Multi-Body Interacting Bosons

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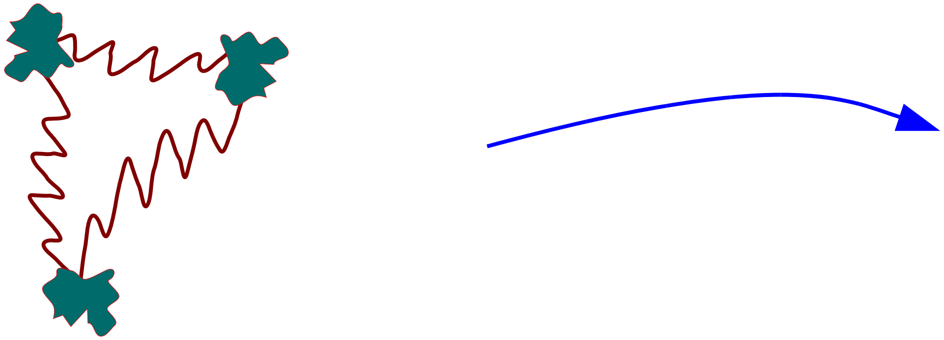
Outline

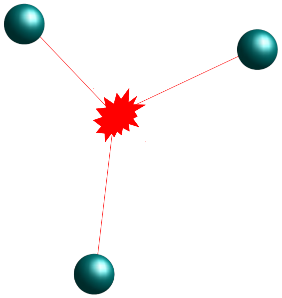
- Effective multi-body interactions
- Multi-body interacting systems. Why interesting?
- How to make (engineer) them?
- 3-body interacting dipolar molecules in bi-layers with tunneling
- 3- and 4-body interacting 39K in optical lattices

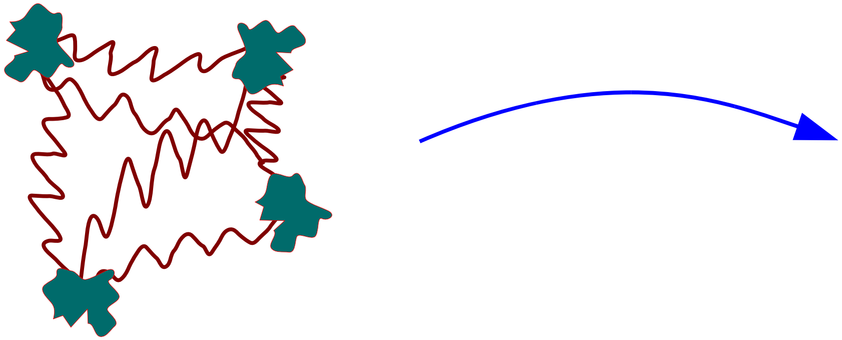
Effective multi-body interactions

Parasites which appear when we want to simplify our life:

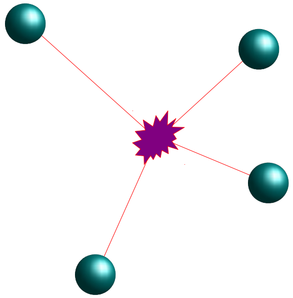


Simple 2-body pot. +





Simple 2-body pot. + 3-body +



⋮

Hammer et al., Rev. Mod. Phys. (2013)

Examples

Lee, Huang, Yang'57, Wu'59 ...

Hard sphere gas

Zero-range interacting gas
+ three-body interaction

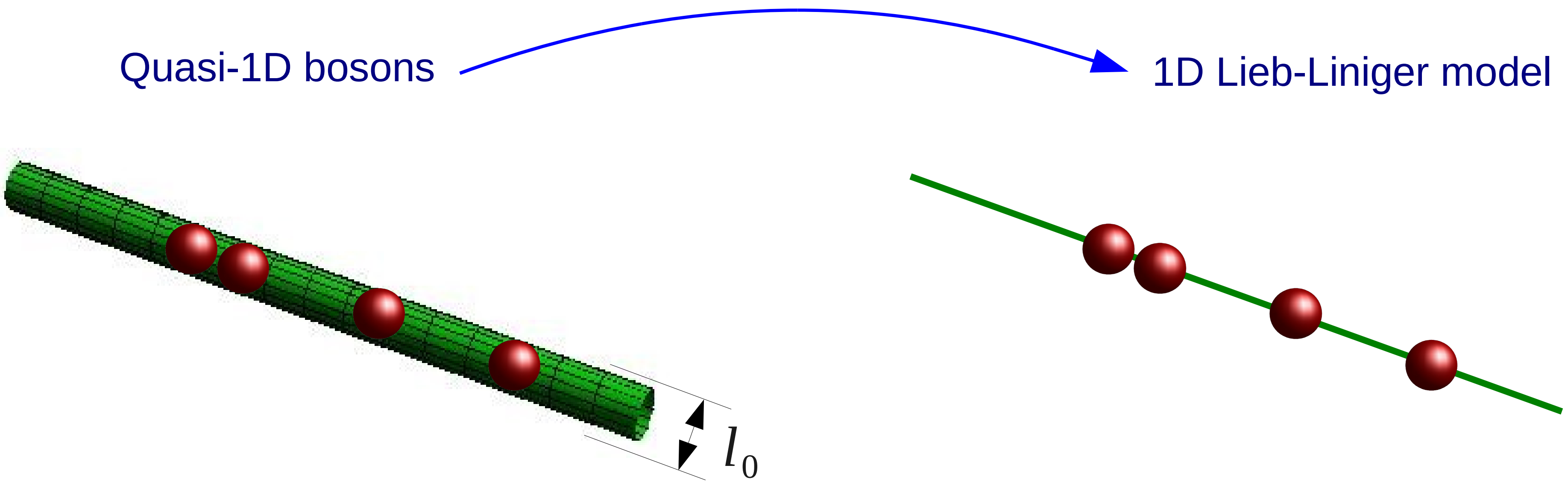
This workshop & program

Bosons with van der
Waals two-body forces

Zero-range interacting gas
+ three-body parameter

Beyond LHY effects.
Three-body force important at
large *a*
Braaten et al.'99, Bulgac'02,...

Examples


Quasi-1D bosons → 1D Lieb-Liniger model

2-body Olshanii'98 + 3-body Muryshev et al.'02

$$H=-\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+\frac{2a}{l_0}\sum_{i<j}\delta(x_i-x_j)-12\log\left(\frac{4}{3}\right)\frac{a^2}{l_0^2}\sum_{i<j<k}\delta(x_i-x_j)\delta(x_j-x_k)$$

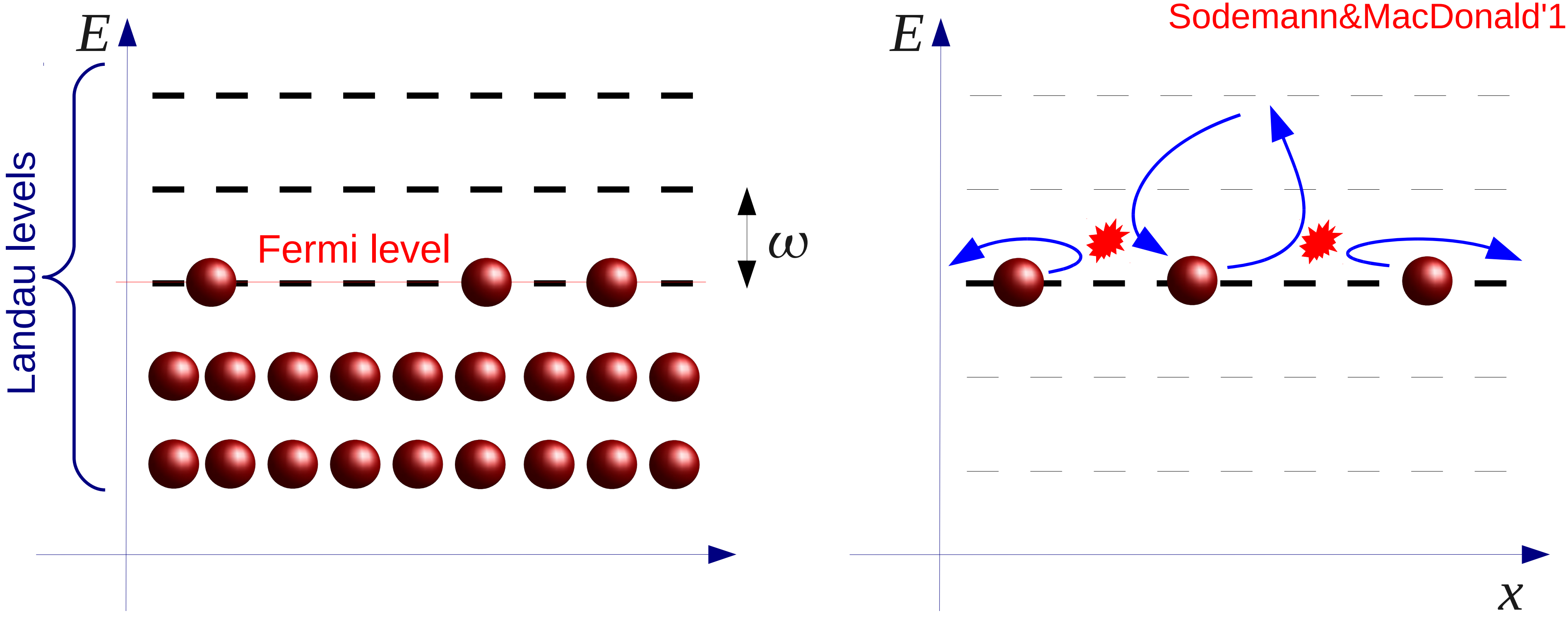
Perturbative, second order, weak,
attractive, but BREAKS INTEGRABILITY!

Examples

All quantum Hall numerics

Quantum Hall layer → Single Landau level model
+ higher order interactions

Nayak et al.'09,'13,
Sodemann&MacDonald'13



Weak, higher order in g/ω , but important due to high Landau level degeneracy!

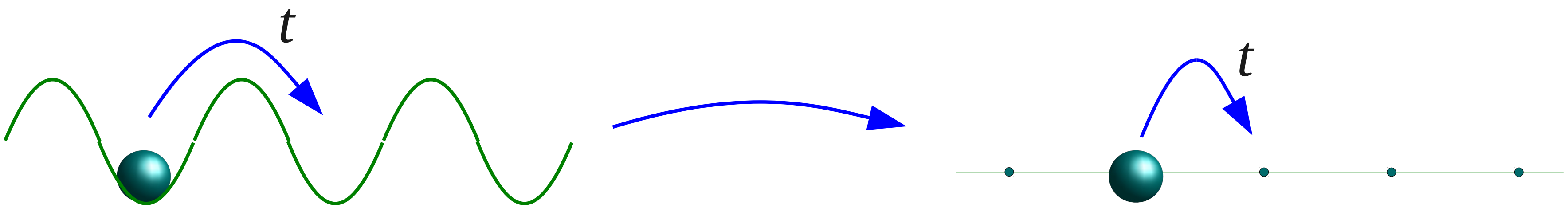
Examples

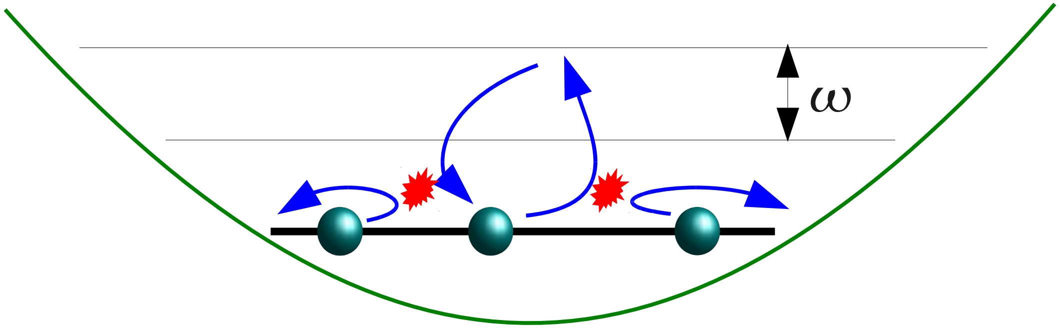
Jaksch et al.'98...

