Efimov physics beyond universality with ultracold atoms

Richard Schmidt

Richard Schmidt, Steffen P. Rath, Wilhelm Zwerger

Eur. Phys. J. B 386 (2012)

INT Program 14-1: Workshop - Universality in Few-Body Systems

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ITAMP

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DFG - FOR 801

Harvard University

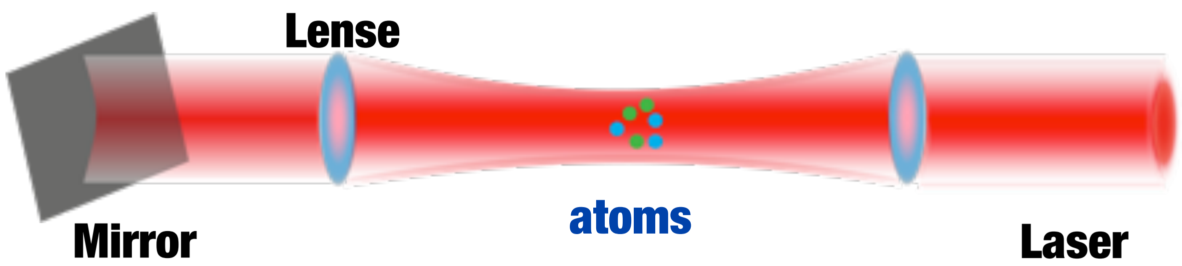
ultracold quantum gases

atoms trapped by laser in harmonic confinement

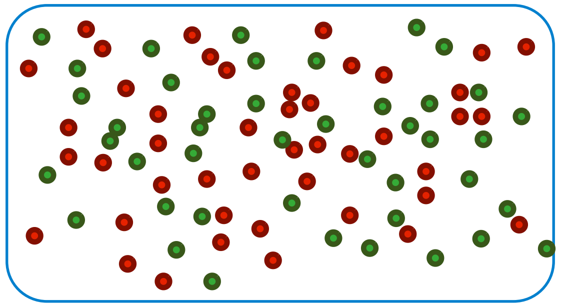
➡ very low temperatures $\sim 100\,\mathrm{nK}$

↳ contact interactions

➡ tunable interaction strength



condensed matter system with *action, we are confident of*



e.g. $$S = \int \varphi^* [-i\hbar\partial_t - \frac{\nabla^2}{2m}]\varphi + \int_x g_\Lambda \, (\varphi^* \varphi)^2(x)$$

⇨ ideal for testing of many-body theories

↳ "Quantum simulator"

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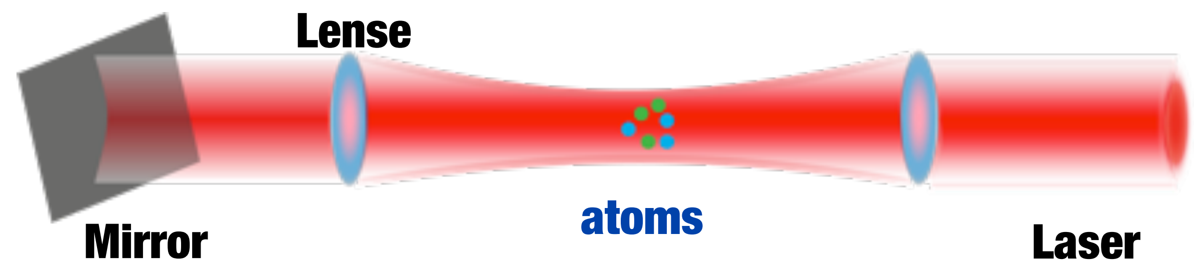

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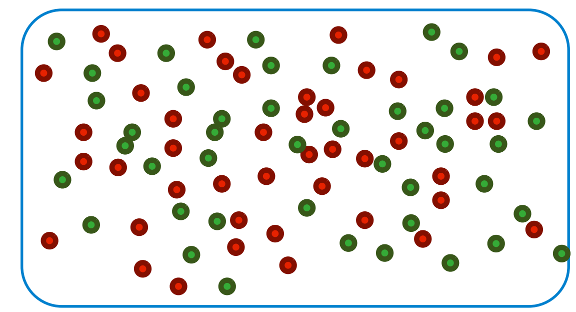

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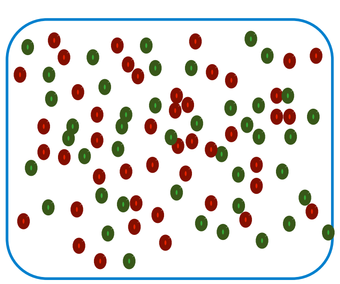
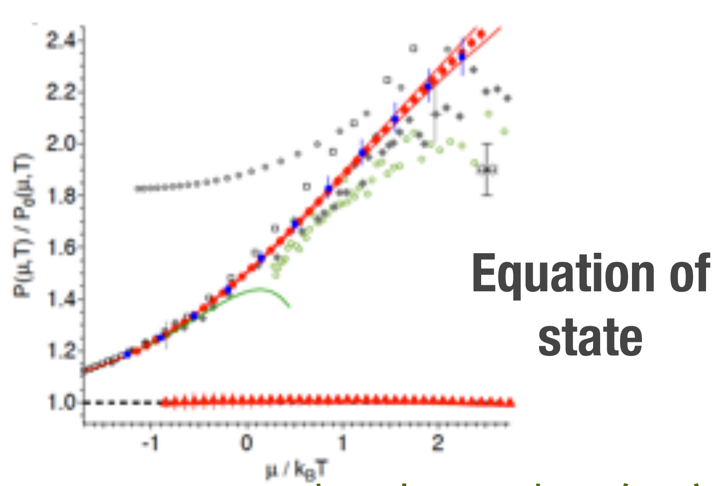
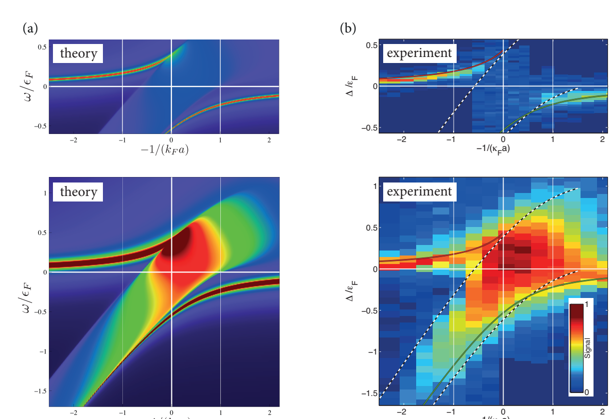


UNITARY FERMI GAS ZWIERLEIN (MIT)

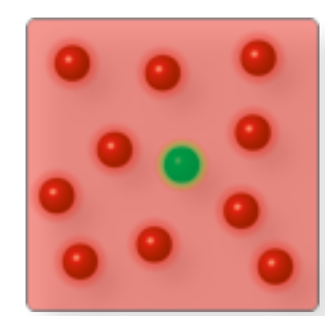


Van Houcke et al. Nature Phys. 8 (2012)

POLARON PHYSICS

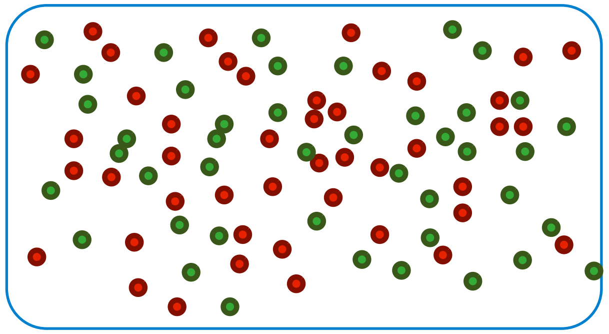


RS, ENSS, PRA 83 (2011); RS, PHD THESIS (2013)
KOHSTALL ET AL., NATURE 485 (2012)



model Hamiltonian

$$H = \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{p}} \hat{c}_{\mathbf{p}} + g \int_x \hat{c}^{\dagger}(\mathbf{x}) c^{\dagger}(\mathbf{x}) \hat{c}(\mathbf{x}) \hat{c}(\mathbf{x})$$



ideal for many-body calculations

however:

- the better the experiments, the more important non-universal details become
- cold atoms beyond condensed matter-simulator:
  - unique system to study interplay between few- and many-body physics
  - exhibits also physics without counterparts in hard condensed-matter

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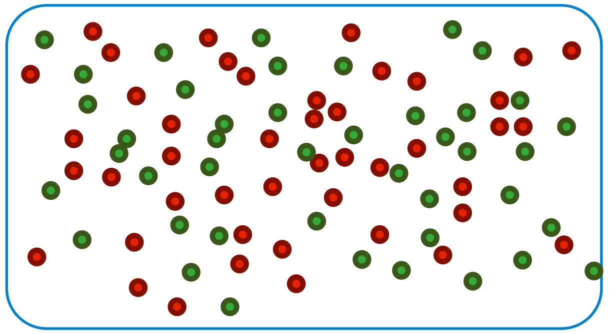

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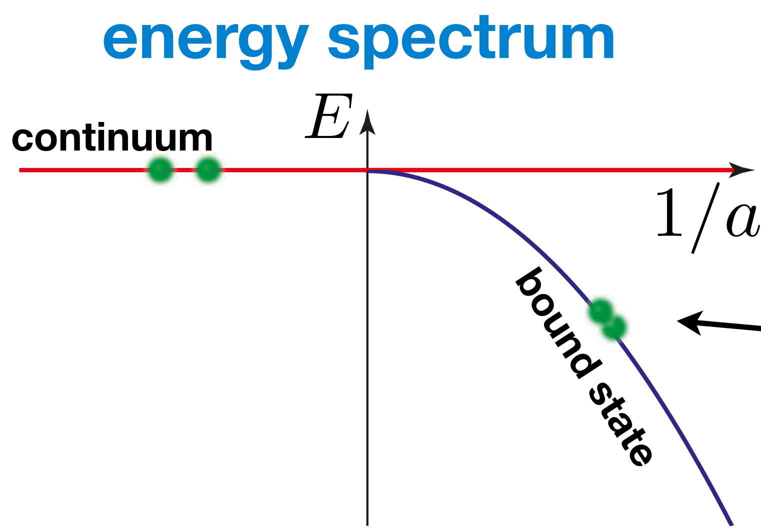
this talk:

Efimov physics combines all of these aspects

two-body universality

- low energy interactions characterized by tunable s-wave scattering length a



at small scattering energies:

$$\epsilon_B = \frac{\hbar^2}{ma^2}$$ universal dimer

“no matter the details of $V(\mathbf{x}' - \mathbf{x})$“

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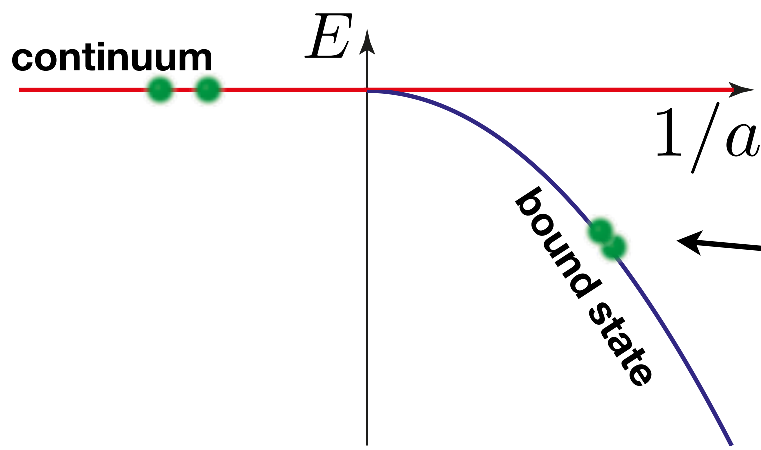
energy spectrum



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1 universality

- scattering length only parameter

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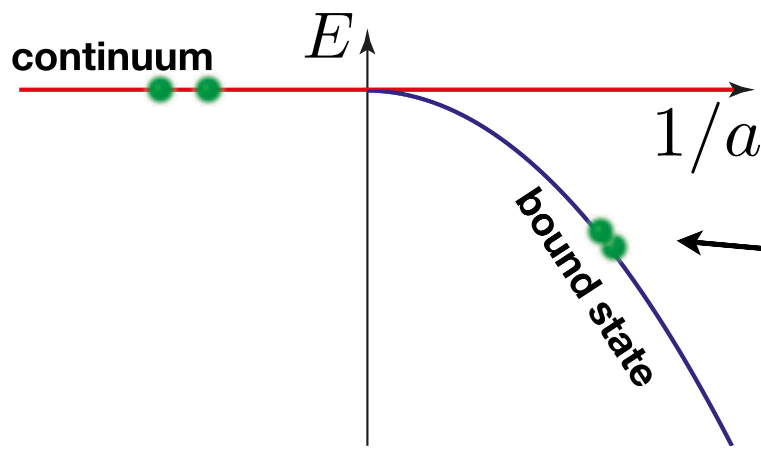

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- at unitarity no scale is left

$$a \to \infty$$

- powerful symmetry

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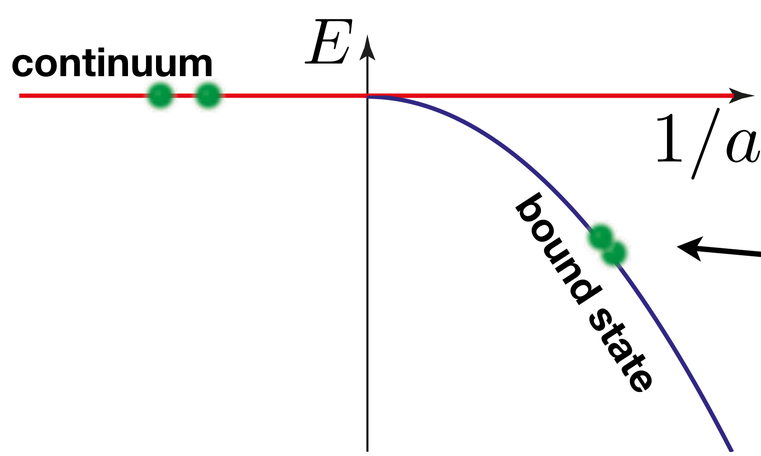

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3 RG fixed point

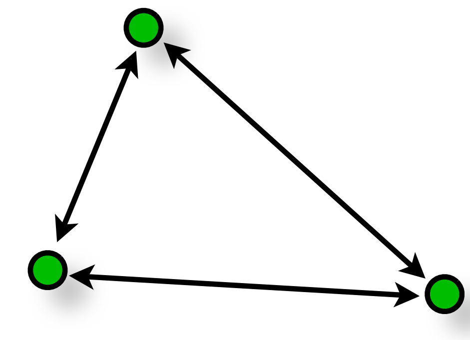
Efimov effect

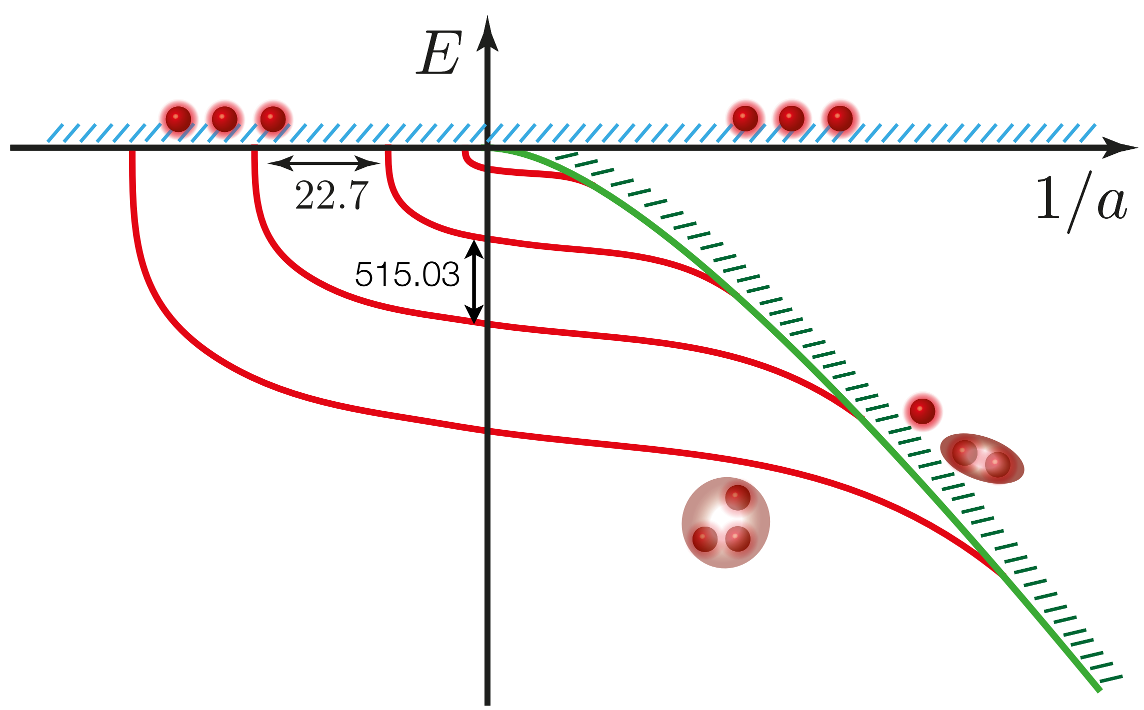
three-body physics - Efimov 1970 Efimov, Phys. Lett. 33 (1970)

bosons, pairwise resonant, *short-range* interactions

- favorable to build **three-body bound states** (trimers)
- trimers even in regime with no two-body bound state
- **infinitely many trimers** Efimov, Phys. Lett. 33 (1970)
- originally predicted for **nuclear matter**
- for the first time observed in **ultracold atoms**

Kraemer et al., Nature 440 (2006)





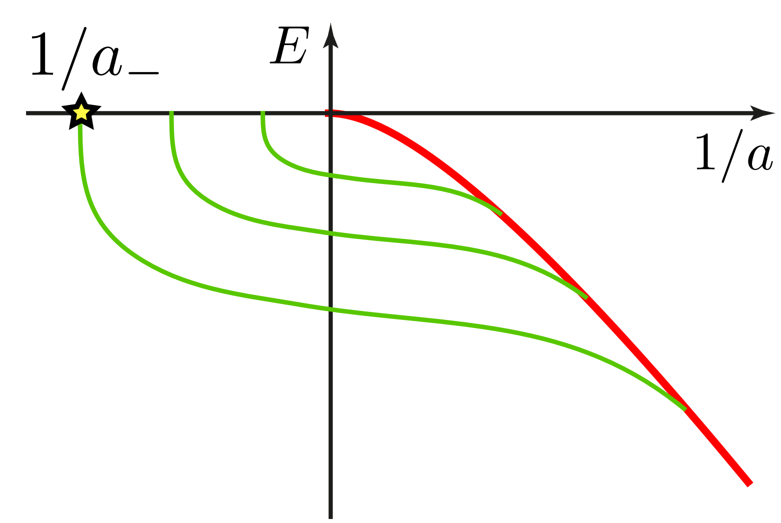
three-body parameter

~~scale invariance~~ ⟹ ***discrete*** **scale invariance**

$$E^{(n)}/E^{(n+1)} = e^{2\pi/s_0} = 515.03$$

$$s_0 \approx 1.00624... \longrightarrow \textit{universal}$$

energy spectrum



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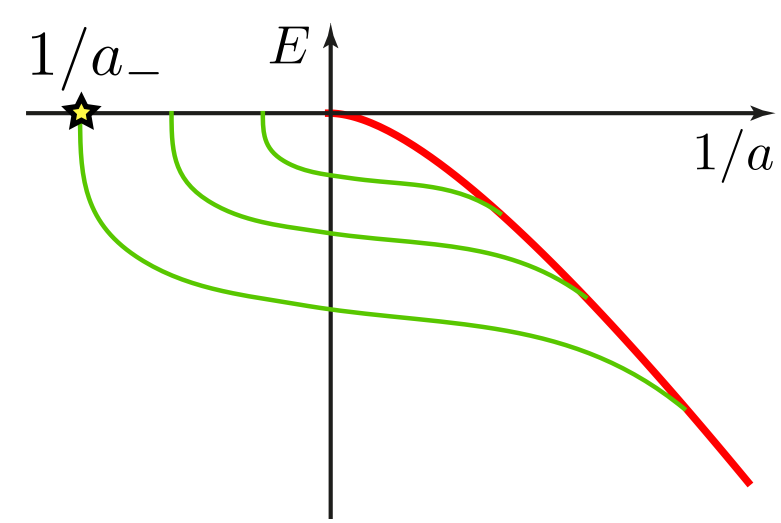

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~~**RG fixed point**~~ ⟹ **RG limit cycle**

~~**universality**~~ ⟹ ***Efimov*** **universality**

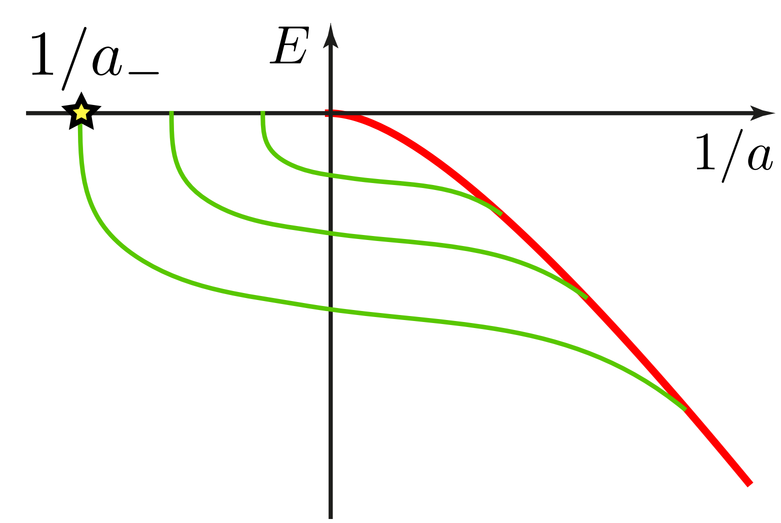
regardless the interaction potential, close to resonance: only one parameter: a

not only a as parameter, ***three-body parameter*** needed to fix overall trimer position

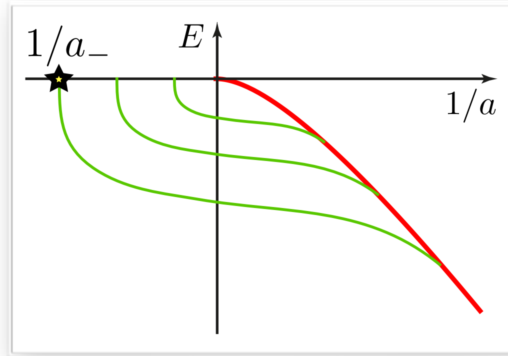
three-body parameter a_-

- determines where lowest trimer enters the atom threshold

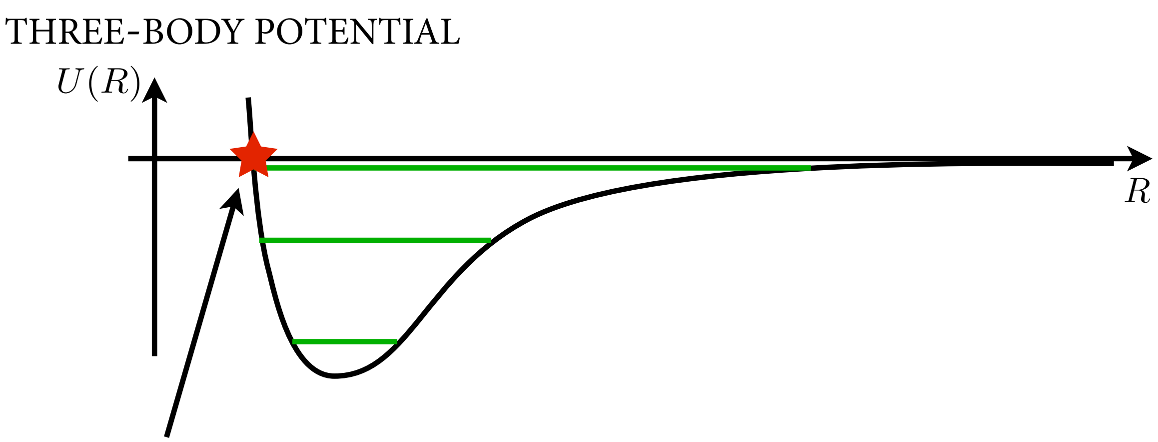
energy spectrum



origin of the three-body parameter

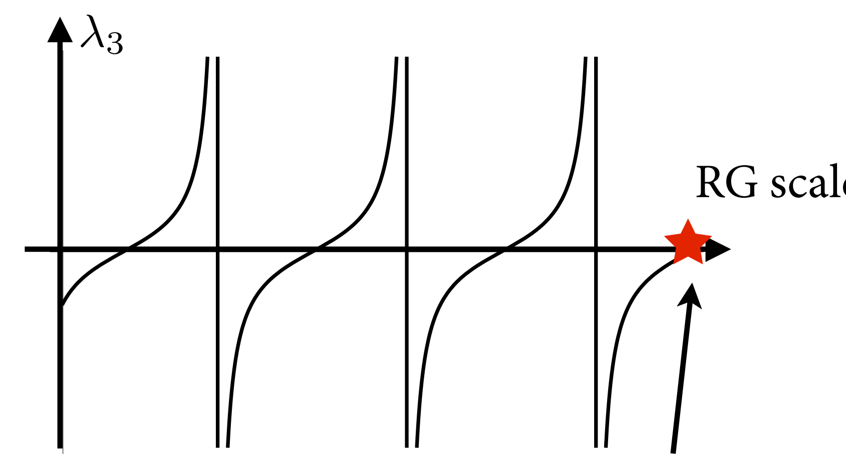


quantum mechanics



R_*: short range regularization

quantum field theory



large momentum (UV) regularization: Λ_*

three-body parameter

$$a_- \Longleftrightarrow R_*,\ \Lambda_*$$

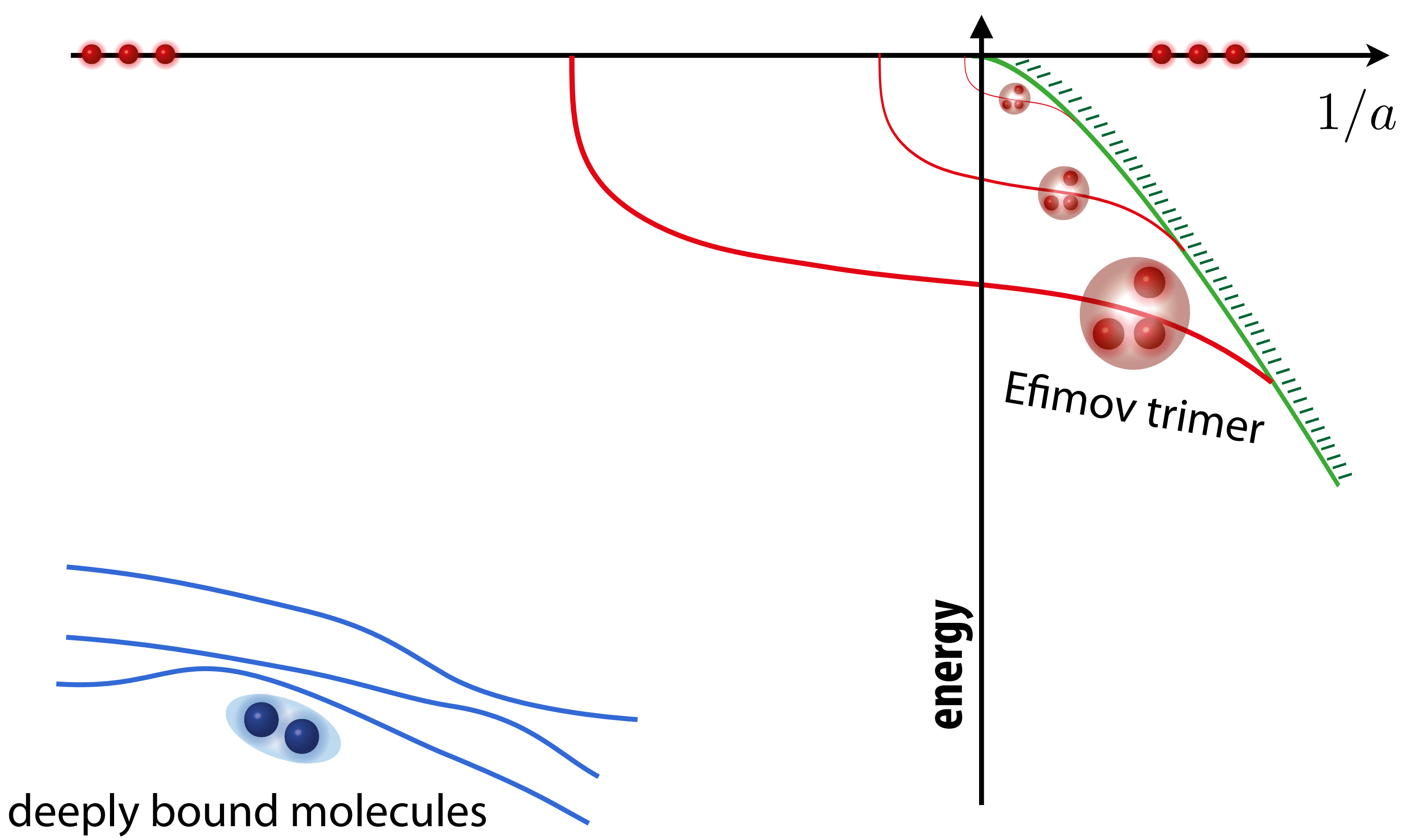
short-range sensitive: non-universal

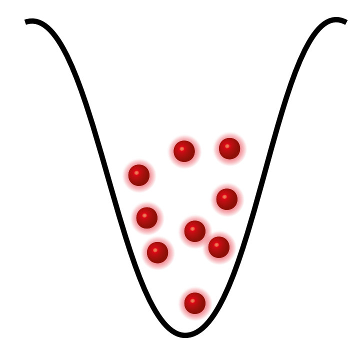
SEE E.G. D'INCAO, GREENE, ESRY, JPB 42 (2009)

how to observe Efimov physics, how to measure a_-?

Observation of Efimov physics in cold atoms

enhanced three-body loss





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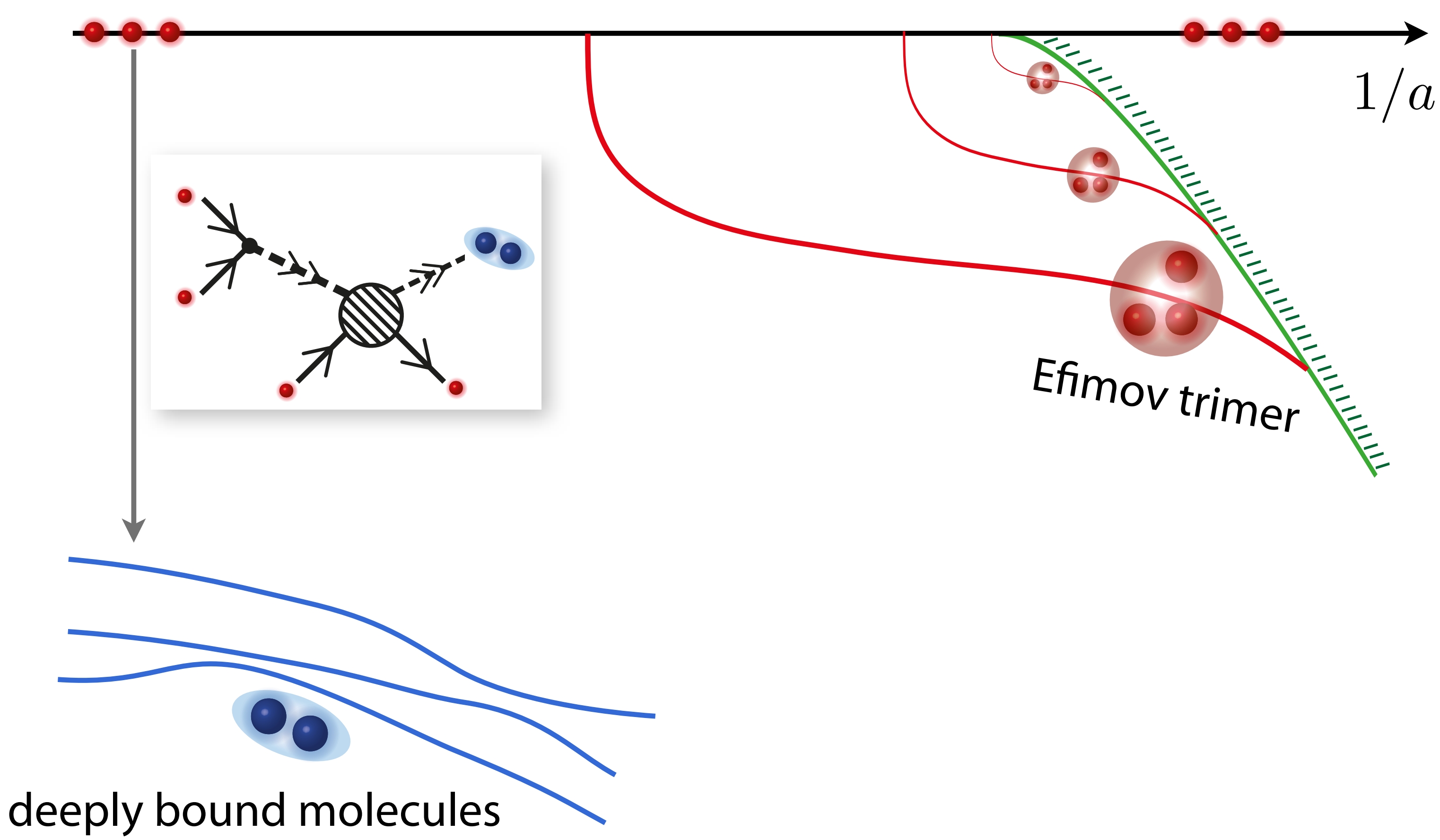
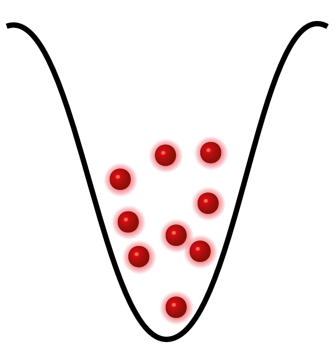
- decay to deeply bound dimers: release of binding energy leads to loss from trap





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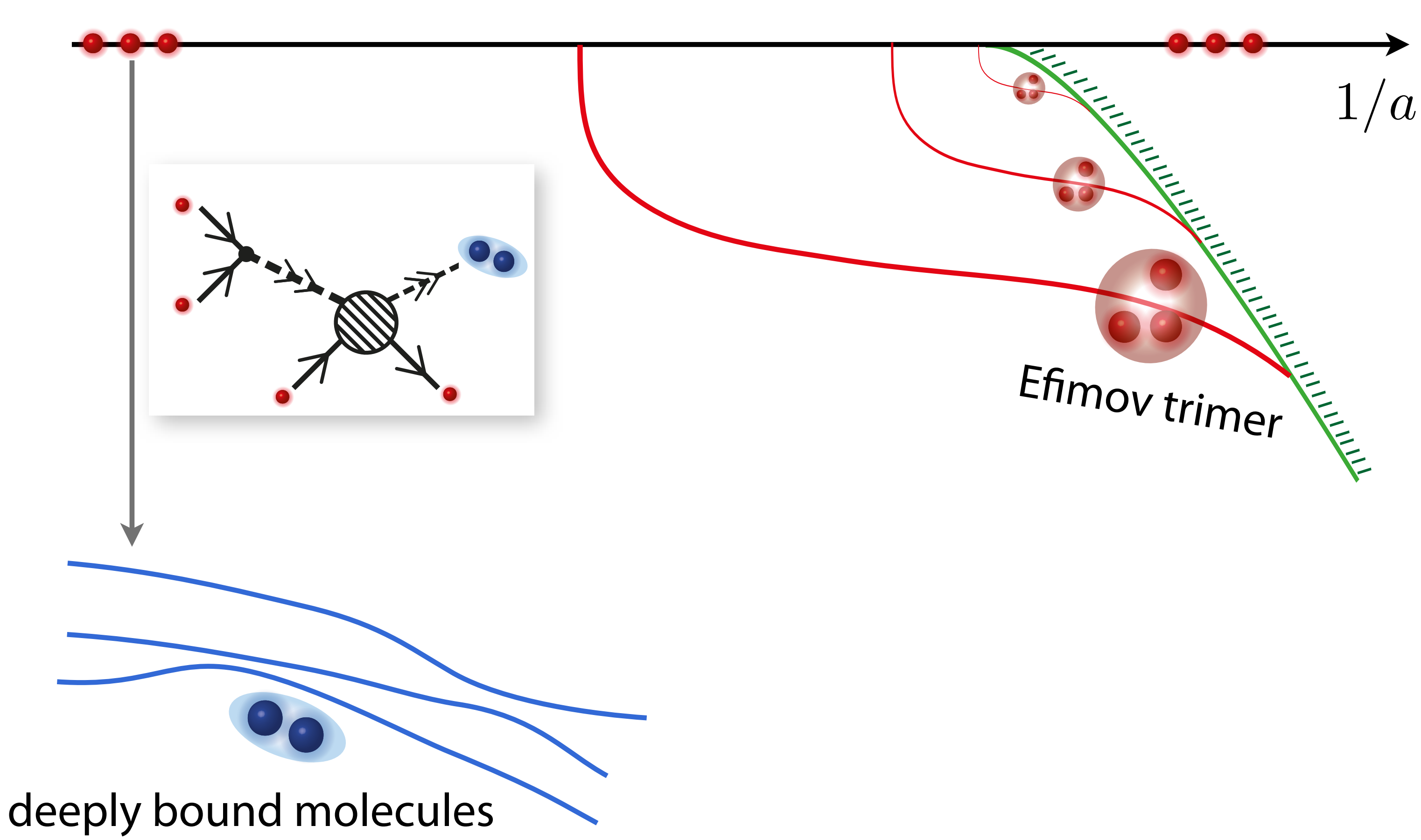

