INT Workshop May 13,
2014

Geometric scaling of three-body collision resonances for a ^{6}Li-^{133}Cs mixture in the Efimov scenario

Funding:

NSF

MRSEC

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AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Colin V. Parker
University of Chicago

SOVIET JOURNAL OF NUCLEAR PHYSICS VOLUME 12, NUMBER 5 MAY, 1971

WEAKLY-BOUND STATES OF THREE RESONANTLY-INTERACTING PARTICLES

V. N. EFIMOV

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

Submitted February 16, 1970

Yad. Fiz. 12, 1080–1091 (November, 1970)

It is shown that if the pair forces of three identical particles are sufficiently resonant, a family of bound states of low energy is produced. The quantum numbers of all the states are the same: for spinless bosons 0^+ and for nucleons $\frac{1}{2}^+$, $T = \frac{1}{2}$. The dimension of the states is larger than the radius of the pair forces. The most favorable conditions for the appearance of a family of levels occur for three spinless neutral bosons: the conditions are less favorable for charged particles and particles with spin and isospin. The possibility of existence of such levels in a system of three particles (in the C^{12} nucleus) and of three nucleons (H^3) is considered.

Hyperspherical equation:

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{m}\frac{\partial^2\phi}{\partial R^2} - \frac{\hbar^2}{m}\frac{s_0^2+1/4}{R^2}\phi$$

⇒ Scale invariant under dilation $R \to \lambda R$ and $t \to \lambda^2 t$

$\lambda = \exp(\pi/s_0) = 22.7$ for 3 identical bosons

Vitaly Efimov

From Nuclear Physics to Cold atoms

Chris Greene (Purdue)

Brett Esry (Kansas State)

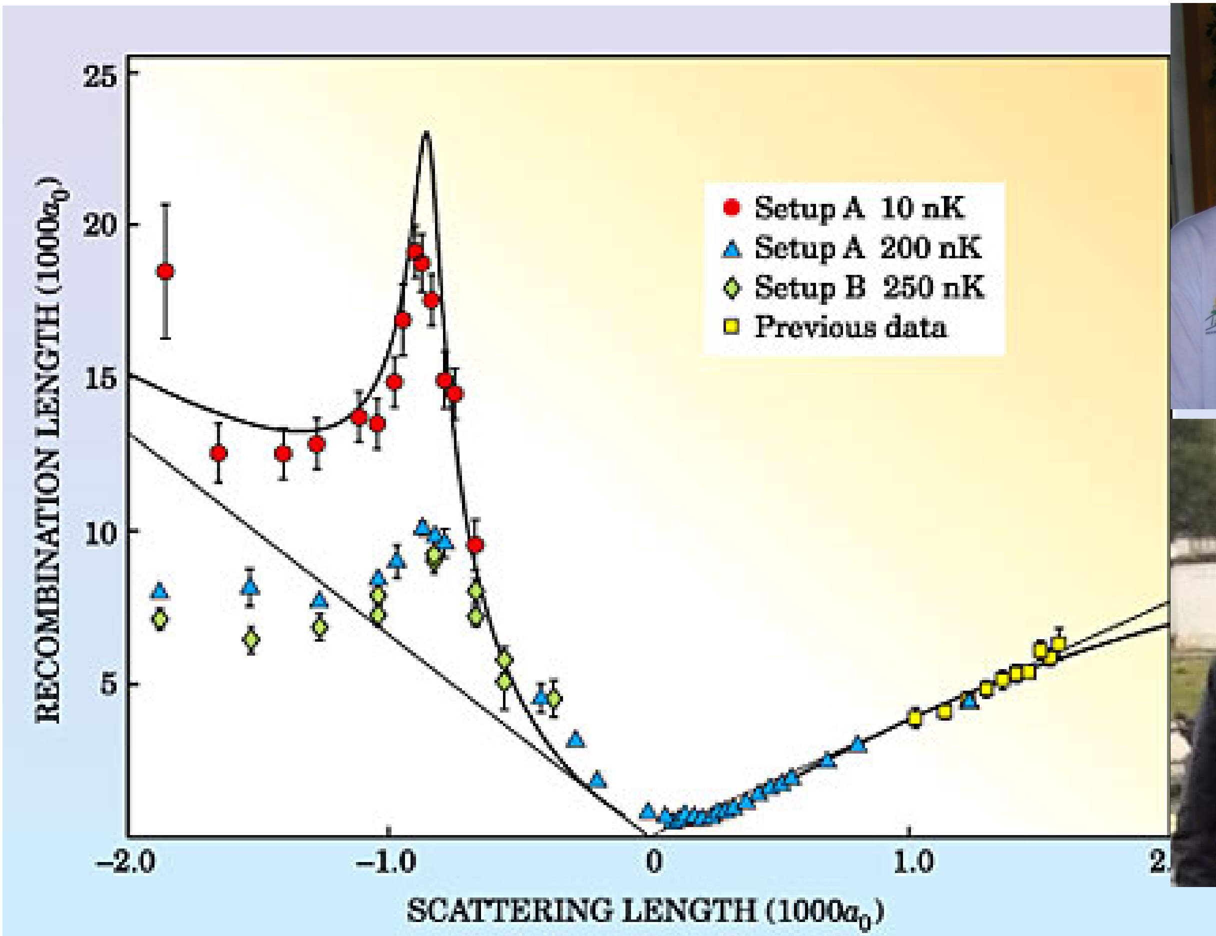
VOLUME 83, NUMBER 9 PHYSICAL REVIEW LETTERS 30 AUGUST 1999

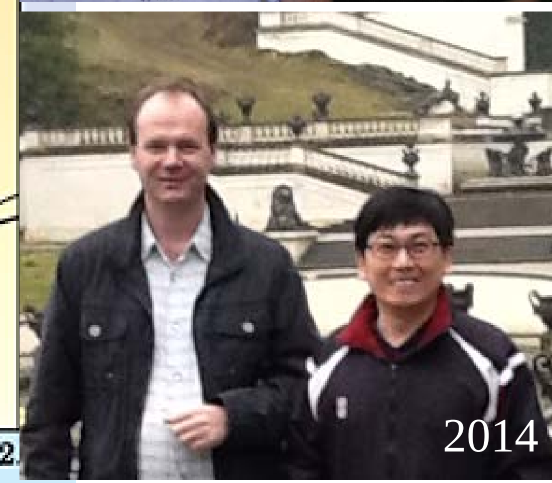
Recombination of Three Atoms in the Ultracold Limit

B. D. Esry
Institute for Theoretical Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

Chris H. Greene and James P. Burke, Jr.
Department of Physics and JILA, University of Colorado, Boulder, Colorado 80309-0440
(Received 19 May 1999)

Observation of Efimov Resonance in Cesium

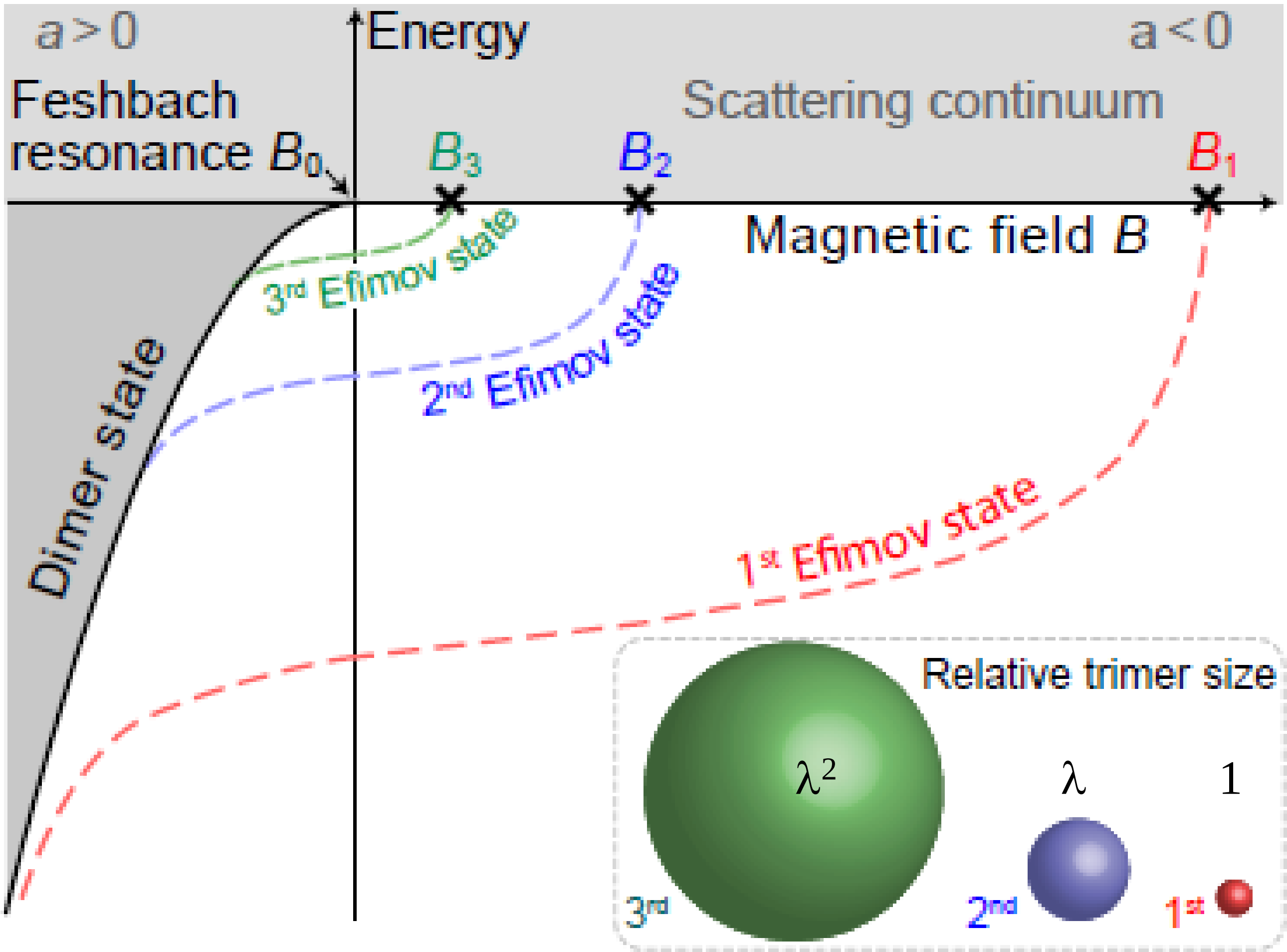




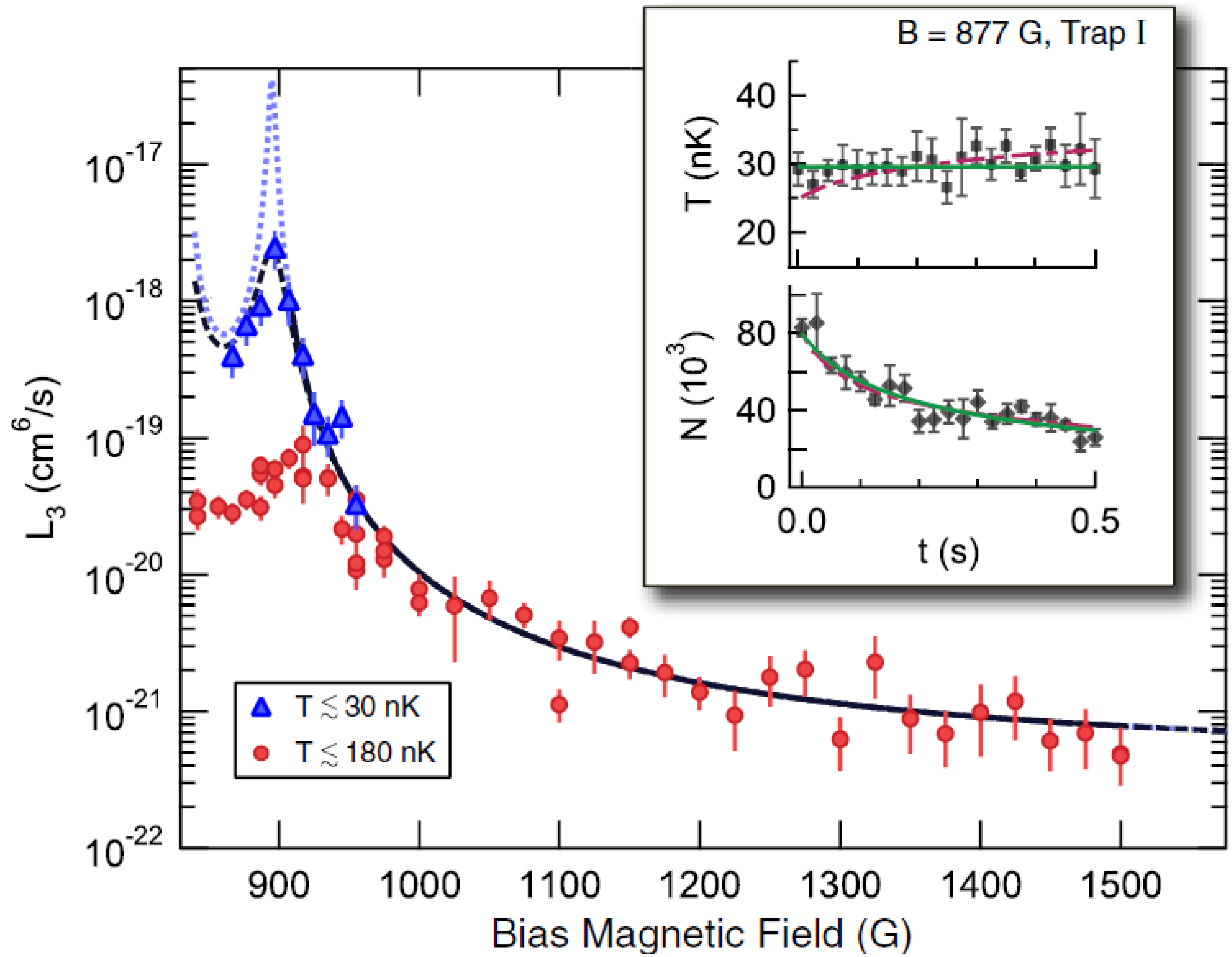
Physics Today 2006

Also found in Li6 (Penn State, Heidelberg), Li7 (Rice, Bar-Ilan), K39 (LENS), Rb85 (JILA)

