

**Few-body universality:
from Efimov effect
to super Efimov effect**

Yusuke Nishida (Tokyo Tech)

**INT workshop on “few-body universality
in atomic and nuclear physics”**

May 12-16 (2014)

Plan of this talk

1. Universality of Efimov effect ⇒ Solid state physics

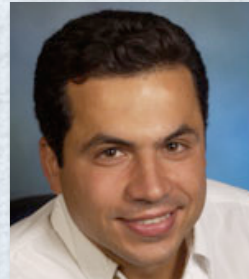
nature
physics

ARTICLES

PUBLISHED ONLINE: 13 JANUARY 2013 | DOI: 10.1038/NPHYS2523

Efimov effect in quantum magnets

Yusuke Nishida^{*}, Yasuyuki Kato and Cristian D. Batista



2. Novel few-body universality ⇒ Super Efimov effect

PRL 110, 235301 (2013)

PHYSICAL REVIEW LETTERS

week ending
7 JUNE 2013



Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

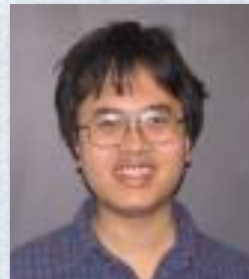
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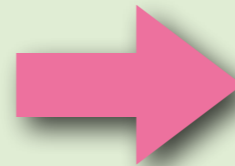




Few-body universality

Efimov effect (1970)

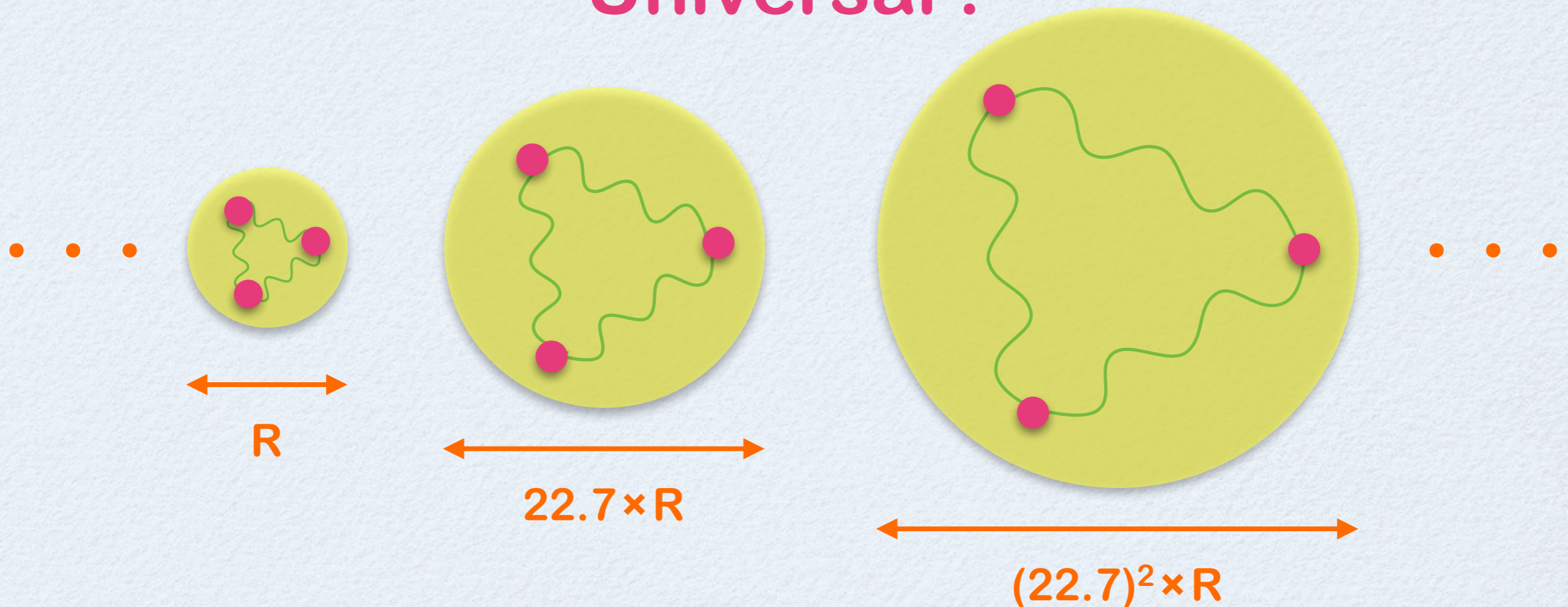
- 3 bosons
- 3 dimensions
- s-wave resonance



Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !