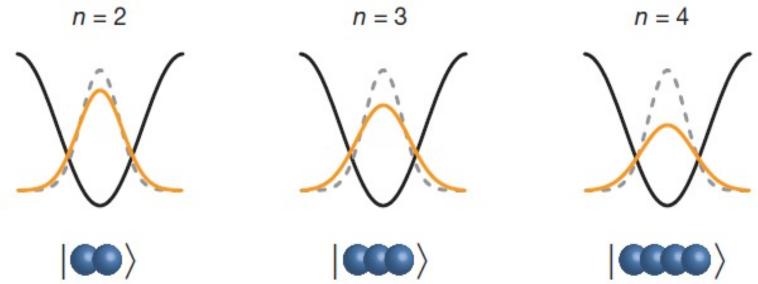
Bosons in an optical lattice

Bose-Hubbard model
+ higher order interactions

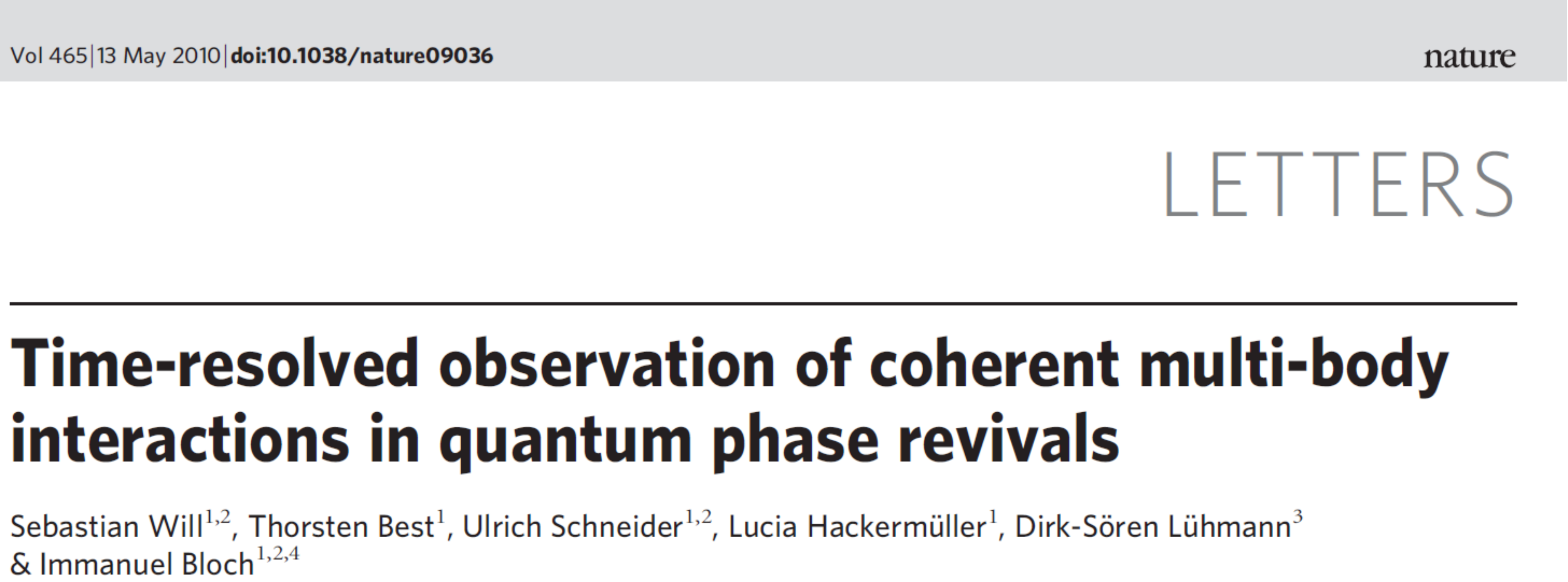
Niu et al.'06, Tiesinga et al.'09,11, Hazzard&Mueller'10







Weak, high order in g/ω, but measurable
Campbell et al.'06, Will et al.'10



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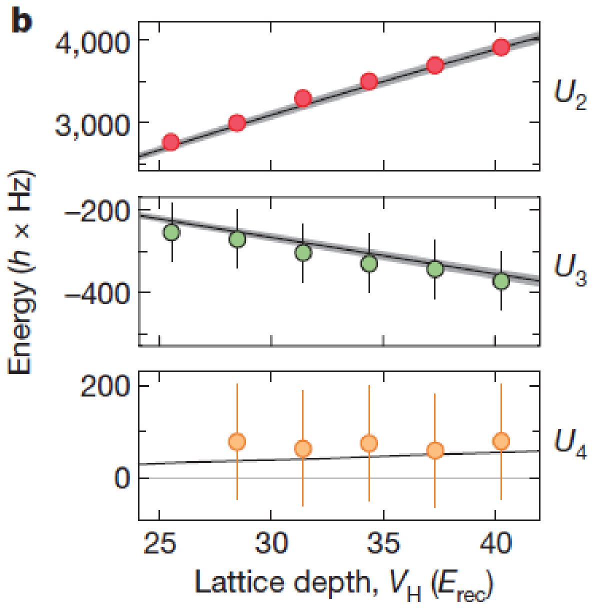
nature

LETTERS

Time-resolved observation of coherent multi-body interactions in quantum phase revivals

Sebastian Will[1,2], Thorsten Best[1], Ulrich Schneider[1,2], Lucia Hackermüller[1], Dirk-Sören Lühmann[3] & Immanuel Bloch[1,2,4]

$$E(N)=\underset{\substack{\wr\\ g\sim a\,\omega^{3/2}}}{U_2}\frac{N(N-1)}{2!}+\underset{\substack{\wr\\ \gg\ g^2/\omega}}{U_3}\frac{N(N-1)(N-2)}{3!}+\underset{\substack{\wr\\ \gg\ g^3/\omega^2}}{U_4}\frac{N(N-1)(N-2)(N-3)}{4!}+\ldots$$



Multi-body interacting systems

“N-body interacting”

⇕

$$\begin{array}{cccccc} U_2 = & U_3 = & \ldots = & U_{N-1} = 0 & \text{AND} & U_N \neq 0 \\ \parallel & \parallel & & \parallel & & \parallel \\ g_2 & g_3 & & g_{N-1} & & g_N \end{array}$$

$$\frac{E}{\text{Volume}} = g_2 \frac{n^2}{2!} + g_3 \frac{n^3}{3!} + g_4 \frac{n^4}{4!} + \ldots$$

Why interesting?

Bosons with $g_2 < 0$ is a bad combination, but if g_2 is small and if we add $g_3 > 0$ ...

Cubic-quintic GP (Schroedinger) equation in a trap, gas-liquid transition Gammal et al.'00-

$$\left[-\nabla_{\vec{r}}^2/2 - |g_2| n(\vec{r}) + g_3 n^2(\vec{r})/2 + V_{ext}(\vec{r}) - \mu \right] \psi(\vec{r}) = 0$$

Free space → self-trapped droplet state Bulgac'02:

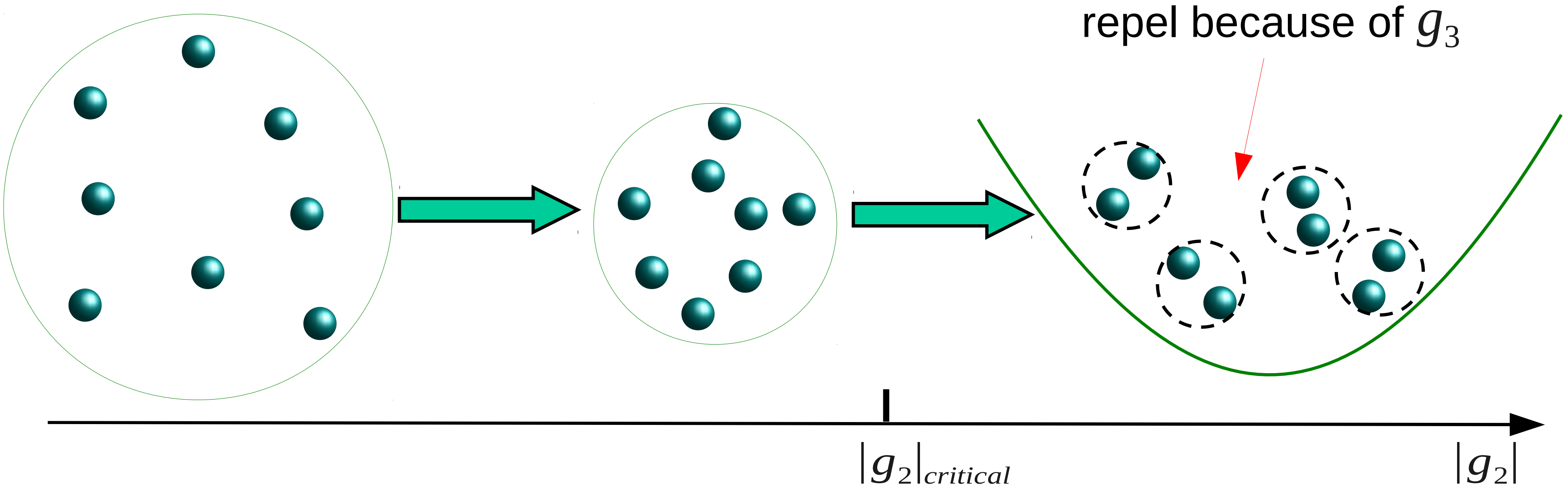
- Neglecting surface tension, flat density profile $n = 3|g_2|/2g_3$
- Including surface tension → surface modes

Staying dilute requires large g_3 and small g_2 close to zero crossing!

Why interesting?

Above critical g_2 bosons pair – topological transition, not crossover!

Nozieres&Saint James'82 Radzihovsky et al., Romans et al., Lee&Lee'04



Pairing on a lattice with three-body constraint:

Daley et al.'09-, Ng&Yang'11, Bonnes&Wessel'12,...

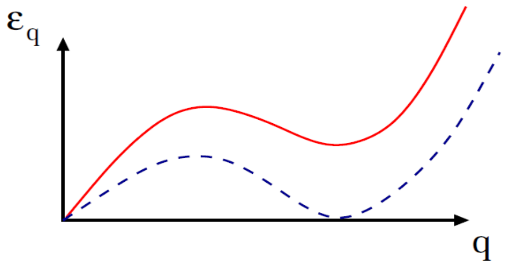
g_3 is necessary! = Pauli pressure in the BCS-BEC crossover!

Why interesting?

Bosons with dipolar interactions

L. Santos, M. Lewenstein, P. Zoller, G. Shlyapnikov
C. Eberlein, S. Giovanazzi, O'Dell
K. Góral, M. Brewekyk, K. Rzatenski
S. Yi and L. You,
S. Giovanazzi, A. Gorliz, T. Pfau

Roton-maxon structure



$g_3 > 0$ stabilizes weakly interacting supersolid phase, Lu et al., to be published

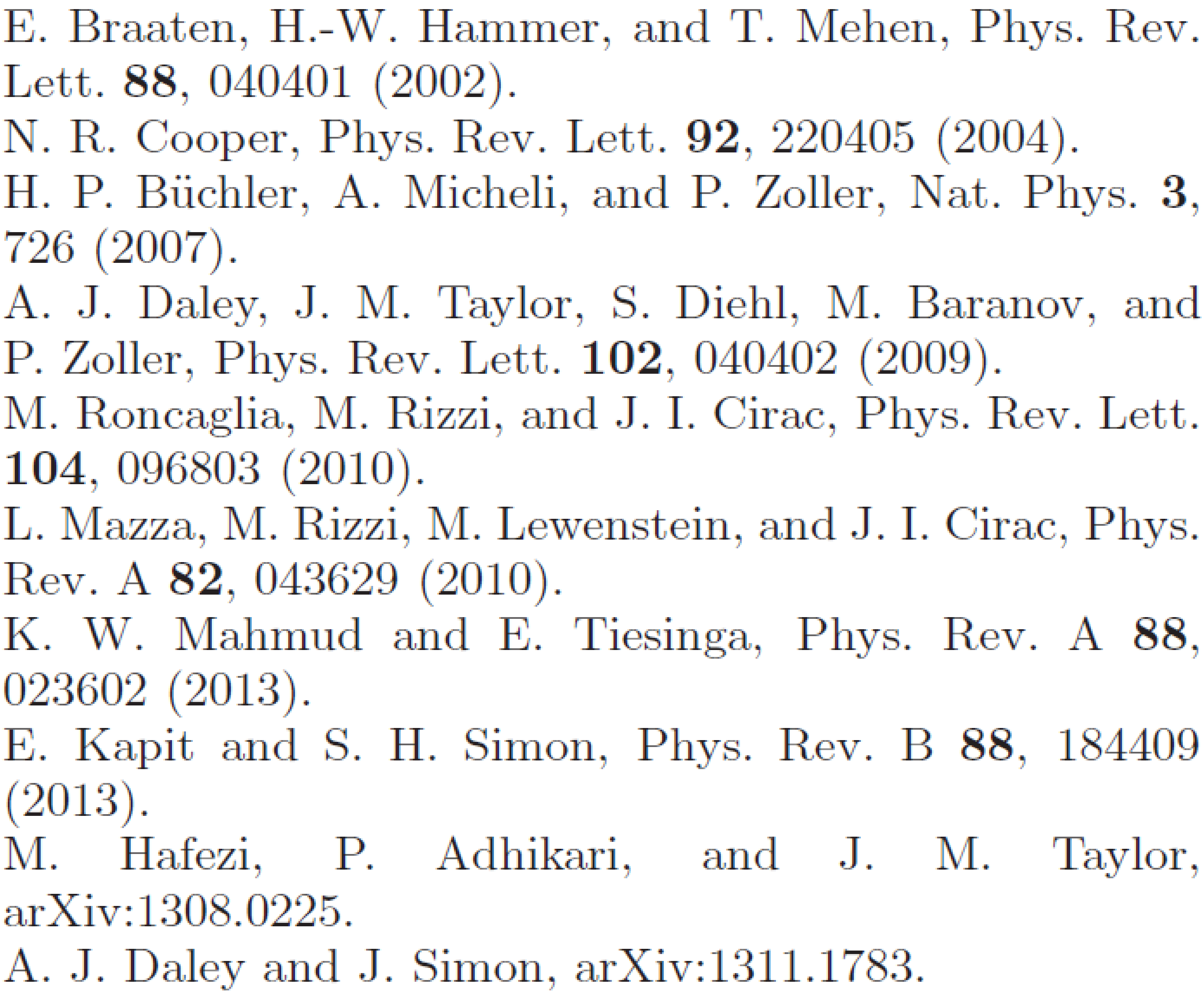
Why interesting?