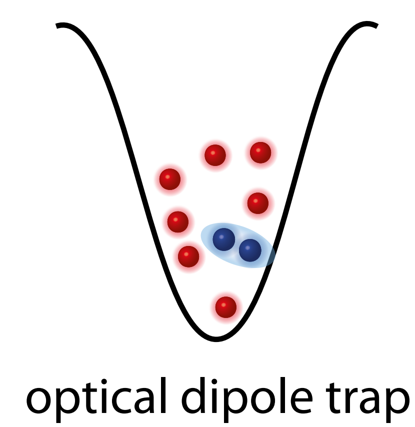

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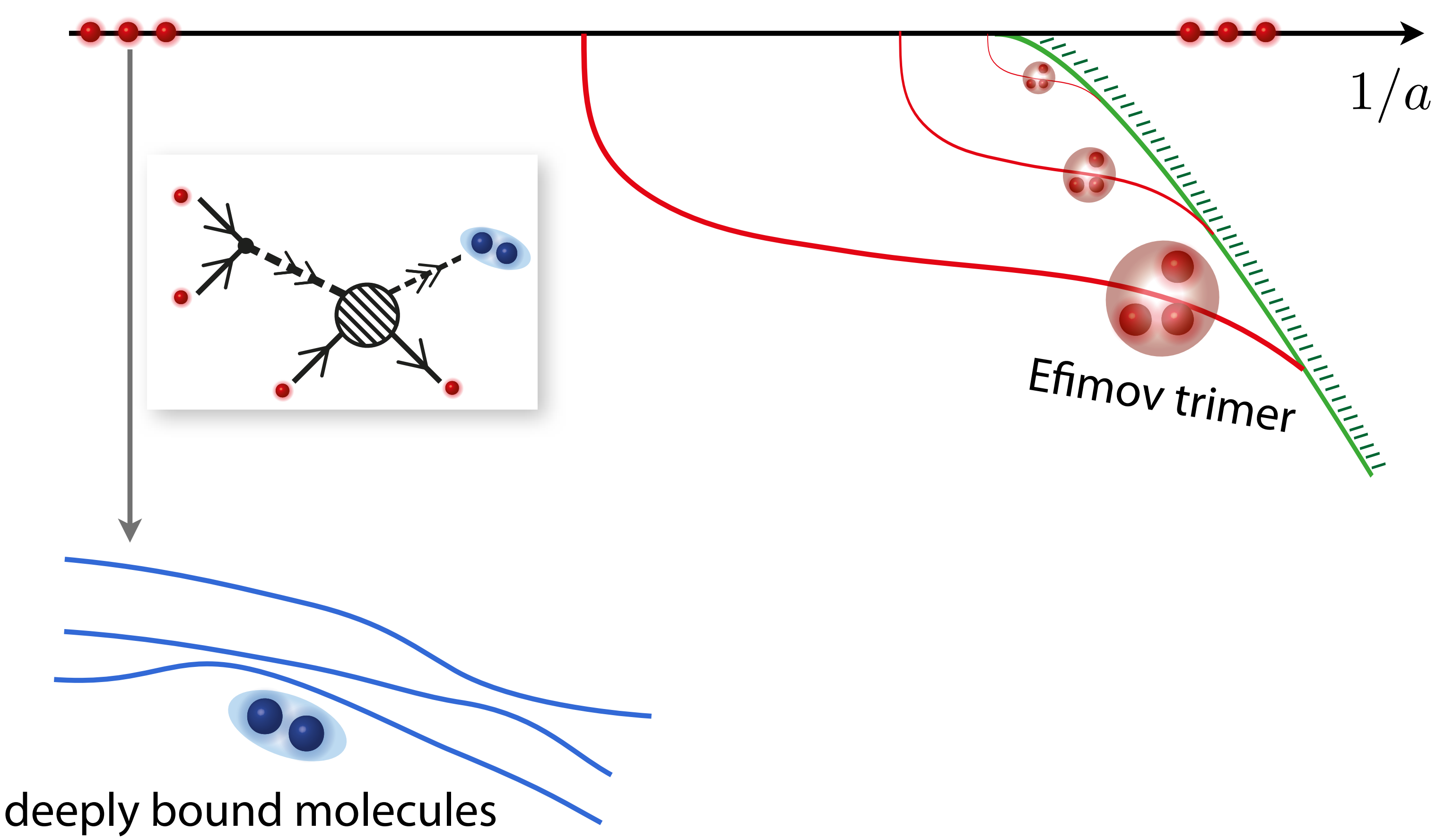

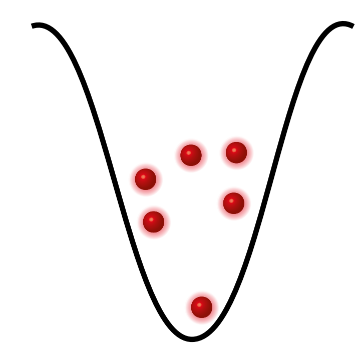

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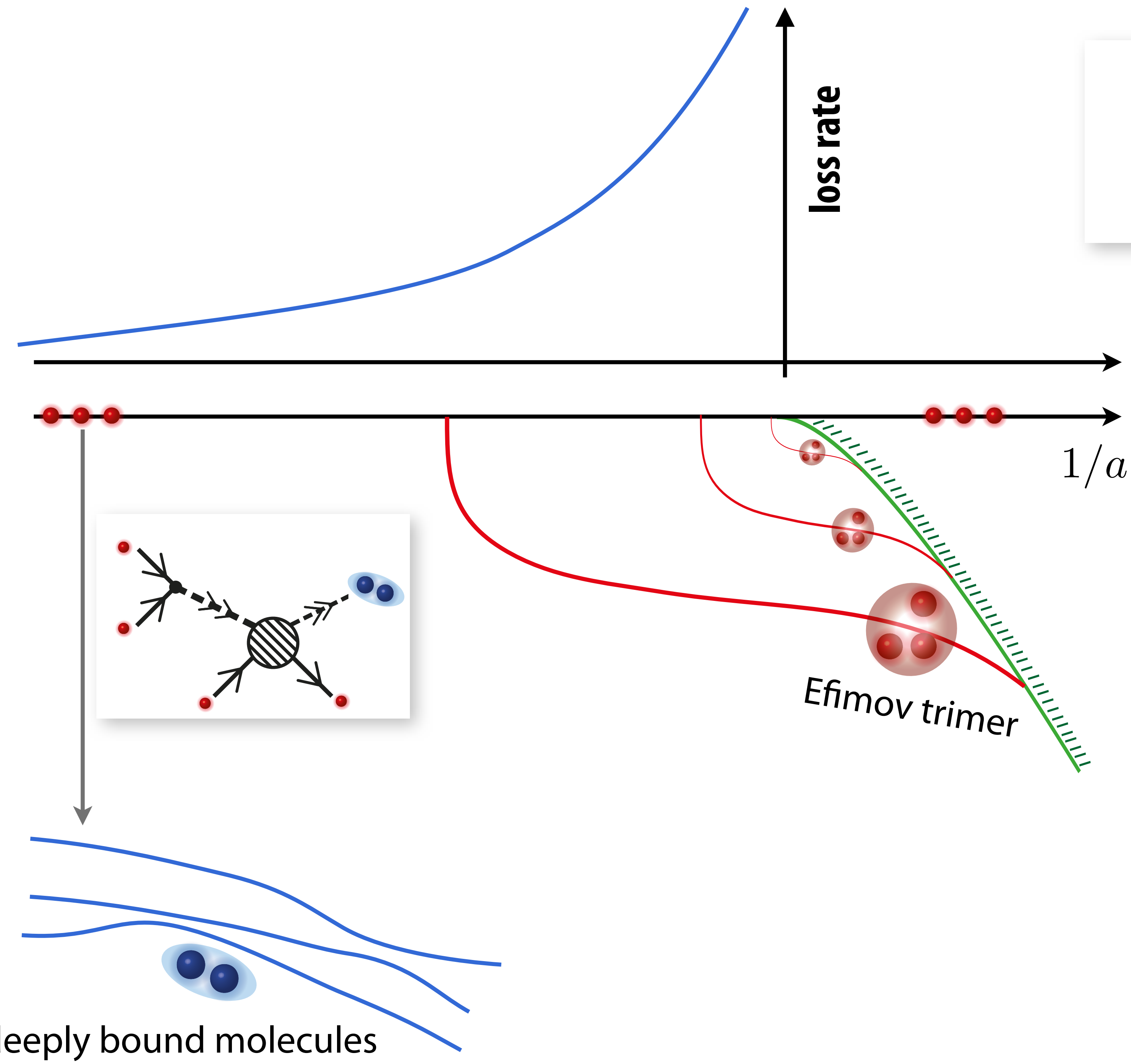

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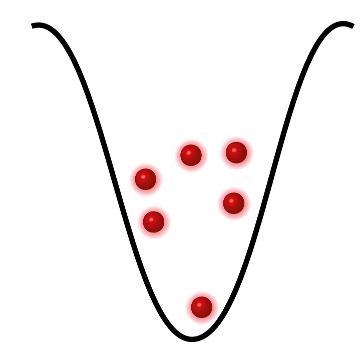
Bedaque, Hammer, van Kolck, PRL 82 (1999)
Braaten, Hammer, PRL 87 (2001)



optical dipole trap

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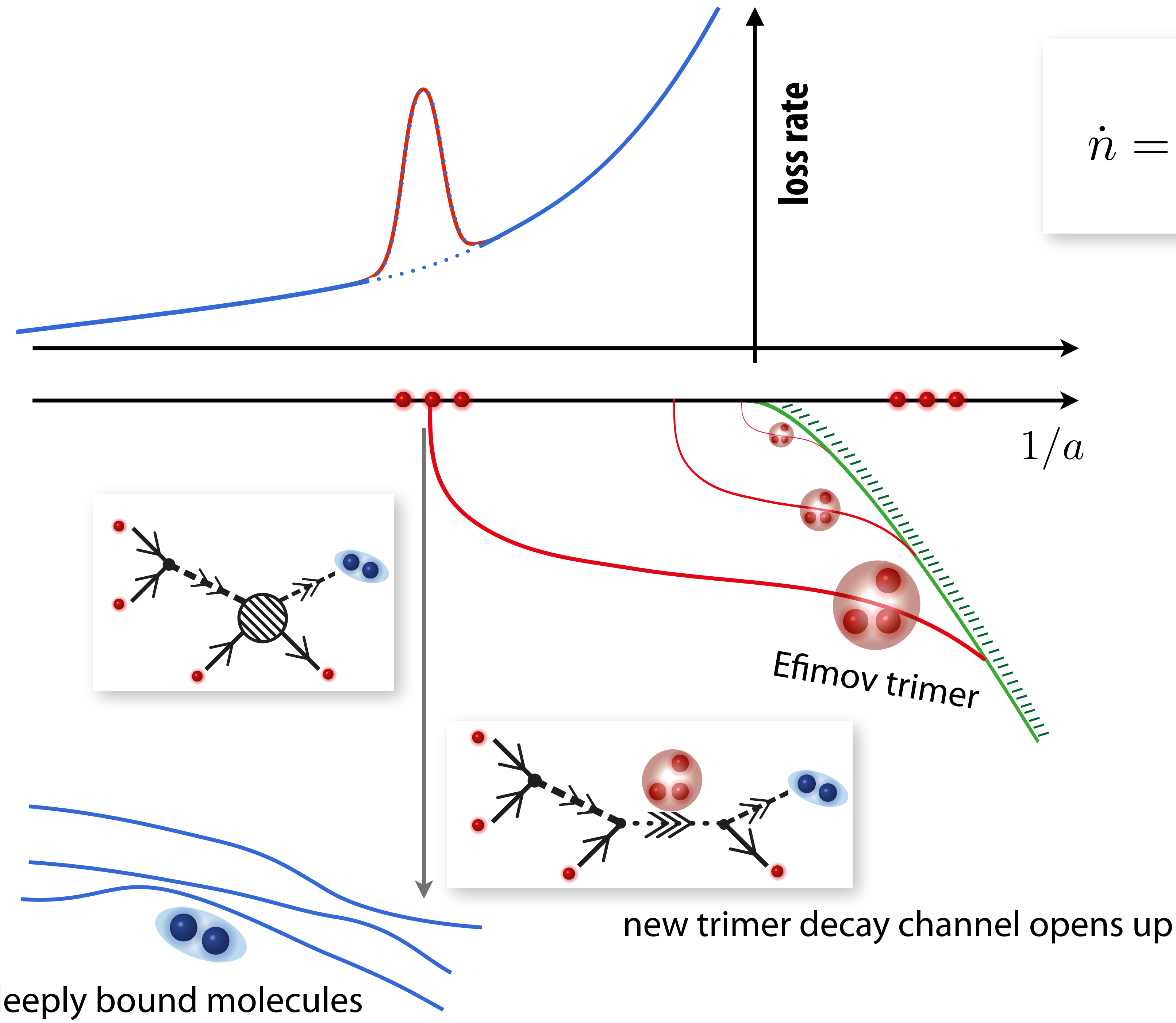

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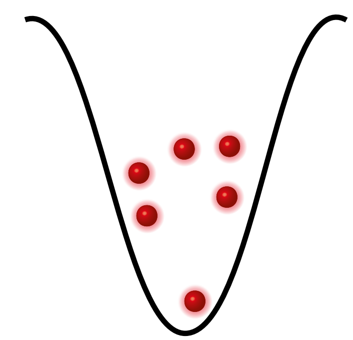

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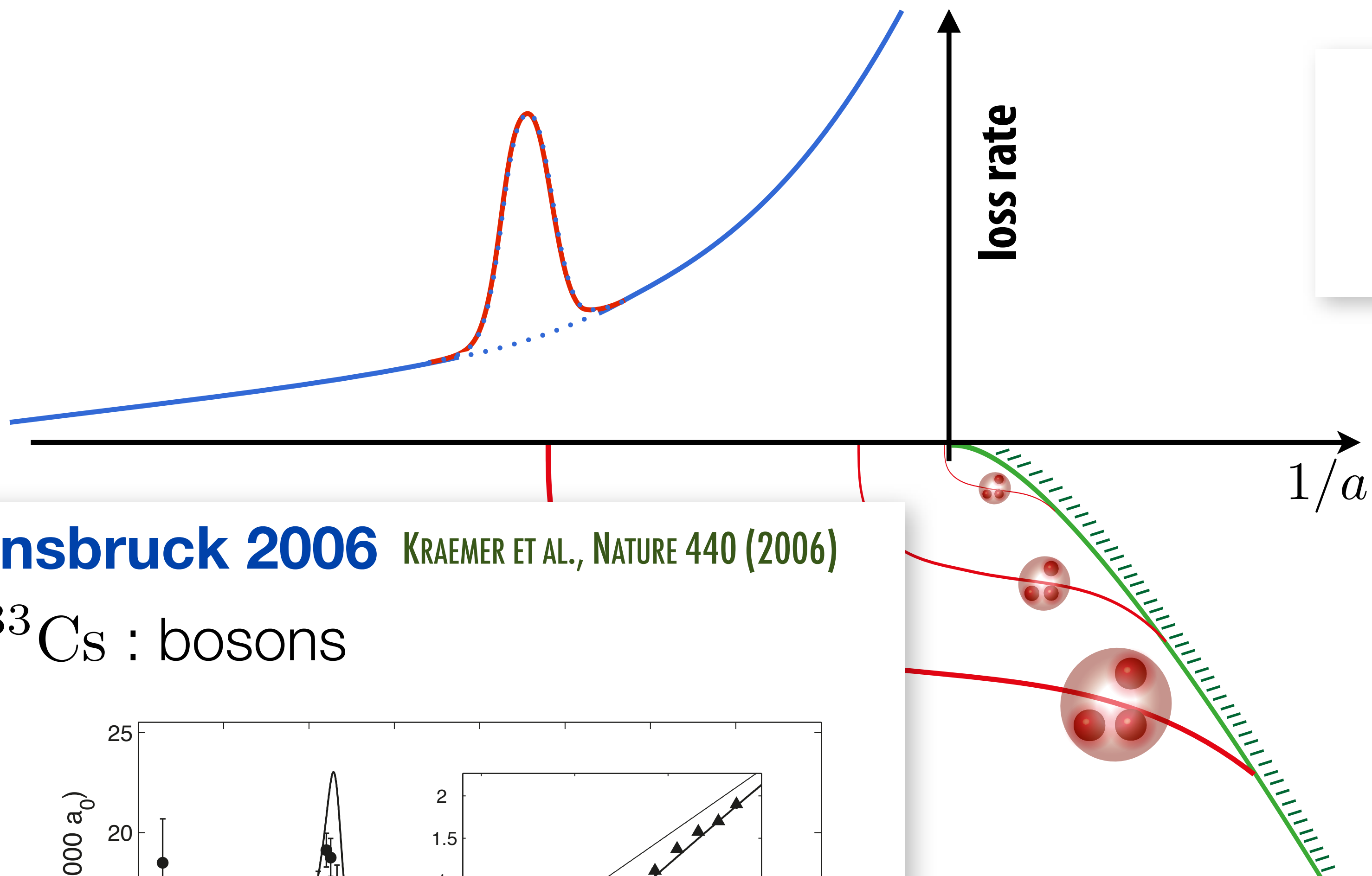

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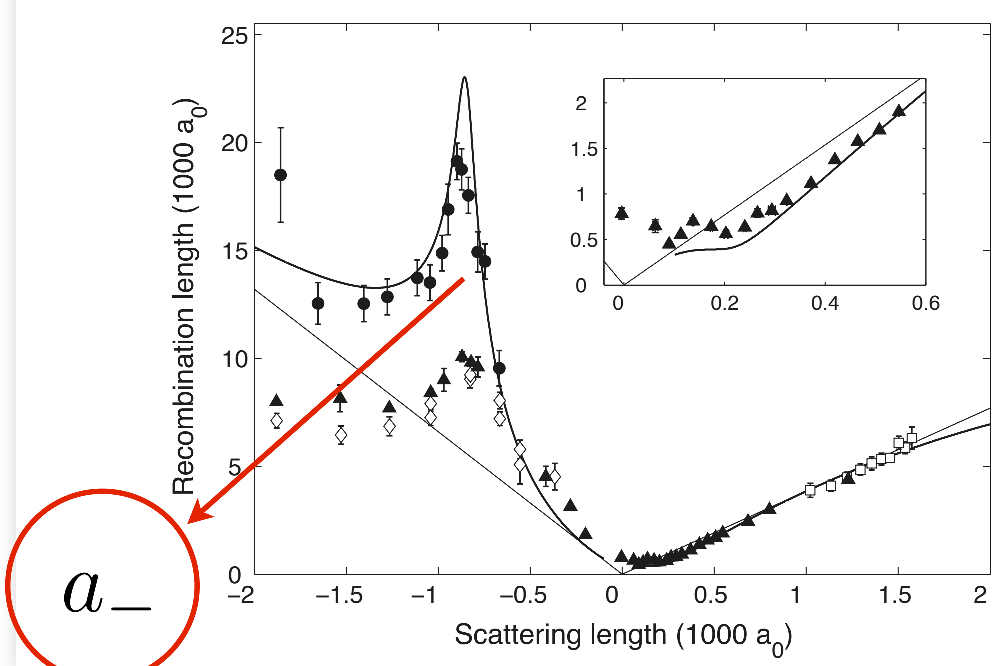
Innsbruck 2006 Kraemer et al., Nature 440 (2006)

^{133}Cs : bosons



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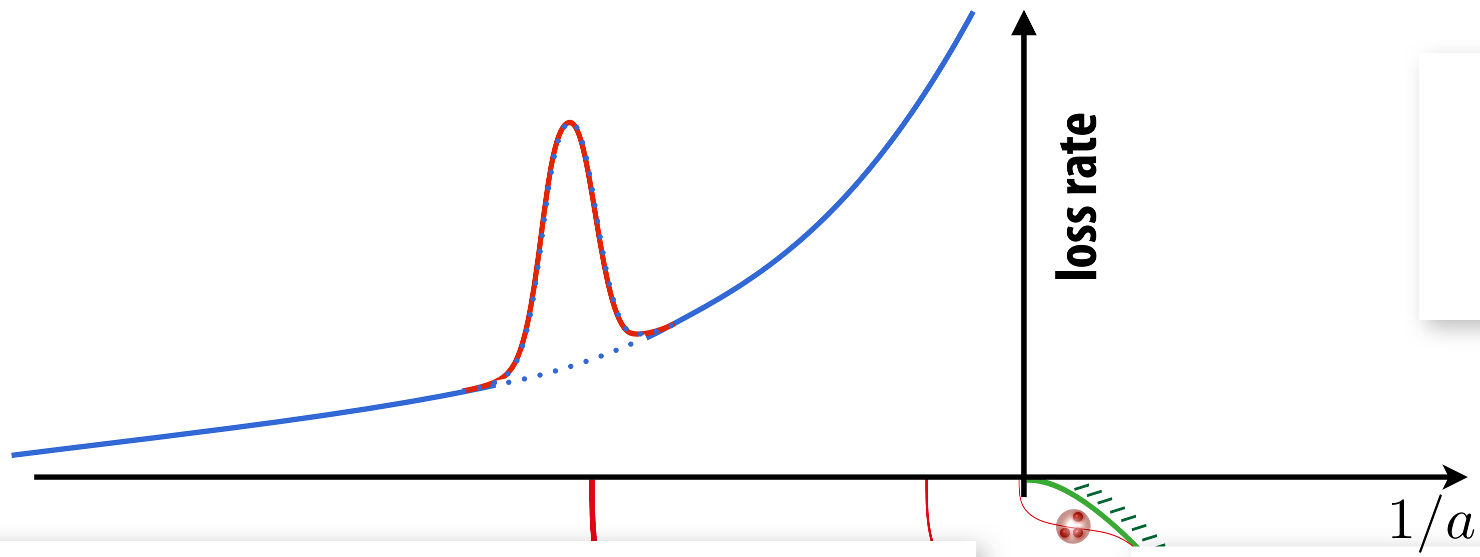

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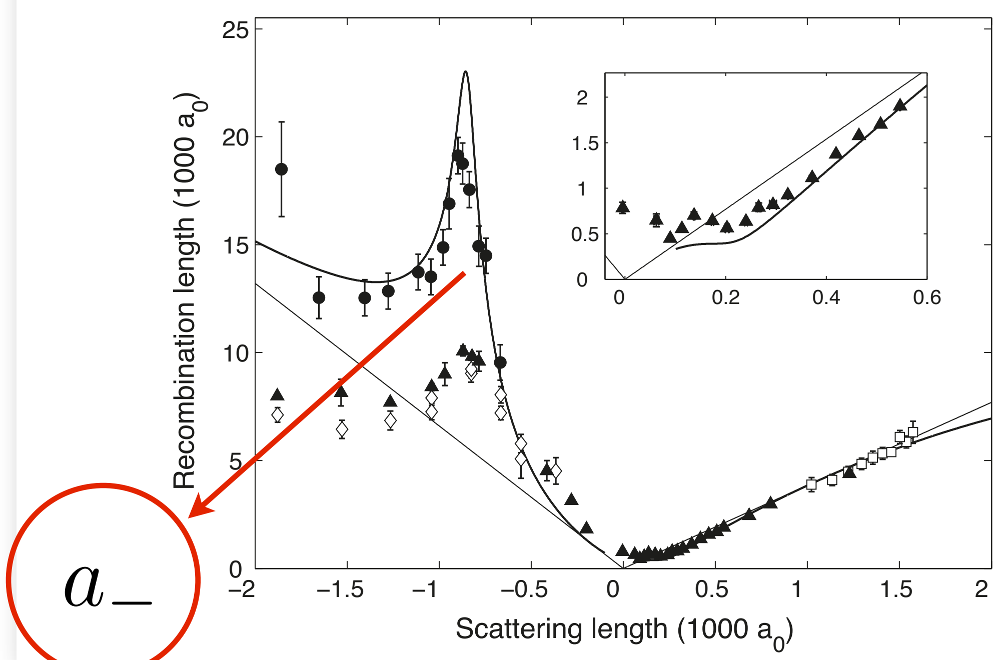
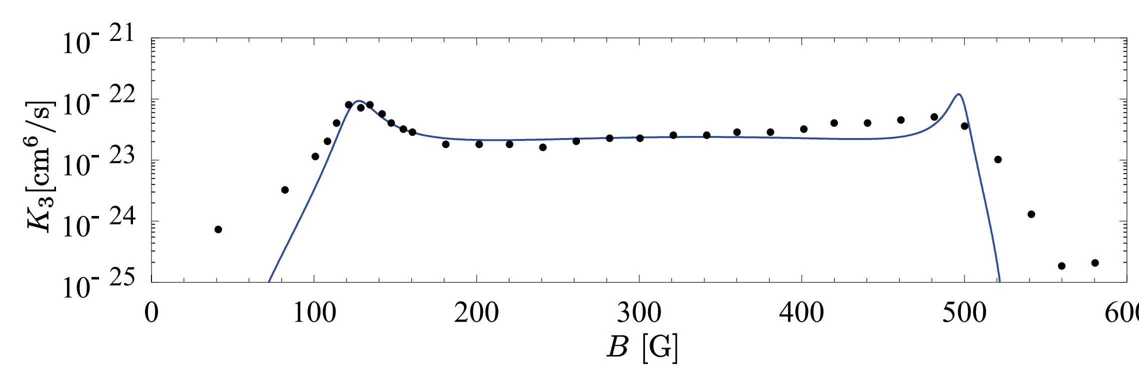


Heidelberg 2008 Ottenstein et al., PRL 101 (2008)

^{6}Li : three-component fermions



fRG computation vs. experiment

Floerchinger, RS, Wetterich, PRA 79 (2009)

Similar results: Braaten et al., PRL 103 (2009)
Naidon, Ueda, PRL 103 (2009)

experimental observation

the combined experimental effort until 2012

Atom	$-a_-^{(1)}/r_{vdW}$	
^{6}Li	9.34	Heidelberg, Penn. State
^{7}Li	9.17(31)	
^{7}Li	8.13(34)	Bar-Ilan, Rice
^{7}Li	8.25(37)	
^{39}K	23.3(1.4)	Florence
^{85}Rb	9.24(07)	JILA, Rice
^{133}Cs	8.63(22)	
^{133}Cs	10.19(57)	Innsbruck
^{133}Cs	9.48(79)	
^{133}Cs	9.46(28)	

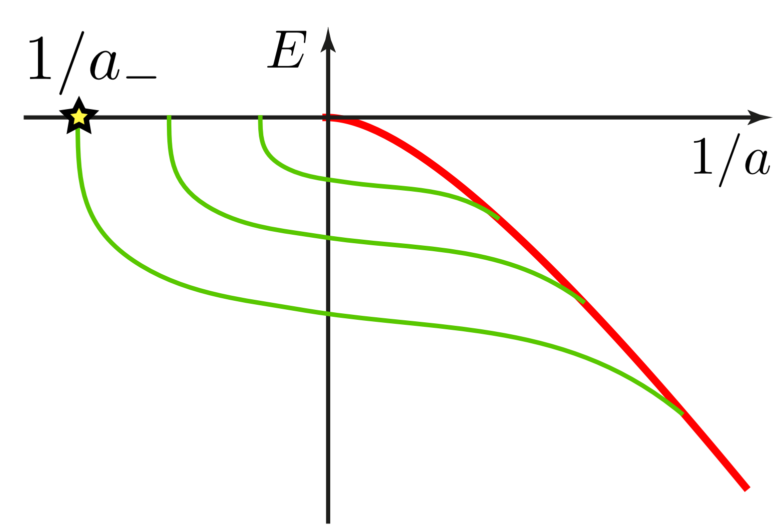
FROM C. CHIN 1111.1484v2

result: BERNINGER ET AL., PRL 107 (2011)

$$a_- \approx -9.2\, l_{\mathrm{vdw}}$$

three-body parameter *universal*?

energy spectrum



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three-body parameter *universal*?

All experiments use ultracold atoms close to *Feshbach resonances*

Our goal:

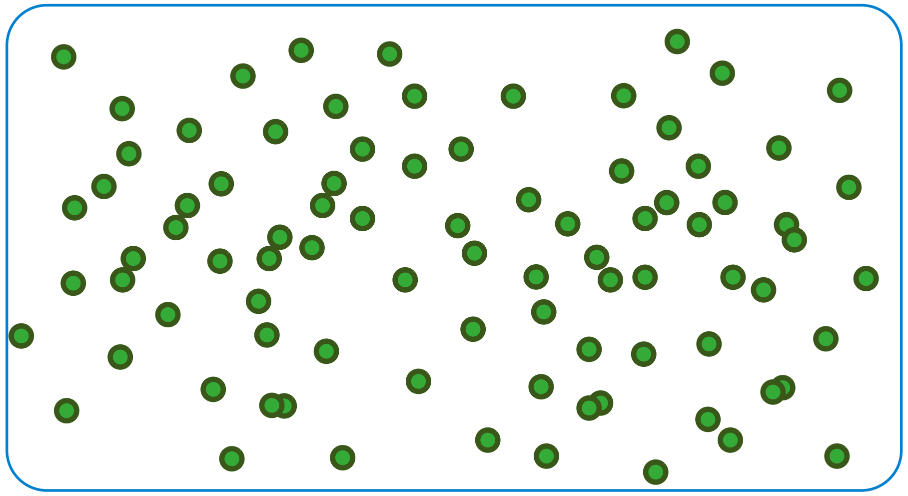
➡ test universality using simple Feshbach two-channel model [w/o fit parameters] using renormalization group methods RS, RATH, ZWERGER, EPJB 85 (2012)

effective action approach

Quantum field theory

definition of theory: **UV scale**

$$S = \int \varphi^* [-i\hbar\partial_t - \frac{\nabla^2}{2m}]\varphi + \int_x g_\Lambda \, (\varphi^*\varphi)^2(x)$$



$$Z[J] = \int D\varphi e^{-S[\varphi] - \int J\varphi}$$
$$= \mathrm{Tr} e^{-\beta \hat{H}}$$

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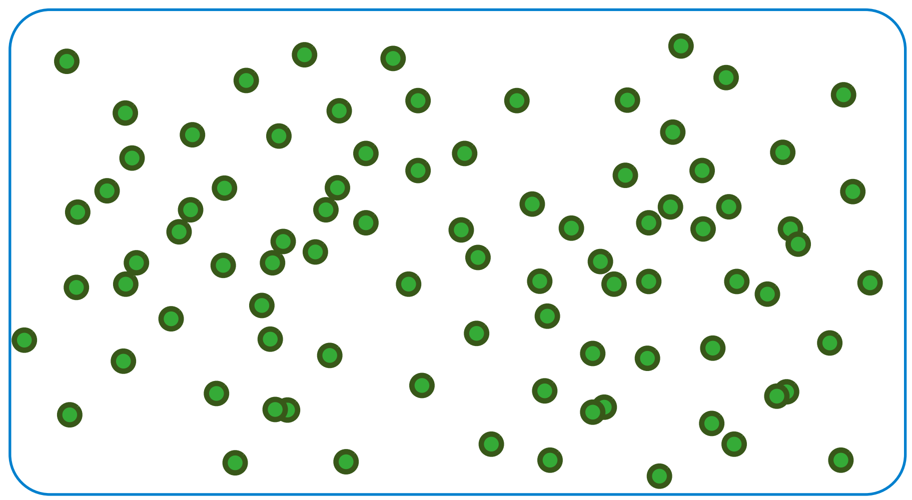


$$Z[J] = \int D\varphi e^{-S[\varphi]-\int J\varphi}$$
$$= \mathrm{Tr} e^{-\beta\hat{H}}$$

experiment: **IR scale**

$$\Gamma[\phi] = \int \phi^*[-i\hbar\partial_t - \frac{\nabla^2}{2m} - \Sigma(\omega,\vec{q})]\phi + \int \Gamma^{(4)}(\{q_j\})(\phi^*\phi)^2 + \ldots$$

effective action - generating functional of 1PI correlation functions

access to: spectral functions, rf response, scattering amplitudes, Fermi liquid parameters...

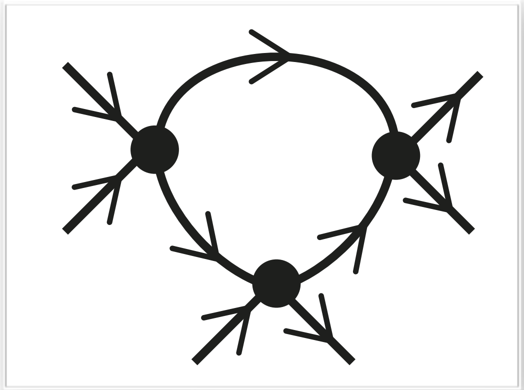
$$f(q) = \frac{1}{-1/a - iq + \ldots}$$

effective action

‚theory‘: **UV**

$$S = \int \varphi^* [-i\hbar\partial_t - \frac{\nabla^2}{2m}]\varphi + \int_x g_\Lambda \, (\varphi^*\varphi)^2(x)$$

problem:
keep track of build up of correlations, e.g.



this talk

‚experiment‘: **IR**

$$\Gamma[\phi] = \int \phi^* [-i\hbar\partial_t - \frac{\nabla^2}{2m} - \Sigma(\omega, \vec{q})]\phi + \int \Gamma^{(4)}(\{q_j\})(\phi^*\phi)^2 + \boxed{\int \lambda_3(E)(\phi^*\phi)^3}$$

$\lambda_3(E)$: three-body scattering amplitude

- poles give bound state spectrum
- relates to hyperspherical wavefunction $f_n(R)$ in momentum space

functional renormalization group

Problem: How to obtain $\Gamma[\phi]$?

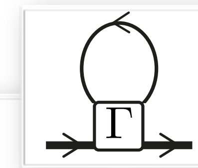
UV $$\Gamma_\Lambda = S = \int \varphi^*[-i\hbar\partial_t - \frac{\nabla^2}{2m}]\varphi + \int_x g_\Lambda\,(\varphi^*\varphi)^2(x)$$

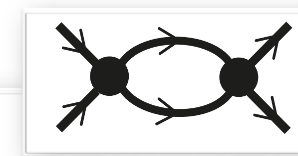
IR $$\Gamma_0 = \Gamma[\phi] = \int \phi^*[-i\hbar\partial_t - \frac{\nabla^2}{2m} - \Sigma(\omega,\vec{q})]\phi + \int \Gamma^{(4)}(\{q_j\})(\phi^*\phi)^2 + \ldots$$

functional renormalization group

Problem: **How to obtain $\Gamma[\phi]$?**

includes successively fluctuations on momentum scales large than k:





functional RG

UV

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Γ_k

define: interpolating effective action

$$\Gamma_k[\phi] = \int \phi^*[-i\hbar\partial_t - \frac{\nabla^2}{2m} - \Sigma_k(\omega,\vec{q})]\phi + \int \Gamma_k^{(4)}(\{q_j\})(\phi^*\phi)^2 + \ldots$$

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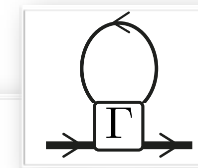

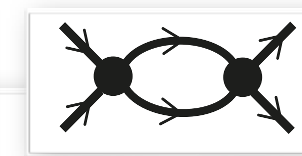

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exact RG equation Wetterich, Phys. Lett. B 301 (1993)

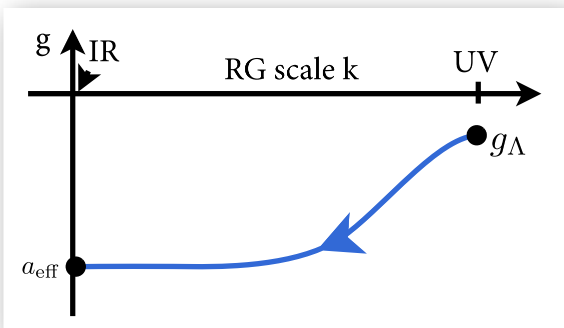
$$\partial_k\Gamma_k[\phi] = \frac{1}{2}\int \frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_k R_k$$

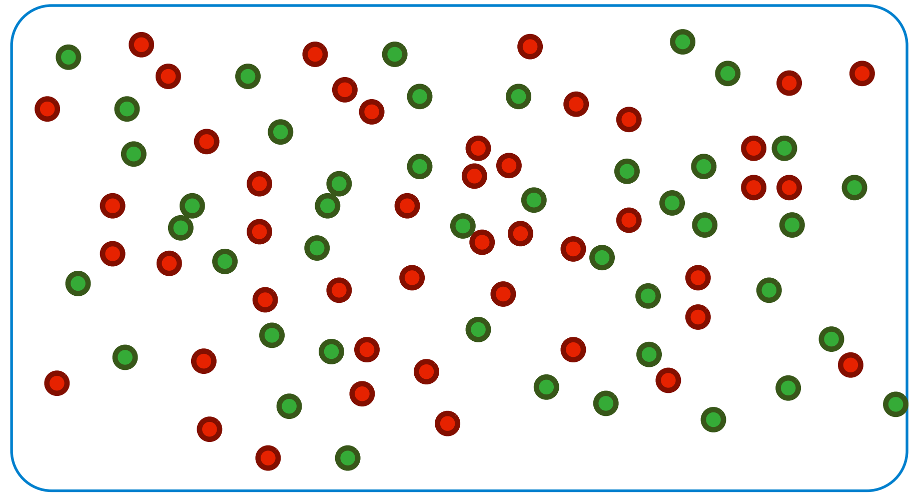
regulator controls inclusion of fluctuations of momenta q>k

build up of correlations: fermions vs. bosons

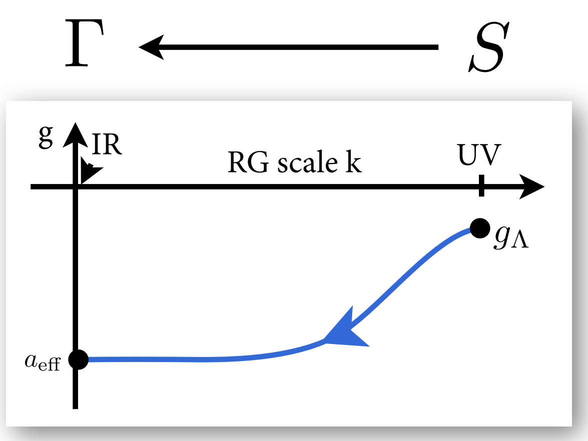
RG flow [illustration]

$\Gamma \longleftarrow S$





build up of correlations: fermions vs. bosons



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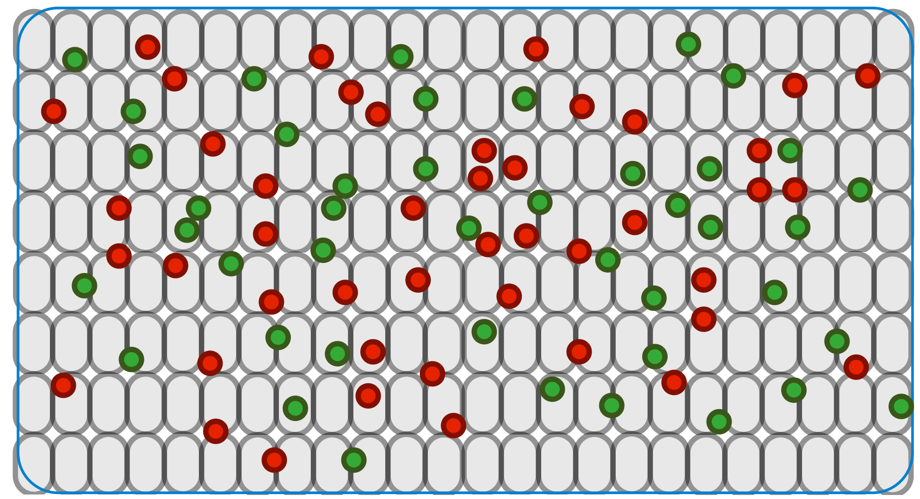
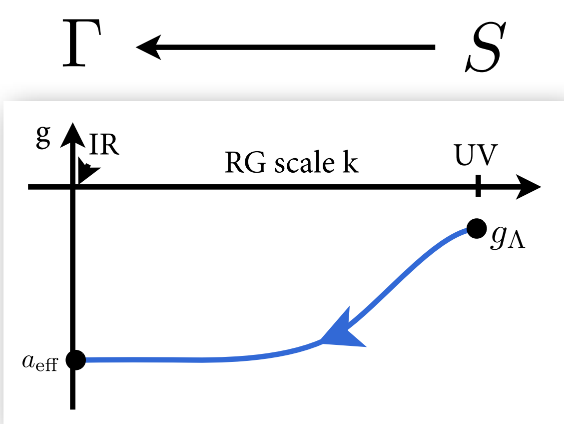


initial RG steps:
determined by few-body physics

build up of correlations: fermions vs. bosons



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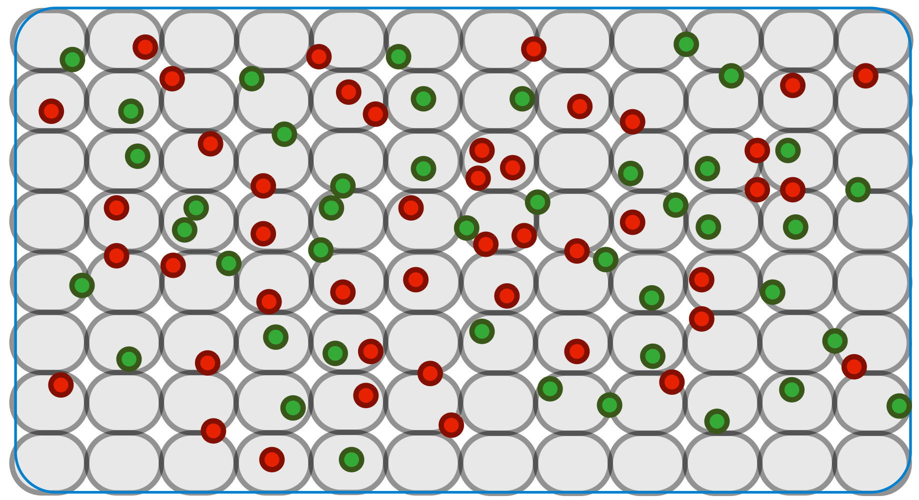