Efimov state structure near a Feshbach resonance

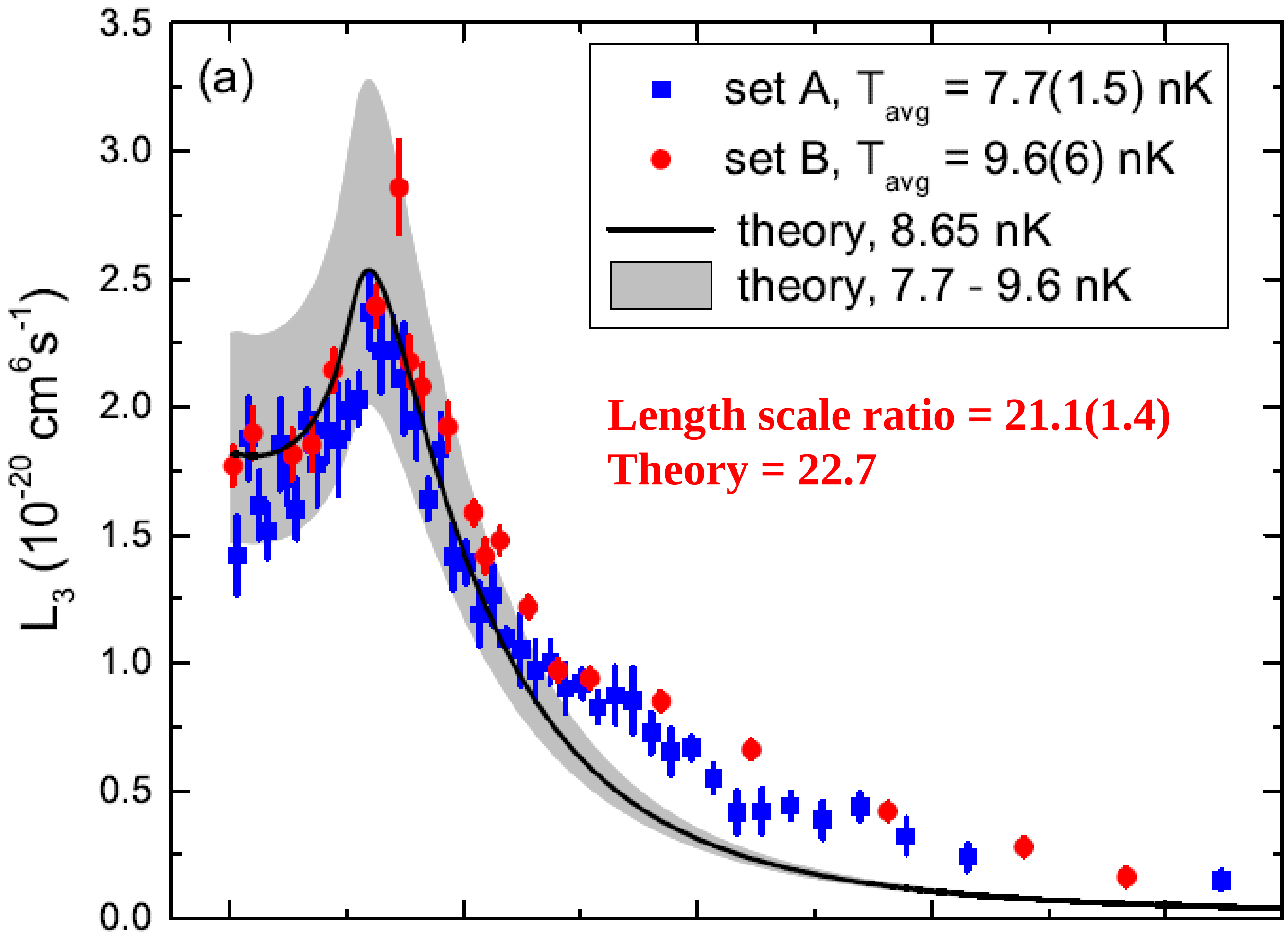


Excited Efimov state in 3-component Fermi gas



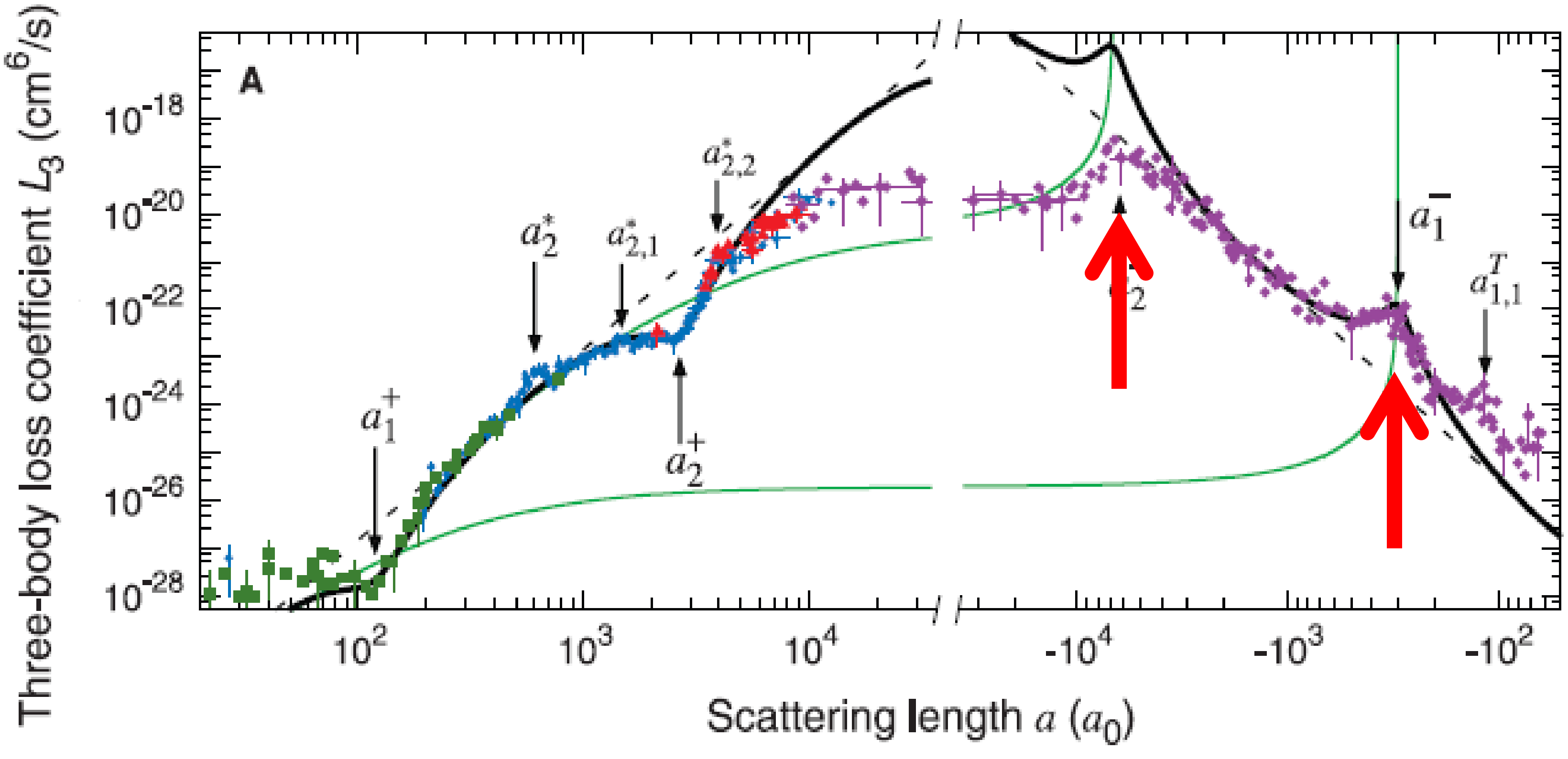
O'Hara Group (Penn State): Williams et al., PRL 103, 130404 (2009)

Second Efimov resonance in Cs!



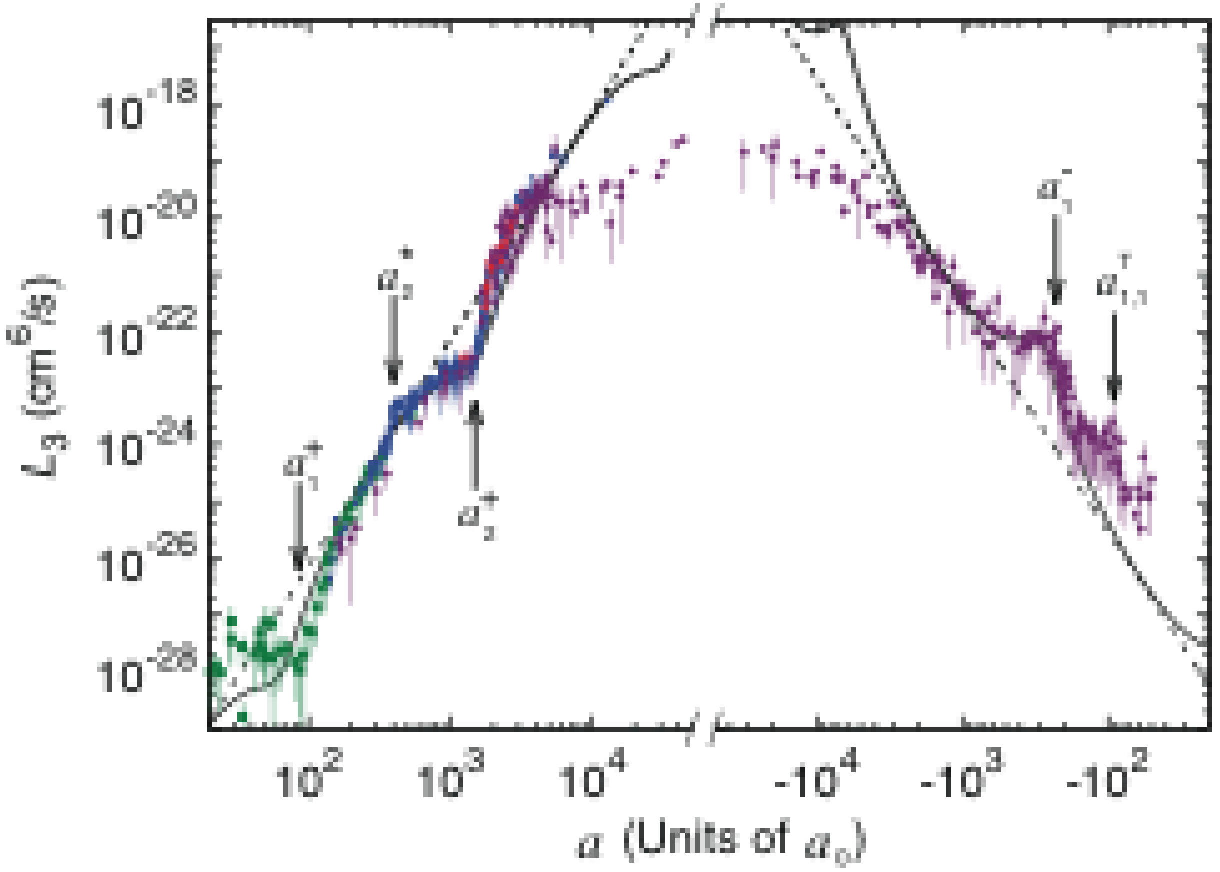
Grimm group (Innsbruck Univ.) B. Huang et al., arXive: 1402.6161

Second Efimov resonance in ^{7}Li?



