

A FRESH APPROACH TO THE THEORY OF ELECTROMAGNETIC INTERACTIONS WITH NUCLEI (BOUND SYSTEMS) AND THE METHOD OF UNITARY CLOTHING TRANSFORMATIONS IN QUANTUM FIELD THEORY

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Gauge Invariance

Gauge invariance (Eicheninvarianz) principle :

- ▶ key point in construction of fundamental interactions
- ▶ its original quantum-mechanical formulation by Fock(26) and Weyl(28) after some generalizations (see, e.g., survey [JacksonOkun(01)])
- ▶ fruitful applications in different branches of physics, including nuclear physics
- ▶ so-called low-energy theorems for EM transitions.

Our departure point in describing electromagnetic (EM) interactions with nuclei (in general, bound systems of charged particles) is to use the so-called Fock-Weyl criterion and a generalization of the Siegert theorem.

Now I would like to show how one can meet this criterion in all orders in charge e and construct EM interaction operators in case of nuclear forces arbitrarily dependent on velocity. Along the guideline we have derived the conserved current density operator for a dicluster system (more precisely, the system of two finite-size clusters with many-body interaction effects included) that could be

The Fock-Weyl criterion and its consequences

Let us write for a quantum-mechanical system that interacts with EM field the Schrödinger equation

$$i \frac{\partial \Psi}{\partial t} = H \{A_\mu\} \Psi \quad (0.1)$$

and consider gradient displacement (gauge transformation after Weyl)

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu G(x) \quad (0.2)$$

of EM potential $A^\mu(x) = (A^0(x), \mathbf{A}(x))$ at space-time point $x = (t, \mathbf{x})$ with an arbitrary function $G(x)$. This point is sufficiently delicate [Kaz(82)]. In particular, one has to consider that function $G(x)$ has well-defined limits at $t = \pm\infty$.

Theory is gauge invariant if there exists a unitary transformation $\Psi \rightarrow \Psi' = U\Psi$ (after Fock(26)) such that Eq. (0.1) remains unchanged in its form, viz.,

$$i \frac{\partial \Psi'}{\partial t} = H \{A'_\mu\} \Psi'$$

This physical requirement is equivalent to the constraint

$$H \{A'_\mu\} = UH \{A_\mu\} U^\dagger + i \frac{\partial U}{\partial t} U^\dagger = UH \{A_\mu\} U^\dagger - iU \frac{\partial U^\dagger}{\partial t} \quad (0.3)$$

imposed upon Hamiltonian $H \{A_\mu\}$ of the system.

Existence of U is demonstrated by assuming that

$$U = e^{i\chi(t)}, \quad \chi(t) = - \int \hat{\rho}(\mathbf{x}) G(\mathbf{x}) d\mathbf{x}, \quad (0.4)$$

where $\hat{\rho}(\mathbf{x})$ is charge-density operator.

In fact, let us consider the field $A_\mu(\mathbf{x})$ minimally coupled with a system of interacting nonrelativistic particles (nucleons). It allows us to split $H\{A_\mu\}$ into unperturbed part $H_0 = H_{rad} + H_{nucl}$ and interaction $H_I\{A_\mu\}$ of the nucleus with the EM field, viz., $H\{A_\mu\} = H_0 + H_I\{A_\mu\}$. In turn,

$$H_I\{A_\mu\} = H_I^{(1)}\{A_\mu\} + \frac{1}{2!} H_I^{(2)}\{A_\mu\} + \dots, \quad (0.5)$$

where superscript denotes order in e . In its canonical form,

$$H_I^{(1)}\{A_\mu\} = \int A_\mu(\mathbf{x}) J^\mu(\mathbf{x}) d\mathbf{x}, \quad (0.6)$$

where $J^\mu(\mathbf{x}) = (\rho(\mathbf{x}), \mathbf{J}(\mathbf{x}))$ is charge-current density operator for system (nucleus). Further, if the operators χ and $\partial\chi/\partial t$ commute, from Eqs. (0.3) it follows that

$$H + H_I\{A_\mu + \partial_\mu G\} = e^{i\chi} H e^{-i\chi} + e^{i\chi} H_I\{A_\mu\} e^{-i\chi} - \frac{\partial\chi}{\partial t}, \quad (0.7)$$

since we are looking for the generator $\chi = \chi^\dagger$ which would be dependent merely on the nuclear variables.

Then in accordance with [KotMelShe(95)]

$$\begin{aligned}
 H^{(1)} \{A_\mu + \partial_\mu G\} &= i[\chi, H] - \frac{\partial \chi}{\partial t} + H^{(1)} \{A_\mu\}, \\
 \frac{1}{2!} H^{(2)} \{A_\mu + \partial_\mu G\} &= \frac{i^2}{2!} [\chi, [\chi, H]] + i[\chi, H^{(1)} \{A_\mu\}] + \frac{1}{2} H^{(2)} \{A_\mu\}, \quad (0.8) \\
 &\dots \quad \dots \quad \dots \quad \dots
 \end{aligned}$$

from which it follows at $A_\mu = 0$ ($\mu = 0, 1, 2, 3$):

$$\int J^\mu(\mathbf{x}) \partial_\mu G(\mathbf{x}) d\mathbf{x} = i[\chi, H] - \frac{\partial \chi}{\partial t}, \quad H^{(n)} \{\partial_\mu G\} = i^n [\chi, H]^{(n)} \quad (0.9)$$

$$(n = 2, 3, \dots)$$

where $[\chi, H]^{(n)} \equiv [\chi, [\chi, \dots [\chi, H] \dots]]$ is the multiple commutator with n brackets. One can see that the first of Eqs. (0.9) is fulfilled for a freely chosen function $G(\mathbf{x})$ with the operator χ determined as in Eq. (0.4). In turn, such a link gives rise to the continuity equation (CE) for the current:

$$i \operatorname{div} \mathbf{J}(\mathbf{x}) = [H, \rho(\mathbf{x})] \quad (0.10)$$

or, equivalently,

$$[\mathbf{P}, \mathbf{J}(0)] = [H, \rho(0)] \quad (0.11)$$

Here \mathbf{P} is total momentum operator of nucleus (system).

Thus we have shown that the Fock-Weyl criterion generates the gauge-invariance conditions in various orders in e at operator level. Following [Kaz(82)], one should stress that even if relations (0.9) are satisfied it is necessary to take care of independence of amplitude for some EM transition from choice of gauge (shortly, its GI). Actually, the GI in the first order in e for a single-photon process with energy transfer ω and momentum transfer \mathbf{q} implies that after the replacement of the photon polarization vector $\varepsilon^\mu = (\varepsilon_0, \boldsymbol{\varepsilon})$ by $q^\mu = (\omega, \mathbf{q})$ the corresponding amplitude (e.g., see below Eq. (0.15)) must be equal zero, i.e.,

$$\boldsymbol{\rho} \langle \mathbf{P}_f; f | \mathbf{J}(0) | \mathbf{P}_i; i \rangle = \omega \langle \mathbf{P}_f; f | \rho(0) | \mathbf{P}_i; i \rangle . \quad (0.12)$$

On other hand, this relation is obtained if one calculates the matrix element of both sides of Eq. (0.11) between the initial and final states $|\mathbf{P}_i; i\rangle$ and $|\mathbf{P}_f; f\rangle$, the H - and \mathbf{P} - eigenvectors:

$$\hat{H} |\mathbf{P}_i; i\rangle = E_i |\mathbf{P}_i; i\rangle , \quad \hat{H} |\mathbf{P}_f; f\rangle = E_f |\mathbf{P}_f; f\rangle \quad (0.13)$$

and

$$\hat{\mathbf{P}} |\mathbf{P}_i; i\rangle = \mathbf{P}_i |\mathbf{P}_i; i\rangle , \quad \hat{\mathbf{P}} |\mathbf{P}_f; f\rangle = \mathbf{P}_f |\mathbf{P}_f; f\rangle , \quad (0.14)$$

keeping in mind energy-momentum conservation with $\omega = E_f - E_i$ ($\mathbf{q} = \mathbf{P}_f - \mathbf{P}_i$) for the photoabsorption and $\omega = E_i - E_f$ ($\mathbf{q} = \mathbf{P}_i - \mathbf{P}_f$) for the photoemission.

In other words, **CE itself is insufficient to ensure GI of EM transition amplitude. It should be augmented with additional requirement, viz., initial and final WFs must be exact solutions of eigenvalue problem by Eqs. (0.13) - (0.14).** Clearly, it is difficult to meet this requirement in practice when handling many-particle WFs (especially, nuclear ones).

An effective way of ensuring gauge independent treatment of single-photon processes on nuclei

As shown in [She(89)], [LevShe(93)], photonuclear reaction amplitude of interest (to be more definite for the photon emission with energy E_γ and momentum \mathbf{k})

$$T_{if} = \left[2(2\pi)^3 E_\gamma \right]^{-1/2} \left\langle \mathbf{P}_i - \mathbf{k}; f \left| \varepsilon^\mu \hat{J}_\mu(0) \right| \mathbf{P}_i; i \right\rangle \quad (0.15)$$

can be expressed through electric ($\mathbf{E}(\mathbf{k})$) and magnetic ($\mathbf{H}(\mathbf{k})$) field strengths,

$$\mathbf{E}(\mathbf{k}) = i \left[2(2\pi)^3 E_\gamma \right]^{-1/2} (E_\gamma \boldsymbol{\varepsilon}(\mathbf{k}) - \mathbf{k} \varepsilon_0(\mathbf{k})), \quad (0.16)$$

$$\mathbf{H}(\mathbf{k}) = i \left[2(2\pi)^3 E_\gamma \right]^{-1/2} \mathbf{k} \times \boldsymbol{\varepsilon}(\mathbf{k}), \quad (0.17)$$

these **manifestly GI quantities**, and the matrix elements $\mathbf{D}_{if}(\mathbf{k})$ and $\mathbf{M}_{if}(\mathbf{k})$ of the so-called generalized electric and magnetic dipole moments of nucleus (system):

$$T_{if} = \mathbf{E}(\mathbf{k}) \mathbf{D}_{if}(\mathbf{k}) + \mathbf{H}(\mathbf{k}) \mathbf{M}_{if}(\mathbf{k}). \quad (0.18)$$

These formulae were derived without separation of the center-of-mass (CM) motion, and thus they can be used in relativistic nuclear models [She(12)] or in problems, where such a separation becomes hardly feasible (see [LevCanShe(04)] and refs. therein).

Note helpful relations such as

i) property of translational invariance

$$J^\mu(\vec{x}) = e^{-i\vec{P}\vec{x}} J^\mu(0) e^{i\vec{P}\vec{x}} \quad (0.19)$$

ii) Foldy representation [Foldy(530)] for

$$\vec{a} e^{i\vec{b}\vec{c}} = \int_0^1 \{ \nabla_{\vec{c}} \left(\vec{a} \cdot \vec{c} e^{i\lambda\vec{b}\vec{c}} \right) + i\lambda\vec{c} \times [\vec{a} \times \vec{b}] e^{i\lambda\vec{b}\vec{c}} \} d\lambda \quad (0.20)$$

with arbitrary vectors \vec{a} , \vec{b} and \vec{c} .

iii) operator identity

$$e^{\hat{a}+\hat{b}} = e^{\hat{b}} e^{\hat{a}} \exp \left\{ \int_0^1 ds e^{-s\hat{a}} \left[e^{-s\hat{b}} \hat{a} e^{s\hat{b}} - \hat{a} \right] e^{s\hat{a}} \right\}, \quad (0.21)$$

that yields

$$e^{\hat{a}+\hat{b}} = e^{\hat{b}} \exp \left\{ \int_0^1 e^{-s\hat{b}} \hat{a} e^{s\hat{b}} ds \right\}$$

if

$$\left[e^{-s\hat{b}} \hat{a} e^{s\hat{b}}, \hat{a} \right] = 0.$$

for any operators \hat{a} and \hat{b} .