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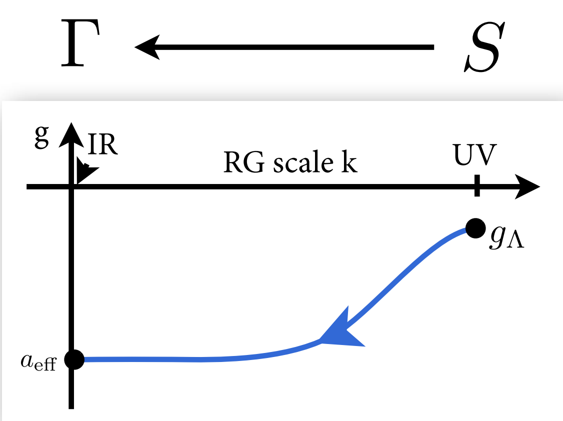

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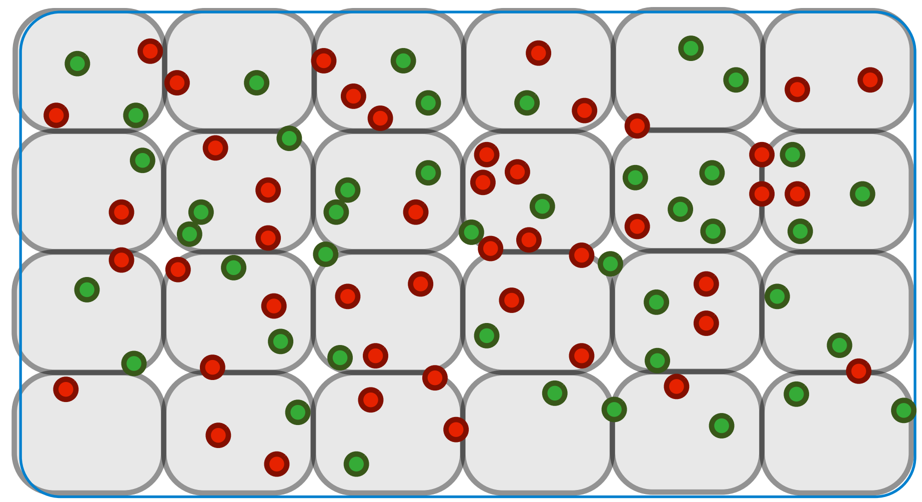

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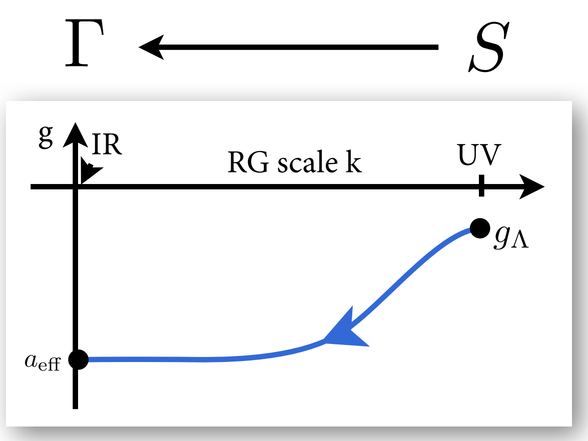
later stage:
realm of many-body, IR physics

build up of correlations: fermions vs. bosons



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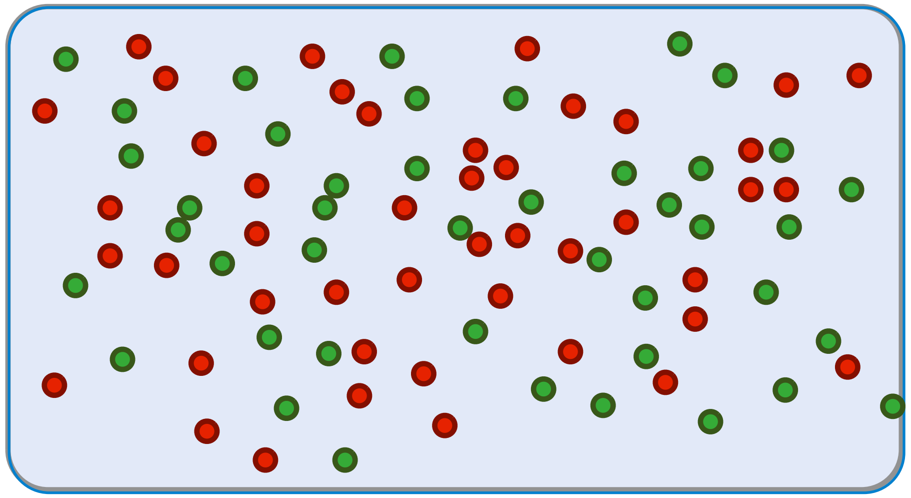

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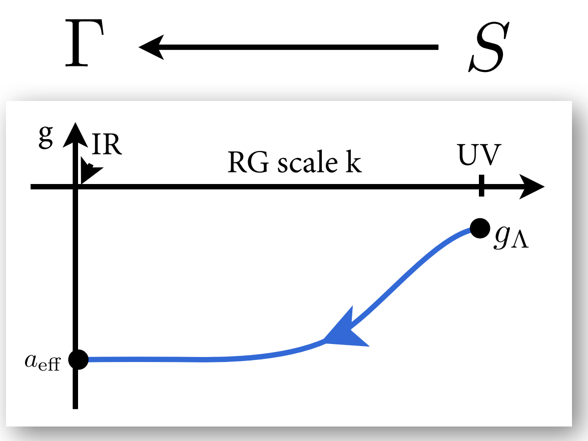

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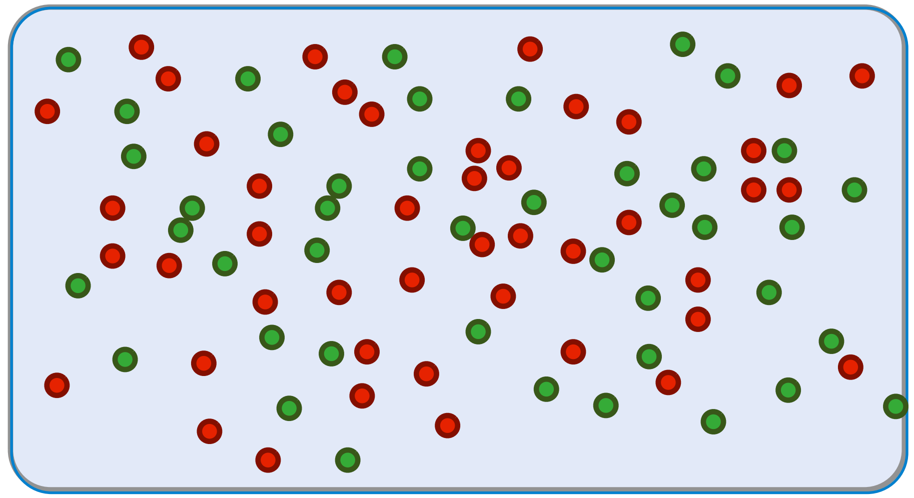

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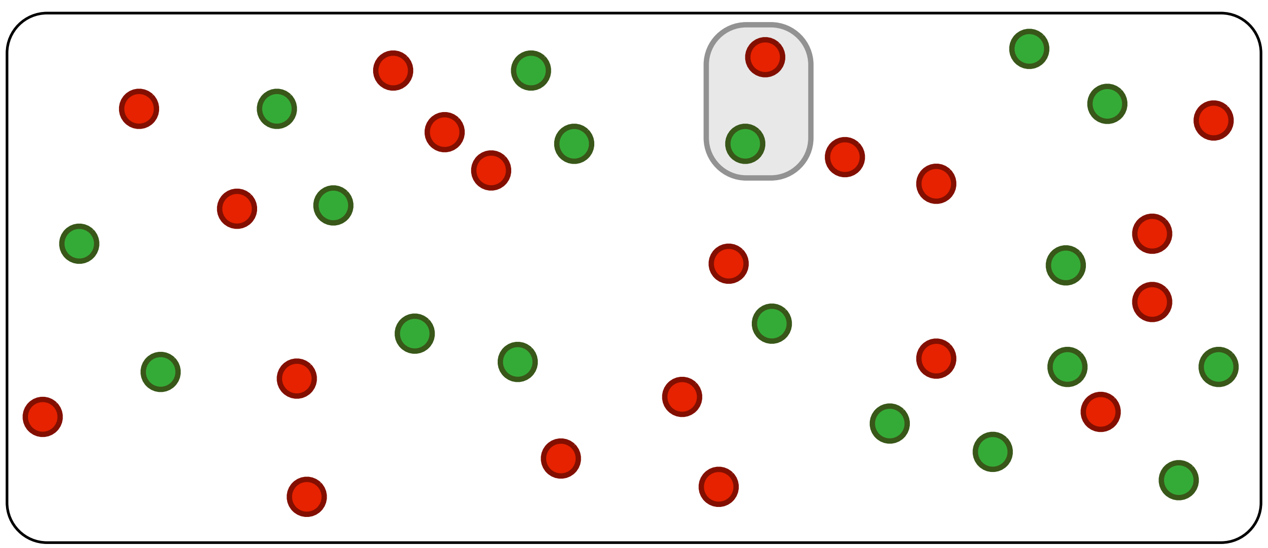
two-component fermions



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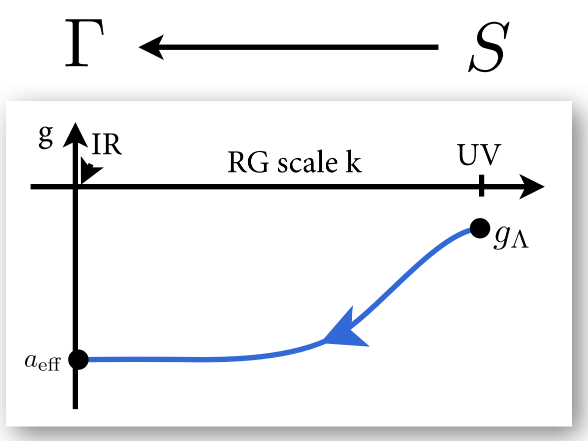

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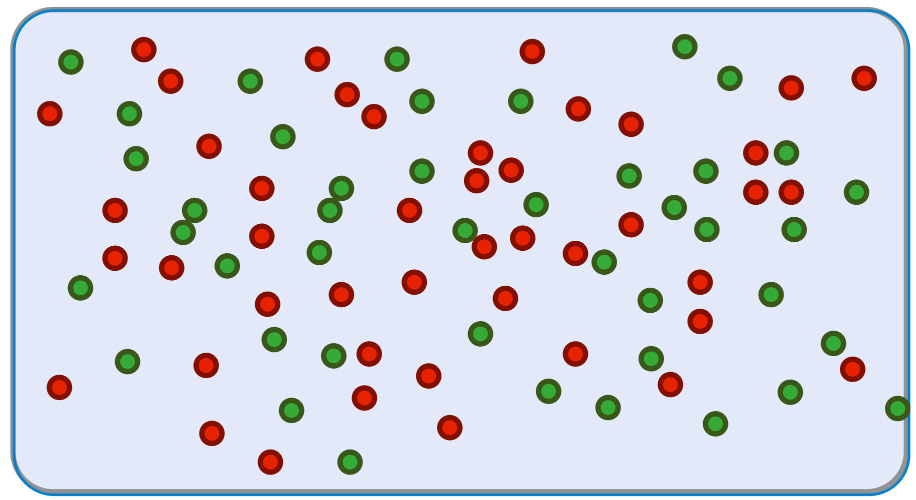

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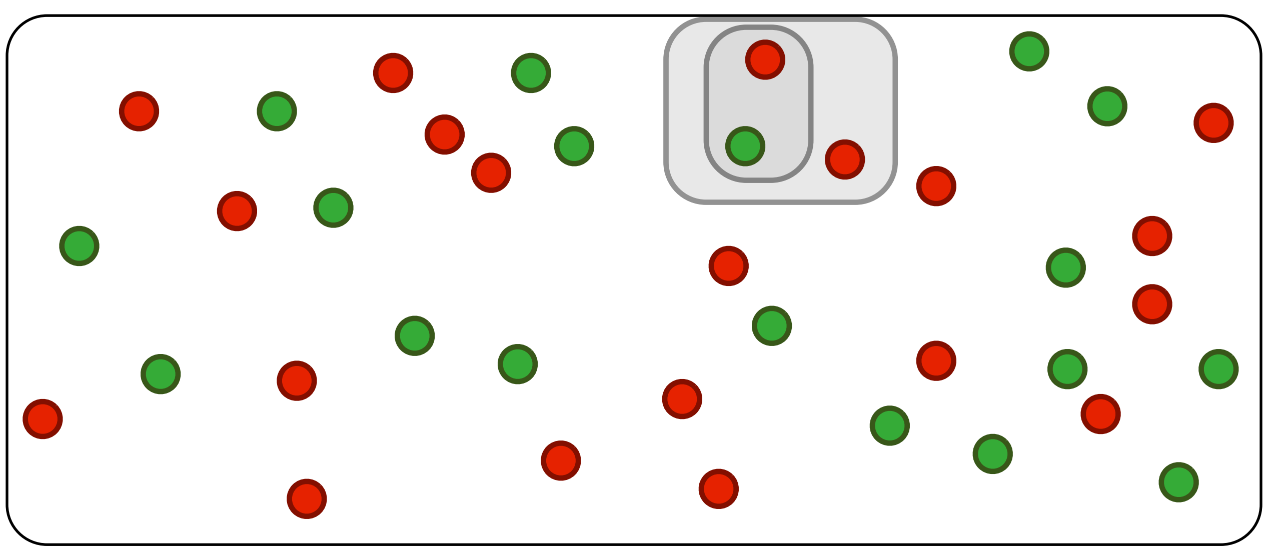
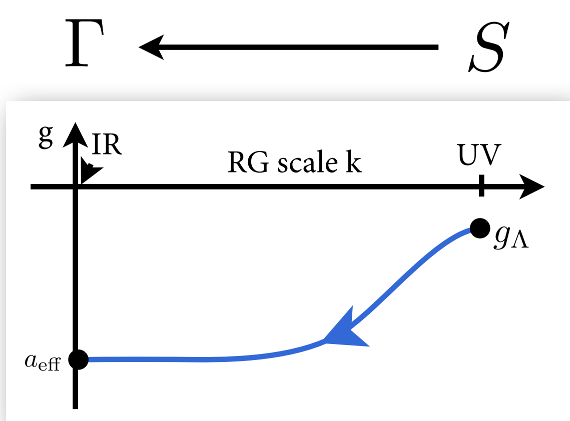


Pauli principle:
three-body correlations suppressed

build up of correlations: fermions vs. bosons



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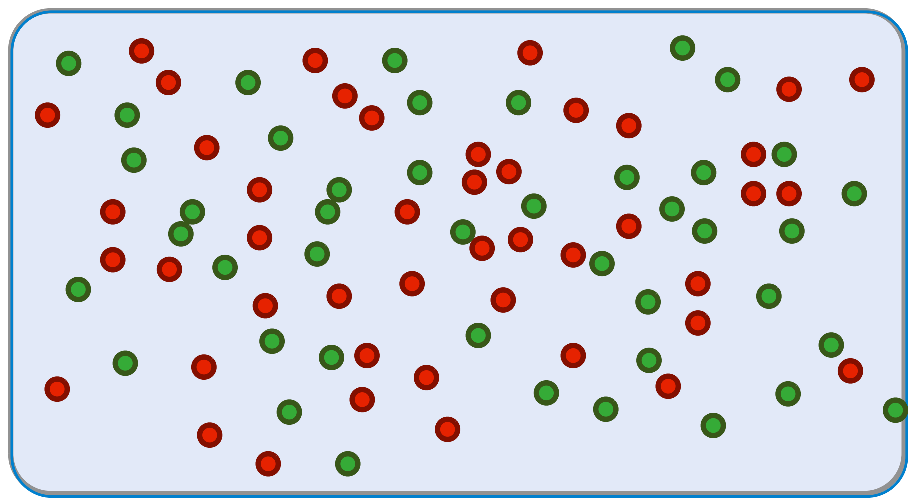

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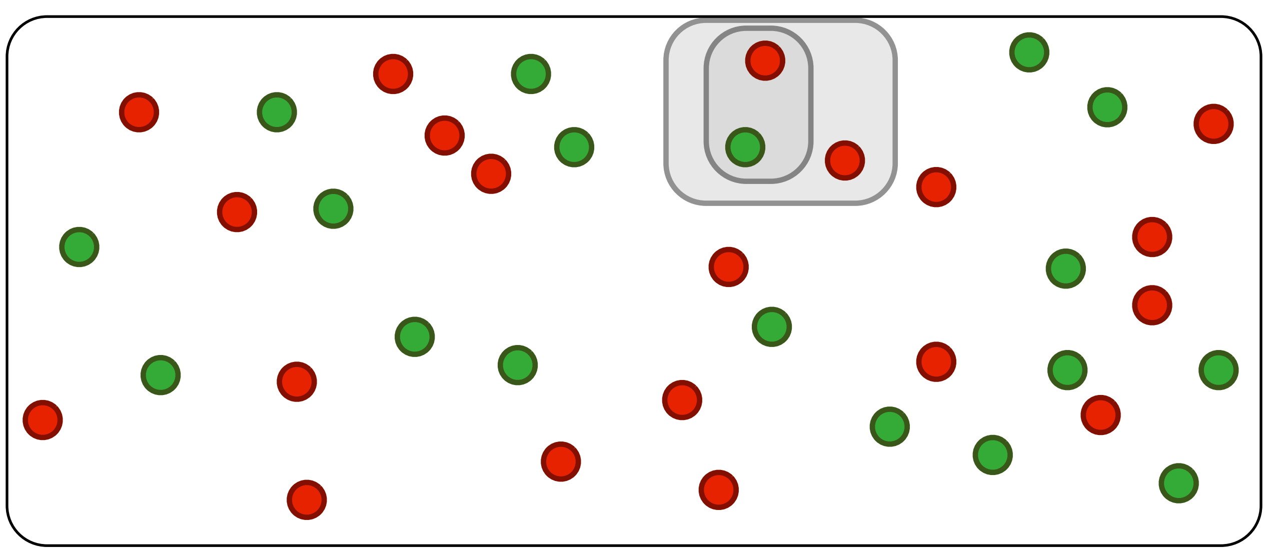
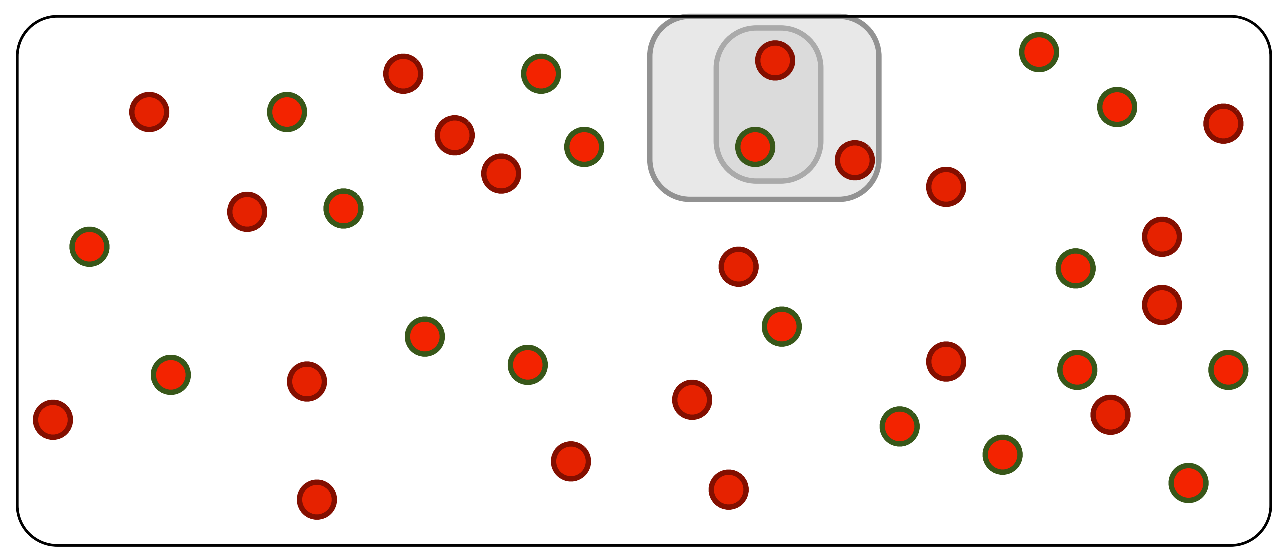


Pauli principle:
three-body correlations suppressed

identical bosons



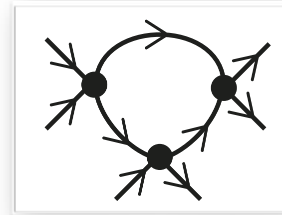
no Pauli principle:
three-body correlations important?

↳ **deep understanding of few-body physics needed for reliable many-body calculation!**

RG flow of the three-body problem

Moroz, Floerchinger, RS, Wetterich PRA 79 (2009)
Moroz, RS, Ann. phys. 325 (2010)
Review: Floerchinger, Moroz, RS, Few-Body. Sys. 51 (2011)

$$S \sim \int g_\Lambda (\varphi^* \varphi)^2 \longrightarrow \Gamma_k \sim \int g_k (\phi^* \phi)^2 + \lambda_3 (\phi^* \phi)^3$$



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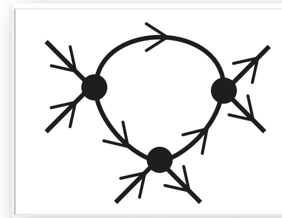

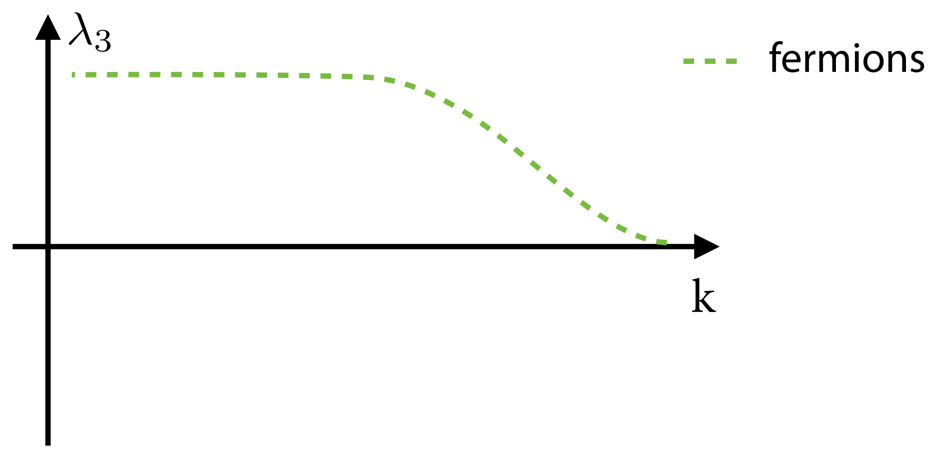

fermions: fixed point

↳ scale invariance preserved

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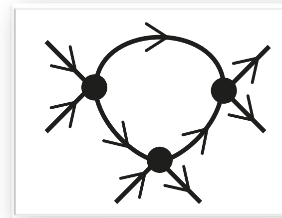
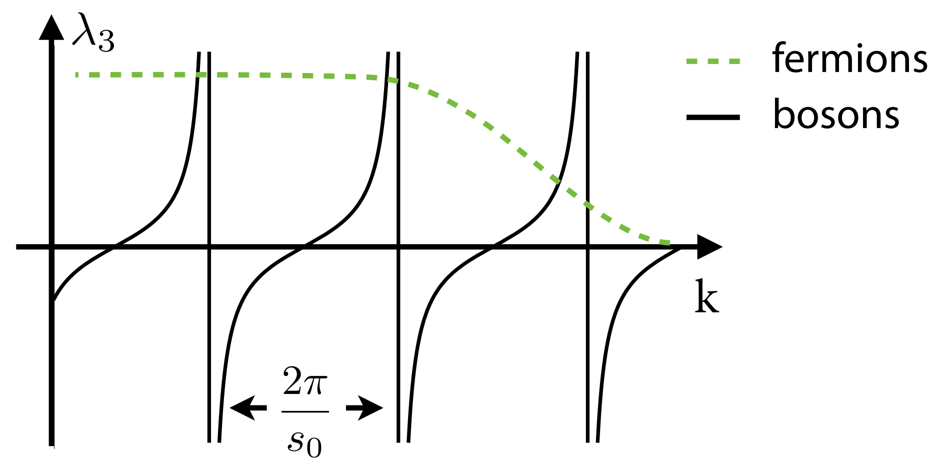
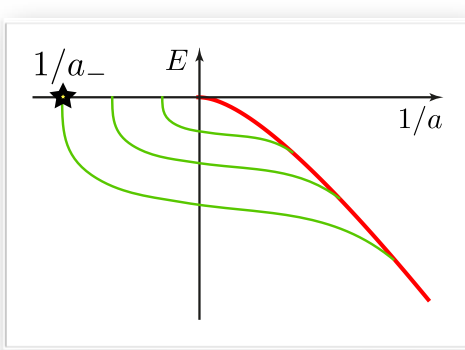




bosons: Limit cycle!

divergencies $\lambda_3(k^*) = \infty$ signal bound states

↳ scale invariance broken



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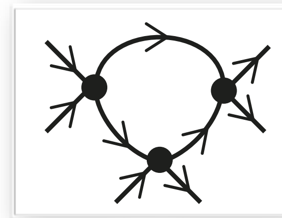

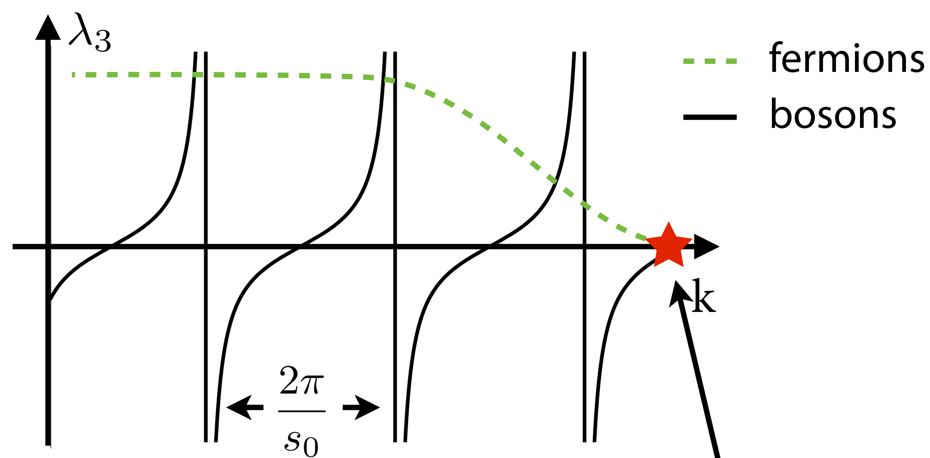

three-body parameter
large momentum regularization: Λ_*

bosons: Limit cycle!

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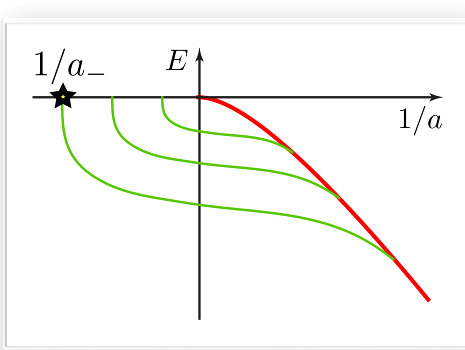

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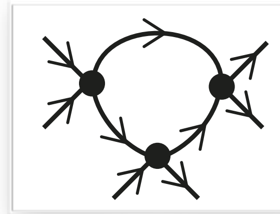


Back to the question:

$$a_- \approx -9.2\, l_{\mathrm{vdw}}$$

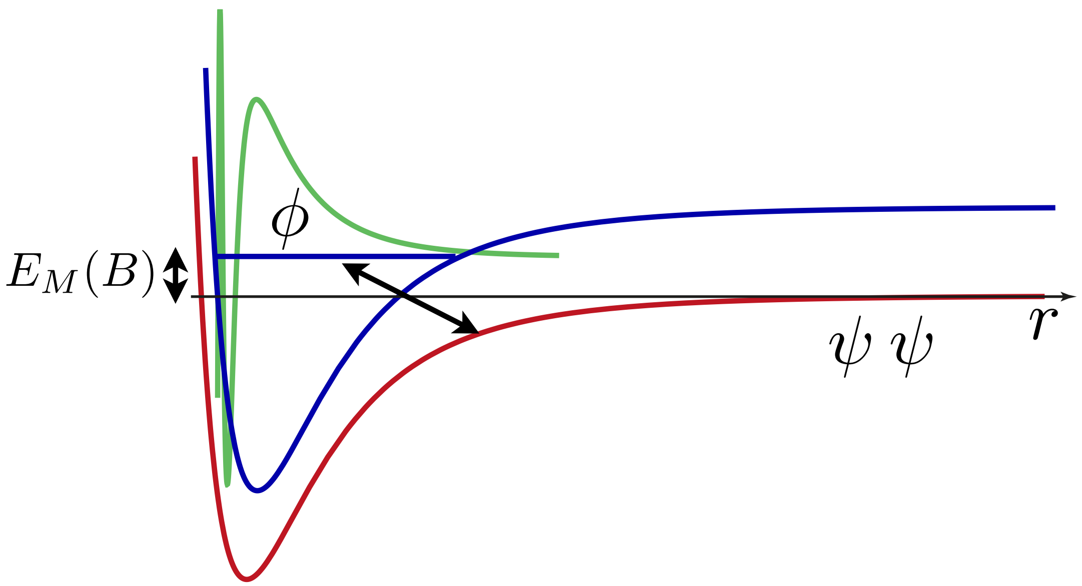
why does the three-body parameter appear to be *universal*?

Two-channel model

extend the „standard two-channel model“ to finite range SIMILAR: MASSIGNAN `08, PRICOUPENKO `10, JONA-LASINIO `10

SEE ALSO: ZINNER ET AL. PRA 86 (2012)

- “minimal model”: still traceable for many-body calculation



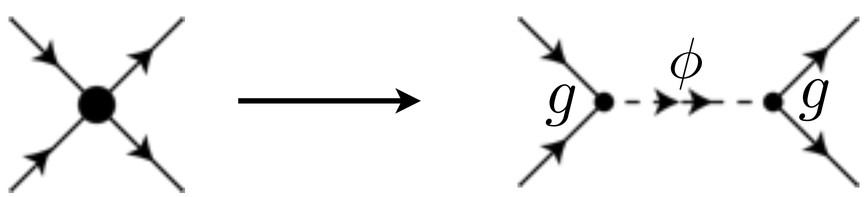
$$S_{\psi,\mathrm{kin}} = \int \psi^*[i\partial_t - \nabla^2]\psi$$

$$S_{\phi,\mathrm{kin}} = \int \phi^*[i\partial_t - \nabla^2/2 + E_M(B)]\phi$$

$E_M(B)$: $\mu\,(B - B_{\mathrm{res}})$

$$S_{\mathrm{int}} = \int g(\mathbf{r}_2 - \mathbf{r}_1)\phi(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}, t)\psi^*(\mathbf{r}_1, t)\psi^*(\mathbf{r}_2, t)$$

- atom-atom interaction *solely* due to exchange of closed-channel molecule



Two-channel model

extend the „standard two-channel model" to finite range Similar: Massignan ` 08, Pricoupenko ` 10, Jona-Lasinio ` 10

See also: Zinner et al. PRA 86 (2012)

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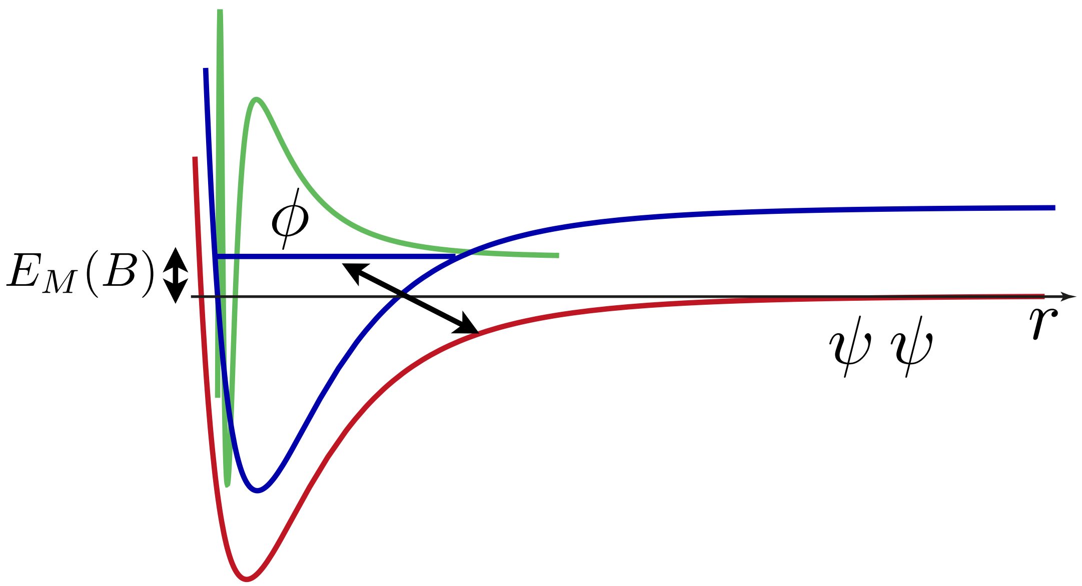

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$g(r)$ **: conversion coupling**

in EFT: usually zero-range model

$$g(r_2 - r_1) = g\,\delta(r_2 - r_1)$$

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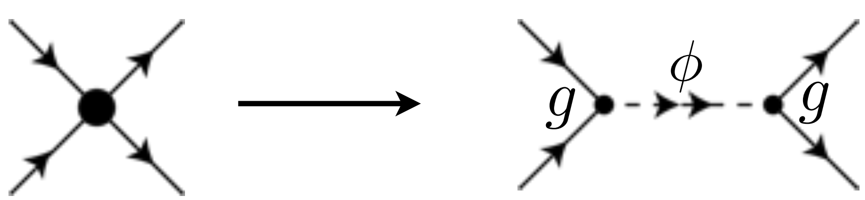

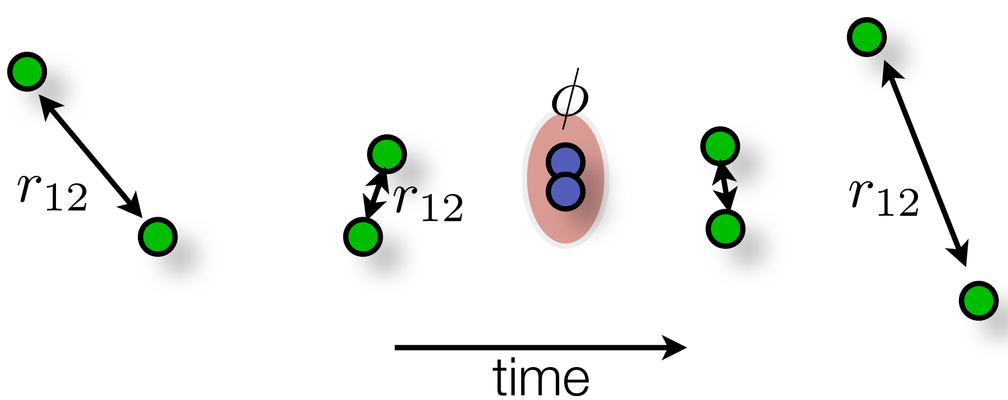

Tuesday, March 25, 14

Two-channel model

extend the „standard two-channel model“ to finite range SIMILAR: MASSIGNAN `08, PRICOUPENKO `10, JONA-LASINIO `10

SEE ALSO: ZINNER ET AL. PRA 86 (2012)

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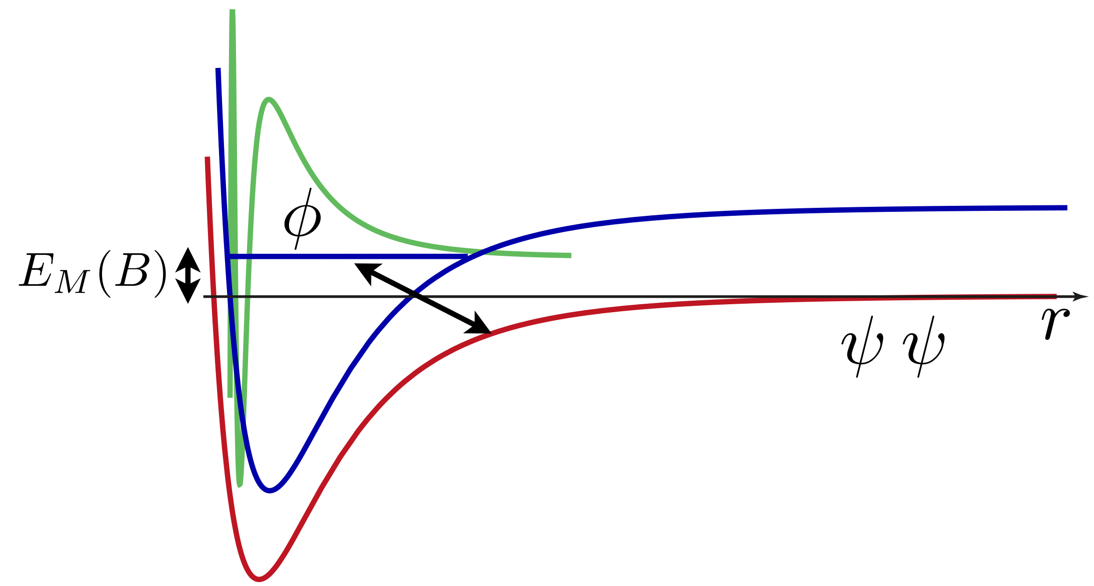

