Equation of state and phase structure of ultracold quantum gases in 2 & 3 dimensions

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Boettcher, JMP, Wetterich, in preparation

Boettcher, JMP, Wetterich, arXiv:1312.0505 [cond-mat.quant-gas]

Boettcher, Diehl, JMP, Wetterich, Phys.Rev. A87, 023606 (2013)

Outline

- **BEC - BCS cross-over & the functional RG**

- **2d & 3d phase structure**

- **Summary and outlook**

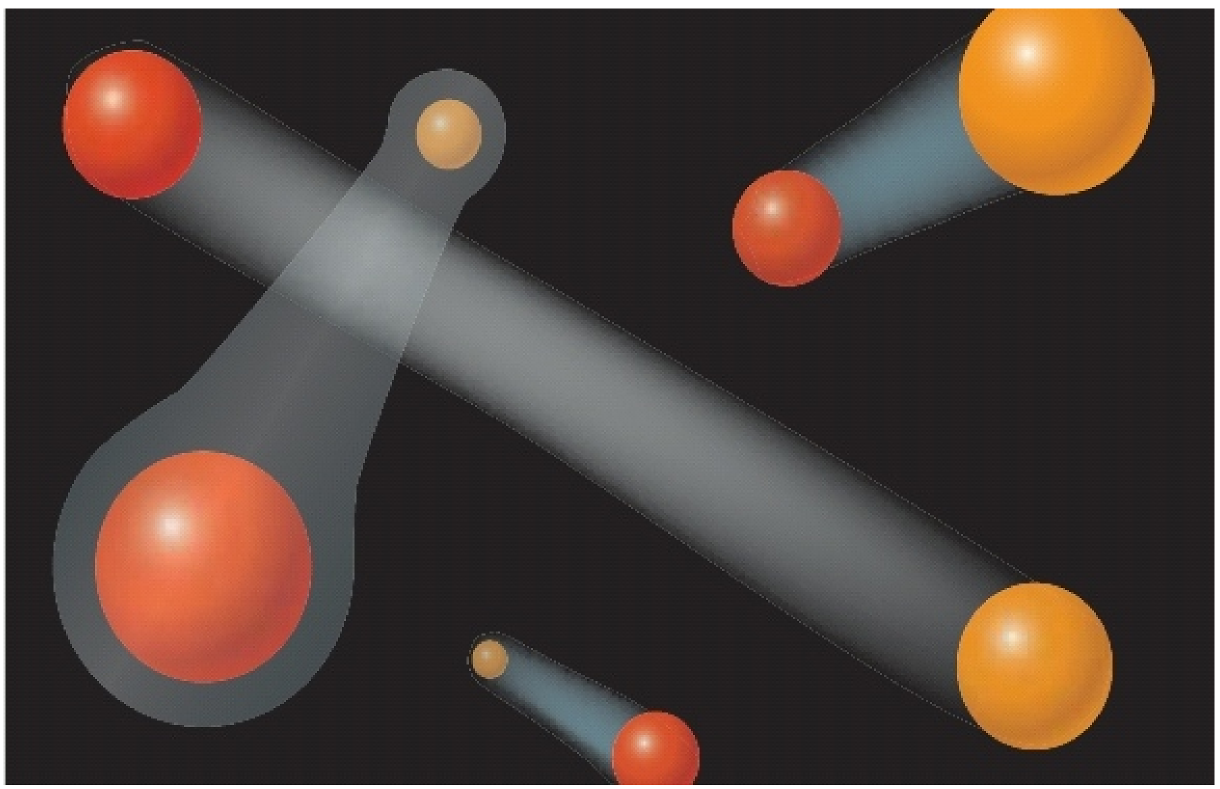
Phase diagram of cold quantum gases

BEC-BCS cross-over

Eagles ’69, Leggett ‘80

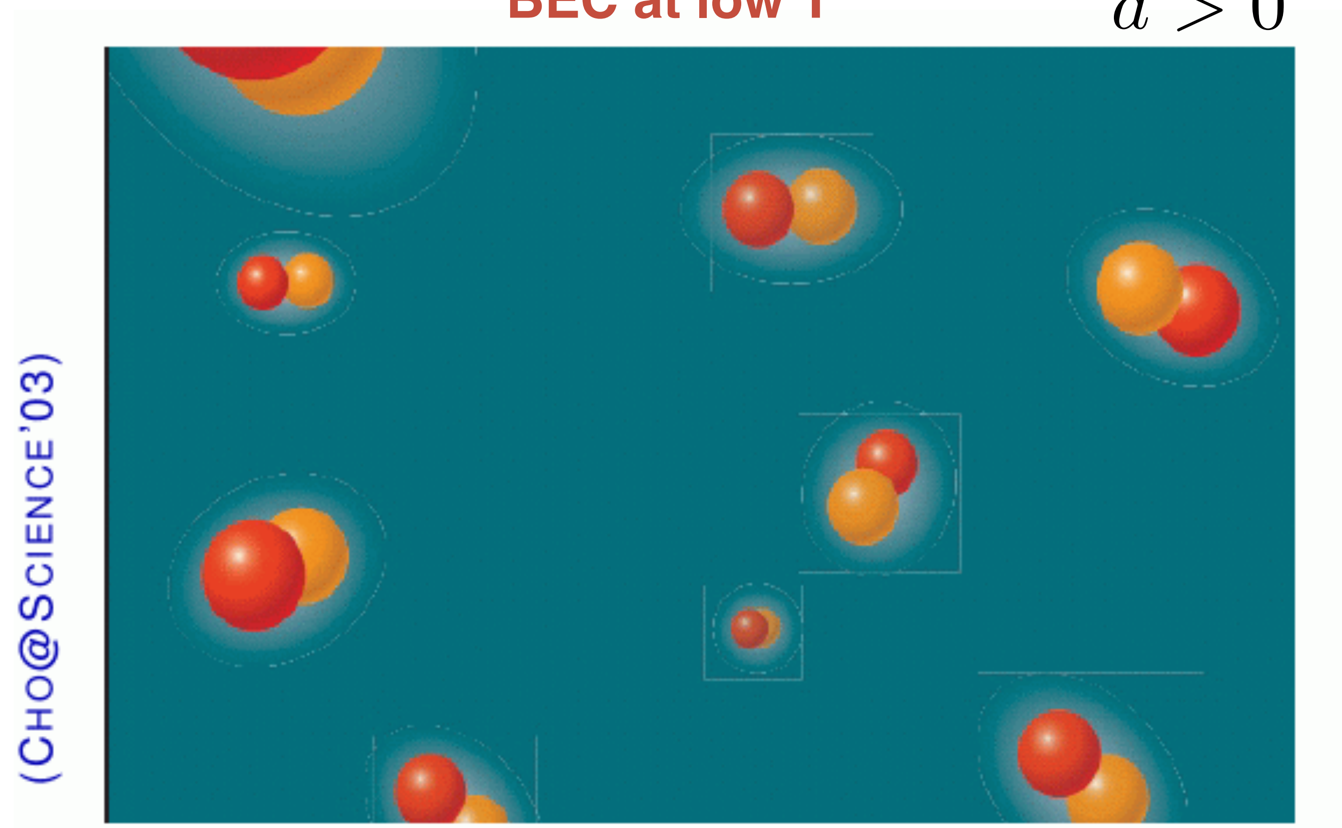
Fermions with attractive interactions

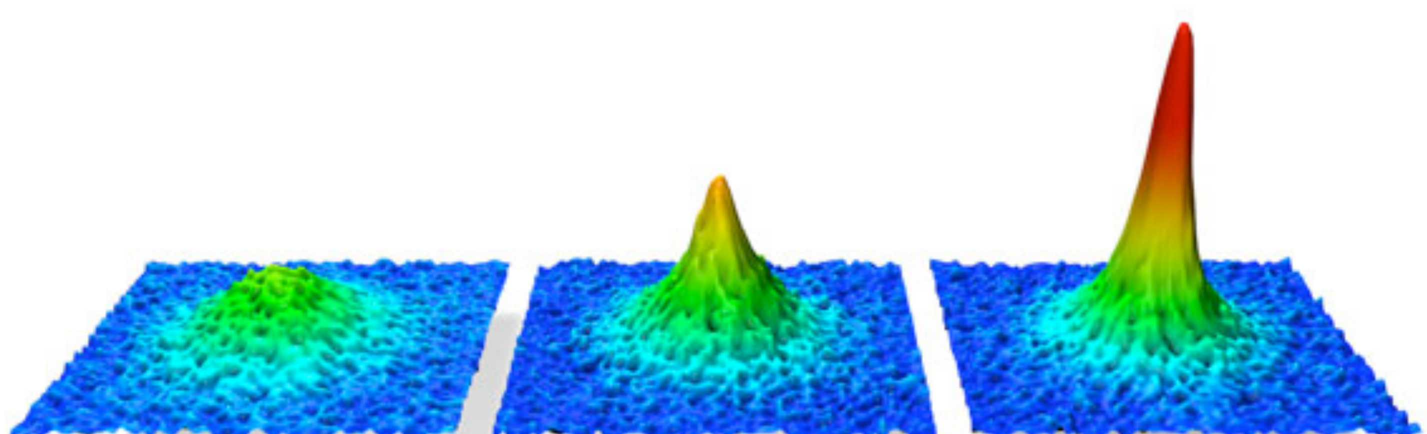
BCS superfluidity at low T $a < 0$



Bound molecules of two atoms on microscopic scale

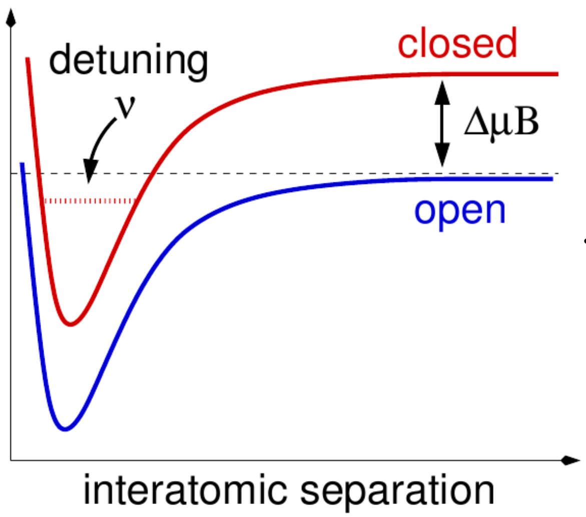
BEC at low T $a > 0$



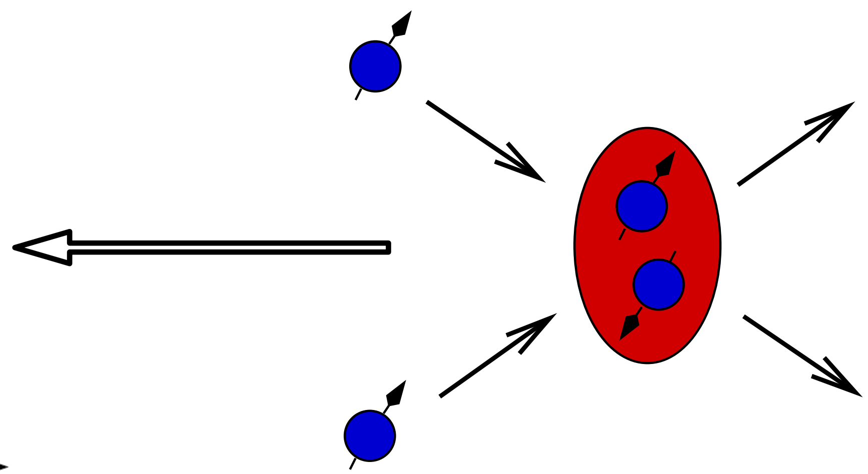


Regal et al ’04

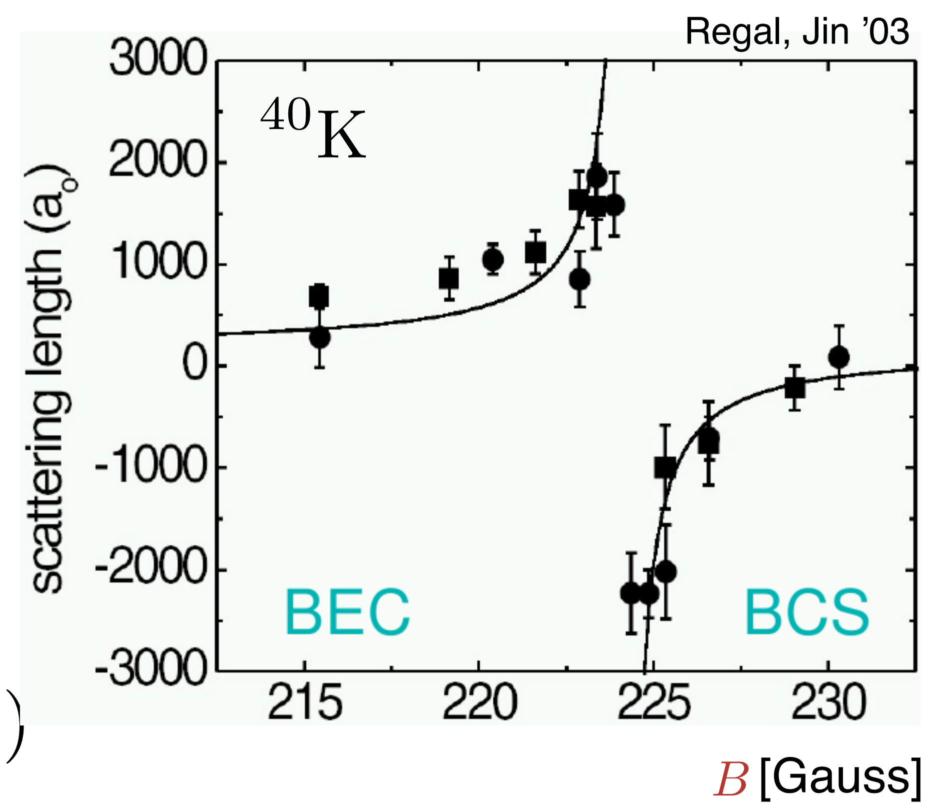
BEC-BCS cross-over



2-atom scattering



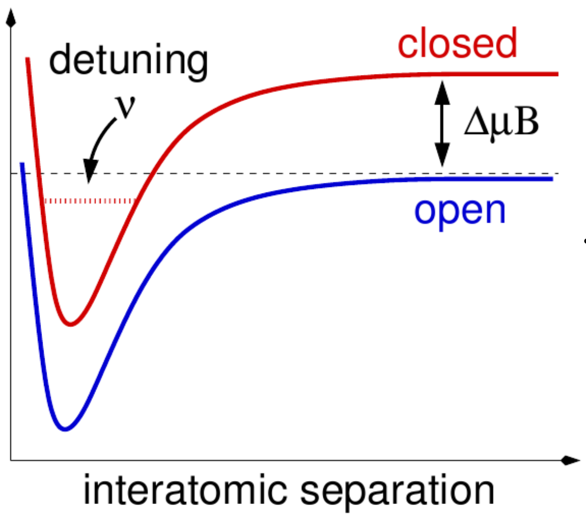
e.g

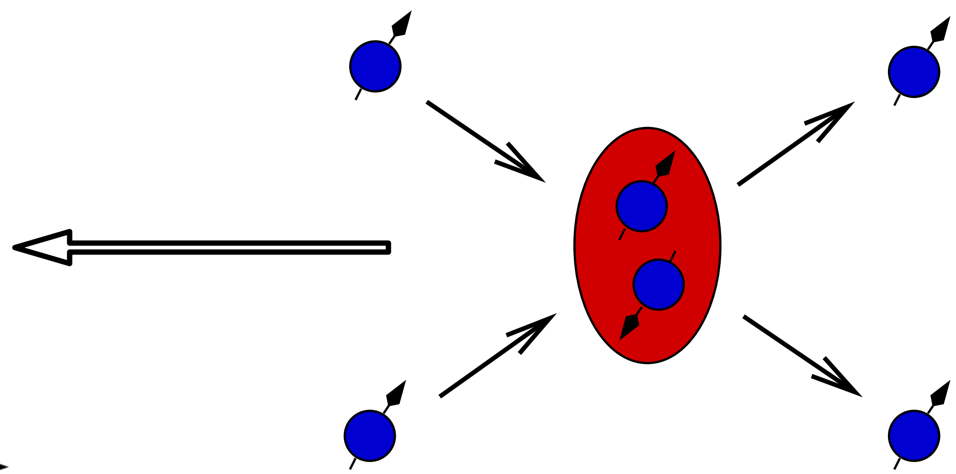


hyperfine interaction between interaction channels ⇒ **s-wave scattering length** $a(B)$

BEC-BCS cross-over

2-atom scattering





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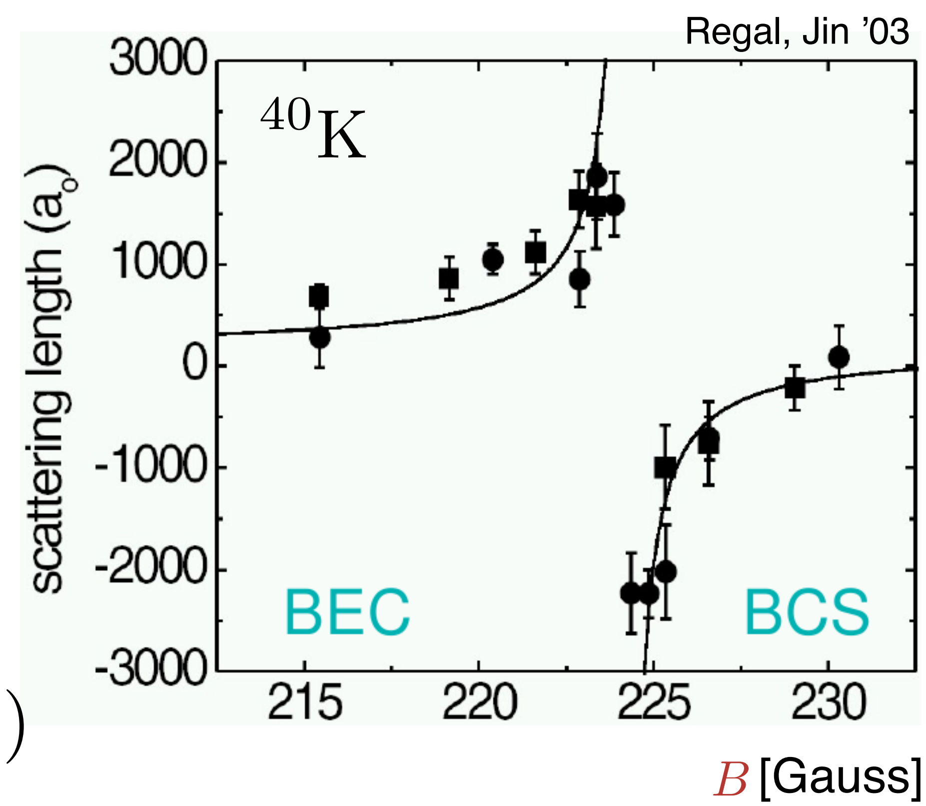
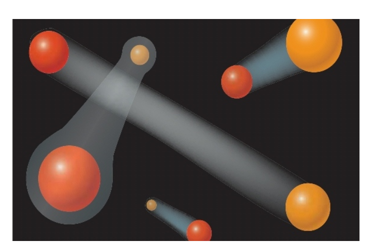


