Strongly-interacting few-fermion systems in a trap

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CHALMERS

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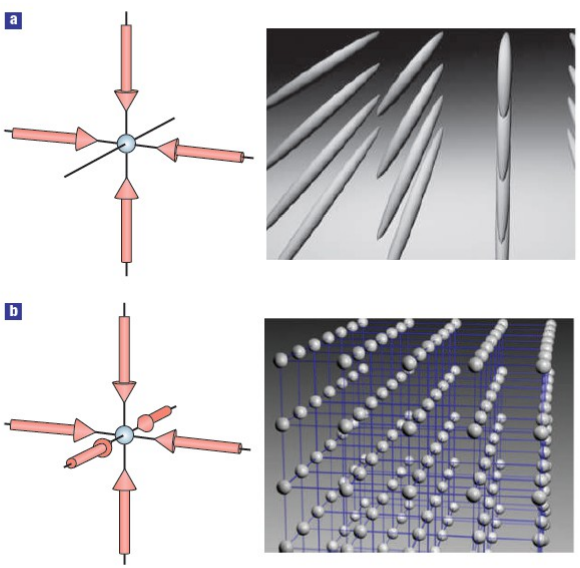
European Research Council
erc

Universality in few-body systems: Theoretical challenges and new directions, INT Program INT-14-1

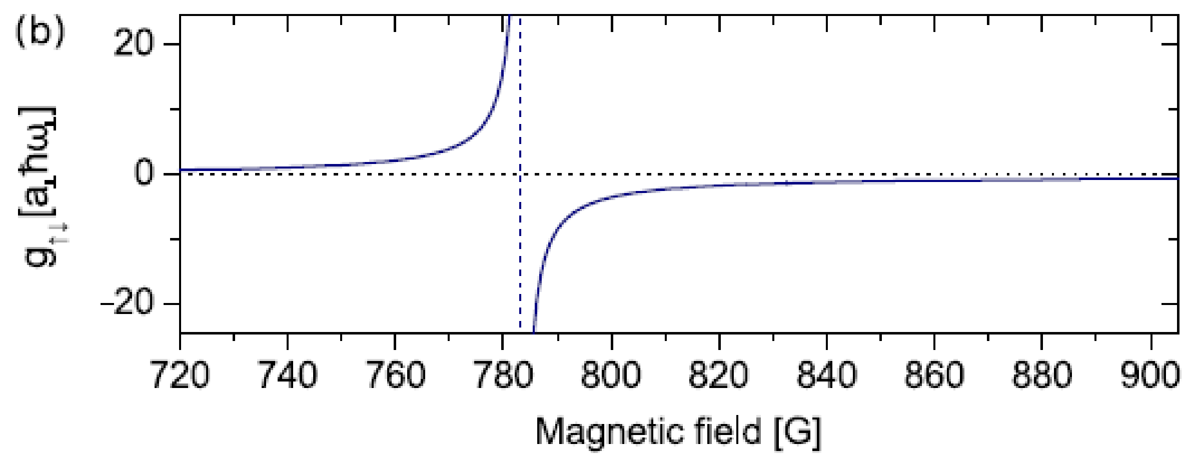
Experimental realization of controllable, strongly-interacting quantum systems.

Optical lattice



Bloch, Nature Physics (2005)

Tunability of interaction via magnetic Feshbach resonance



Zürn et al, PRL 108 (2012)

* Cold, dilute systems

$$\rho R^3 \ll 1$$

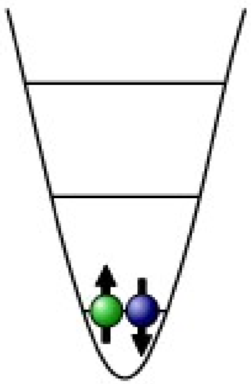
R : range of the potential, Van der Waals length

* Detailed knowledge of underlying interaction is not important for these systems:

$$k\cot(\delta) = -\frac{1}{a_2}$$ for $k \ll 1/R$

$$V^\delta = \frac{2\pi\hbar^2}{\mu} a_2 \delta(\mathbf{r}) \frac{\partial}{\partial r} r$$ zero-range pseudopotential

Two particles in a Harmonic Oscillator (HO) trap



-> two distinguishable fermions
-> two neutrons/protons in singlet channel
……

-> for b/R >> 1, the spectrum is given by the Busch formula :

HO length $b = \sqrt{\frac{\hbar}{\mu\omega}}$

$$\frac{\Gamma(3/4 - E/2\hbar\omega)}{\Gamma(1/4 - E/2\hbar\omega)} = \frac{b}{2a_2}$$

Busch et al, Found. Phys (1998)

For A>2, besides the unitary case at A=3, one has to rely on numerical approaches.

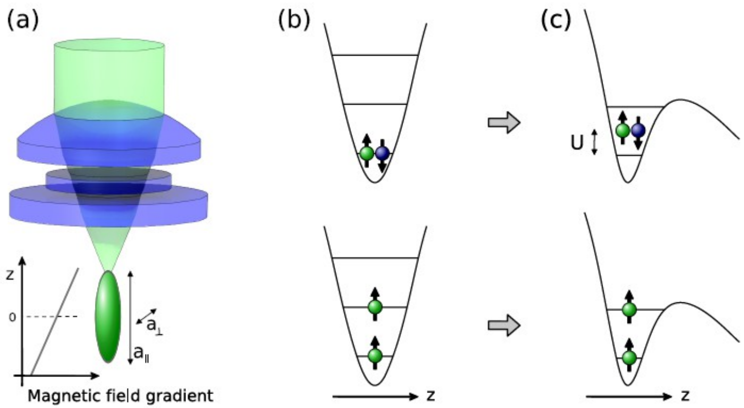
Outline

i) Derivation of an effective interaction for few-body systems in a trap

ii) 3-D systems of two-component identical fermions

iii) Fermionization of (distinguishable) fermions in 1-D

iv) Tunneling theory for two-particles



taken from Zürn et al, PRL 108 (2012)

Few (two-component) fermions in a 3D HO trap

$$H = \sum_{i<j} \frac{(\vec{p_i} - \vec{p_j})^2}{2mA} + \frac{m\omega^2}{2A} \sum_{i<j} (\vec{r_i} - \vec{r_j})^2 + \sum_{i<j} V_{ij}$$

Resolution of the Schrödinger equation by expansion of the solution in a *finite* HO basis

$$|\Psi\rangle = \sum_i c_i |u_1 \ldots u_A\rangle_i \quad \begin{cases} |u\rangle \equiv |nlj\rangle \\ E_{nl} = (2n + l + 3/2)\hbar\omega \end{cases}$$

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?

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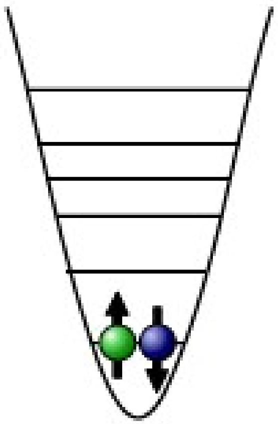
* Effective Field Theory : interaction tailored to the Hilbert space, improvable order by order.	Stetcu et al, PRA 76 (2007); J. R et al, PRA 82 (2010); S. Tölle et al J.Phys. G40 (2013)...
* Separable interaction fitted such that the exact two-body spectrum is reproduced in a truncated (two-body) space.	Y. Alhassid et al; PRL 100 (2008)
* Short-range interactions used with Monte Carlo techniques.	S. Chang et al, PRA 76 (2007); D. Blume et al, PRL 99 (2007); N.T. Zinner et al, PRA 80 (2009)....

Our approach : Unitary transformation of the exact two-body spectrum

$$\frac{\Gamma(3/4 - E/2\hbar\omega)}{\Gamma(1/4 - E/2\hbar\omega)} = \frac{b}{2a_2}$$



$$\phi(r) = Are^{-\frac{r^2}{2b^2}} U\left(\frac{3/4 - E/2}{\hbar\omega}, 3/2, r^2/b^2\right)$$

$E^{(2)}$ and X matrices formed with the energies and eigenvectors in the infinite Hilbert space

$$H^{(2)} = X^\dagger E^{(2)} X$$

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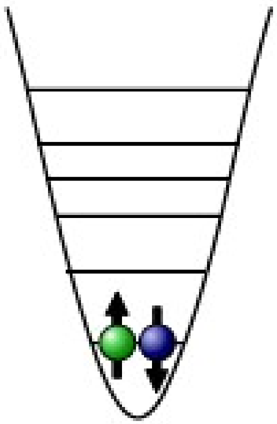


$E^{(2)}$ and X matrices formed with the energies and eigenvectors in the infinite Hilbert space

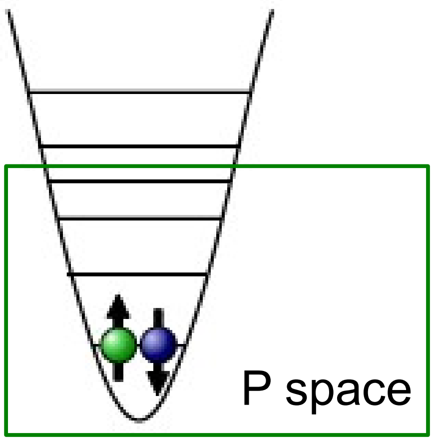
$$H^{(2)} = X^\dagger E^{(2)} X$$

Effective interaction in a two-body truncated space P:

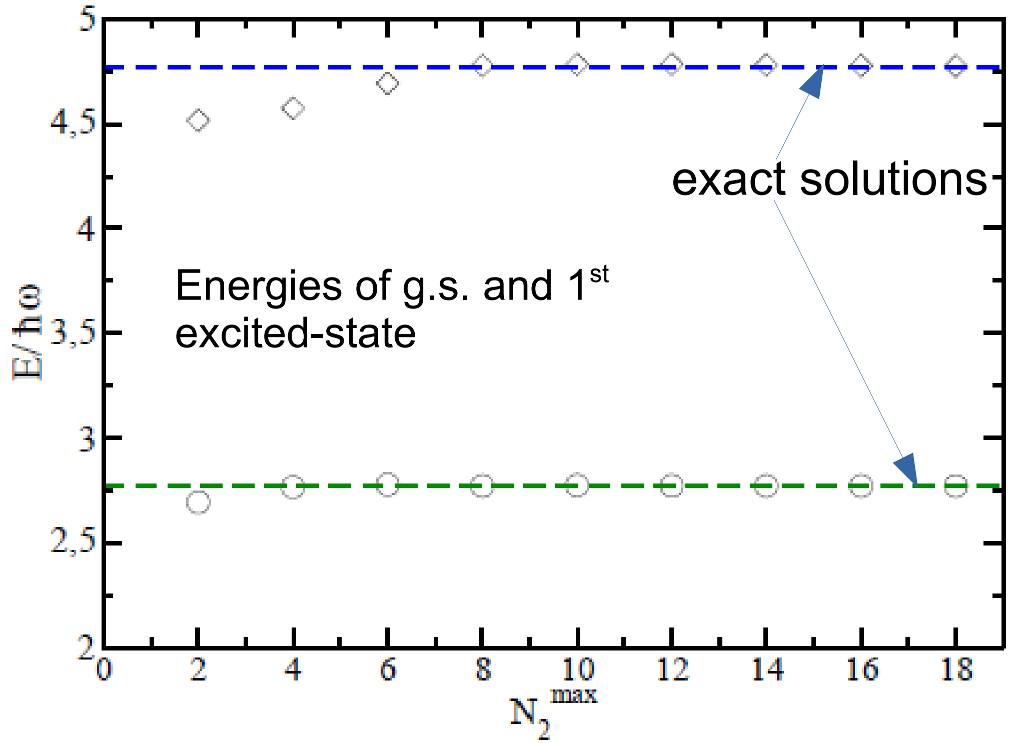
$$H_P^{eff} = \frac{X_P^\dagger}{\sqrt{(X_P^\dagger X_P)}} E_P^{(2)} \frac{X_P}{\sqrt{(X_P^\dagger X_P)}}$$