Generalized electric dipole moment operator with one-body charge density

For the exposition we will confine ourselves to nonrelativistic approach, in which nuclear Hamiltonian H

$$H = \frac{\mathbf{P}^2}{2M} + H_{int} \equiv K_{CM} + H_{int}, \quad (0.22)$$

where M total mass of nuclear system, is divided into kinetic energy operator K_{CM} of center-of-mass (CM) motion and intrinsic Hamiltonian H_{int} that depends on internal variables of interacting nucleons. The respective eigenvectors can be factorized as $|\mathbf{P}_i; i\rangle = |\mathbf{P}_i\rangle |i\rangle$, $|\mathbf{P}_i - \mathbf{k}; f\rangle = |\mathbf{P}_i - \mathbf{k}\rangle |f\rangle$, where the bracket $| \rangle$ is used to represent a vector in the space of the CM coordinate \mathbf{R} so that $\hat{\mathbf{P}} |\mathbf{P}\rangle = \mathbf{P} |\mathbf{P}\rangle$ and $H_{int} |i(f)\rangle = E_{i(f)}^{int} |i(f)\rangle$.

Then the reaction amplitude can be written in the form $T_{if} = \langle f | T_{int} | i \rangle$ with operator T_{int} , which acts in the space of internal variables,

$$T_{int} = \mathbf{E}(\mathbf{k})\mathbf{D}(\mathbf{k}) + \mathbf{H}(\mathbf{k})\mathbf{M}(\mathbf{k}), \quad (0.23)$$

where we have introduced operator $\mathbf{D}(\mathbf{k})$ of generalized electric dipole moment (GEDM)

$$\mathbf{D}(\mathbf{k}) = \frac{1}{E_\gamma} \int_0^1 (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{R} [H, \rho(0)] | \mathbf{P}_i) d\lambda, \quad (0.24)$$

and operator $\mathbf{M}(\mathbf{k})$ of generalized magnetic dipole moment (GMDM)

$$\mathbf{M}(\mathbf{k}) = - \int_0^1 (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{R} \times \mathbf{J}(0) | \mathbf{P}_i) \lambda d\lambda. \quad (0.25)$$

Further, with the help the relation $[H, \mathbf{R}] = -i\mathbf{P}/M$ we find

$$(2\pi)^3 \mathbf{D}(\mathbf{k}) = \frac{1}{E_\gamma} \int_0^1 d\lambda \left\{ [\mathbf{D}_{int}(\lambda \mathbf{k}), H_{int}] + \lambda \frac{\mathbf{k} \cdot (2\mathbf{P}_i - \lambda \mathbf{k})}{2M} \mathbf{D}_{int}(\lambda \mathbf{k}) + i \frac{\mathbf{P}_i - \lambda \mathbf{k}}{M} \rho_{int}(\lambda \mathbf{k}) \right\} \quad (0.26)$$

with

$$\rho_{int}(\lambda \mathbf{k}) = (2\pi)^3 (\mathbf{P}_i - \lambda \mathbf{k} | \rho(0) | \mathbf{P}_i), \quad (0.27)$$

$$\mathbf{D}_{int}(\lambda \mathbf{k}) = -(2\pi)^3 (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{R} \rho(0) | \mathbf{P}_i) = \frac{i}{\lambda} \nabla_{\mathbf{k}} \rho_{int}(\lambda \mathbf{k}). \quad (0.28)$$

In the phenomenological treatment of EM interactions with nuclei one considers the one-body (additive in constituents) charge density operator,

$\rho(\mathbf{x}) = \rho^{[1]}(\mathbf{x}) \equiv \sum_{\alpha} \rho_{\alpha}(\mathbf{x})$, where the summation runs over all the constituents (protons and neutrons) and the charge distribution referred to one nucleon

$$\rho_{\alpha}(\mathbf{x}) = \rho_p(\mathbf{x} - \hat{\mathbf{r}}_{\alpha})\pi_p(\alpha) + \rho_n(\mathbf{x} - \hat{\mathbf{r}}_{\alpha})\pi_n(\alpha) \quad (0.29)$$

with arbitrary (for a moment) functions $\rho_{p,n}(\mathbf{y})$ normalized as

$\int \rho_p(\mathbf{y})d\mathbf{y} = e_p = e$, $\int \rho_n(\mathbf{y})d\mathbf{y} = 0$. Here $\hat{\mathbf{r}}_{\alpha}$ nucleon coordinate operator and $\pi_p(\alpha) = \frac{1}{2}[1 + \tau_3(\alpha)]$ ($\pi_n(\alpha) = \frac{1}{2}[1 - \tau_3(\alpha)]$) is the projection operator onto proton (neutron) states.

Thus within this one-body model we obtain

$$\rho_{int}^{[1]}(\lambda\mathbf{k}) = \sum_{\alpha} \rho_{\alpha}(\lambda\mathbf{k}) = \sum_{\alpha} f_{\alpha}^N(\lambda\mathbf{k}) e^{-i\lambda\mathbf{k}\mathbf{r}'_{\alpha}} \quad (0.30)$$

and

$$\mathbf{D}_{int}^{[1]}(\lambda\mathbf{k}) = \mathbf{d}_{int}^{[1]}(\lambda\mathbf{k}) + i\frac{\mathbf{k}}{k} \sum_{\alpha} e^{-i\lambda\mathbf{k}\mathbf{r}'_{\alpha}} \left. \frac{df_{\alpha}^N(q)}{dq} \right|_{q=\lambda k}, \quad (0.31)$$

$$\mathbf{d}_{int}^{[1]}(\lambda\mathbf{k}) = \sum_{\alpha} \mathbf{d}_{\alpha}(\lambda\mathbf{k}) = \sum_{\alpha} f_{\alpha}^N(\lambda\mathbf{k}) \mathbf{r}'_{\alpha} e^{-i\lambda\mathbf{k}\mathbf{r}'_{\alpha}}, \quad (0.32)$$

where $\mathbf{r}'_{\alpha} = \mathbf{r}_{\alpha} - \mathbf{R}$ relative coordinate (Jacobi variable) and $f_{\alpha}^N(\vec{q})$ nucleon FF.

Generalized magnetic dipole moment operator for systems with two-body forces between their constituents.

As to the GMDM, we are looking for current density operator $\mathbf{J}(\mathbf{x})$ in form

$$\mathbf{J}(\mathbf{x}) = \mathbf{J}^{[1]}(\mathbf{x}) + \mathbf{J}^{[2]}(\mathbf{x}) \quad (0.33)$$

to be consistent with **nonrelativistic** Hamiltonian

$$H = \sum_{\alpha} \frac{\mathbf{p}_{\alpha}^2}{2m_{\alpha}} + \sum_{\alpha < \beta} V(\alpha, \beta) \equiv K + V \quad (0.34)$$

via CE, viz.,

$$[\mathbf{P}, \mathbf{J}^{[1]}(0)] = [K, \rho^{[1]}(0)], \quad (0.35)$$

$$[\mathbf{P}, \mathbf{J}^{[2]}(0)] = [V, \rho^{[1]}(0)], \quad (0.36)$$

where \mathbf{p}_{α} momentum operator of nucleon with label α , $m_{\alpha} = m$ nucleon (constituent) mass and $V(\alpha, \beta)$ interaction between two constituents.

Respectively,

$$(2\pi)^3 \mathbf{M}(\mathbf{k}) = \mathbf{M}_{1BD}(\mathbf{k}) + \mathbf{M}_{2BD}(\mathbf{k}), \quad (0.37)$$

$$\mathbf{M}_{1BD}(\mathbf{k}) \equiv i(2\pi)^3 \nabla_{\mathbf{k}} \times \int_0^1 d\lambda (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{J}^{[1]}(0) | \mathbf{P}_i) = \mathbf{M}_C(\mathbf{k}) + \mathbf{M}_S(\mathbf{k}), \quad (0.38)$$

and let me be concerned with two-body part

$$\mathbf{M}_{2BD}(\mathbf{k}) = i(2\pi)^3 \nabla_{\mathbf{k}} \times \int_0^1 d\lambda (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{J}^{[2]}(0) | \mathbf{P}_i) \quad (0.39)$$

At this point, if we put $\hat{\mathbf{J}}_{int}(\lambda \mathbf{k}) = (2\pi)^3 (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{J}(0) | \mathbf{P}_i)$ by analogy with the relation (0.27) and consider the Fourier transform $\hat{\mathbf{J}}_{int}(\mathbf{q}) = \int e^{-i\mathbf{q}\mathbf{x}} \hat{\mathbf{J}}_{int}(\mathbf{x}) d\mathbf{x}$, it gives rise to quantum analog

$$\hat{\mathbf{M}}(0) \equiv \hat{\mathbf{M}} = \int d\mathbf{x} \mathbf{x} \times \hat{\mathbf{J}}_{int}(\mathbf{x}) \quad (0.40)$$

of the Biot-Savart formula for magnetic dipole moment from classical magnetodynamics. Such a continuity justifies the terminology adopted here. In its turn, operator $\mathbf{M}_{2BD}(\mathbf{k})$, being generated by two-body current density $\mathbf{J}^{[2]}(\mathbf{x})$, occurs if commutator in the r.h.s. of Eq. (0.36) is nonzero. In this application to nuclear physics we will show a possible way of constructing the density taking into exchange and nonlocal properties of nuclear forces.

In this context, following [KoShe(84)], we employ the representation

$$\hat{V}(\alpha, \beta) = \int d\mathbf{x} \exp(-i\hat{\mathbf{p}}_{\alpha\beta}\mathbf{x}) V(\hat{\mathbf{r}}_{\alpha\beta} + \mathbf{x}, \hat{\mathbf{r}}_{\alpha\beta}) \quad (0.41)$$

of translationally invariant interaction $V(\alpha, \beta)$ that can depend on $\mathbf{r}_{\alpha\beta} = \mathbf{r}_{\alpha} - \mathbf{r}_{\beta}$ $\mathbf{p}_{\alpha\beta} = (m_{\beta} \mathbf{p}_{\alpha} - m_{\alpha} \mathbf{p}_{\beta}) / (m_{\alpha} + m_{\beta})$ ($\mathbf{p}_{\alpha\beta} = \frac{1}{2} (\mathbf{p}_{\alpha} - \mathbf{p}_{\beta})$ for identical particles) in an arbitrary manner. Of course, one needs to keep in mind that quantities $V(\mathbf{r}'_{\alpha\beta}, \mathbf{r}_{\alpha\beta}) \equiv \langle \mathbf{r}'_{\alpha\beta} | \hat{V}(\alpha, \beta) | \mathbf{r}_{\alpha\beta} \rangle$ are operators in spin space of two particles (nucleons) and depend on their charge state.