hyperfine interaction between interaction channels ⇨ **s-wave scattering length** $a(B)$

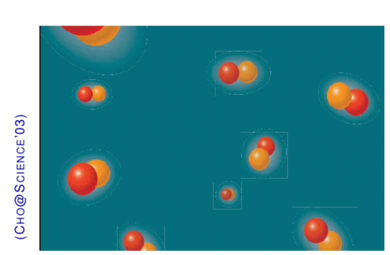
Strongly-correlated set-up

Relevant degrees of freedom

stable fermionic atom field ψ



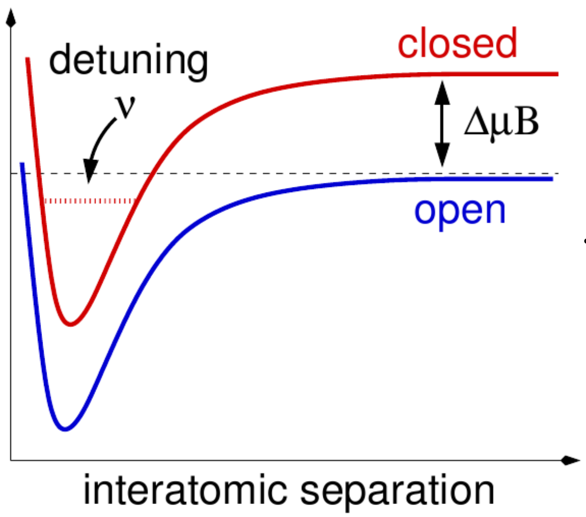
bosonic molecule field /Cooper pair ϕ

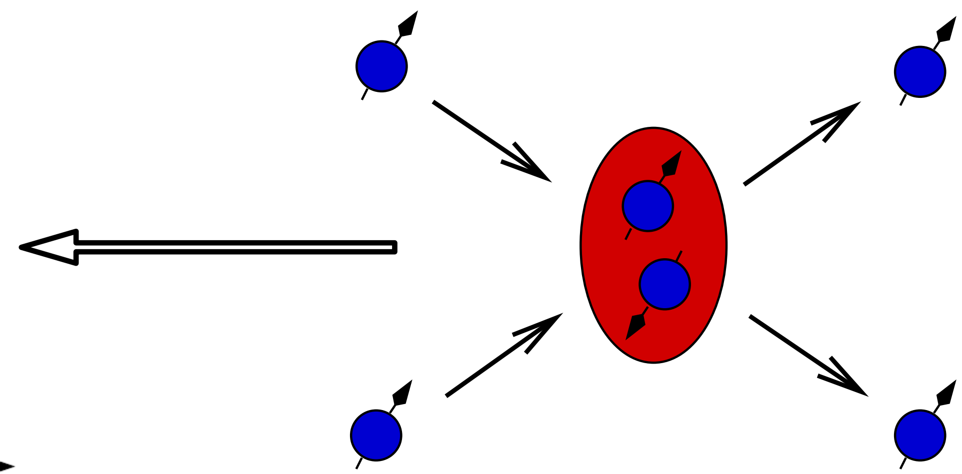


$$\hbar = k_B = 2M = 1$$

BEC-BCS cross-over

2-atom scattering





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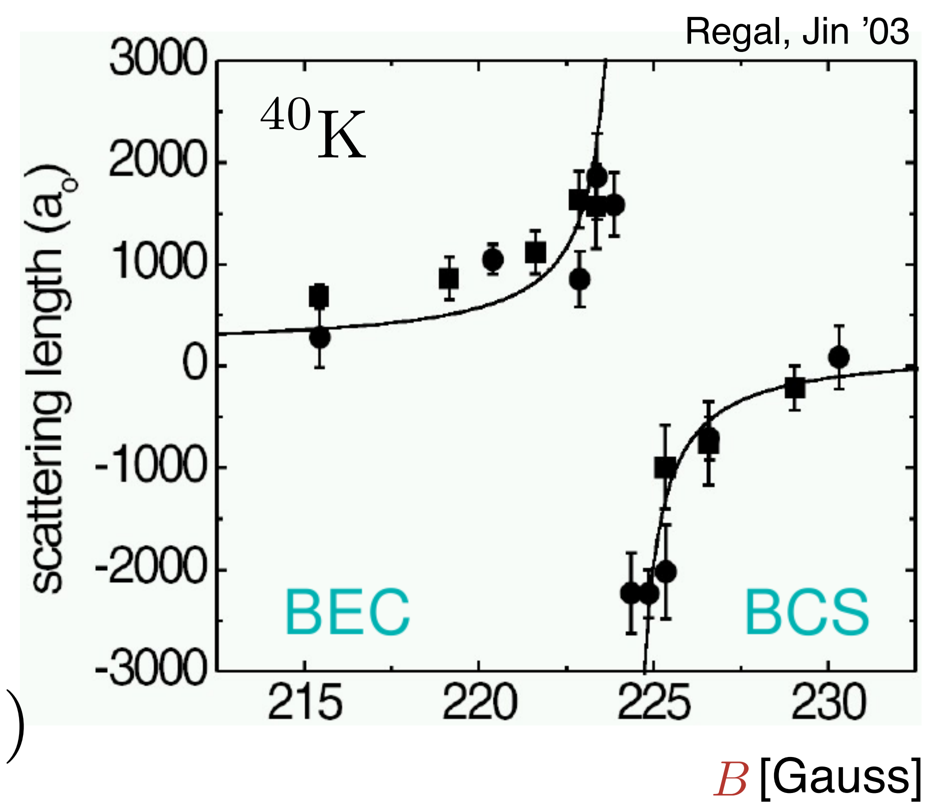

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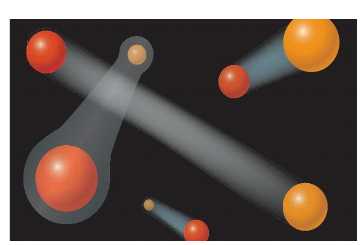

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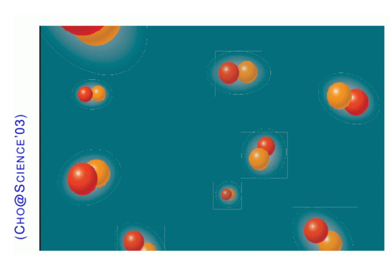


Effective action

$$\Gamma[\psi,\phi] = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger \left(Z_\psi \partial_\tau - A_\psi \nabla^2 - \mu \right) \psi$$

$$+\phi^* \left(Z_\phi \partial_\tau - A_\phi \frac{\nabla^2}{2} \right) \phi$$

$$+\lambda_\psi (\psi^\dagger \psi)^2 + U(\phi)$$

$$-\frac{h_\phi}{2} (\phi^* \psi^{\,\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \ldots \Big\}$$

BEC-BCS cross-over

Effective action

$$\dots + m_\phi^2 \phi^* \phi + \lambda_\psi (\psi^\dagger \psi)^2 - \frac{h_\phi}{2}(\phi^* \psi^{\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \dots$$