→ two-body energies reproduced in P (by construction)

→ eigenfunctions converge to “true” eigenfunctions as P grows

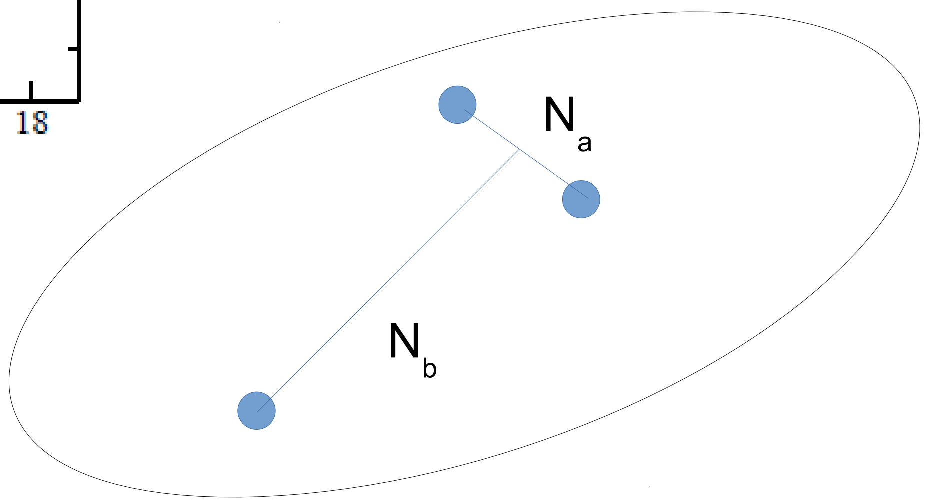


Three (two-component) fermions at Unitarity



exact solutions from F. Werner et al, PRL 97 (2006))

For $N_a > N_2^{max}$, the effective interaction gives no contribution.

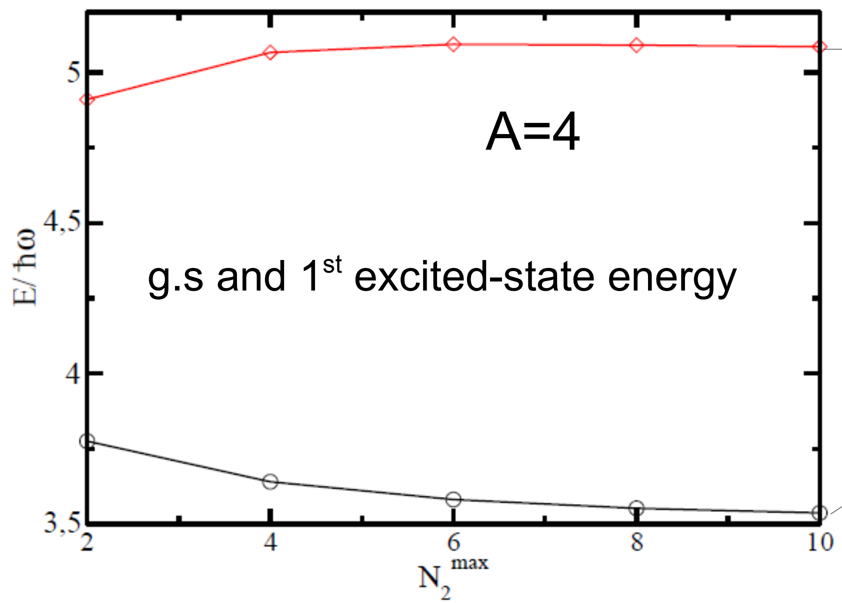


For a fixed N_2^{max} the three-body Hilbert is made larger, i.e., N_3 is increased, until convergence is reached.

In Jacobi coordinates: $N_3 = N_a + N_b$

J.R; EPJD 67 (2013)

A=4,5 fermions at Unitarity :

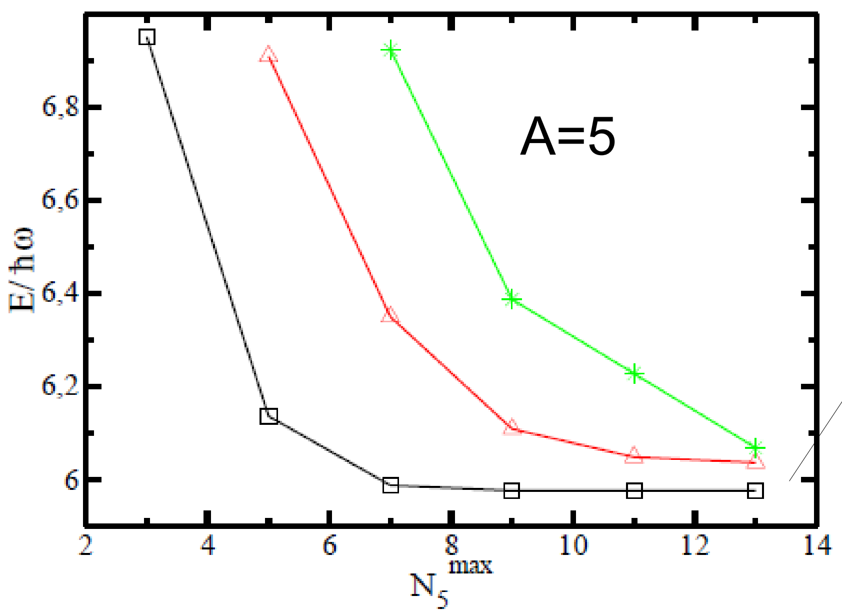


E=5.085

Other approach:
*5.07 (N^2LO EFT)

E=3.537

Other approaches:
*3.52 (N^2LO EFT)
*3.545 ∓0.003 (separable interaction)



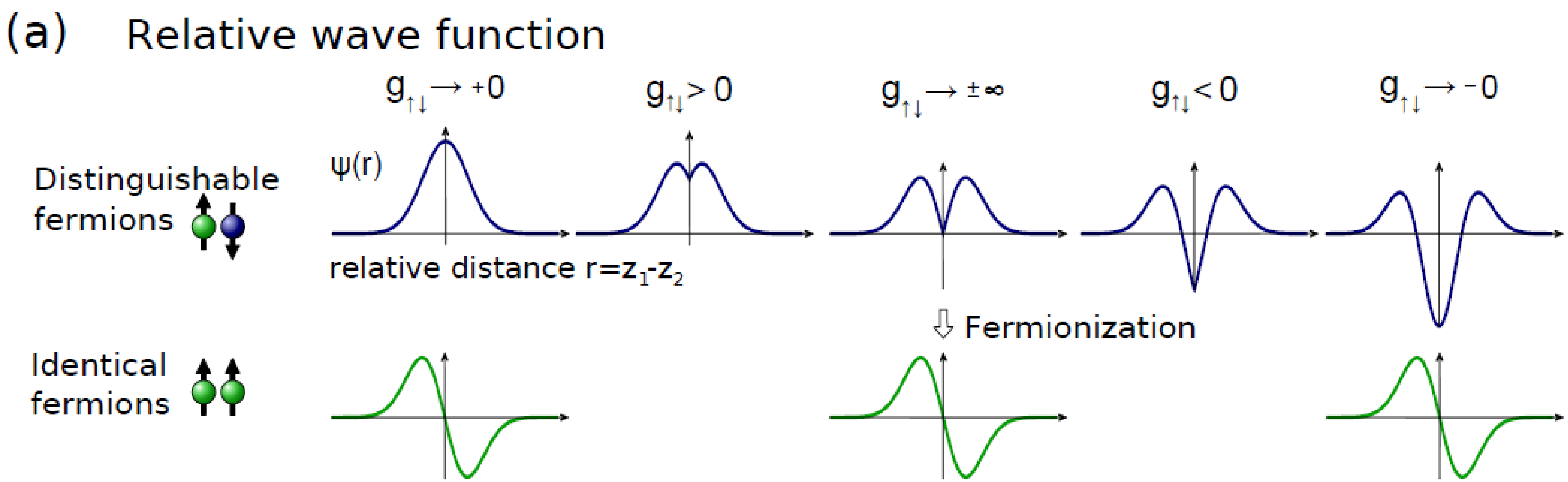
5.976 <E<6.068

Other approach:
*6.1 (Monte Carlo)

*excellent agreement with exact solutions (A=3) and other approaches (A=4,5)
*similar agreement for finite a_2,r_2

J.R, EPJD 67 (2013)