Local repulsive g_{k+1} is the ``parent'' Hamiltonian for the *k*-th state of the Read-Rezayi series of quantum Hall states, Nayak et al., Rev. Mod. Phys. (2008)

- *k*=1 (*2-body int.*) → Laughlin state (abelian anyons)
- *k*=2 (*3-body int.*) → Moore-Read state (non-abelian anyons, some topologically protected operations)
- *k*=3 (*4-body int.*) → Read-Rezayi state (non-abelian anyons, universal quantum computing)

Ground state degeneracy protected by gap ~ g_{k+1} Important to maximize !

Some previous work



E. Braaten, H.-W. Hammer, and T. Mehen, Phys. Rev. Lett. **88**, 040401 (2002).

N. R. Cooper, Phys. Rev. Lett. **92**, 220405 (2004).

H. P. Büchler, A. Micheli, and P. Zoller, Nat. Phys. **3**, 726 (2007).

A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).

M. Roncaglia, M. Rizzi, and J. I. Cirac, Phys. Rev. Lett. **104**, 096803 (2010).

L. Mazza, M. Rizzi, M. Lewenstein, and J. I. Cirac, Phys. Rev. A **82**, 043629 (2010).

K. W. Mahmud and E. Tiesinga, Phys. Rev. A **88**, 023602 (2013).

E. Kapit and S. H. Simon, Phys. Rev. B **88**, 184409 (2013).

M. Hafezi, P. Adhikari, and J. M. Taylor, arXiv:1308.0225.

A. J. Daley and J. Simon, arXiv:1311.1783.

3-body interacting case: perturb. prosp.

$$\boxed{E(N)=U_2\frac{N(N-1)}{2!}+U_3\frac{N(N-1)(N-2)}{3!}+U_4\frac{N(N-1)(N-2)(N-3)}{4!}+\dots}$$

Perturbative approach Johnson et al.'09

$$\boxed{E(2)=U_2=\langle\psi_0|V|\psi_0\rangle+\underbrace{\sum_{\nu}\frac{|\langle\psi_\nu|V|\psi_0\rangle|^2}{\epsilon_0-\epsilon_\nu}+\dots}_{\text{Higher order terms}}}$$

$$\boxed{E(3)=3E(2)\underbrace{-\underset{\overset{\uparrow}{\text{Double counting compensation}}}{O(V^2)}+\underbrace{\sum_{\bar{\nu}}\frac{|\langle\bar{\psi}_{\bar{\nu}}|V|\bar{\psi}_0\rangle|^2}{\bar{\epsilon}_0-\bar{\epsilon}_{\bar{\nu}}}+\dots}_{\text{Higher order terms}}}_{U_3\sim V^2/|\epsilon_0-\epsilon_1|}}$$

3-body interacting case: perturb. prosp.

$$\boxed{E(N)=U_2\frac{N(N-1)}{2!}+U_3\frac{N(N-1)(N-2)}{3!}+U_4\frac{N(N-1)(N-2)(N-3)}{4!}+\ldots}$$

Perturbative approach Johnson et al.'09

$$E(2)=U_2=\langle\psi_0|V|\psi_0\rangle+\underbrace{\sum_{\nu}\frac{|\langle\psi_\nu|V|\psi_0\rangle|^2}{\epsilon_0-\epsilon_\nu}}_{\text{Higher order terms}}+\ldots$$

$$E(3)=3E(2)-O(V^2)+\underbrace{\sum_{\bar{\nu}}\frac{|\langle\bar{\psi}_{\bar{\nu}}|V|\bar{\psi}_0\rangle|^2}{\bar{\epsilon}_0-\bar{\epsilon}_{\bar{\nu}}}}_{\text{Additional non-additive higher order terms}}+\ldots$$

Double counting compensation

Additional non-additive higher order terms

$$\underbrace{\text{Double counting compensation} \qquad \text{Additional non-additive higher order terms}}_{U_3\sim V^2/|\epsilon_0-\epsilon_1|}$$

This talk →

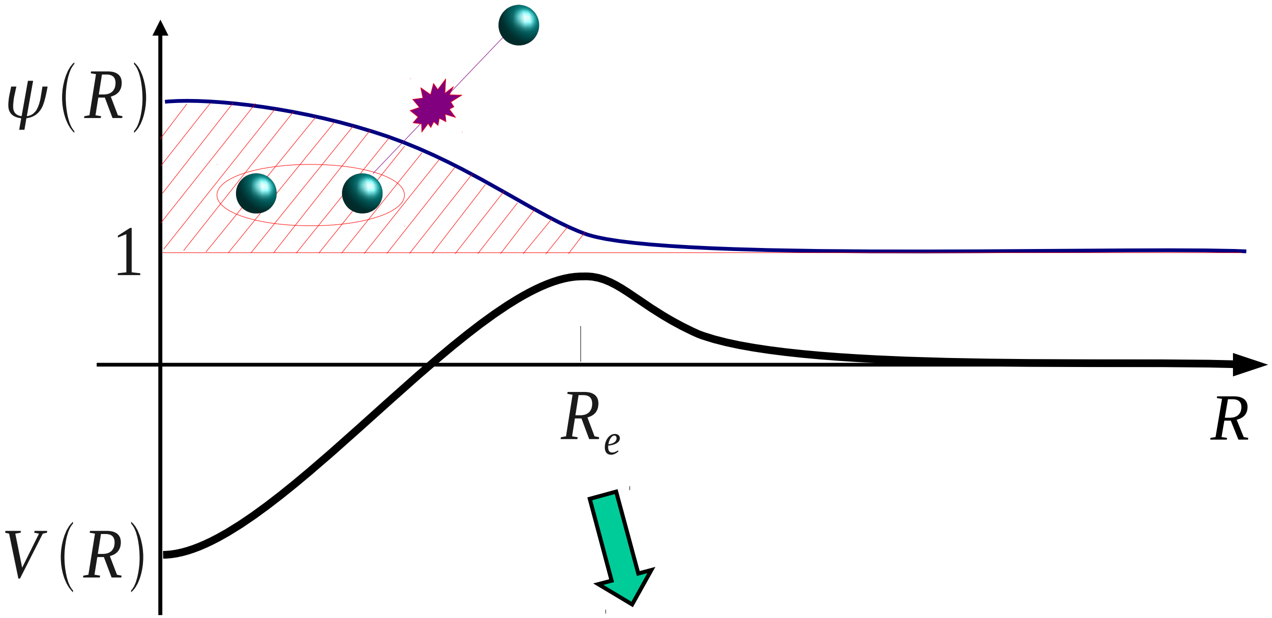
$$\boxed{U_2=0 \quad \text{AND} \quad U_3>0 \quad \text{AND} \quad \text{STRONG!}}$$

Where to look?

$\langle \psi_0 | V | \psi_0 \rangle \approx 0$ → vanishing on-shell scattering... OK

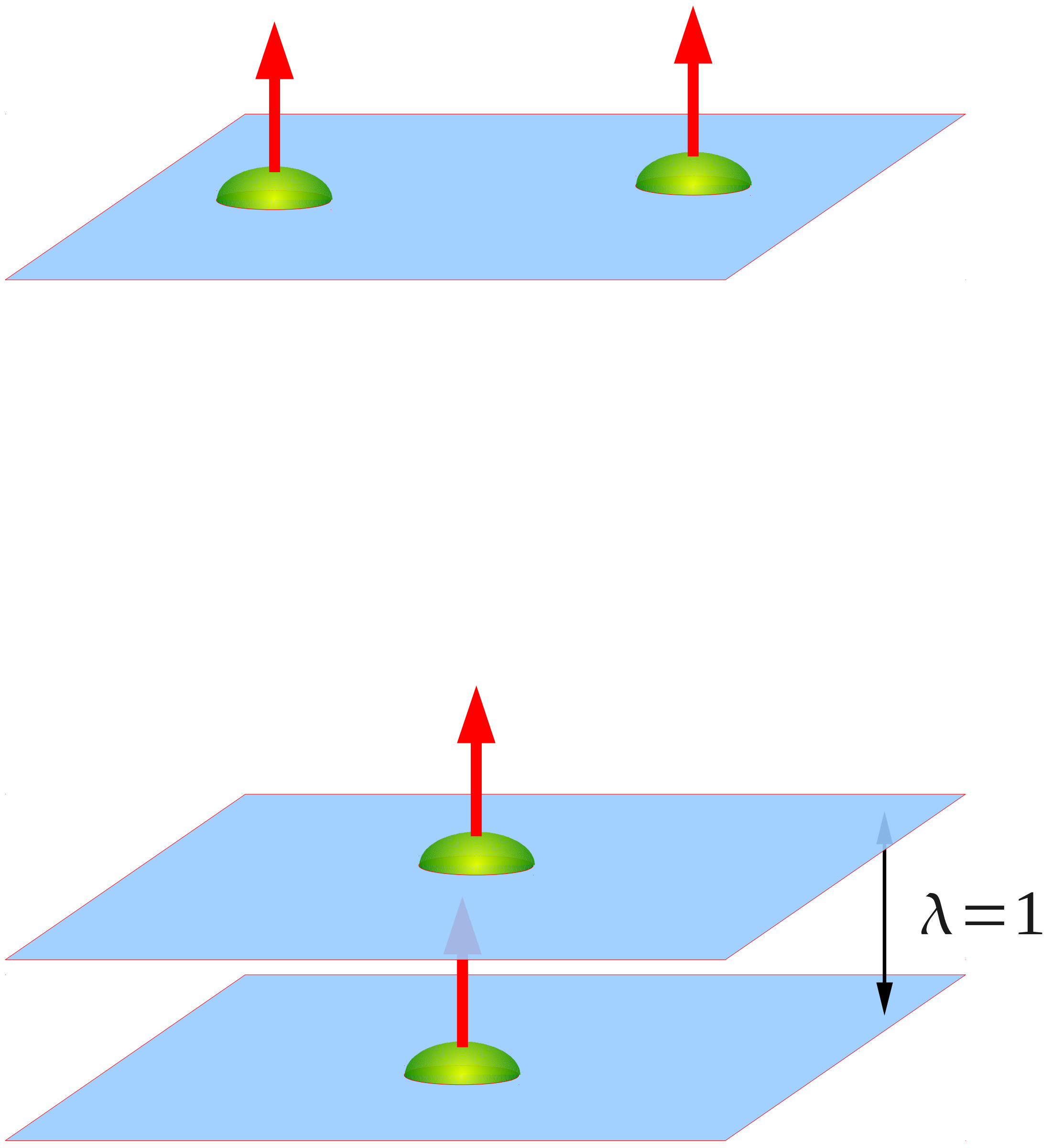
$\langle \psi_\nu | V | \psi_0 \rangle \neq 0$ ↔ large off-shell contribution...

which should repel the third particle... ???



$g_3^{(3D)} \sim R_e^3 a \sim R_e^4$ Just two-body zero crossing is not enough !