Functional $H\{A_\mu\}$ can be built up if we include EM field via minimal substitution:

$$\hat{\mathbf{p}}_\alpha \rightarrow \hat{\mathbf{p}}_\alpha - \int d\mathbf{x} \mathbf{A}(\mathbf{x}) \hat{\rho}_\alpha(\mathbf{x}), \quad i \frac{\partial}{\partial t} \rightarrow i \frac{\partial}{\partial t} - \int d\mathbf{x} A_0(\mathbf{x}) \hat{\rho}(\mathbf{x}). \quad (0.42)$$

Omitting details we arrive to two-body current density operator

$$\mathbf{J}^{[2]}(\mathbf{x}) = -\frac{i}{2} \sum_{\alpha < \beta} \int d\mathbf{y} \int_0^1 ds \left\{ \hat{\rho}_\alpha \left(\mathbf{x} + \frac{1}{2} \mathbf{y} s \right) - \hat{\rho}_\beta \left(\mathbf{x} - \frac{1}{2} \mathbf{y} s \right) \right\} \mathbf{y} e^{-i\hat{\mathbf{p}}_{\alpha\beta} \mathbf{y}} V(\hat{\mathbf{r}}_{\alpha\beta} + \mathbf{y}, \hat{\mathbf{r}}_{\alpha\beta}) \quad (0.43)$$

From Eq. (0.43) for a local NN interaction $V(\mathbf{r} + \mathbf{y}, \mathbf{r}) = V(\mathbf{r}) \delta(\mathbf{y})$ it follows that $\hat{\mathbf{J}}^{[2]}(\mathbf{x}) = 0$. For nonlocal interactions $\hat{V}(\alpha, \beta)$ the commutator $[\hat{\mathbf{r}}_{\alpha\beta}, \hat{V}(\alpha, \beta)] \neq 0$. Among them we encounter spin-orbit potentials, forces of Majorana type, $\hat{V}_M(\alpha, \beta) = P_M(\alpha, \beta) V_M(\hat{\mathbf{r}}_{\alpha\beta})$, for which $V_M(\mathbf{r} + \mathbf{y}, \mathbf{r}) = V_M(\mathbf{r}) \delta(2\mathbf{r} + \mathbf{y})$, separable interactions and so on. Explicit analytic expressions for contribution $\hat{\mathbf{J}}_M^{[2]}(\mathbf{x})$ that stems from product $P_M(\alpha, \beta) = -P_\sigma(\alpha, \beta) P_\tau(\alpha, \beta)$ of spin and isospin exchange operators, $P_\sigma(\alpha, \beta) = \frac{1}{2} [1 + \sigma(\alpha) \cdot \sigma(\beta)]$ and $P_\tau(\alpha, \beta) = \frac{1}{2} [1 + \tau(\alpha) \cdot \tau(\beta)]$, to the two-particle current $\mathbf{J}^{[2]}(\mathbf{x})$ can be written as

$$\hat{\mathbf{J}}_M^{[2]}(\mathbf{x}) = -\frac{i}{2} \sum_{\alpha < \beta} \int_0^1 ds \{ \hat{\rho}_\alpha(\mathbf{x} + \hat{\mathbf{r}}_{\alpha\beta} \mathbf{s}) - \hat{\rho}_\beta(\mathbf{x} - \hat{\mathbf{r}}_{\alpha\beta} \mathbf{s}) \} [\hat{\mathbf{r}}_{\alpha\beta}, \hat{V}_M(\alpha, \beta)] \quad (0.44)$$

or within isospin formalism

$$\hat{\mathbf{J}}_M^{[2]}(\mathbf{x}) = \frac{1}{2} \sum_{\alpha < \beta} [\tau(\alpha) \times \tau(\beta)]_3 P_\sigma(\alpha, \beta) \hat{\mathbf{r}}_{\alpha\beta} V_M(\hat{\mathbf{r}}_{\alpha\beta}) \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_v(\mathbf{x} - \hat{\mathbf{R}}_{\alpha\beta} + \hat{\mathbf{r}}_{\alpha\beta} t) dt, \quad (0.45)$$

where function $\rho_v(\mathbf{x})$ determines isovector part of distribution (0.29). For this derivation we have used the relation $[\tau_3(\alpha) - \tau_3(\beta)] P_\tau(\alpha, \beta) = -\frac{1}{2} i [\tau(\alpha) \times \tau(\beta)]_3$.

After this, by introducing the so-called exchange contribution

$$\hat{\mathbf{M}}_{exc}(\mathbf{k}) \equiv -(2\pi)^3 \int_0^1 \lambda d\lambda (\mathbf{P}_i - \lambda \mathbf{k} | \mathbf{J}_M^{[2]}(0) | \mathbf{P}_i) \quad (0.46)$$

to the GMDM we get with point-like nucleons

$$\hat{\mathbf{M}}_{exc} = \hat{\mathbf{M}}_{exc}(0) = -\frac{e}{8} \sum_{\alpha < \beta} [\tau(\alpha) \times \tau(\beta)]_3 \mathbf{r}'_{\alpha} \times \mathbf{r}'_{\beta} P_{\sigma}(\alpha, \beta) V_M(\hat{\mathbf{r}}_{\alpha\beta}) \quad (0.47)$$

to be compared with the magnetic dipole moment of system with any nonlocal two-body interaction,

$$\hat{\mathbf{M}}_{2BD} = -\frac{i}{4} \sum_{\alpha < \beta} (\hat{\mathbf{d}}_{\alpha}(0) - \hat{\mathbf{d}}_{\beta}(0)) \times [\hat{\mathbf{r}}_{\alpha\beta}, \hat{V}(\alpha, \beta)] \quad (0.48)$$

that follows from Eq.(0.43) and property

$[\hat{\mathbf{r}}_{\alpha\beta}, \exp(-i\hat{\mathbf{p}}_{\alpha\beta}\mathbf{y})V(\hat{\mathbf{r}}_{\alpha\beta} + \mathbf{y}, \hat{\mathbf{r}}_{\alpha\beta})] = \mathbf{y} \exp(-i\hat{\mathbf{p}}_{\alpha\beta}\mathbf{y})V(\hat{\mathbf{r}}_{\alpha\beta} + \mathbf{y}, \hat{\mathbf{r}}_{\alpha\beta})$. The operator (0.47) becomes equivalent to that from [Sachs(48)] if we replace vector product $\mathbf{r}'_{\alpha} \times \mathbf{r}'_{\beta}$ by $\mathbf{r}_{\alpha} \times \mathbf{r}_{\beta}$. With such a proviso we could call operator (0.47) the Sachs exchange moment.

Finally, we would like to emphasize that the factor $[\tau(\alpha) \times \tau(\beta)]_3$ in the isospin dependence of $\mathbf{J}_M^{[2]}(\mathbf{x})$ arises from the exchange $\tau(\alpha) \cdot \tau(\beta)$ nonlocality, being typical of realistic NN forces (cf., the so-called pionic and sea-gull MECs).

Some Recollections

We will now prove the fundamental theorem:
any operator O may be expressed as a sum of
products of creation and annihilation operators ...

S. Weinberg

Quantum Theory of Fields, Vol. I, 1995, p. 175.

In accordance with the motto each of ten generators of the Poincaré group Π may be expressed as a sum of products of creation and annihilation operators $a^\dagger(n)$ and $a(n)$ ($n = 1, 2, \dots$) for free particles, e.g., bosons and/or fermions.

In the framework of such a corpuscular picture Hamiltonian of a system of interacting mesons and nucleons can be written as

$$H = \sum_{C=0}^{\infty} \sum_{A=0}^{\infty} H_{CA},$$

$$H_{CA} = \sum H_{CA}(1', 2', \dots, n'_C; 1, 2, \dots, n_A) a^\dagger(1') a^\dagger(2') \dots a^\dagger(n'_C) a(n_A) \dots a(2) a(1),$$

$C(A)$ – particle-creation (annihilation) number for operator substructure H_{CA} and

$$H_{CA}(1', 2', \dots, C; 1, 2, \dots, A) = \delta(\vec{p}'_1 + \vec{p}'_2 + \dots + \vec{p}'_C - \vec{p}_1 - \vec{p}_2 - \dots - \vec{p}_A) \\ \times h_{CA}(p'_1 \mu'_1 \xi'_1, p'_2 \mu'_2 \xi'_2, \dots, p'_C \mu'_C \xi'_C; p_1 \mu_1 \xi_1, p_2 \mu_2 \xi_2, \dots, p_A \mu_A \xi_A),$$

c -number coefficients h_{CA} do not contain! delta function.

“To free ourselves from any dependence on pre-existing field theories” (after S.Weinberg), boost operators $\vec{N} = (N^1, N^2, N^3)$

$$\vec{N} = \sum_{C=0}^{\infty} \sum_{A=0}^{\infty} \vec{N}_{CA},$$

$$\vec{N}_{CA} = \int \vec{N}_{CA}(1', 2', \dots, n'_C; 1, 2, \dots, n_A) a^\dagger(1') a^\dagger(2') \dots a^\dagger(n'_C) a(n_A) \dots a(2) a(1)$$

one of our purposes is to find some links between coefficients H_{CA} and \vec{N}_{CA} , compatible with commutations

$$[P_i, P_j] = 0, \quad [J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, P_j] = i\epsilon_{ijk} P_k,$$

$$[\vec{P}, H] = 0, \quad [\vec{J}, H] = 0, \quad [J_i, N_j] = i\epsilon_{ijk} N_k, \quad [P_i, N_j] = i\delta_{ij} H,$$

$$[H, \vec{N}] = i\vec{P}, \quad [N_i, N_j] = -i\epsilon_{ijk} J_k,$$

$$(i, j, k = 1, 2, 3),$$

$\vec{P} = (P^1, P^2, P^3)$ and $\vec{J} = (J^1, J^2, J^3)$ linear and angular momentum operators. For instant form of relativistic dynamics after Dirac only Hamiltonian and boost operators carry interactions,

$$H = H_F + H_I$$

$$\vec{N} = \vec{N}_F + \vec{N}_I$$

while $\vec{P} = \vec{P}_F$ and $\vec{J} = \vec{J}_F$.

In turn,

with density

$$H_{CA} = \int H_{CA}(\vec{x}) d\vec{x} \quad \text{so} \quad H = \int H(\vec{x}) d\vec{x}$$

$$H(\vec{x}) = \sum_{C=0}^{\infty} \sum_{A=0}^{\infty} H_{CA}(\vec{x}).$$

For instance, in case with $C = A = 2$,

$$H_{22}(1', 2'; 1, 2) = \delta(\vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2) h(1' 2'; 12)$$

$$H_{22}(\vec{x}) = \frac{1}{(2\pi)^3} \int \exp[-i(\vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2)\vec{x}] h(1' 2'; 12) a^\dagger(1') a^\dagger(2') a(2) a(1).$$

As usually $a(n) = a(\vec{p}_n, \mu_n, \xi_n)$. Further, transformation properties with respect to Π in case of massive particle with spin j :

$$U_F(\Lambda, b) a^\dagger(p, \mu) U_F^{-1}(\Lambda, b) = e^{i\Lambda pb} D_{\mu'\mu}^{(j)}(W(\Lambda, p)) a^\dagger(\Lambda p, \mu'),$$

$$\forall \Lambda \in L_+ \quad \text{and arbitrary spacetime shifts } b = (b^0, \vec{b}),$$

with D -function whose argument is Wigner rotation $W(\Lambda, p)$, L_+ the homogeneous (proper) orthochronous Lorentz group, $(\Lambda, b) \rightarrow U_F(\Lambda, b)$ unitary irreducible representation of Π in Hilbert space, e.g. hadronic states, for operators $a(p, \mu) = a(\vec{p}, \mu) \sqrt{p_0}$ that meet covariant commutation relations

$$[a(p', \mu'), a^\dagger(p, \mu)]_{\pm} = p_0 \delta(\vec{p} - \vec{p}') \delta_{\mu'\mu},$$

$$[a(p', \mu'), a(p, \mu)]_{\pm} = [a^\dagger(p', \mu'), a^\dagger(p, \mu)]_{\pm} = 0.$$

Here $p_0 = \sqrt{\vec{p}^2 + m^2}$ is fourth component of 4-momentum $p = (p_0, \vec{p})$.

