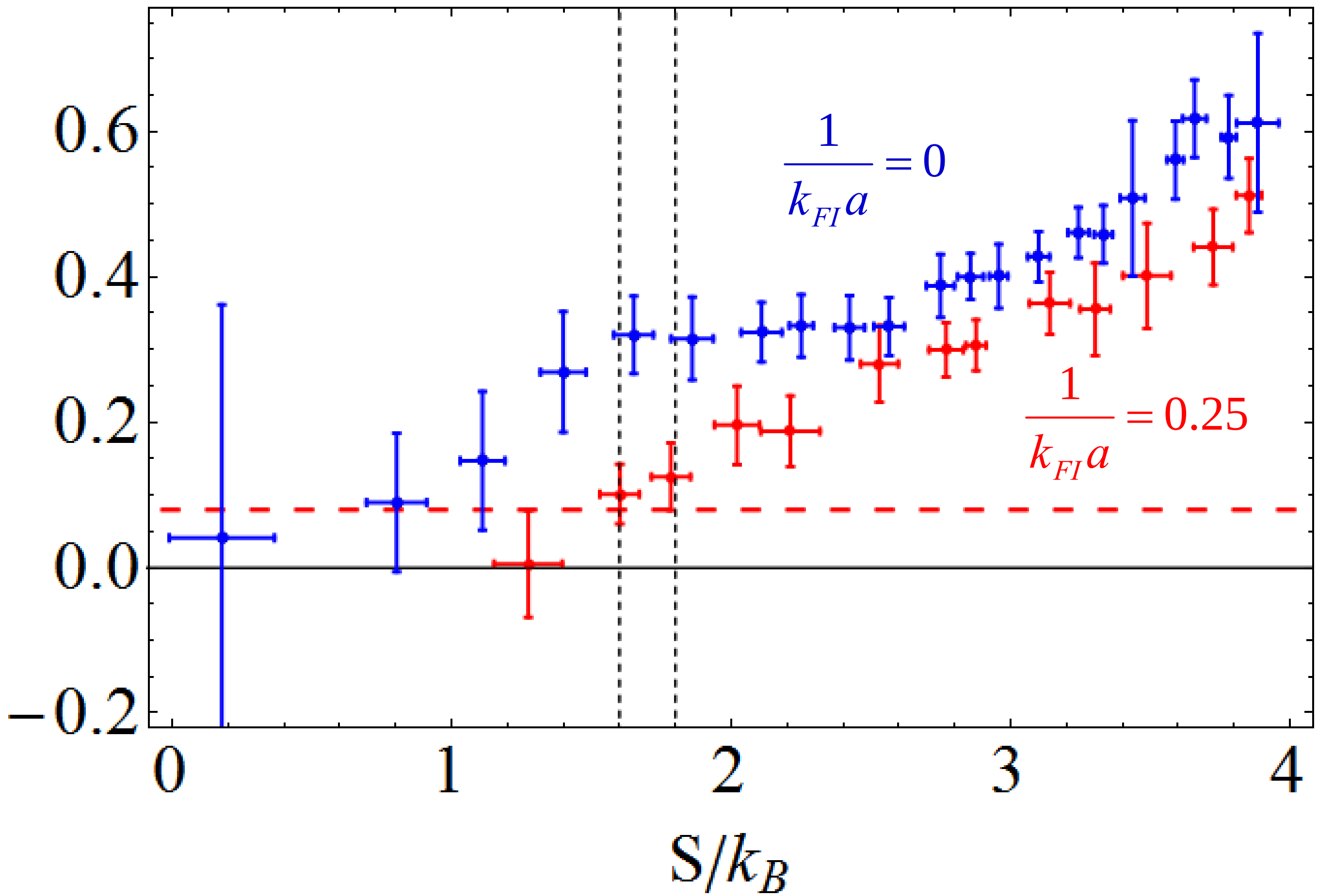
Resonance

Shear Viscosity versus Interaction Strength

$$\langle \alpha_S \rangle = \langle \alpha_S \rangle_0 + \frac{c_1 \Gamma^{1/3}(t)}{k_{FI} a} + \frac{c_2 \Gamma^{2/3}(t)}{(k_{FI} a)^2}$$



Shear Viscosity/Entropy versus Entropy: Perfect Fluid?