Fermionization of (distinguishable) fermions in 1-D

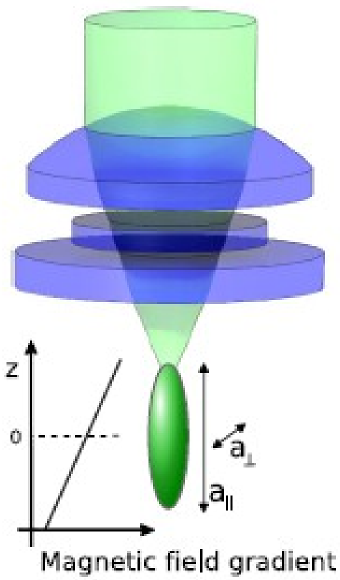


Experiment with the two lowest hyperfine states in ^{6}Li

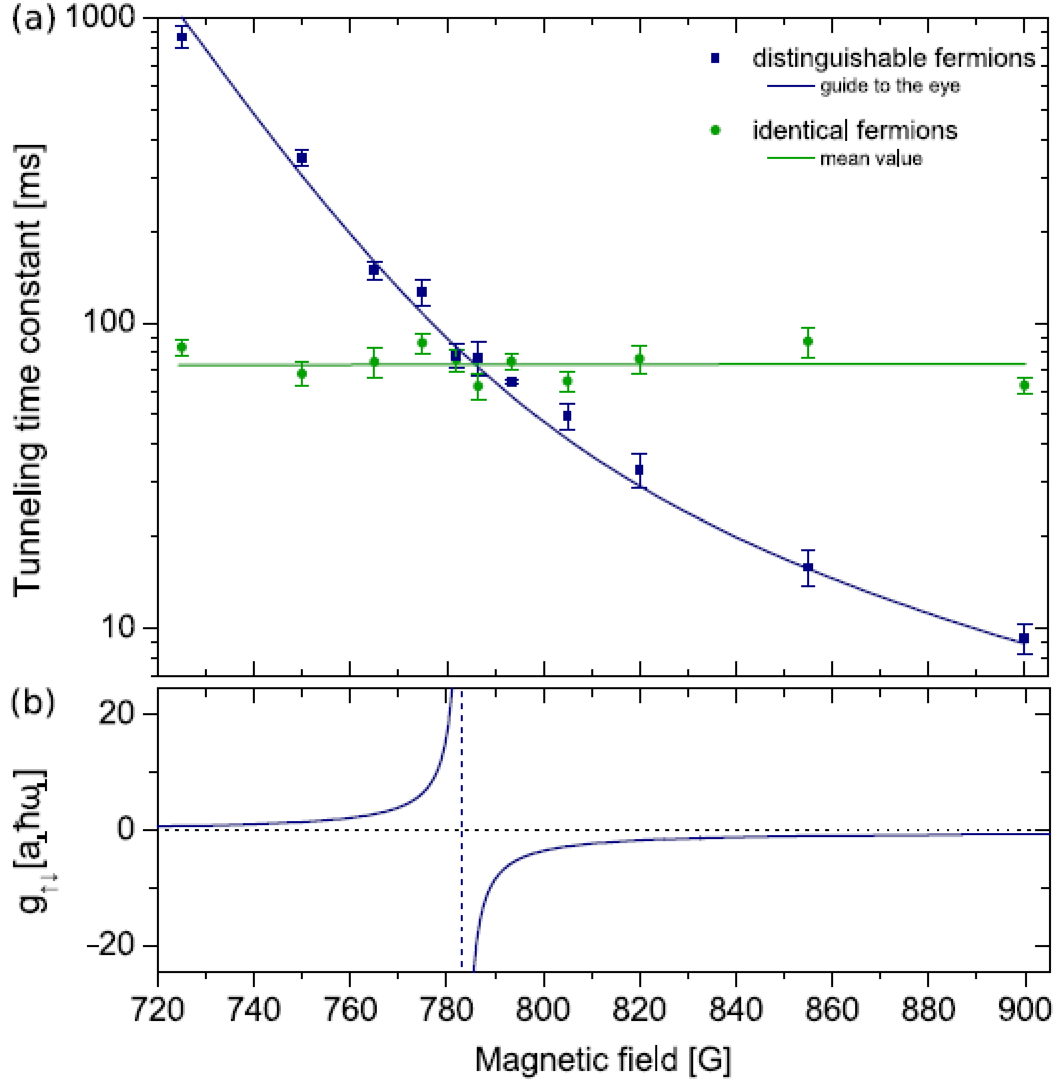
$$|F=\frac{1}{2},m_f=\frac{1}{2}\rangle$$

$$|F=\frac{1}{2},m_f=-\frac{1}{2}\rangle$$

1:10 asymmetric
Opto- Magnetic trap.



taken from G. Zürn et al., PRL 108 (2012).



Experimental observation of fermionization

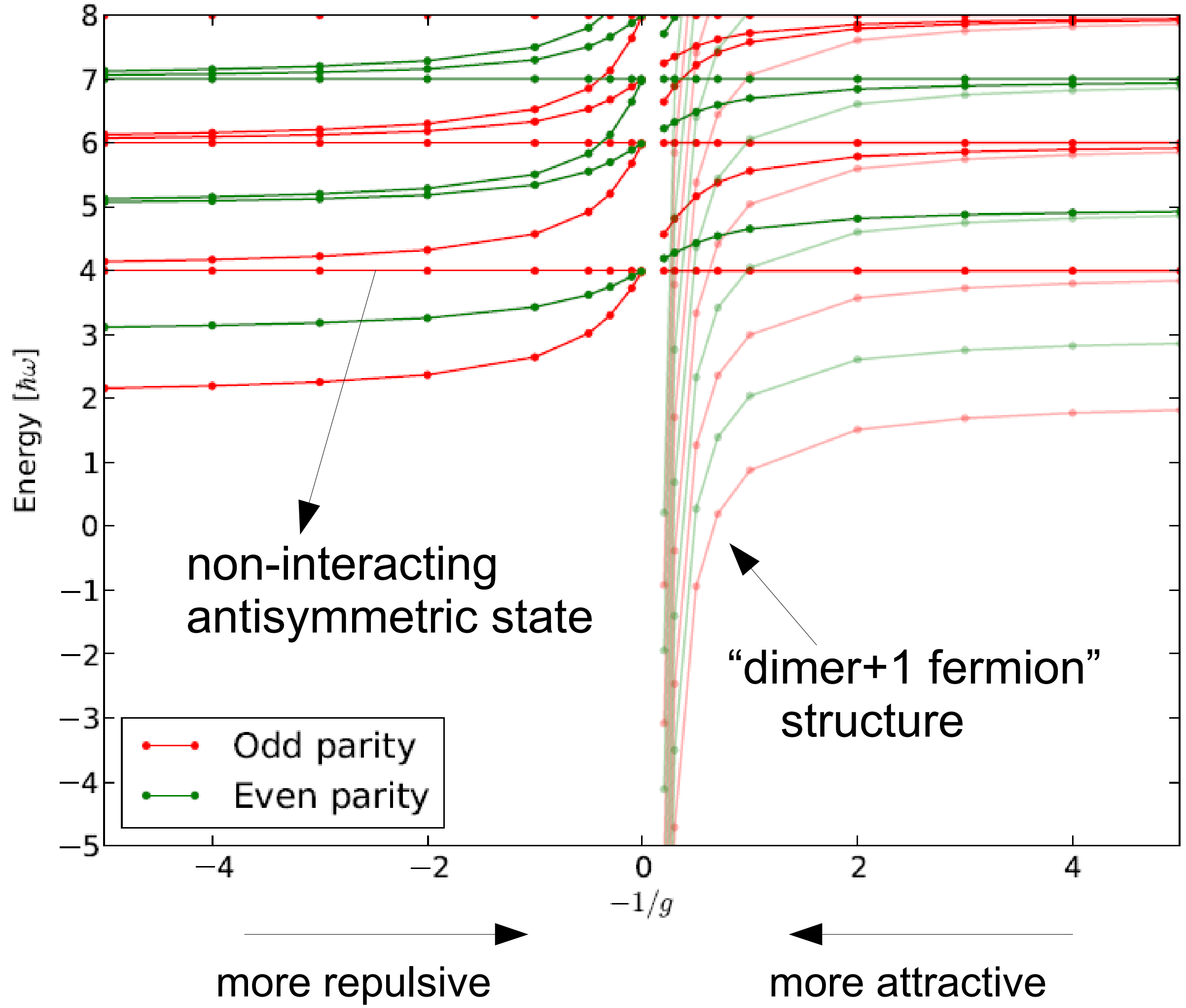
G. Zürn et al., PRL 108 (2012).

(2+1) fermions in 1D trap

* $V(x_1-x_2)=g\,\delta(x_1-x_2)$

* no interaction between the 2 identical fermions (Pauli)