Often one has to deal with field models where in Dirac (D) picture

$$U_F(\Lambda, b)H_I(x)U_F^{-1}(\Lambda, b) = H_I(\Lambda x + b), \quad \forall x = (t, \vec{x}).$$

For interaction density

$$H_{22}(x) = \frac{1}{(2\pi)^3} \int \exp[i(p'_1 + p'_2 - p_1 - p_2)x] \times h(1'2'; 12) a^\dagger(1') a^\dagger(2') a(2) a(1)$$

it means

$$D_{\eta'_1 \mu'_1}^{(j'_1)}(W(\Lambda, p'_1)) D_{\eta'_2 \mu'_2}^{(j'_2)}(W(\Lambda, p'_2)) D_{\eta_1 \mu_1}^{(j_1)*}(W(\Lambda, p_1)) D_{\eta_2 \mu_2}^{(j_2)*}(W(\Lambda, p_2)) \\ \times h(p'_1 \mu'_1, p'_2 \mu'_2; p_1 \mu_1, p_2 \mu_2) = h(\Lambda p'_1 \eta'_1, \Lambda p'_2 \eta'_2; \Lambda p_1 \eta_1, \Lambda p_2 \eta_2).$$

Of course, summations over all dummy labels are implied.

After these preliminaries we will show how one can build up interaction parts in Hamiltonian and boosts.

Recall that angular momentum $\vec{J} = \vec{J}_F = \vec{J}_\pi + \vec{J}_{ferm}$ with

$$\vec{J}_\pi = \frac{i}{2} \int d\vec{k} \vec{k} \times \left(\frac{\partial a^\dagger(\vec{k})}{\partial \vec{k}} a(\vec{k}) - a^\dagger(\vec{k}) \frac{\partial a(\vec{k})}{\partial \vec{k}} \right)$$

and $\vec{J}_{ferm} = \vec{L}_{ferm} + \vec{S}_{ferm}$, where

$$\vec{L}_{ferm} = \frac{i}{2} \int d\vec{p} \vec{p} \times \left(\frac{\partial b^\dagger(\vec{p}_\mu)}{\partial \vec{p}} b(\vec{p}_\mu) - b^\dagger(\vec{p}_\mu) \frac{\partial b(\vec{p}_\mu)}{\partial \vec{p}} + \frac{\partial d^\dagger(\vec{p}_\mu)}{\partial \vec{p}} d(\vec{p}_\mu) - d^\dagger(\vec{p}_\mu) \frac{\partial d(\vec{p}_\mu)}{\partial \vec{p}} \right),$$

$$\vec{S}_{ferm} = \frac{1}{2} \int d\vec{p} \chi^\dagger(\mu') \vec{\sigma} \chi(\mu) (b^\dagger(\vec{p}_{\mu'}) b(\vec{p}_\mu) - d^\dagger(\vec{p}_{\mu'}) d(\vec{p}_\mu)),$$

boosts $\vec{N}_F = \vec{N}_\pi + \vec{N}_{ferm}$ with

$$\vec{N}_\pi = \frac{i}{2} \int d\vec{k} \omega_{\vec{k}} \left(\frac{\partial a^\dagger(\vec{k})}{\partial \vec{k}} a(\vec{k}) - a^\dagger(\vec{k}) \frac{\partial a(\vec{k})}{\partial \vec{k}} \right)$$

and $\vec{N}_{ferm} = \vec{N}_{ferm}^{orb} + \vec{N}_{ferm}^{spin}$, where

$$\vec{N}_{ferm}^{orb} = \frac{i}{2} \int d\vec{p} E_{\vec{p}} \left(\frac{\partial b^\dagger(\vec{p}_\mu)}{\partial \vec{p}} b(\vec{p}_\mu) - b^\dagger(\vec{p}_\mu) \frac{\partial b(\vec{p}_\mu)}{\partial \vec{p}} + \frac{\partial d^\dagger(\vec{p}_\mu)}{\partial \vec{p}} d(\vec{p}_\mu) - d^\dagger(\vec{p}_\mu) \frac{\partial d(\vec{p}_\mu)}{\partial \vec{p}} \right),$$

$$\vec{N}_{ferm}^{spin} = -\frac{1}{2} \int d\vec{p} \vec{p} \times \frac{\chi^\dagger(\mu) \vec{\sigma} \chi(\mu)}{E_{\vec{p}} + m} (b^\dagger(\vec{p}_\mu) b(\vec{p}_\mu) + d^\dagger(\vec{p}_\mu) d(\vec{p}_\mu)),$$

$\omega_{\vec{k}} = \sqrt{\vec{k}^2 + m_\pi^2}$ ($E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$) pion (nucleon) energy and $\chi(\mu)$ Pauli spinor.

Clothed Particle Representation (CPR) of Hamiltonian and Other Generators of the Poincaré Group

At this point, one can address the so-called Belinfante ansatz

$$\vec{N}_{bel} = - \int \vec{x} H(\vec{x}) d\vec{x}$$

which is helpful for a simultaneous blockdiagonalization of Hamiltonian and boost [2,3], viz., both of them, being dependent on primary operators $\{\alpha\}$ (such as $a^\dagger(a)$, $b^\dagger(b)$ and $d^\dagger(d)$ for mesons and nucleons) in bare particle representation (BPR), are expressed through corresponding operators $\{\alpha_c\}$ for particle creation and annihilation in CPR via unitary clothing transformations (UCTs) $W(\alpha) = W(\alpha_c)$

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$$

A key point of clothing procedure in question is to remove so-called bad terms from Hamiltonian

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c),$$

By definition, such terms prevent physical vacuum $|\Omega\rangle$ (H lowest eigenstate) and one-clothed-particle states $|n\rangle_c = a_c^\dagger(n)|\Omega\rangle$ to be H eigenvectors for all n included.

Bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')\dots a^\dagger(n'_c)a(n_A)\dots a(2)a(1)$$

of class [C.A] embodies, at least, one substructure $\in [k.0]$ ($k = 1, 2, \dots$) or/and $[k.1]$ ($k = 2, 3, \dots$).

Respectively, let us write for boson–fermion system

$$H_I(\alpha) = V(\alpha) + V_{ren}(\alpha)$$

with primary (trial) interaction

$$V(\alpha) = V_{bad} + V_{good}$$

”good” (e.g., $\in [k.2]$) as antithesis of ”bad” while $V_{ren}(\alpha) \sim [1.1] + [0.2] + [2.0]$ **”mass renormalization counterterms”**. Latter are important to ensure relativistic invariance (RI) in Dirac sense.

In its turn, $V = \sum_b V_b$ comprises separate boson–fermion couplings V_b . In order to compare our calculations with those by Bonn group (Machleidt, Holinde, Elster) we have employed

$$V(\alpha) = V_s + V_{ps} + V_v$$

$$V_s = g_s \int d\vec{x} \bar{\psi}(\vec{x}) \psi(\vec{x}) \varphi_s(\vec{x}) \quad V_{ps} = ig_{ps} \int d\vec{x} \bar{\psi}(\vec{x}) \gamma_5 \psi(\vec{x}) \varphi_{ps}(\vec{x})$$

$$V_v = V_v^{(1)} + V_v^{(2)}, \quad V_v^{(1)} = \int d\vec{x} H_{sc}(\vec{x}), \quad V_v^{(2)} = \int d\vec{x} H_{nonsc}(\vec{x})$$

$$H_{sc}(\vec{x}) = g_v \bar{\psi}(\vec{x}) \gamma_\mu \psi(\vec{x}) \varphi_v^\mu(\vec{x}) + \frac{f_v}{4m} \bar{\psi}(\vec{x}) \sigma_{\mu\nu} \psi(\vec{x}) \varphi_v^{\mu\nu}(\vec{x})$$

$$H_{nonsc}(\vec{x}) = \frac{g_v^2}{2m_v^2} \bar{\psi}(\vec{x}) \gamma_0 \psi(\vec{x}) \bar{\psi}(\vec{x}) \gamma_0 \psi(\vec{x}) + \frac{f_v^2}{4m^2} \bar{\psi}(\vec{x}) \sigma_{0i} \psi(\vec{x}) \bar{\psi}(\vec{x}) \sigma_{0i} \psi(\vec{x})$$

$\varphi_v^{\mu\nu}(\vec{x}) = \partial^\mu \varphi_v^\nu(\vec{x}) - \partial^\nu \varphi_v^\mu(\vec{x})$ tensor of vector field in Schrödinger (S) picture.

Here we encounter scalar H_{sc} and nonscalar H_{nonsc} contributions to interaction densities of ρNN and ωNN couplings

$$U_F(\Lambda, \mathbf{a})H_{sc}(\mathbf{x})U_F^{-1}(\Lambda, \mathbf{a}) = H_{sc}(\Lambda\mathbf{x} + \mathbf{a})$$

$$U_F(\Lambda, \mathbf{a})H_{nonsc}(\mathbf{x})U_F^{-1}(\Lambda, \mathbf{a}) \neq H_{nonsc}(\Lambda\mathbf{x} + \mathbf{a})$$

Therefore, in order to apply our approach to local field models with derivatives and/or spin $j \geq 1$ and also to their nonlocal extensions in framework of such a corpuscular picture we have developed clothing procedure [2,3] removing from V_{bad} only its scalar part V_{sc} , if any. Clothing itself (cf. our talks at ISHEPP'02 and ISHEPP'04), as illustration for ρNN and ωNN couplings, is prompted by

$$H(\alpha) = K(\alpha_c) = W(\alpha_c)[H_F(\alpha_c) + V_v(\alpha_c) + V_{ren}(\alpha_c)]W^\dagger(\alpha_c)$$

or putting $W = \exp R$ with $R = -R^\dagger$ so

$$\begin{aligned} K(\alpha_c) = & H_F(\alpha_c) + V_v^{(1)}(\alpha_c) + [R, H_F] + V_v^{(2)}(\alpha_c) \\ & + [R, V_v^{(1)}] + \frac{1}{2}[R, [R, H_F]] + [R, V_v^{(2)}] + \frac{1}{2}[R, [R, V_v^{(1)}]] + \dots \end{aligned}$$

and requiring $[R, H_F] = -V_v^{(1)}$ (*) for the operator R of interest to get

$$H = K(\alpha_c) = K_F + K_I$$

with a new free part $K_F = H_F(\alpha_c) \sim a_c^\dagger a_c$ and interaction

$$K_I = \frac{1}{2}[R, V_v^{(1)}] + V_v^{(2)} + \frac{1}{3}[R, [R, V_v^{(1)}]] + \dots$$

After a simple algebra we find

$$\frac{1}{2} \left[R, V_v^{(1)} \right] (NN \rightarrow NN) = K_v(NN \rightarrow NN) + K_{cont}(NN \rightarrow NN)$$

Operator $K_{cont}(NN \rightarrow NN)$ may be associated with a contact interaction since it does not contain any propagators (details see in Refs. [6,7]). It has turned out that this operator cancels completely non-scalar operator $V^{(2)}$. In our opinion, such a cancellation, first discussed here, is a pleasant feature of the CPR.

Moreover, using property $V_{sc}(x)$ to be Lorentz scalar one can show that Lie algebra of Π is satisfied with

$$\vec{N}_I = \vec{N}_{Bel} + \vec{D} \equiv \int \vec{x} V_v^{(1)}(\vec{x}) d\vec{x} + \vec{D}$$

and get recursive formulae for finding contributions $\vec{D}^{(n)}$ to $\vec{D} = \sum_{n=2}^{\infty} \vec{D}^{(n)}$, label (n) – n 'th order in coupling constants. It differs from expansion by Krueger and Gloeckle (1999).