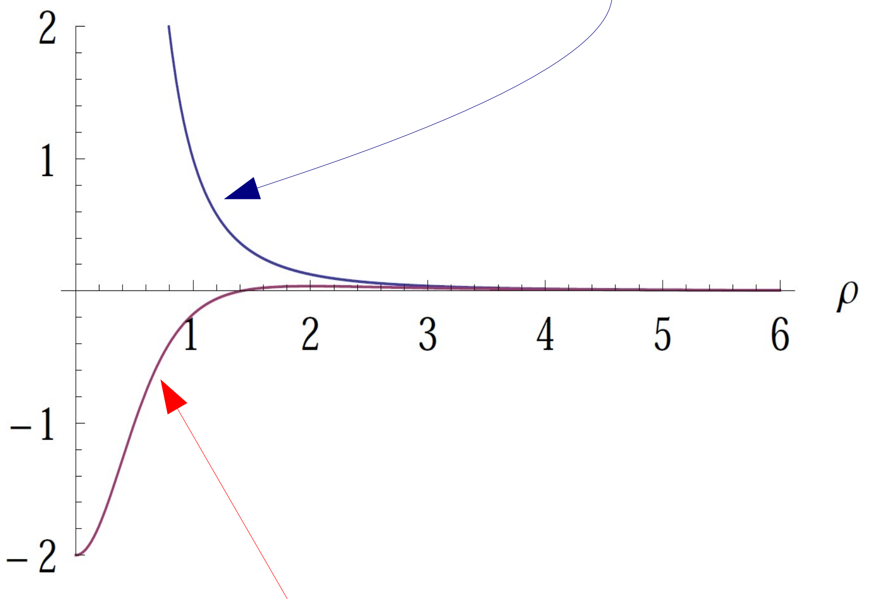
Dipoles on layers



Repulsive intralayer potential

$$V_{\uparrow\uparrow}(\rho)=r_*/\rho^3$$

No bound state



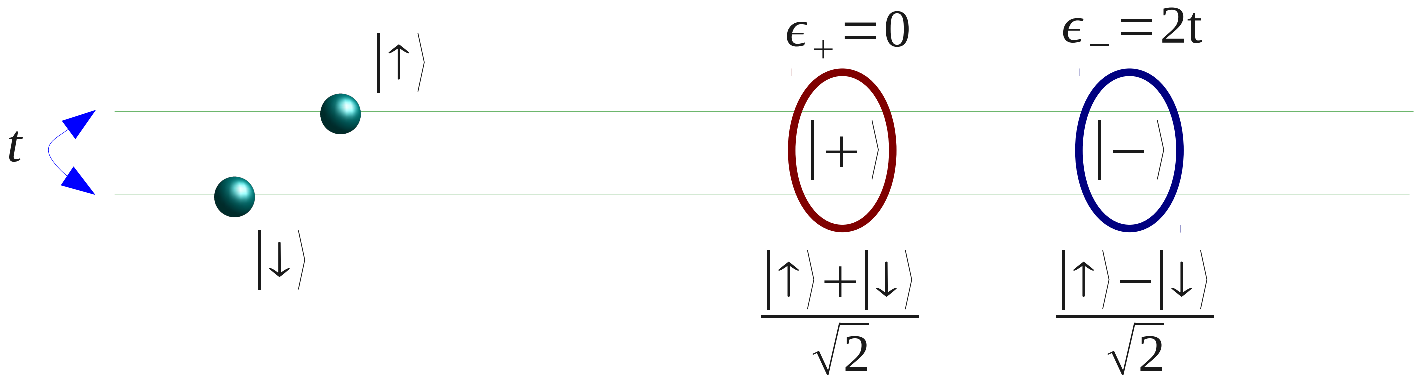
Interlayer potential averages to zero

$$V_{\uparrow\downarrow}(\rho)=r_*(\rho^2-2)/(\rho^2+1)^{5/2}$$

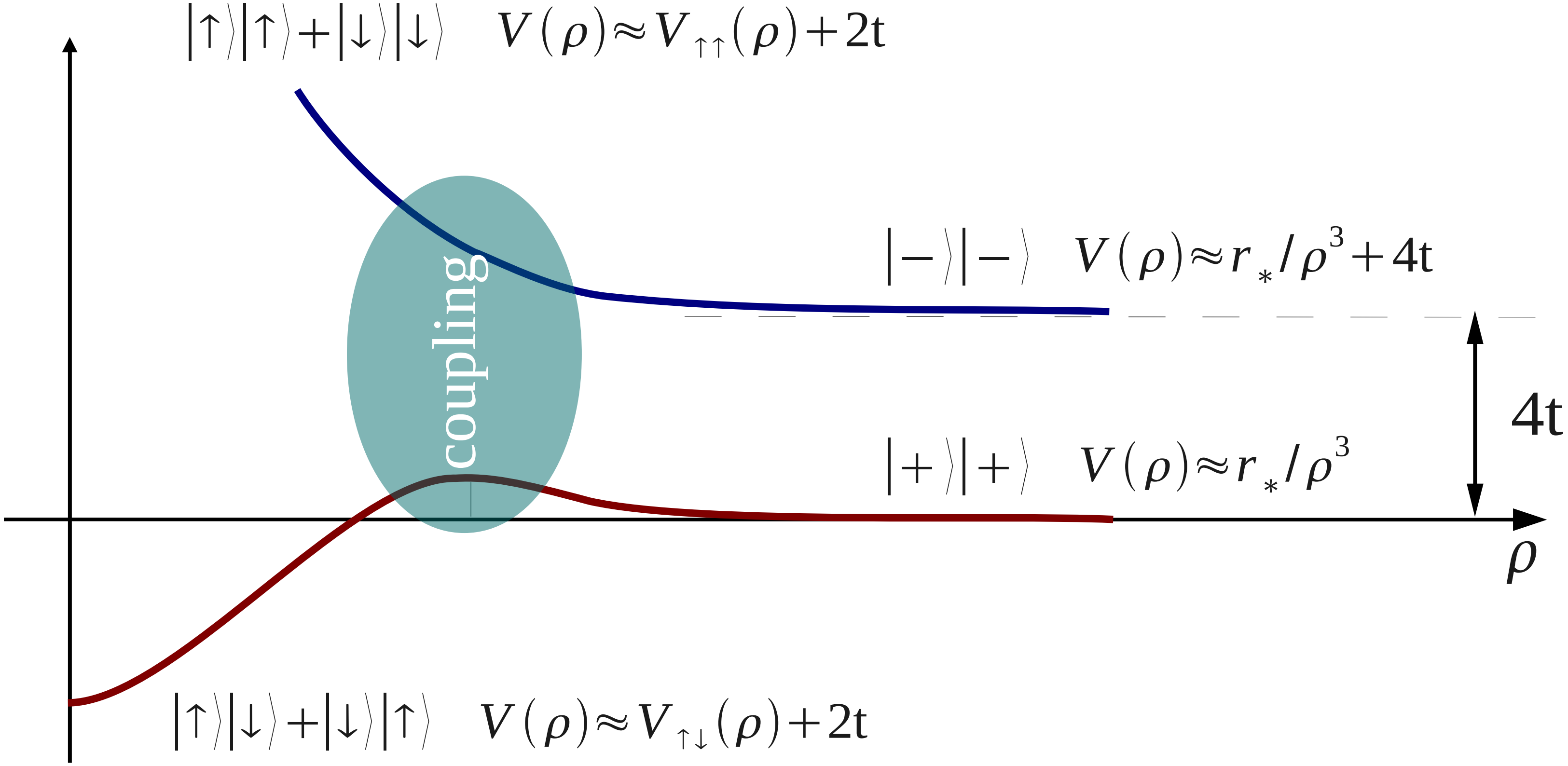
At least one bound state

Bilayer with tunneling

One-body problem



Two-body problem



Vertex function and bound state

Vertex function for 2D scattering with weakly-bound state + dipolar tails:

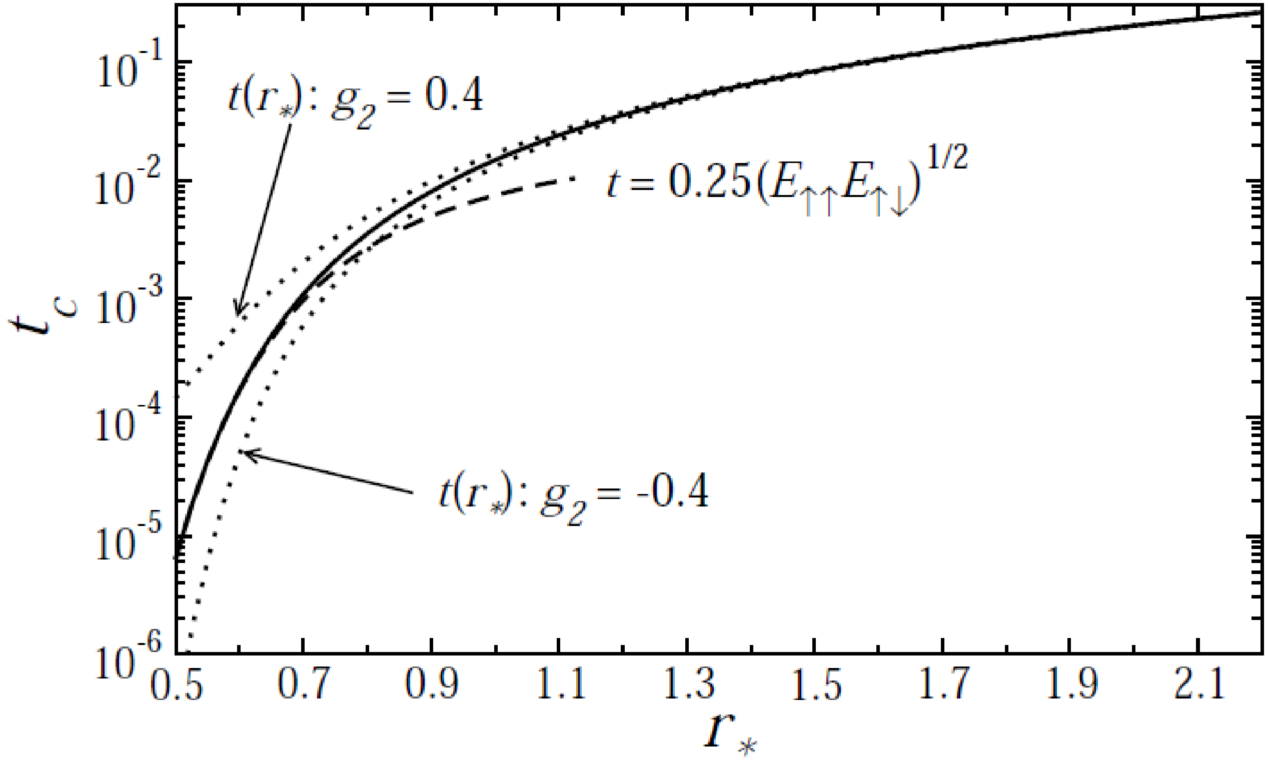
Baranov et al.'11

$$\Gamma(E,\vec{k},\vec{k}')\approx\frac{4\pi}{\ln(4t/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}'|$$