* Effective interaction from the Busch formula in 1D

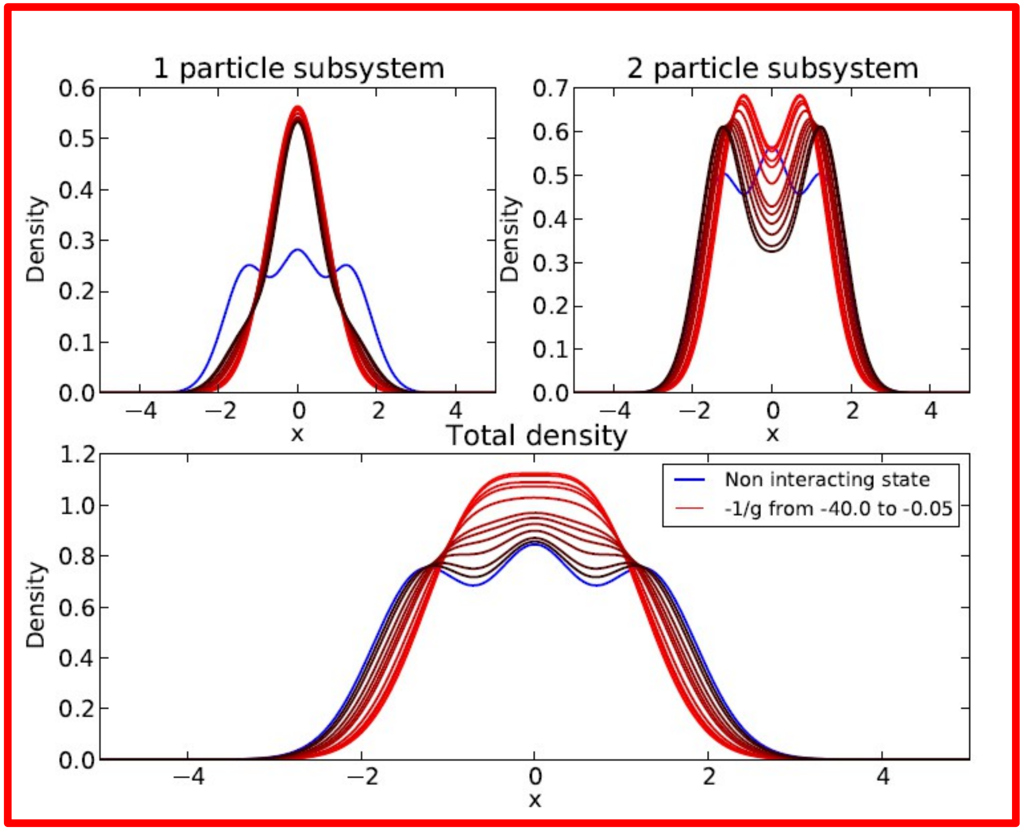


J. Lindgren et al., arXiv:1304.2992 (2013), to be published in NJP.

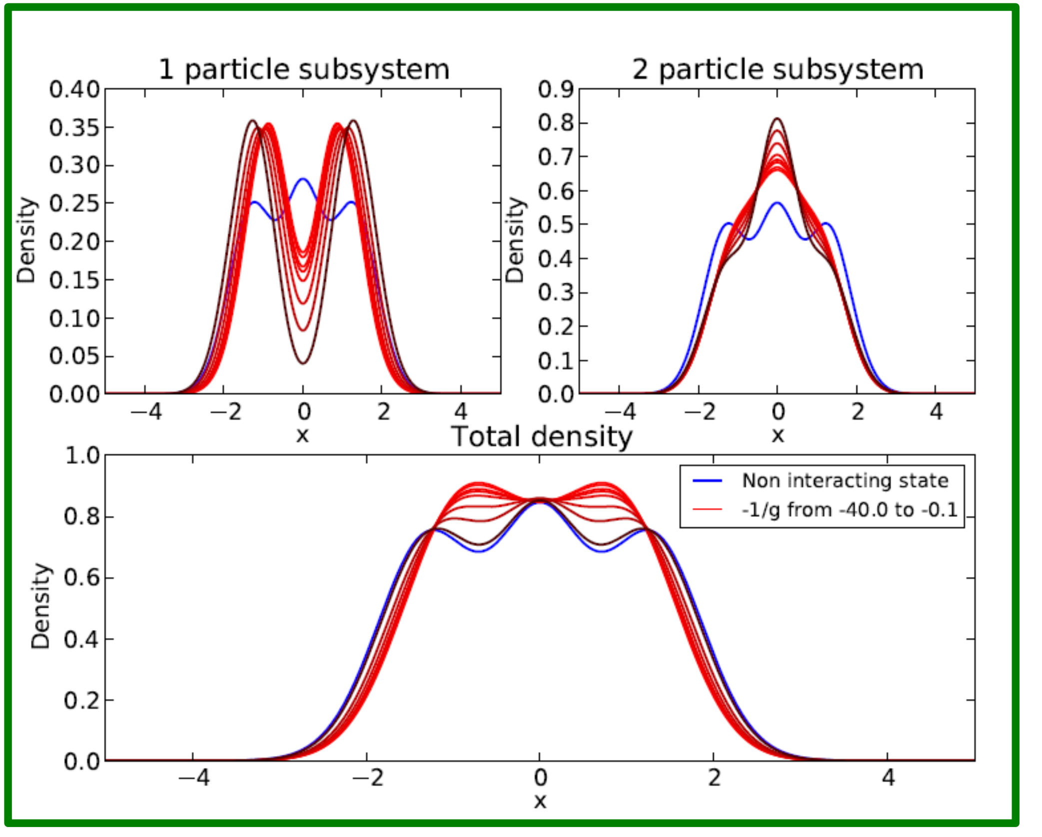
S.E. Gharashi and D. Blume, PRL 111 (2013).

Density distribution for the (2+1) g.s and 1st excited state

Ground state



1st excited state



* Interaction from weakly to strongly repulsive.

* For infinite repulsion, the total density is the same as for three identical (non-interacting) fermions.

Density distribution for the ground state in the (3+1), (6+1) and (9+1) systems

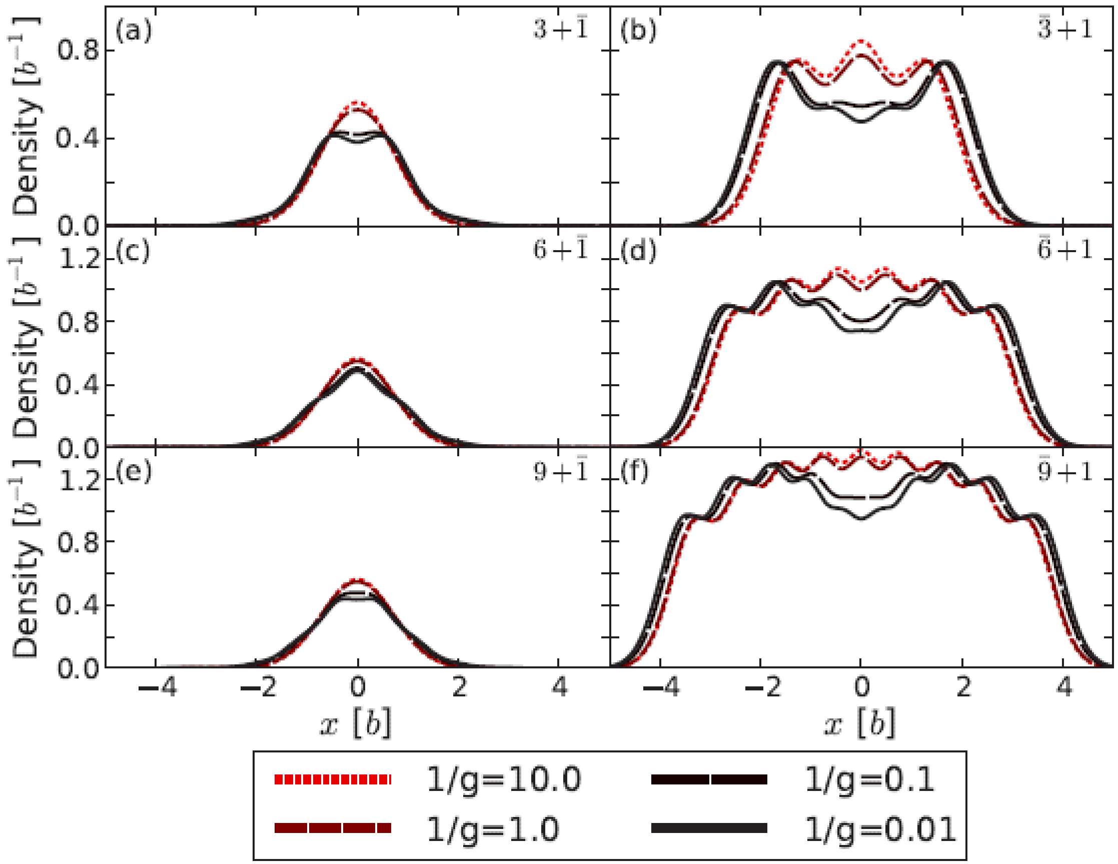
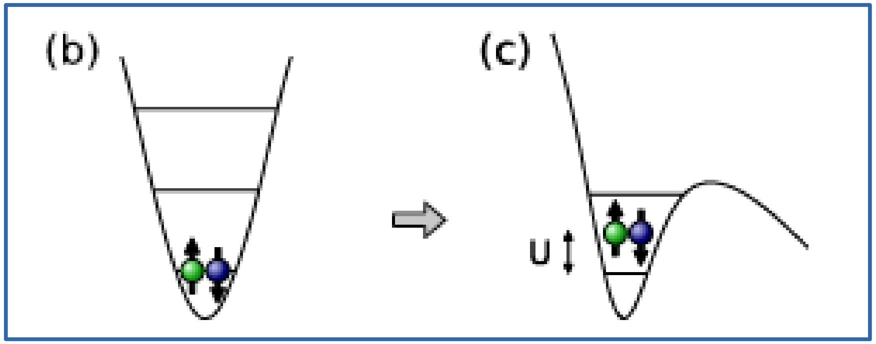
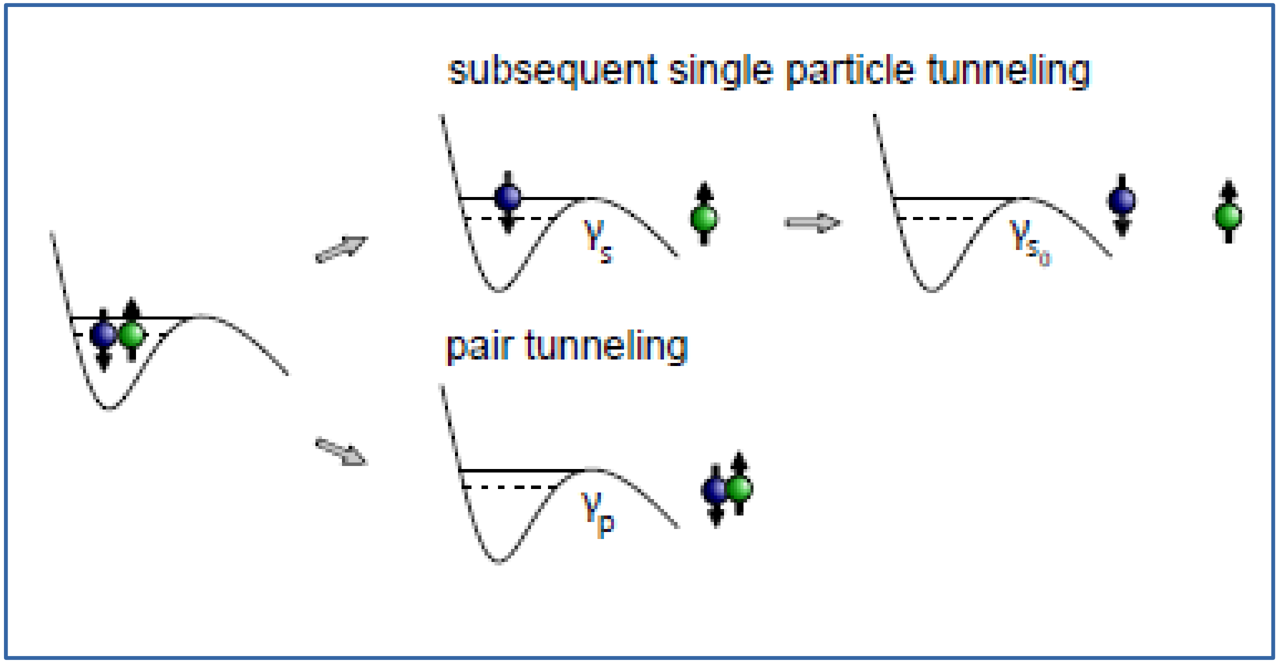


Figure 4. Spin-resolved densities for the 3+1, 6+1, and 9+1 systems, cf. Fig. 3. Panels (a), (c) and (e) show the distribution of the impurity particle, while panels (b), (d) and (f) show the majority density.

Tunneling theory for two particles escaping from the trap

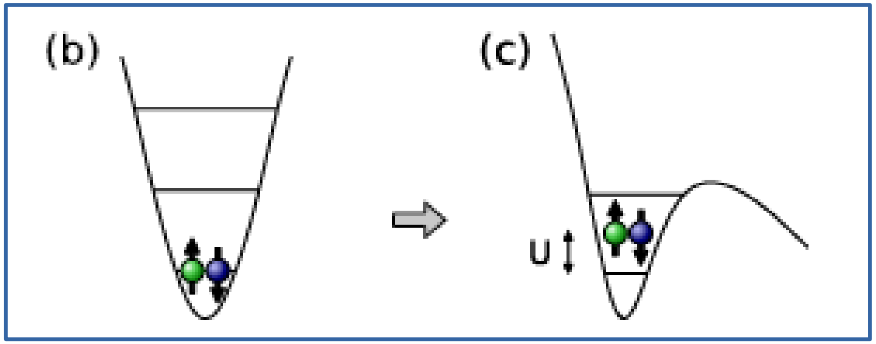


-> How do the two atoms tunnel out ?
-> How is the decay mechanism affected by the "pairing" interaction ?



taken from G. Zürn et al., PRL 108 (2012).