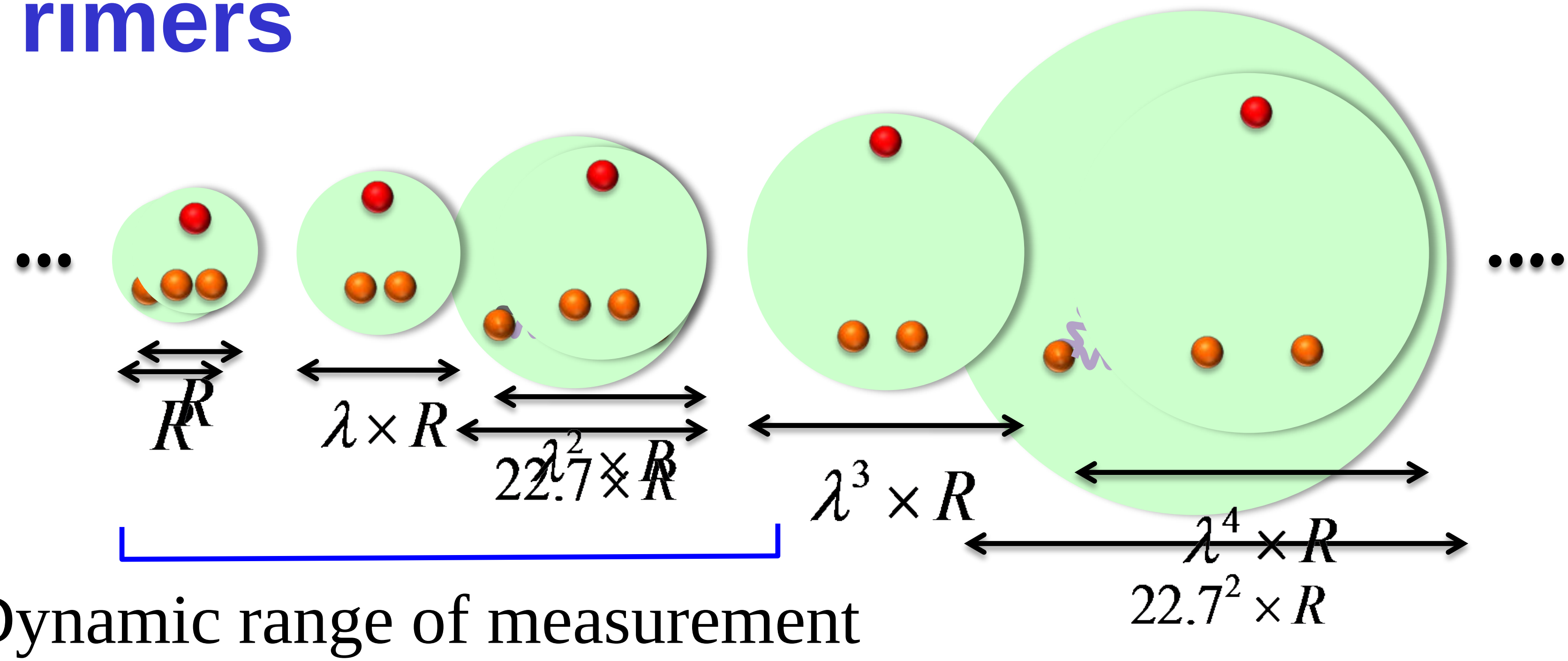
Hulet group (Rice University): Science 326, 1683 (2009)
New interpretation: Physical Review A 88, 023625 (2013).

Second Efimov resonance in ^{7}Li?



Hulet group (Rice University): Science 326, 1683 (2009)
New interpretation: Physical Review A 88, 023625 (2013).

Homonuclear vs. Heteronuclear Trimers



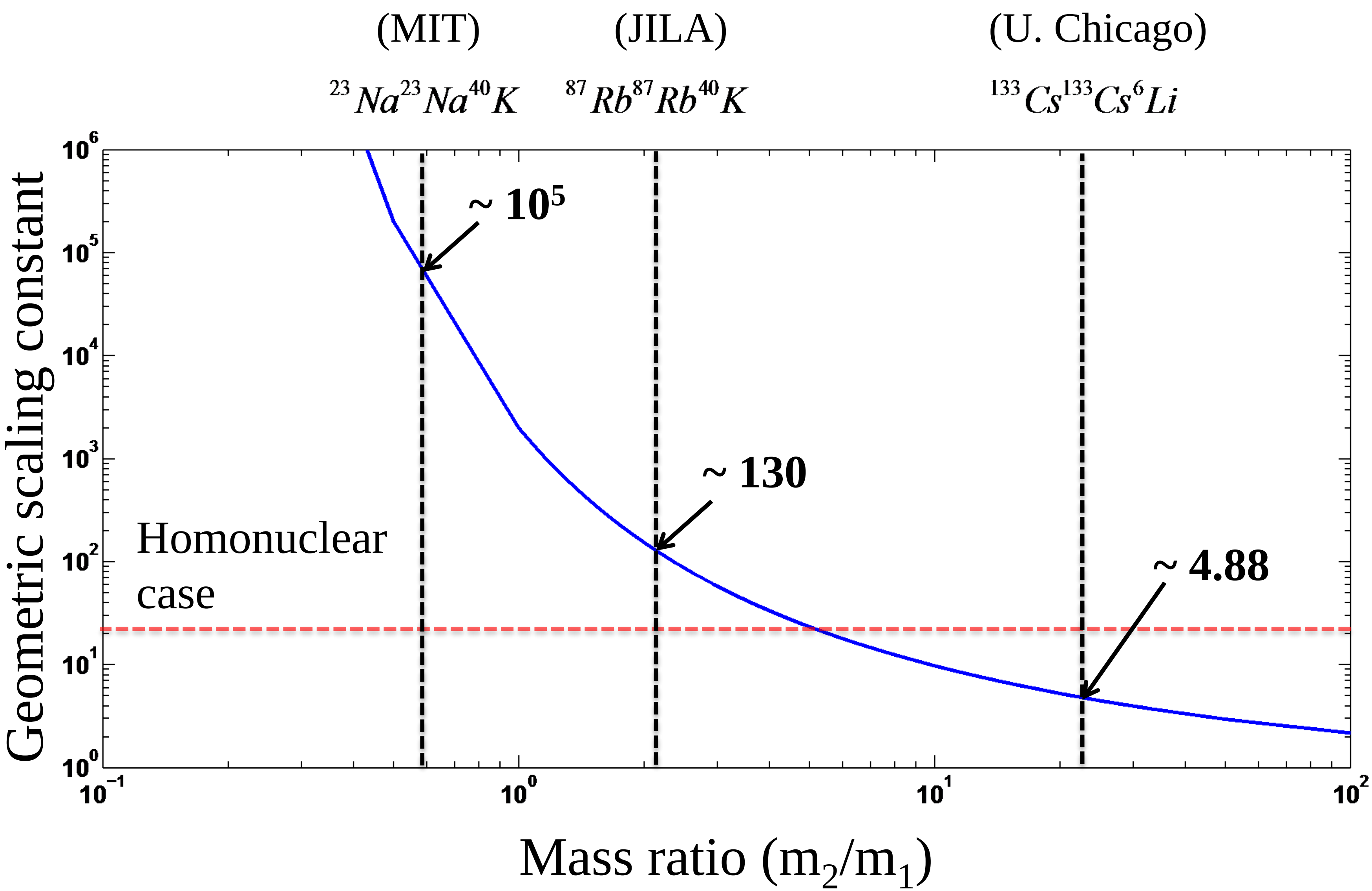
λ: Determined by

(1) Mass ratio of constituent atoms

(2) Number of resonant pair interactions

(3) Symmetry of particle statistics

Mass Dependence



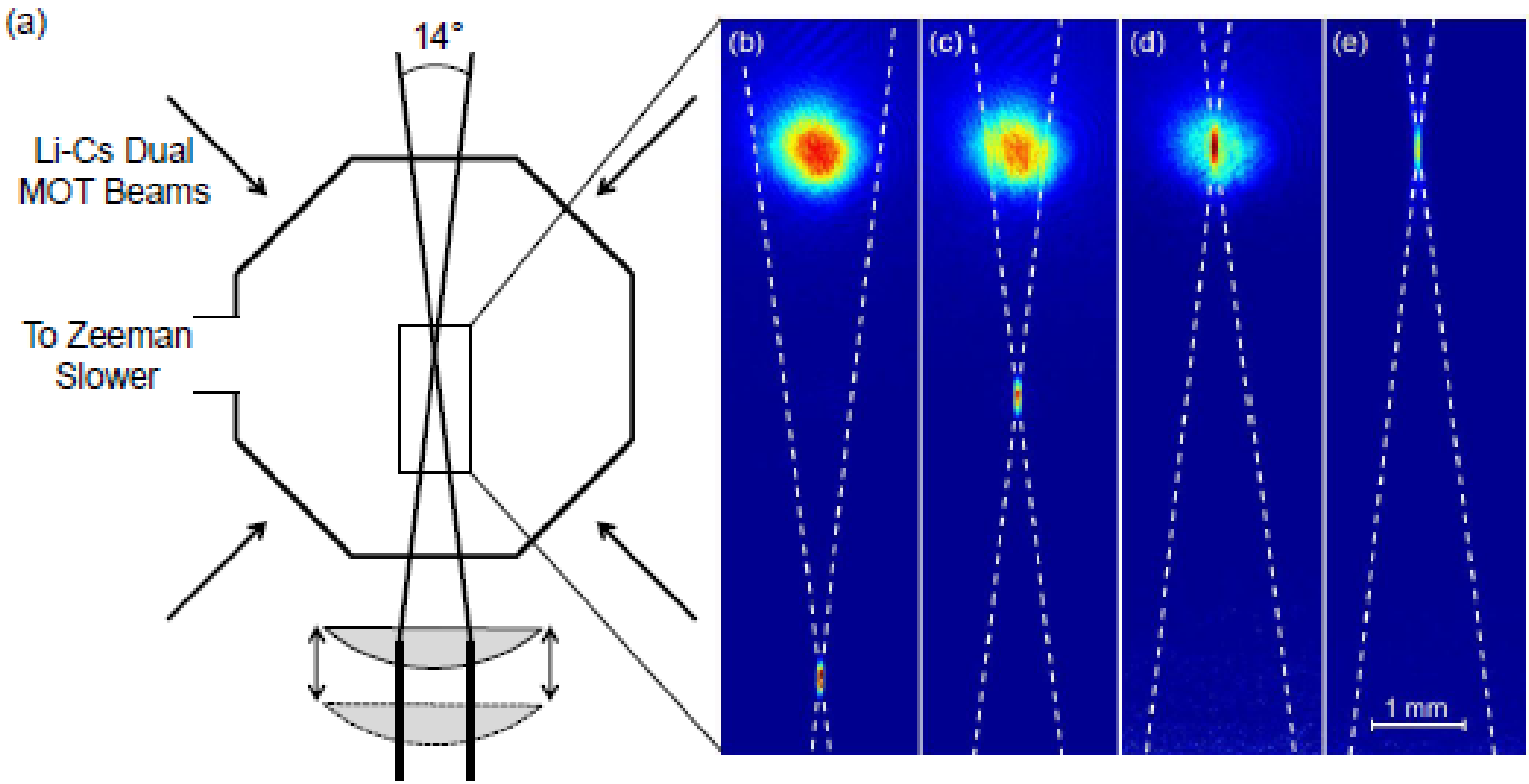
Efimov states in atomic mixtures

2 heavy Bosons + 1 light atom

		Two features		Three features	
B-F	e^{π/s_0}	$\|a_{\min}\|$	$E_{\max}$(nK)	$\|a_{\min}\|$	$E_{\max}$(nK)
^{133}Cs-^{6}Li	4.877	3×10^3	1500	2×10^4	60.0
^{87}Rb-^{6}Li	6.856	8×10^3	230	6×10^4	5.00
^{23}Na-^{6}Li	36.28	9×10^5	$\leqslant0.1$	3×10^7	$\leqslant0.1$
^{7}Li-^{6}Li	$>10^2$	$\geqslant10^8$	$\leqslant0.1$	$\geqslant10^8$	$\leqslant0.1$
^{133}Cs-^{40}K	47.02	2×10^6	$\leqslant0.1$	9×10^7	$\leqslant0.1$
^{87}Rb-^{40}K	$>10^2$	$\geqslant10^8$	$\leqslant0.1$	$\geqslant10^8$	$\leqslant0.1$
^{23}Na-^{40}K	$>10^2$	$\geqslant10^8$	$\leqslant0.1$	$\geqslant10^8$	$\leqslant0.1$
^{7}Li-^{40}K	$>10^2$	$\geqslant10^8$	$\leqslant0.1$	$\geqslant10^8$	$\leqslant0.1$

J.P. D'Incao and B.D. Esry, PRA (2006)