⇑ **relevant terms**

$$\lambda_\psi (\psi^\dagger \psi)^2 = \frac{1}{2} \lambda_\psi (\psi^\dagger \epsilon \psi^*)(\psi^{\mathrm{T}} \epsilon \psi)$$

Fierz transformation

BEC-BCS cross-over

Effective action

$$\cdots + m_\phi^2 \phi^* \phi + \lambda_\psi (\psi^\dagger \psi)^2 - \frac{h_\phi}{2}(\phi^* \psi^{\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \dots$$

relevant terms

Relation to microphysics via Hubbard-Stratonovich

$$m_\phi^2 = \bar{\mu}(B - B_0) - 2\mu$$

chemical potential of molecule

$$\lambda_\psi = \frac{4\pi a_{\mathrm{bg}}}{M}$$

$$\lambda_{\psi,\mathrm{eff}} = \lambda_\psi - \frac{h^2}{m_\phi^2} \quad \Longrightarrow \quad a(B) = \frac{M}{4\pi}\left(\lambda_\psi - \frac{h^2}{m_\phi^2}\right)$$

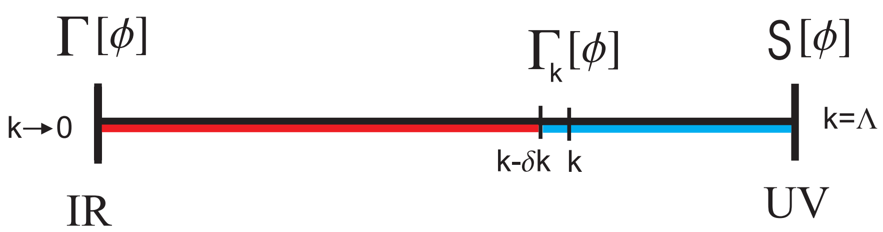
$$h_\phi^2 = \Delta B$$

Functional Methods for ultracold atoms

Functional RG

Boettcher, JMP, Diehl, Nucl.Phys.Proc.Suppl. 228 (2012) 63-135

free energy at momentum scale k



RG-scale k: $t = \ln k$

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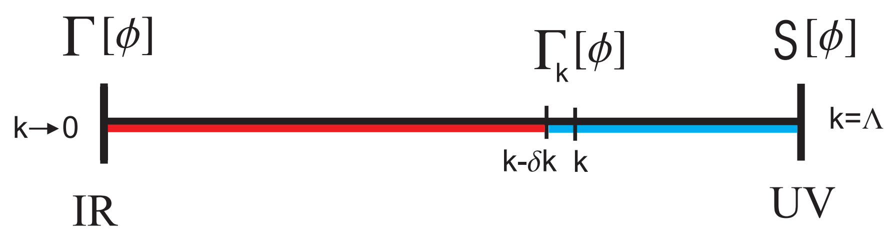

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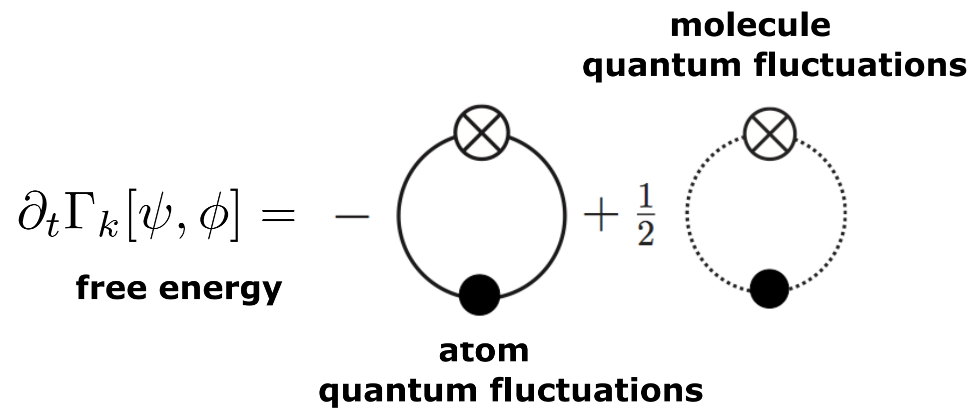

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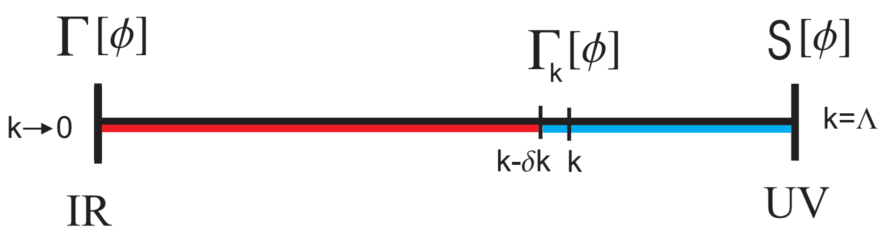
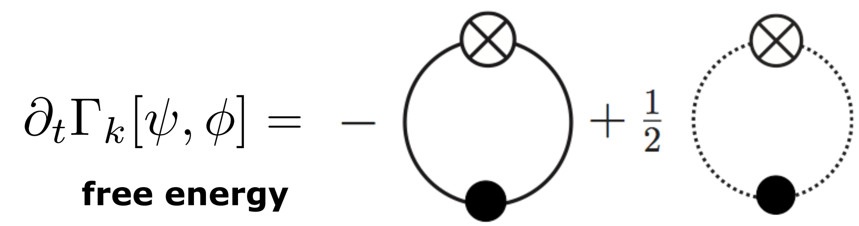


RG-scale k: $t = \ln k$

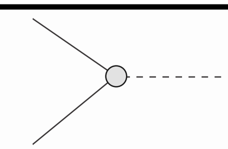
molecule quantum fluctuations



free energy

atom quantum fluctuations

Dynamical bosonisation



dynamical

Gies, Wetterich ’01
JMP ’05
Flörchinger, Wetterich ’09

Functional Methods for ultracold atoms

Functional RG

Boettcher, JMP, Diehl, Nucl.Phys.Proc.Suppl. 228 (2012) 63-135

ultracold atoms

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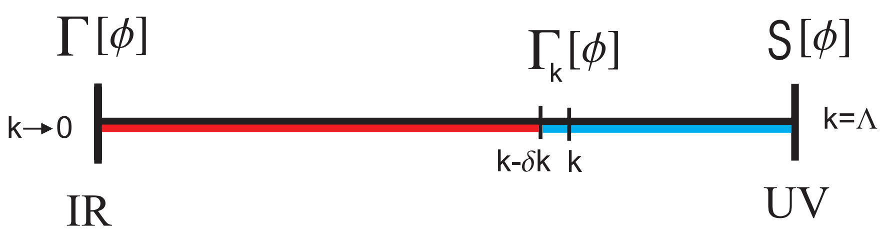
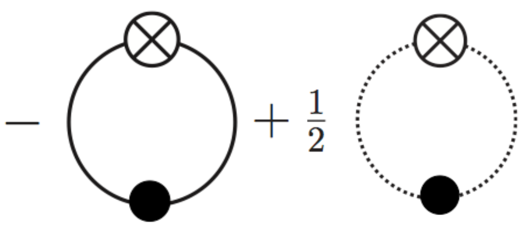


RG-scale k: $t = \ln k$

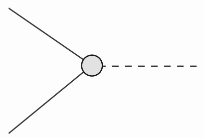
molecule quantum fluctuations

$\partial_t \Gamma_k[\psi,\phi] = -$ 

free energy

atom quantum fluctuations

Dynamical bosonisation



dynamical

Gies, Wetterich '01
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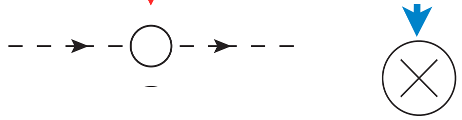
Bosons

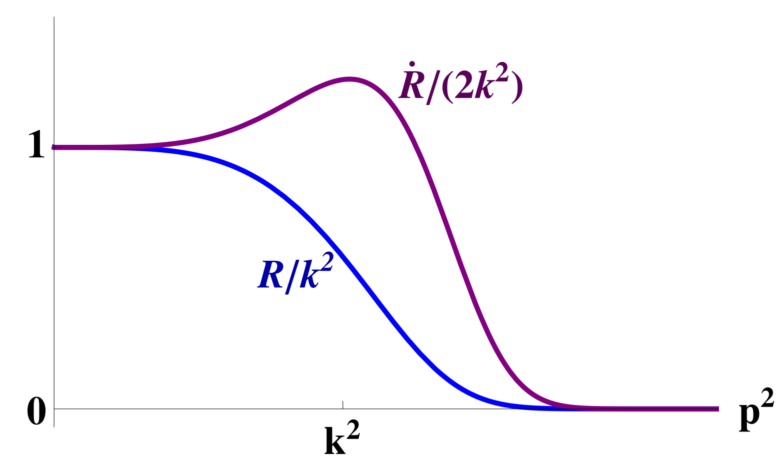
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[\phi]+R_k}\,\partial_t R_k\right\} - \partial_t C_k$$

$\partial_t = k\,\partial_k$

full propagator

regulator





BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi]=\int_{\tau,\vec{x}}\Big\{\psi^\dagger\left(Z_\psi\partial_\tau-A_\psi\nabla^2-\mu\right)\psi+\phi^*\left(Z_\phi\partial_\tau-A_\phi\frac{\nabla^2}{2}\right)\phi\ +\lambda_\psi(\psi^\dagger\psi)^2+U(\phi)-\frac{h_\phi}{2}(\phi^*\psi^{\,\mathrm{T}}\epsilon\psi-\phi\psi^\dagger\epsilon\psi^*)\Big\}\ +\ \dots$$

Approximation

Fermion propagator

$A_{\psi,k}(\omega_0,\vec{q}^2)$

$Z_{\psi,k}(\omega_0,\vec{q}^2)$

Molecule propagator

$A_{\phi,k}+B_{\phi,k}\frac{p_0^2}{\vec{p}^2}\qquad Z_{\phi,k}$

Molecule effective potential

$U_k(\phi)$

Molecule-atom coupling

$h_{\phi,k}$

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Why?

BEC-BCS cross-over

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Why? density!

$$n(\mu, T) = 2 \int_{\vec{p}} \left(\frac{1}{2} - \int_{p_0} G_{\psi^*\psi}(P) \right)$$

BEC-BCS cross-over

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$$n(\mu, T) = 2 \int_{\vec{p}} \left(\frac{1}{2} - \int_{p_0} \frac{[i\,p_0 + \vec{p}^{\,2} - \mu + \Sigma_{\psi^*\psi}(P)]^*}{|i\,p_0 + \vec{p}^{\,2} - \mu + \Sigma_{\psi^*\psi}(P)|^2 + |\Sigma_{\psi^T\psi}|^2} \right)$$

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Contact

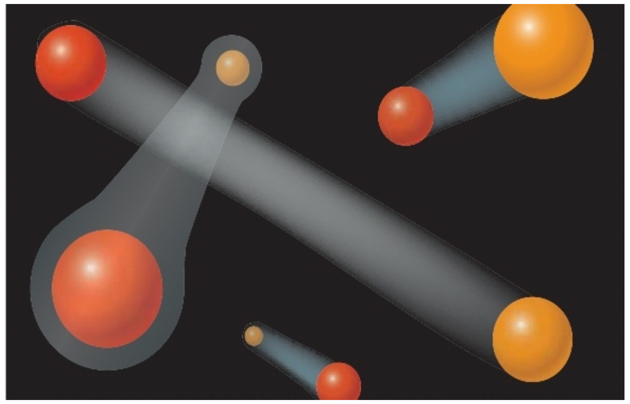
$$\Sigma_{\psi^*\psi}(P)] \simeq \frac{4C}{-ip_0 + \vec{p}^{\,2} - \mu} - \delta\mu$$