Tunneling theory for the two atoms escaping from the trap



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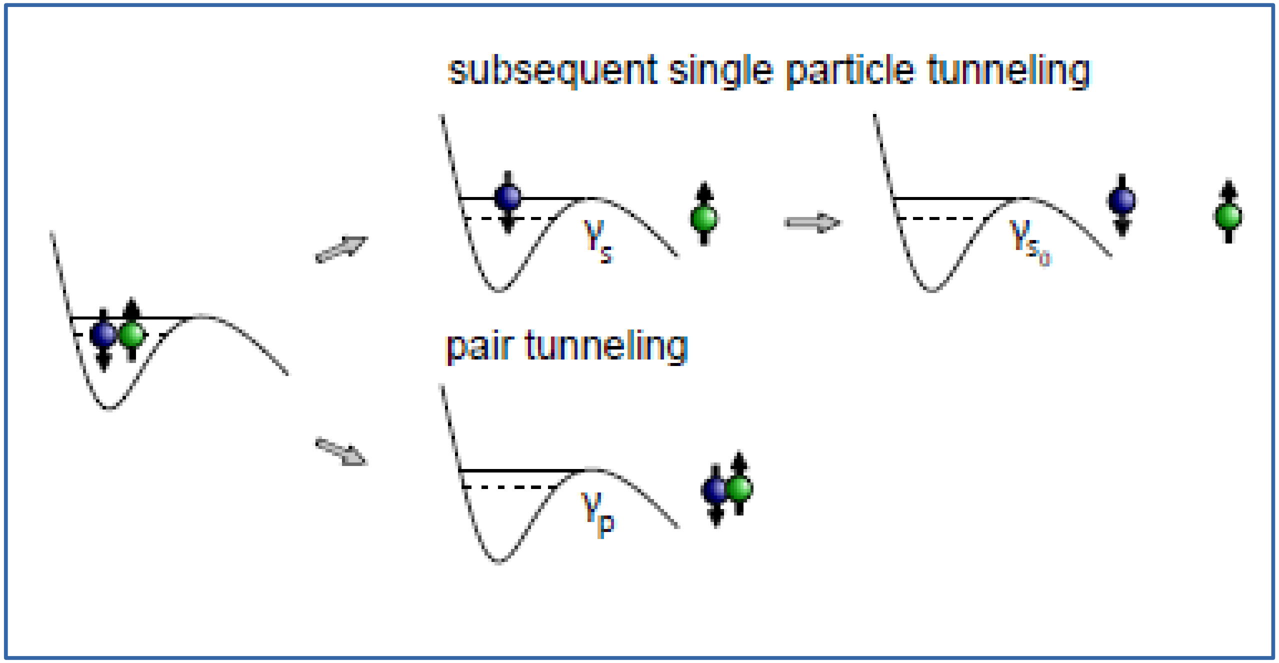
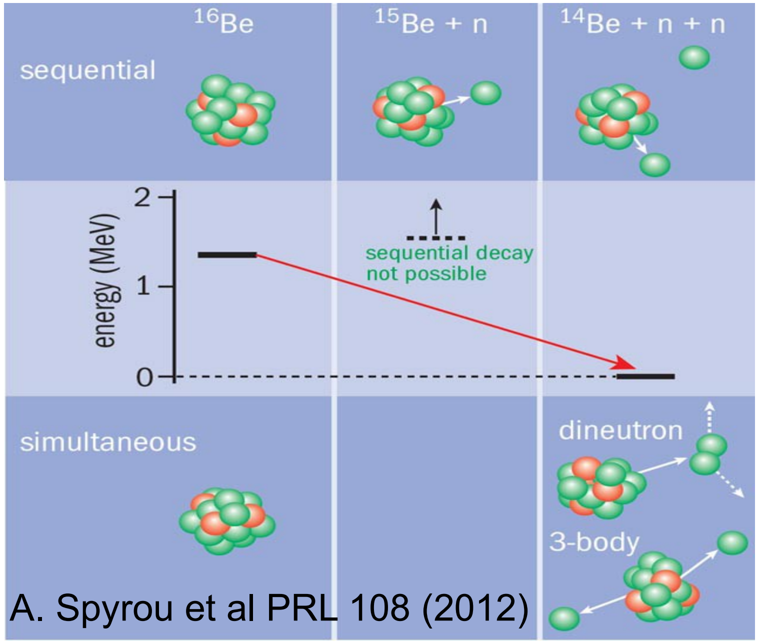


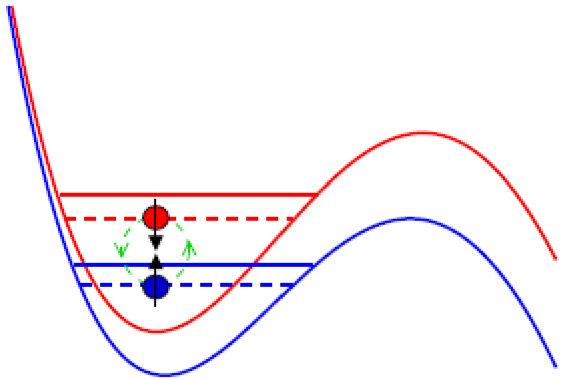
taken from G. Zürn et al., PRL 108 (2012).

Similar questions as for the decay of some exotic nuclei by emission of two neutrons (protons) !



A. Spyrou et al PRL 108 (2012)

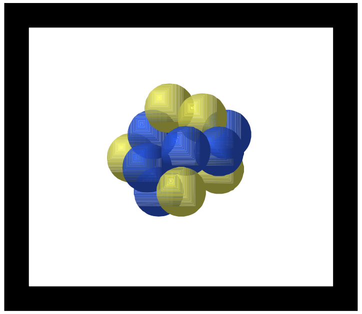
$$V(z) = pV_0\left(1 - \frac{1}{1+(z/z_R)^2}\right) - c_{B|state\rangle}\mu_B B' z$$



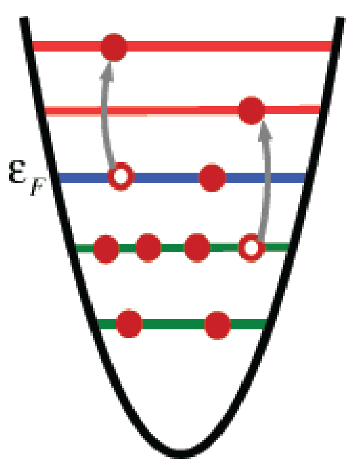
"The Trap"

Parameter	Value	Designation
V_0	$3.326\mu\mathrm{K}\cdot\mathrm{k_B}$	Potential depth.
z_R	$9.975\mu\mathrm{m}^2$	Rayleigh range of trapping beam.
μ_B	$6.717\,138\,8\cdot 10^5\,\mu\mathrm{K}\cdot\mathrm{k_B}/\mathrm{T}$	Bohr magneton.
B'	$18.92\cdot 10^{-8}\,\mathrm{T}/\mu\mathrm{m}$	Magnetic field gradient.
$c_{B\lvert\mathrm{state}\rangle}$	≈ 1	

Closed quantum systems



infinite well

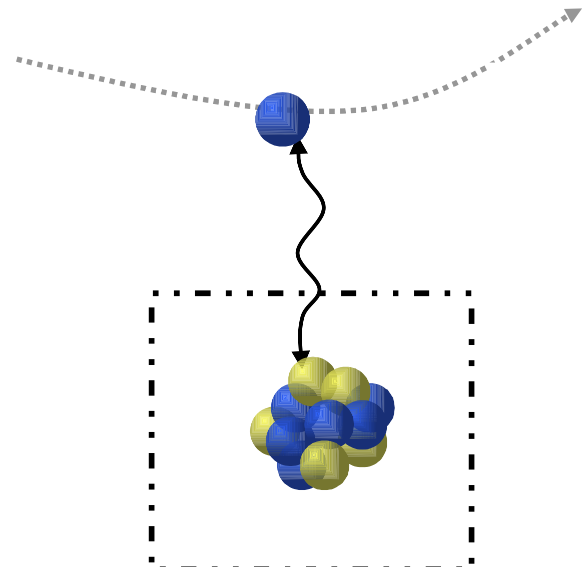


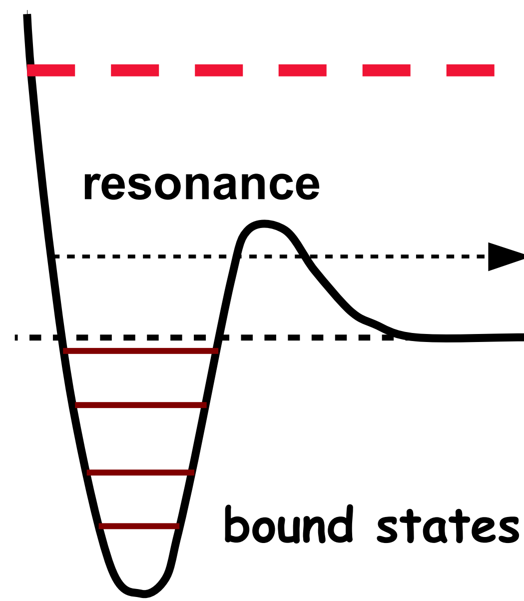
discrete states only, HO basis usually

exact treatment of the c.m, analytical solution…

Open quantum systems

(nuclei far from stability)



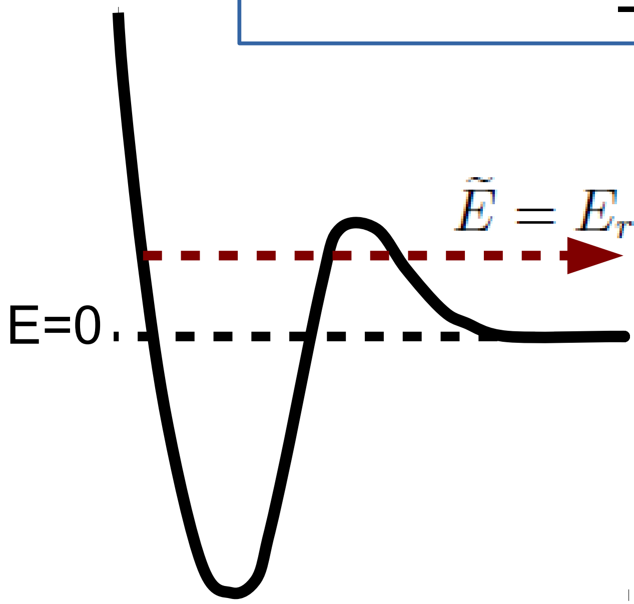


Gamow States

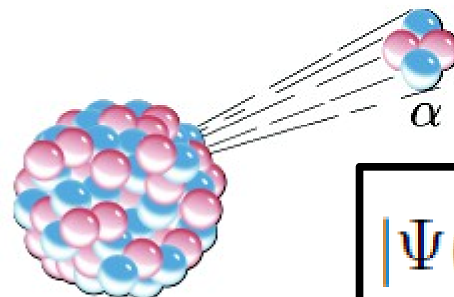
G. Gamow, Z. Phys. 51 (1928) 204

How to describe decay in a (quasi) stationary formalism ?