A Translatable Crossed Dipole Trap



Li : T = 3 μK; N_{Li} = ~ 10^5

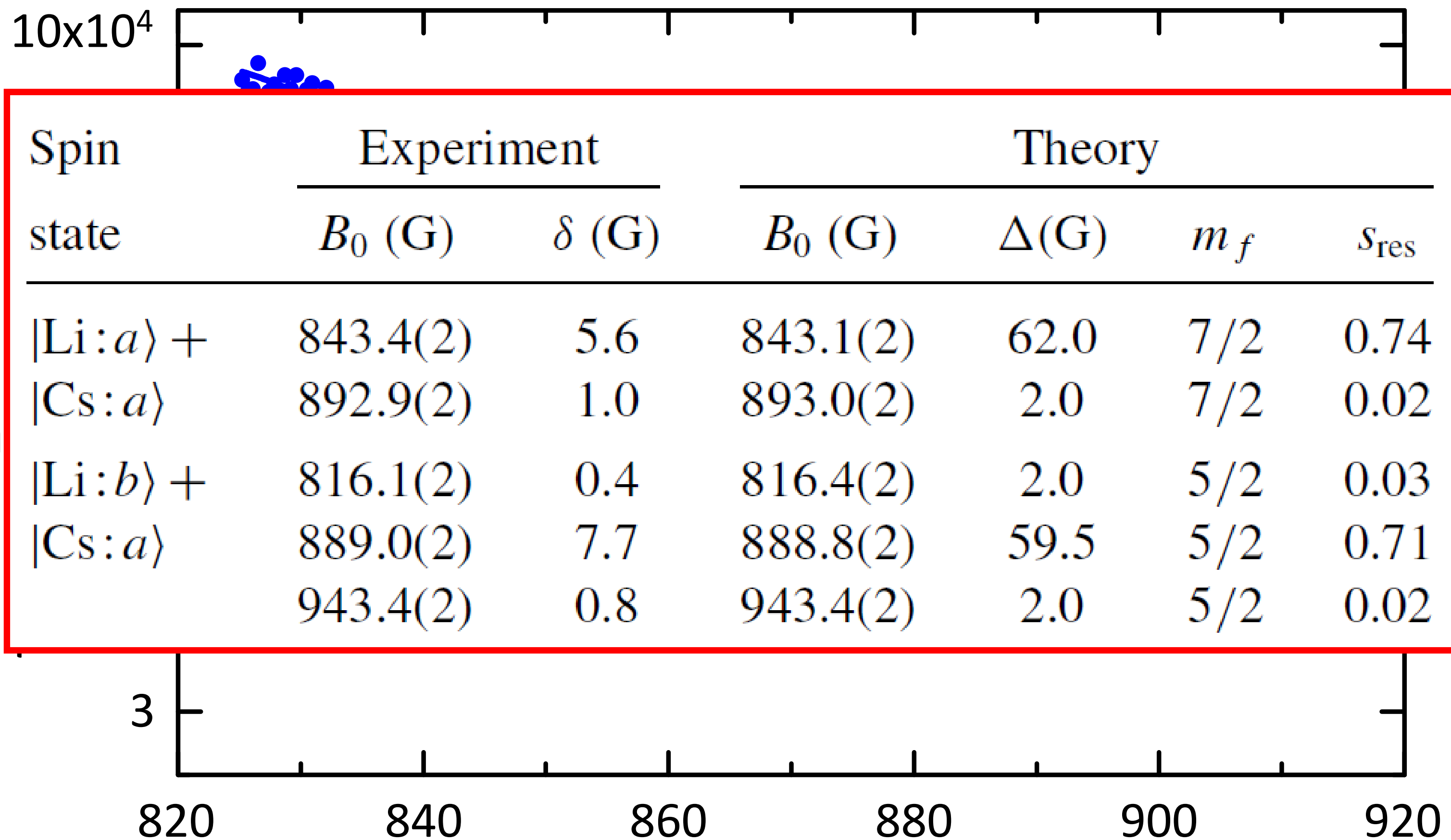
Cs: T = 16 μK; N_{Cs} = ~10^5

S. Tung et. al., PRA 87, 010702(R) (2013)

Trap Loss of Li-a State + Cs-a State Mixture

Lithium number

Tempera

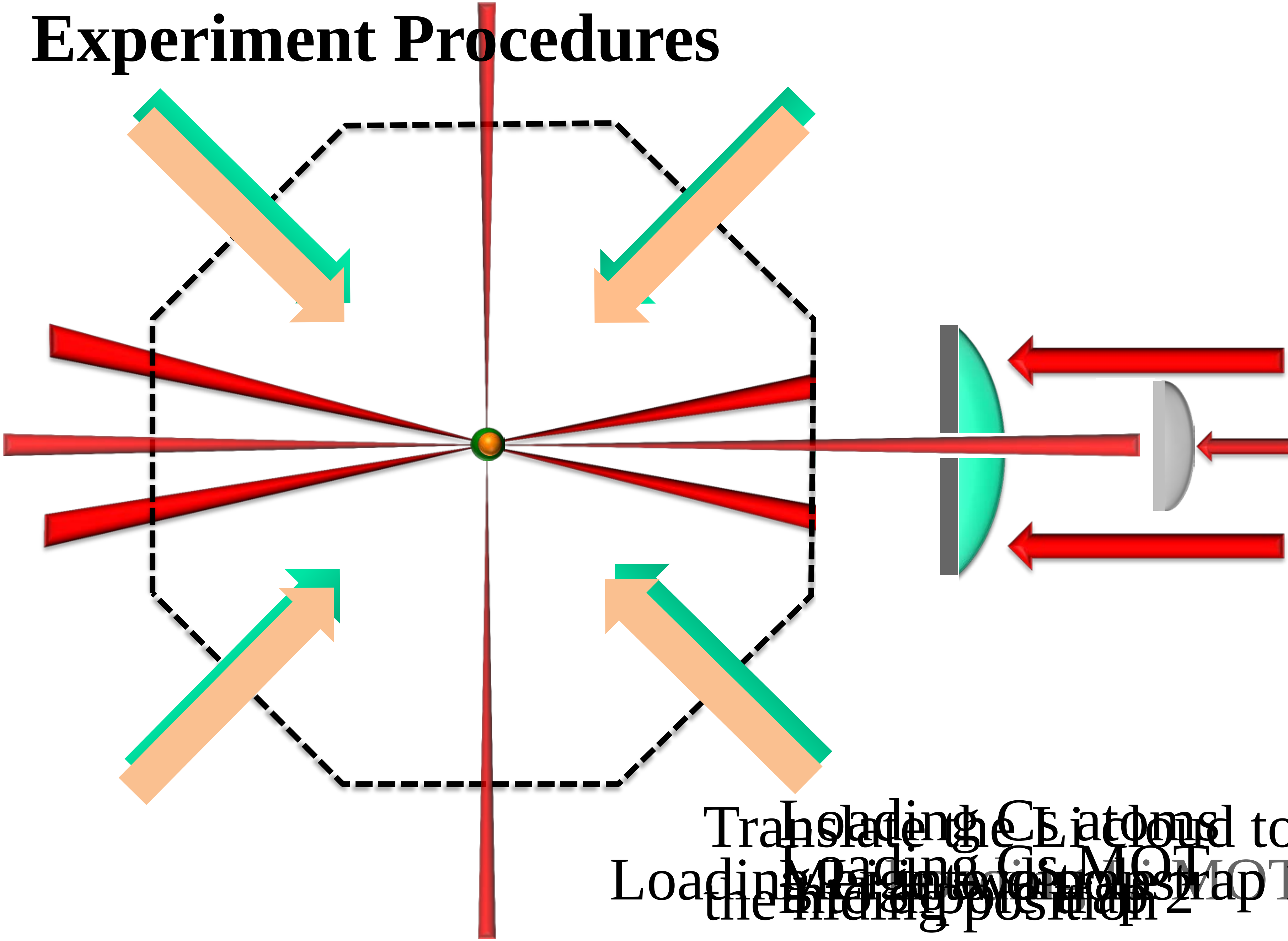


Spin state	Experiment B_0 (G)	Experiment δ (G)	Theory B_0 (G)	Theory Δ(G)	Theory m_f	Theory s_{res}
$\vert Li:a\rangle +$	843.4(2)	5.6	843.1(2)	62.0	7/2	0.74
$\vert Cs:a\rangle$	892.9(2)	1.0	893.0(2)	2.0	7/2	0.02
$\vert Li:b\rangle +$	816.1(2)	0.4	816.4(2)	2.0	5/2	0.03
$\vert Cs:a\rangle$	889.0(2)	7.7	888.8(2)	59.5	5/2	0.71
	943.4(2)	0.8	943.4(2)	2.0	5/2	0.02

Magnetic field B (G)

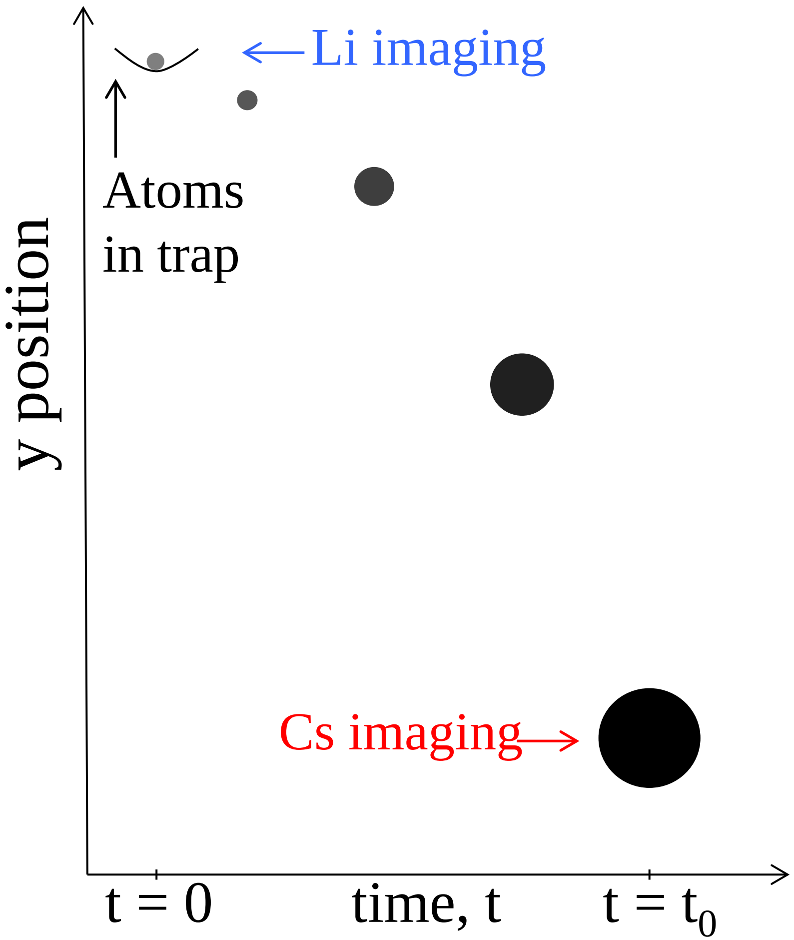
Tung et al., PRA 87 010702 (2013)
Repp et al., PRA 87 010701 (2013)