for $\vec{p}^{\,2} \to \infty$

Bound molecules of two atoms on microscopic scale

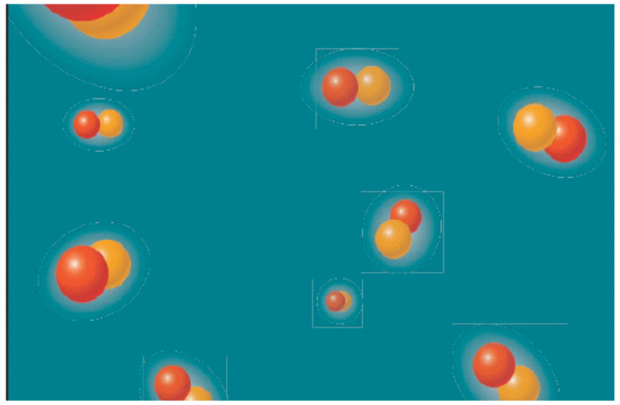
Fermions with attractive interactions

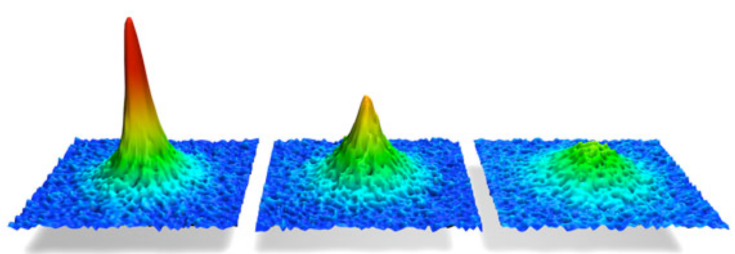
BEC at low T $a > 0$



BCS superfluidity at low T $a < 0$

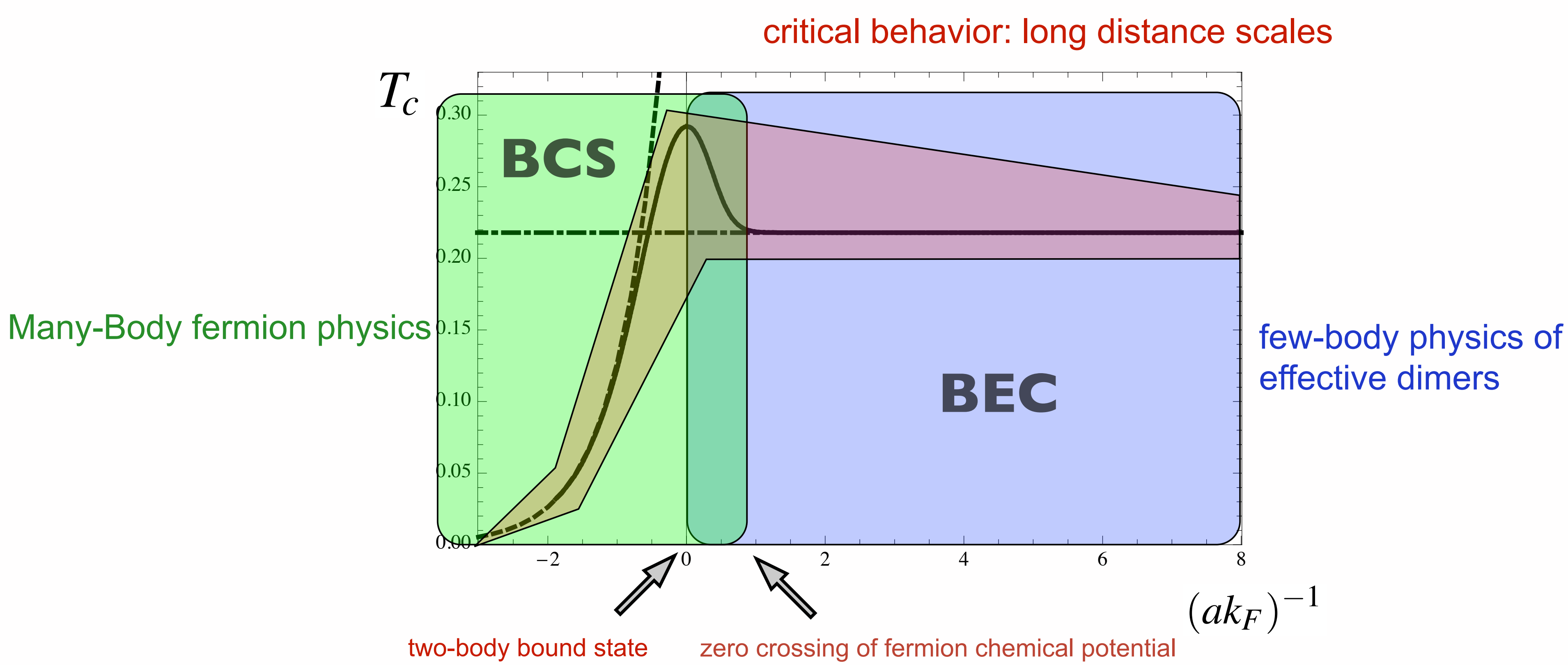
(Cho@Science'03)





Regal et al '04

Phase diagram of cold quantum gases



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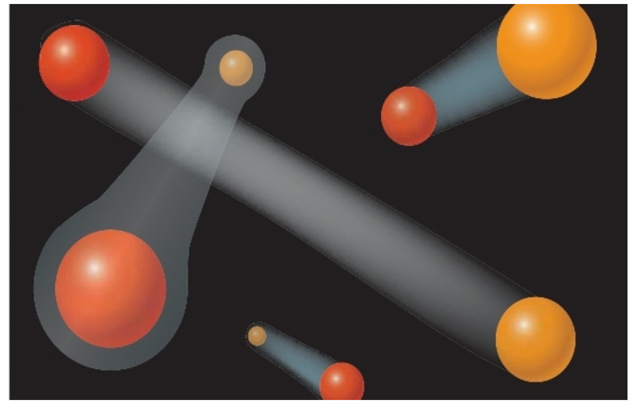

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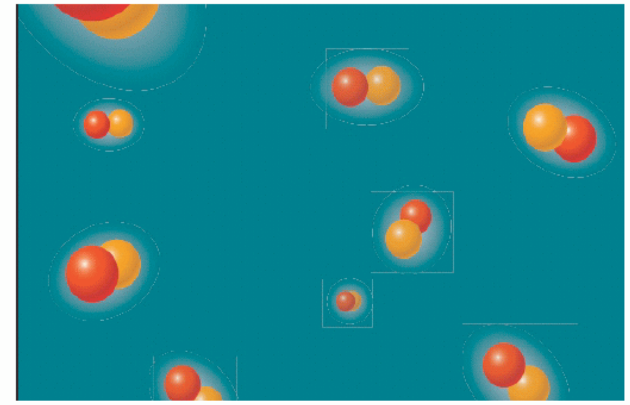

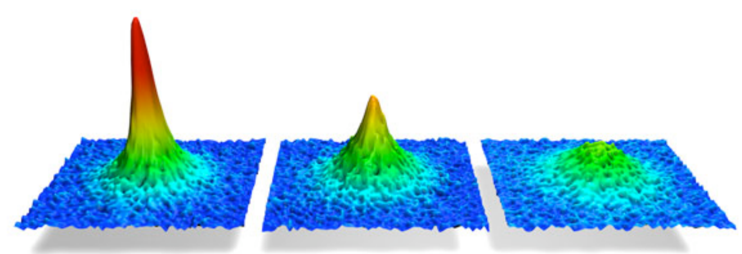

Regal et al '04

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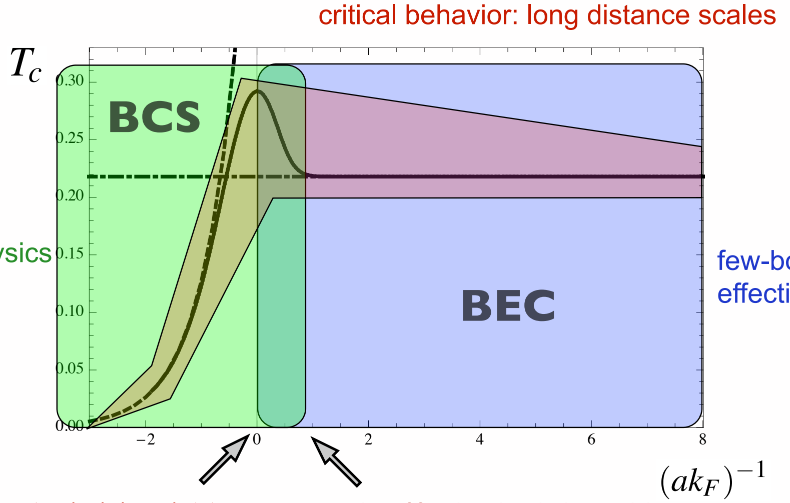
Boettcher, JMP, Wetterich, arXiv:1312.0505 [cond-mat.quant-gas]

Phase diagram of cold quantum gases

critical behavior: long distance scales

Many-Body fermion physics



few-body physics of effective dimers

two-body bound state

zero crossing of fermion chemical potential

three & four-body

Floerchinger, Moroz, Schmidt, Wetterich, Phys. Rev. A 79, 013603; 042705 (2009)

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Floerchinger, Moroz, Schmidt, Few-Body Syst. 51, 153 (2011)

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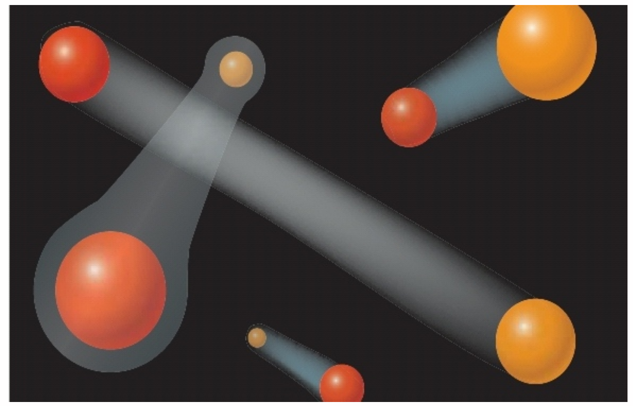

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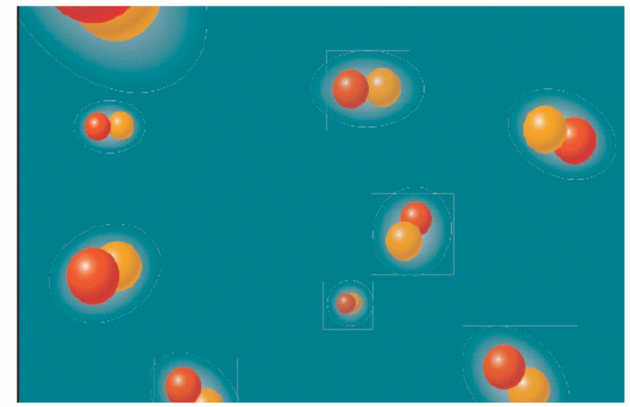

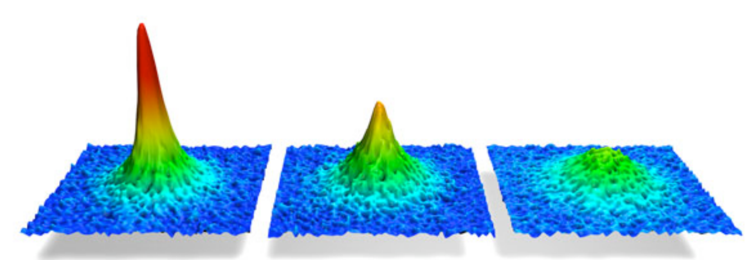

Regal et al '04

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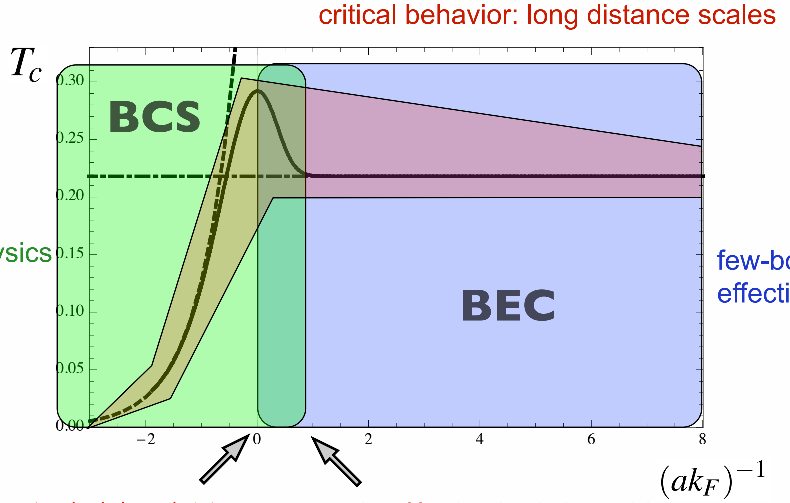
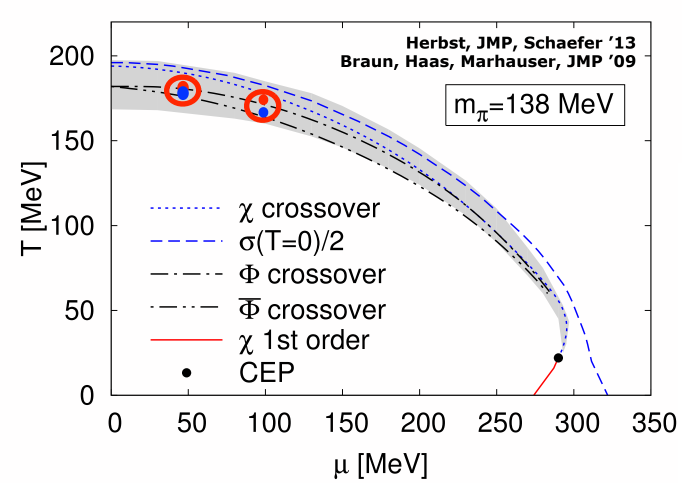