In parallel, we have

$$\vec{N}(\alpha) = \vec{B}(\alpha_c) = W(\alpha_c) \{ \vec{N}_F(\alpha) + \vec{N}_I(\alpha) + \vec{N}_{ren}(\alpha) \} W^\dagger(\alpha_c)$$

with

$$\vec{N}_I = - \int \vec{x} V_v(\vec{x}) d\vec{x} = - \int \vec{x} \{ V_v^{(1)}(\vec{x}) + V_v^{(2)}(\vec{x}) \} d\vec{x} = \vec{N}_I^{(1)} + \vec{N}_I^{(2)}$$

As before (see Refs. [2,3]) we find

$$[R, \vec{N}_F] = -\vec{N}_I^{(1)},$$

once operator meets condition (*) so boost generators in CPR acquire structure similar to $K(\alpha_c)$

$$\vec{B}(\alpha_c) = \vec{B}_F + \vec{B}_I.$$

Here $\vec{B}_F = \vec{N}_F(\alpha_c)$ the boost operator for noninteracting clothed particles (in our case fermions and vector mesons) and \vec{B}_I includes the contributions induced by interactions between them

$$\vec{B}_I = +\frac{1}{2}[R, \vec{N}_I^{(1)}] + \frac{1}{3}[R, [R, \vec{N}_I^{(1)}]] + \dots$$

Relativistic Interactions in Meson–Nucleon Systems

Interaction operators

$$K_I \sim a_c^\dagger b_c^\dagger a_c b_c (\pi N \rightarrow \pi N) + b_c^\dagger b_c^\dagger b_c b_c (NN \rightarrow NN) + d_c^\dagger d_c^\dagger d_c d_c (\bar{N}\bar{N} \rightarrow \bar{N}\bar{N}) \\ + b_c^\dagger b_c^\dagger b_c^\dagger b_c b_c b_c (NNN \rightarrow NNN) + \dots + [a_c^\dagger a_c^\dagger b_c d_c + H.c.](N\bar{N} \leftrightarrow 2\pi) + \dots \\ + [a_c^\dagger b_c^\dagger b_c^\dagger b_c b_c + H.c.](NN \leftrightarrow \pi NN) + \dots$$

Pion-nucleon interaction operator

$$K(\pi N \rightarrow \pi N) = \int d\vec{p}_1 d\vec{p}_2 d\vec{k}_1 d\vec{k}_2 V_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1) a_c^\dagger(\vec{k}_2) b_c^\dagger(\vec{p}_2) a_c(\vec{k}_1) b_c(\vec{p}_1),$$

$$V_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1) = \frac{g^2}{2(2\pi)^3} \frac{m}{\sqrt{\omega_{\vec{k}_1} \omega_{\vec{k}_2} E_{\vec{p}_1} E_{\vec{p}_2}}} \delta(\vec{p}_1 + \vec{k}_1 - \vec{p}_2 - \vec{k}_2)$$

$$\bar{u}(\vec{p}_2) \left\{ \frac{1}{2} \left[\frac{1}{\hat{p}_1 + \hat{k}_1 + m} + \frac{1}{\hat{p}_2 + \hat{k}_2 + m} \right] \right. \\ \left. + \frac{1}{2} \left[\frac{1}{\hat{p}_1 - \hat{k}_2 + m} + \frac{1}{\hat{p}_2 - \hat{k}_1 + m} \right] \right\} u(\vec{p}_1)$$

πN quasipotential in momentum space is:

$$\tilde{V}_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1) = \langle a_c^\dagger(\vec{k}_2) b_c^\dagger(\vec{p}_2) \Omega | K(\pi N \rightarrow \pi N) | a_c^\dagger(\vec{k}_1) b_c^\dagger(\vec{p}_1) \Omega \rangle$$

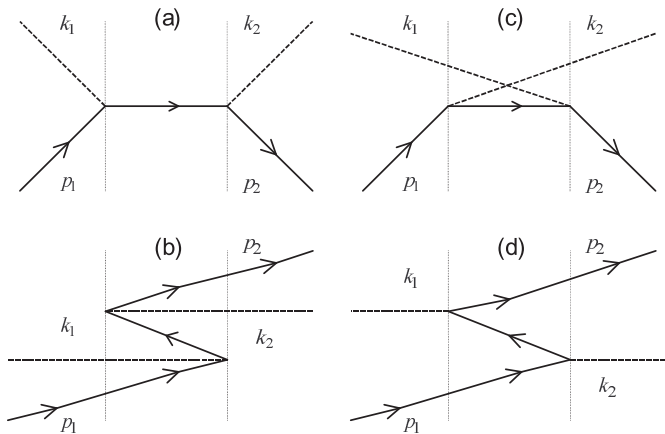


Figure 1: Different contributions to πN quasipotential.

Graphs in Fig. 1 are topologically equivalent to well-known time-ordered Feynman diagrams. However, in Schrödinger picture used here, where all events are related to one and the same instant $t = 0$, such an analogy could be misleading: line directions in Fig. 1 are given with the sole scope to discriminate between nucleon and antinucleon states.

Energy conservation is not assumed in constructing this and other quasipotentials. Indeed, coefficients in front of $a_c^\dagger b_c^\dagger a_c b_c$ generally do not fulfill on-energy-shell condition

$$E_{\vec{p}_1} + \omega_{\vec{k}_1} = E_{\vec{p}_2} + \omega_{\vec{k}_2},$$

In this connection, "left" four-vector s_1 is not necessarily equal to "right" Mandelstam vector $s_2 = p_2 + k_2$.

Nucleon-nucleon interaction operator

After normal ordering of fermion operators we derive $NN \rightarrow NN$ interaction operator:

$$K_{NN} = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}'_1 d\vec{p}'_2 V_{NN}(\vec{p}'_1, \vec{p}'_2; \vec{p}_1, \vec{p}_2) b_c^\dagger(\vec{p}'_1) b_c^\dagger(\vec{p}'_2) b_c(\vec{p}_1) b_c(\vec{p}_2),$$

$$V_{NN}(\vec{p}'_1, \vec{p}'_2; \vec{p}_1, \vec{p}_2) = -\frac{1}{2} \frac{g^2}{(2\pi)^3} \frac{m^2}{\sqrt{E_{\vec{p}_1} E_{\vec{p}_2} E_{\vec{p}'_1} E_{\vec{p}'_2}}} \delta(\vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2) \\ \times \bar{u}(\vec{p}'_1) \gamma_5 u(\vec{p}_1) \frac{1}{(p_1 - p'_1)^2 - \mu^2} \bar{u}(\vec{p}'_2) \gamma_5 u(\vec{p}_2),$$

Corresponding relativistic and properly symmetrized NN interaction

$$\tilde{V}_{NN}(\vec{p}'_1, \vec{p}'_2; \vec{p}_1, \vec{p}_2) = \left\langle b_c^\dagger(\vec{p}'_1) b_c^\dagger(\vec{p}'_2) \Omega \mid K_{NN} \mid b_c^\dagger(\vec{p}_1) b_c^\dagger(\vec{p}_2) \Omega \right\rangle$$

or through covariant (Feynman-like) “propagators”,

$$\begin{aligned} \tilde{V}_{NN}(\vec{p}'_1, \vec{p}'_2; \vec{p}_1, \vec{p}_2) = & -\frac{1}{2} \frac{g^2}{(2\pi)^3} \frac{m^2}{2\sqrt{E_{\vec{p}_1} E_{\vec{p}_2} E_{\vec{p}'_1} E_{\vec{p}'_2}}} \delta(\vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2) \\ & \times \bar{u}(\vec{p}'_1) \gamma_5 u(\vec{p}_1) \frac{1}{2} \left\{ \frac{1}{(p_1 - p'_1)^2 - \mu^2} \right. \\ & \left. + \frac{1}{(p_2 - p'_2)^2 - \mu^2} \right\} \bar{u}(\vec{p}'_2) \gamma_5 u(\vec{p}_2) - (1 \leftrightarrow 2). \quad (*) \end{aligned}$$

Formula (*) determines NN part of OBE interaction derived earlier via Okubo transformation method by Korchin, Shebeko [Phys. At. Nucl. **56** (1993) 1663] (cf. Fuda, Zhang. Phys. Rev. C **51** (1995) 23) taking into account pion exchange and heavy-meson exchanges.

Distinctive feature of potential (*) is the presence of covariant (Feynman-like) “propagator”,

$$\frac{1}{2} \left\{ \frac{1}{(p_1 - p'_1)^2 - \mu^2} + \frac{1}{(p_2 - p'_2)^2 - \mu^2} \right\}.$$

On the energy shell for NN scattering, that is

$$E_i \equiv E_{\vec{p}_1} + E_{\vec{p}_2} = E_{\vec{p}'_1} + E_{\vec{p}'_2} \equiv E_f,$$

this expression is converted into genuine Feynman propagator.

$NN \leftrightarrow \pi NN$ transition operators

$$K(NN \rightarrow \pi NN) = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}'_1 d\vec{p}'_2 d\vec{k} V_{\pi NN}(\vec{p}'_1, \vec{p}'_2, \vec{k}; \vec{p}_1, \vec{p}_2) a_c^\dagger(\vec{k}) b_c^\dagger(\vec{p}'_1) b_c^\dagger(\vec{p}'_2) b_c(\vec{p}_1) b_c(\vec{p}_2)$$

$$V_{\pi NN}(\vec{p}'_1, \vec{p}'_2, \vec{k}; \vec{p}_1, \vec{p}_2) = V_{\pi NN}(\text{Feynman-like}) + V_{\pi NN}(\text{off-energy-shell}),$$

where

$$V_{\pi NN}(\text{Feynman-like}) = -i \frac{g^3}{(2\pi)^{9/2}} \frac{m^2 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}'_1 - \vec{p}'_2 - \vec{k})}{\sqrt{2\omega_{\vec{k}} E_{\vec{p}_1} E_{\vec{p}_2} E_{\vec{p}'_1} E_{\vec{p}'_2}}} \times \frac{\bar{u}(\vec{p}'_2) \gamma_5 u(\vec{p}_2)}{(\hat{p}_2 - \hat{p}'_2)^2 - \mu^2} \bar{u}(\vec{p}'_1) \left[\frac{1}{\hat{p}'_1 + \hat{k} + m} + \frac{1}{\hat{p}_1 - \hat{k} + m} \right] u(\vec{p}_1),$$

Then we introduce quasipotential

$$\tilde{V}_{\pi NN}(\vec{p}'_1, \vec{p}'_2, \vec{k}; \vec{p}_1, \vec{p}_2) = \langle a_c^\dagger(\vec{k}) b_c^\dagger(\vec{p}'_1) b_c^\dagger(\vec{p}'_2) \Omega | K(NN \rightarrow \pi NN) | b_c(\vec{p}_1) b_c(\vec{p}_2) \Omega \rangle$$

and draw respective graphs

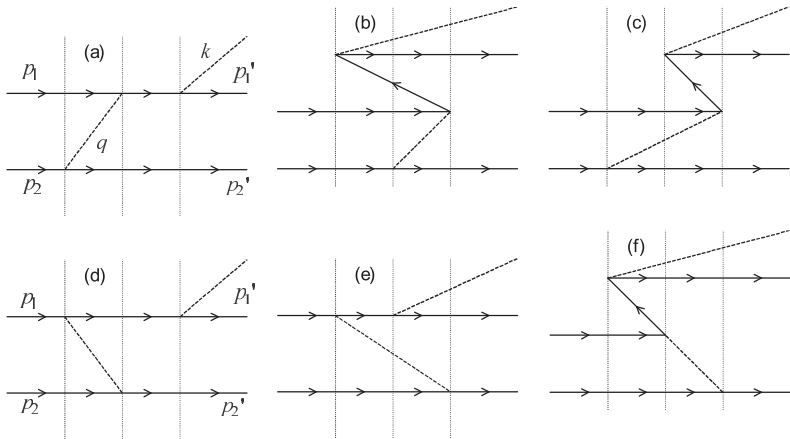


Figure 2: Illustration of the "retarded" pion production mechanisms on the NN pair in the g^3 -order.