Experiment Procedures



Dual Resonant Absorption Imaging

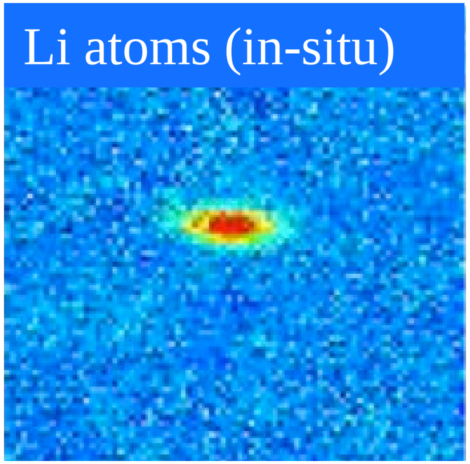
The time of flight method



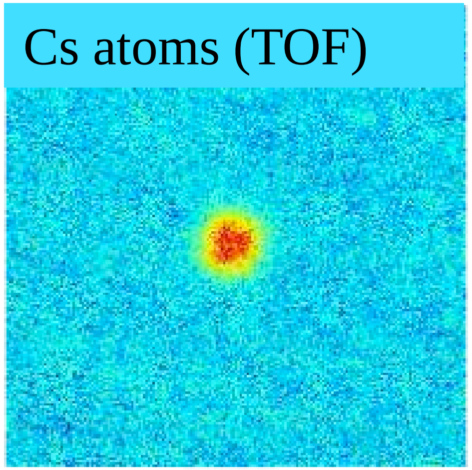
Switch-off
trapping potential

Probe atoms
after expansion

Resonant absorption imaging



30×10^3

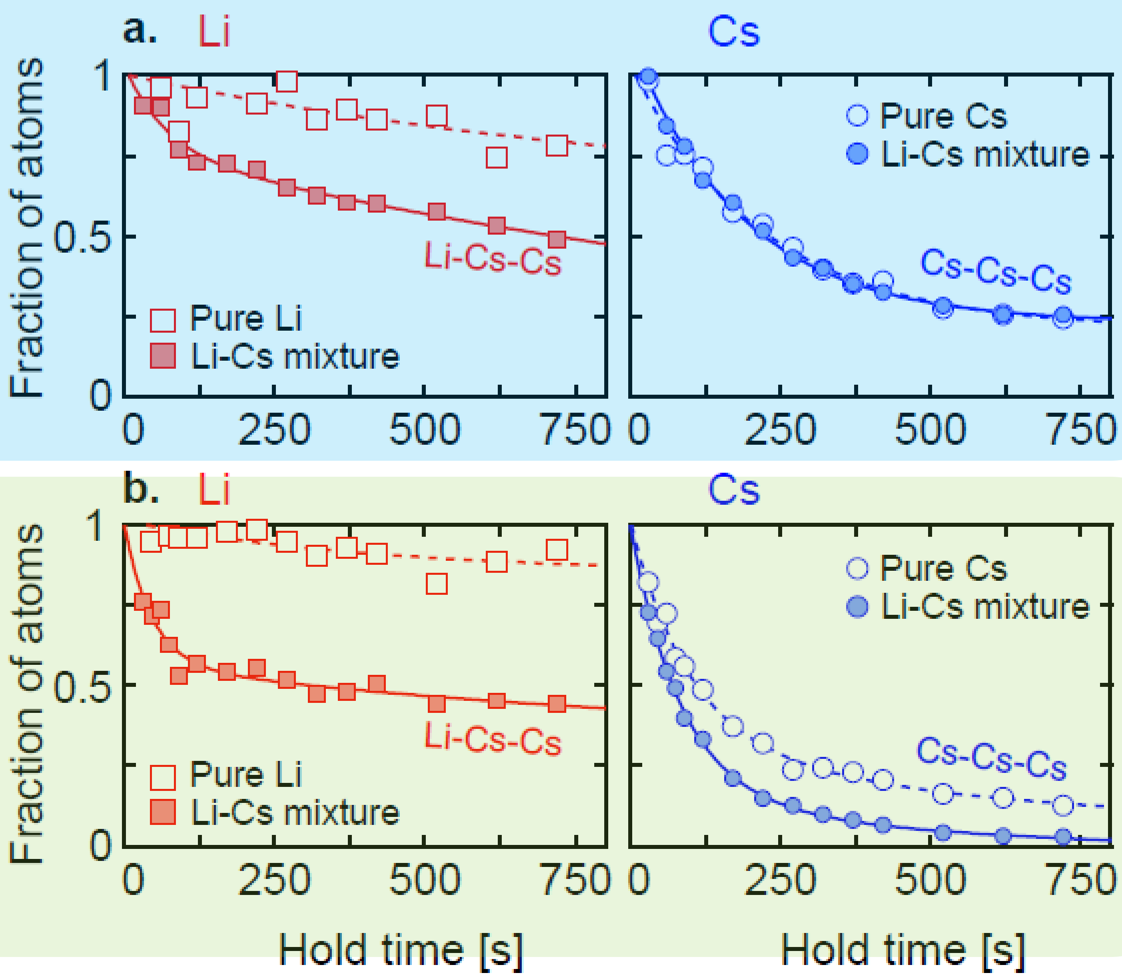


15 ms 30×10^3

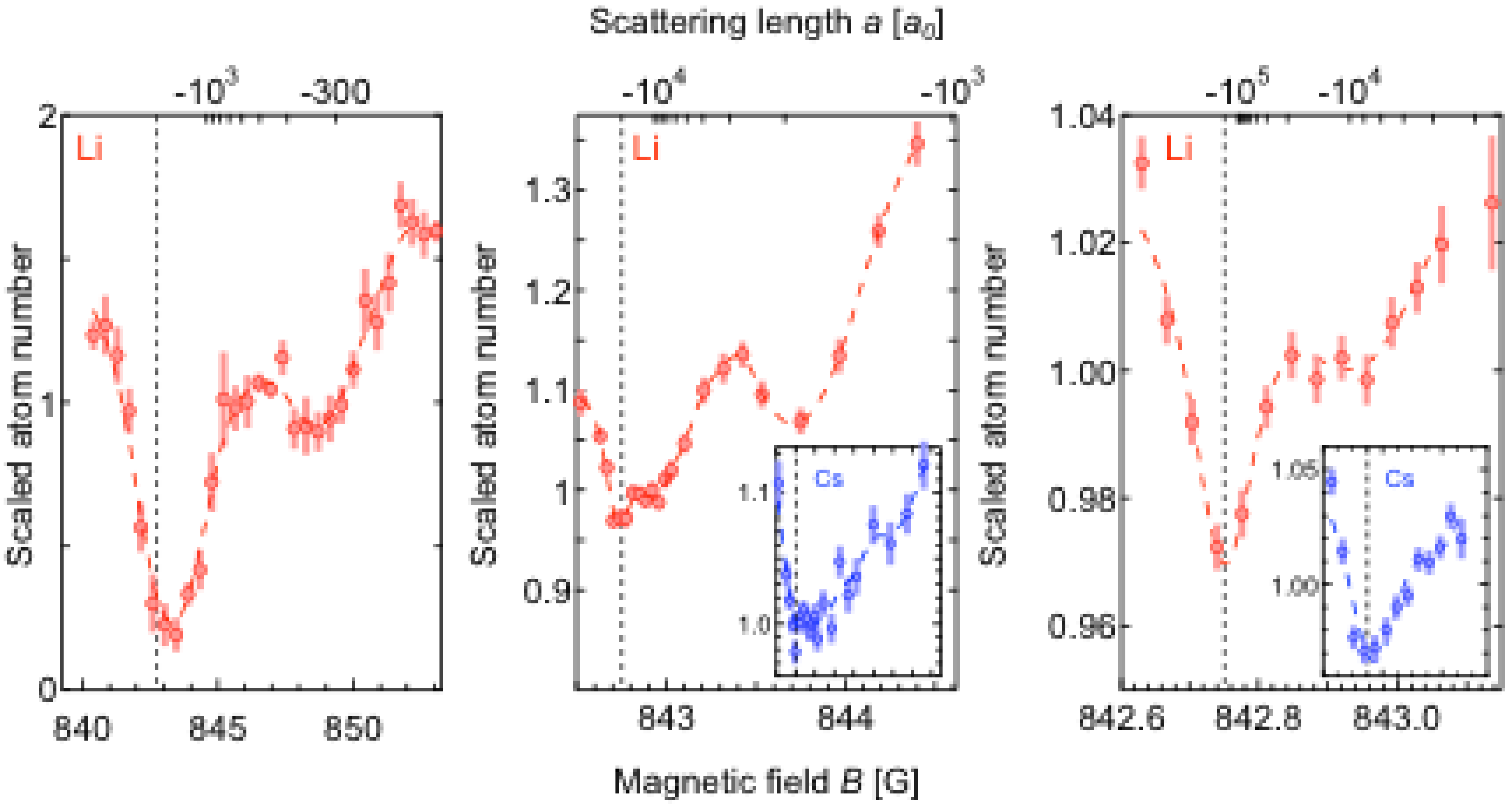
Trap loss measurement: Cs+Cs+Cs vs. Li+Cs+Cs

5G away
(-300 a_0)