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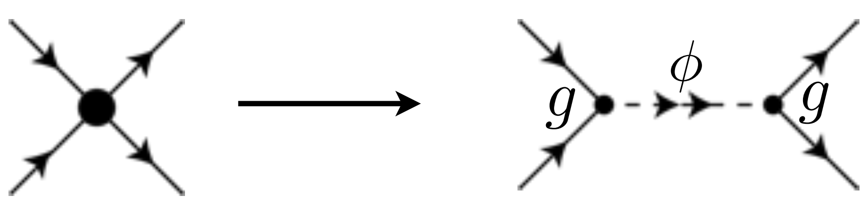

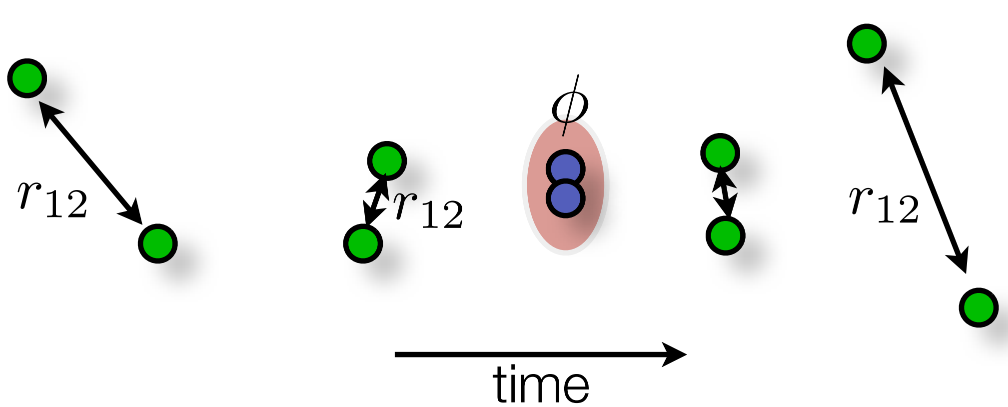

$g(r)$ **: conversion coupling**

in EFT: usually zero-range model

$$g(r_2 - r_1) = g\,\delta(r_2 - r_1)$$

here: finite range, due to Frank-Condon overlap

$$g(r) = g\,\underbrace{e^{-r/\sigma}/r}_{\chi(r)}$$

Two-channel model

extend the „standard two-channel model“ to finite range SIMILAR: MASSIGNAN `08, PRICOUPENKO `10, JONA-LASINIO `10

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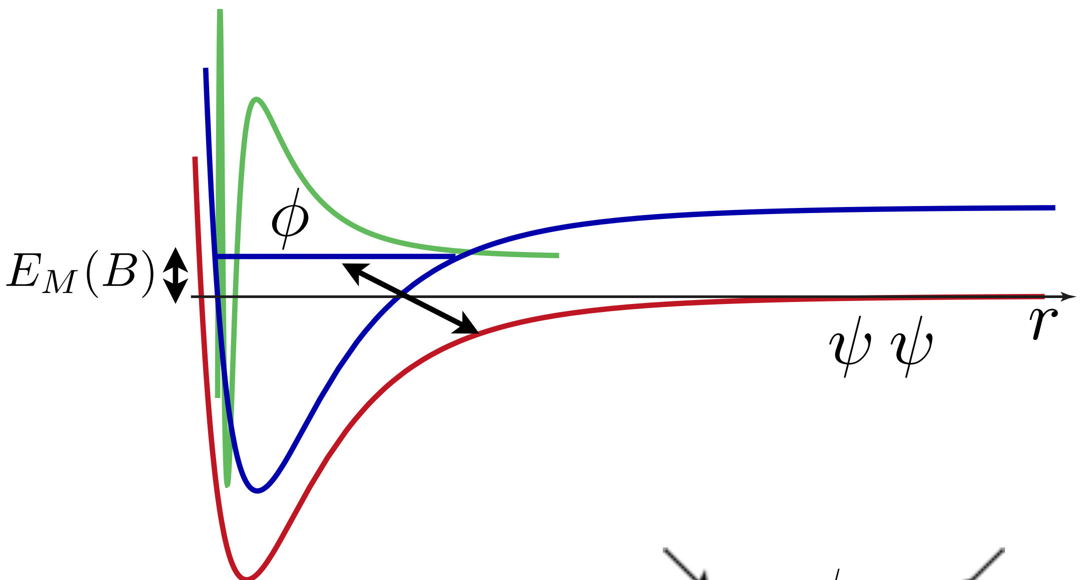
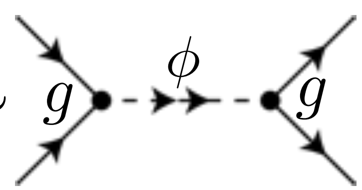


$a(B) \sim$ 

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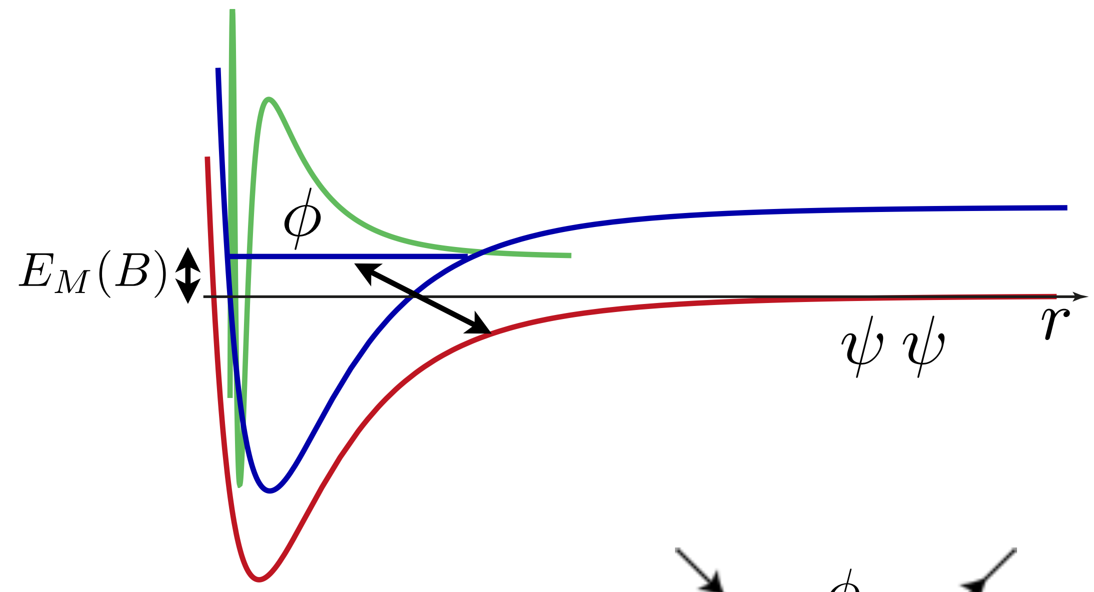

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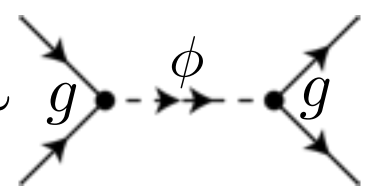
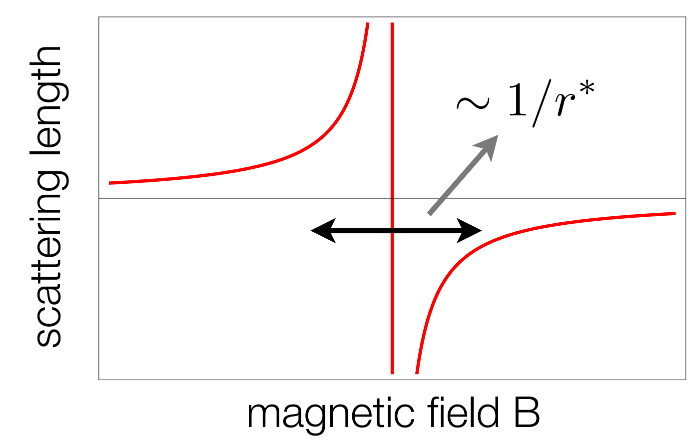


parameters from two-body physics

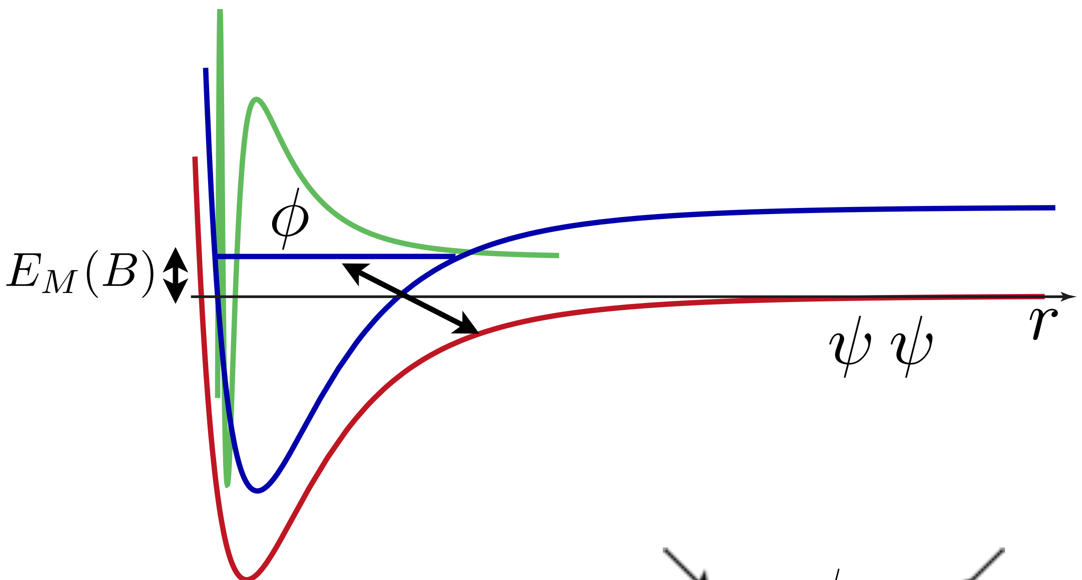
Feshbach resonances



$$a(B) = -\frac{\hbar^2}{r^* \mu (B - B_0)}$$

Two-channel model

extend the „standard two-channel model“ to finite range SIMILAR: MASSIGNAN `08, PRICOUPENKO `10, JONA-LASINIO `10
SEE ALSO: ZINNER ET AL. PRA 86 (2012)



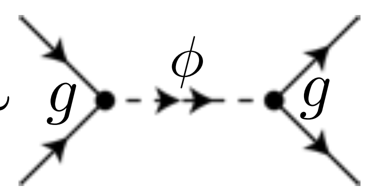
$$S_{\psi,\mathrm{kin}} = \int \psi^*[i\partial_t - \nabla^2]\psi$$

$\mu\,(B - B_{\mathrm{res}})$

$$S_{\phi,\mathrm{kin}} = \int \phi^*[i\partial_t - \nabla^2/2 + E_M(B)]\phi$$

$$S_{\mathrm{int}} = \int g(\mathbf{r}_2 - \mathbf{r}_1)\phi(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, t)\psi^*(\mathbf{r}_1, t)\psi^*(\mathbf{r}_2, t)$$

$a(B) \sim$

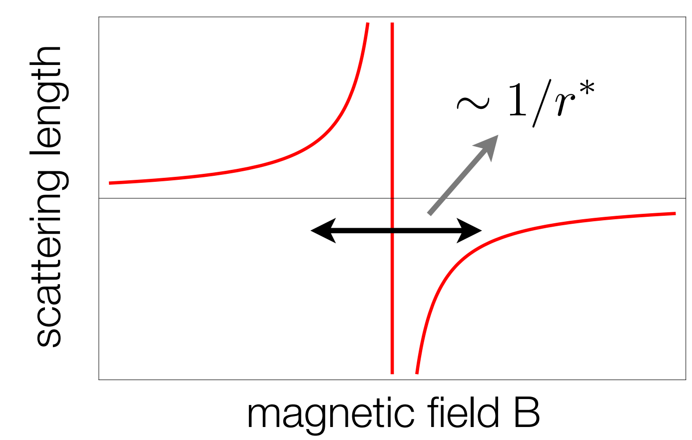


$$g(r) = g\, e^{-r/\sigma}/r$$

parameters from two-body physics

1 g from „width of Feshbach resonance“ r^*

Feshbach resonances



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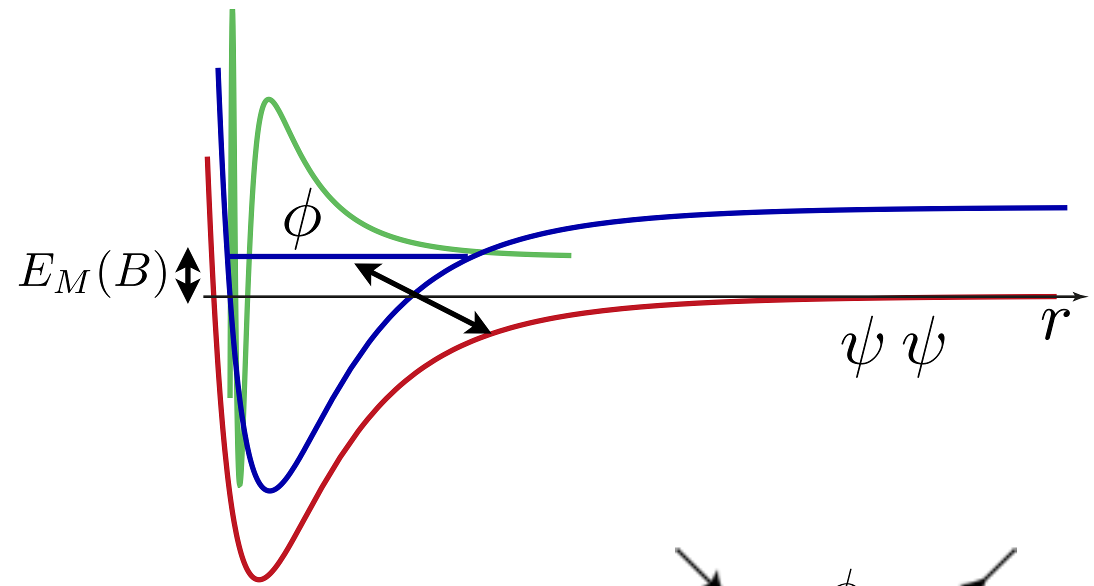

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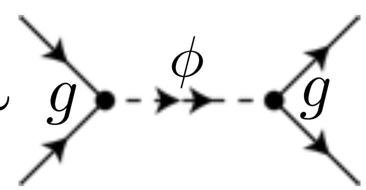
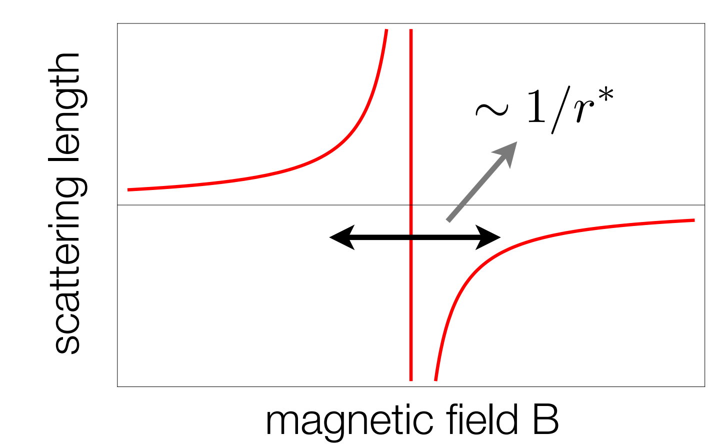
$a(B) \sim$ 

parameters from two-body physics

1. g from „width of Feshbach resonance" r^*
2. B_{res} from Feshbach resonance position B_0

Feshbach resonances



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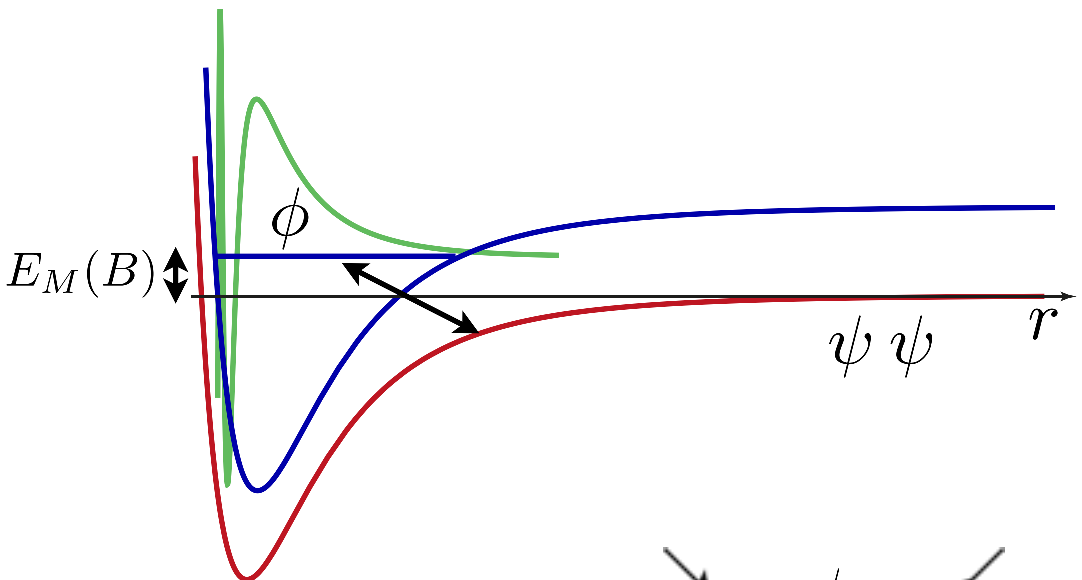
Two-channel model

extend the „standard two-channel model“ to finite range SIMILAR: MASSIGNAN ` 08, PRICOUPENKO ` 10, JONA-LASINIO ` 10

SEE ALSO: ZINNER ET AL. PRA 86 (2012)



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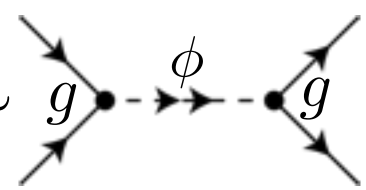
$a(B) \sim$ 

parameters from two-body physics

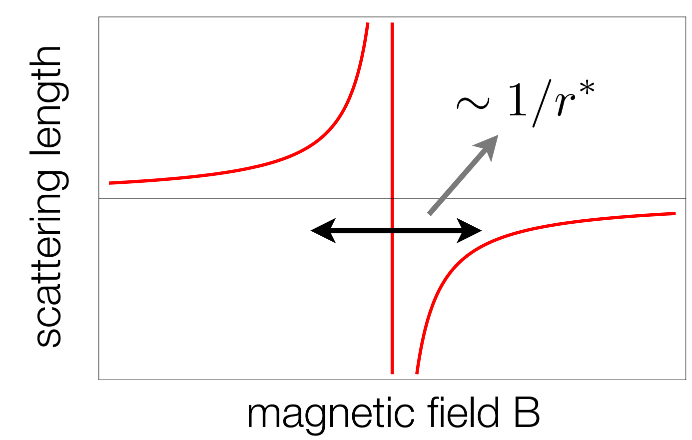
1. g from „width of Feshbach resonance“ r^*
2. B_{res} from Feshbach resonance position B_0
3. σ determined from QdT calc. of resonance shift

$$\sigma = \bar{a} = 0.95\, l_{\mathrm{vdw}}$$

GORAL ET AL., JPB 37 (2004)

➡ all model parameters are fixed, **no fit parameter**

Feshbach resonances



$$a(B) = -\frac{\hbar^2}{r^* \mu\,(B - B_0)}$$

Feshbach resonance strength

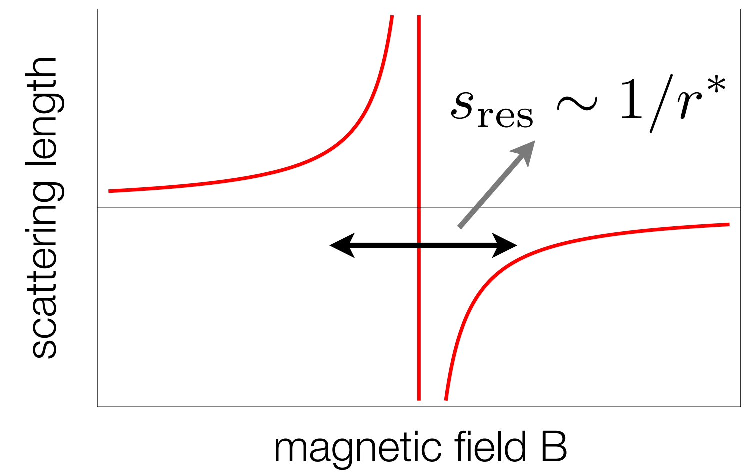
Strength of Feshbach resonance

$$s_{\rm res} = \bar{a}/r^* \sim g^2$$

$$\bar{a} = 0.95\, l_{\rm vdw}$$

$s_{\rm res},\ g^2 \ll 1$: *closed-channel dominated resonance*, *'narrow'*

Feshbach resonances



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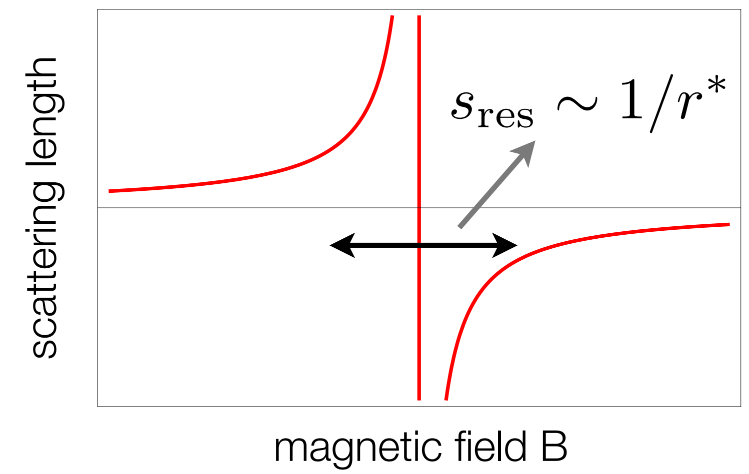
↳ up to now most experiments in this limit

Feshbach resonances



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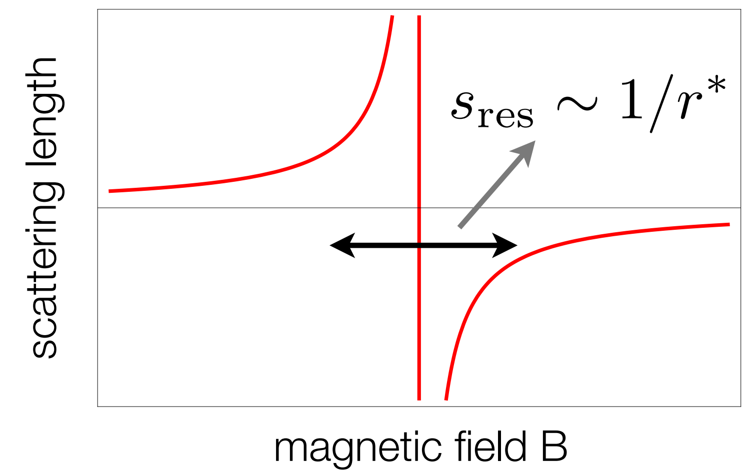
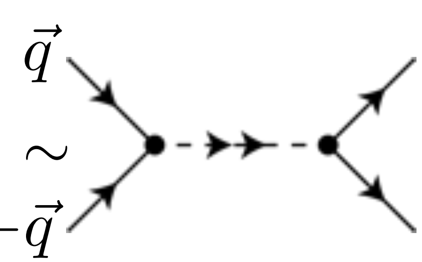
Feshbach resonances



$$a(B) = -\frac{\hbar^2}{r^* \mu\,(B - B_0)}$$

$a(B) \sim$  $\sim \dfrac{g^2}{\mathcal{P}_\phi}$

$$\mathcal{P}_\phi(E, \mathbf{q}) = -E + \mathbf{q}^2/2 + E_M(B) - \frac{g^2/(32\pi)}{\sigma\left[1 + \sigma\sqrt{-\frac{E}{2} + \frac{\mathbf{q}^2}{4} - i\epsilon}\right]^2}$$

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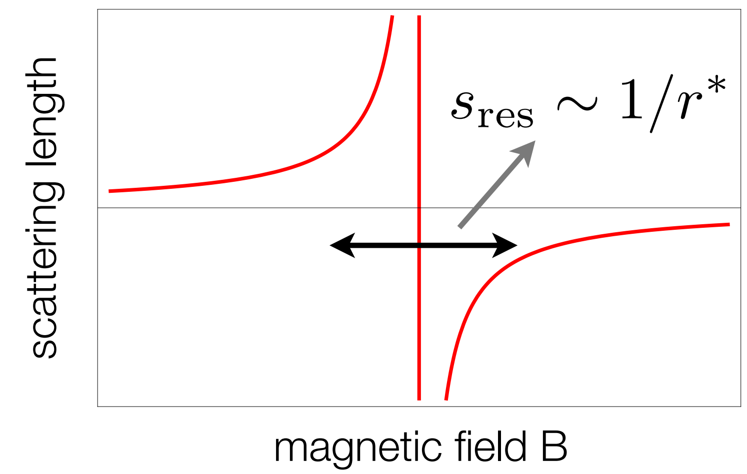
Feshbach resonances



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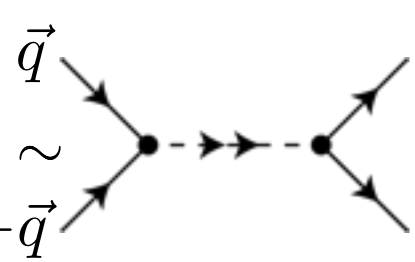
$a(B) \sim$



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closed channel ($-E + \mathbf{q}^2/2 + E_M(B)$)

open channel / quantum (the second term)

truncation for exact solution with RG

systematic vertex expansion

SIMILAR FRG FOR ZERO-RANGE: MOROZ, FLOERCHINGER, RS, WETTERICH, PRA 79 (2009)

- includes all possible correlations generated in three-body problem

$$\Gamma_k = \sum_{n=0}^{\infty} \Gamma_k(n) = \Gamma_k(2) + \Gamma_k(3) + \Gamma_k(4) + ...,$$

$$\Gamma_k(2) = \int \psi^*[i\partial_t - \Delta]\psi + \int \phi^* \overbrace{[i\partial_t - \Delta - E_M(B) + \Sigma_{\phi,k}(\partial_t, \Delta)]}^{1/\mathcal{G}_\phi(\partial_t, \nabla^2)}\phi$$

$$\Gamma_k(3) = g\int \chi(\mathbf{r}_2 - \mathbf{r}_1)\left[\phi(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, t)\psi^*(\mathbf{r}_1, t)\psi^*(\mathbf{r}_2, t) + h.c.\right]$$

$$\Gamma_k(4) = -\int \lambda_k^{(3)}(Q_1, Q_2, Q_3)\phi(Q_1)\psi(Q_2)\phi^*(Q_3)\psi^*(Q_4)\delta(Q_1 + Q_2 - Q_3 - Q_4)$$

↳ atom-dimer scattering vertex, mediates three-body scattering

truncation for exact solution with RG

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$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \int \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_k R_k$$

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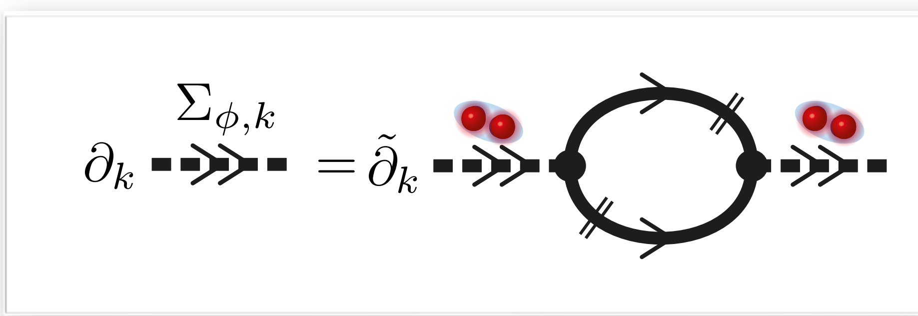
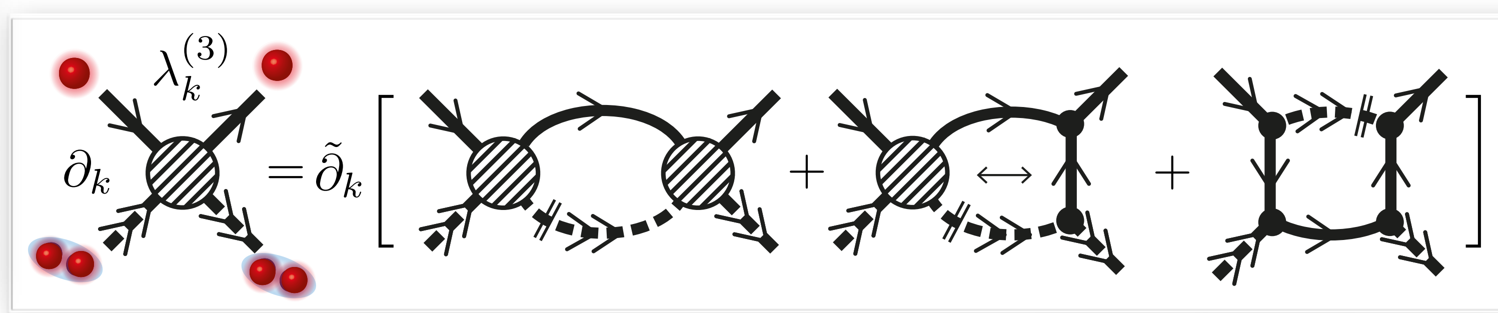
rg flow equations



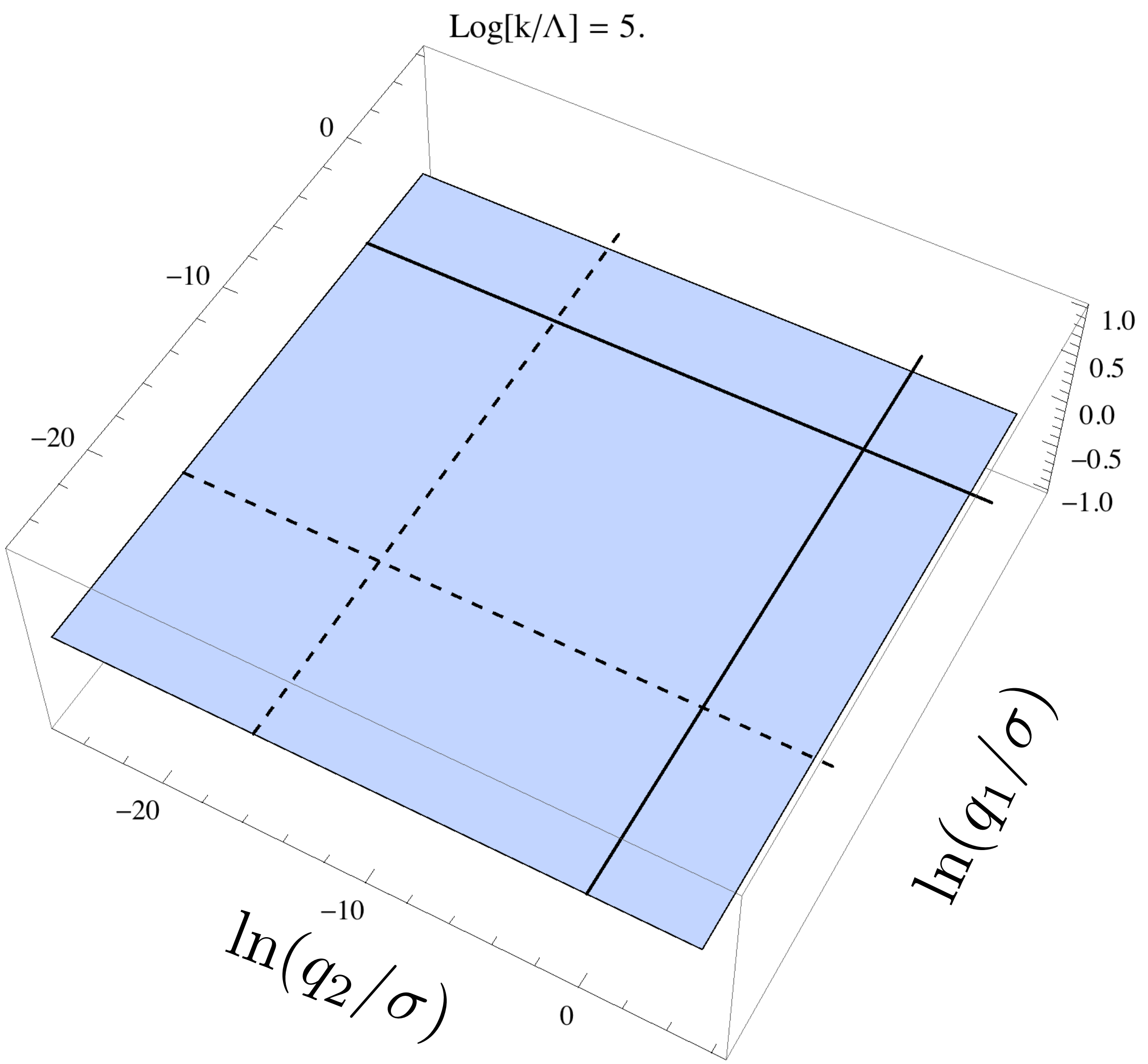


rg scheme chosen yields

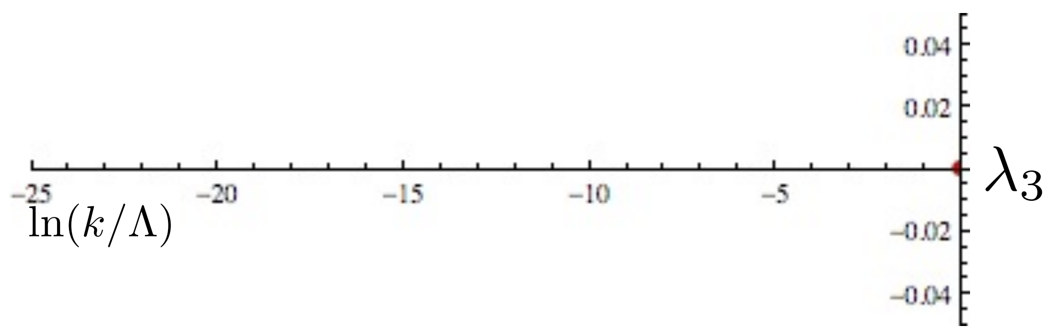
- exact solution of three-body flow equations
  - ↳ IR: Lippmann-Schwinger and modified STM equation

RG flow - limit cycle

exact RG flow of $\lambda_k^{(3)}(q_1, q_2; E)$ SIMILAR FRG FOR ZERO-RANGE: MOROZ, FLOERCHINGER, RS, WETTERICH, PRA 79 (2009)



RG flow - gradient expansion



FLOERCHINGER, RS, MOROZ, WETTERICH, PRA 79 (2009)

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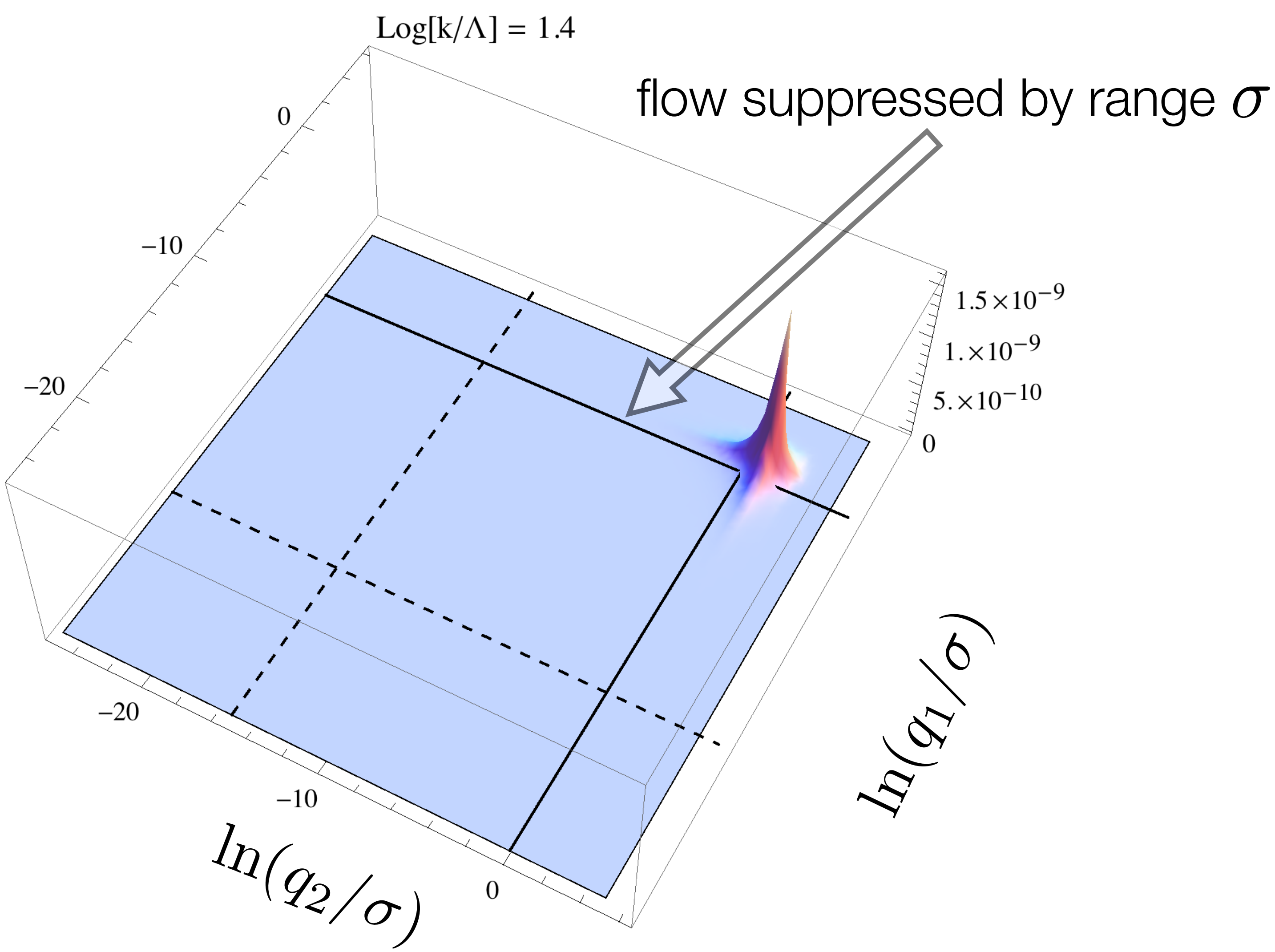