-> complex energy



$$\tilde{E} = E_r - i\frac{\Gamma}{2}$$



$$\Psi(t,r) = e^{\frac{-i\tilde{E}t}{\hbar}}\psi(r)$$

$$|\Psi(t,r)| \sim e^{-\frac{\Gamma t}{2\hbar}} e^{k_1 r}, r \to \infty$$

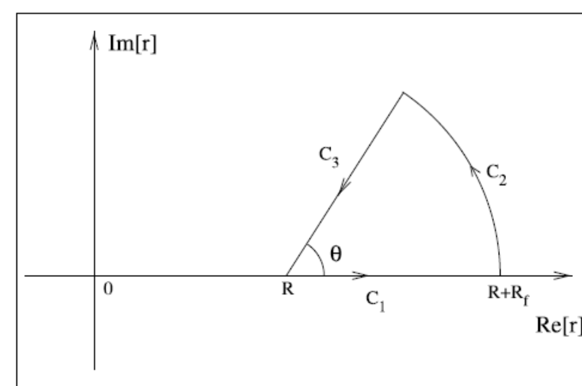
Radial Schrödinger Equation in 3D

$$\left(-\frac{d^2}{dr^2} + \frac{2\mu}{\hbar^2}V(r) + \frac{l(l+1)}{r^2} - k^2\right)\psi(r) = 0$$

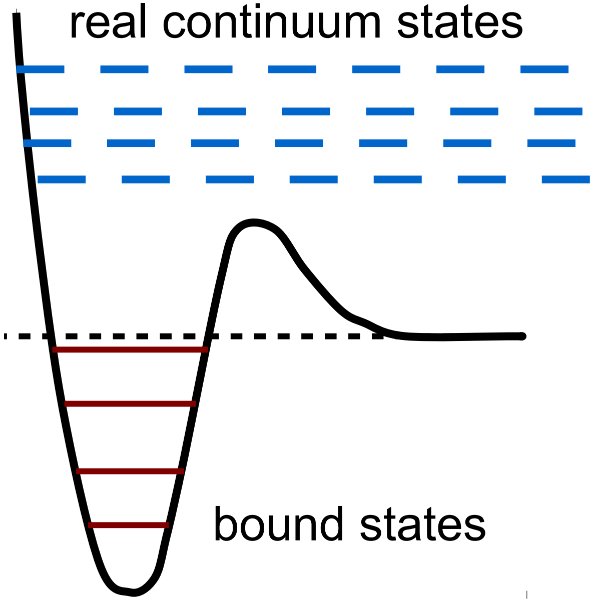
Asymptotic behavior of the Gamow states

$$\psi(r) \sim \mathcal{O}_l(kr) \sim e^{ikr}, r \to \infty$$

Normalization with the complex scaling



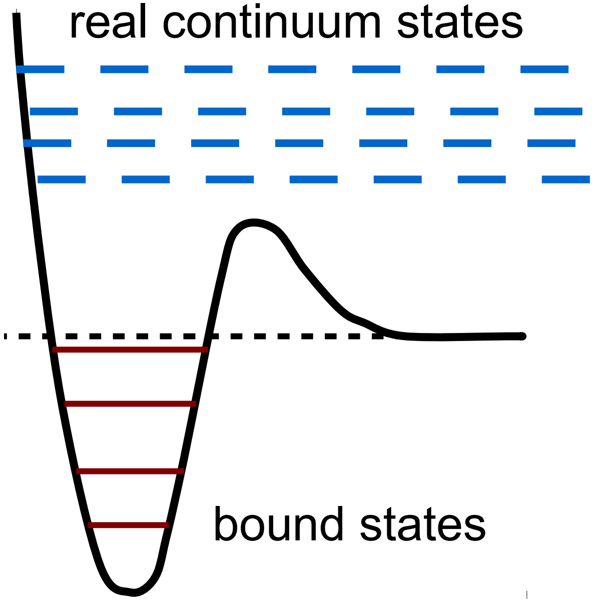
$$N_i = \sqrt{\int_0^R u_i^2(r)\,dr + \int_0^{+\infty} u_i^2(R + x \cdot e^{i\theta})\,e^{i\theta}\,dx}$$



Newton Completeness relation

$$\sum_b |u_b\rangle\langle u_b| + \int_0^{+\infty} dk|u_k\rangle\langle u_k| = 1$$

How to include resonance in this expansion ?



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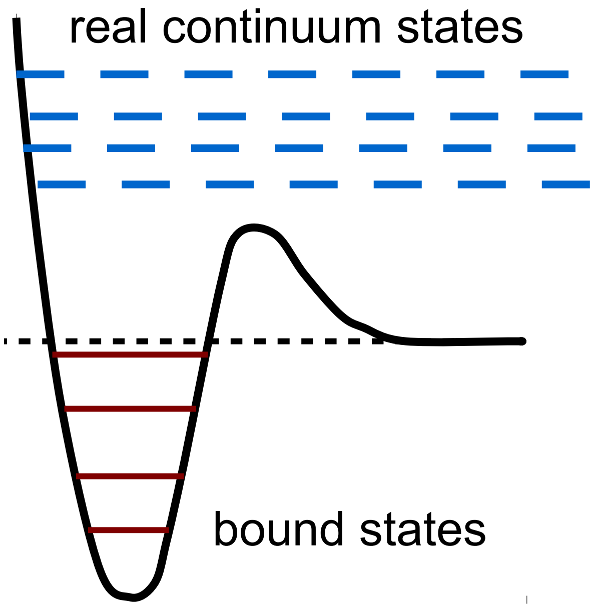

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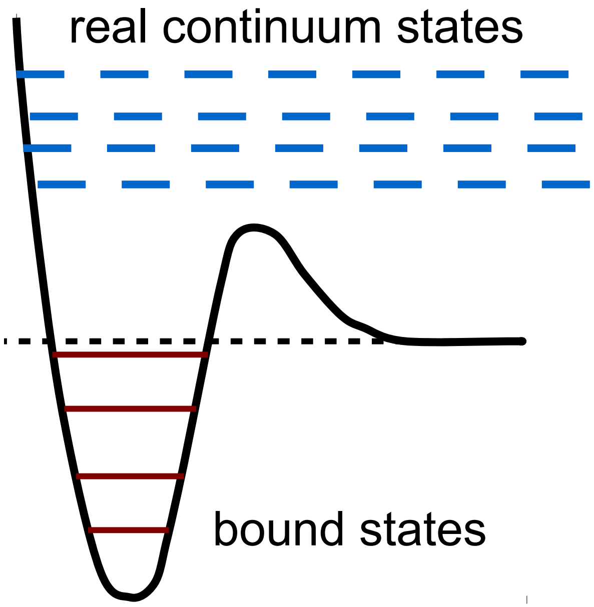
$$= 1 + \sum_{res} |u_{res}\rangle\langle u_{res}| \neq 1$$



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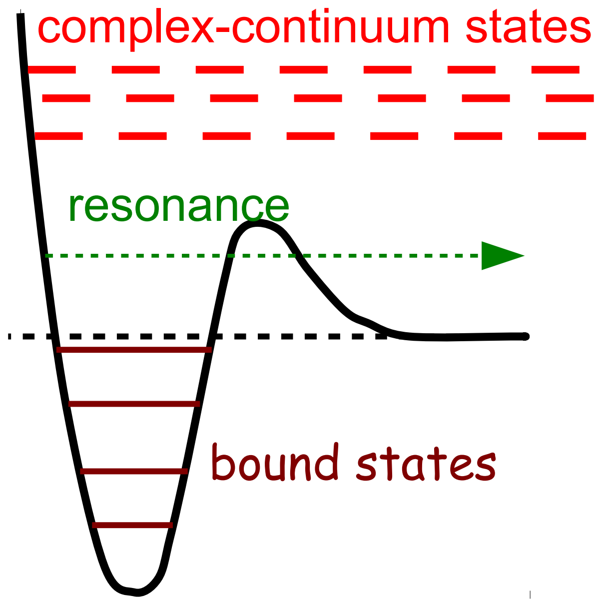
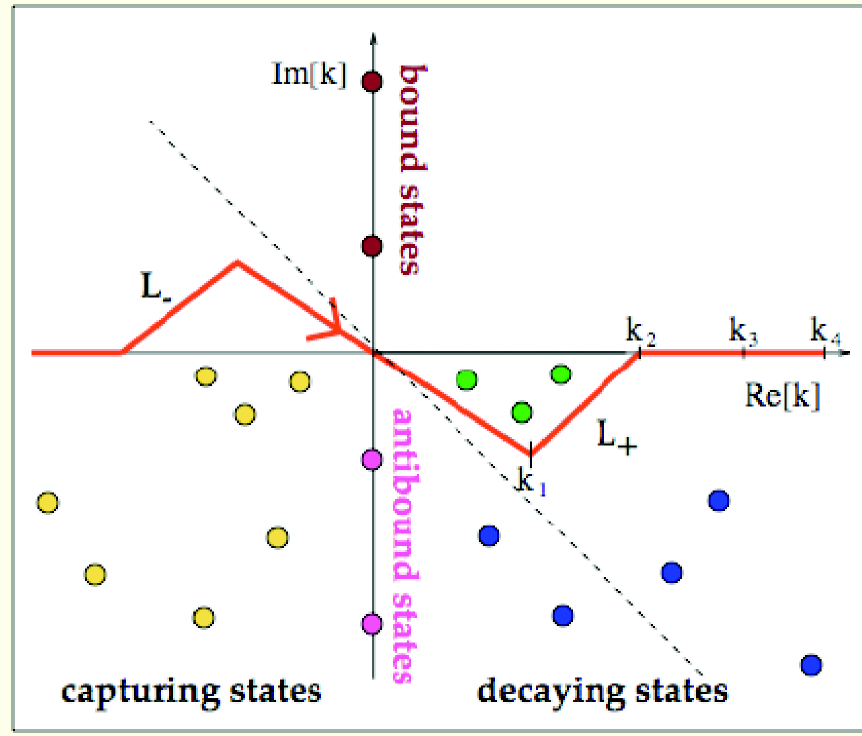
Gamow states and completeness relations

T. Berggren, Nucl. Phys. A109, 265 (1968); Nucl. Phys. A389, 261 (1982)
T. Lind, Phys. Rev. C47, 1903 (1993)





Berggren completeness relation

$$\sum_b |u_b\rangle\langle u_b| + \sum_{res} |u_{res}\rangle\langle \tilde{u}_{res}| + \int_{L^+} dk |u_k\rangle\langle \tilde{u}_k| = 1$$

Gamow Shell Model (GSM)