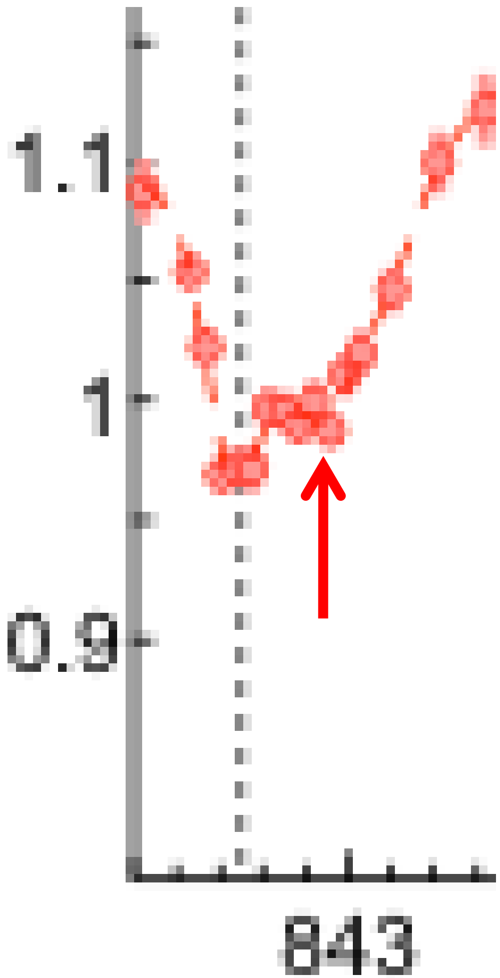
Feshbach
resonance

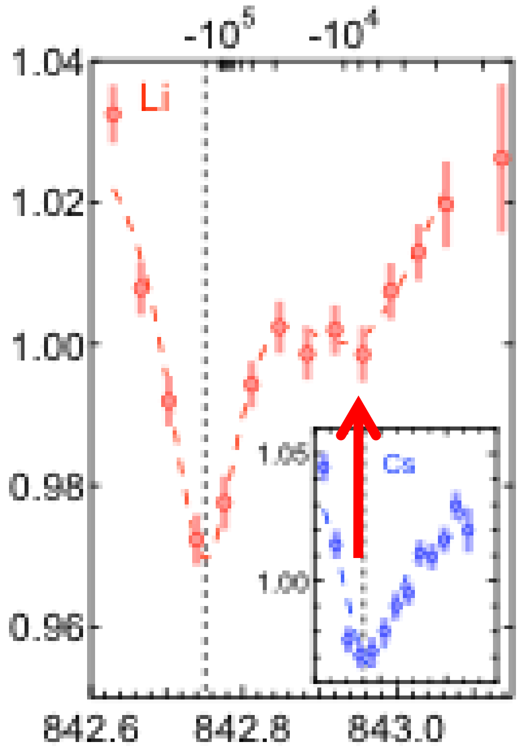


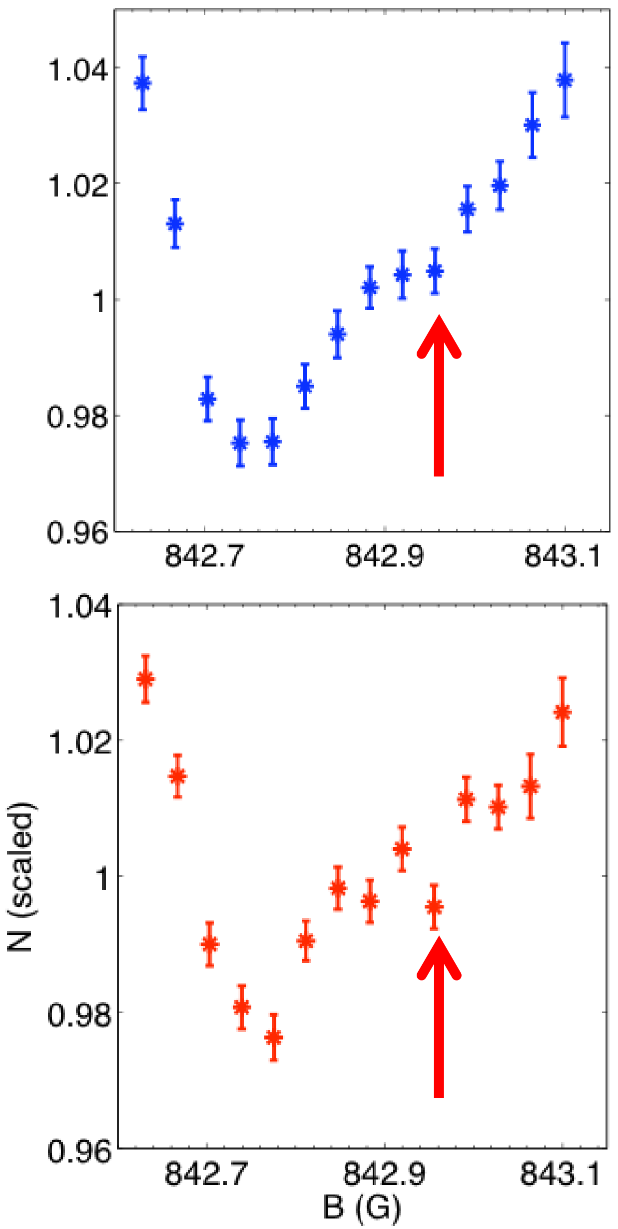
Trap loss measurement: Field scans



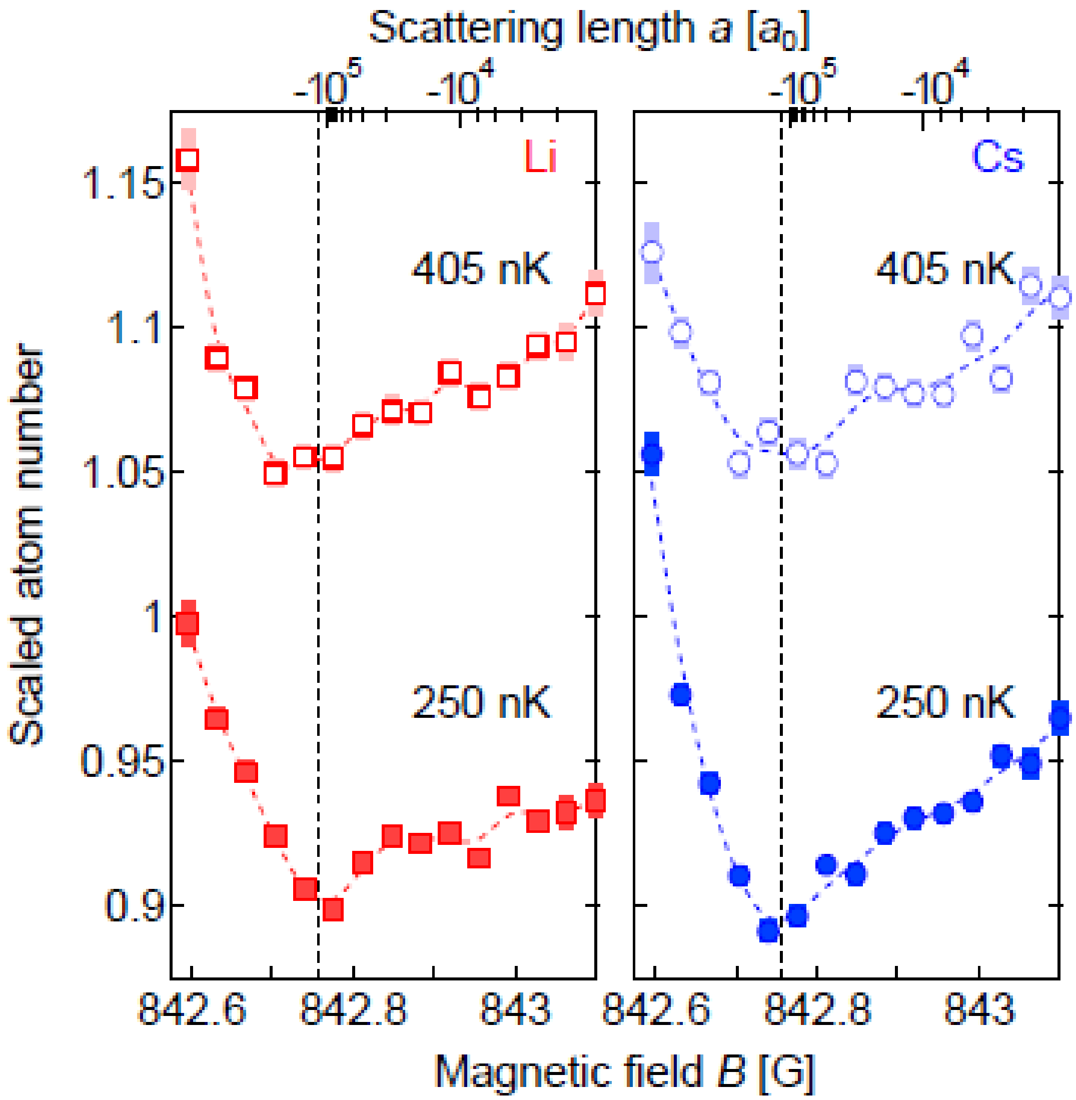
Finest Scans







Temperature dependence?



< T > = 405 nK
209 averages

< T >=250 nK
328 averages

Result

- Three Efimov resonances:

First resonance:	+5.6(2) G,	a_1 = -337(9) Bohr
Second resonance:	+1.07(2) G,	a_2 = -1650(30) Bohr
Third resonance:	+0.22(4) G,	a_3 = -7900(1400) Bohr
Feshbach:	*842.75(1)G*	

- Scaling ratio: $a_1 : a_2 = 1 : 4.90(16)$

  $a_2 : a_3 = 1 : 4.79(87)$

  Weighted ratio: 4.85(44)

  Theory: 4.88 D'Incao and B.Esry, PRA (2006)

Systematics (Thanks to R. Grimm and C. Salomon)

- Finite temperature shift: < 8% in 0 ~ 1μK (Y. Wang)
- Feshbach resonance position: < 10 mG (Y. Wang)
- Finite size: E_3 = 500nK, Cs trap freq. = 4 nK, LiCs freq. = 4.5 nK

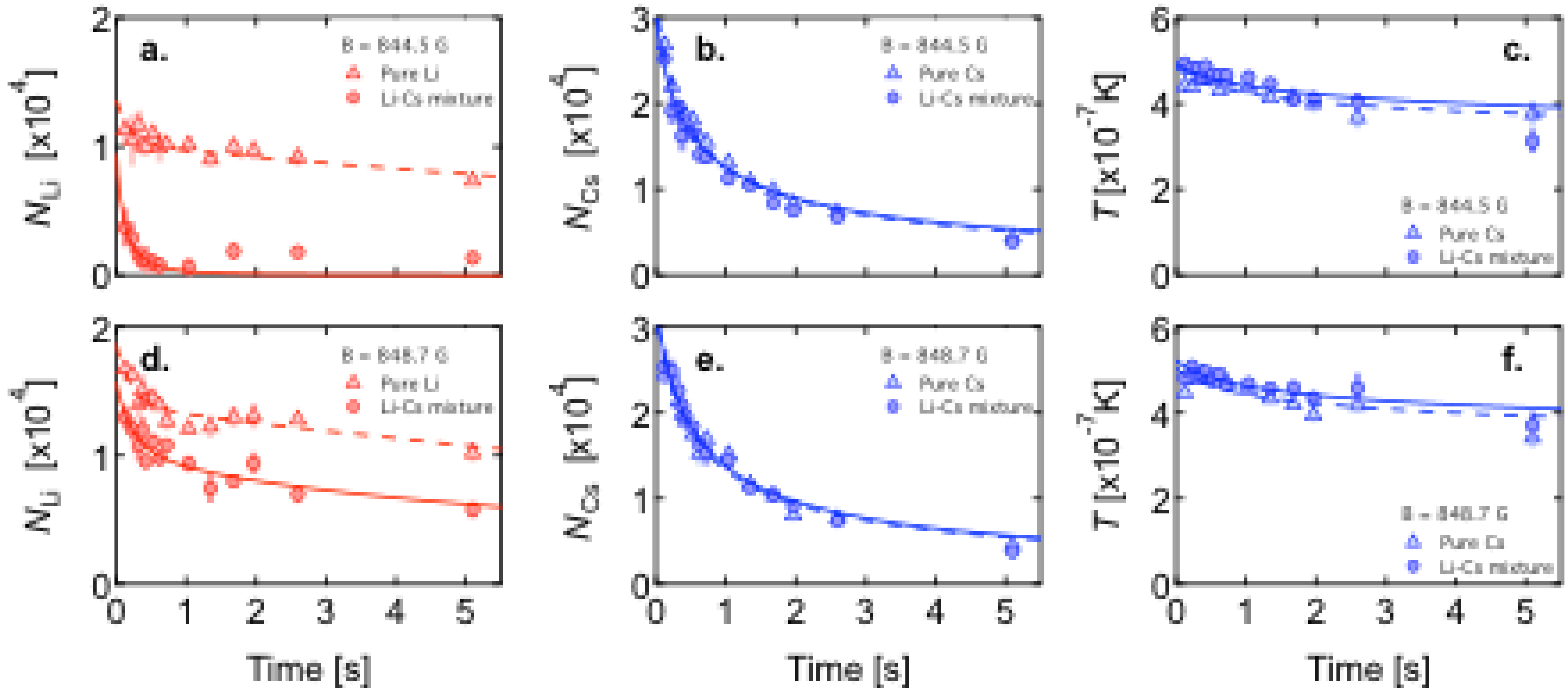
Trap independent coefficients (a best guess)

$$\frac{dN_{\mathrm{Li}}}{dt} = -K_3 X(T) N_{\mathrm{Cs}}^2 N_{\mathrm{Li}} - A N_{\mathrm{Li}} - B N_{\mathrm{Li}} e^{-t/C}$$

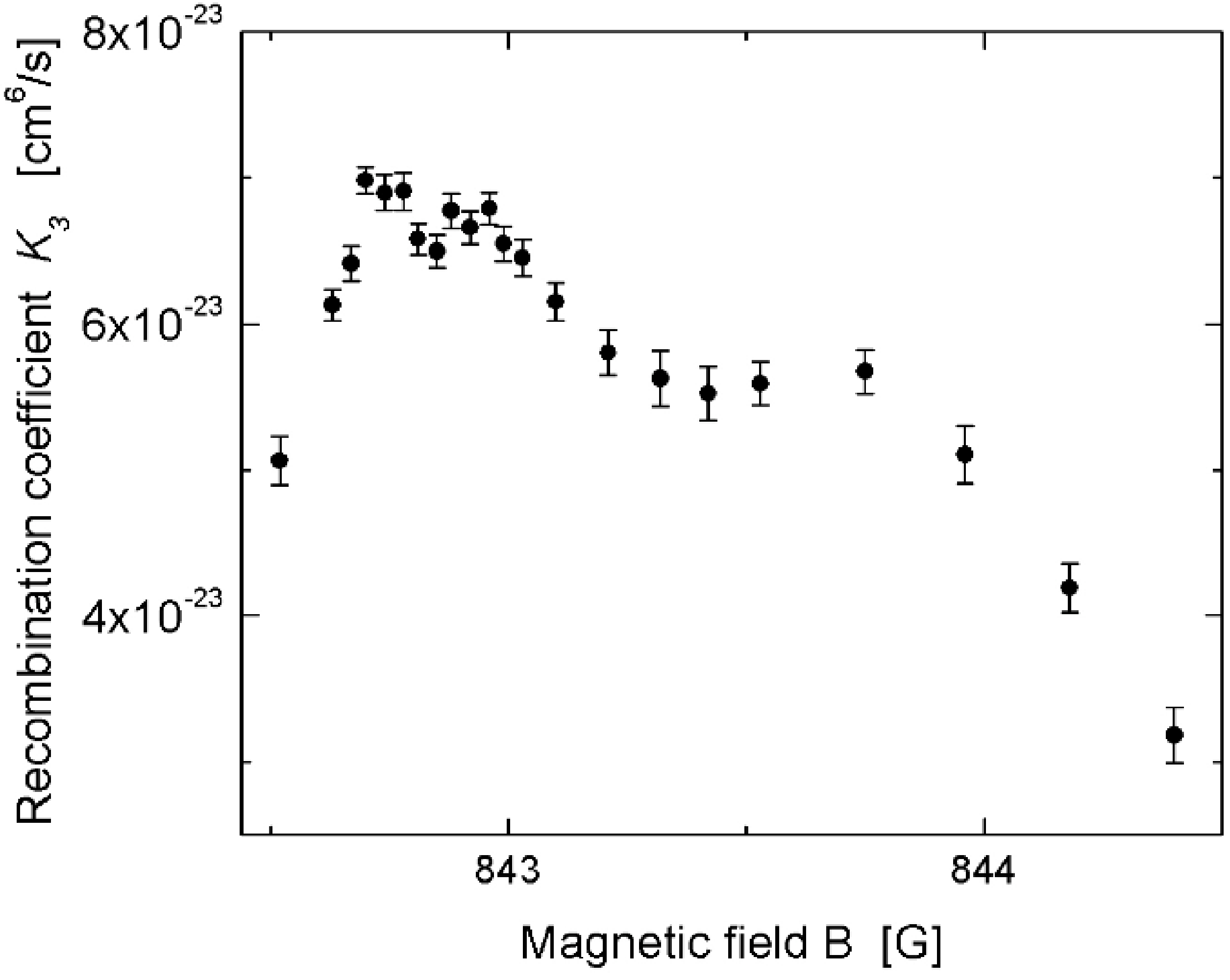
$$\frac{dN_{\mathrm{Cs}}}{dt} = -2K_3' X(T) N_{\mathrm{Cs}}^2 N_{\mathrm{Li}} - D N_{\mathrm{Cs}}^3 T^{-3}$$

$$\frac{dT_{\mathrm{Cs}}}{dt} = -F N_{\mathrm{Cs}}^2 T_{\mathrm{Cs}}^{-3/2},$$

$$X(T) N_{\mathrm{Cs}}^2 N_{\mathrm{Li}} = \int \left[n_{\mathrm{Cs}}(\mathbf{x},T)\right]^2 \left[n_{\mathrm{Li}}(\mathbf{x},T)\right] \, d^3\mathbf{x},$$