$$\varepsilon_0=4t\exp(4\pi/g_2)\longleftarrow$$

Exponentially weakly bound state for small negative g_2

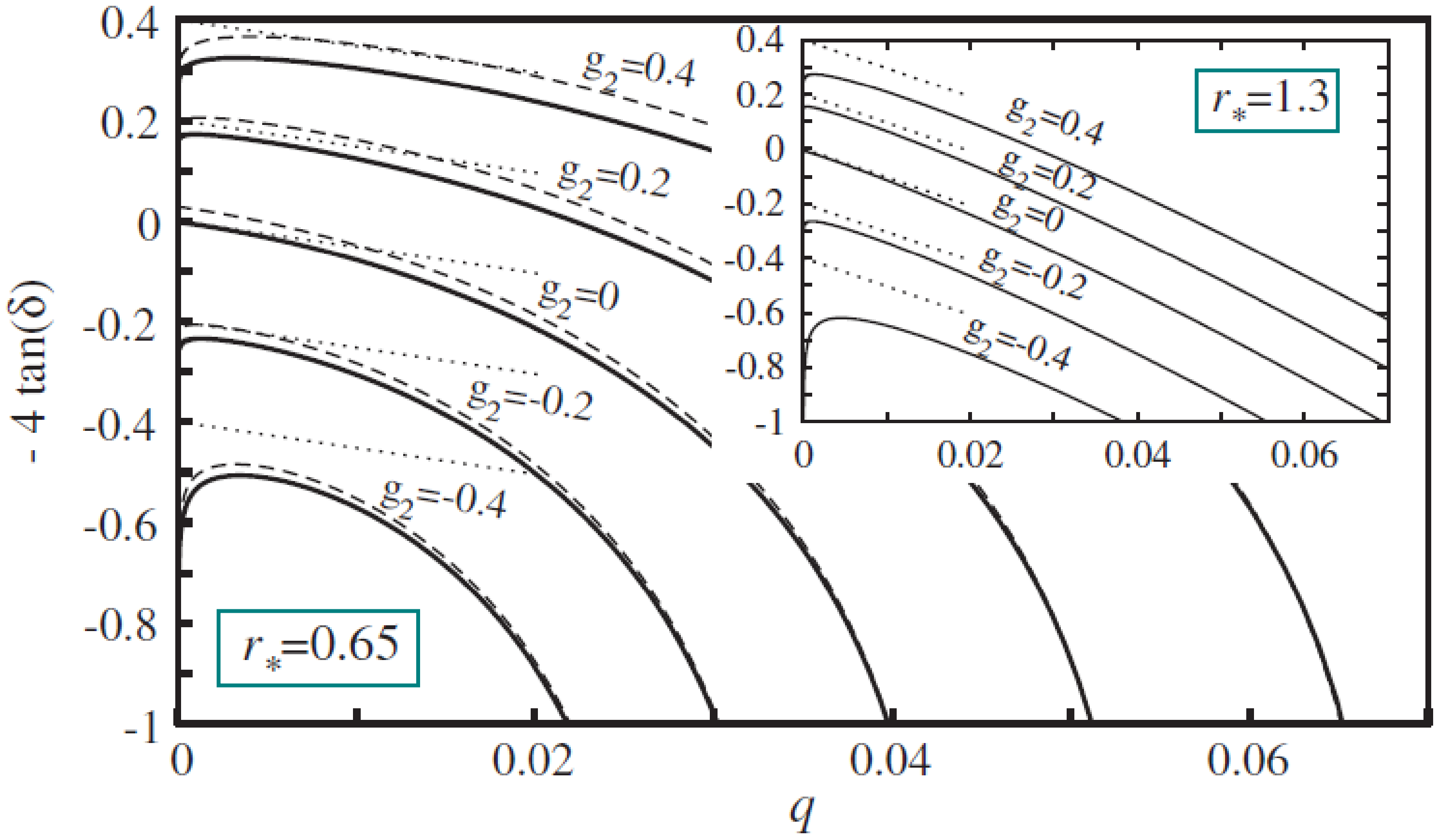
Critical *t*



S-wave scattering at finite collision energy

$$\Gamma(E,\vec{k},\vec{k}\,')\approx\frac{4\pi}{\ln(4\mathrm{t}/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}\,'| \longrightarrow -4\tan\delta_s(q)\approx g_2-8r_*q$$

s-wave and on shell

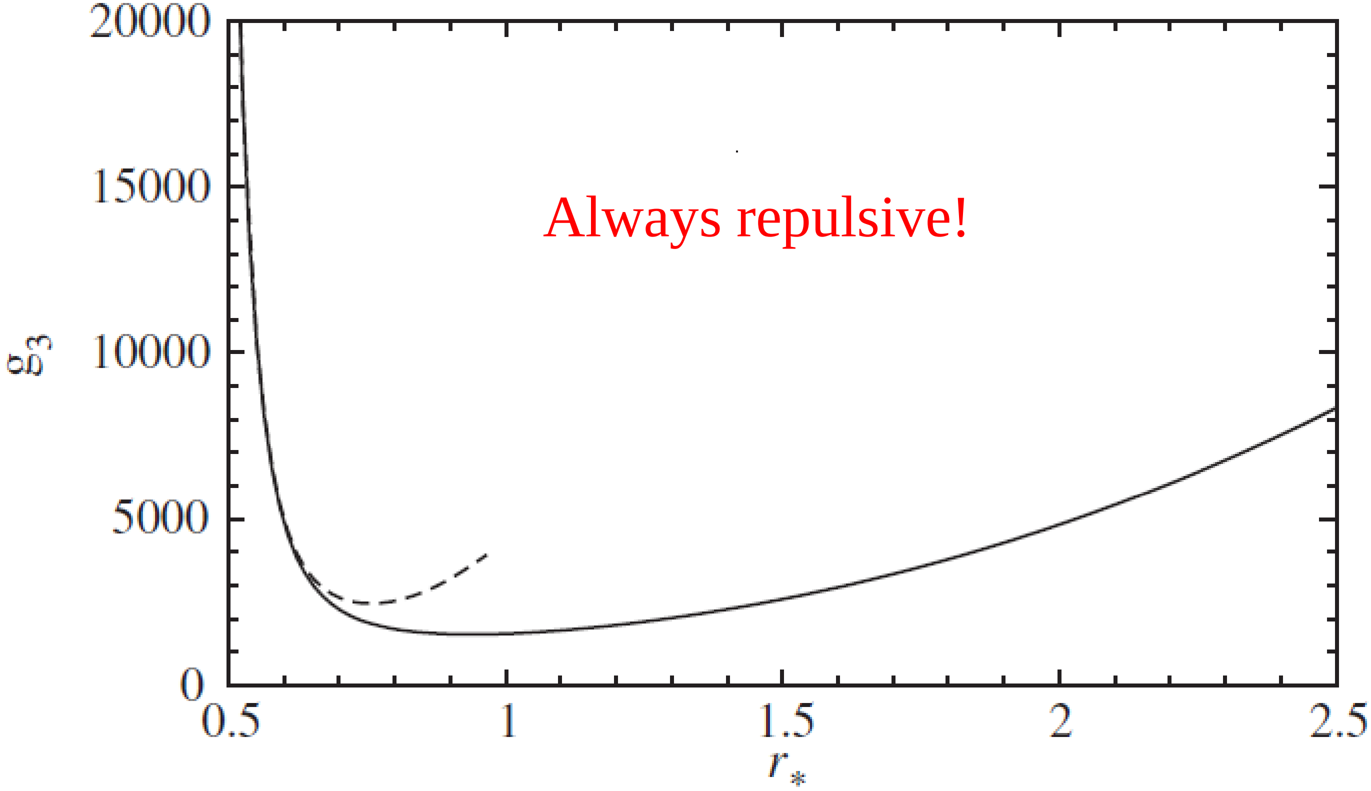


Two-body zero crossing + dipolar tail → rotonization, density wave, etc

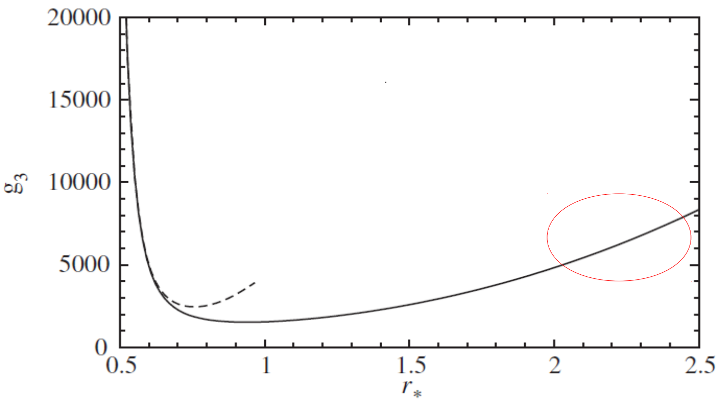
3-body coupling constant

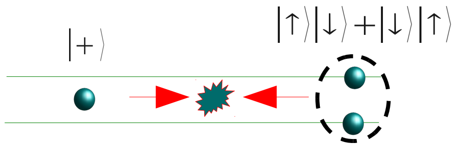
$$g_3 = \langle free_3 | \sum V | true_3 \rangle - \langle free_2 | V | true_2 \rangle$$

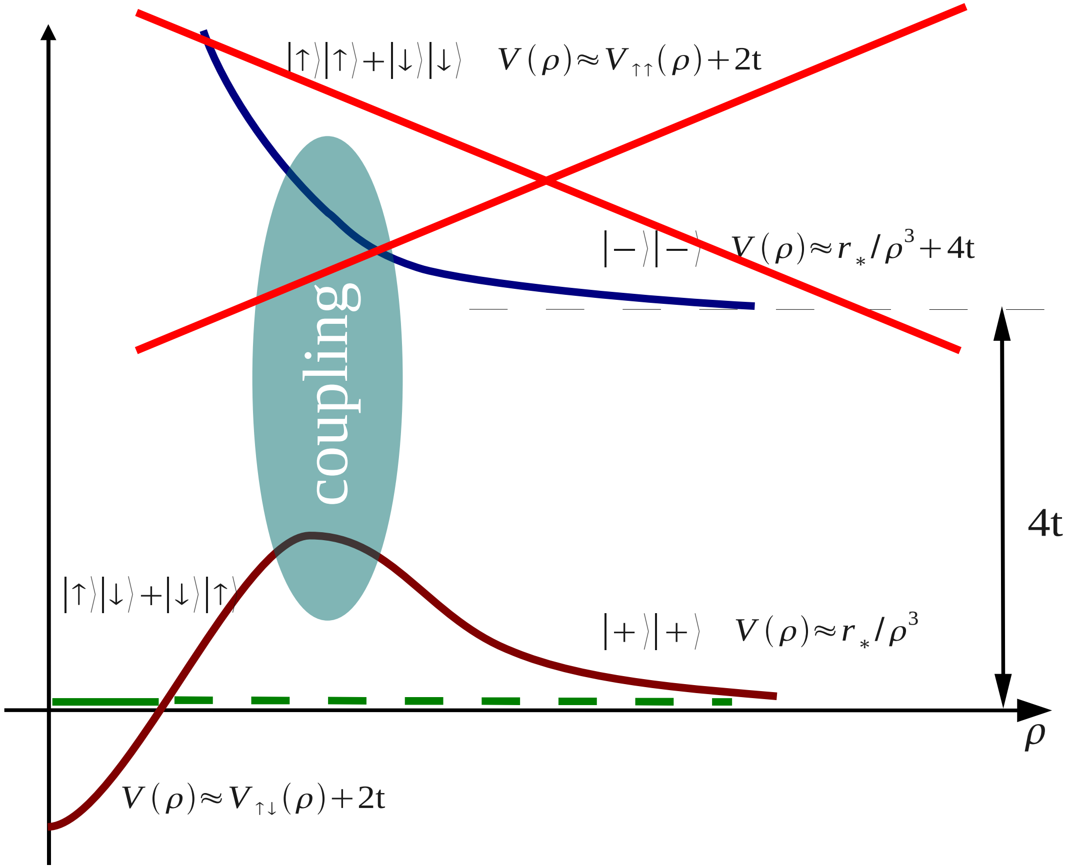
adiabatic hyperspherical method
on the line g_2=0



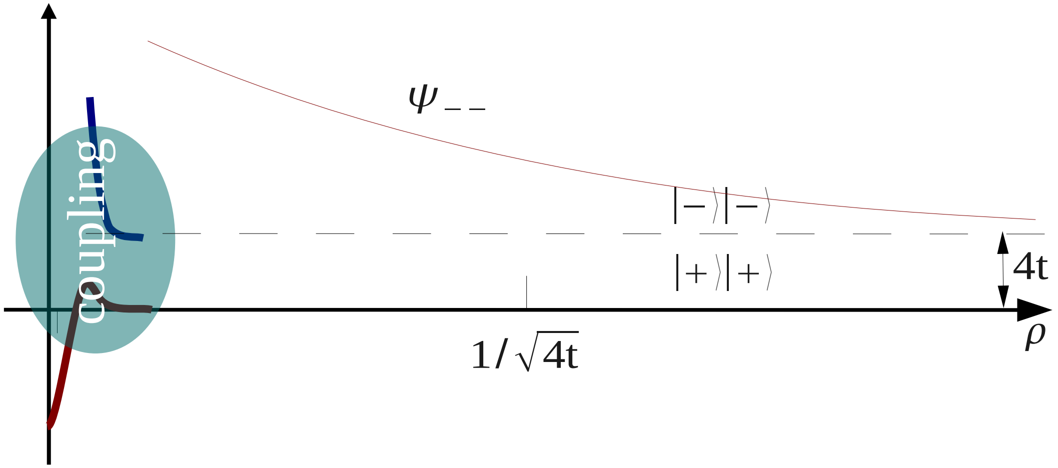
Large r_* – dipolar frustration (cf. Volosniev et al.'12)