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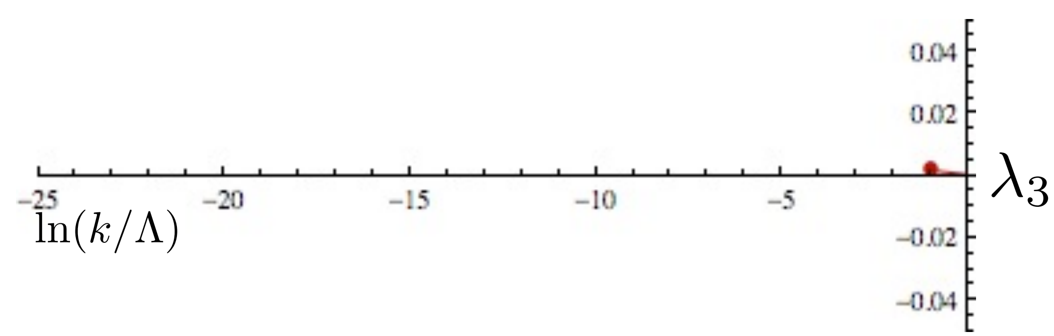

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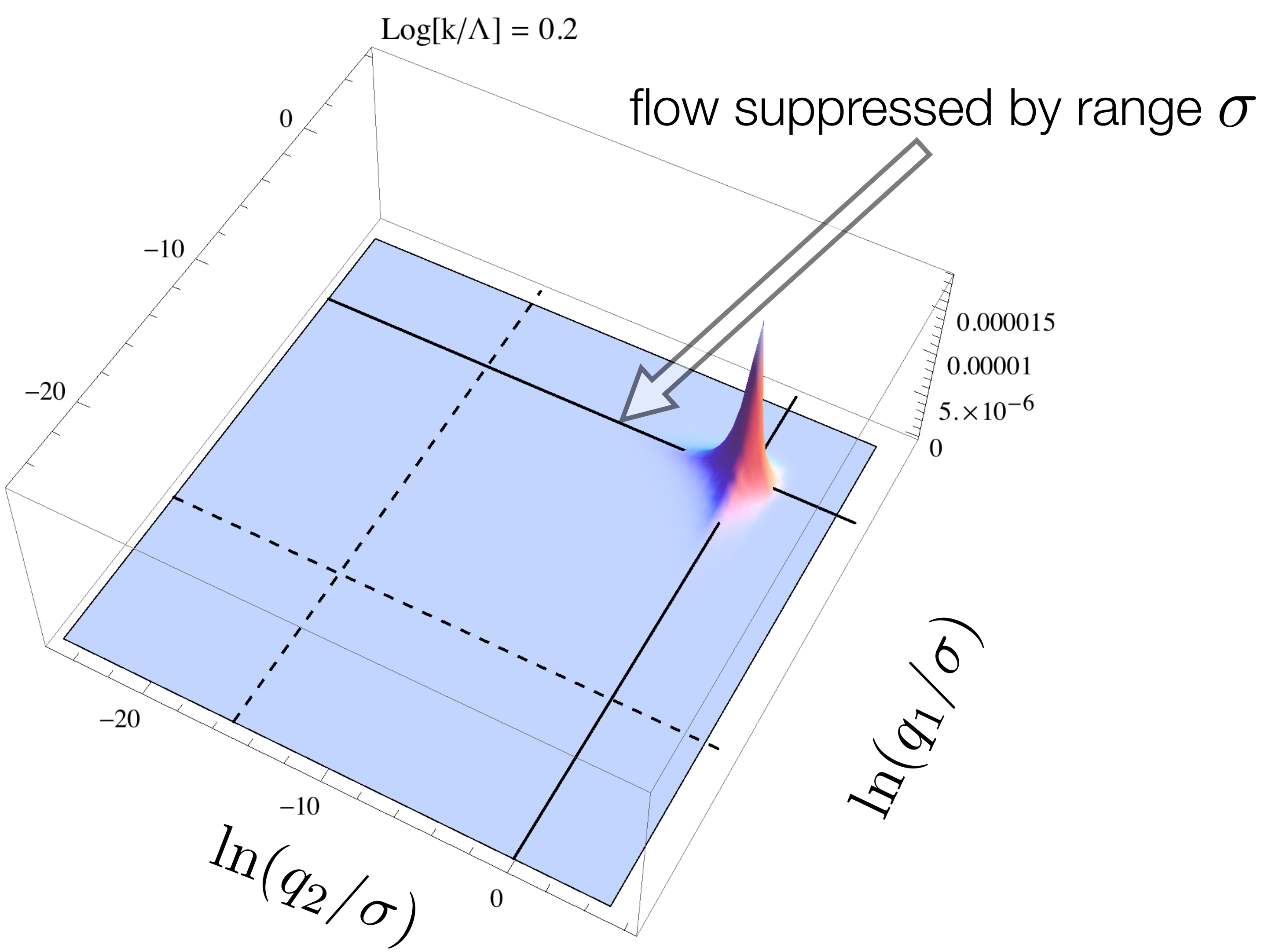

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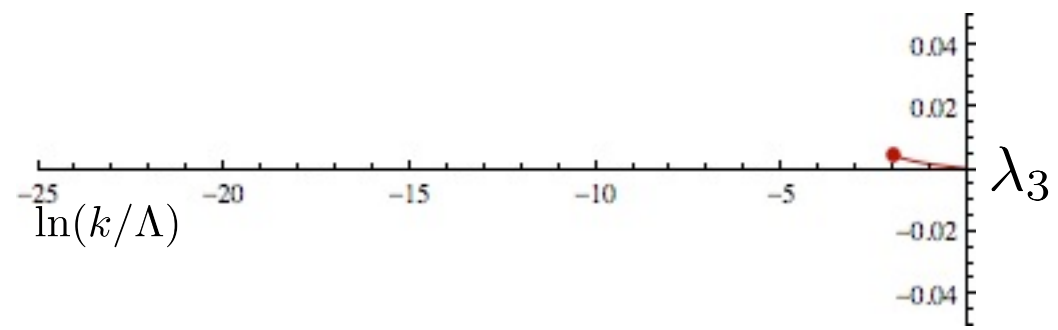

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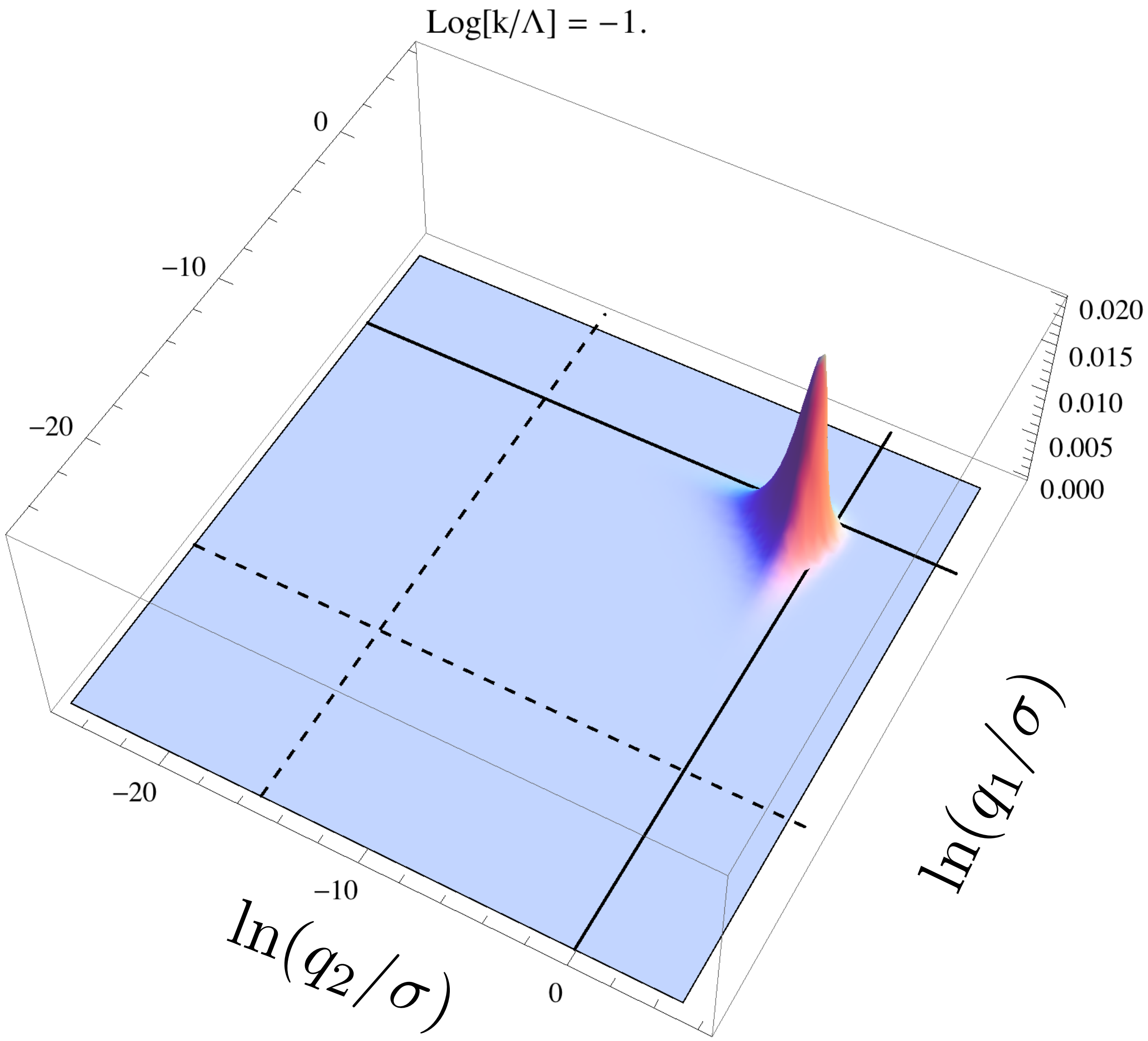

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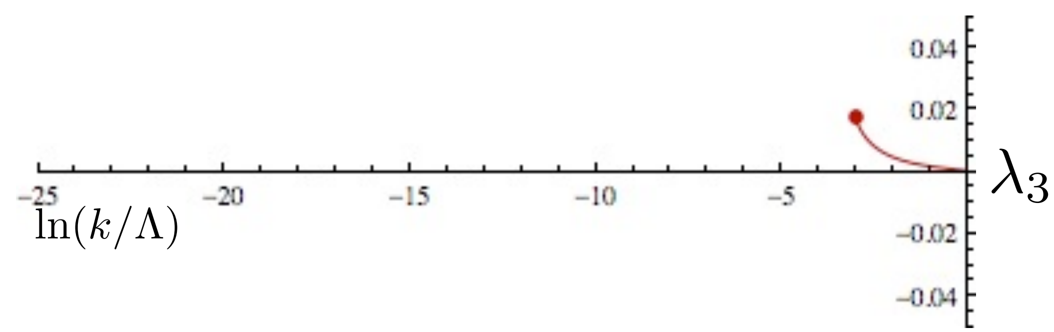

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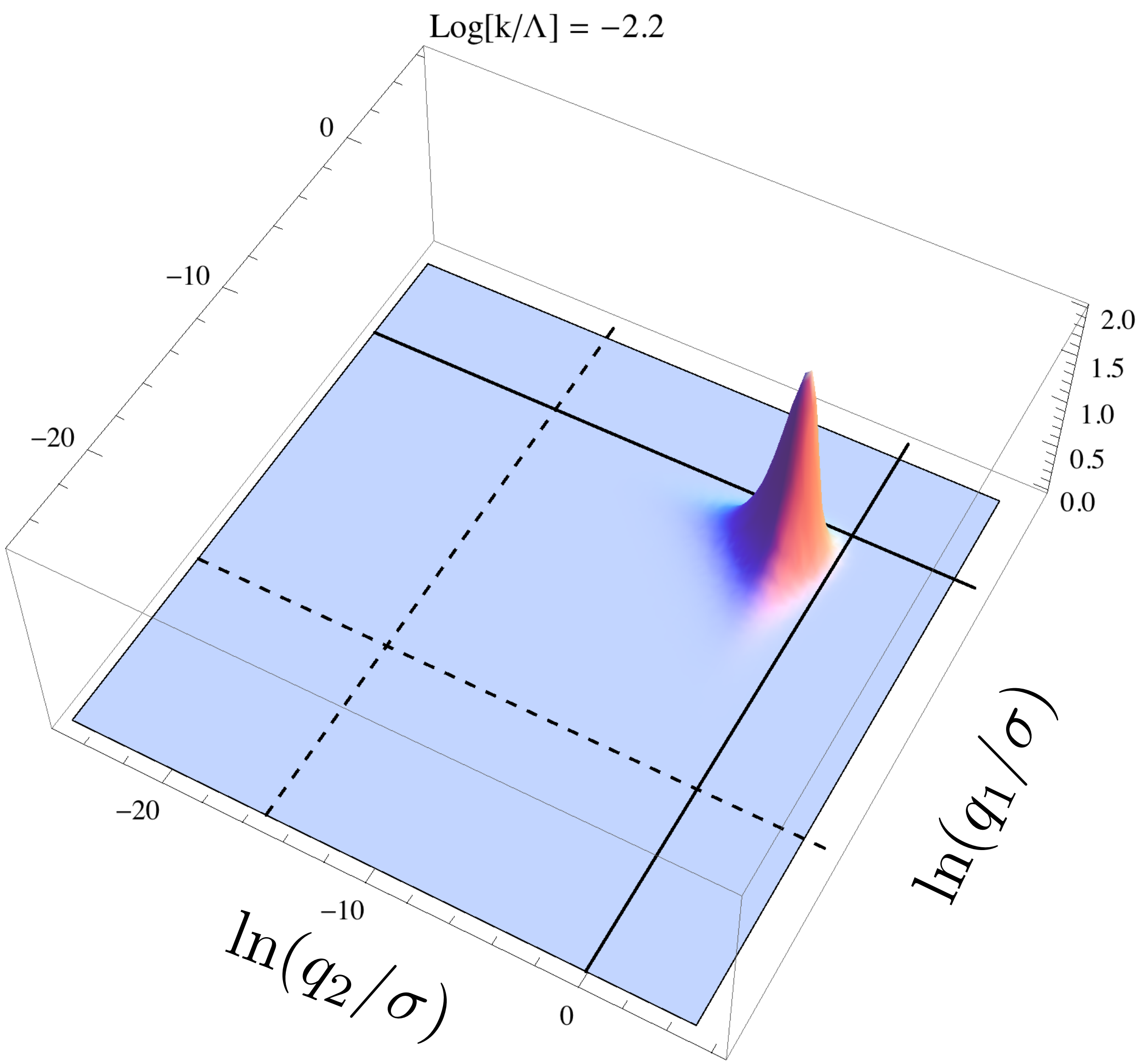

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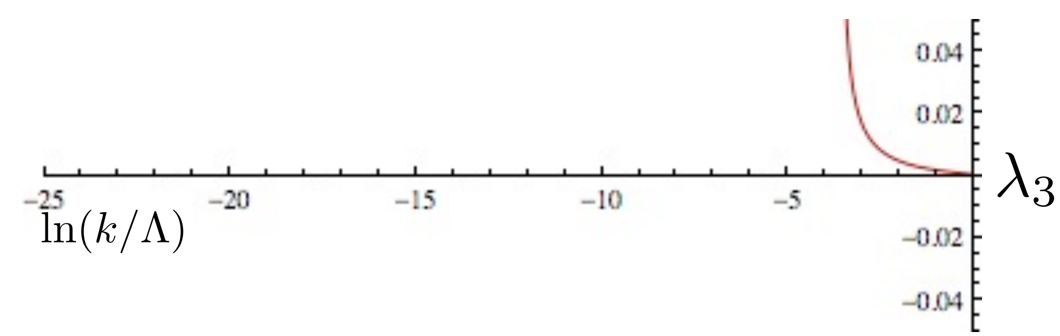

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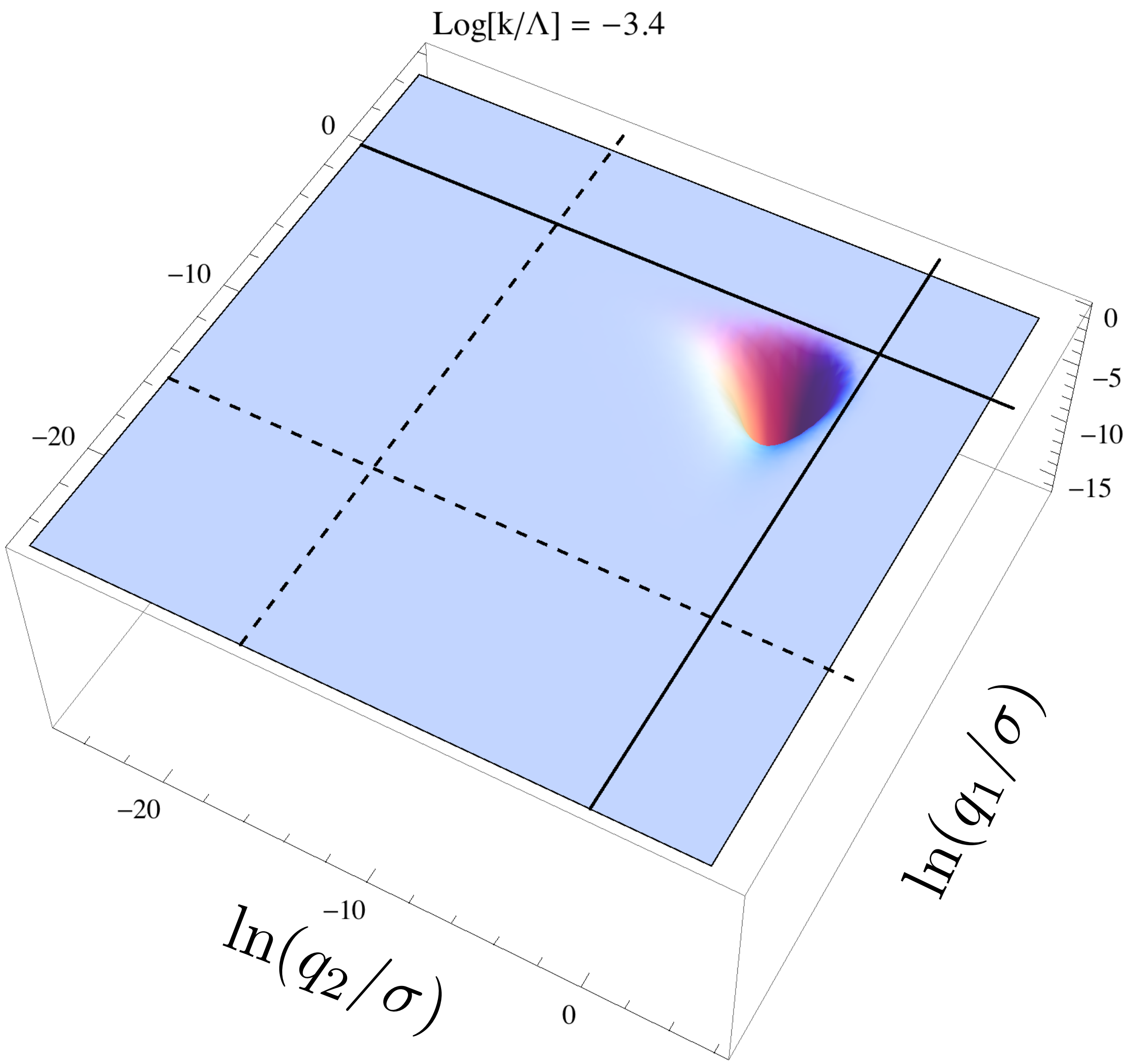

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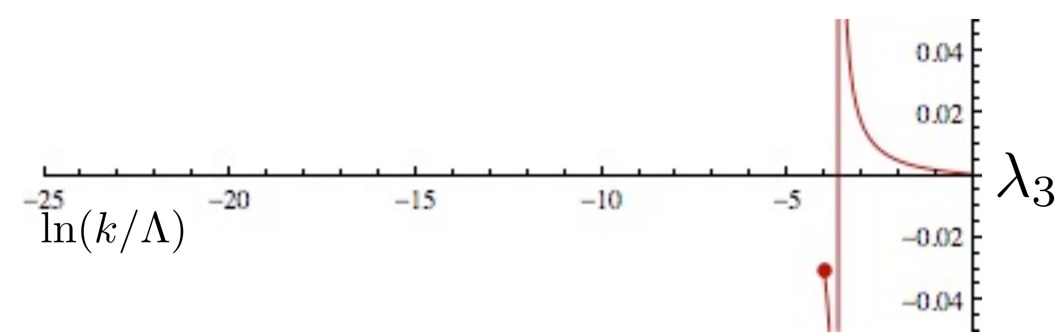

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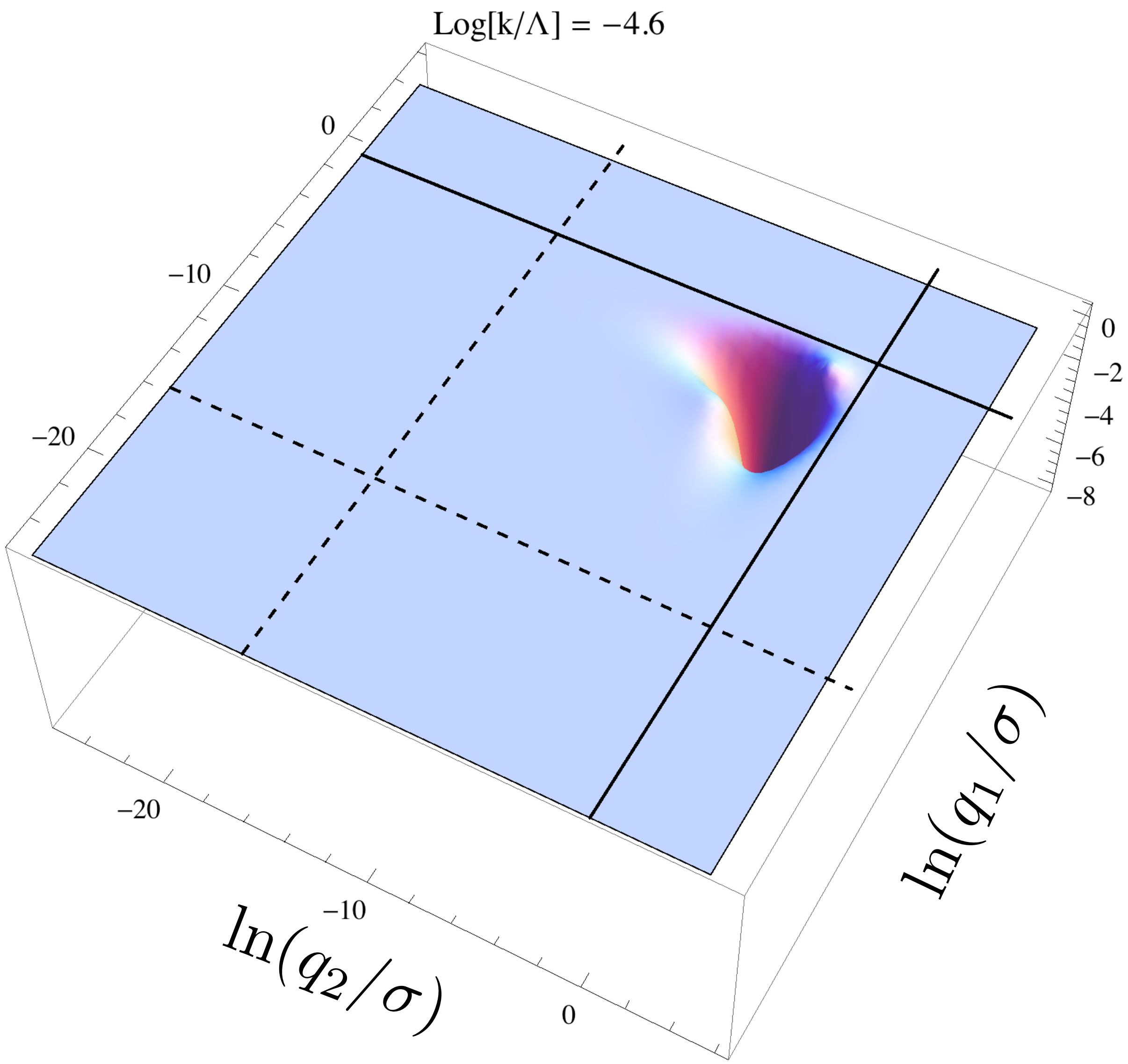

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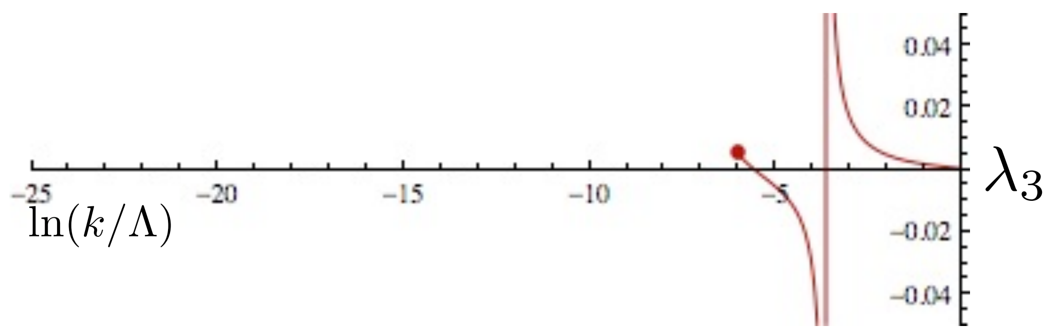
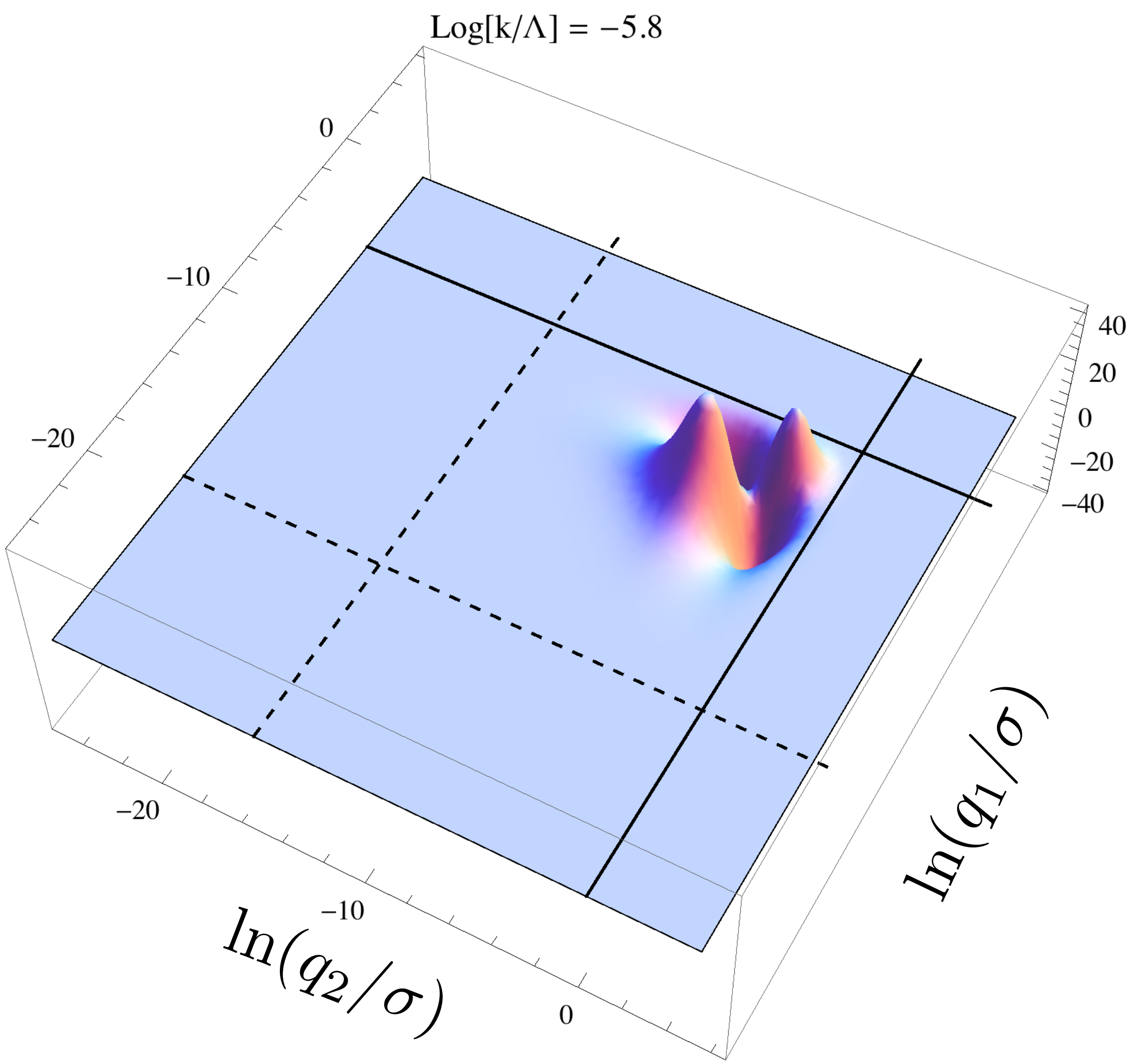


FLOERCHINGER, RS, MOROZ, WETTERICH, PRA 79 (2009)

RG flow - limit cycle

exact RG flow of $\lambda_k^{(3)}(q_1, q_2; E)$

SIMILAR FRG FOR ZERO-RANGE: MOROZ, FLOERCHINGER, RS, WETTERICH, PRA 79 (2009)



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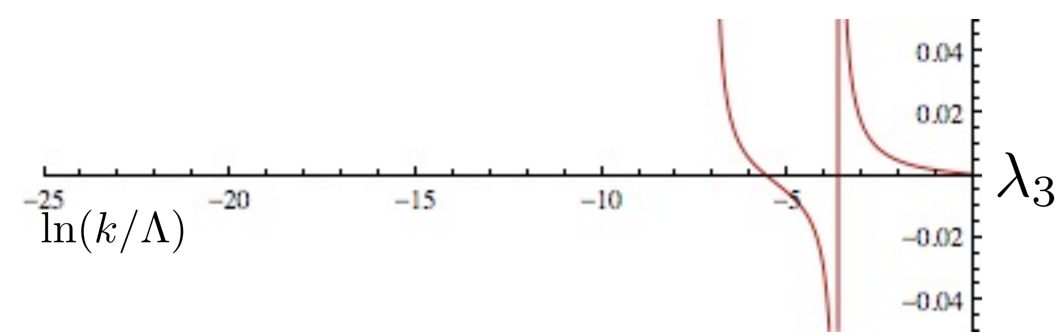

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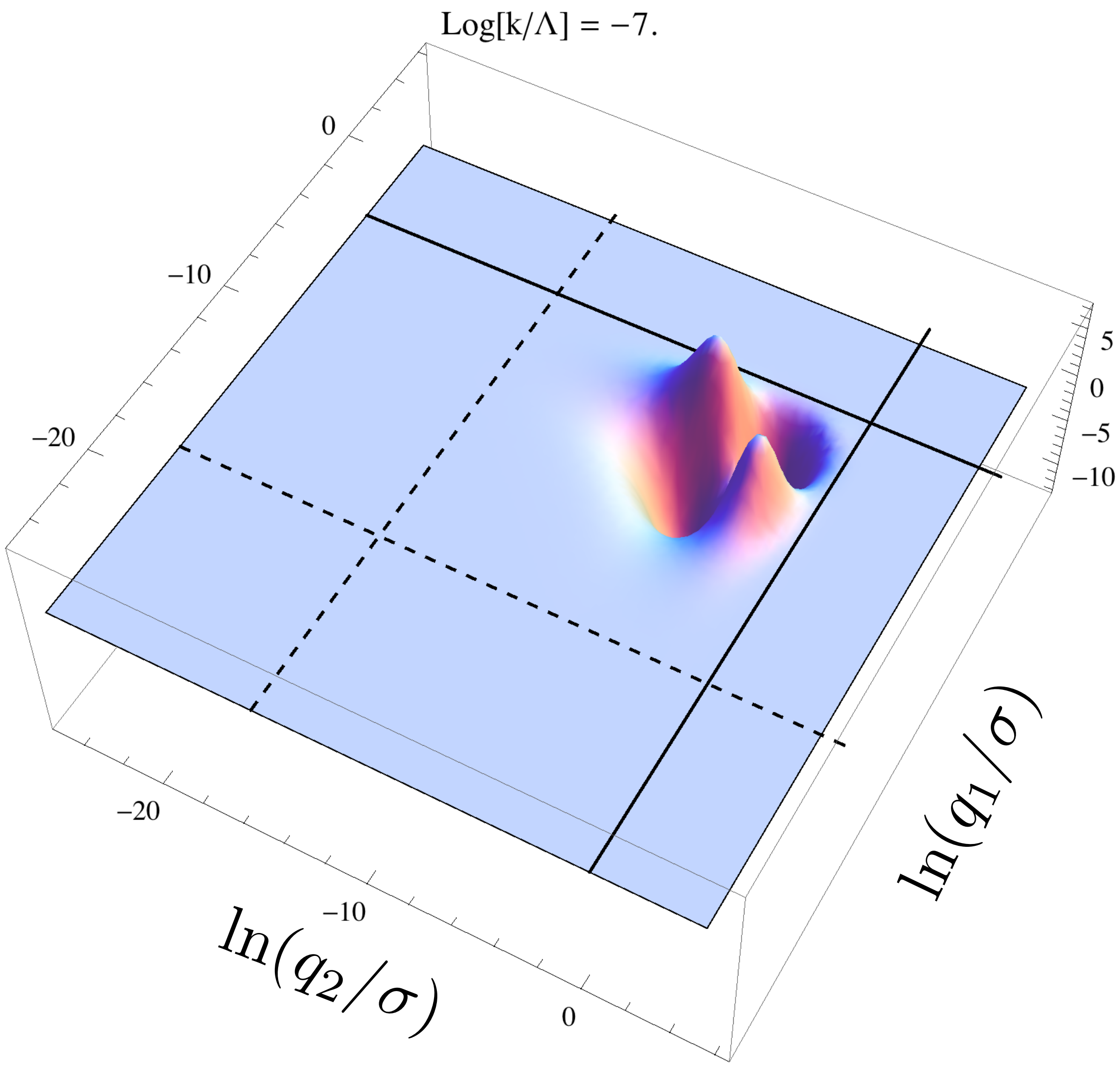
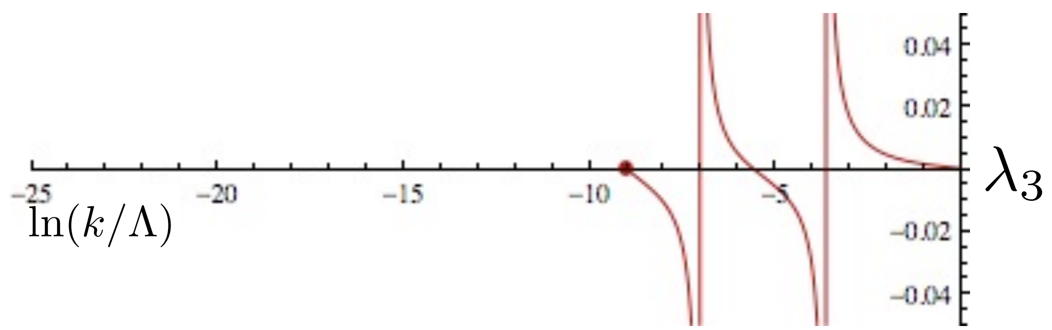


RG flow - gradient expansion



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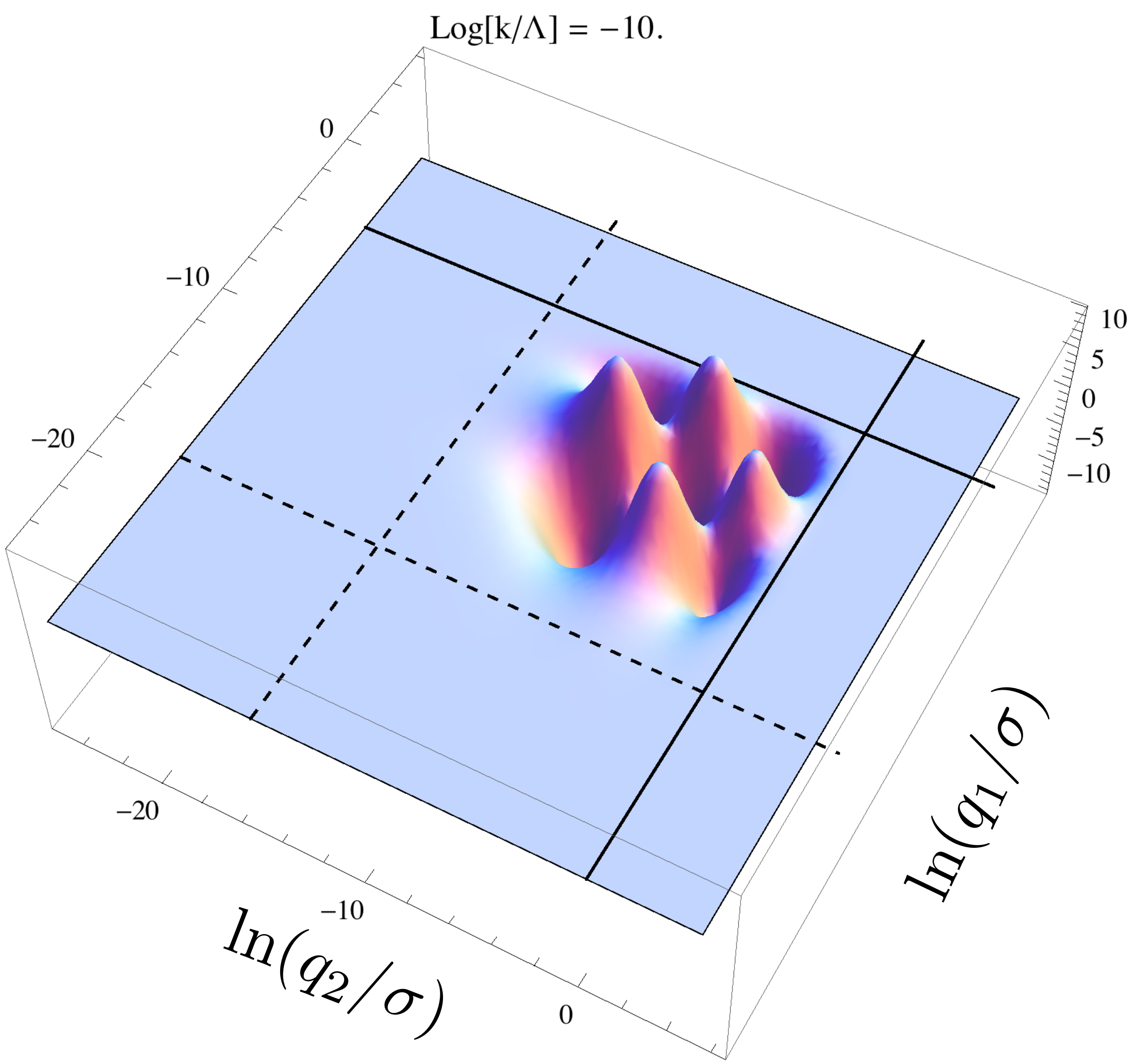