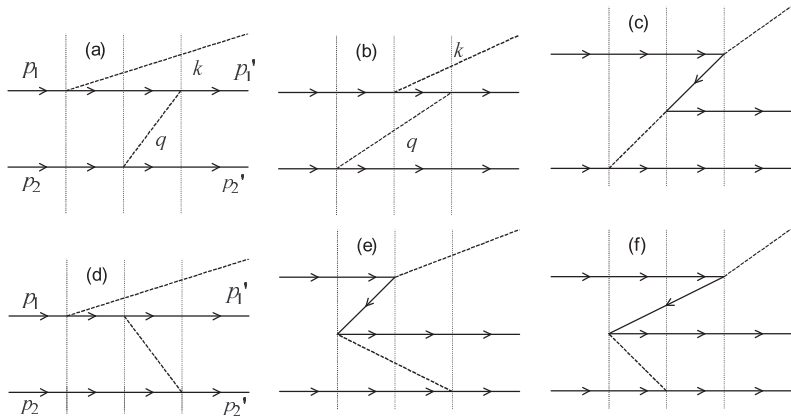


Figure 3: Illustration of the "advanced" pion production mechanisms on the NN pair in the g^3 -order.

Three-Nucleon Forces

Normal ordering of fermion operators in $[R, [R, [R, V]]]$ leads to $NNN \rightarrow NNN$ interaction operator (antiparticle degrees of freedom are neglected),

$$K(3N \rightarrow 3N) = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}_3 d\vec{p}'_1 d\vec{p}'_2 d\vec{p}'_3 V_{3N}(\vec{p}'_1, \vec{p}'_2, \vec{p}'_3; \vec{p}_1, \vec{p}_2, \vec{p}_3) \\ \times b_c^\dagger(\vec{p}'_1) b_c^\dagger(\vec{p}'_2) b_c^\dagger(\vec{p}'_3) b_c(\vec{p}_1) b_c(\vec{p}_2) b_c(\vec{p}_3),$$

$$V_{3N}(\vec{p}'_1, \vec{p}'_2, \vec{p}'_3; \vec{p}_1, \vec{p}_2, \vec{p}_3) \\ = -\frac{1}{8} \frac{g^4 m^4}{(2\pi)^6} \frac{\delta(\vec{p}'_1 + \vec{p}'_2 + \vec{p}'_3 - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)}{\sqrt{E_{\vec{p}_1} E_{\vec{p}_2} E_{\vec{p}_3} E_{\vec{p}'_1} E_{\vec{p}'_2} E_{\vec{p}'_3}}} D_{\vec{p}_1, \vec{p}_2, \vec{p}_3}^{\vec{p}'_1, \vec{p}'_2, \vec{p}'_3} \frac{1}{E_{\vec{q}}} \bar{u}(\vec{p}'_1) \gamma_5 u(\vec{p}_1) \\ \times \bar{u}(\vec{p}'_2) \frac{m - \hat{q}}{2m} u(\vec{p}_2) \bar{u}(\vec{p}'_3) \gamma_5 u(\vec{p}_3), D_{\vec{p}_1, \vec{p}_2, \vec{p}_3}^{\vec{p}'_1, \vec{p}'_2, \vec{p}'_3} = \frac{E_{\vec{p}'_2} - E_{\vec{q}} + E_{\vec{p}_1} - E_{\vec{p}'_1}}{[(p_1 - p'_1)^2 - \mu^2][(p'_2 - q)^2 - \mu^2]} \\ \times \left[\frac{3}{(p_3 - p'_3)^2 - \mu^2} + \frac{1}{(p_2 - q)^2 - \mu^2} \right] + \frac{E_{\vec{p}_2} - E_{\vec{q}} + E_{\vec{p}'_3} - E_{\vec{p}_3}}{[(p_3 - p'_3)^2 - \mu^2][(p_2 - q)^2 - \mu^2]} \\ \times \left[\frac{3}{(p_1 - p'_1)^2 - \mu^2} + \frac{1}{(p'_2 - q)^2 - \mu^2} \right], \vec{q} = \vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 = \vec{p}_2 + \vec{p}_3 - \vec{p}'_3$$

In static limit for nucleons the quasipotential appears as a correction of nucleon-recoil order.

S Operator, Equivalence Theorem for S Matrix and Its Application to Elastic NN Scattering

By definition, with $H = H_F(\alpha) + H_I(\alpha)$

$$S = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iH_F t_2} e^{-iH(t_2-t_1)} e^{-iH_F t_1}$$

Let us introduce S operator for decomposition $H = K(\alpha_c) = K_F(\alpha_c) + K_I(\alpha_c)$,

$$S_{cloth} = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iK_F t_2} e^{-iK(t_2-t_1)} e^{-iK_F t_1}$$

One can show that if $W_D(t) = \exp(iK_F t) W \exp(-iK_F t)$ meets condition

$$\lim_{t \rightarrow \pm\infty} W_D(t) = 1 \quad \text{or} \quad \lim_{t \rightarrow \pm\infty} R_D(t) = 0$$

then

$$S_{cloth} = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iK_F(\alpha_c)t_2} e^{-iH(\alpha_c)(t_2-t_1)} e^{-iK_F(\alpha_c)t_1}$$

Matrix elements of $S = S(\alpha)$ between *bare* states $\alpha^\dagger \dots \Omega_0$ with $H_F \Omega_0 = 0$,

$$\left\langle \alpha^\dagger \dots \Omega_0 \left| S(\alpha) \right| \alpha^\dagger \dots \Omega_0 \right\rangle$$

and matrix elements of $S_{cloth} = S(\alpha_c)$ between *clothed* states $\alpha_c^\dagger \dots \Omega$ with $K_F \Omega = 0$,

$$\left\langle \alpha_c^\dagger \dots \Omega \left| S(\alpha_c) \right| \alpha_c^\dagger \dots \Omega \right\rangle$$

are equal to each other since α_c -algebra with physical vacuum Ω is isomorphic to α -algebra with bare vacuum Ω_0 , i.e.,

$$S_{fi} \equiv \langle f | S | i \rangle = \langle f; c | S_{cloth} | i; c \rangle$$

Application to Elastic NN Scattering

This result (ISHEPP'02, FB'03) has allowed us to reduce extremely complicated problem of describing NN scattering in QFT to solution of integral equation

$$\langle 1', 2' | T_{NN}(E + i0) | 1, 2 \rangle = \langle 1', 2' | K_{NN} | 1, 2 \rangle + \langle 1', 2' | K_{NN}(E + i0 - K_F)^{-1} T_{NN}(E + i0) | 1, 2 \rangle$$

$|12\rangle = b_c^\dagger b_c^\dagger |\Omega\rangle$ any clothed two-nucleon state, once we will confine ourselves to approximation $K_f = K_{NN}$ or equation for R -matrix

$$\langle 1' 2' | \bar{R}(E) | 12 \rangle = \langle 1' 2' | \bar{K}_{NN} | 12 \rangle + \sum_{34} \langle 1' 2' | \bar{K}_{NN} | 34 \rangle \frac{\langle 34 | \bar{R}(E) | 12 \rangle}{E - E_3 - E_4}$$

with $\bar{R}(E) = R(E)/2$ ($\bar{K}_{NN} = K_{NN}/2$), symbol \sum_{34} implies the *p.v.* integration.

After angular-momentum decomposition in c.m.s

$$\bar{R}_{L'L}^{JST}(p', p) = \bar{V}_{L'L}^{JST}(p', p) + \frac{1}{2} \sum_{L''} P \int_0^\infty \frac{q^2 dq}{E_p - E_q} \bar{V}_{L'L''}^{JST}(p', q) \bar{R}_{L''L}^{JST}(q, p)$$

$$\bar{R}_{L'L}^{JST}(p', p) \equiv \bar{R}_{L'L}^{JST}(p', p; 2E_p)$$

In our case such a decomposition means transition to matrix elements between states

$$|pJ(LS)M_J\rangle = \sum \left(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 |SM_S\rangle (Lm_L SM_S |JM_J) \right. \\ \left. \times \int d\Omega_{\vec{p}} Y_{Lm_L}(\hat{\vec{p}}) b_c^\dagger(\vec{p}\mu_1) b_c^\dagger(-\vec{p}\mu_2) |\Omega\rangle \right)$$

A careful exploration shows that our equation for T -matrix with cutoff functions

$$F_b(p', p) = \left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 - (p' - p)^2} \right]^{n_b} \equiv F_b[(p' - p)^2]$$

has much common with equation by Bonn group in JST -representation (in particular, for their Potential B). Nevertheless, one needs to keep in mind some distinctions, viz., Potential B by Bonn group can be obtained from UCT quasipotentials with help of following transformations

- ▶ for boson propagators

$$[(p' - p)^2 - m_b^2]^{-1} \longrightarrow -[\vec{p}' - \vec{p}]^2 + m_b^2]^{-1}$$

- ▶ for cutoff functions

$$\left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 - (p' - p)^2} \right]^{n_b} \longrightarrow \left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 + (\vec{p}' - \vec{p})^2} \right]^{n_b}$$

- ▶ omitting off-energy-shell correction in tensor-tensor term

$$\frac{f_v^2}{4m^2} (E_{p'} - E_p)^2 \bar{u}(\vec{p}') [\gamma_0 \gamma_\nu - g_{0\nu}] u(\vec{p}) \bar{u}(-\vec{p}') [\gamma^0 \gamma^\nu - g^{0\nu}] u(-\vec{p}) \longrightarrow 0$$

Table 1: The best-fit parameters for the two models. All masses are in *MeV*, and $n_b = 2$ except for $n_\rho = n_\omega = 4$.

Meson		Potential B	UCT
π	$g_\pi^2/4\pi$	14.4	14.574
	Λ_π	1700	2200
	m_π	138.03	138.03
η	$g_\eta^2/4\pi$	3	2.1
	Λ_η	1500	1200
	m_η	548.8	548.8
ρ	$g_\rho^2/4\pi$	0.9	1.3
	Λ_ρ	1850	1450
	f_ρ/g_ρ	6.1	5.953
	m_ρ	769	769
ω	$g_\omega^2/4\pi$	24.5	25.325
	Λ_ω	1850	2143.8
	m_ω	782.6	782.6
δ	$g_\delta^2/4\pi$	2.488	2.923
	Λ_δ	2000	2092.2
	m_δ	983	983
$\sigma, T = 0, T = 1$	$g_\sigma^2/4\pi$	18.3773, 8.9437	16.081, 10.089
	Λ_σ	2000, 1900	2012.4, 2200
	m_σ	720, 550	693.66, 562.07

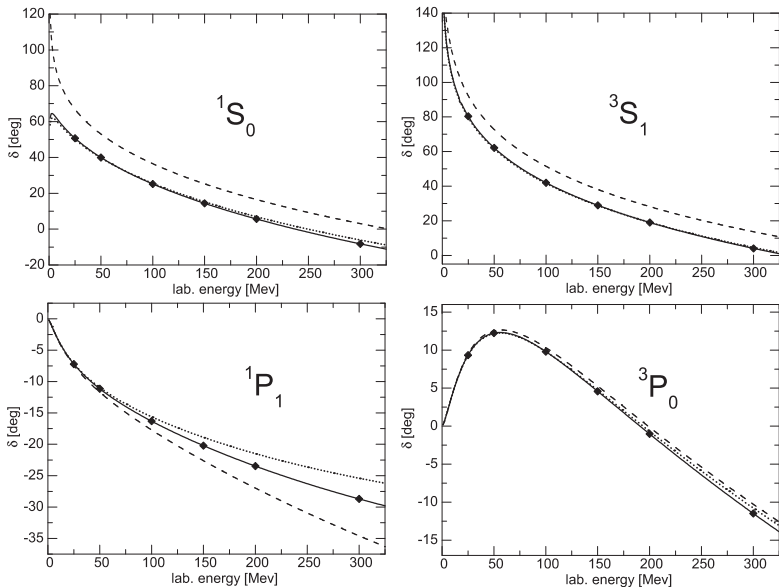


Figure 4: Neutron-proton phase parameters plotted versus nucleon kinetic energy in lab. system. Solid curves calculated for Potential B. Dashed (dotted) - for UCT potential with Potential B (UCT) parameters from Table 1. The rhombs show original OBEP results.