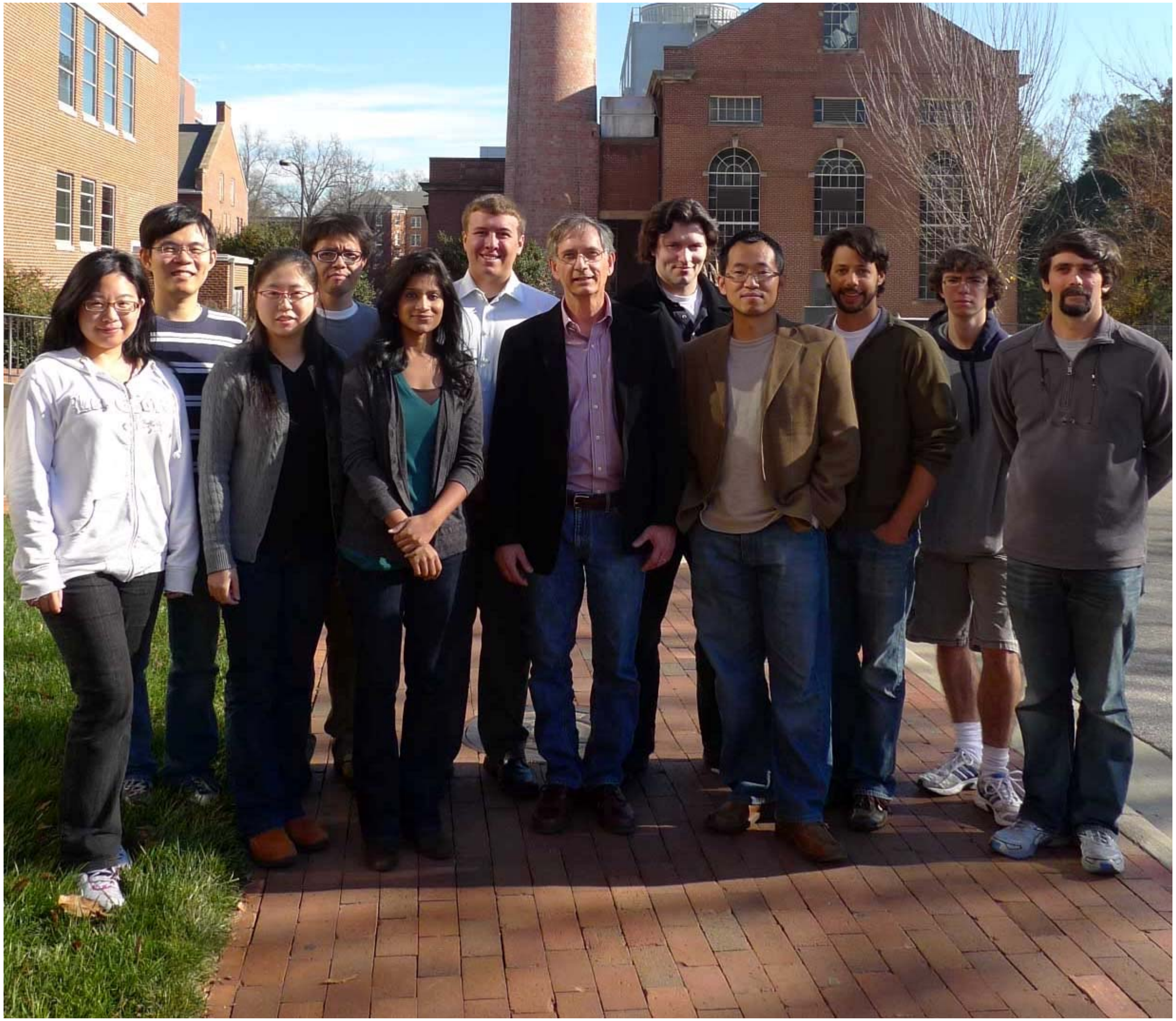
String theory limit

Summary

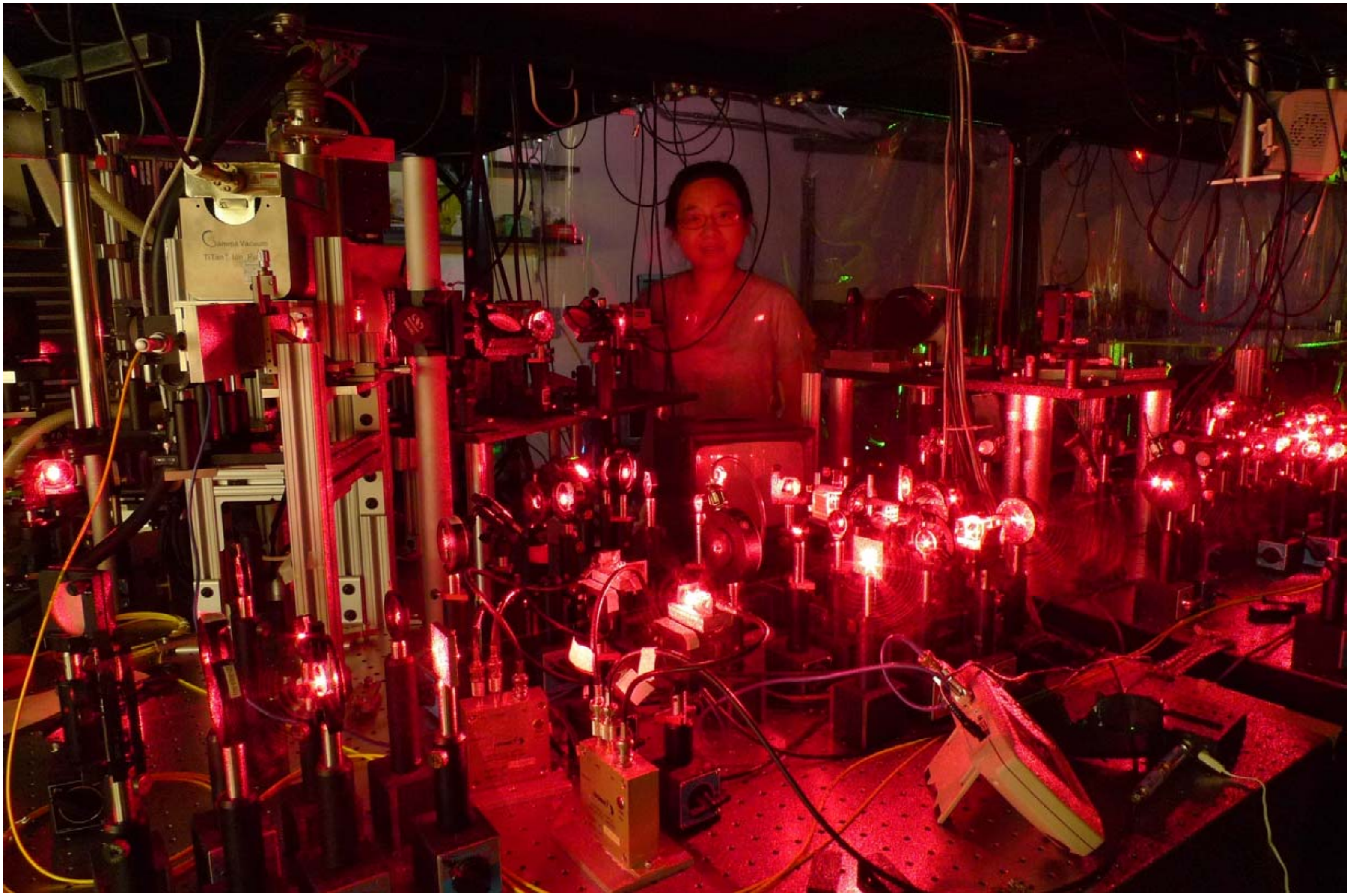
- Scale invariance in expanding Fermi gases:
  - Ballistic flow of resonant, hydrodynamic gas
  - Bulk viscosity vanishes
- Shear viscosity:
  - Shows phase transition below T_c on resonance
  - Minimum on BEC side of resonance
  - Origin of shift? Pauli blocking/atom pairs?
  - More perfect fluids?

JETLAB—NCSU Team

PHYSICS



Atom Cooling Lab



December 2010 Celebration!