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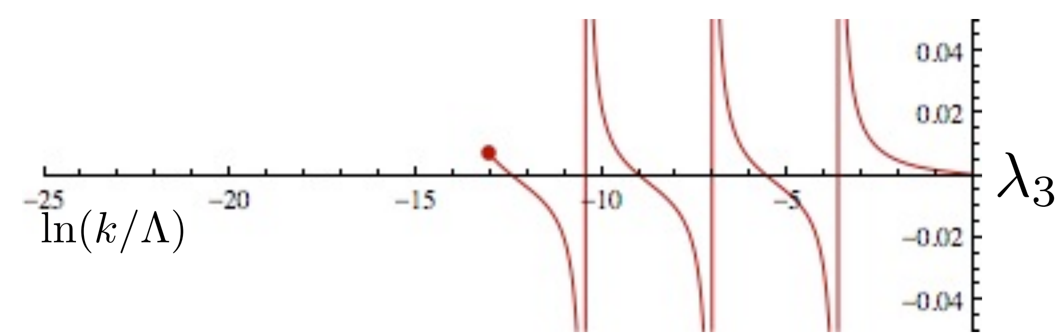

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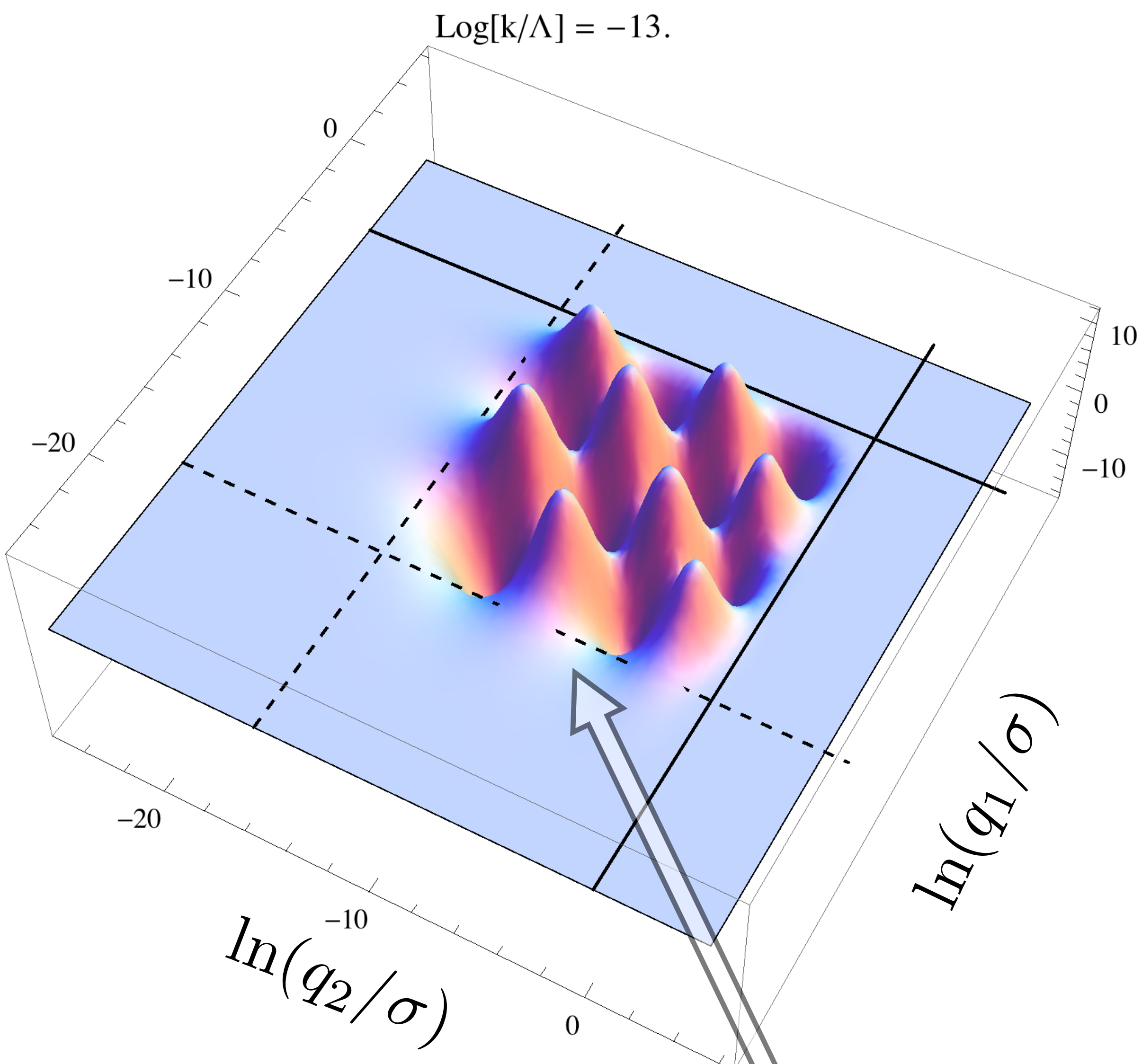

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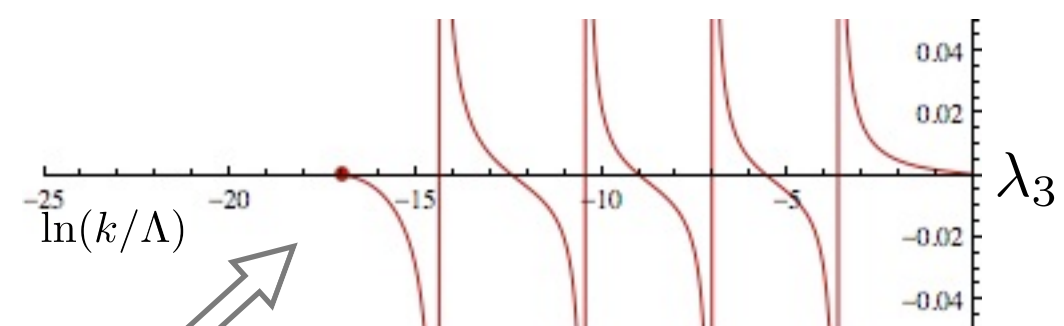

FLOERCHINGER, RS, MOROZ, WETTERICH, PRA 79 (2009)

flow suppressed by scattering length a

RG flow - limit cycle

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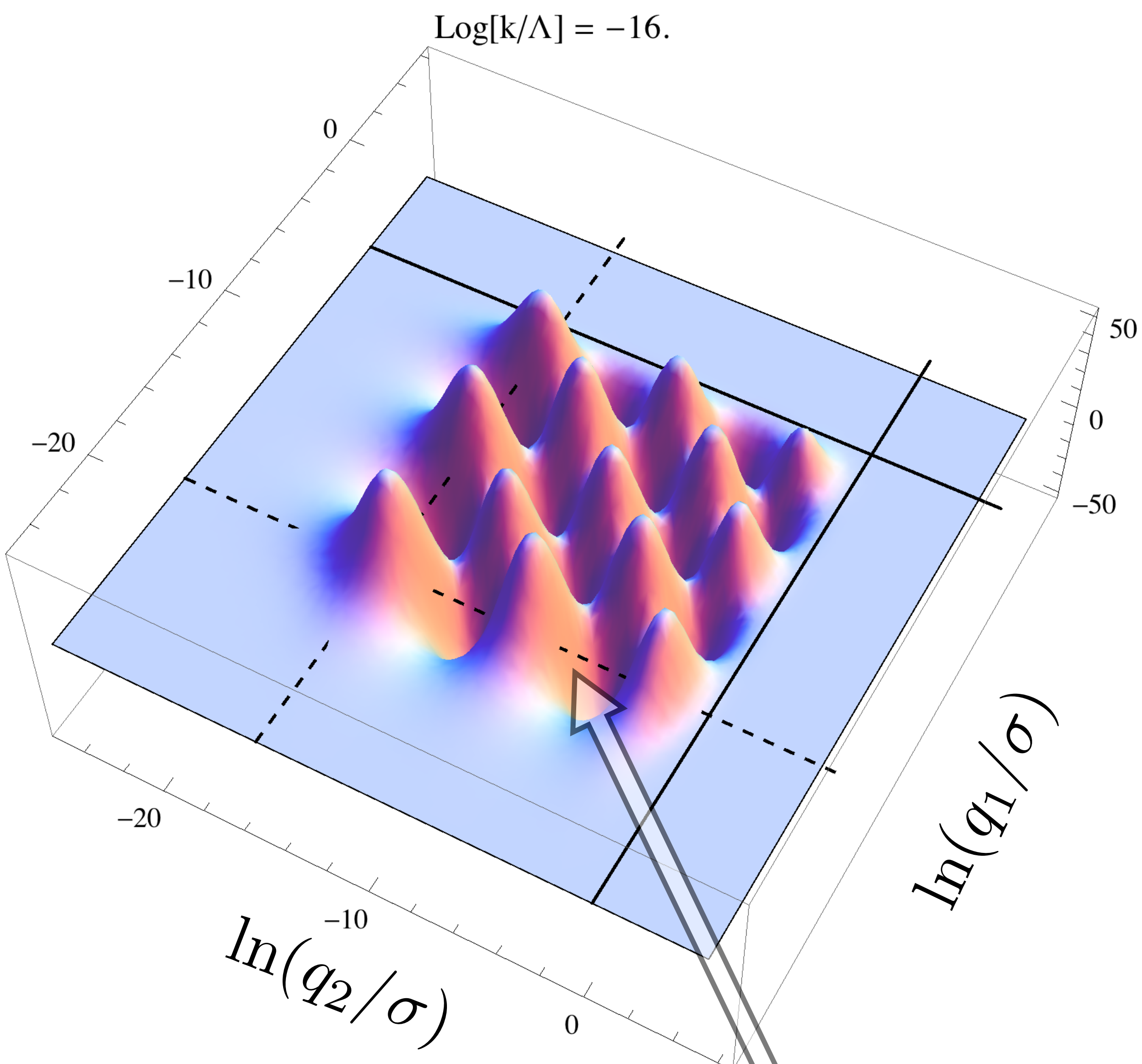
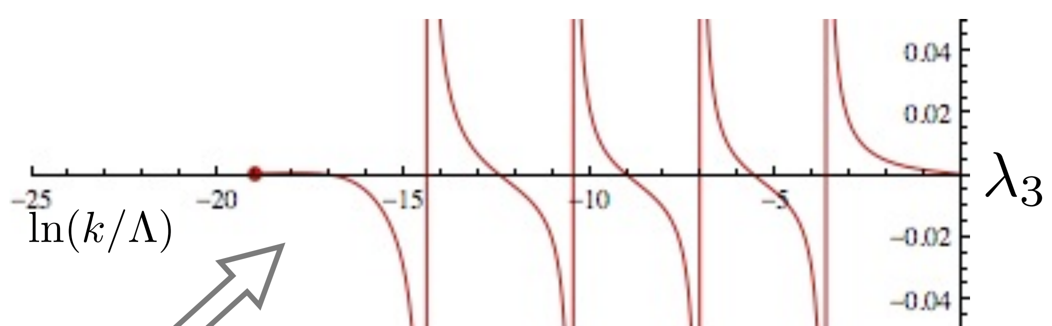


RG flow - gradient expansion

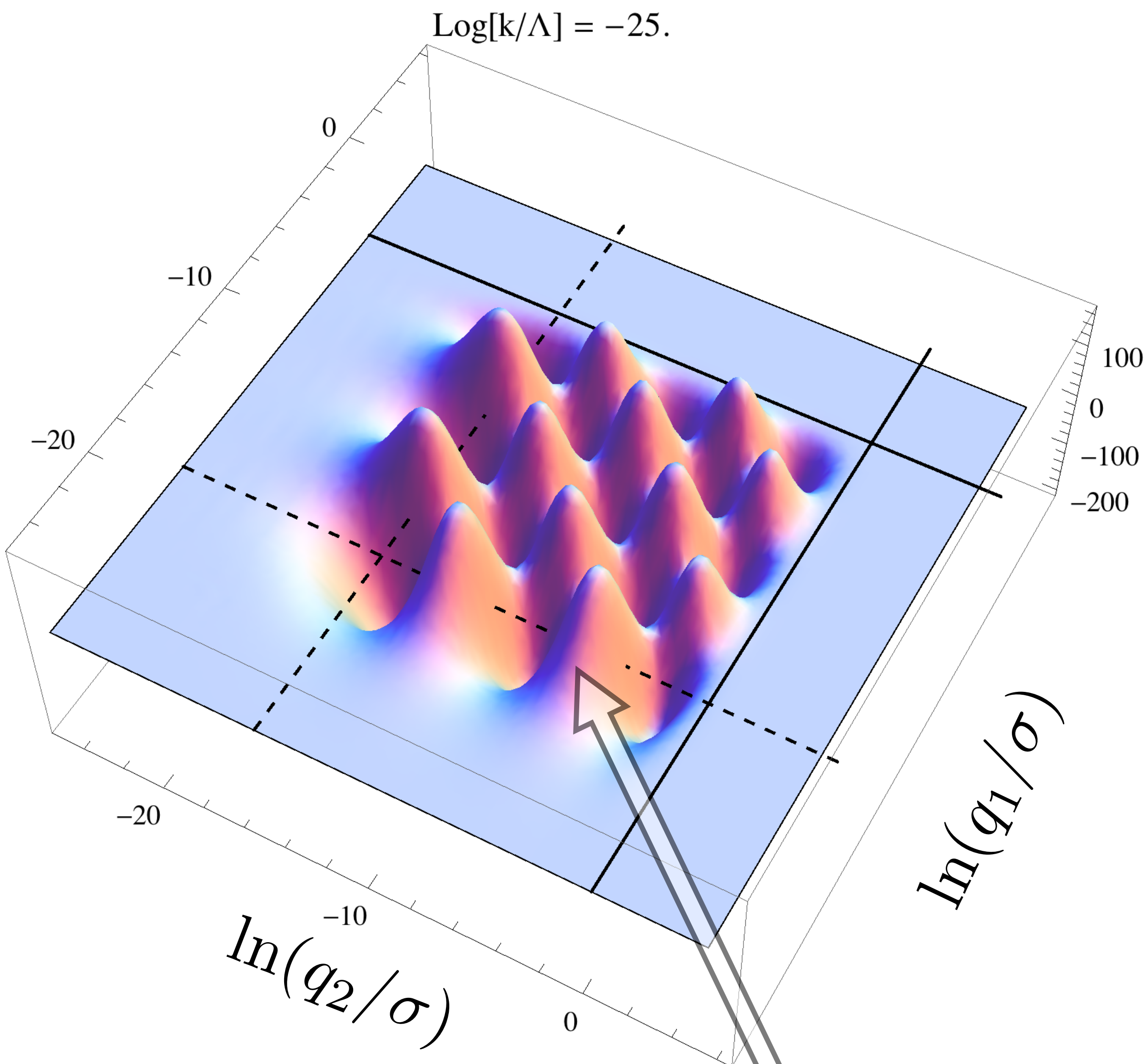


FLOERCHINGER, RS, MOROZ, WETTERICH, PRA 79 (2009)

flow suppressed by scattering length a

RG flow - limit cycle

exact RG flow of $\lambda_k^{(3)}(q_1, q_2; E)$ SIMILAR FRG FOR ZERO-RANGE: MOROZ, FLOERCHINGER, RS, WETTERICH, PRA 79 (2009)



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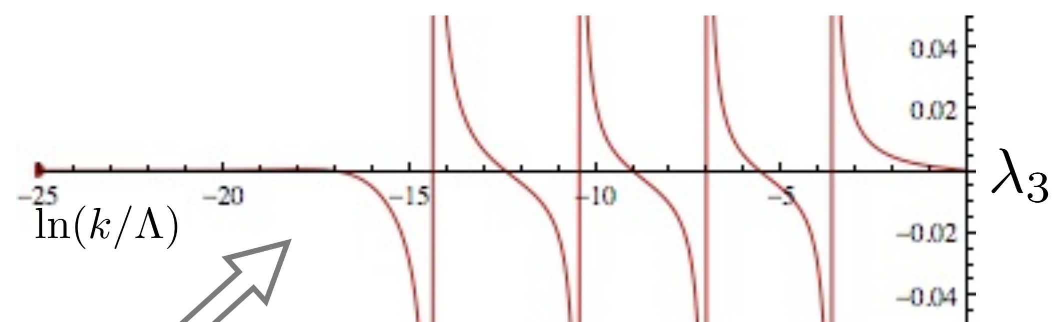

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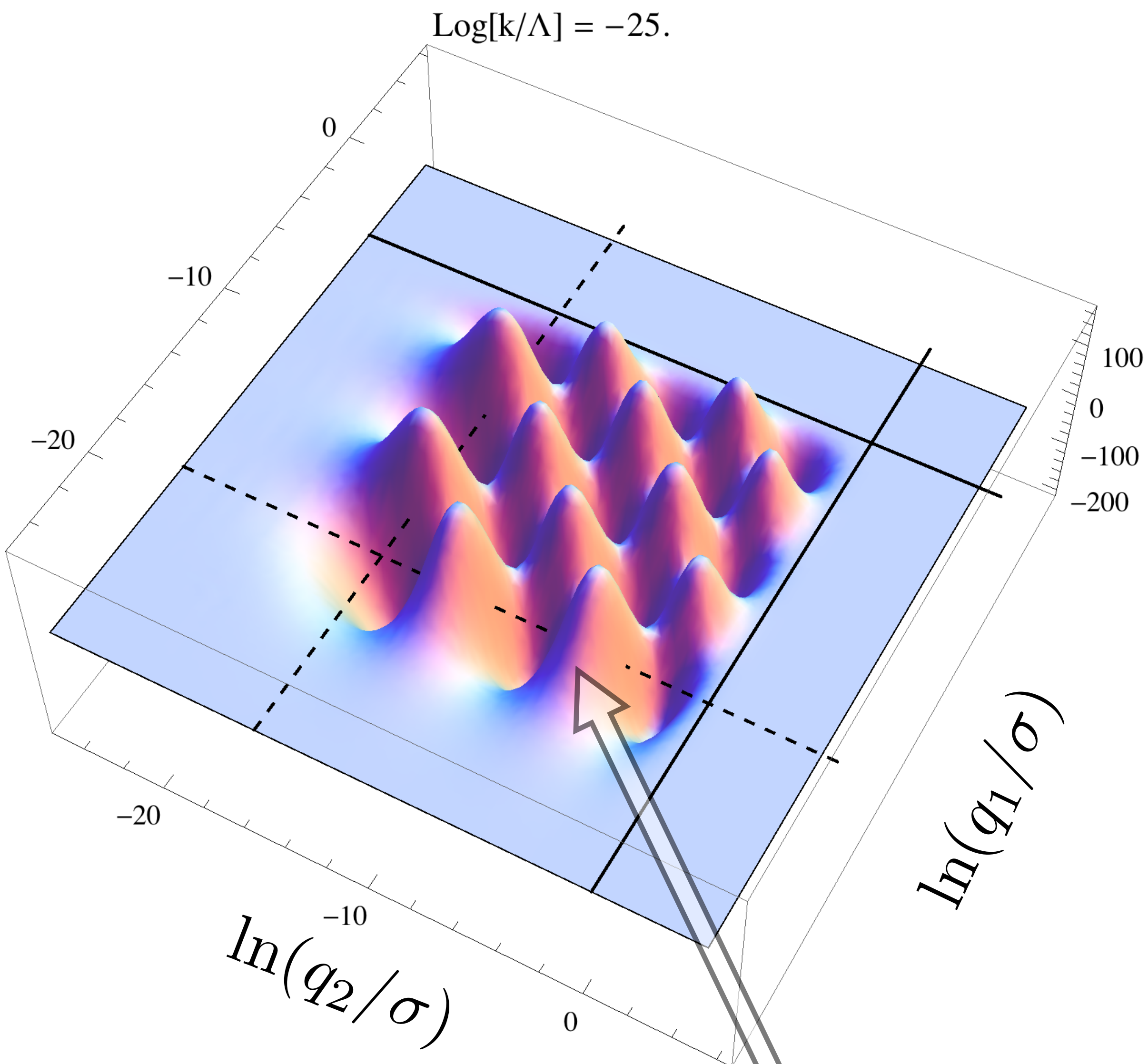

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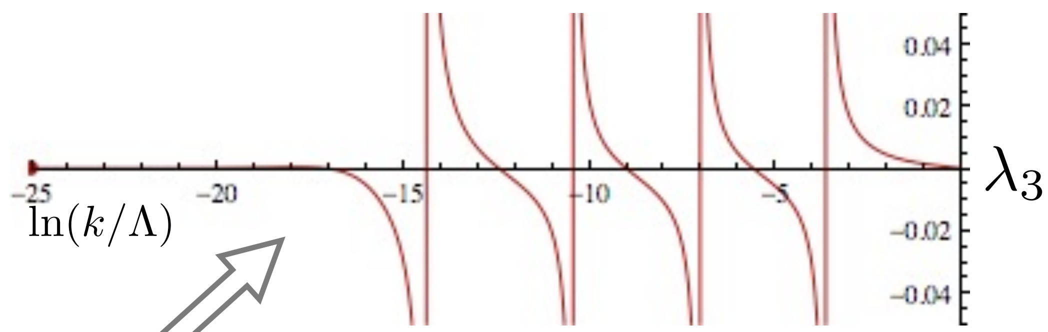


FLOERCHINGER, RS, MOROZ, WETTERICH, PRA 79 (2009)

flow suppressed by scattering length a

IR value of $\lambda_3(q_1, q_2; E)$ carries all information about three-body problem

bound state spectrum

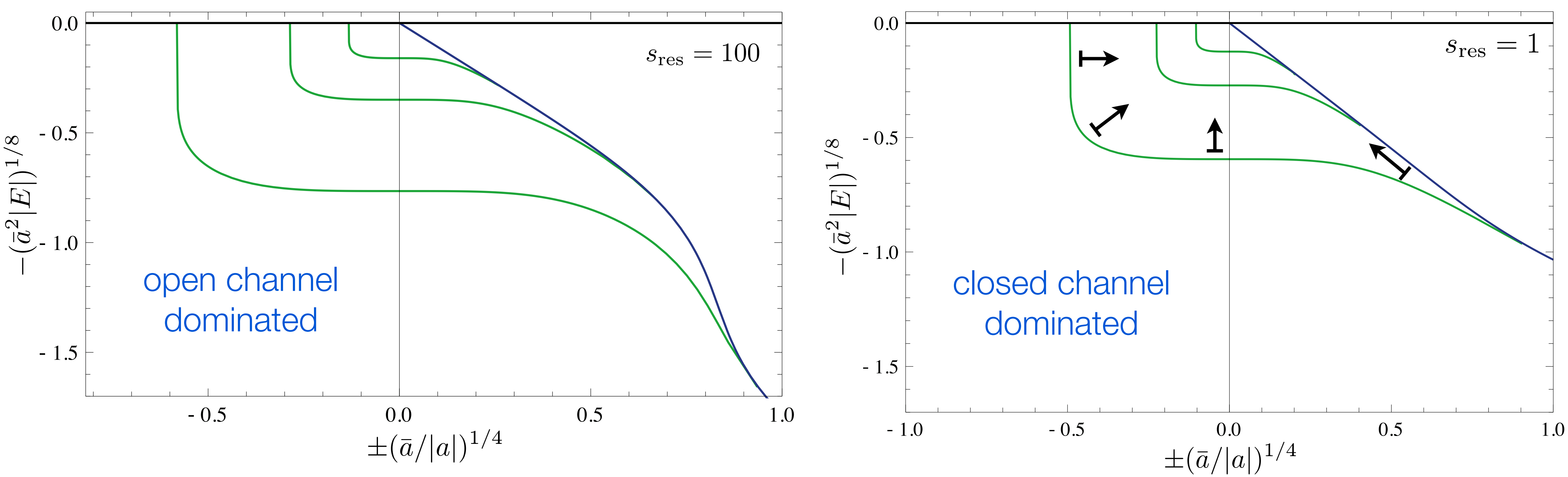
IR value of $\lambda_3(q_1, q_2; E)$ carries all information about three-body problem

bound state spectrum, pole expansion

$$\lambda_3(q_1, q_2; E) \approx \frac{\mathcal{B}(q_1, q_2)}{E - E^{(n)} + i\Gamma^{(n)}}$$

SEE E.G.: BRAATEN, HAMMER, PHYS. REP. 428 (2006)

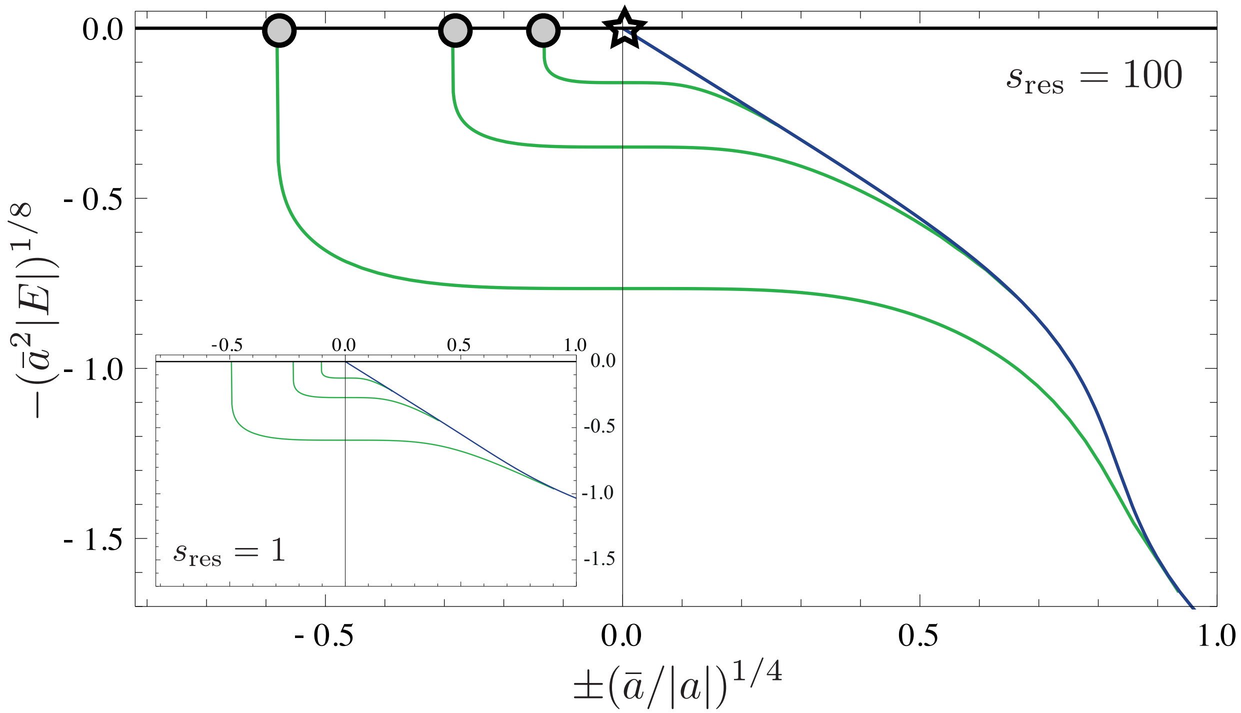
energy spectrum



- spectrum reaches maximal extent for open-channel dominated resonances
- spectrum pushed towards unitarity point for closed-channel dom. resonances
- atom-dimer threshold: highly non-universal, model dependent

approach of universality

energy spectrum



exact results for non-universal corrections

level number

l_{vdw}/r^*	n	0	1	2	$n \gg 1$
100	$a_-^{(n+1)}/a_-^{(n)}$	17.083	21.827	22.654	22.694
1	$a_-^{(n+1)}/a_-^{(n)}$	22.869	22.650	22.690	22.694
0.1	$a_-^{(n+1)}/a_-^{(n)}$	26.230	22.964	22.71	22.694

RS, Rath, Zwerger, EPJB 85 (2012)

univ. scaling $n \gg 1$

approach of universality

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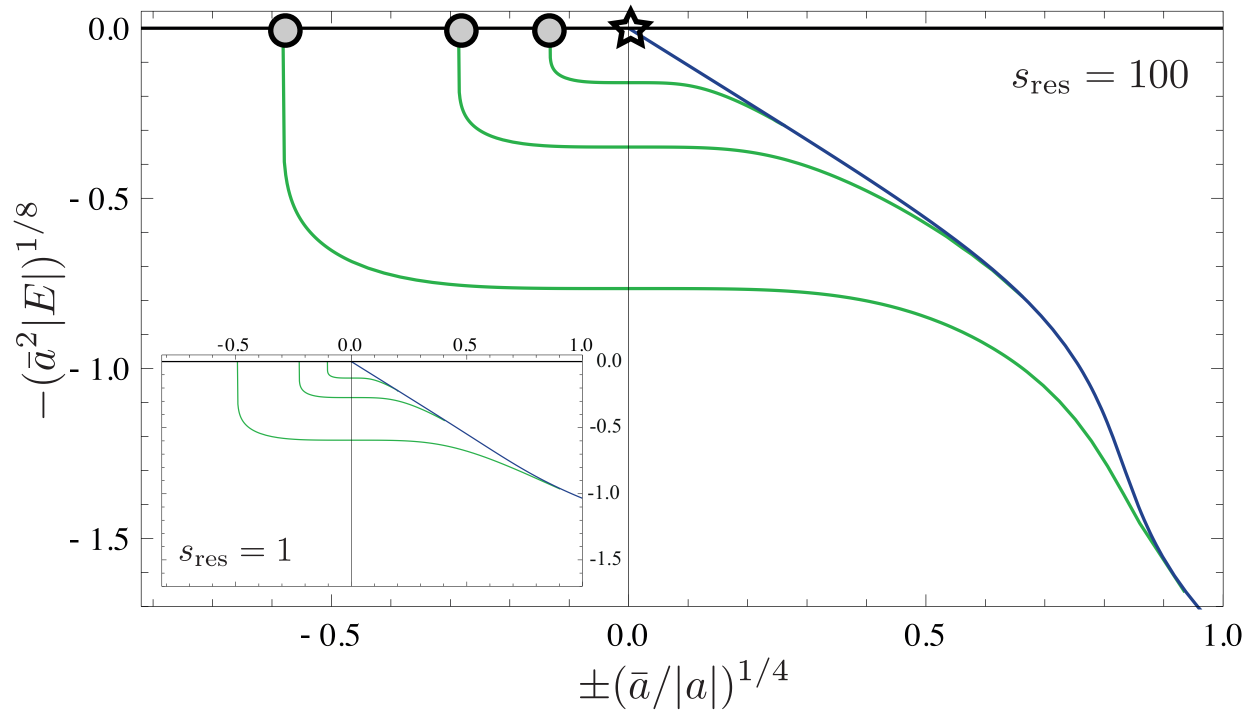

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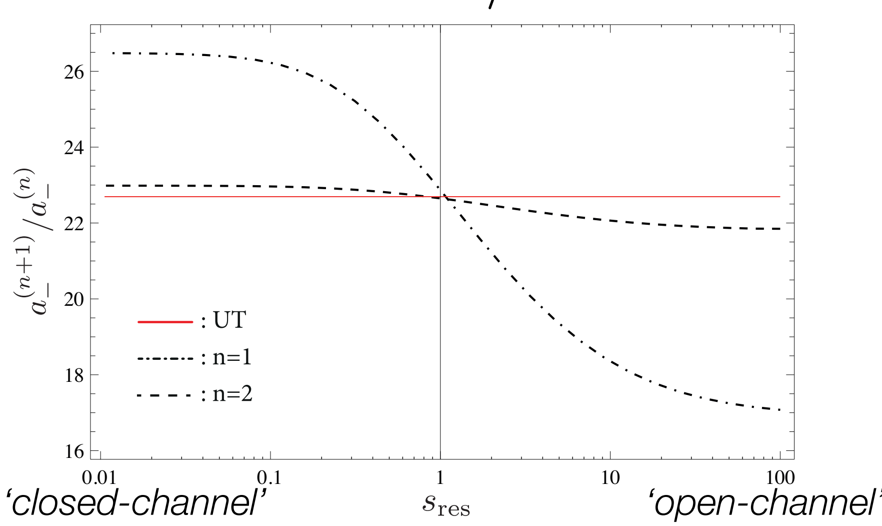
crossover of observables

$$a_-^{(n+1)}/a_-^{(n)}$$



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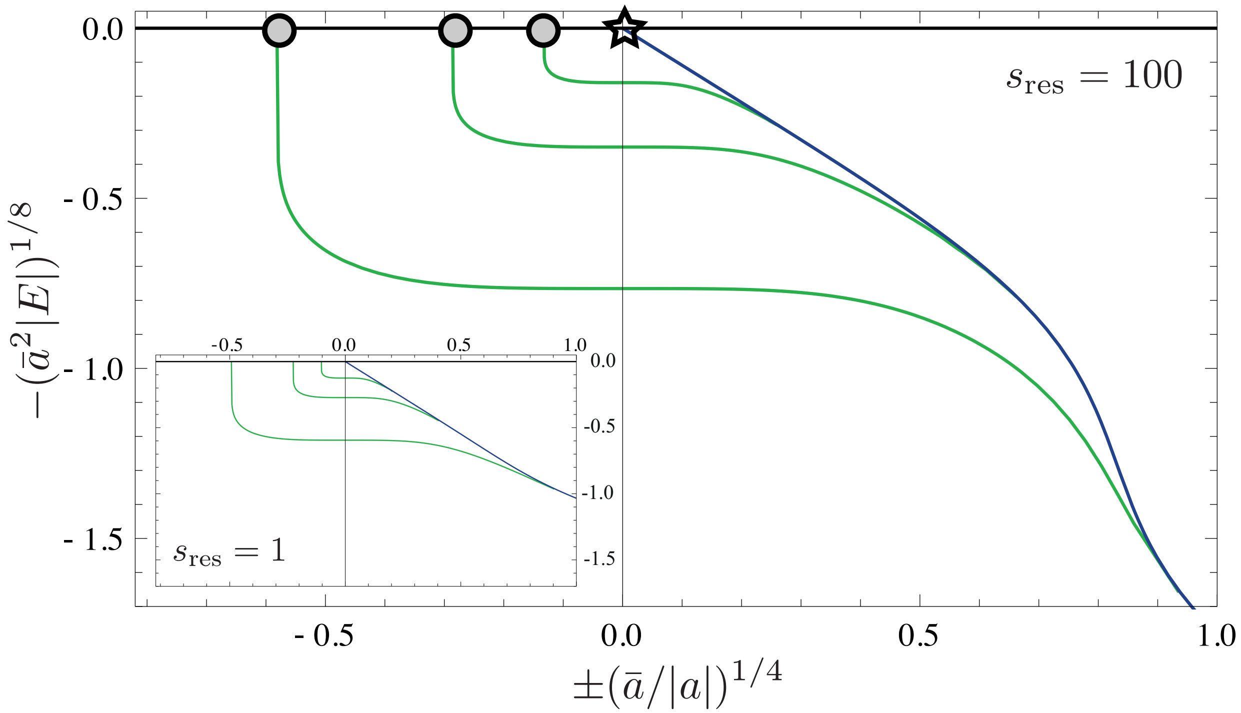

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RS, Rath, Zwerger, EPJB 85 (2012)

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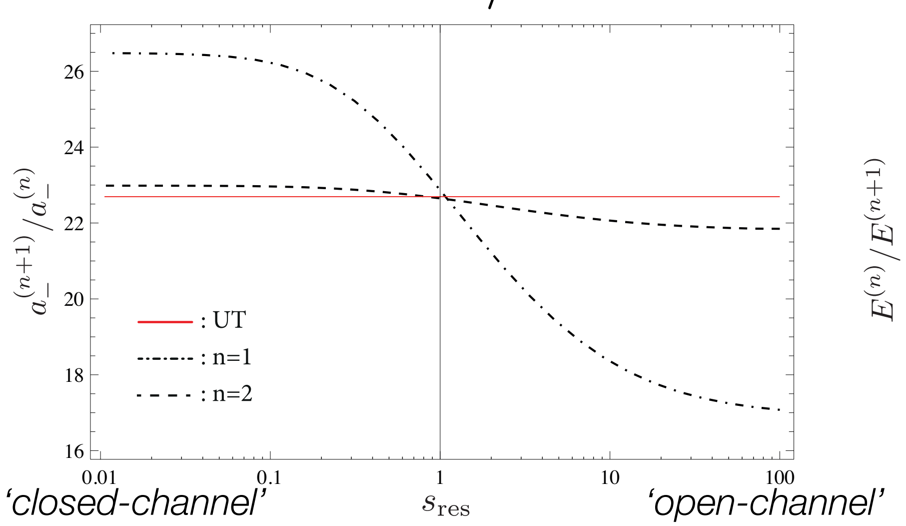
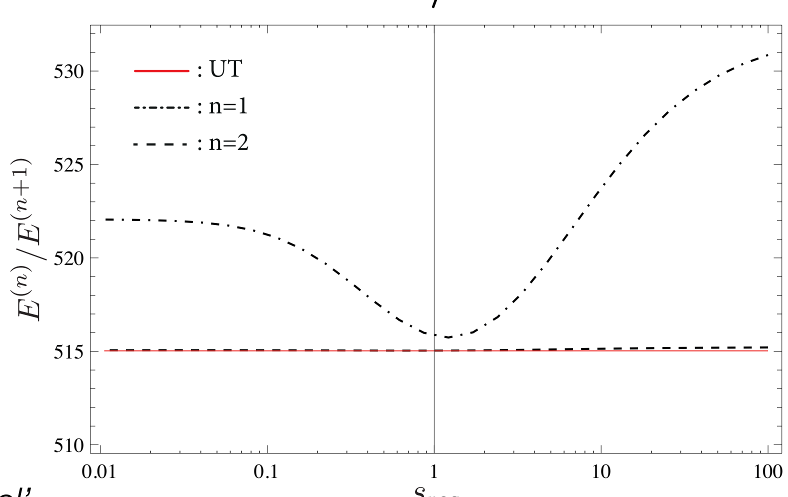
$a_-^{(n+1)}/a_-^{(n)}$