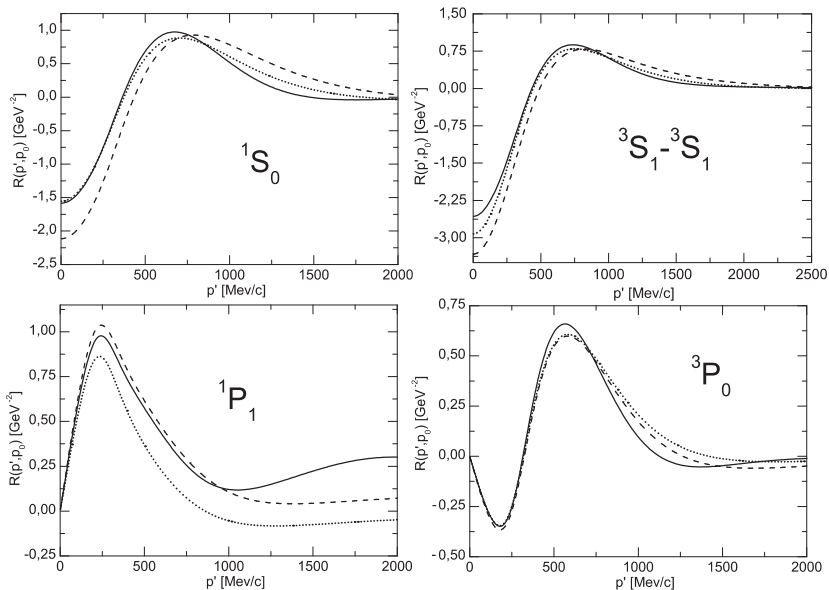


Figure 5: Half-off-shell R -matrices at laboratory energy equal to 150 MeV ($p_0 = 265 \text{ MeV}$). Other notations as in Fig. 1.

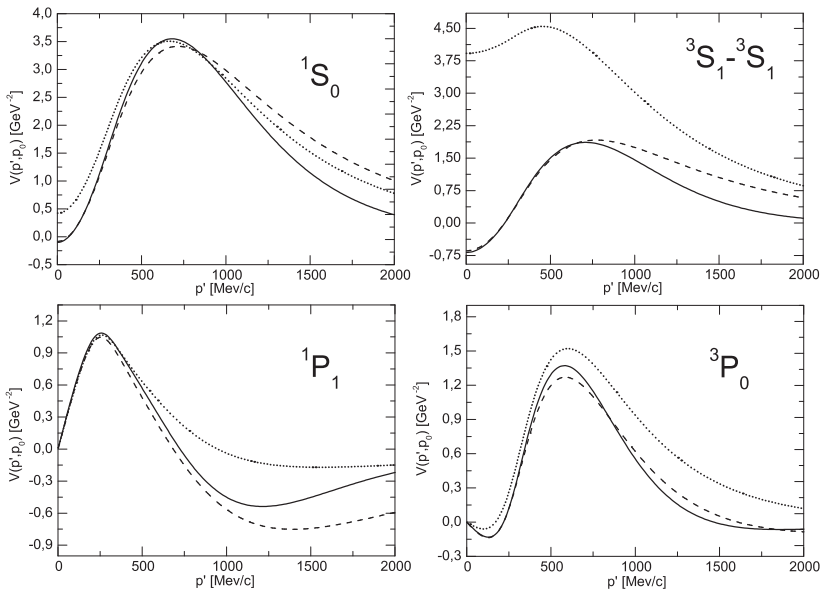


Figure 6: Off-shell potentials with the momentum p_0 fixed as in Fig. 2. Other notations in Fig. 1.

Clothing Procedure in the Theory of EM Interactions with Nuclei.

Deuteron Properties

The Deuteron Equation

Now, we consider a $K(\alpha_c)$ eigenstate from the NN sector

$$|\psi_{NN}\rangle = \sum_{\mu_1 \mu_2} \int d\vec{p}_1 d\vec{p}_2 \psi_{NN}(\vec{p}_1 \mu_1, \vec{p}_2 \mu_2) b^\dagger(\vec{p}_1 \mu_1) b^\dagger(\vec{p}_2 \mu_2) |\Omega\rangle$$

In the approximation $K_l = K_l^{(2)}$, the eigenvalue equation has the form

$$[K_F + K_{NN}] |\psi_{NN}\rangle = E |\psi_{NN}\rangle$$

In turn the deuteron state at rest can be written as the superposition

$$|\psi_d^M\rangle = \sum_{l=0,2} \int_0^\infty dq q^2 |q(l)1M\rangle \psi_l^d(q)$$

with coefficients $\psi_l^d(q) = \langle q(l)1M | \psi_{NN} \rangle$ that satisfy the equations

$$\psi_l^d(p) = \frac{1}{M_d - 2E_{\vec{p}}} \sum_{l'} \int_0^\infty dq q^2 \bar{V}_{l'l'}^{J=1, S=1, T=0}(p, q) \psi_{l'}^d(q)$$

where $M_d = 2m - \varepsilon_d$ deuteron mass, ε_d deuteron binding energy.

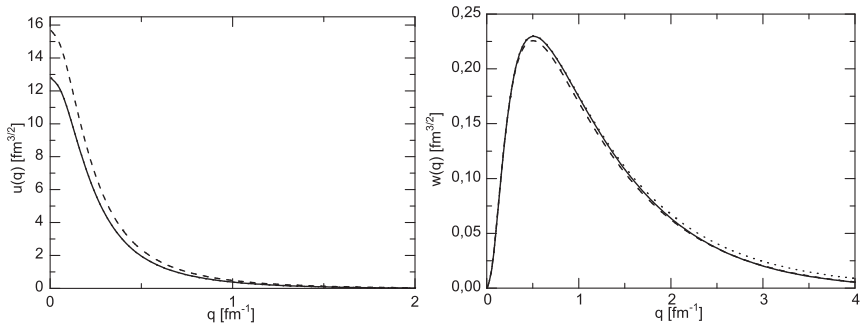


Figure 7: Deuteron wave functions $\psi_0^d(q) = u(q)$ and $\psi_2^d(q) = w(q)$. Solid curves for Bonn Potential B. Dashed (dotted) - for UCT potential with *Potential B (UCT)* parameters from Table 1.

In case of the UCT potential after parameters fitting we have for the deuteron binding energy $\varepsilon_d = 2.224$ MeV and for the D-state probability $P_D = 5.494\%$ vs Bonn values $\varepsilon_d = 2.223$ MeV and $P_D = 4.986\%$).

Deuteron Properties

In its most general form, the relativistic deuteron electromagnetic current can be written as

$$\begin{aligned} \langle P' M' | J^\mu(0) | P M \rangle = & - \left\{ G_1(Q^2) [\xi_{M'}^*(P') \cdot \xi_M(P)] (P' + P)^\mu \right. \\ & + G_2(Q^2) [\xi_M(P) [\xi_{M'}^*(P') \cdot q] - \xi_{M'}^*(P') [\xi_M(P) \cdot q]] \\ & \left. - G_3(Q^2) \frac{1}{2m_d^2} [\xi_{M'}^*(P') \cdot q] [\xi_M(P) \cdot q] (P' + P)^\mu \right\} \end{aligned}$$

$\xi_M(P)$ ($\xi_{M'}(P')$) - polarizations of incoming (outgoing) deuteron.

$$G_C(Q^2) = G_1(Q^2) + \frac{2}{3}\eta G_Q(Q^2), \quad G_M(Q^2) = G_2(Q^2),$$

$$G_Q(Q^2) = G_1(Q^2) - G_M(Q^2) + (1+\eta)G_3(Q^2), \quad q = P' - P, \quad Q^2 = -q^2, \quad \eta = \frac{Q^2}{4m_d^2}$$

At $Q^2 = 0$, form factors G_C , G_M and G_Q give charge, magnetic and quadrupole moments of deuteron:

$$Q_C(0) = 1, \quad Q_M(0) = \frac{m_d}{m_p} \mu_d, \quad G_Q(0) = m_d^2 Q_d$$

For example, in case of deuteron magnetic moment we have

$$\mu_d \sim \lim_{\eta \rightarrow 0} \frac{\langle P' M' = 1 | J^x(0) | P M = 0 \rangle}{\sqrt{\eta} \sqrt{1 + \eta}} = \lim_{\eta \rightarrow 0} \frac{\langle P' M' = 1 | J^x(0) | P = (m_d, \vec{0}) M = 0 \rangle}{\sqrt{\eta} \sqrt{1 + \eta}}$$

Deuteron state in moving frame can be built up as

$$|P' M'\rangle = e^{-i\vec{\beta}(P')\vec{B}} |\vec{0} M'\rangle$$

where boost operator

$$\vec{B} = \vec{B}_F + \vec{B}_I$$

contains interaction part and

$$\vec{\beta} = \beta \vec{n}, \quad \vec{n} = \frac{\vec{v}}{v}, \quad \tanh \beta = v, \quad \vec{v} = \frac{\vec{P}'}{m_d}$$

Choosing $\vec{P}' = (0, 0, q)$ we have

$$\mu_d \sim \langle \vec{0} M' = 1 | (B_F^z + B_I^z) J^x(0) | \vec{0} M = 0 \rangle$$

Current Operator

For brevity, we omit any addressing to the Fock–Weyl criterion to satisfy the gauge independence principle, e.g., for reaction amplitude

$$T(\gamma d \rightarrow pn) = \epsilon_\mu \langle pn; out | J^\mu(0) | d \rangle$$

and local analog of Siegert theorem based on transformation property of current density operator $J_\mu(x)$ with respect to Poincaré group (Shebeko Sov. J. Nucl. Phys. 90). For this illustration,

$$J^\mu(0) = J_N^\mu(0) + J_M^\mu(0)$$

where, for instance, $J_N^\mu(0) = \bar{\psi}(0) \frac{1 + \tau_3}{2} \gamma^\mu \psi(0)$ and $J_M^\mu(0) = [\vec{\phi} \times \partial^\mu \vec{\phi}]_3$. In CPR

$$J(0) = J_{eff}(0) \equiv W J_c(0) W^\dagger = J_c(0) + [R, J_c(0)] + \frac{1}{2} [R, [R, J_c(0)]] + \dots$$

$J_c(0)$ initial current in which “bare” operators are replaced by clothed ones. This decomposition involves one–body, two–body and more complicated effective currents if one uses terminology customary in the theory of meson exchange currents (MEC).