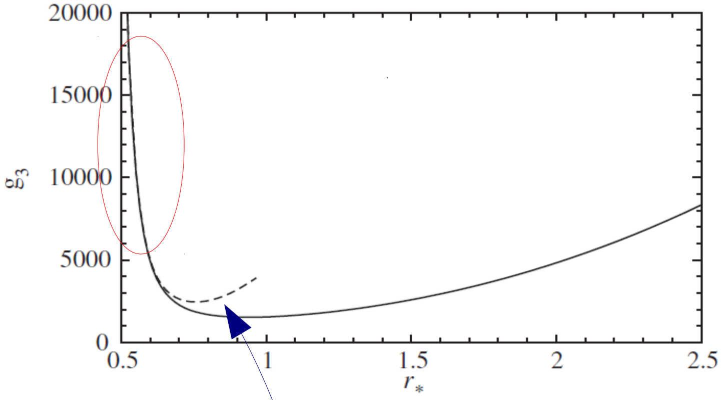


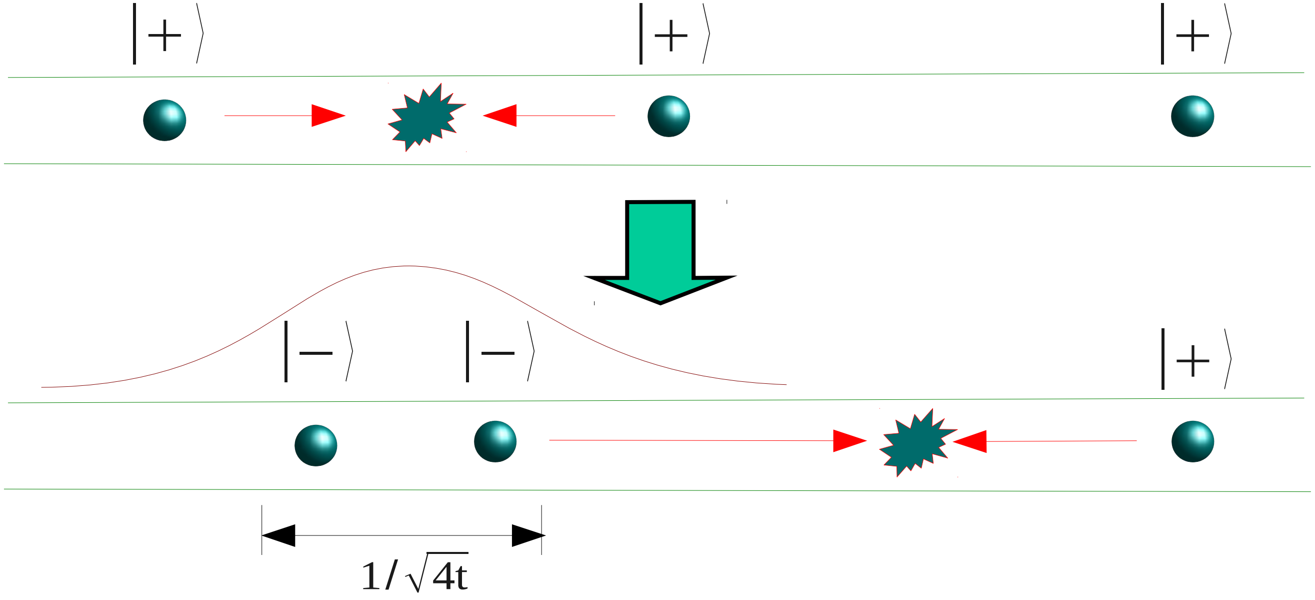




Small r_* – large off shell contribution







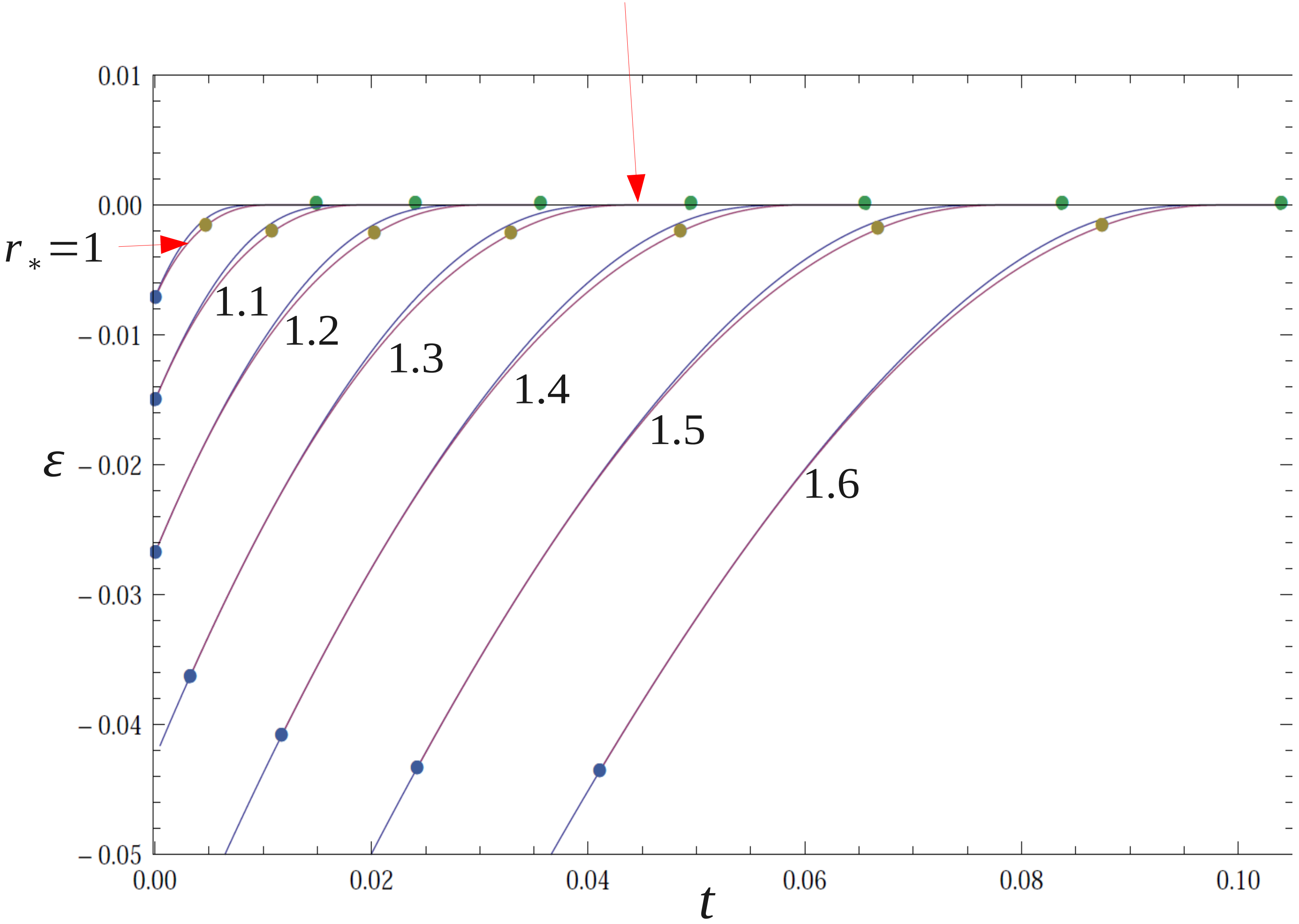
Zero-range model →

$$g_3 = \frac{24\pi^2}{t_c}\left[\frac{1}{\ln^3\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} - \frac{3\ln(4/3)}{\ln^4\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} + \ldots\right]$$

Three-body repulsion and trimers

$$\boxed{B_3^{(0)} = 16.522\,688\,(1)\,B_2}$$

Bruch&Tjon'79,Nielsen et al.'99,
Hammer&Son'04



Frustration is good!

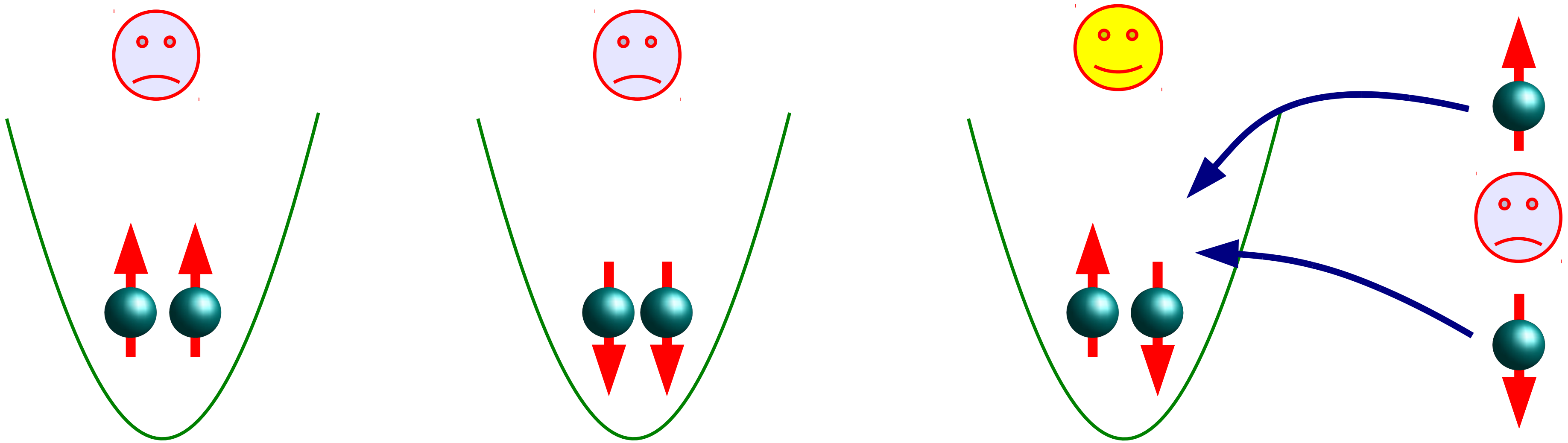
Lattice bosons with on-site Hamiltonian

$$H=\frac{\Delta}{2}(b_\downarrow^+ b_\downarrow - b_\uparrow^+ b_\uparrow) - \frac{\Omega}{2}(b_\uparrow^+ b_\downarrow + b_\downarrow^+ b_\uparrow) + \sum_{\sigma,\sigma'} \frac{g_{\sigma\sigma'}}{2} b_\sigma^+ b_{\sigma'}^+ b_\sigma b_{\sigma'}$$

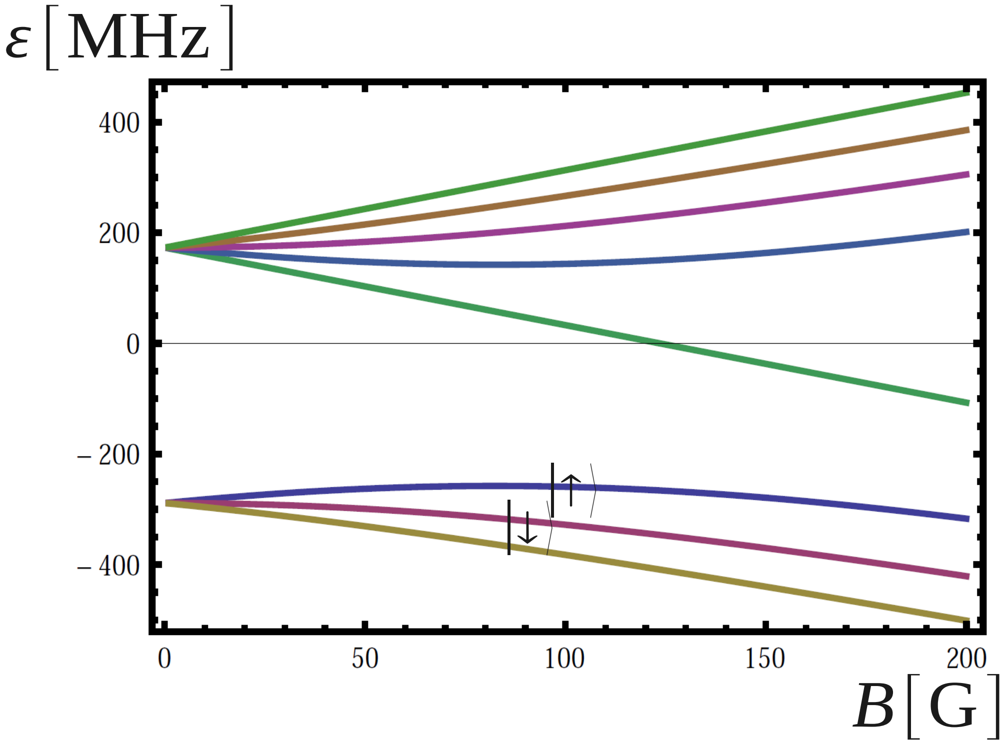
Bilayer dipoles → $\Delta=0,\ \Omega=2\mathrm{t},\ g_{\uparrow\uparrow}=g_{\downarrow\downarrow}>0,$ and $g_{\uparrow\downarrow}<0$

Simple example

$$\Delta=\Omega=g_{\uparrow\downarrow}=0,\ \ g_{\uparrow\uparrow}=g_{\downarrow\downarrow}>0$$



^{39}K: |F=1,m_F=0> and |F=1,m_F=-1>



Couple them with RF (~50MHz)

$\Omega=$Rabi frequency (~kHz)

$\Delta=$Detuning (~kHz)

$$\sqrt{\Omega^2+\Delta^2}=\text{spin splitting}\gg T$$

Spin is virtually excited during collisions!

$$\boxed{g_{\sigma\sigma'}=?}$$

→ D'Errico et al.'07, Lysebo&Veseth'10

Many thanks to A. Simoni and M. Lysebo for the data!