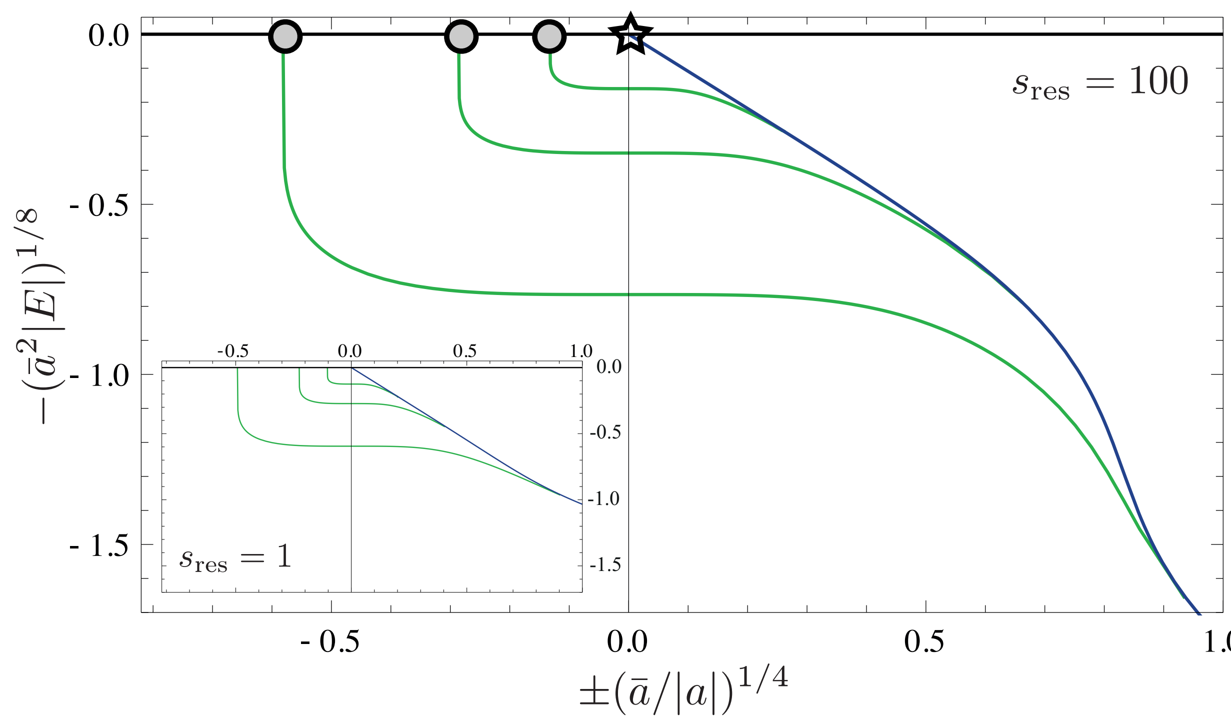


$E^{(n)}/E^{(n+1)}$



approach of universality

energy spectrum



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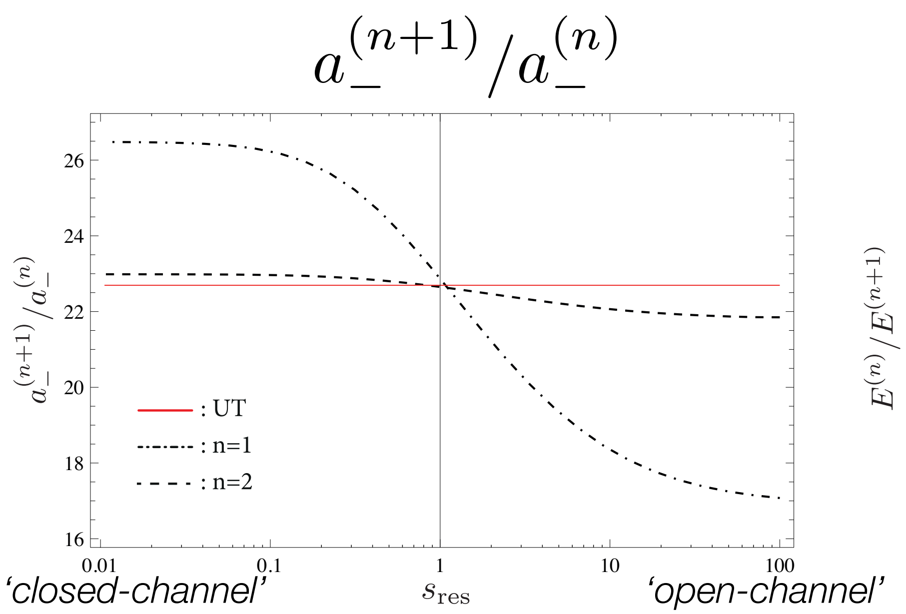
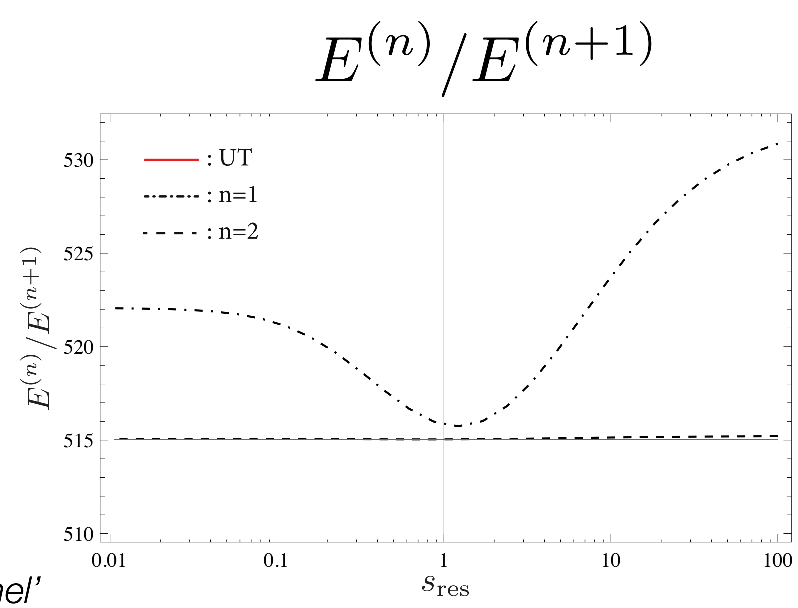
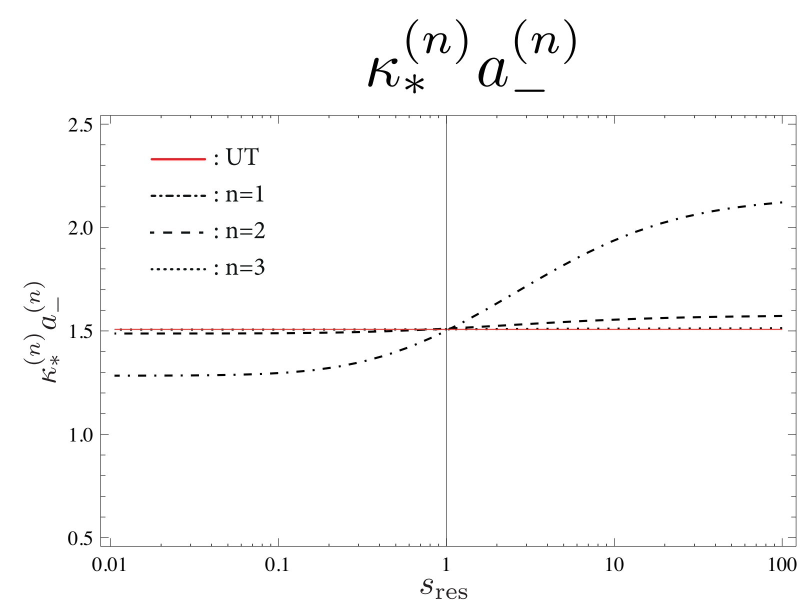
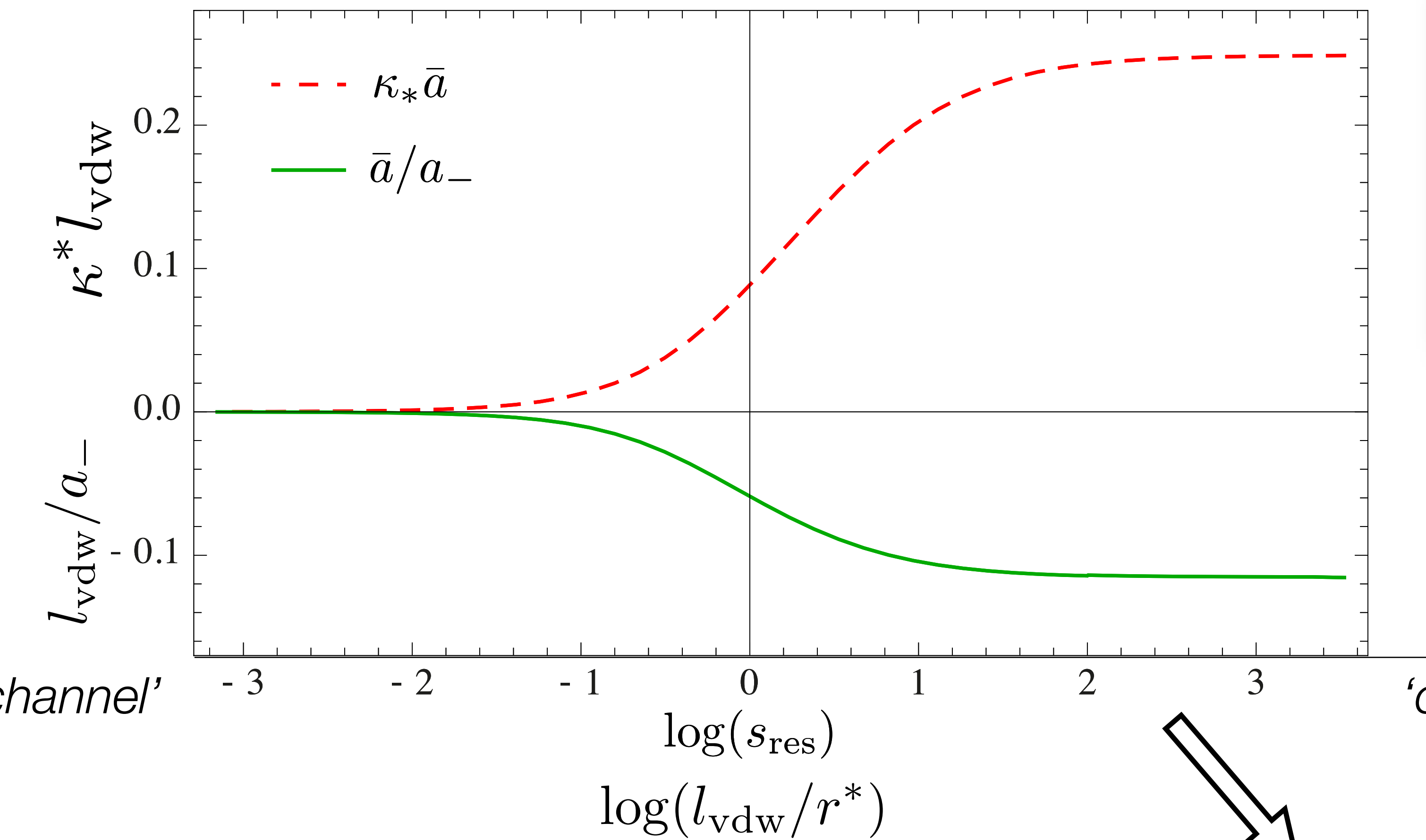




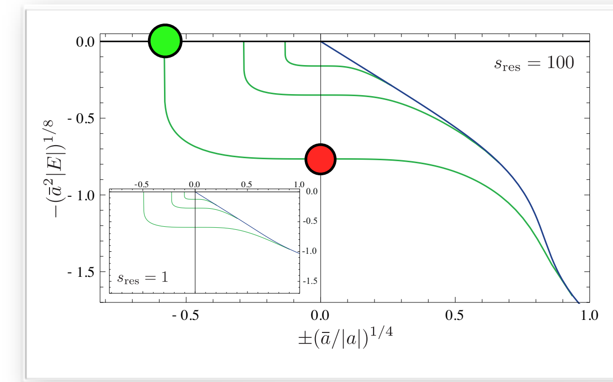


crossover of a_-

RS, Rath, Zwerger, EPJB 85 (2012)



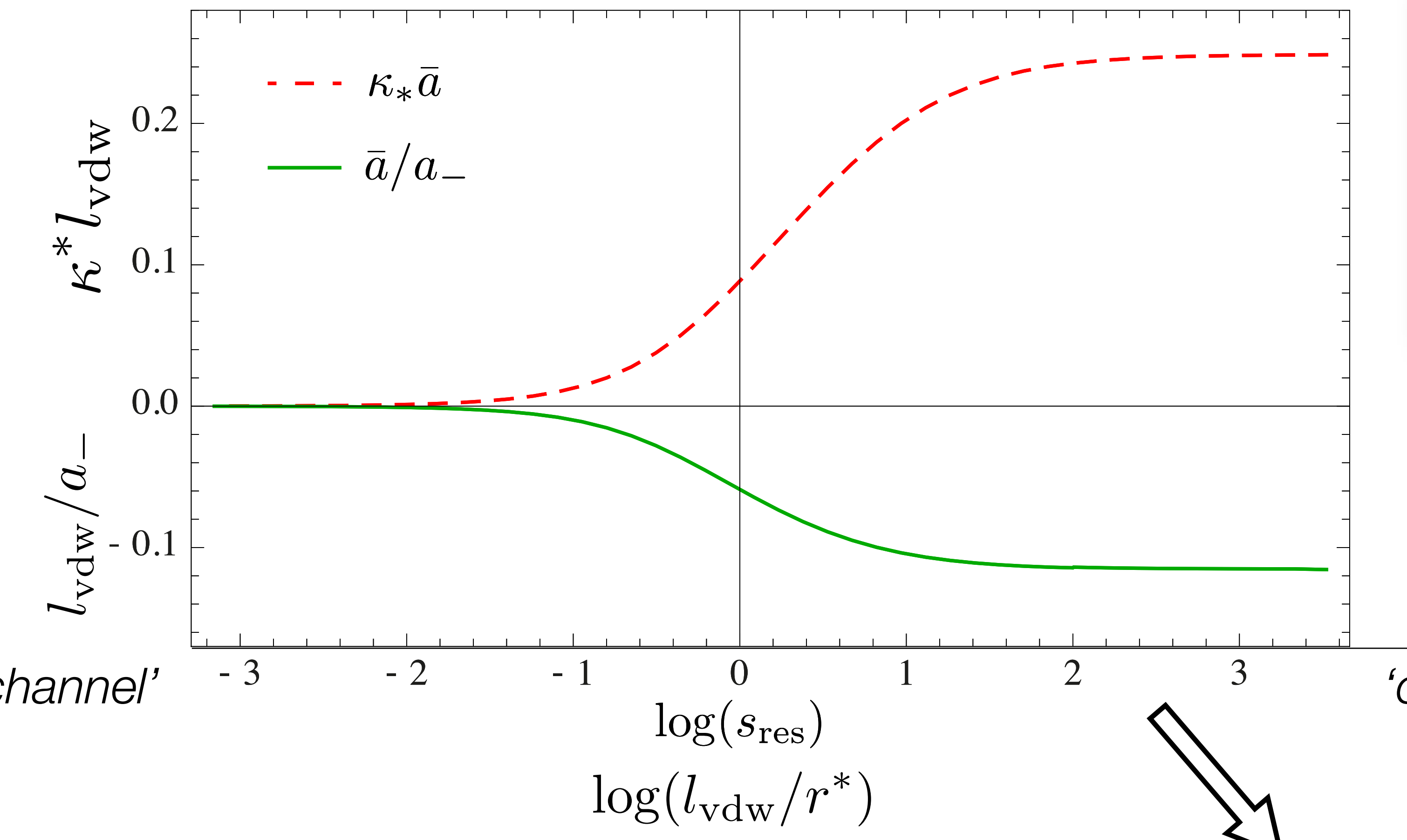
'closed-channel' 'open-channel'

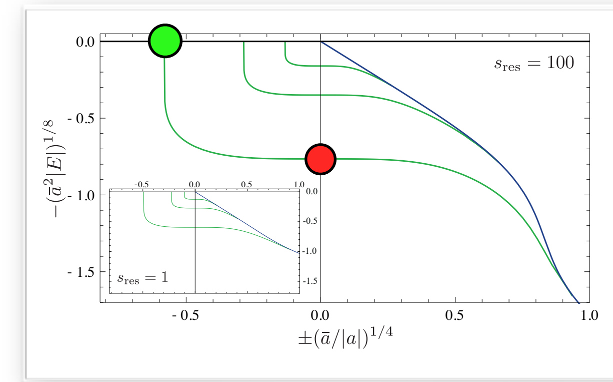


$l_{\rm vdw}$ **sets relevant scale**

crossover of a_-

RS, Rath, Zwerger, EPJB 85 (2012)





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$l_{\rm vdw}$ **sets relevant scale**

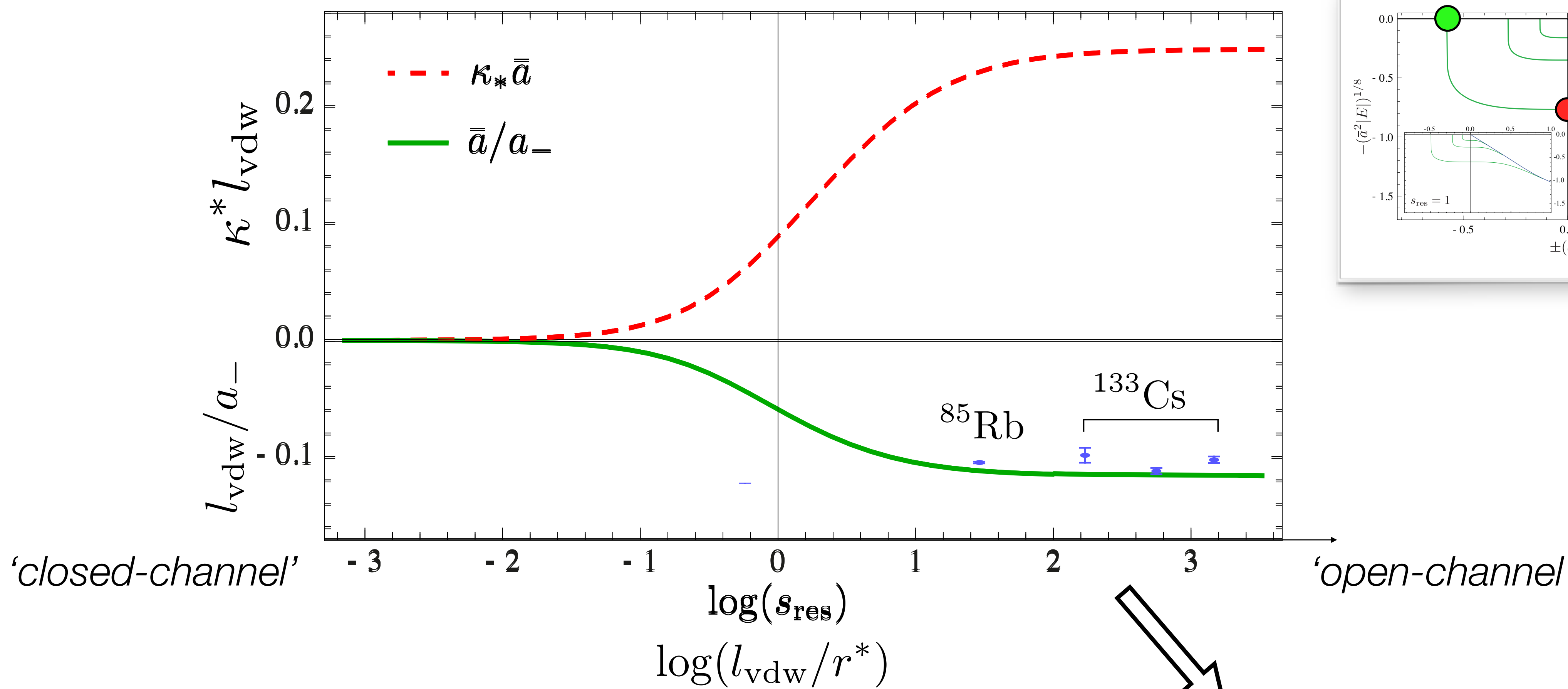
dependence on form factor

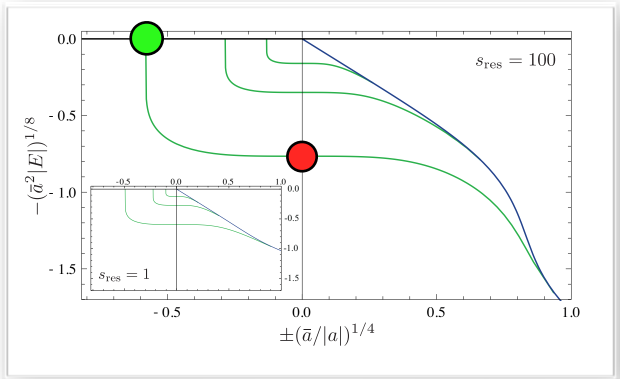
two-channel: $a_- \approx -8.2\, l_{\rm vdw} (\pm\, 10\%)$

RS et al. EPJB 85 (2012)

crossover of a_-

RS, Rath, Zwerger, EPJB 85 (2012)





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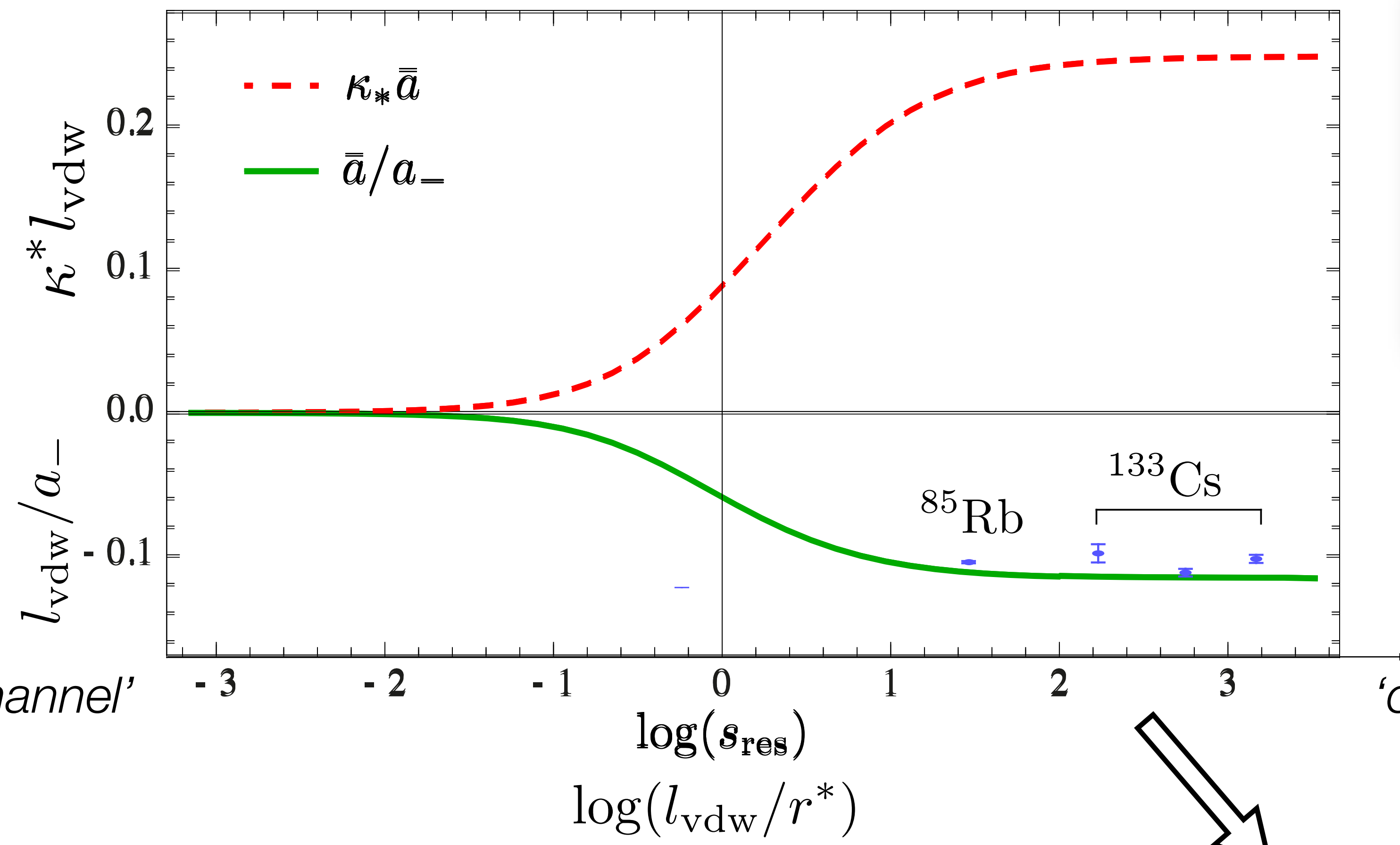
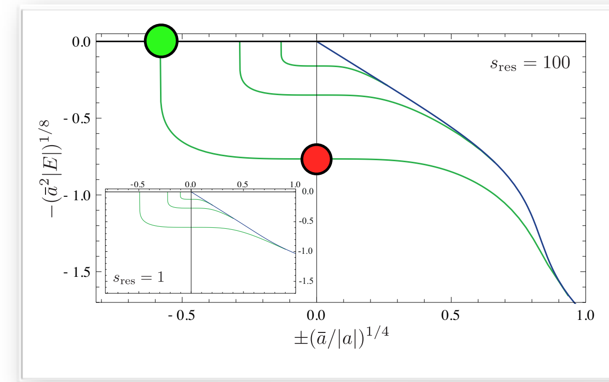


‘open-channel’



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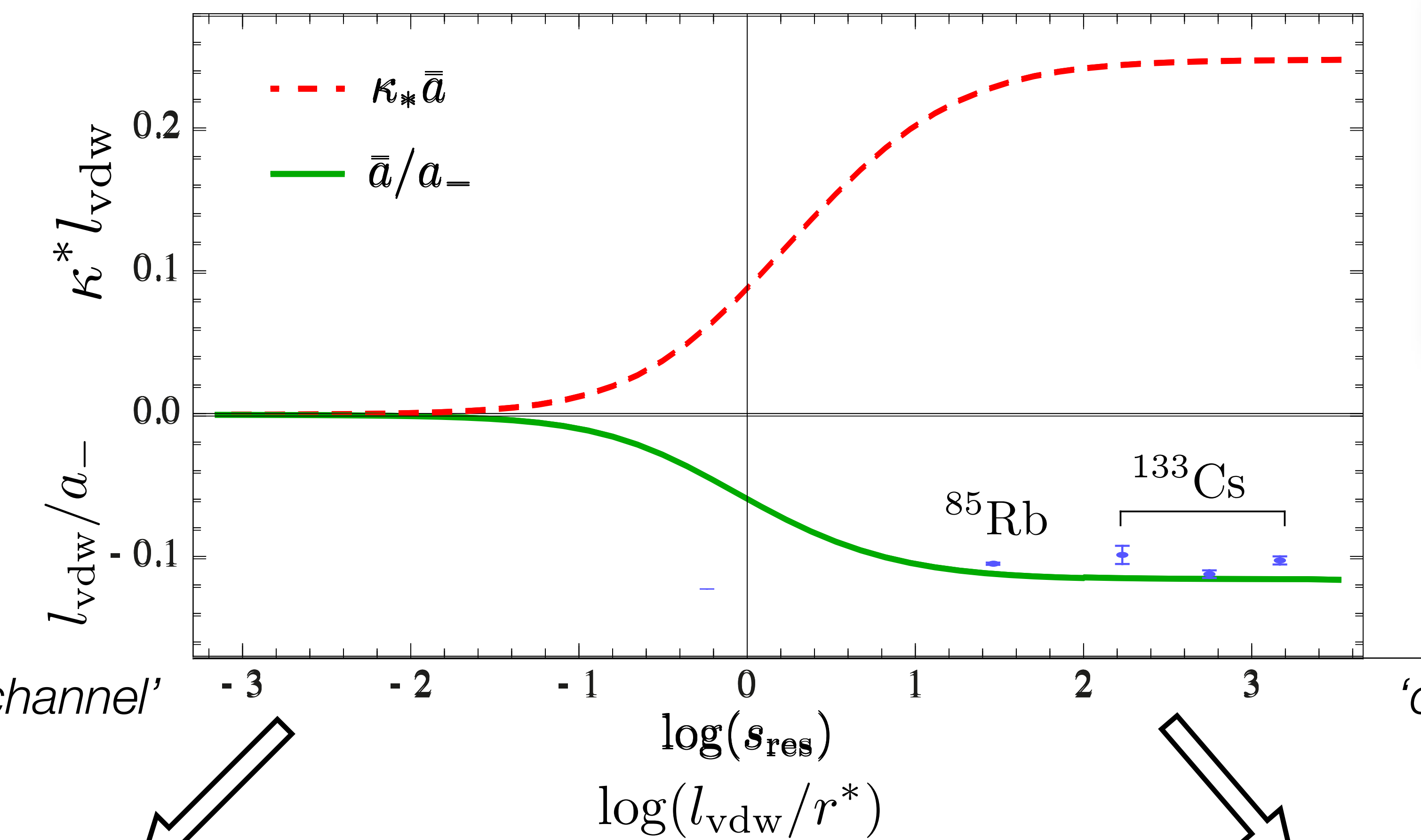

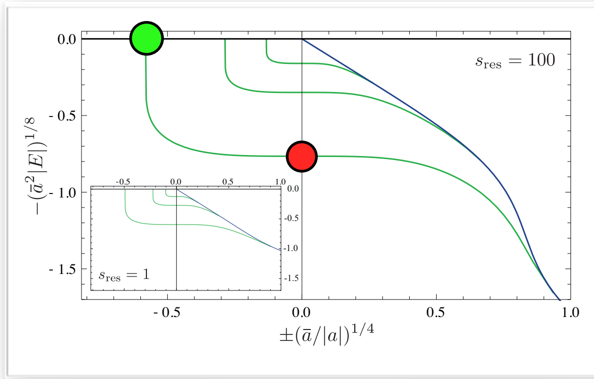

'closed-channel'

'open-channel'

r^* **sets relevant scale**

$$a_- = -10.9\, r^*$$

$l_{\rm vdw}$ **sets relevant scale**

dependence on form factor

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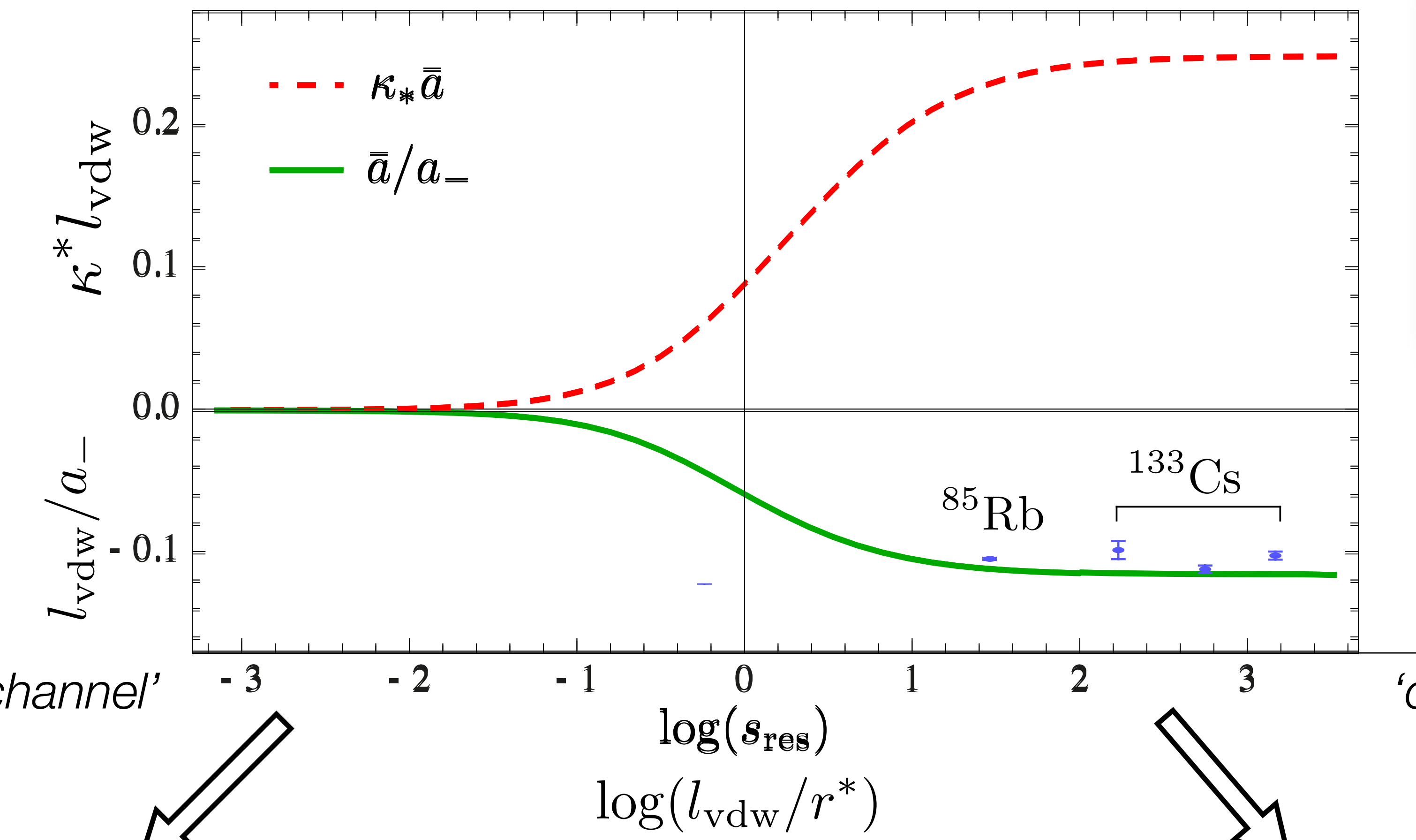

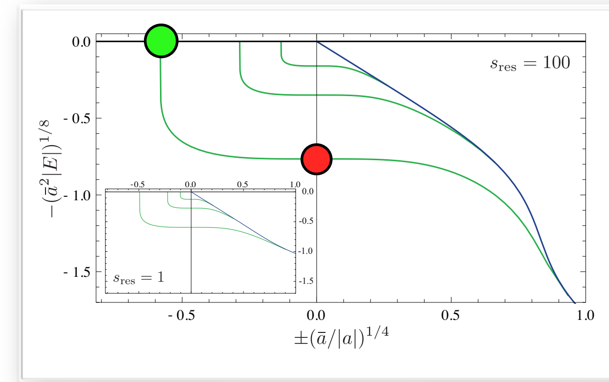

'closed-channel' 'open-channel'

r^* sets relevant scale

$$a_- = -10.9\, r^*$$

- recover exact result for excited states $n \gg 1$

$$a_- = -12.90 r^*$$ PETROV ` 04, GOGOLIN ` 08

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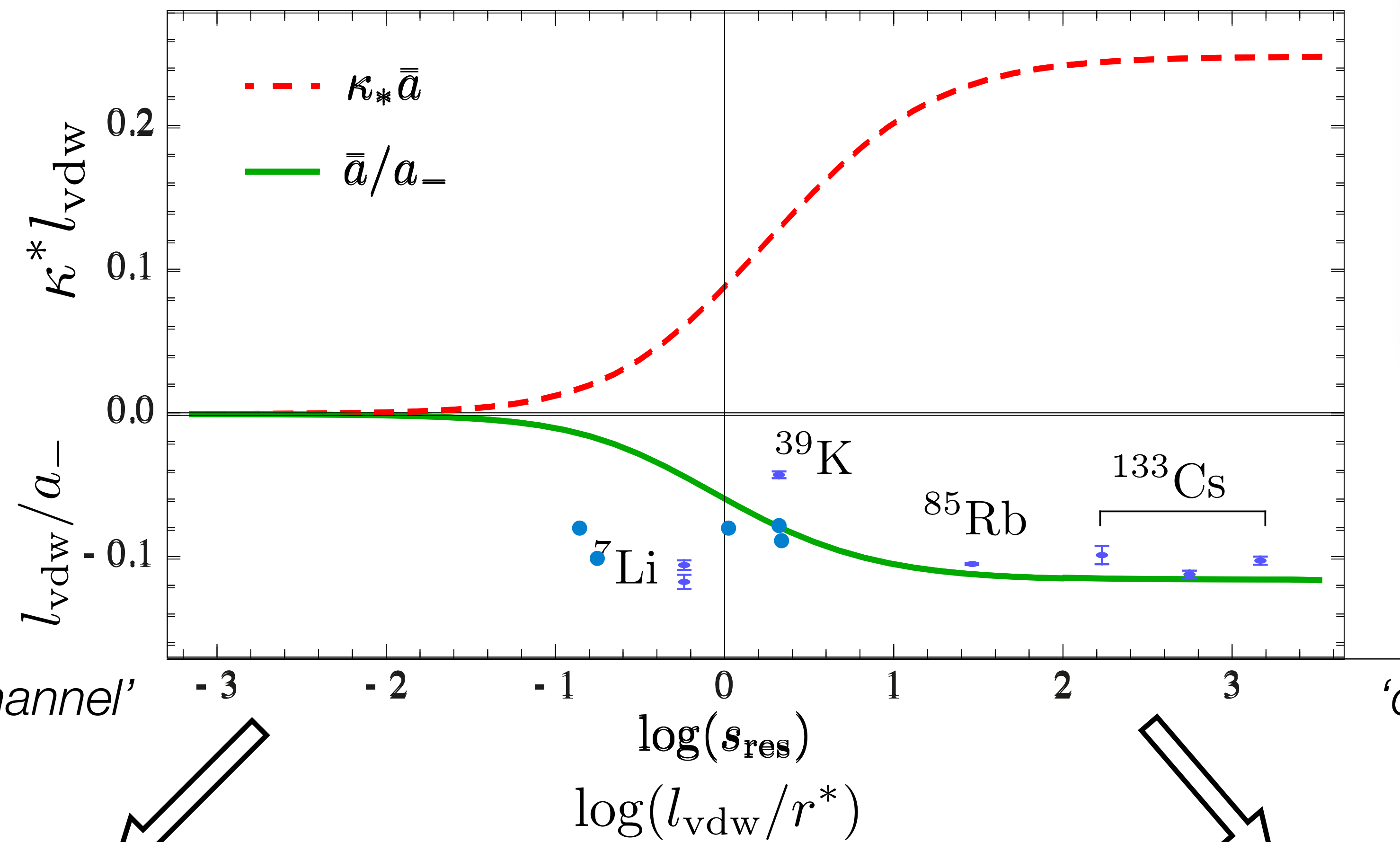
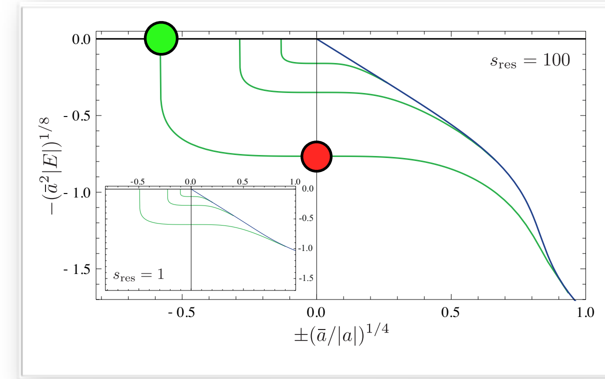
crossover of a_-

> *a puzzle:*
> • ^{39}K
> Roy et al., PRL 111 (2013)

RS, Rath, Zwerger, EPJB 85 (2012)





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l_{vdw} sets relevant scale

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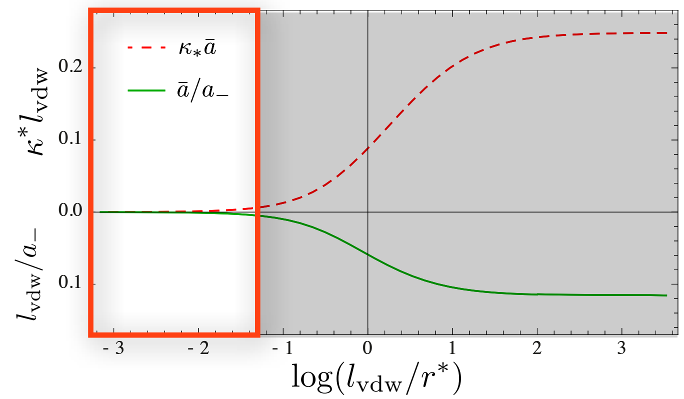
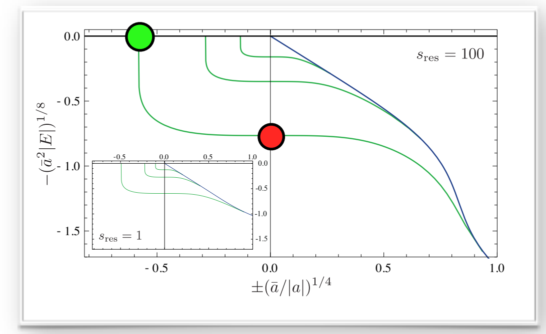
Extended universality for ‘narrow’ resonances





in limit of closed channel dominated resonances $g \to 0$ an extended universality appears

RS, PhD Thesis (2013)

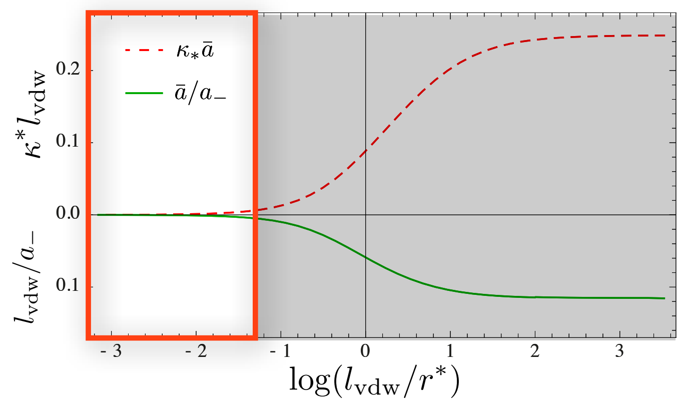
prediction of *universal* deviations away from unitarity

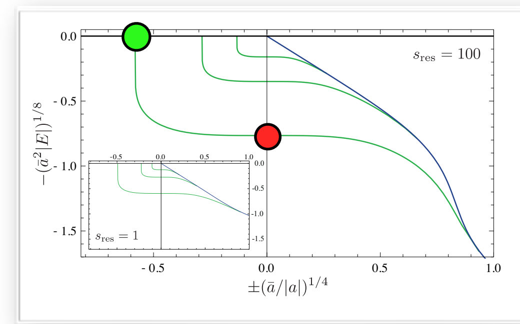
n	0	1	2	3	4
$a_-^{(n)}/r^*$	-10.90	-12.72	-12.89	-12.897	-12.899
$\kappa^{(n)} r^*$	0.118	0.117	0.117	0.117	0.117
$a_-^{(n+1)}/a_-^{(n)}$	26.48	22.98	22.713	22.698	-

RS, PhD Thesis (2013)

↳ ratios independent of form factor or regularization chosen!