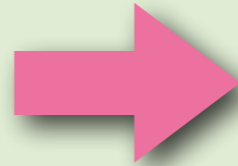




Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance



Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !

atomic
physics

- helium atoms
- cold atoms
- ...

nuclear
physics

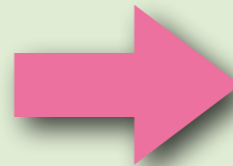
- nucleons
- halo nucleus
- ...



Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance



Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !

atomic
physics

condensed
matter

nuclear
physics

Efimov effect in **solid states** ?

- × electrons (fermions with long-range repulsion)
- bosonic collective excitations !?

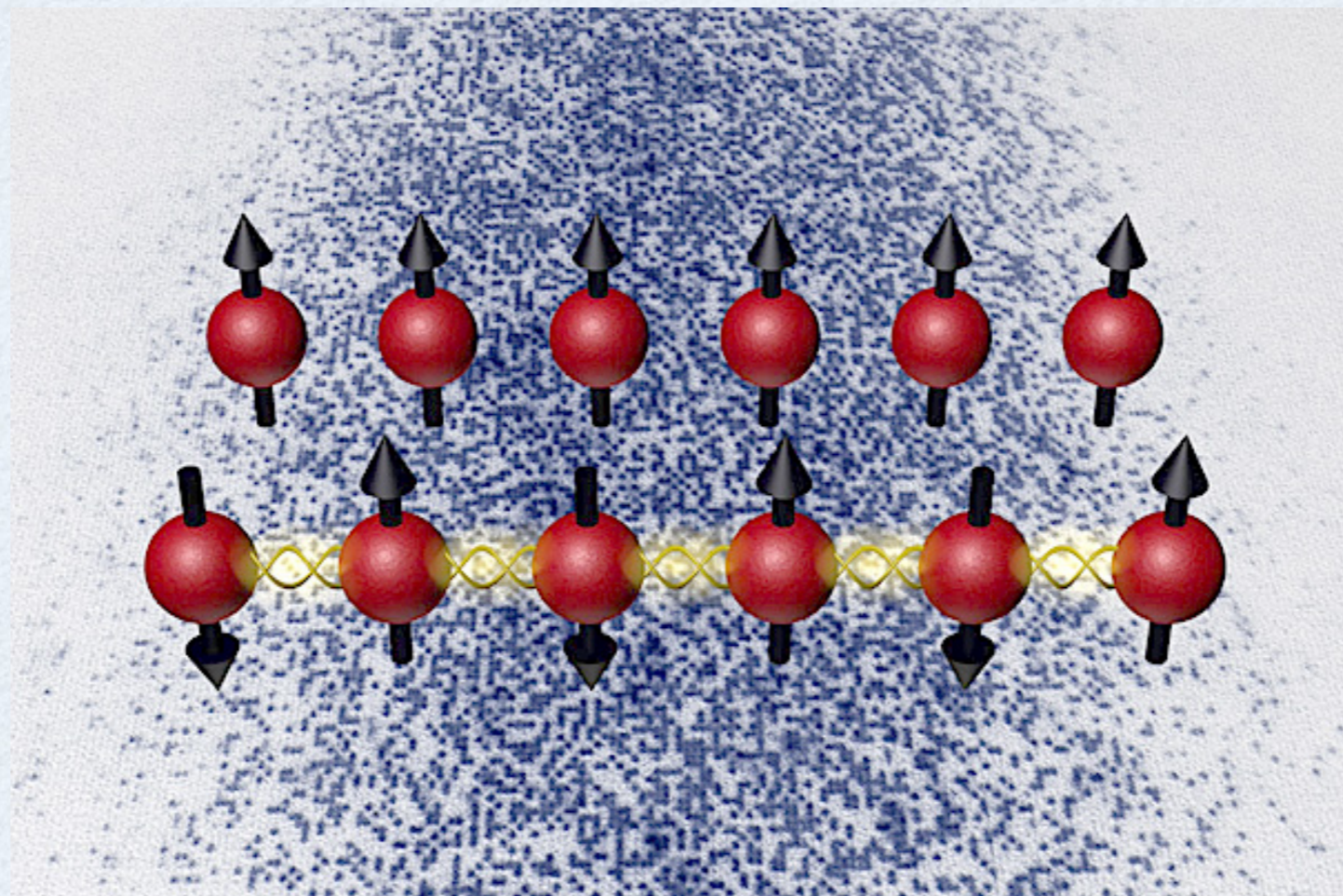
Efimov effect in quantum magnets



Quantum magnet

Anisotropic Heisenberg model on a **3D** lattice

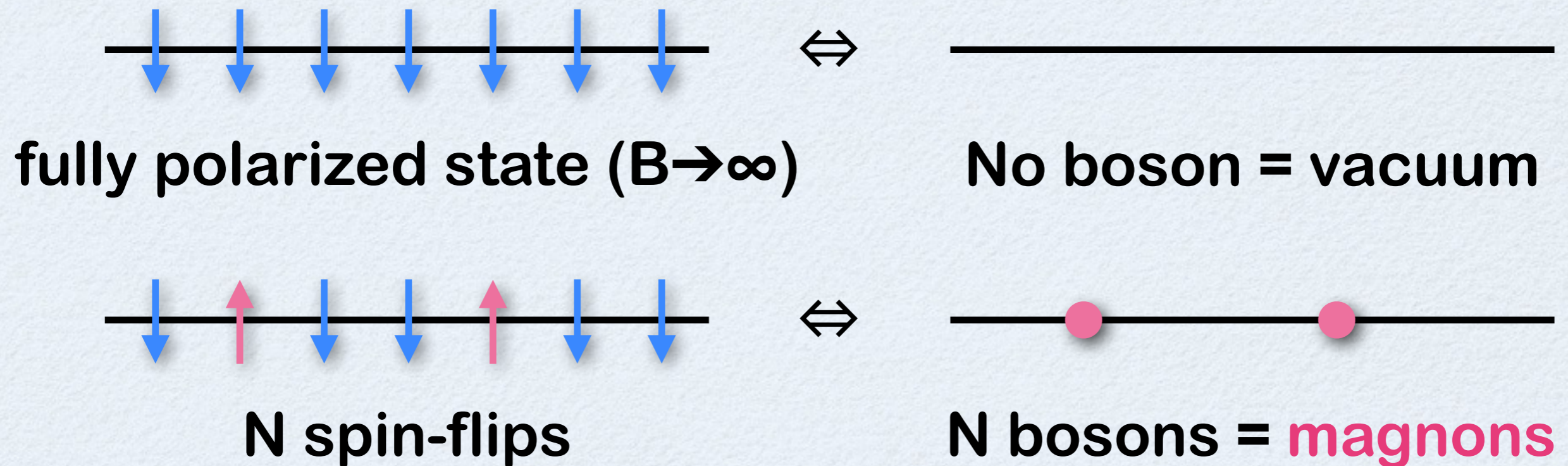
$$H = - \sum_r \left[\sum_{\hat{e}} \left(\underset{\substack{\uparrow \\ \text{exchange anisotropy}}}{J} S_r^+ S_{r+\hat{e}}^- + \underset{\substack{\uparrow \\ \text{exchange anisotropy}}}{J_z} S_r^z S_{r+\hat{e}}^z \right) + \underset{\substack{\uparrow \\ \text{single-ion anisotropy}}}{D} (S_r^z)^2 - B S_r^z \right]$$



Anisotropic Heisenberg model on a **3D** lattice

$$H = - \sum_r \left[\sum_{\hat{e}} \left(\underset{\substack{\uparrow \\ \text{exchange anisotropy}}}{J} S_r^+ S_{r+\hat{e}}^- + \underset{\substack{\uparrow \\ \text{exchange anisotropy}}}{J_z} S_r^z S_{r+\hat{e}}^z \right) + \underset{\substack{\uparrow \\ \text{single-ion anisotropy}}}{D} (S_r^z)^2 - B S_r^z \right]$$

Spin-boson correspondence



Anisotropic Heisenberg model on a **3D** lattice

$$H = - \sum_r \left[\sum_{\hat{e}} \left(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z \right) + D (S_r^z)^2 - B S_r^z \right]$$

xy-exchange coupling
 \Leftrightarrow hopping

single-ion anisotropy
 \Leftrightarrow on-site attraction

z-exchange coupling
 \Leftrightarrow neighbor attraction



N spin-flips



N bosons = magnons

Anisotropic Heisenberg model on a **3D** lattice

$$H = - \sum_r \left[\sum_{\hat{e}} \left(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z \right) + D (S_r^z)^2 - B S_r^z \right]$$