Optimization problem

Find *E*(2) and *E*(3) by diagonalizing

$$H=\frac{\Delta}{2}(b_\downarrow^+ b_\downarrow - b_\uparrow^+ b_\uparrow)-\frac{\Omega}{2}(b_\uparrow^+ b_\downarrow + b_\downarrow^+ b_\uparrow)+\sum_{\sigma,\sigma'}\frac{g_{\sigma\sigma'}}{2} b_\sigma^+ b_{\sigma'}^+ b_\sigma b_{\sigma'}$$

$$U_2 \text{ and } U_3$$

Find parameters for which $\boxed{U_2=0 \text{ and } U_3 \rightarrow max}$

The result is:

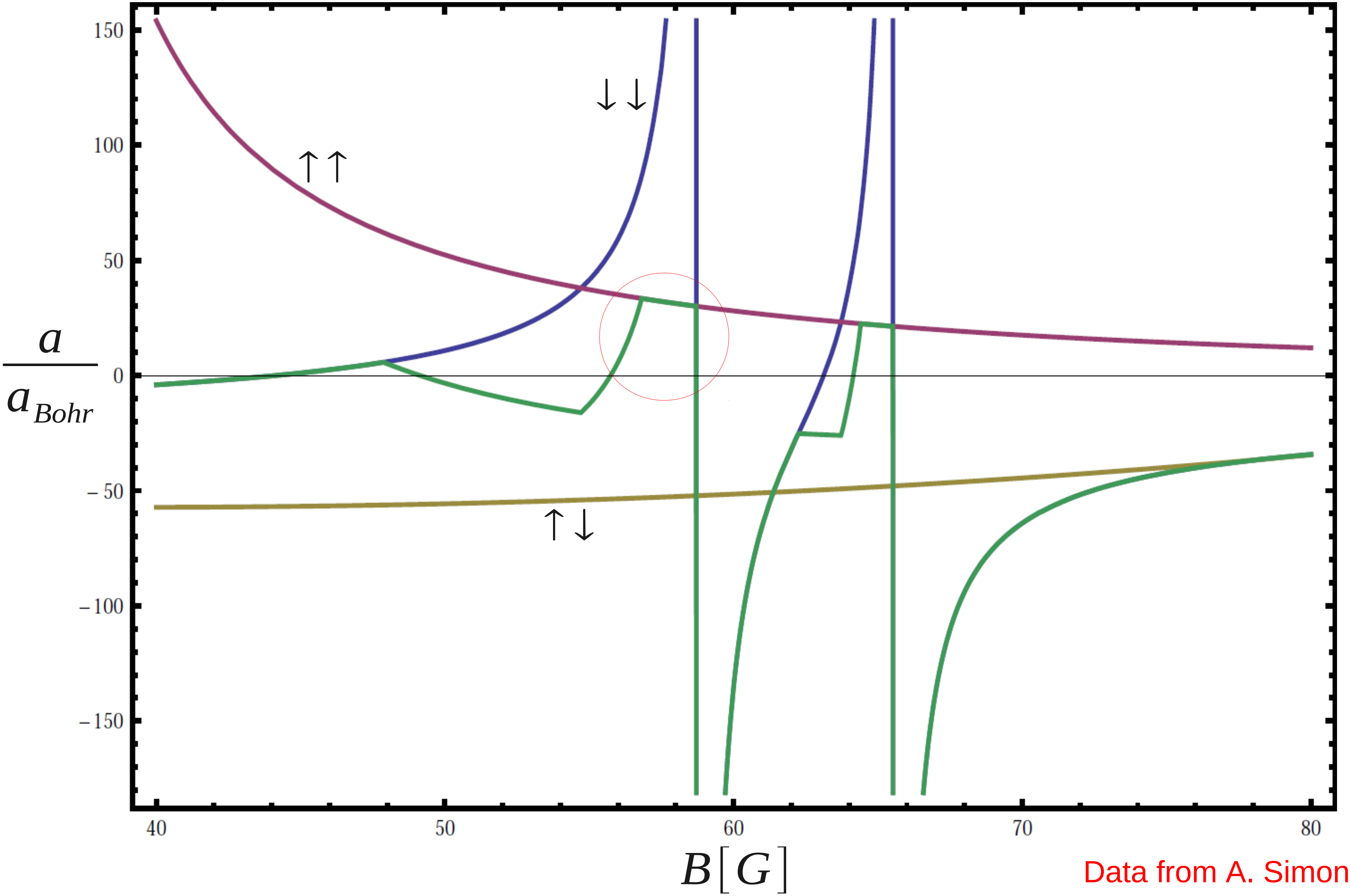
$$U_{3,\,max}=\begin{cases} min(g_{\downarrow\downarrow},g_{\uparrow\uparrow}), & |g_{\downarrow\downarrow}-g_{\uparrow\uparrow}|>-g_{\uparrow\downarrow} \\ max(g_{\downarrow\downarrow},g_{\uparrow\uparrow})+g_{\uparrow\downarrow}, & |g_{\downarrow,\downarrow}-g_{\uparrow,\uparrow}|<-g_{\uparrow\downarrow} \end{cases}$$

reached for

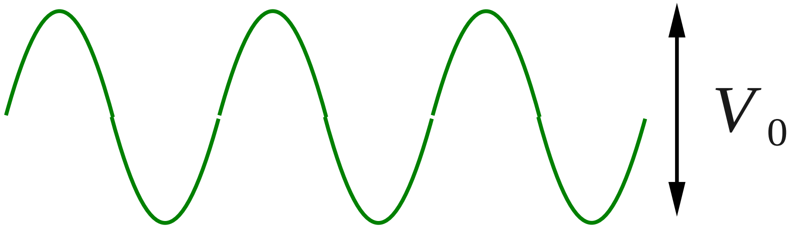
$$\Omega=0 \;\; \&\& \;\; \Delta=g_{\uparrow\downarrow}sign(g_{\downarrow\downarrow}-g_{\uparrow\uparrow})$$

$$\boxed{U_{3,\,max}>0 \;\; \text{requires} \;\; min(g_{\downarrow\downarrow},g_{\uparrow\uparrow})>0 \;\; \&\& \;\; -max(g_{\downarrow\downarrow},g_{\uparrow\uparrow})<g_{\uparrow\downarrow}<0}$$

Nice window of B



Data from A. Simoni



lattice constant $=532\,\mathrm{nm}$

$V_0=15\,E_R$

on-site osc. freq. $=2\pi\times 35\,\mathrm{kHz}$

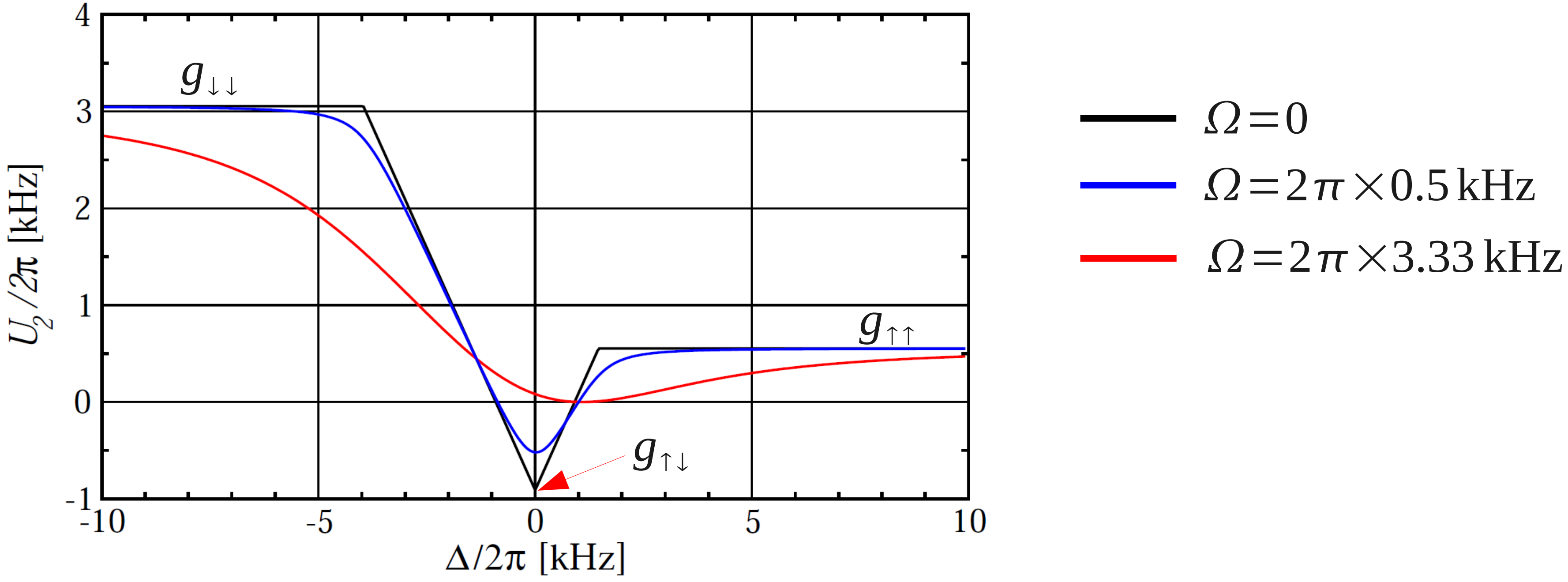
$l_x=l_y=l_z=86\,\mathrm{nm}$

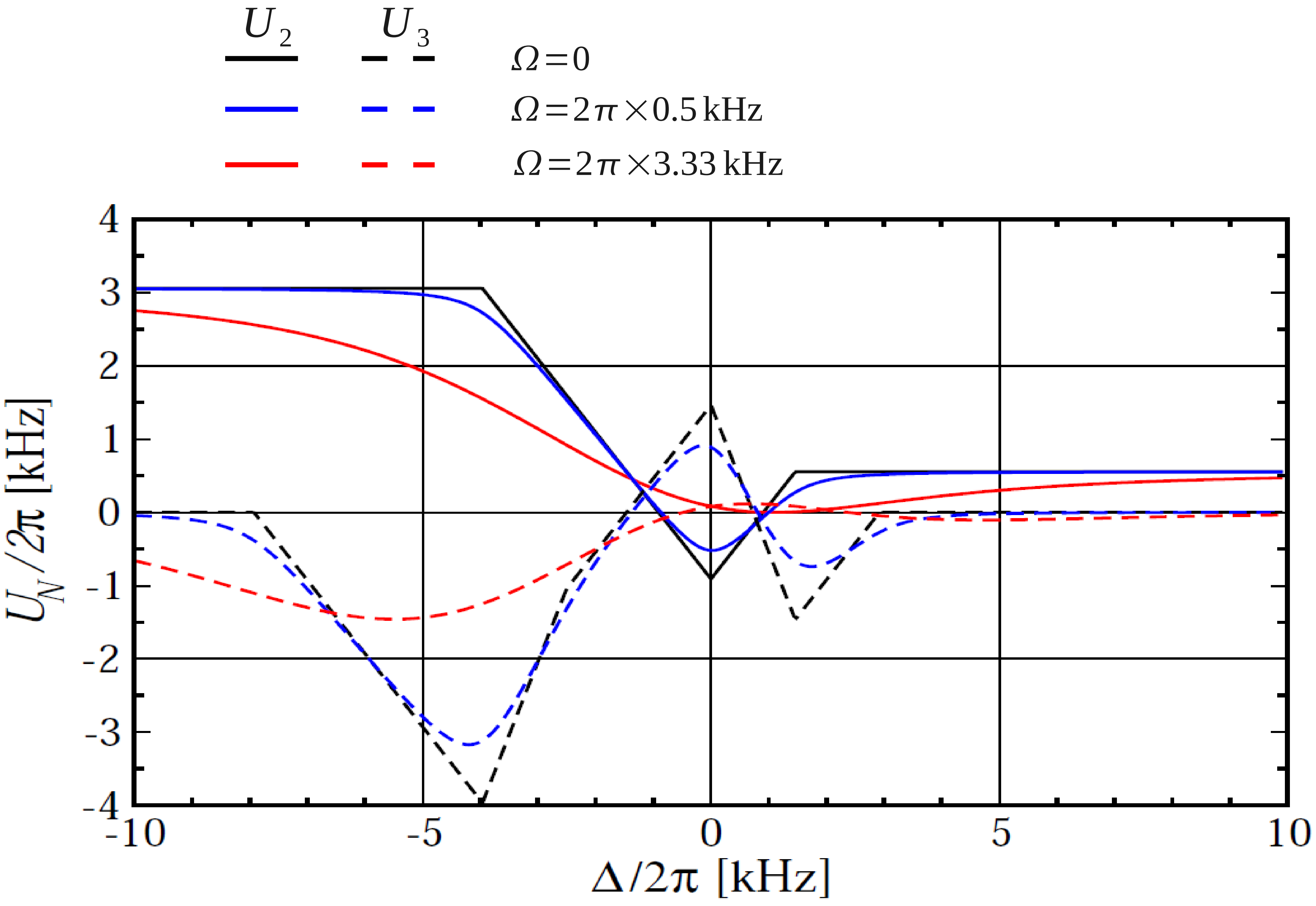
tunneling amp. $=2\pi\times 30\,\mathrm{Hz}$