Extended universality for 'narrow' resonances





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RS, PhD Thesis (2013)

↳ ratios independent of form factor or regularization chosen!

indication universal scaling of deviations with trimer level

$$\frac{a_-^{(n+1)}}{a_-^{(n)}} \approx 22.694 + \gamma_1 e^{-\gamma_2 n}, \qquad \gamma_1 = 63(20) \text{ and } \gamma_2 = 2.7(3)$$

RS, PhD Thesis (2013)

Test of universality I

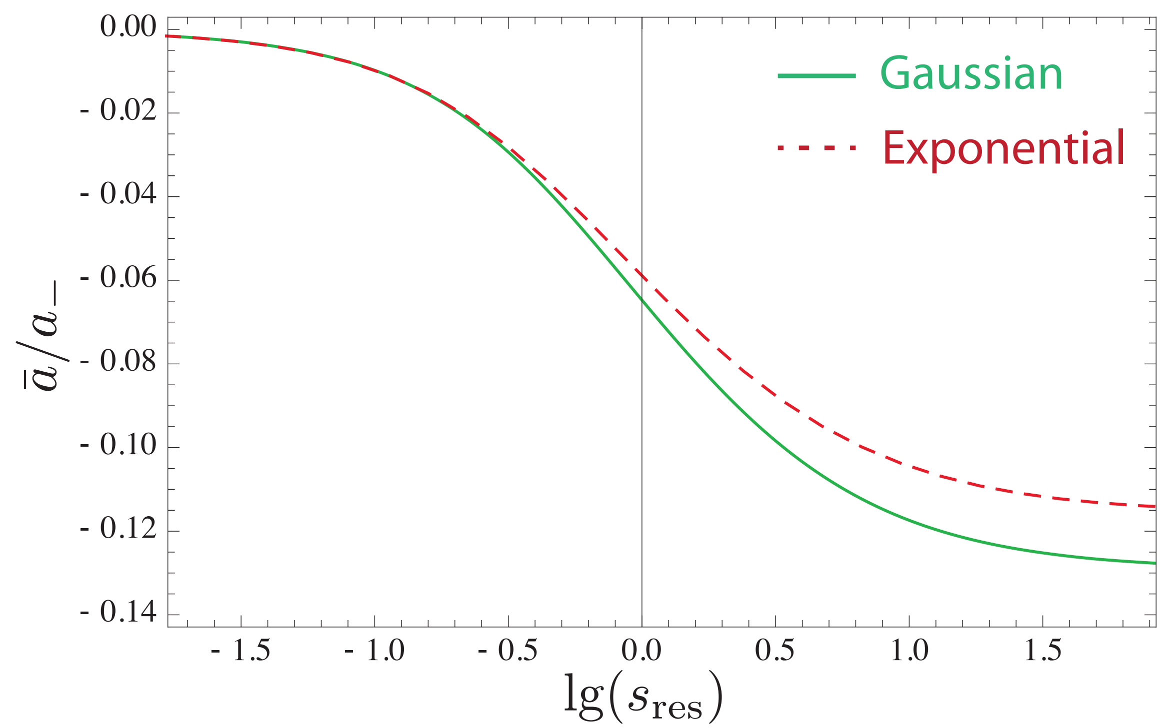
change microphysics ⇨ effect on observables?

Test of universality I

change microphysics ⇒ effect on observables?

choice of atom-dimer conversion coupling

$$S_{\mathrm{int}} = \int g(\mathbf{r}_2 - \mathbf{r}_1)\phi(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, t)\psi^*(\mathbf{r}_1, t)\psi^*(\mathbf{r}_2, t)$$



$$\chi(r) \sim e^{r/\sigma}/r$$

$$\chi(r) \sim e^{r^2/(2\sigma^2)}$$

RS, Rath, Zwerger, EPJB 85 (2012)

RS, PhD Thesis (2013)

- deviations around 10%
- similar to study of varying single-channel potential by Wang, D'Incao, Esry, Greene, PRL 108 (2012)

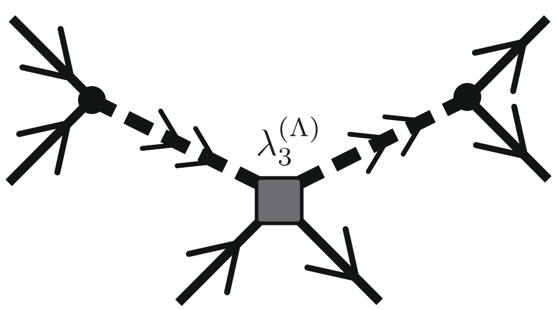
Test of universality II

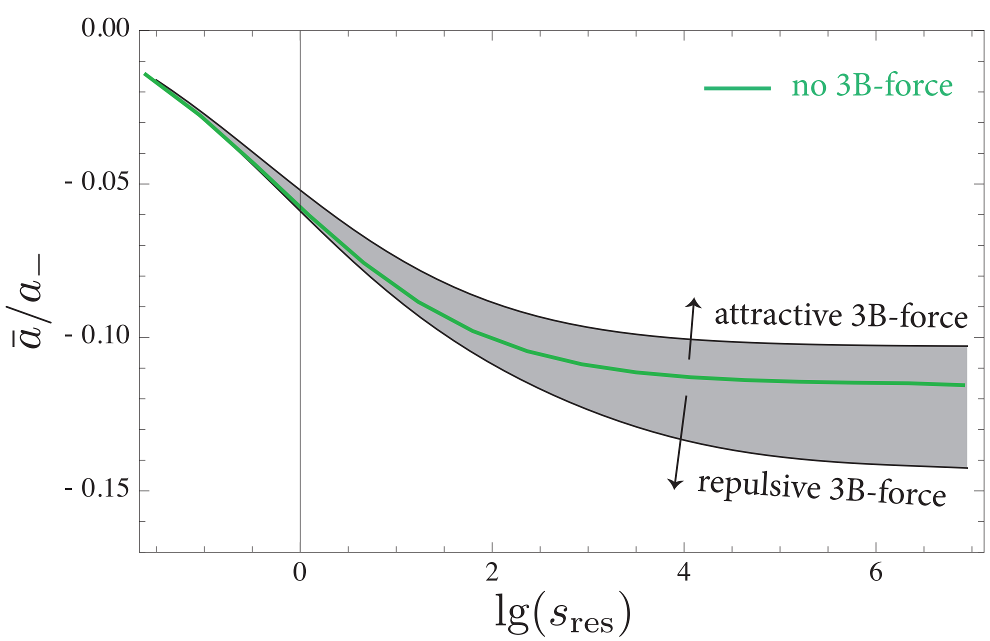
microscopic three-body force

- 3rd order pert. theory in dipole-dipole interaction: Axilrod-Teller three-body potential

$$W_{\mathrm{AT}} = \gamma \frac{1 + 3\cos\theta_{12}\cos\theta_{23}\cos\theta_{31}}{r_{12}^3 r_{23}^3 r_{31}^3}$$

- qualitative study assuming simplified three-body force





RS, Rath, Zwerger, EPJB 85 (2012)
RS, PhD Thesis (2013)

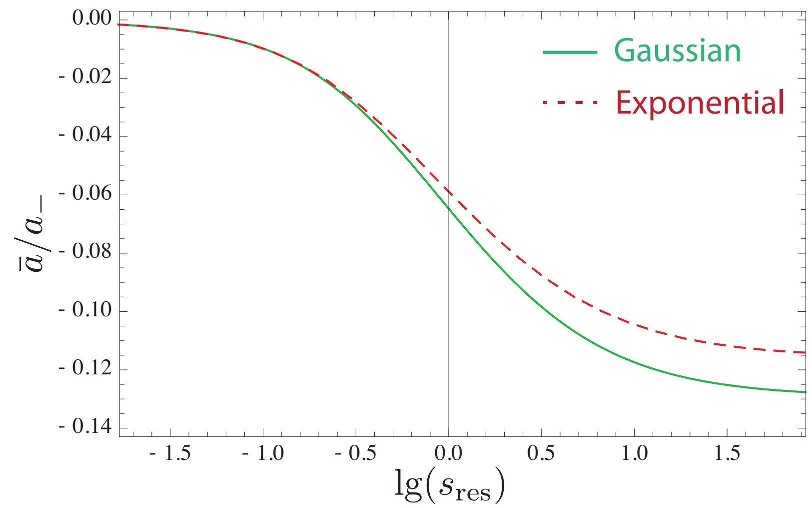
- even for infinite attraction only 10% change
- similar to: single-channel model + Axilrod-Teller yields also 10% deviation D'Incao, Greene, Esry, JPB 42 (2009)

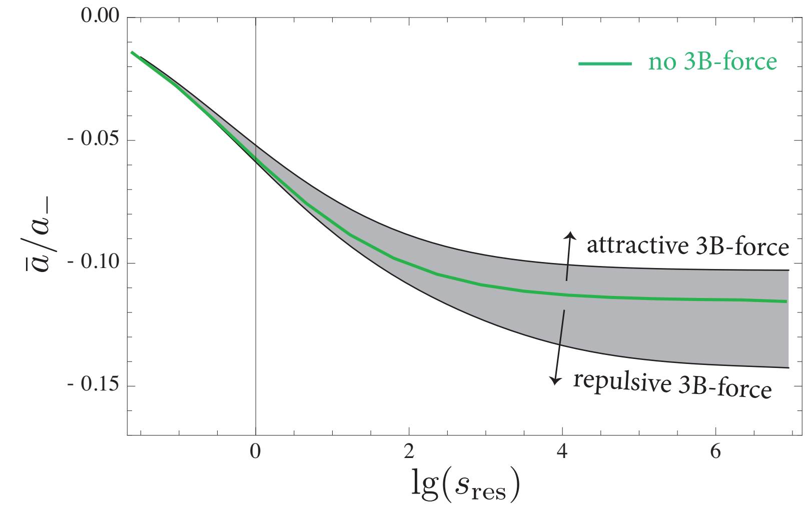
Test of universality III

two-channel model

RS, Rath, Zwerger, EPJB 85 (2012)

see also: Zinner et al. PRA 86 (2012)

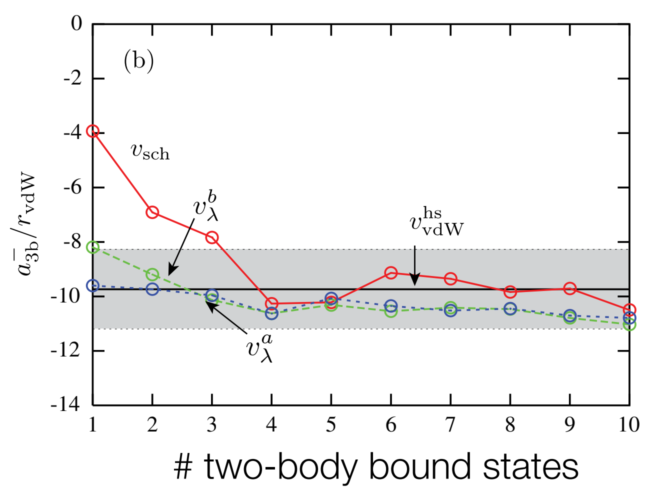


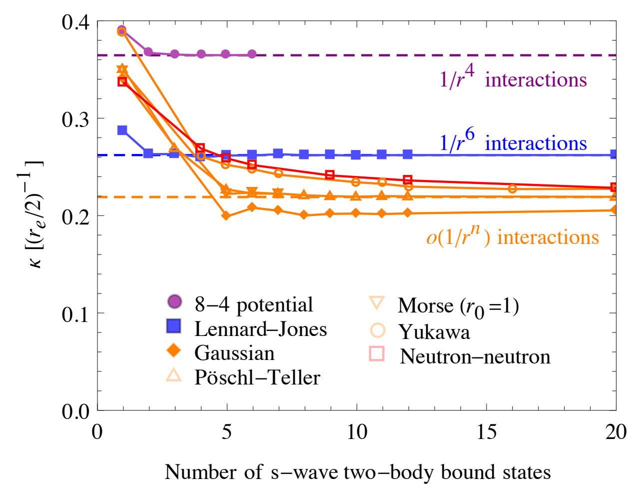


pure single-channel models

Wang, D'Incao, Esry, Greene, PRL 108 (2012)

Naidon et al. 1403.0294 (2014)



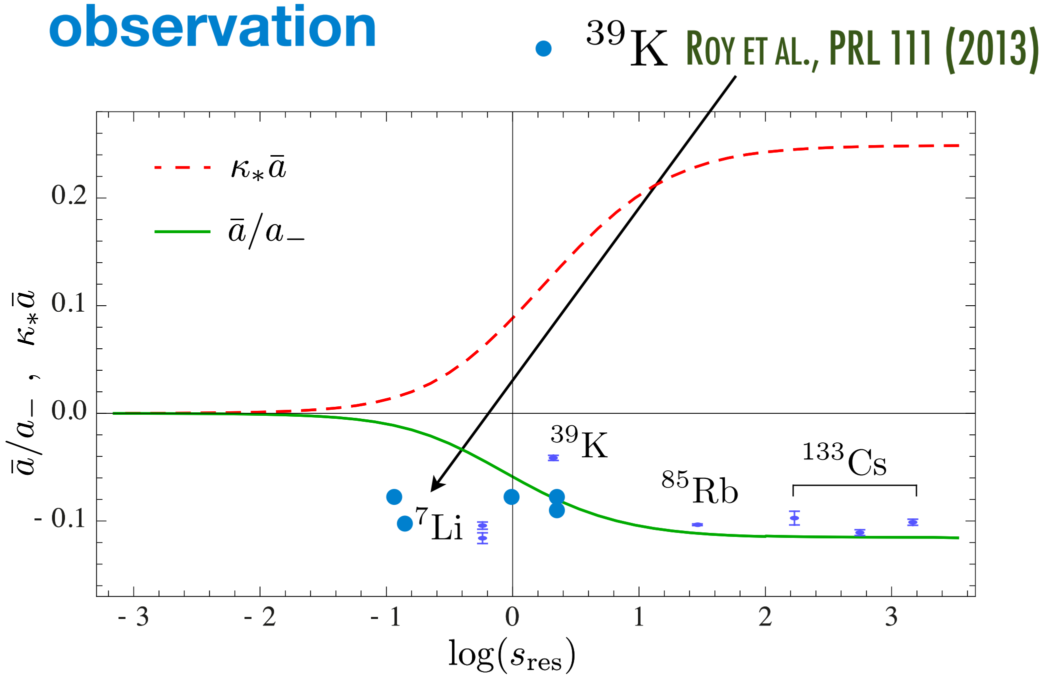


no true universality but consistent picture of an
robust approximate universality

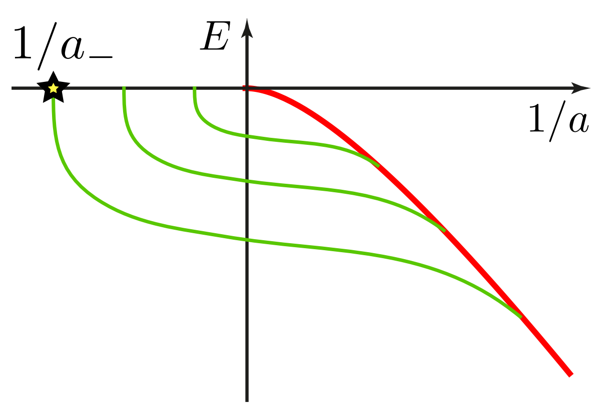
$$a_- \approx -(7.5\ldots10.5)\, l_{\rm vdw}$$

outlook: the Florence puzzle

observation



still follows single-channel prediction!

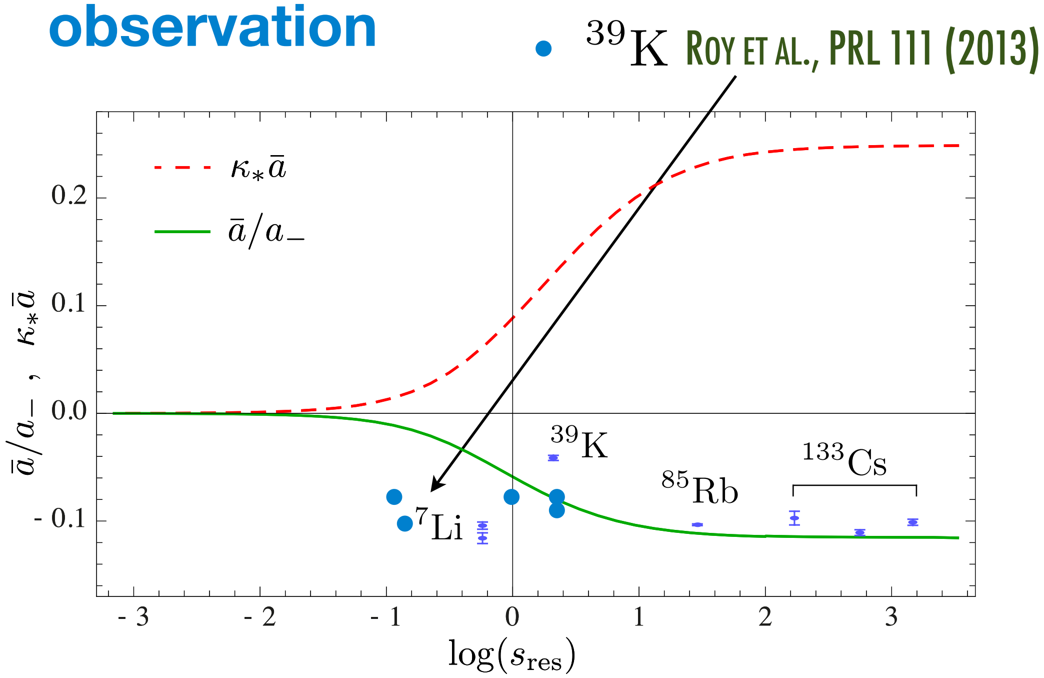


the puzzle

- narrow resonance: single-channel model insufficient
- but also: our ‚pure‘ two-channel model insufficient

outlook: the Florence puzzle

observation



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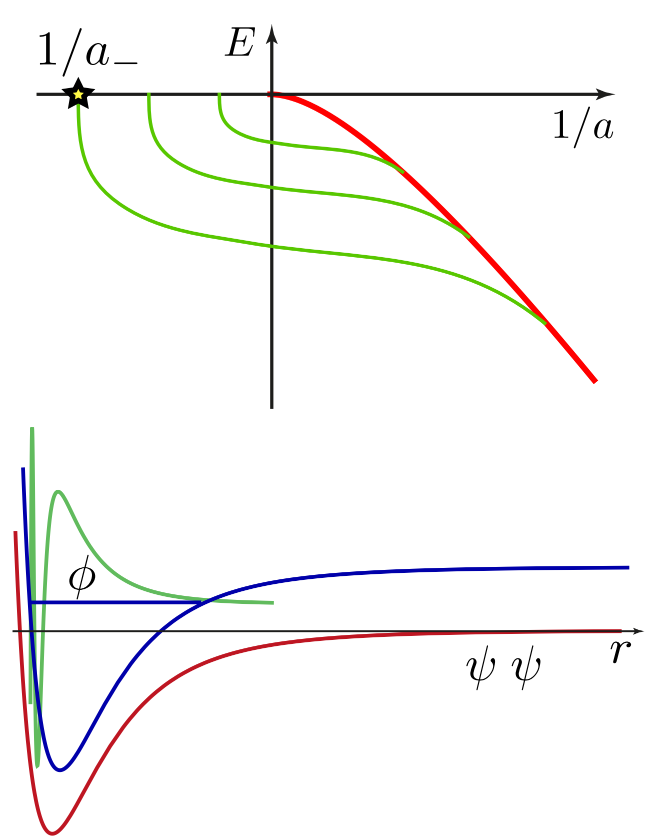


the puzzle

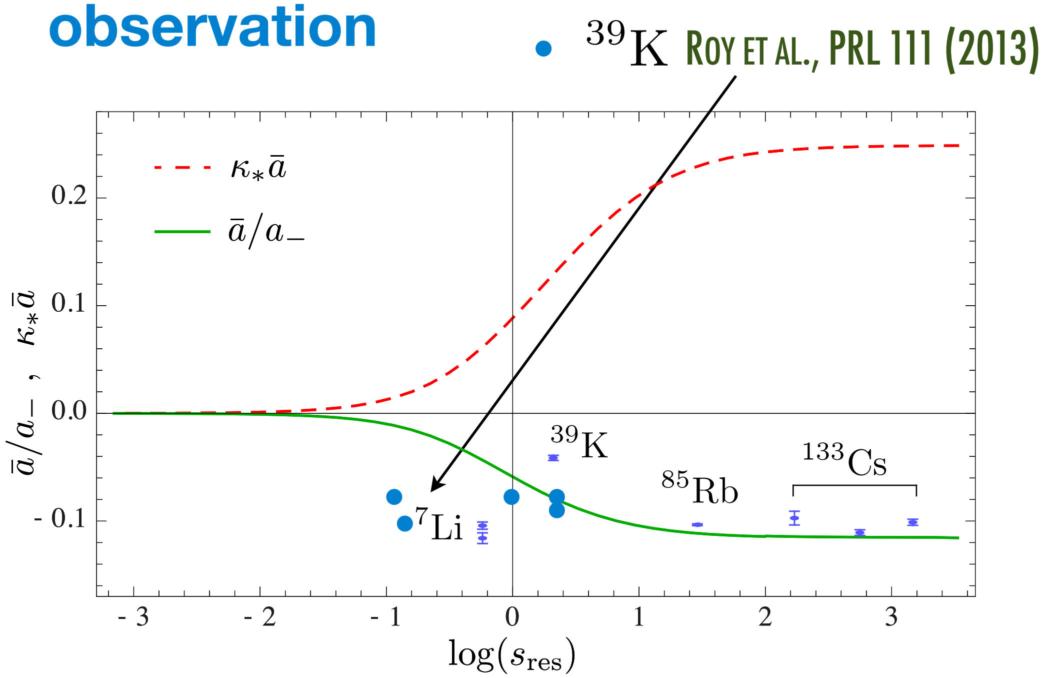
- narrow resonance: single-channel model insufficient
- but also: our ‚pure‘ two-channel model insufficient

possible solution

calculation with *realistic two-channel potential*
including weak open-channel interaction
[in *density channel]*

outlook: the Florence puzzle

observation



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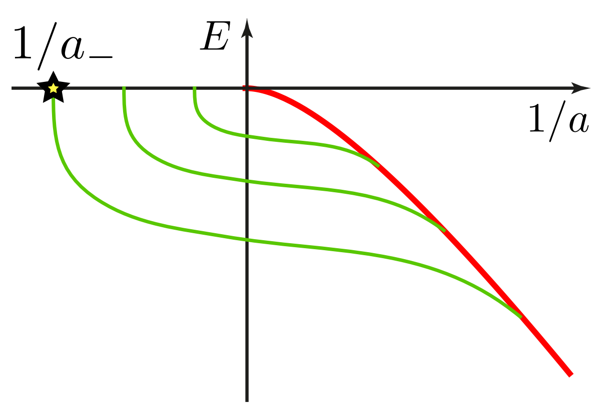

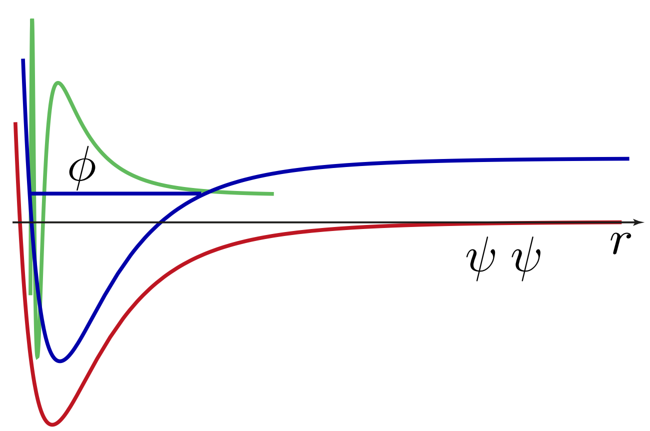

the puzzle

- narrow resonance: single-channel model insufficient
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possible solution

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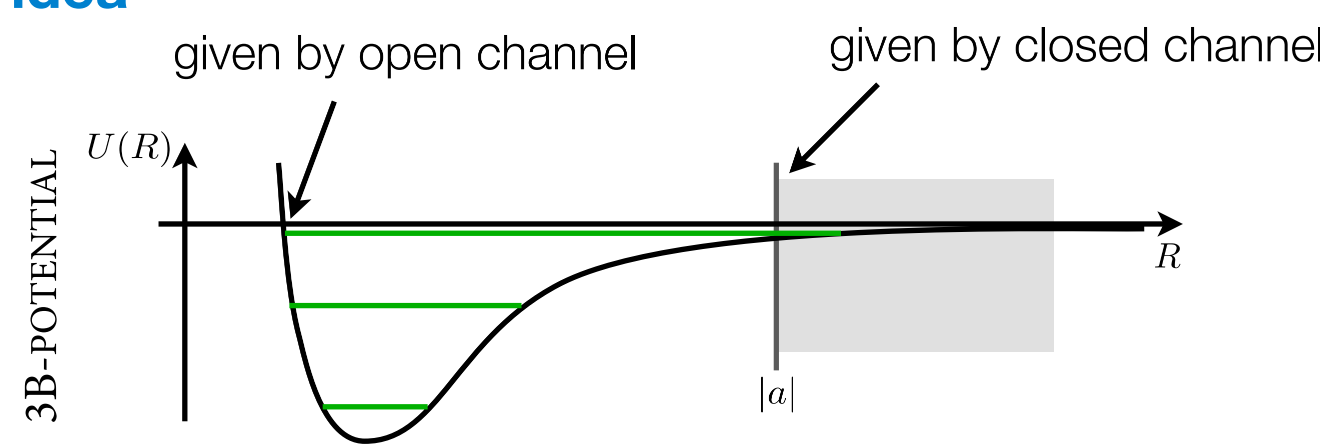
idea



- even weak background scattering gives 3-body potential

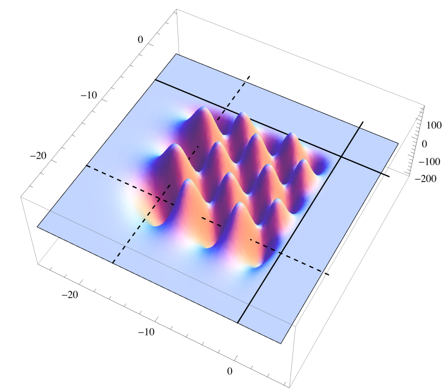
  short-range cutoff $\rightarrow l_{\rm vdw}$

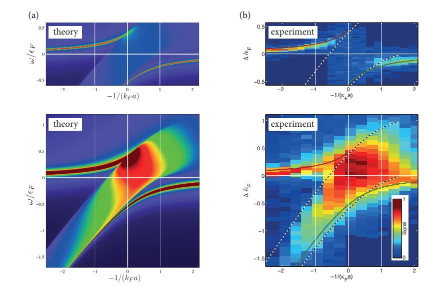
- closed-channel scattering gives large a

  only for large enough a : scaling with r^*

summary

- the functional renormalization group as unified approach for few- and many-body problems





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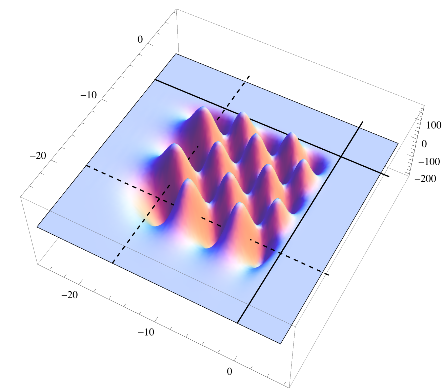
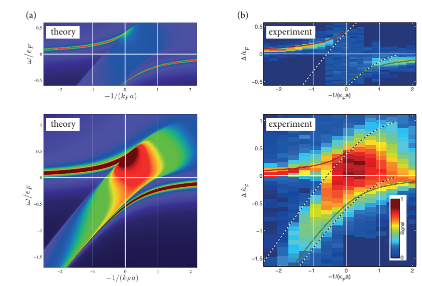
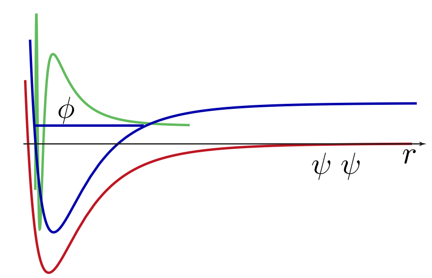




- derivation of an exact solution for Efimov physics for simple two-channel model



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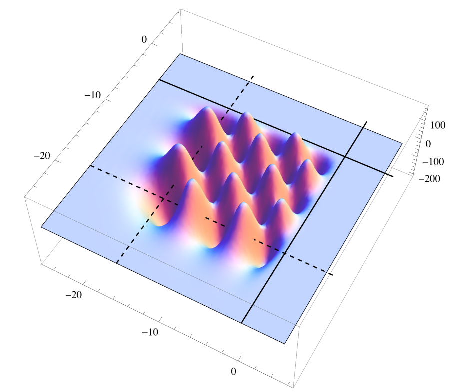

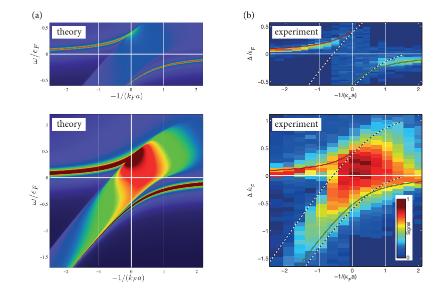

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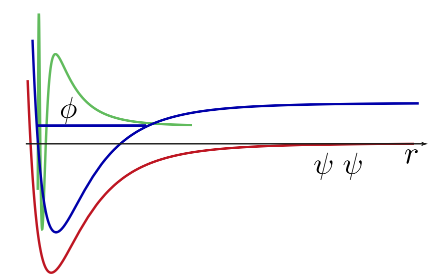
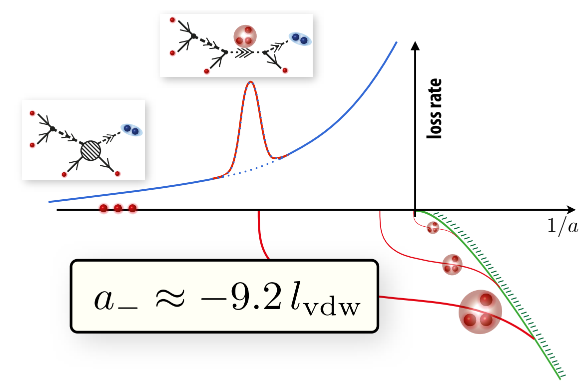
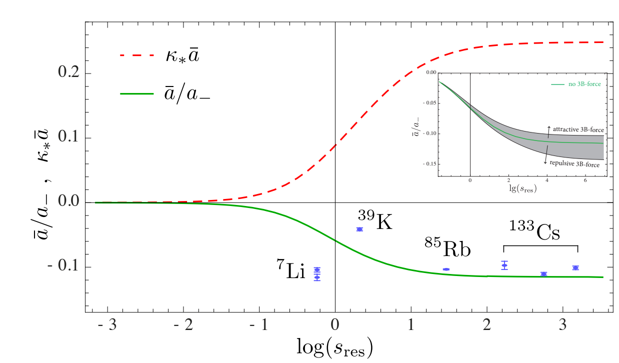


- the model explains observed 'universality of the three-body parameter'
  - no fit parameter
  - extendable to many-body physics





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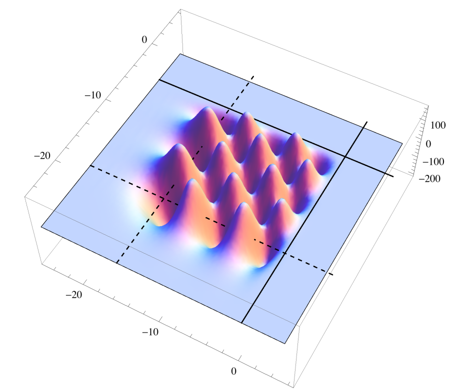

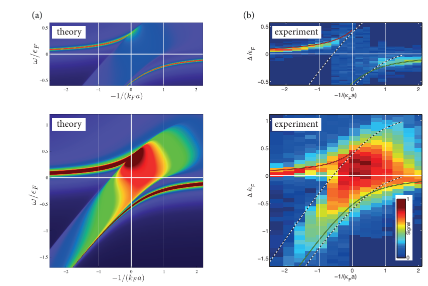

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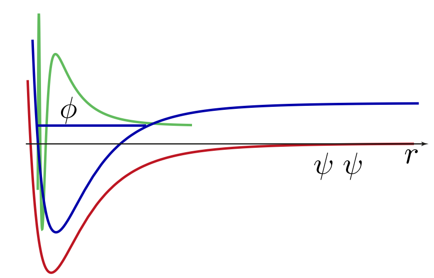

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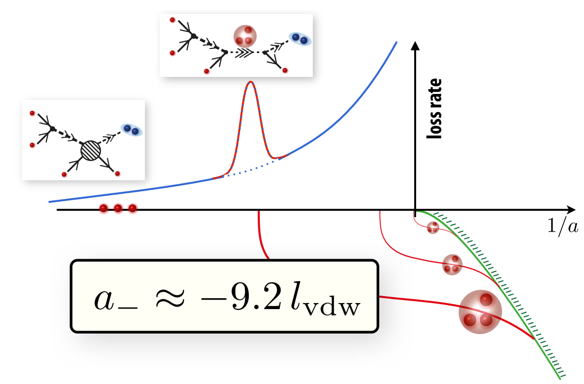
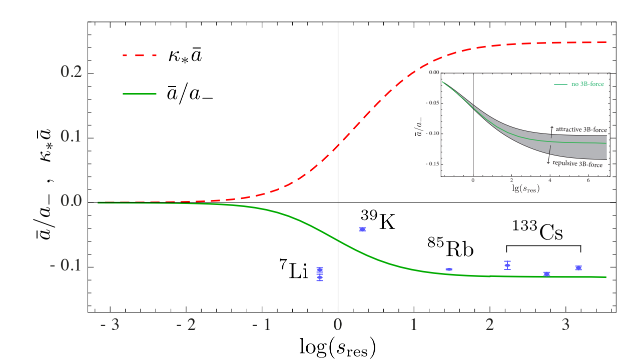
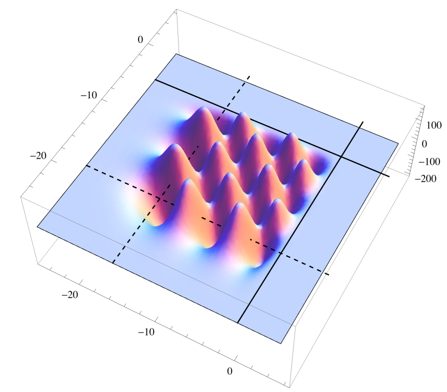


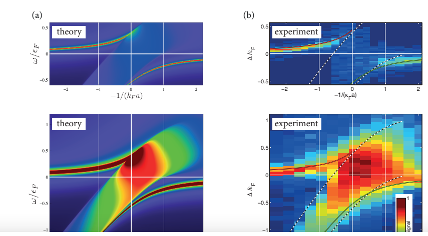


- predictions for extended class of universal ratios for 'narrow' resonances
- we find rather robust universality of the three-body parameter for 'broad' resonances
- open question: closed channel dominated resonances have still to be understood in more detail

summary

- the functional renormalization group as unified approach for few- and many-body problems





Thank you!

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