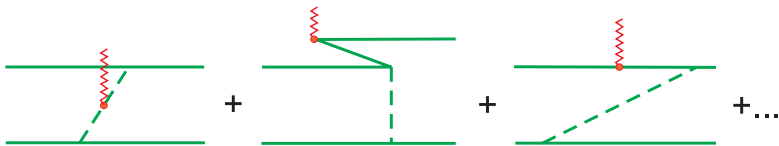
Following clothing procedure current operator $J^{\text{eff}}(0)$ can be written as

$$J_{\text{eff}}^{\mu}(0) = J_N^{\mu}(0) + J_{\text{MEC}}^{\mu}(0) + \dots = \int d\vec{p}' d\vec{p} \mathbf{F}_N^{\mu}(\vec{p}', \vec{p}) b_c^{\dagger}(\vec{p}') b_c(\vec{p}) \\ + \int d\vec{p}'_1 d\vec{p}'_2 d\vec{p}_1 d\vec{p}_2 \mathbf{F}_{\text{MEC}}^{\mu}(\vec{p}'_1, \vec{p}'_2; \vec{p}_1, \vec{p}_2) b_c^{\dagger}(\vec{p}'_1) b_c^{\dagger}(\vec{p}'_2) b_c(\vec{p}_1) b_c(\vec{p}_2) + \dots$$

First term is contained nucleon form factors

$$\langle \vec{q}', p[n] | J_N^{\mu}(0) | \vec{q}, p[n] \rangle = \frac{e}{(2\pi)^3} \bar{u}(\vec{q}') \left\{ F_1^{p[n]}[(q' - q)^2] \gamma^{\mu} \right. \\ \left. + i\sigma^{\mu\nu} (q' - q)_{\nu} F_2^{p[n]}[(q' - q)^2] \right\} u(\vec{q}),$$

second – so-called interaction (or meson exchange) currents



Conclusions and Prospects

- ▶ Our departure point in describing EM interactions with nuclei (in general, bound systems of charged particles) is to use the Fock-Weyl criterion and a generalization of the Siegert theorem.

It has been shown **how one can meet the gauge invariance principle in all orders in the charge and construct the corresponding EM interaction operators in case of nuclear forces arbitrarily dependent on velocity.** Along the guideline we have derived the conserved current density operator for a dicluster system (more precisely, the system of two finite-size clusters with many-body interaction effects included).

Being expressed through **electric and magnetic field strengths** and matrix elements of the generalized electric and magnetic dipole moments of the system **single-photon transition amplitudes attain a manifestly gauge-independent (GI) form.**

- ▶ Starting from a total Hamiltonian for interacting meson and nucleon fields, we come to Hamiltonian and boost generator in CPR whose interaction parts consist of new relativistic interactions responsible for physical (not virtual) processes, particularly, in system of bosons (π -, η -, ρ -, ω -, δ - and σ -mesons) and fermions (nucleons and antinucleons).
The corresponding quasipotentials (these essentially nonlocal objects) for binary processes $NN \rightarrow NN$, $\bar{N}N \rightarrow \bar{N}N$, etc. are **Hermitian and energy independent**. It makes them attractive for various applications in nuclear physics. They embody the off-shell effects in a natural way without addressing to **any off-shell extrapolations of the S-matrix** for the NN scattering.
- ▶ Using unitary equivalence of CPR to BPR, we have seen how in approximation $K_I = K_I^{(2)}$ NN scattering problem in QFT can be reduced to three-dimensional LS-type equation for the T -matrix in momentum space. The equation kernel is given by clothed two-nucleon interaction of class [2.2]. Such a conversion becomes possible owing to property of $K_I^{(2)}$ to leave two-nucleon sector and its separate subsectors to be invariant.
- ▶ Special attention has been paid to the elimination of auxiliary field components. We encounter such a necessity for interacting vector and fermion fields when in accordance with the canonical formalism the interaction Hamiltonian density embodies not only a scalar contribution but nonscalar terms too. It has proved (at least, for primary ρN and ωN couplings) that the UCT method allows us to remove such noncovariant terms (sometimes called contact ones) directly in the Hamiltonian.

- ▶ Being concerned with constructing two–nucleon states from \mathcal{H} and their angular–momentum decomposition we have not used the so–called **separable ansatz**, where every such state is a direct product of corresponding one–nucleon (particle) states. The clothed two–nucleon partial waves have been built up as common eigenstates of the field total angular–momentum generator and its polarization (fermionic) part expressed through the clothed creation/destruction operators and their derivatives in momentum space.
- ▶ Our calculations of deuteron magnetic and quadrupole moments have been carried out using the clothed particle representation (CPR) of the Hamiltonian, the boost and EM current density operators for the n–p system.
In the course of our current work we are trying to understand to what extent the deuteron quenching in flight affects the deuteron electromagnetic form factors. In our opinion, the exposed approach has promising prospects, e.g., in the theory of decaying states (after evident refinements), certainly in quantum electrodynamics see an attachment to this presentation and, we believe, in quantum chromodynamics too. Such endeavors are under way.

We have not tried to attain a global treatment of modern precision data. But a fair agreement with the earlier analysis by Bonn group and reasonable treatment of deuteron properties makes sure that our approach may be useful for a more advanced analysis. In the context, to have a more convincing argumentation one needs to do at least the following:

- 1) consider triple commutators $[R, [R, [R, V_b]]]$ to extract two-boson-two-nucleon interaction operators of the same class [2.2] in fourth order in coupling constants.
- 2) extend our approach for describing the NN scattering above pion production threshold.

As a whole, the persistent clouds of virtual particles are no longer explicitly contained in CPR, and their influence is included in properties of clothed particles (these quasiparticles of UCT method). In addition, we would like to stress that problem of mass and vertex renormalizations is intimately interwoven with constructing the interactions between clothed nucleons. Renormalized quantities are calculated step by step in course of clothing procedure unlike some approaches, where they are introduced by "hands".

Supplement 1: The UT Method in Scalar Field Model

From Appendix C in [survey \[3\]](#) in order to illustrate key points of clothing approach with $H = H_0 + V$,

$$H = m_0 B(0) + \int \omega_{\mathbf{k}} a^\dagger(\mathbf{k}) a(\mathbf{k}) d\mathbf{k}, \quad \omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \mu^2},$$

$$V = g \int \omega_{\mathbf{k}} [B(\mathbf{k}) a(\mathbf{k}) + \text{H.c.}] h(\mathbf{k}^2) d\mathbf{k}, \quad h(\mathbf{k}^2) = \frac{f(\mathbf{k}^2)}{\sqrt{2(2\pi\omega_{\mathbf{k}})^3}}$$

$$B(\mathbf{k}) \equiv \int b^\dagger(\mathbf{p} + \mathbf{k}) b(\mathbf{p}) d\mathbf{p} = B^\dagger(-\mathbf{k}),$$

where $a(\mathbf{k})$ and $b(\mathbf{p})$ are destruction operators for bosons and fermions, respectively,

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}'),$$

$$\{b(\mathbf{p}), b^\dagger(\mathbf{p}')\} = \delta(\mathbf{p} - \mathbf{p}').$$

The cut-off factor $f(x)$ is assumed to fall off rapidly enough for large x to make finite all integrals involved.

Generator of the corresponding unitary clothing transformation W is given by

$$R = -g \int [B(\mathbf{k})a(\mathbf{k}) - \text{H.c.}] h(\mathbf{k}^2) d\mathbf{k} \equiv g(X^\dagger - X)$$

with

$$X = \int h(\mathbf{k}^2) B(\mathbf{k}) a(\mathbf{k}) d\mathbf{k}$$

and after simple algebra we find

$$W = \exp[g(X^\dagger - X)] = \exp(gX^\dagger) \exp(-gX) \exp\left(-\frac{g^2}{2}[X, X^\dagger]\right)$$

so

$$H \equiv H(a, b) = K(a_c, b_c) \equiv WH(a_c, b_c)W^\dagger = K_F + K_I,$$

$$K_F = mB_c(0) + \int \omega_{\mathbf{k}} a_c^\dagger(\mathbf{k}) a_c(\mathbf{k}) d\mathbf{k} \equiv K_{ferm} + K_{boson},$$

with radiative correction (renormalization) to bare fermion mass

$$m = m_0 - g^2 \int \omega_{\mathbf{k}} h^2(\mathbf{k}^2) d\mathbf{k},$$

$$K_{\text{I}} = \int d\mathbf{x} \int d\mathbf{x}' \psi_c^\dagger(\mathbf{x}) \psi_c^\dagger(\mathbf{x}') V_{\text{ff}}(|\mathbf{x} - \mathbf{x}'|) \psi_c(\mathbf{x}) \psi_c(\mathbf{x}') ,$$

$$V_{\text{ff}}(|\mathbf{r}|) = -g^2 \int \omega_{\mathbf{k}} \hbar^2(\mathbf{k}^2) e^{i\mathbf{k}\mathbf{r}} d\mathbf{k} ,$$

where in agreement with secondary quantization prescriptions we have introduced ψ_c - field for clothed fermions in the Schrödinger picture assuming

$$\psi_c(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int b_c(\mathbf{p}) e^{i\mathbf{p}\mathbf{x}} d\mathbf{p}$$

$$\{\psi_c(\mathbf{x}), \psi_c^\dagger(\mathbf{x}')\} = \delta(\mathbf{x} - \mathbf{x}') .$$

One should point out that new interaction Hamiltonian K_I expressed through clothed operators no longer contains any self-interaction and leads merely to an interaction between pairs of clothed fermions.

At last, we have relations

$$a(\mathbf{k}) = W a_c(\mathbf{k}) W^\dagger = a_c(\mathbf{k}) - gh(\mathbf{k}^2) B_c^\dagger(\mathbf{k}) ,$$

$$b_c^\dagger(\mathbf{p}) = W^\dagger b^\dagger(\mathbf{p}) W = \int \mathcal{F}(\mathbf{p} - \mathbf{p}') b^\dagger(\mathbf{p}') d\mathbf{p}' ,$$

$$\mathcal{F}(\mathbf{q}) = \frac{1}{(2\pi)^3} \int e^{-i\mathbf{q}\mathbf{x}} \exp\{-g \int \exp[-i\mathbf{k}\mathbf{x}] (a^\dagger(\mathbf{k}) - a^\dagger(-\mathbf{k})) h(\mathbf{k}^2) d\mathbf{k}\} d\mathbf{x} .$$

Factor $\mathcal{F}(\mathbf{q})$ characterizes boson distribution in a cloud.

In free case with ($g = 0$) $\mathcal{F}(\mathbf{q}) = \delta(\mathbf{q})$.

Supplement 2: Clothed Particle Approach vs Renormalized QED

A weird description of physical reality, inherent in the very popular interpretation of departure points of different QFT models (including QED) with its postulate of 'bare' particles (e.g., electrons) which were never observed having arbitrary (sometimes infinite masses, etc.), seems to be unnecessary in the framework of clothing procedure put forward by Greenberg and Schweber and developed in works of the Dubna-Kharkov collaboration.

Why? Even if in the well-known renormalization program **ultraviolet divergences were removed from the S -operator, they again appeared in the total Hamiltonian in form of infinite counterterms** unlike the clothing procedure exposed here.

Many prominent scientists, such as Dirac and Landau, concerned inconsistencies of the renormalization approach. For example, Rohrlich wrote in his monograph "The theory of the electron":

Thus, the present quantum electrodynamics is one of the strangest achievements of the human mind. No theory has been confirmed by experiment to higher precision; and no theory has been plagued by greater mathematical difficulties which have withstood repeated attempts at their elimination.