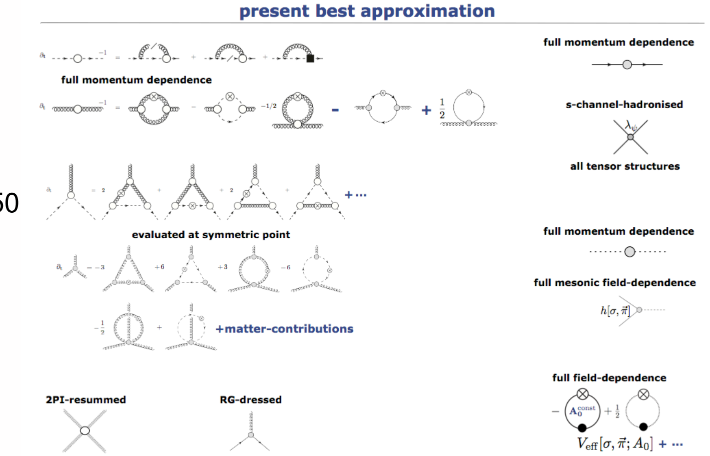
FRG-UCG

Phase diagram of QCD



FRG-QCD

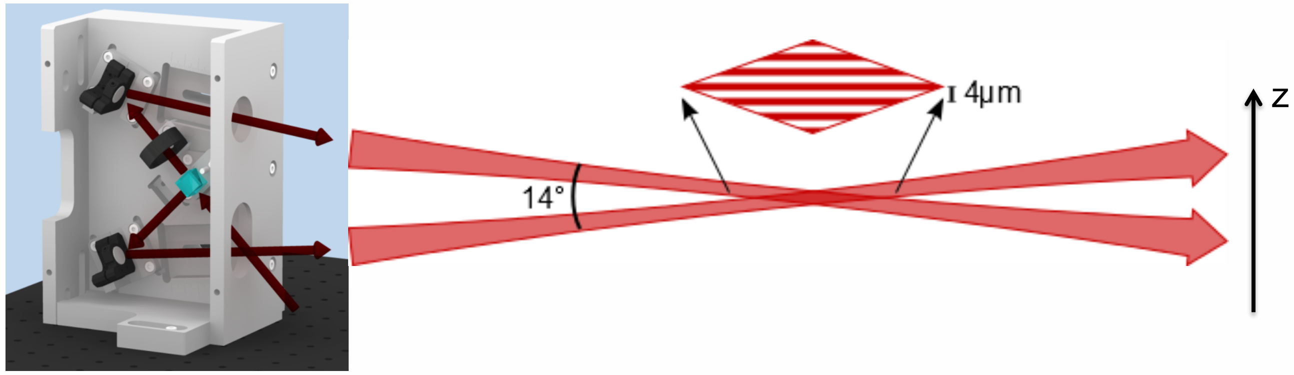
Fister, Herbst, Mitter, Rennecke, Strodthoff, JMP



ultracold quantum gases in 2 dimensions

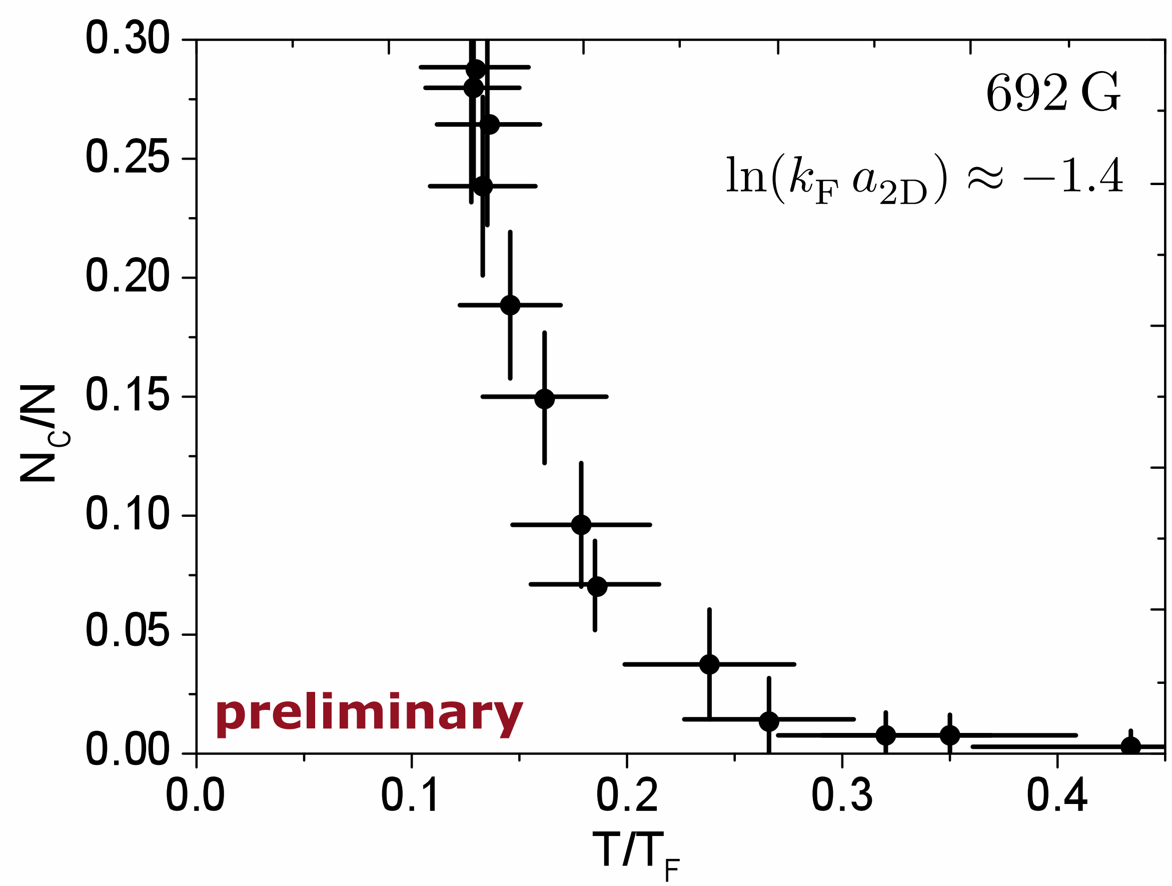
Experimental realisation

Jochim group in Heidelberg



$\omega_r = 2\pi \times$ 19Hz
$\omega_z = 2\pi \times$ 5900Hz

$\omega_r / \omega_z \approx 1:310$ ⇨ ~ 50 000 radial states in transversal ground state



ultracold quantum gases in 2 dimensions

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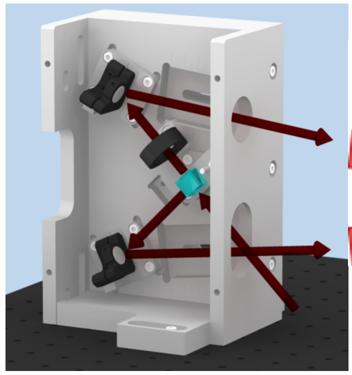

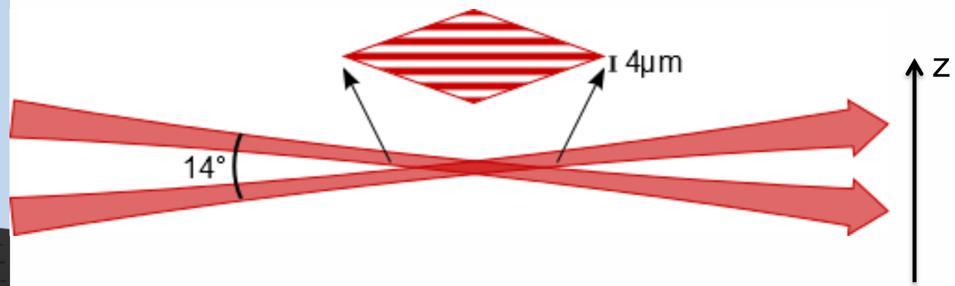

for details ask Thomas Lompe

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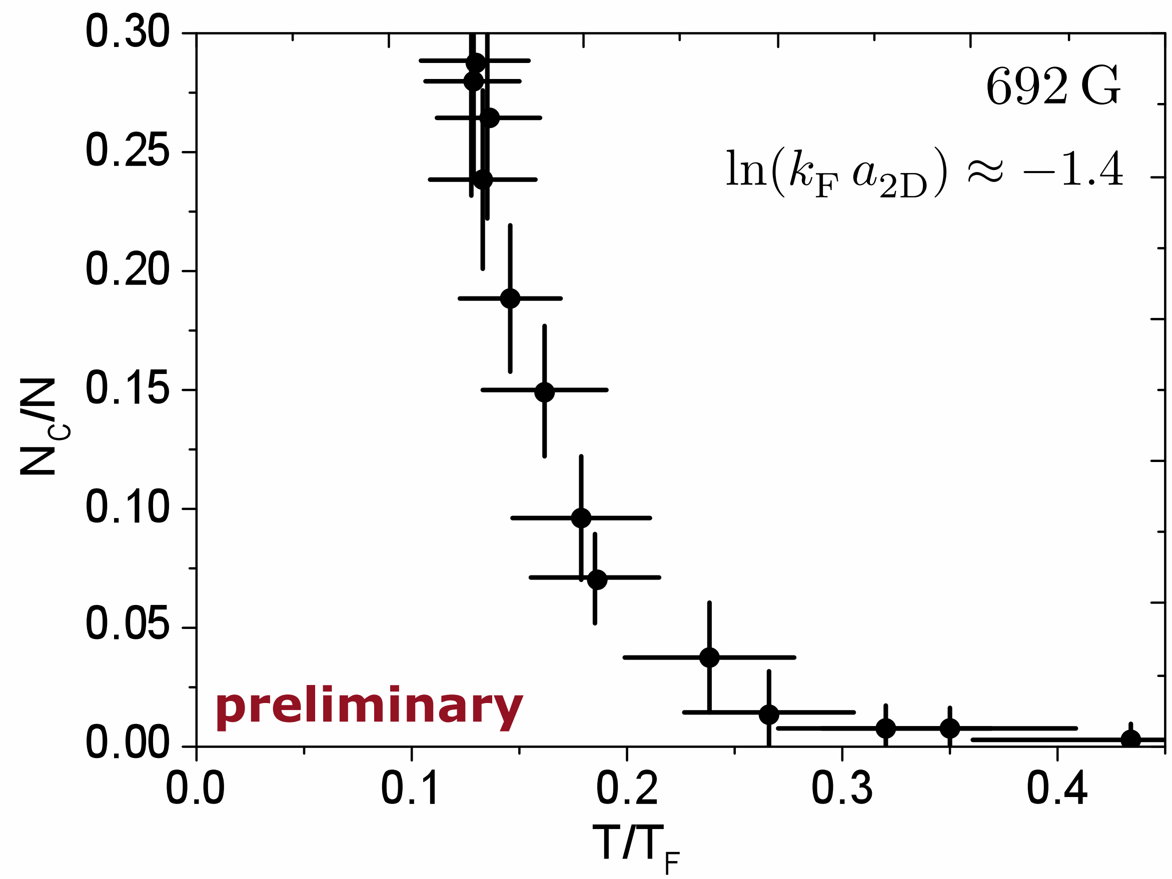
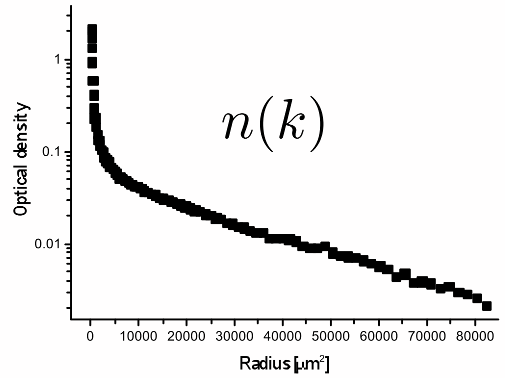


ultracold quantum gases in 2 dimensions

Measurements

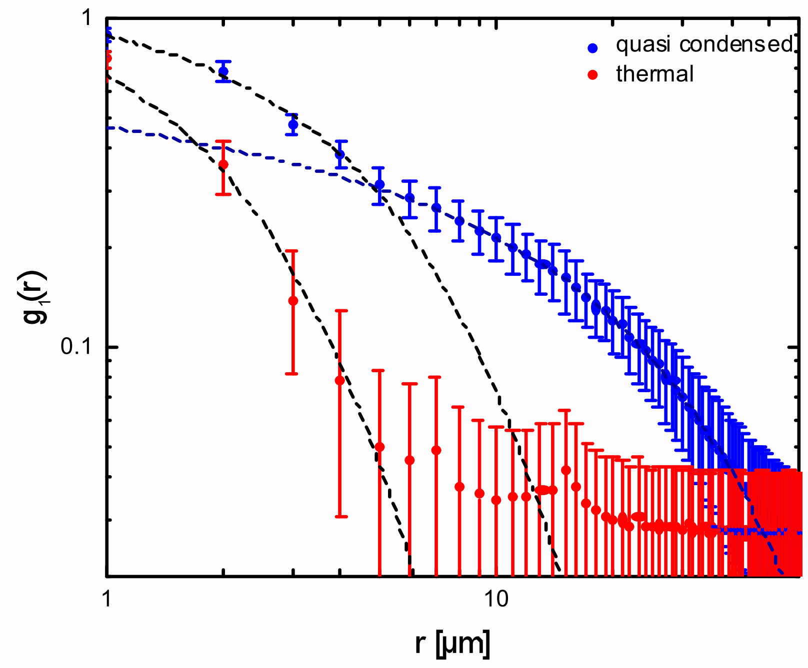
Jochim group in Heidelberg

$$\langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle \propto r^{-\eta}$$