N. Michel *et al,* PRL 89 (2002) ; PRC70 (2004)
G. Hagen *et al*, PRC71 (2005)
J.R *et al,* PRL 97 (2006)
N. Michel *et al*, JPG (2009)
G.Papadimitriou et al, PRC(R) 84 (2011)

i) *discretization* of continuum contour

$$\sum_{res} |u_{res}\rangle\langle\tilde{u}_{res}| + \sum_{i} |u_{ki}\rangle\langle\tilde{u}_{ki}| \simeq 1$$

ii) construction of many-body basis

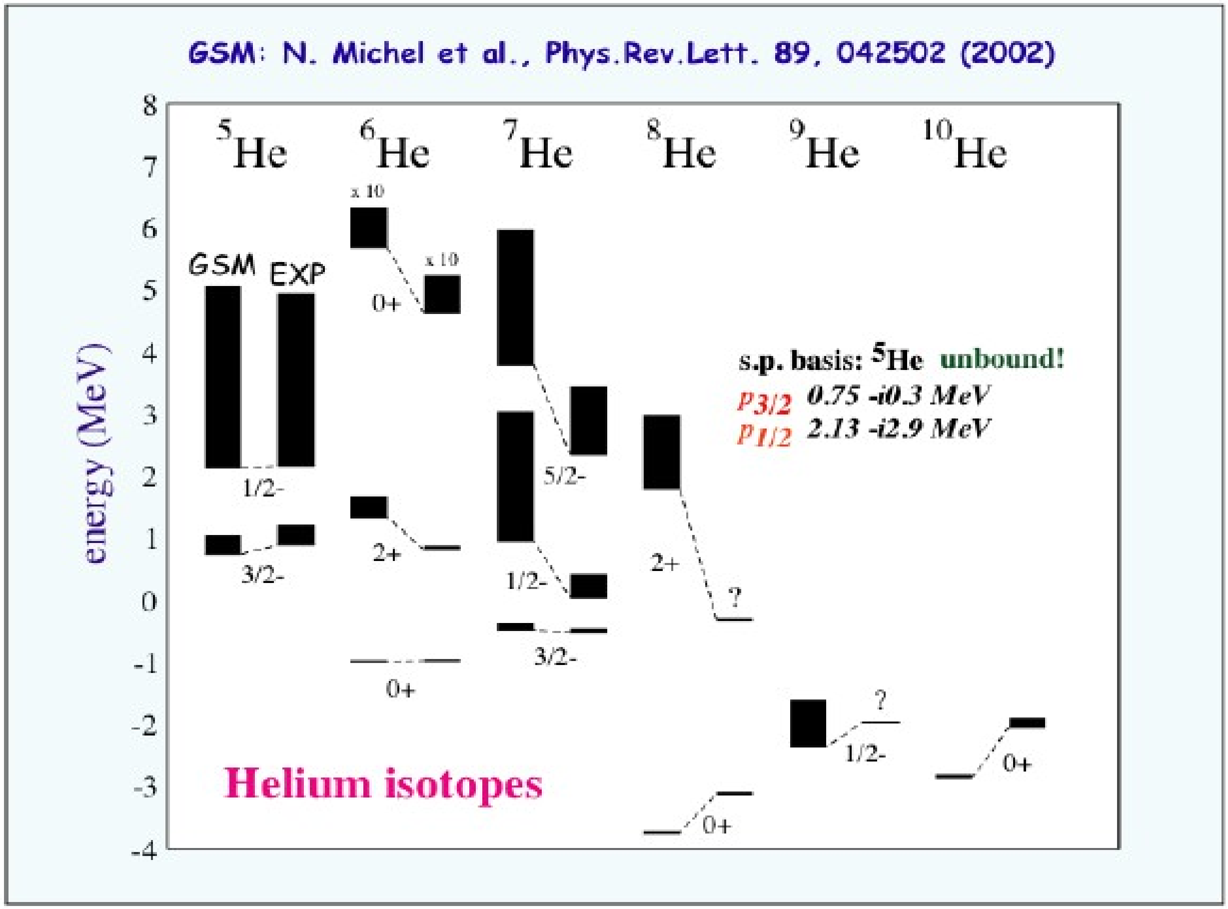
$$|SD_n\rangle = |u_1 \ldots u_A\rangle$$

iii) construction of Hamiltonian matrix

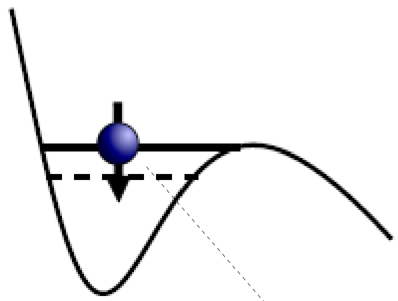
$$\langle SD_i|H|SD_j\rangle$$

(complex-symmetric matrix)

iv) -> many-body bound,resonant and continuum states

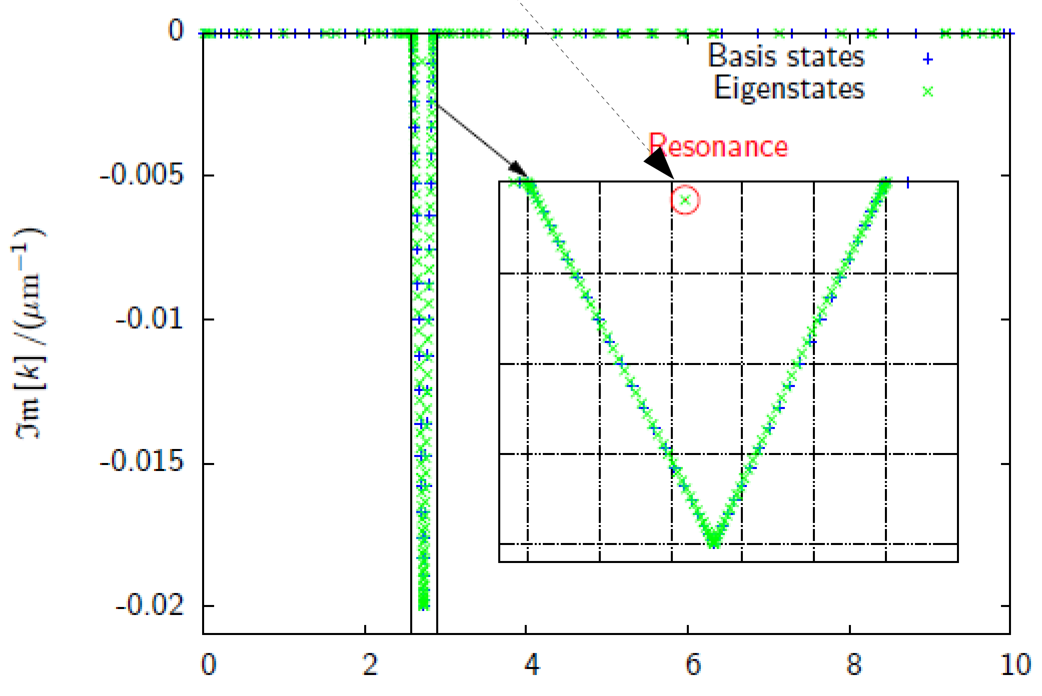


Tunneling of a single atom

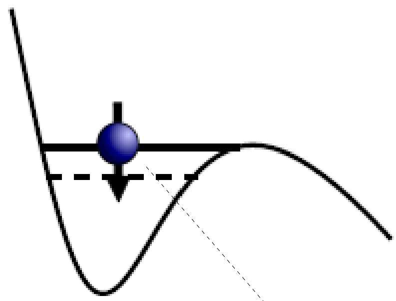


$$V(z) = pV_0\left(1 - \frac{1}{1+(z/z_R)^2}\right) - c_{B|state\rangle}\mu_B B'z$$

Resolution of the Schrödinger equation in the Berggren Basis

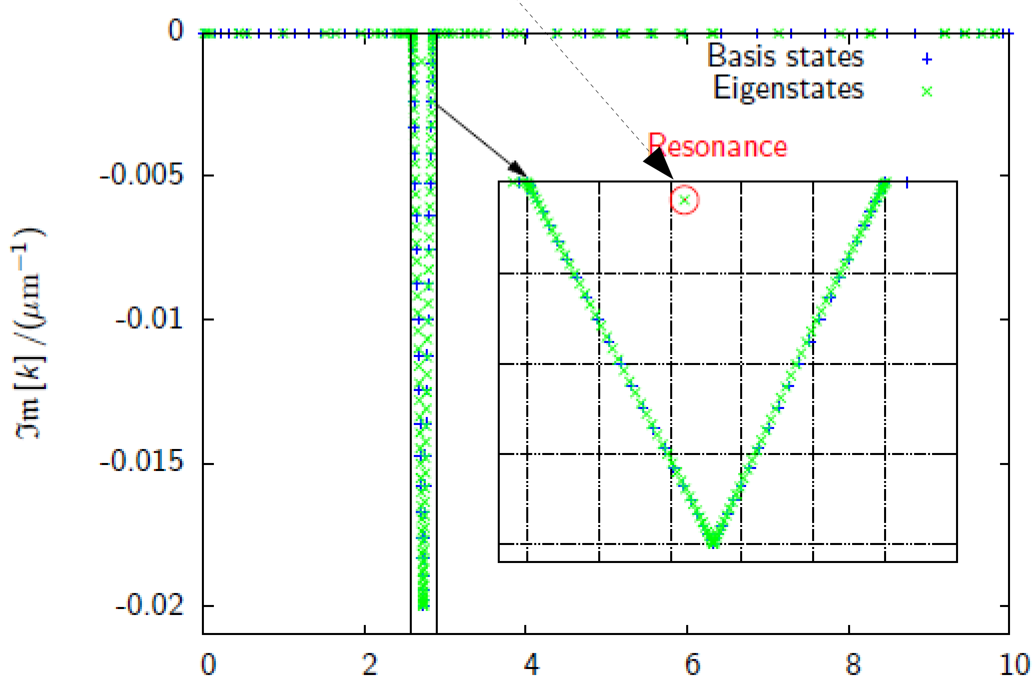


Tunneling of a single atom



$$V(z) = pV_0 \left(1 - \frac{1}{1 + (z/z_R)^2}\right) - c_{B|state\rangle} \mu_B B' z$$

Resolution of the Schrödinger equation in the Berggren Basis

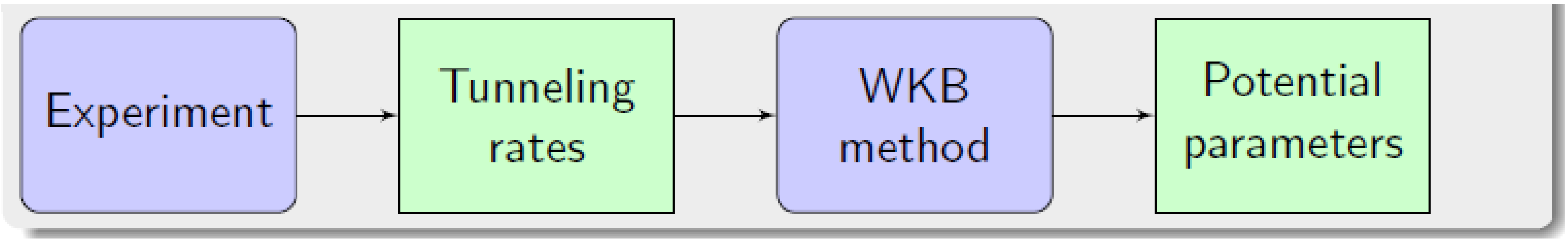


-> calculated decay rates off by almost a factor 2 compared to experimental values

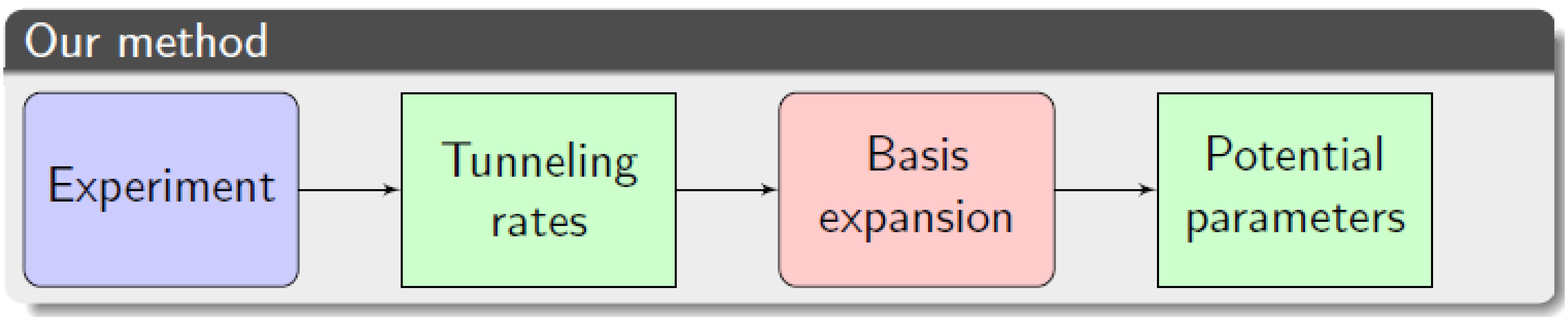
-> No approximation in our approach

-> Why such discrepancy ?