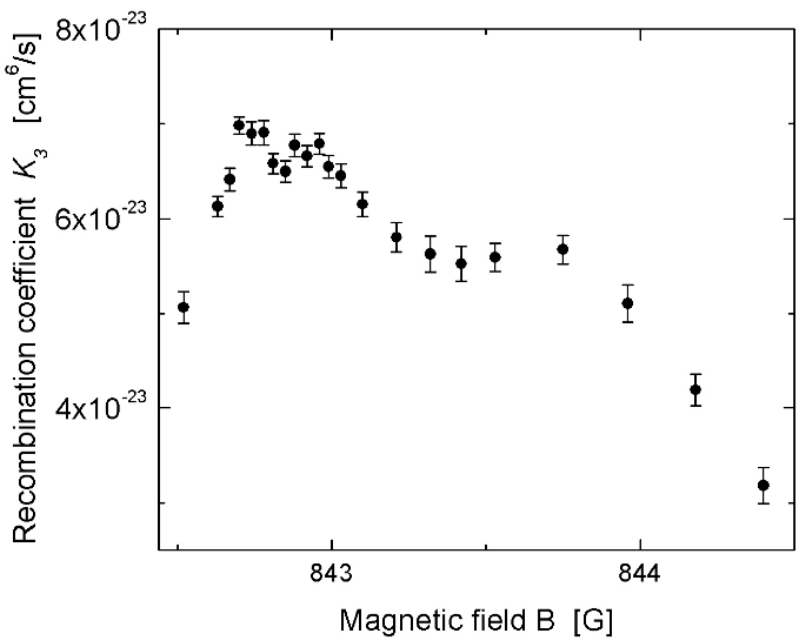


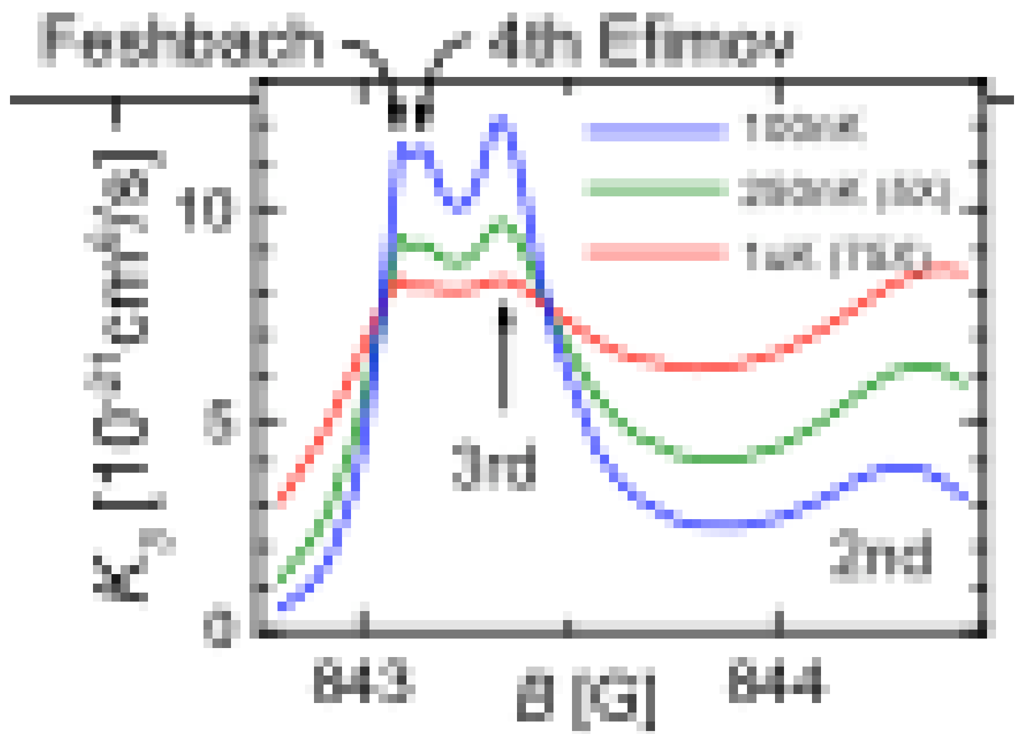
Trap independent coefficients (a best guess)



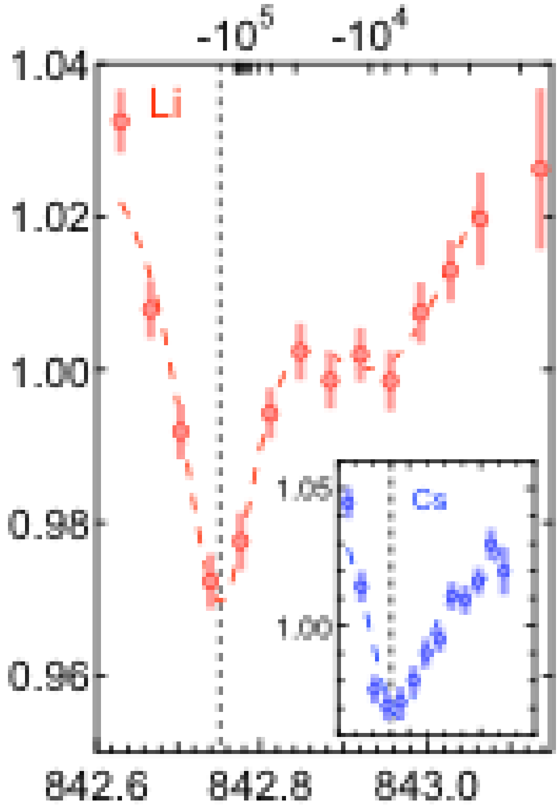
Theory:

(Yujun Wang, NIST and Kansas State)





Scattering length determination

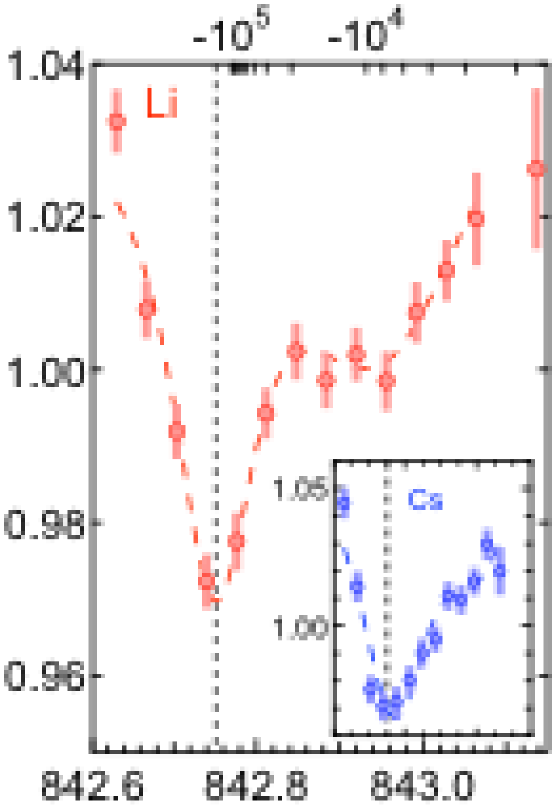



```
FB:   842.75(1)  G
E3:    +0.22(4)  G
E2:    +1.07(2)  G
E1:    +5.60(20) G
```

Within the resonance width,
scattering length ~ $1/(B-B_0)$

```
(E1-FB)/(E2-FB): 5.2(2)
(E2-FB)/(E3-FB): 4.9(9)
```

Scattering length determination



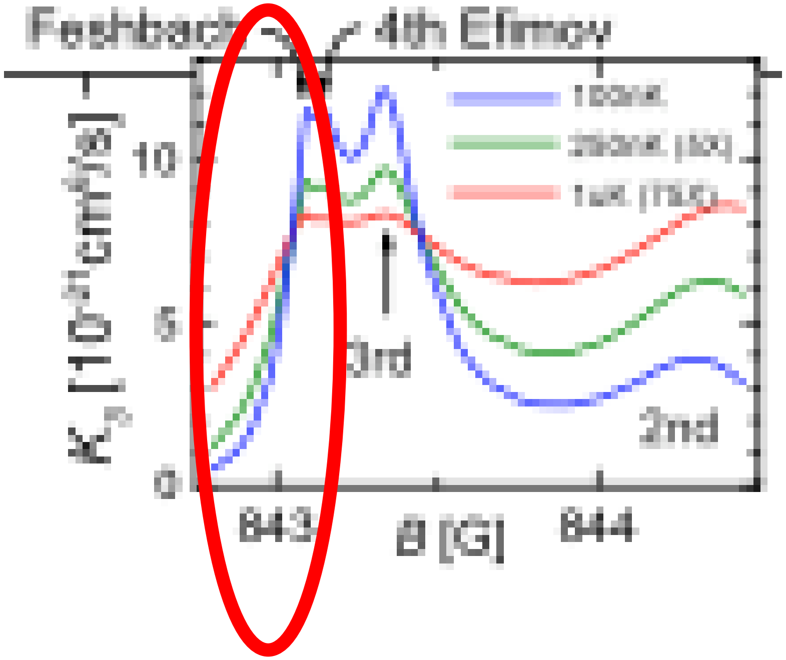
~~FB~~: E4? 842.75(1) G
E3: +0.22(4) G
E2: +1.07(2) G
E1: +5.60(20) G

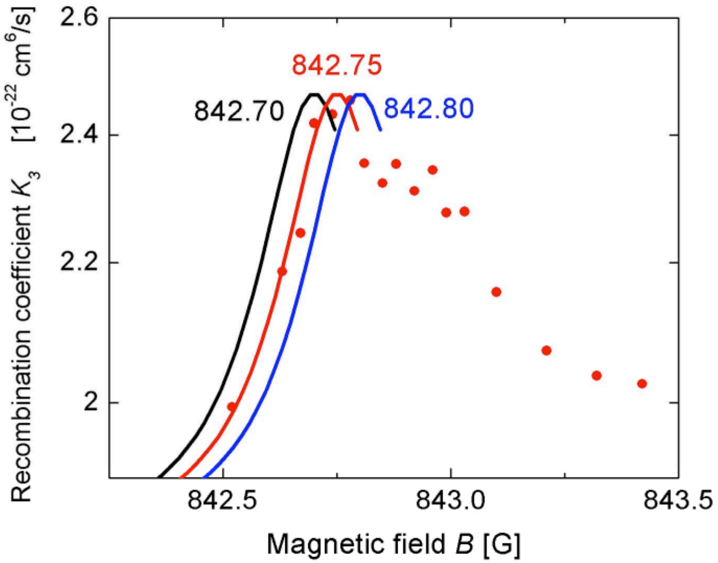
Within the resonance width,
scattering length ~ $1/(B-B_0)$

(E1-E2)/(E2-E3): 5.3(4)
(E2-E3)/(E3-E4): 3.9(7)

Back to Theory:

(Yujun Wang, NIST and Kansas State)





$$K_3 \propto \text{const} + \frac{1}{(B-B_0)^2 + \Delta B^2}$$

The Crew

PI

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Karina Jiménez-García

Undergrads

Nicholas Kowalski

Dylan O.A.B. Sabulsky

Grads

Logan Clark

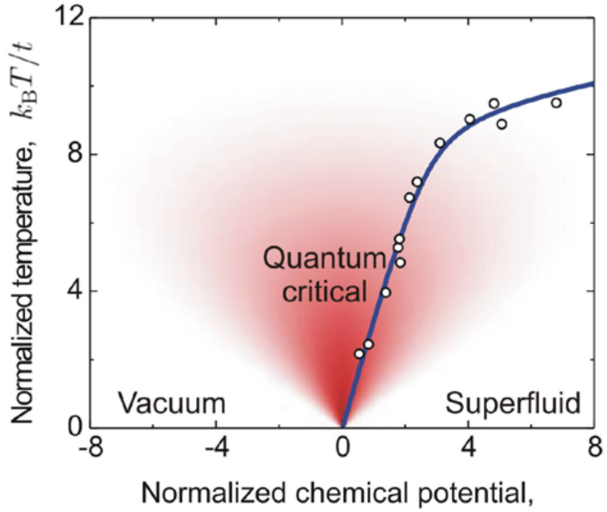
Gustaf Downs

Lei Fang

Jacob Johansen

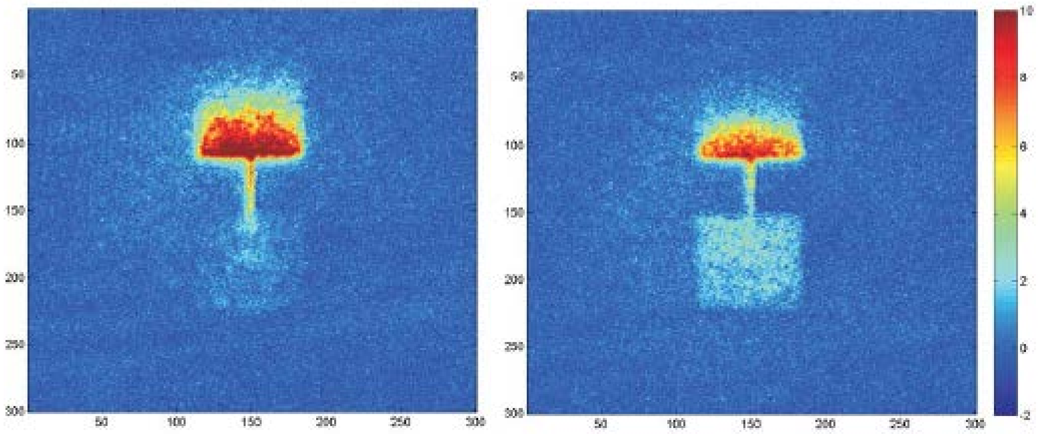
Li-Chung Ha

Quantum criticality



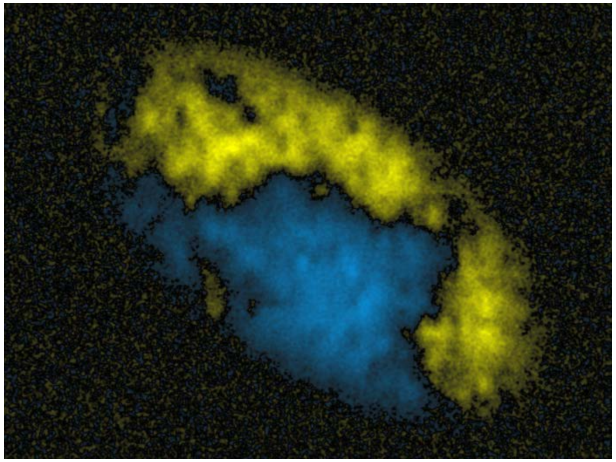
Science 335, 1070 (2012)