Fourier transform ⇒

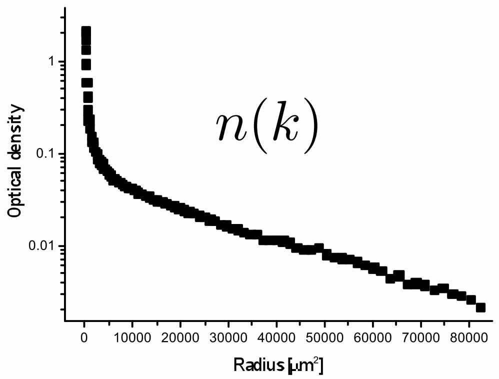
$$g_1(r) \sim \langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle$$



ultracold quantum gases in 2 dimensions

Measurements

Jochim group in Heidelberg



Fourier transform

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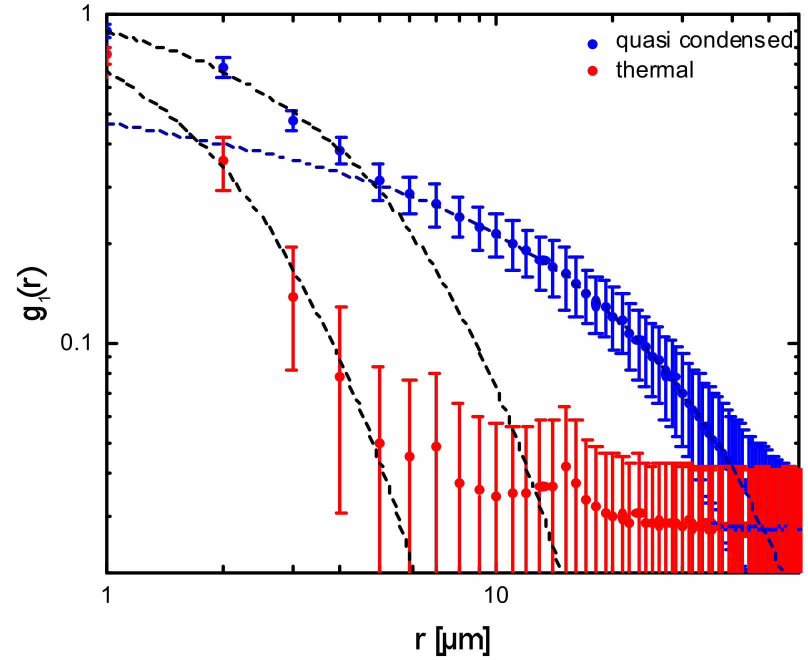
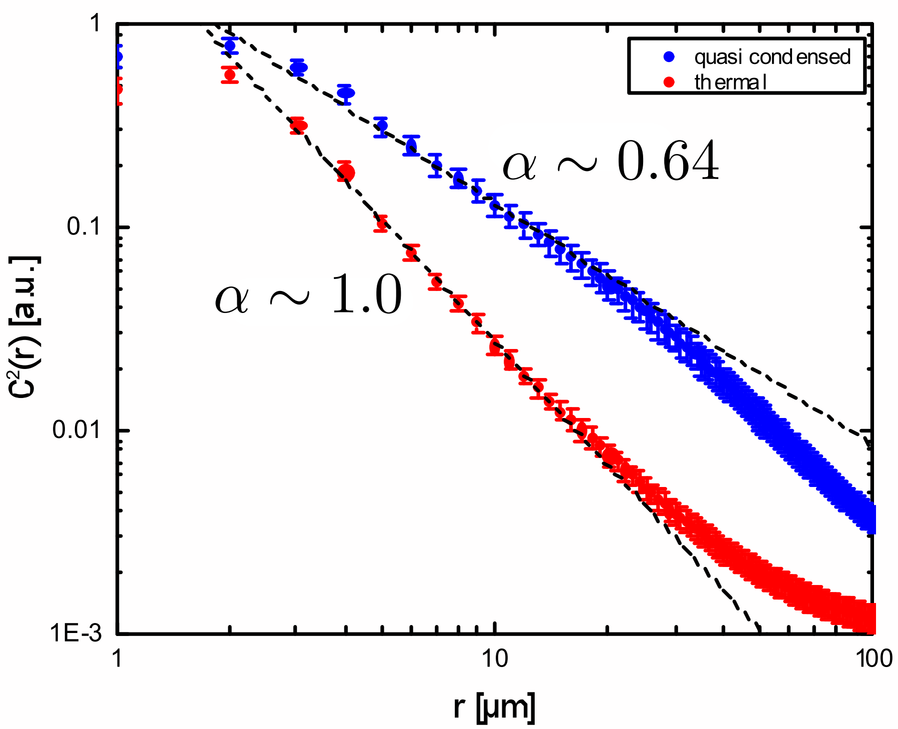


$$g_1(r) \sim r^{-\eta}$$

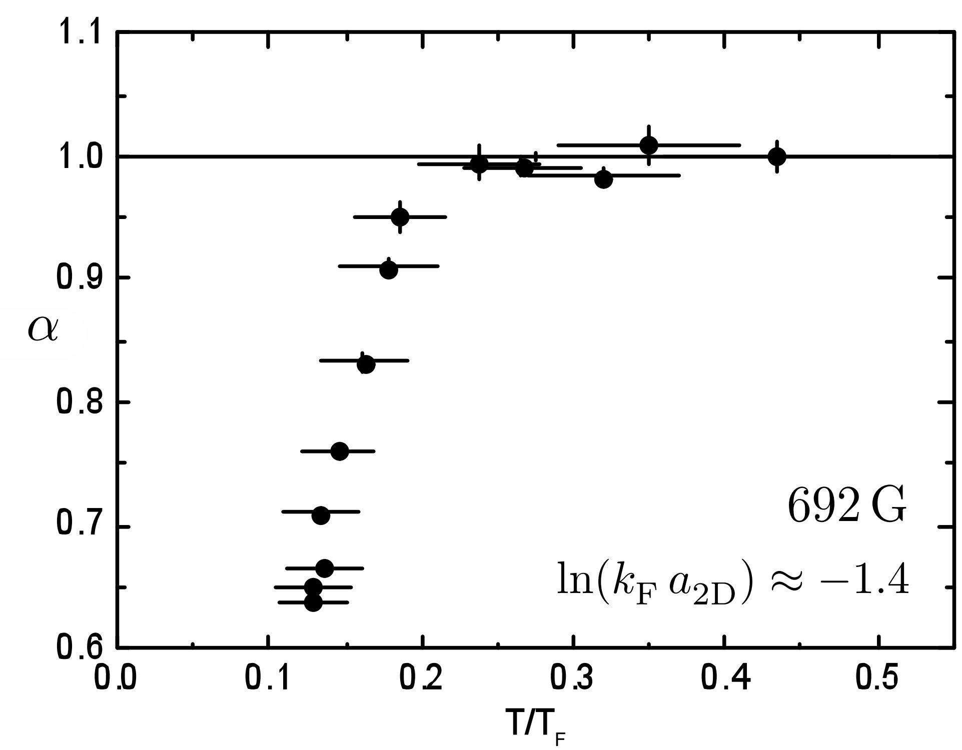
$$C^2(r) = \frac{1}{\pi r^2}\int_0^r d^2r' \, |g_1(r')|^2 \propto r^{-2\alpha}$$



thermal: $\alpha = 1$

hom. BEC: $\alpha = 0$

hom. BKT: $\alpha = 0.25$



ultracold quantum gases in 2 dimensions

Measurements

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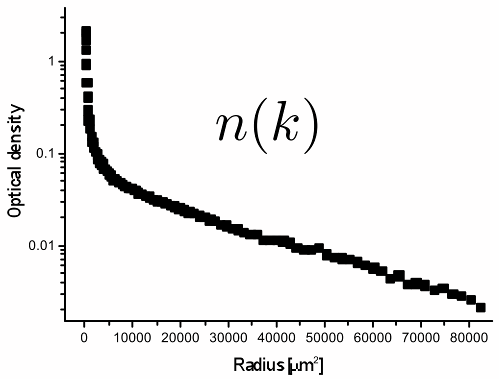

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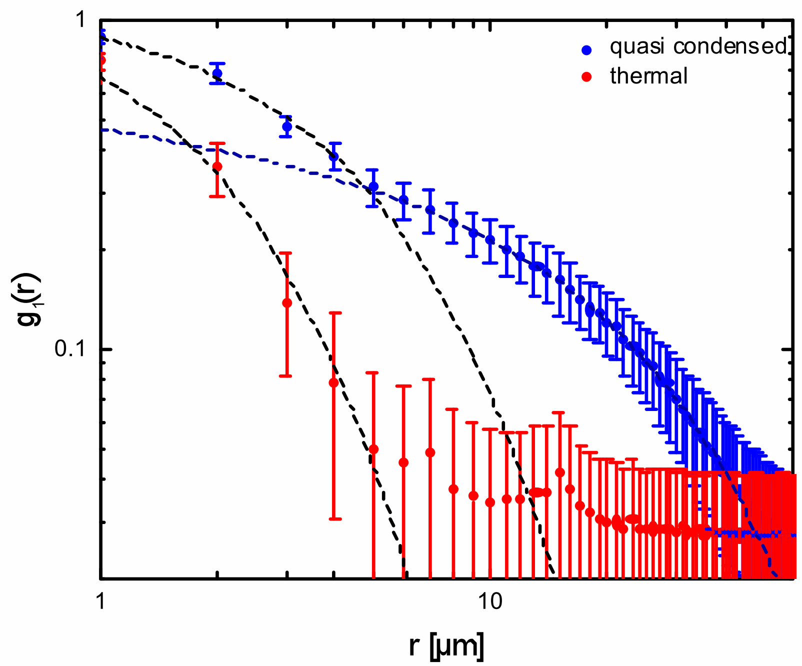

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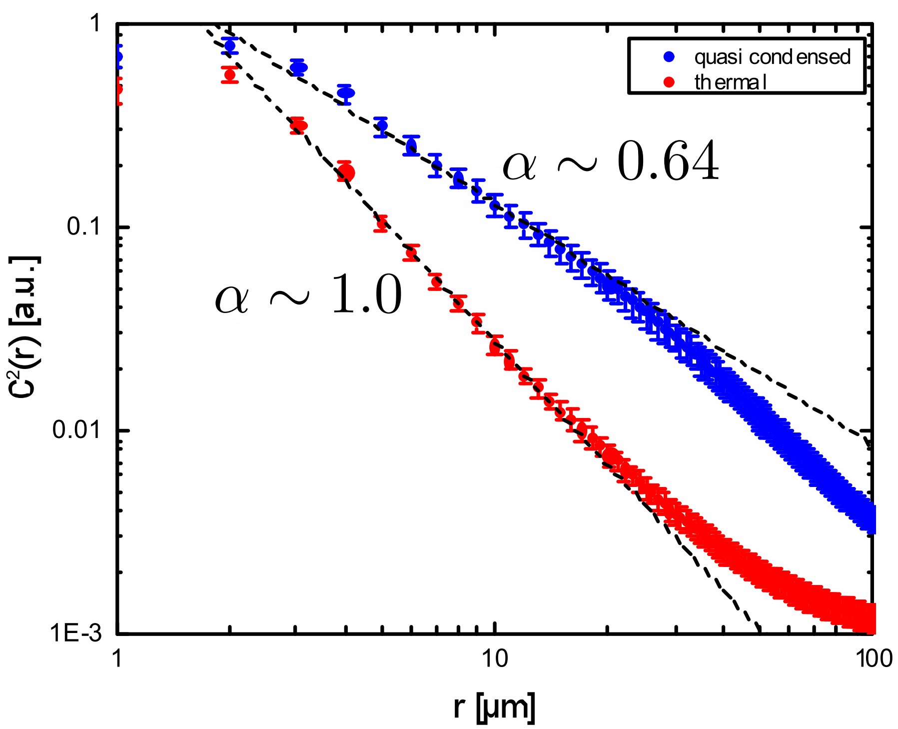
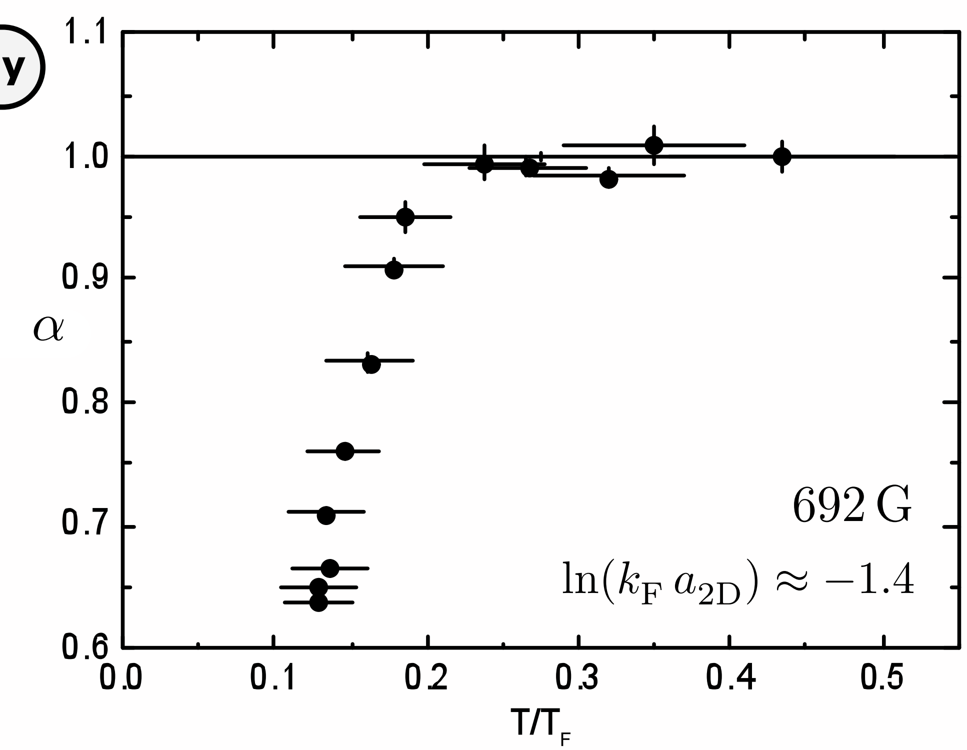