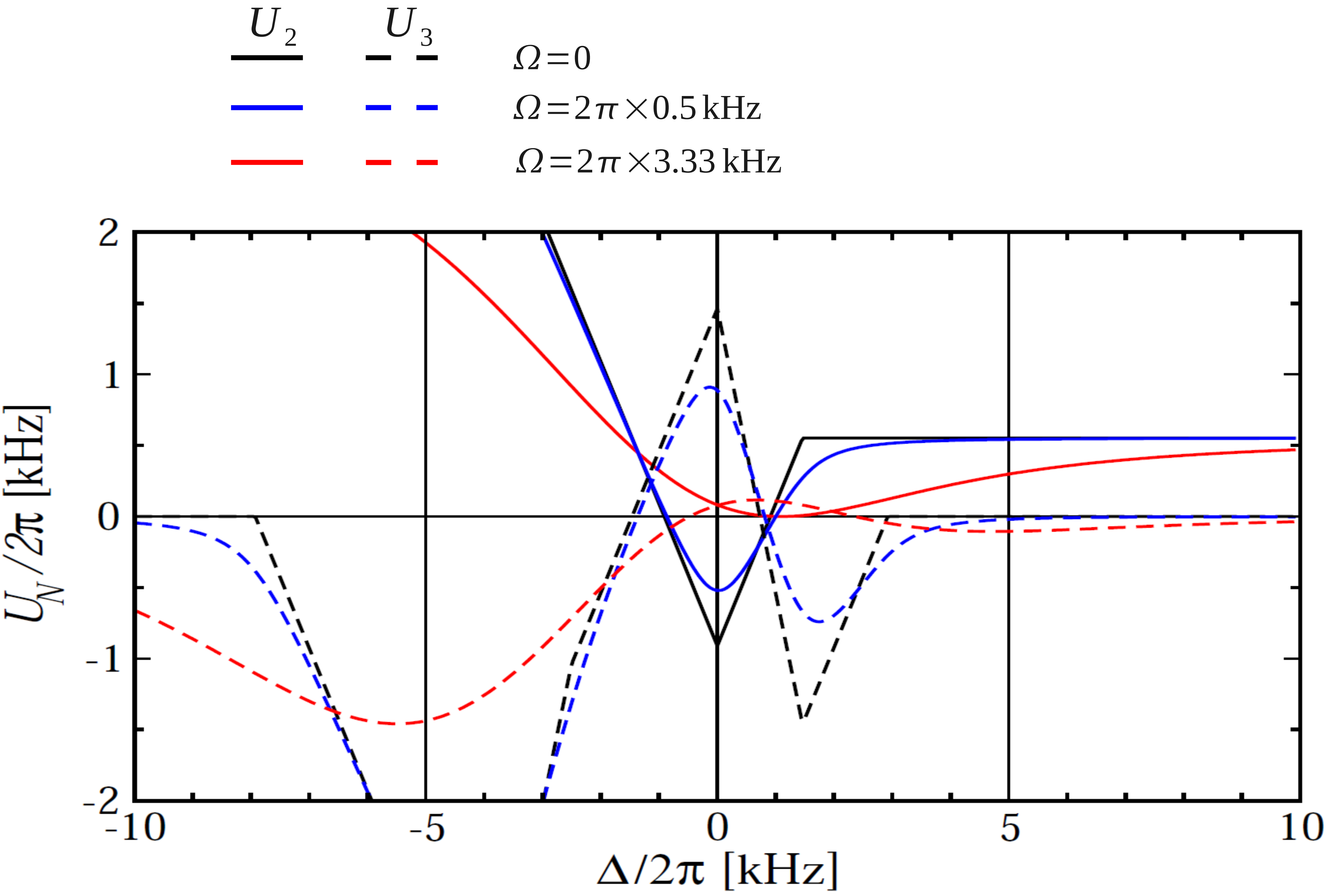
$a_{\downarrow\downarrow}=9.4\,\mathrm{nm}\to g_{\downarrow\downarrow}=2\pi\times 3.05\,\mathrm{kHz}$

$a_{\uparrow\uparrow}=1.7\,\mathrm{nm}\to g_{\uparrow\uparrow}=2\pi\times 0.55\,\mathrm{kHz}$

$a_{\uparrow\downarrow}=-2.8\,\mathrm{nm}\to g_{\uparrow\downarrow}=-2\pi\times 0.91\,\mathrm{kHz}$



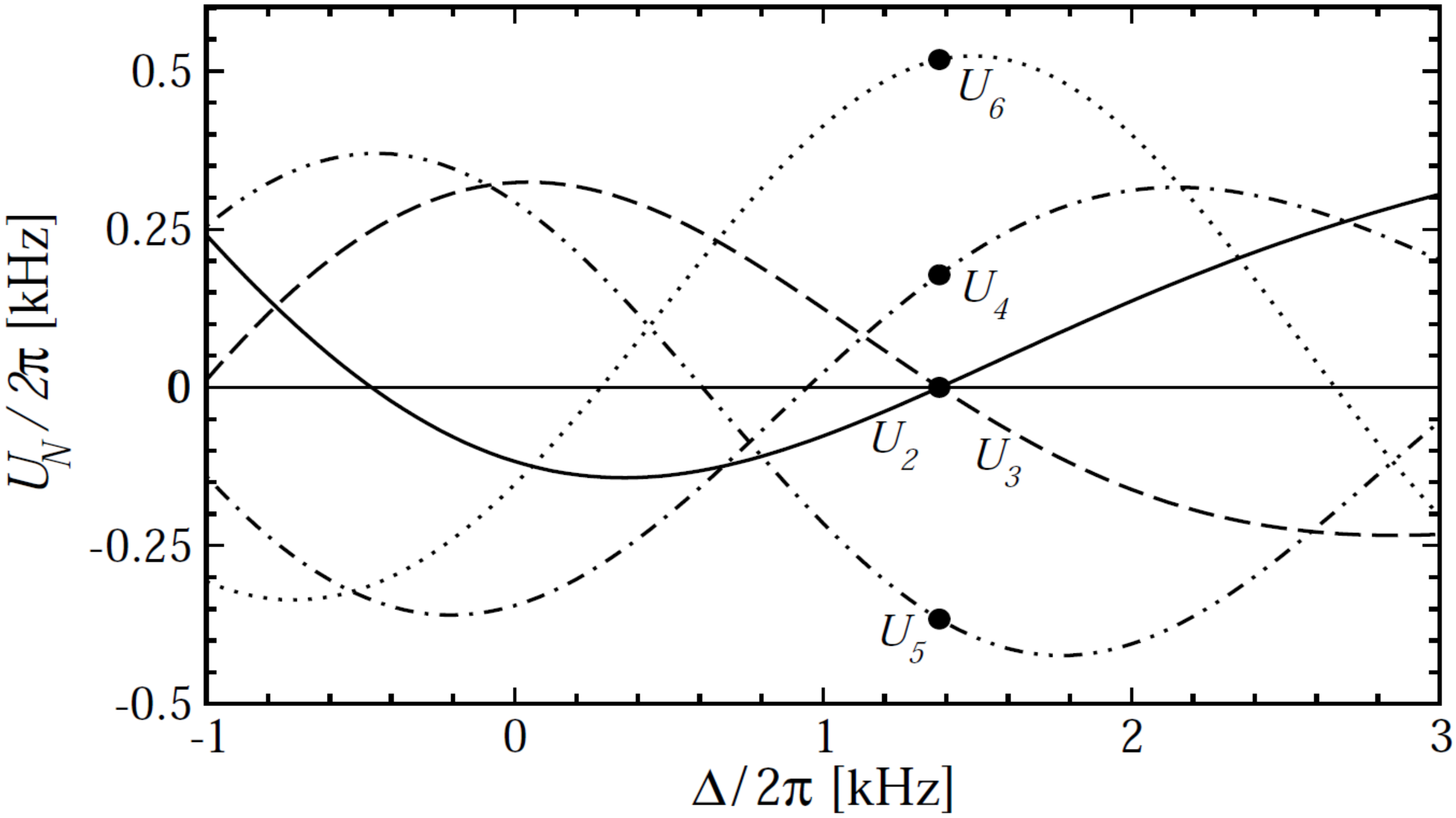

U_2
U_3
$\Omega=0$
$\Omega=2\pi\times0.5\,$kHz
$\Omega=2\pi\times3.33\,$kHz
$U_N/2\pi$ [kHz]
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10
$\Delta/2\pi$ [kHz]



U_2
U_3
$\Omega=0$
$\Omega=2\pi\times0.5\,\mathrm{kHz}$
$\Omega=2\pi\times3.33\,\mathrm{kHz}$
$U_N/2\pi$ [kHz]
$\Delta/2\pi$ [kHz]
2
1
0
-1
-2
-10
-5
0
5
10

4-body interacting case

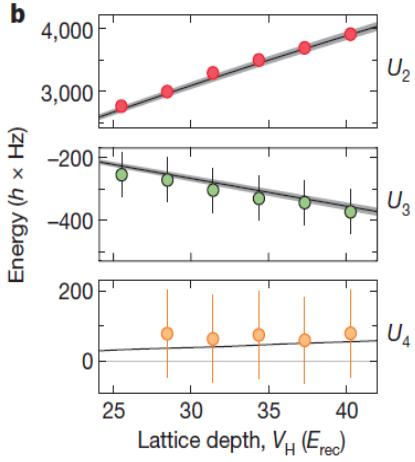
$\Omega = 2\pi \times 1.7\,\mathrm{kHz}$

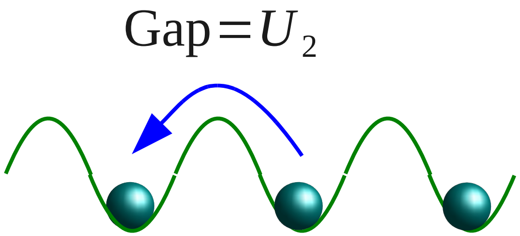


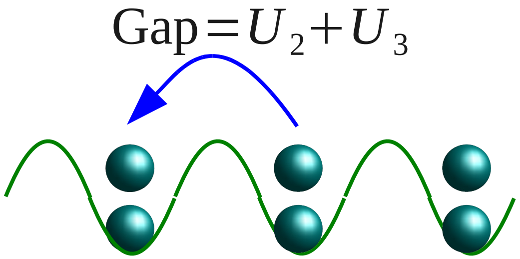
Perspectives

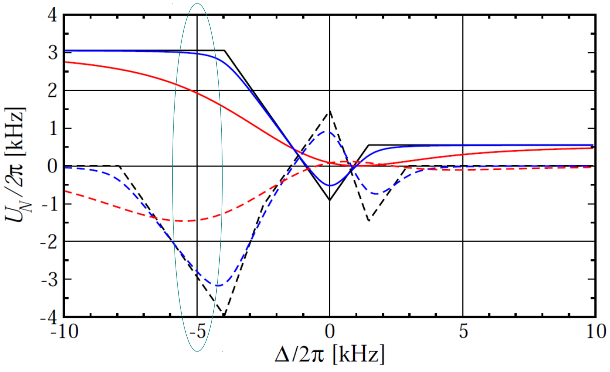
- Bosonic dipoles... Have to wait a bit...
- ^{39}K on a lattice

  - collapse and revivals
  - solitonic-like self-trapping
  - Mott lobes Chen et al.'08









- Frustration is local, large-size off-shell effects are cooler!

  requires small *t* (bilayers) or small $\sqrt{\Omega^2+\Delta^2}$ (RF coupling)

Thank you!
Merci beaucoup!