Transport



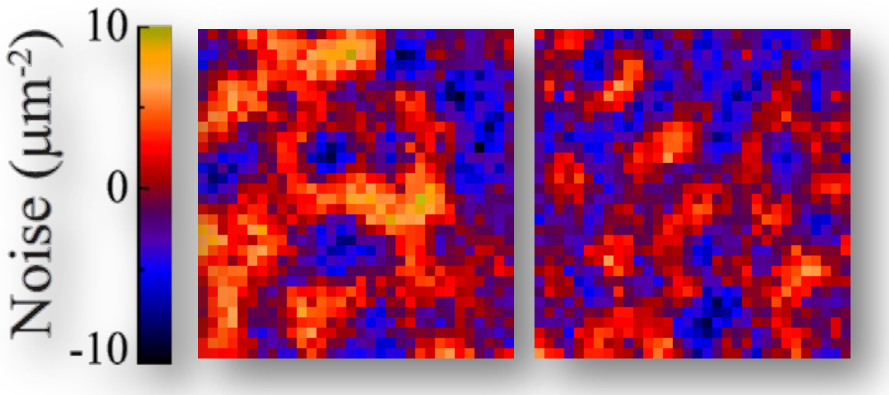
Postdoc positions available

Arbitrary potential



Nature Physics 9, 769 (2013)

Sakharov oscillations



Science 341, 1213 (2013)

Postdoc positions available

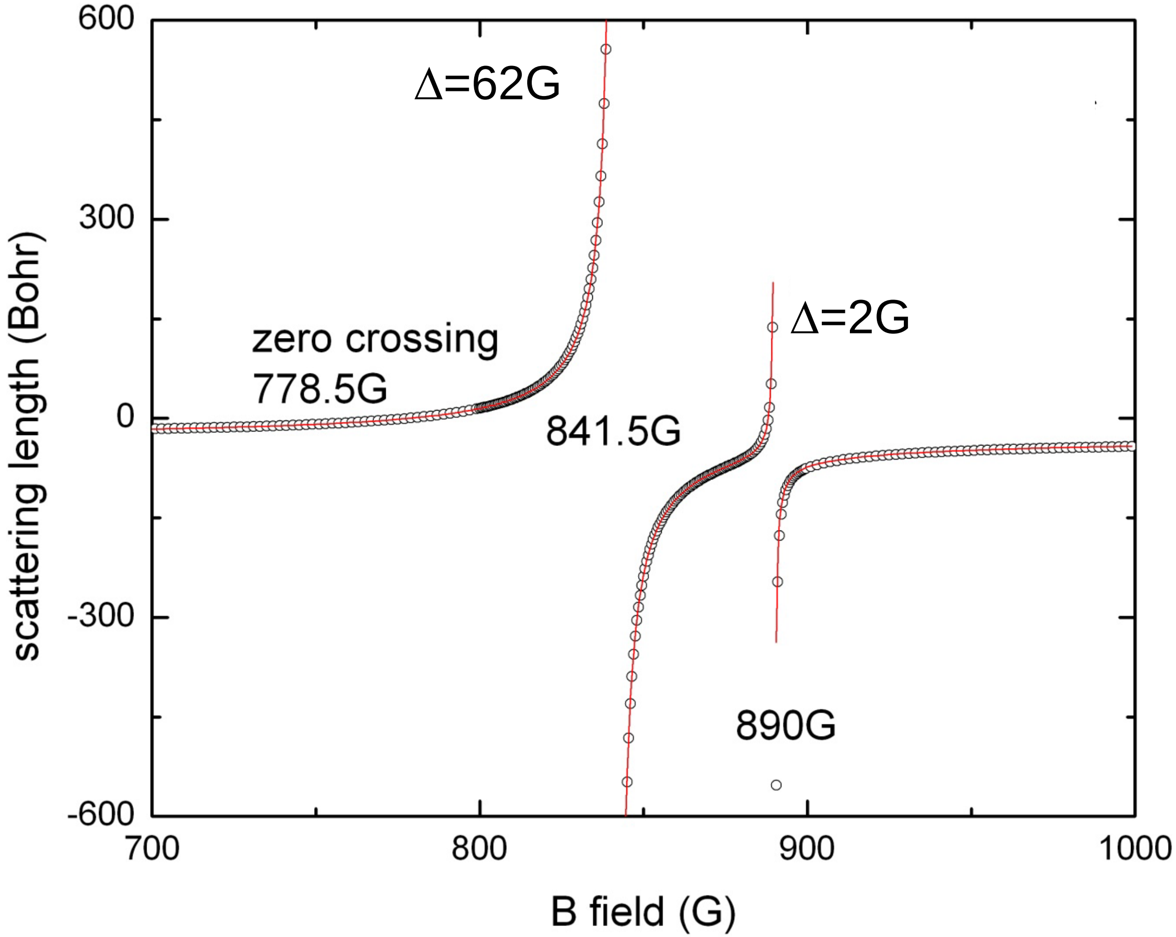
Which resonance should we choose?

TABLE I. Scattering length dependence for three-body collision rates in BBX and XYZ systems. For BBX systems both a_{BX} and a_{BB} scattering lengths are resonant, while for XYZ systems only a_{XY} and a_{XZ} are resonant. The notation $|a|$ indicates $a < 0$, and no entry indicates that the associated process is not possible. Expressions for M and P are given in Eqs. (2)–(5).

		$a_{BX} \ll a_{BB}$	$a_{BX} \gg a_{BB}$	$\|a_{BX}\| \gg a_{BB}$	$\|a_{BX}\| \ll a_{BB}$
	$BX^* + B \to BB^* + X$	$\cdots$	$P_{s_0^*}(\frac{a_{BX}}{a_{BB}})M_{s_0}(\frac{a_{BB}}{r_0})a_{BX}$	$\cdots$	$\cdots$
	$\to BB + X, BX + B$	$P_{s_0}(\frac{a_{BX}}{r_0})a_{BX}$	$P_{s_0^*}(\frac{a_{BX}}{a_{BB}})a_{BX}$	$\cdots$	$\cdots$
	$BB^* + X \to BX^* + B$	$M_{s_0}(\frac{a_{BX}}{r_0})a_{BX}^2/a_{BB}$	$\cdots$	$\cdots$	$\cdots$
	$\to BB + X, BX + B$	a_{BX}^2/a_{BB}	$P_{s_0^*}(\frac{a_{BB}}{r_0})a_{BB}$	$P_{s_0^*}(\frac{a_{BB}}{r_0})a_{BB}$	$P_{s_0}(\frac{a_{BX}}{r_0})a_{BX}^2/a_{BB}$
ba	$B + B + X \to BX^* + B$	$M_{s_0}(\frac{a_{BX}}{r_0})a_{BX}^2 a_{BB}^2$	$M_{s_0^*}(\frac{a_{BX}}{a_{BB}})a_{BX}^4$	$\cdots$	$\cdots$
	$\to BB^* + X$	a_{BB}^4	$M_{s_0}(\frac{a_{BB}}{r_0})a_{BX}^4$	$P_{s_0^*}(\frac{a_{BX}}{a_{BB}})M_{s_0}(\frac{a_{BB}}{r_0})a_{BX}^4$	a_{BB}^4
	$\to BB + X, BX + B$	$a_{BX}^2 a_{BB}^2$	a_{BX}^4	$P_{s_0^*}(\frac{a_{BX}}{a_{BB}})a_{BX}^4$	$a_{BX}^2 a_{BB}^2$
		$a_{BX} \ll \|a_{BB}\|$	$a_{BX} \gg \|a_{BB}\|$	$\|a_{BX}\| \gg \|a_{BB}\|$	$\|a_{BX}\| \ll \|a_{BB}\|$
	$BX^* + B \to BB + X, BX + B$	$P_{s_0}(\frac{a_{BX}}{r_0})a_{BX}$	$P_{s_0}^{s_0^*}(\frac{a_{BB}}{r_0}, \frac{a_{BX}}{a_{BB}})a_{BX}$	$\cdots$	$\cdots$
aa	$B + B + X \to BX^* + B$	$M_{s_0}(\frac{a_{BX}}{r_0})a_{BX}^2 a_{BB}^2$	$M_{s_0}^{s_0^*}(\frac{a_{BB}}{r_0}, \frac{a_{BX}}{a_{BB}})a_{BX}^4$	$\cdots$	$\cdots$
	$\to BB + X, BX + B$	$a_{BX}^2 a_{BB}^2$	a_{BX}^4	$P_{s_0}^{s_0^*}(\frac{a_{BB}}{r_0}, \frac{a_{BX}}{a_{BB}})a_{BX}^4$	$P_{s_0}(\frac{a_{BX}}{r_0})a_{BX}^2 a_{BB}^2$
		$a_{XY} \gg a_{XZ}$	$\|a_{XY}\| \gg a_{XZ}$	$a_{XY} \gg \|a_{XZ}\|$	$\|a_{XY}\| \gg \|a_{XZ}\|$
	$XY^* + Z \to XZ^* + Y$	$M_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}^2/a_{XY}$	$\cdots$	$\cdots$	$\cdots$
	$\to XY + Z, XZ + Y, \ldots$	a_{XZ}^2/a_{XY}	$\cdots$	$P_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}^2/a_{XY}$	$\cdots$
	$XZ^* + Y \to XY + Z, XZ + Y, \ldots$	$P_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}$	$P_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}$	$\cdots$	$\cdots$
	$X + Y + Z \to XY^* + Z$	a_{XY}^4	$\cdots$	a_{XY}^4	$\cdots$
	$\to XZ^* + Y$	$M_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}^2 a_{XY}^2$	$M_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}^2 a_{XY}^2$	$\cdots$	$\cdots$
	$\to XY + Z, XZ + Y, \ldots$	$a_{XZ}^2 a_{XY}^2$	$a_{XZ}^2 a_{XY}^2$	$a_{XZ}^2 a_{XY}^2$	$P_{s_0}(\frac{a_{XZ}}{r_0})a_{XZ}^2 a_{XY}^2$

Choose the one with $a_{BB} < 0$. *J*. P. D'Incao and B. D. Esry, PRL 103 083202 (2009)

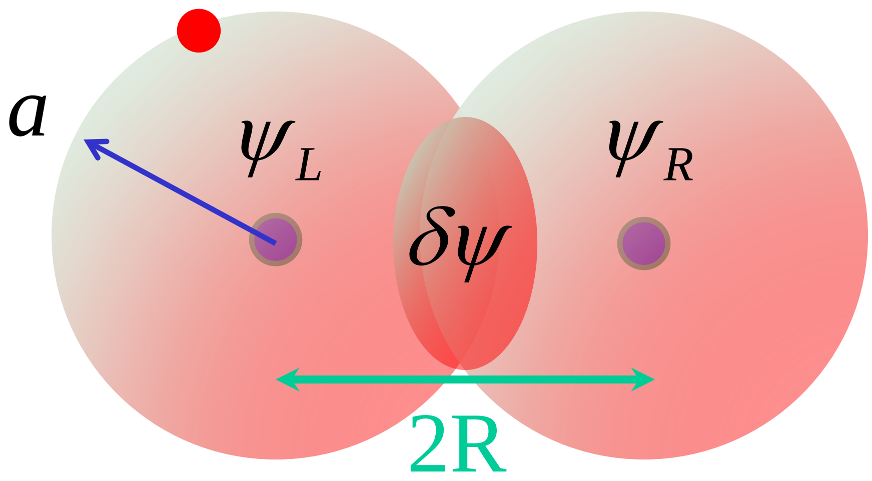
Broad (62G) ^{6}Li- ^{133}Cs Feshbach resonance



Theory: Y. Wang and P. Julienne (NIST)

S.K. Tung et al., PRA 87, 010702 (2013)

Picture of Efimov potential



$$\psi_{\pm}(r) = \psi_L(r) \pm \psi_R(r)$$

$$-\frac{\hbar^2}{2m}\text{È}''(\text{r}) + \text{V}\text{È}(\text{r}) = \text{E}\text{È}(\text{r}) \;\Rightarrow\; \frac{\hbar^2}{2m}\frac{\delta\text{È}(\text{r})}{R^2} + \text{V}_{\text{eff}}\;\delta\text{È}(\text{r}) = 0$$

$$\Rightarrow \text{V}_{\text{eff}} \propto -\frac{\hbar^2}{2\text{mR}^2} \qquad \text{when } R < |a|$$

$$\approx 0 \qquad \text{when } R > |a|$$

Geometric scaling of Efimov states