thermal: $\alpha = 1$ **exp. decay**

hom. BEC: $\alpha = 0$ $\eta = \alpha$

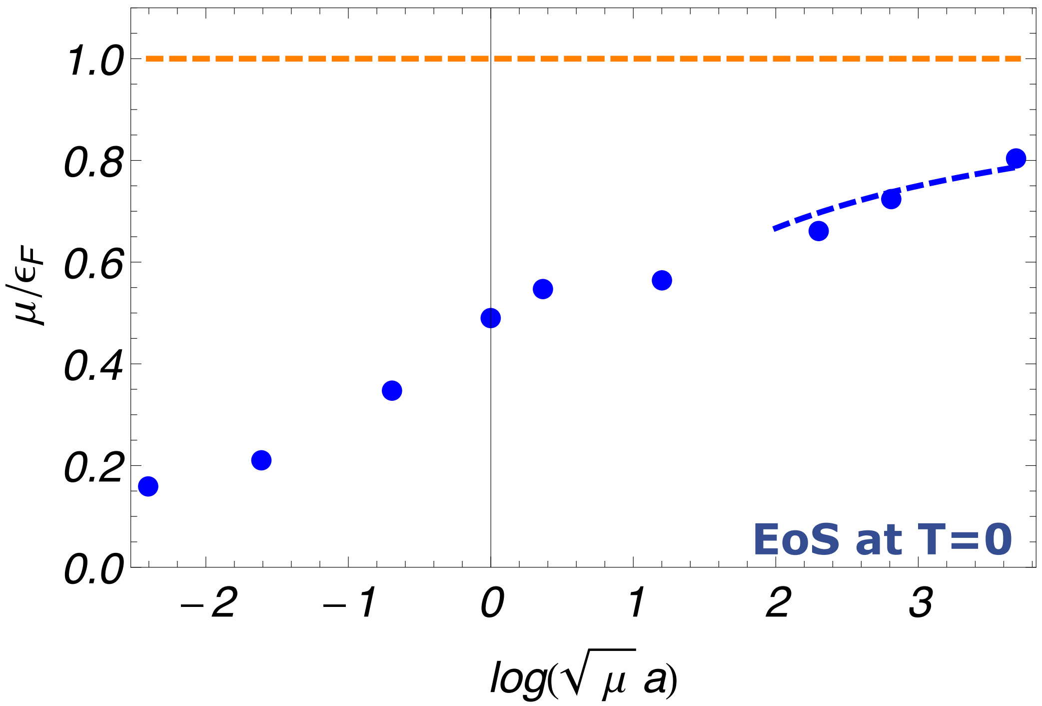
hom. BKT: $\alpha = 0.25$

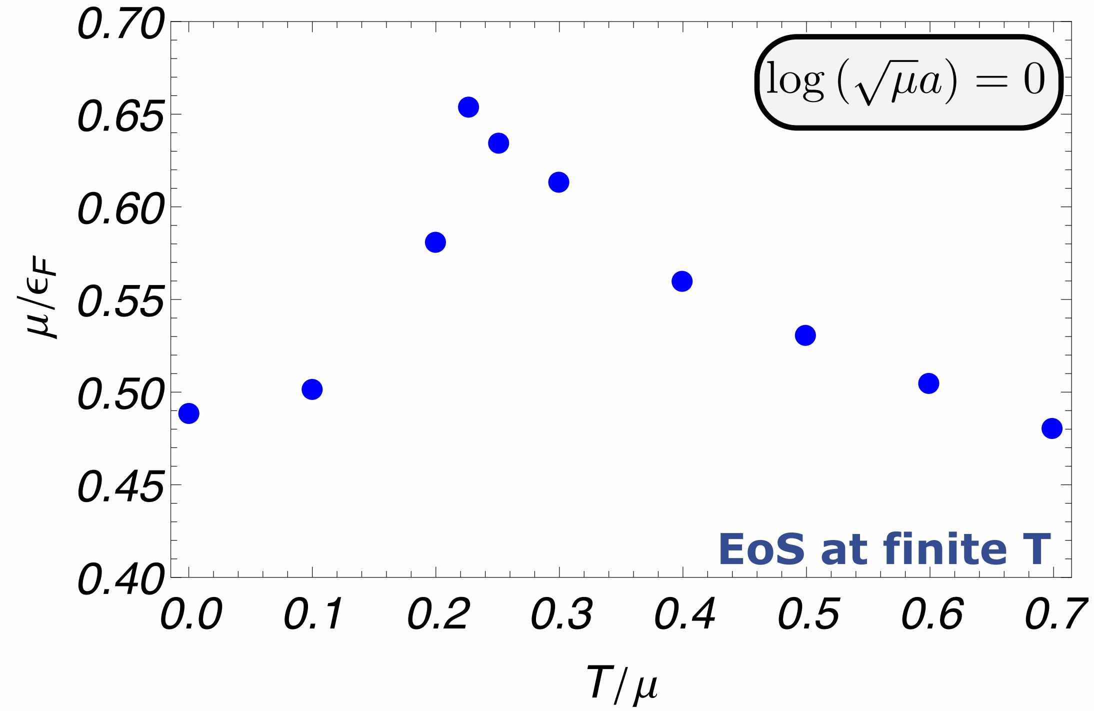


ultracold quantum gases in 2 dimensions

EoS & phase structure

Boettcher, JMP, Wetterich, in preparation





- - - - - - : **mean field**

- - - - - - : $\frac{\mu}{\epsilon_F} = \frac{\log(k_F\, a)}{1 + \log(k_F\, a)}$

$$\mu \to \mu_{\rm mb} = \mu - \frac{\epsilon_b}{2} = \mu + \frac{1}{a^2}$$

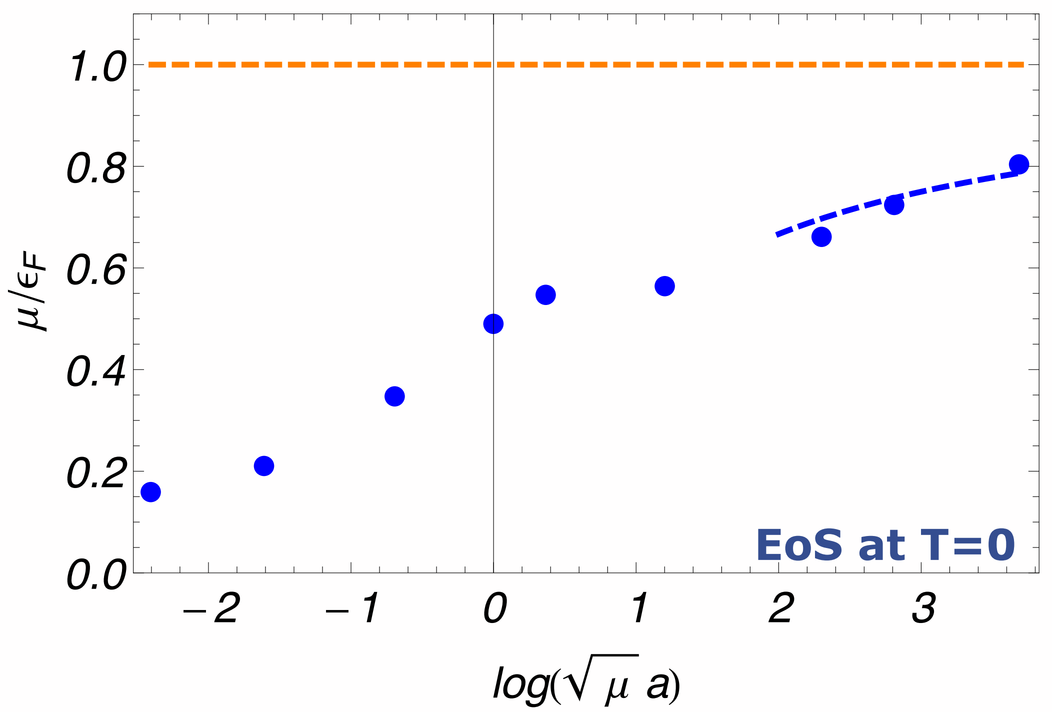
$$\epsilon_F = 2\pi\, n(\mu, T)$$

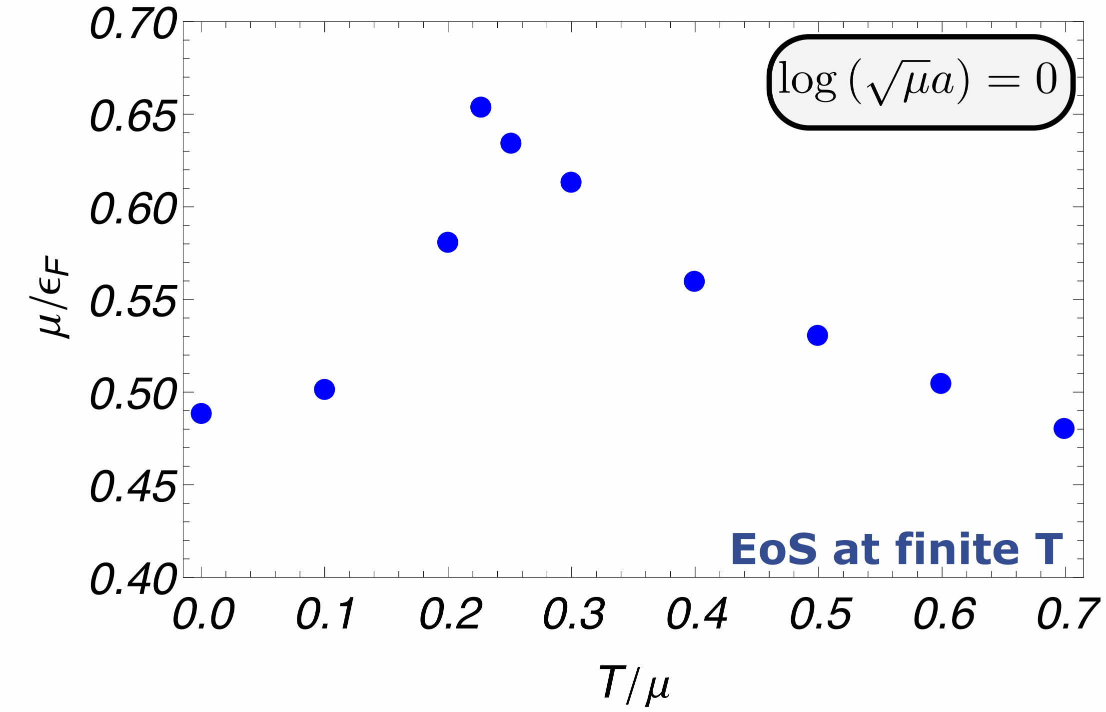
$$k_F = \sqrt{\epsilon_F}$$

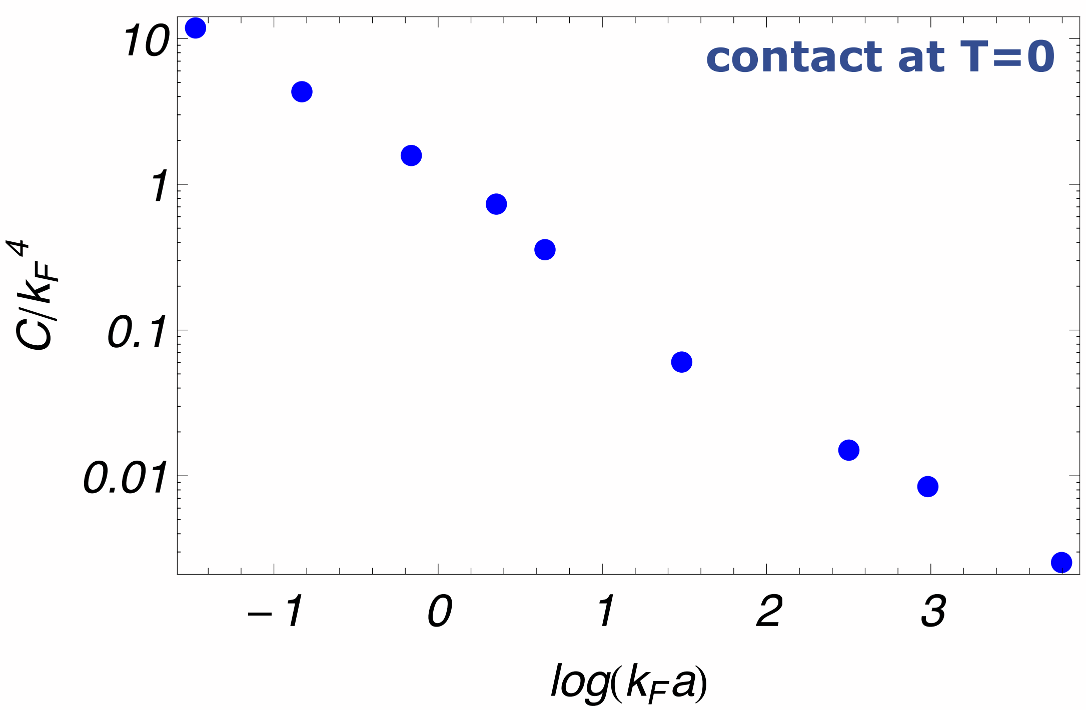
ultracold quantum gases in 2 dimensions

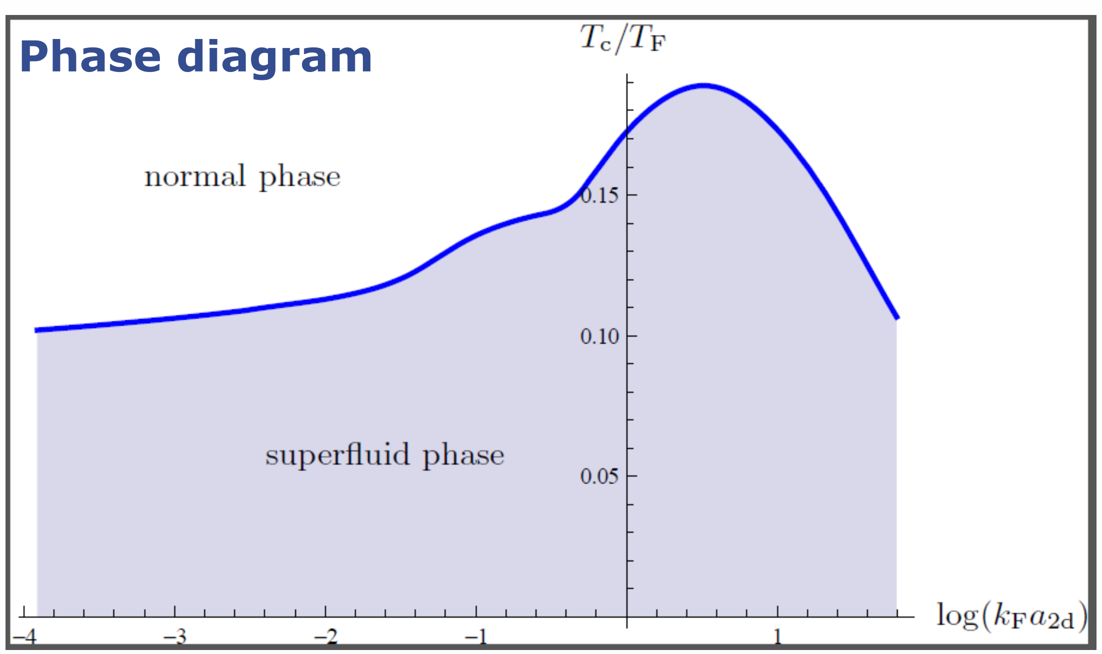
EoS & phase structure

Boettcher, JMP, Wetterich, in preparation