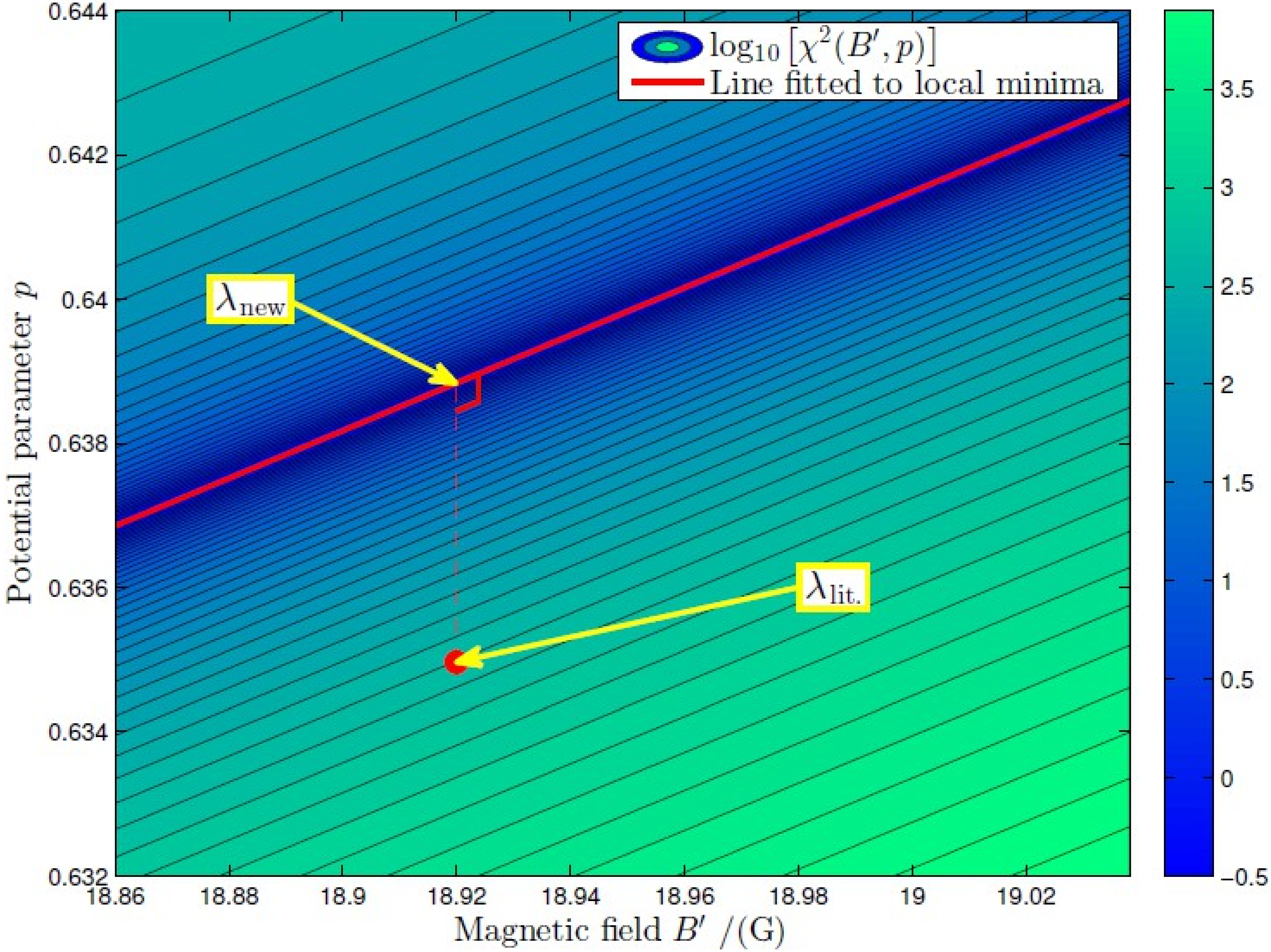
Some input parameters are extracted from experimental results using WKB method



We want to refit parameters without WKB



Minimization of $\chi^2(B', p) \equiv \sum_{B|\text{state}\rangle} \left(\frac{\gamma^{\text{calc}}_{B|\text{state}\rangle} - \gamma^{\text{exp}}_{B|\text{state}\rangle}}{\sigma_{\gamma_{B|\text{state}\rangle}}} \right)^2$



$\log_{10}[\chi^2(B', p)]$
Line fitted to local minima
λ_{new}
$\lambda_{\text{lit.}}$
Potential parameter p
Magnetic field B' /(G)
0.644
0.642
0.64
0.638
0.636
0.634
0.632
18.86
18.88
18.9
18.92
18.94
18.96
18.98
19
19.02
3.5
3
2.5
2
1.5
1
0.5
0
−0.5

Tunneling of the two atoms

g $/(\mathrm{nK}\cdot\mathrm{k_B}\cdot\mu\mathrm{m})$	$E_{\mathrm{int}}^{\mathrm{calc}}$ $/(\mathrm{nK}\cdot\mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{calc}}$ $/(\mathrm{s}^{-1})$	$E_{\mathrm{int}}^{\mathrm{WKB}}$ $/(\mathrm{nK}\cdot\mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{exp}}$ $/(\mathrm{s}^{-1})$
$-30.969\,33$	-8.45	19.19	-3.010 ± 0.215	[illegible]2.20 $\pm$ 1.0
$-41.527\,05$	-	-	-3.932 ± 0.369	1[illegible].84 $\pm$ 1.0
$-45.046\,30$	-13.59	25.81	-4.484 ± 0.3[illegible]	9.7[illegible] $\pm$ 0.3
$-99.946\,47$	-37.02	(0.44)	(-9.829 [illegible]44)	2.14 $\pm$ 0.2
$-104.169\,56$	-39.28	(0.56)	(-10.04[illegible] $\pm$ 0.8[illegible])	1.93 ± 0.1
$-110.504\,19$	-42.79	(1.12)	([illegible]10.9[illegible]5 $\pm$ [illegible].768)	1.23 ± 0.1
$-123.877\,31$	-50.55	(0.34)	($-$[illegible].532 $\pm$ 0.921)	0.51 ± 0.0

Berggren basis approach | G. Zürn et al., PRL 111 (2013).

-> Tunneling rate becomes smaller as the interaction becomes more attractive.

Tunneling of the two atoms

g $/(\mathrm{nK}\cdot\mathrm{k_B}\cdot\mu\mathrm{m})$	$E_{\mathrm{int}}^{\mathrm{calc}}$ $/(\mathrm{nK}\cdot\mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{calc}}$ $/(\mathrm{s}^{-1})$	$E_{\mathrm{int}}^{\mathrm{WKB}}$ $/(\mathrm{nK}\cdot\mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{exp}}$ $/(\mathrm{s}^{-1})$
$-30.969\,33$	-8.45	19.19	-3.010 ± 0.215	22.20 ± 1.0
$-41.527\,05$	-	-	-3.932 ± 0.369	13.84 ± 1.0
$-45.046\,30$	-13.59	25.81	-4.484 ± 0.307	9.70 ± 0.3
$-99.946\,47$	-37.02	(0.44)	(-9.829 ± 1.044)	2.14 ± 0.2
$-104.169\,56$	-39.28	(0.56)	(-10.044 ± 0.860)	1.93 ± 0.1
$-110.504\,19$	-42.79	(1.12)	(-10.965 ± 0.768)	1.23 ± 0.1
$-123.877\,31$	-50.55	(0.34)	(-12.532 ± 0.921)	0.51 ± 0.0

Berggren basis approach

G. Zürn et al., PRL 111 (2013).

-> Tunneling rate becomes smaller as the interaction becomes more attractive.

Tunneling of the two atoms

g $/(\mathrm{nK}\cdot \mathrm{k_B}\cdot \mu\mathrm{m})$	$E_{\mathrm{int}}^{\mathrm{calc}}$ $/(\mathrm{nK}\cdot \mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{calc}}$ $/(\mathrm{s}^{-1})$	$E_{\mathrm{int}}^{\mathrm{WKB}}$ $/(\mathrm{nK}\cdot \mathrm{k_B})$	$\gamma_{\mathrm{total}}^{\mathrm{exp}}$ $/(\mathrm{s}^{-1})$
-30.96933	-8.45	19.19	-3.010 ± 0.215	22.20 ± 1.0
-41.52705	-	-	-3.932 ± 0.369	13.84 ± 1.0
-45.04630	-13.59	25.81	-4.484 ± 0.307	9.70 ± 0.3
-99.94647	-37.02	(0.44)	(-9.829 ± 1.044)	2.14 ± 0.2
-104.16956	-39.28	(0.56)	(-10.044 ± 0.860)	1.93 ± 0.1
-110.50419	-42.79	(1.12)	(-10.965 ± 0.768)	1.23 ± 0.1
-123.87721	-50.55	(0.34)	(-12.532 ± 0.921)	0.51 ± 0.0

Berggren basis approach

G. Zürn et al., PRL 111 (2013).

-> Tunneling rate becomes smaller as the interaction becomes more attractive

For each g, the single particle potential is slightly different

g	-0.70385	-30.96933	-41.52705	-45.04630
$c_{B\|\uparrow\rangle}$	1.00457	1.00407	1.00356	1.00311
$c_{B\|\downarrow\rangle}$	0.99968	0.99806	0.99512	0.98989

Strongly-interacting few-fermion systems in a trap

✔ Derivation of effective interaction for few-atom systems in HO trap

✔ Applications for 3D/1D systems, excellent agreement with other numerical and exact approaches

✔ Tunneling theory for two particles using the Berggren basis

Future plans:

Tunneling in heavier systems, repulsive interactions, excited states..