simpler approximation

ultracold quantum gases in 2 dimensions

Scaling

Boettcher, JMP, Wetterich, in preparation

$$\Gamma_k[\psi,\phi] = \cdots + \int_{\tau,\vec{x}} \phi^* \left(Z_{\phi,k}\partial_\tau - A_{\phi,k}\frac{\nabla^2}{2}\right)\phi + \cdots$$

$$\eta_k = -\frac{\partial_t A_{\phi,k}}{A_{\phi,k}}$$

$$g_1(r) \sim \langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle$$

$$C^2(r) = \frac{1}{\pi r^2}\int_0^r d^2r'\,|g_1(r')|^2 \propto r^{-2\eta}$$

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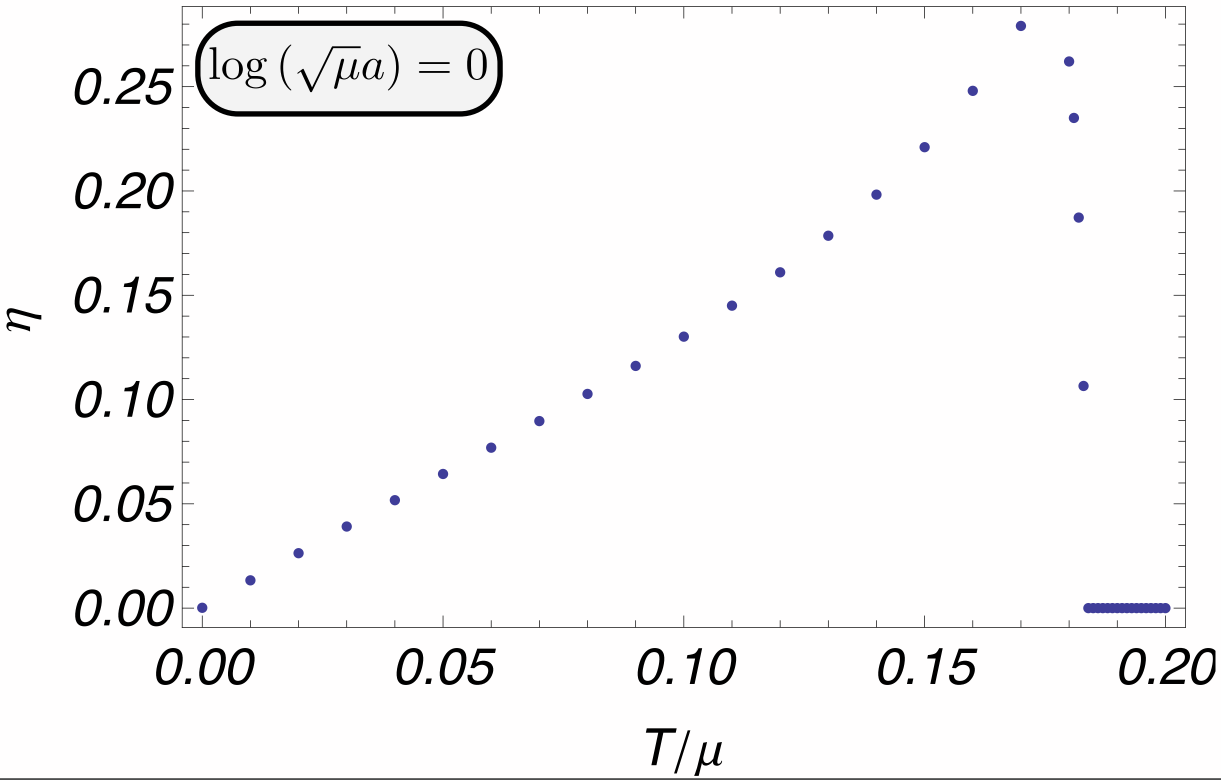
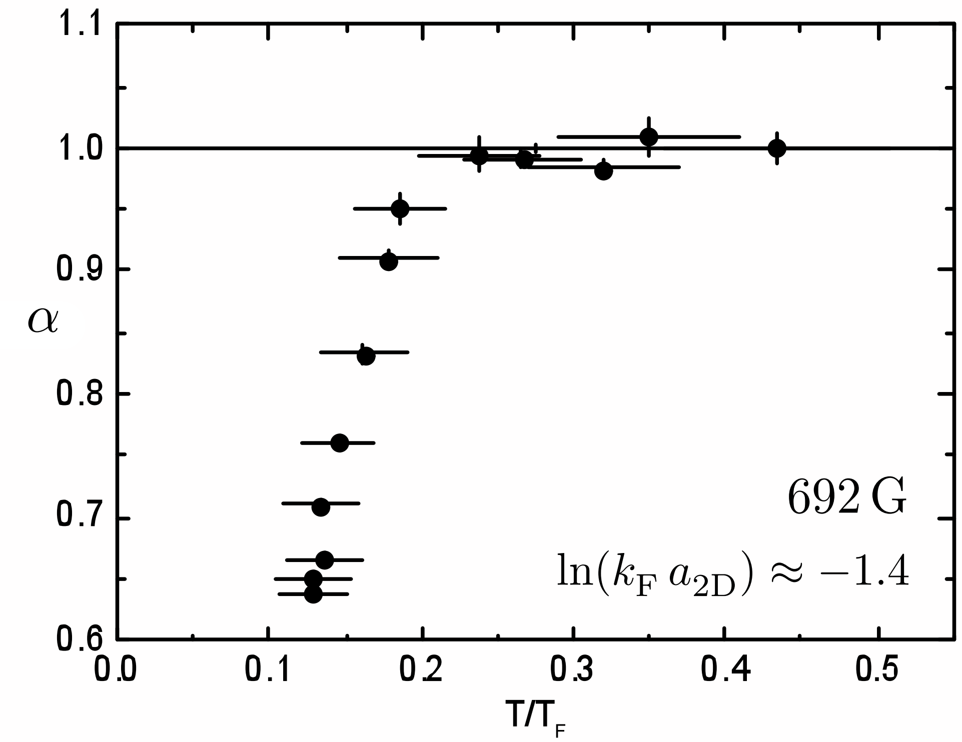
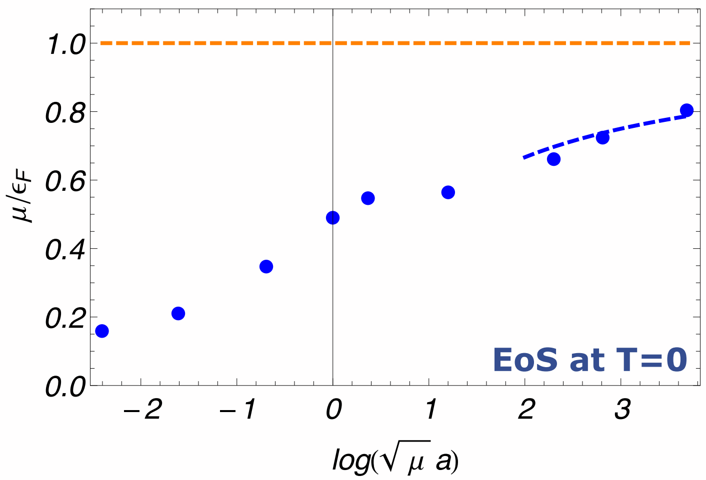


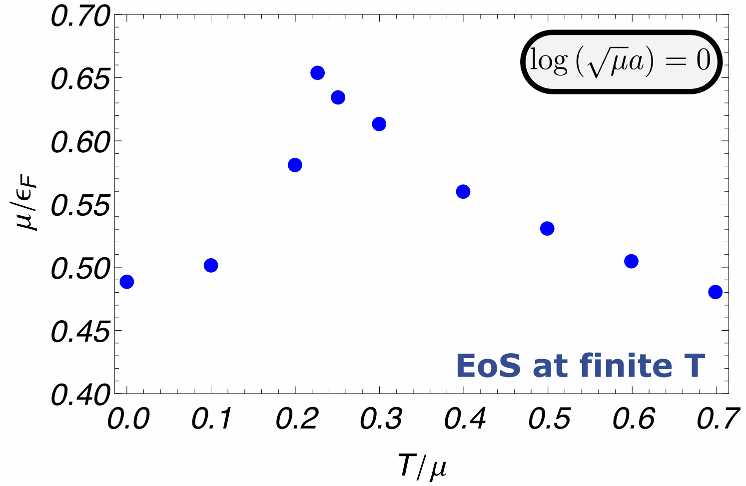


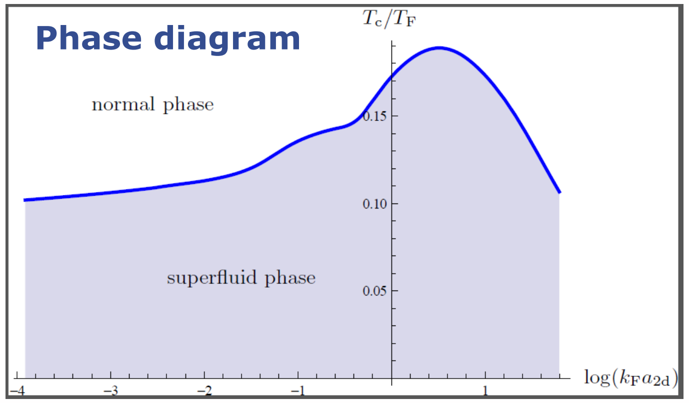
Summary & Outlook

Summary & outlook

▪Eos & phase structure in two dimensions







▪Tan contact & BKT scaling