xy-exchange coupling
 \Leftrightarrow hopping

single-ion anisotropy
 \Leftrightarrow on-site attraction

z-exchange coupling
 \Leftrightarrow neighbor attraction

Tune these couplings to induce scattering resonance between two magnons

\Rightarrow Three magnons show the Efimov effect

Two-magnon resonance

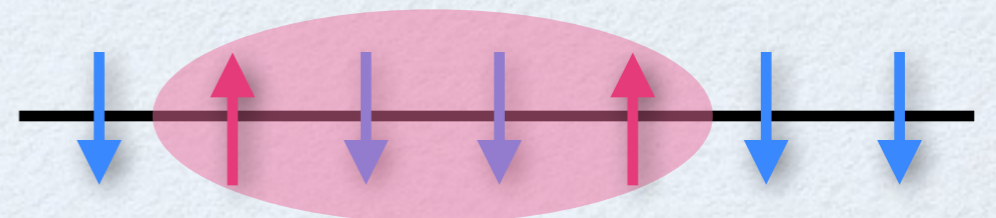
Schrödinger equation for two magnons

$$E\Psi(r_1, r_2) = \left[SJ \sum_{\hat{e}} (2 - \nabla_{1\hat{e}} - \nabla_{2\hat{e}}) \leftarrow \text{hopping} \right. \\ \left. + J \sum_{\hat{e}} \delta_{r_1, r_2} \nabla_{2\hat{e}} - J_z \sum_{\hat{e}} \delta_{r_1, r_2 + \hat{e}} - 2D\delta_{r_1, r_2} \right] \Psi(r_1, r_2)$$

neighbor/on-site attraction

Scattering length between two magnons

$$\lim_{|r_1 - r_2| \rightarrow \infty} \Psi(r_1, r_2) \Big|_{E=0} \rightarrow \frac{1}{|r_1 - r_2|} - \frac{1}{a_s}$$



Two-magnon resonance

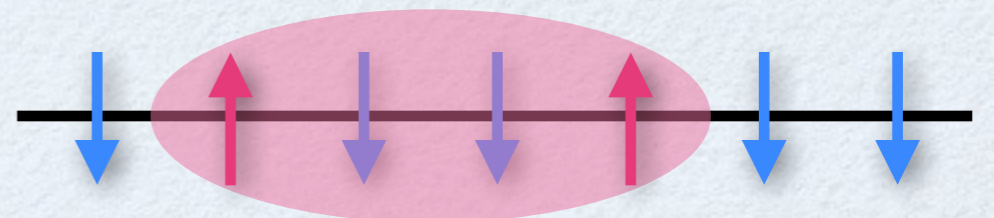
Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi} \left[1 - \frac{D}{3J} - \frac{J_z}{J} \left(1 - \frac{D}{6SJ} \right) \right]}{2S - 1 + \frac{J_z}{J} \left(1 - \frac{D}{6SJ} \right) + 1.52 \left[1 - \frac{D}{3J} - \frac{J_z}{J} \left(1 - \frac{D}{6SJ} \right) \right]}$$



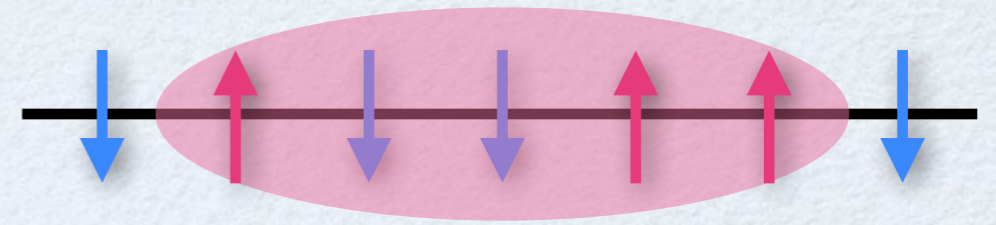
Two-magnon resonance ($a_s \rightarrow \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, $D=0$)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- ...



Three-magnon spectrum

At the resonance, **three magnons** form bound states with binding energies E_n



- Spin-1/2

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- Spin-1, $D=0$

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- Spin-1, $J_z=J>0$

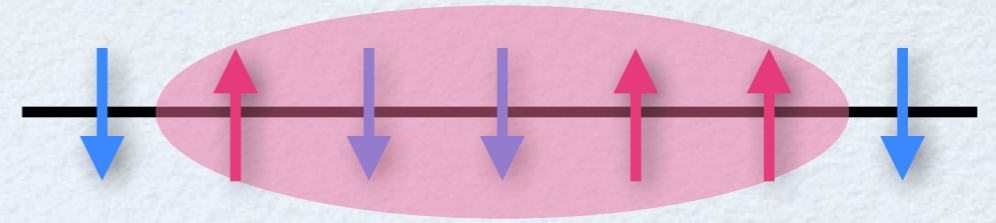
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- Spin-1, $J_z=J<0$

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, **three magnons** form bound states with binding energies E_n



- Spin-1/2

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	
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- Spin-1, D=0

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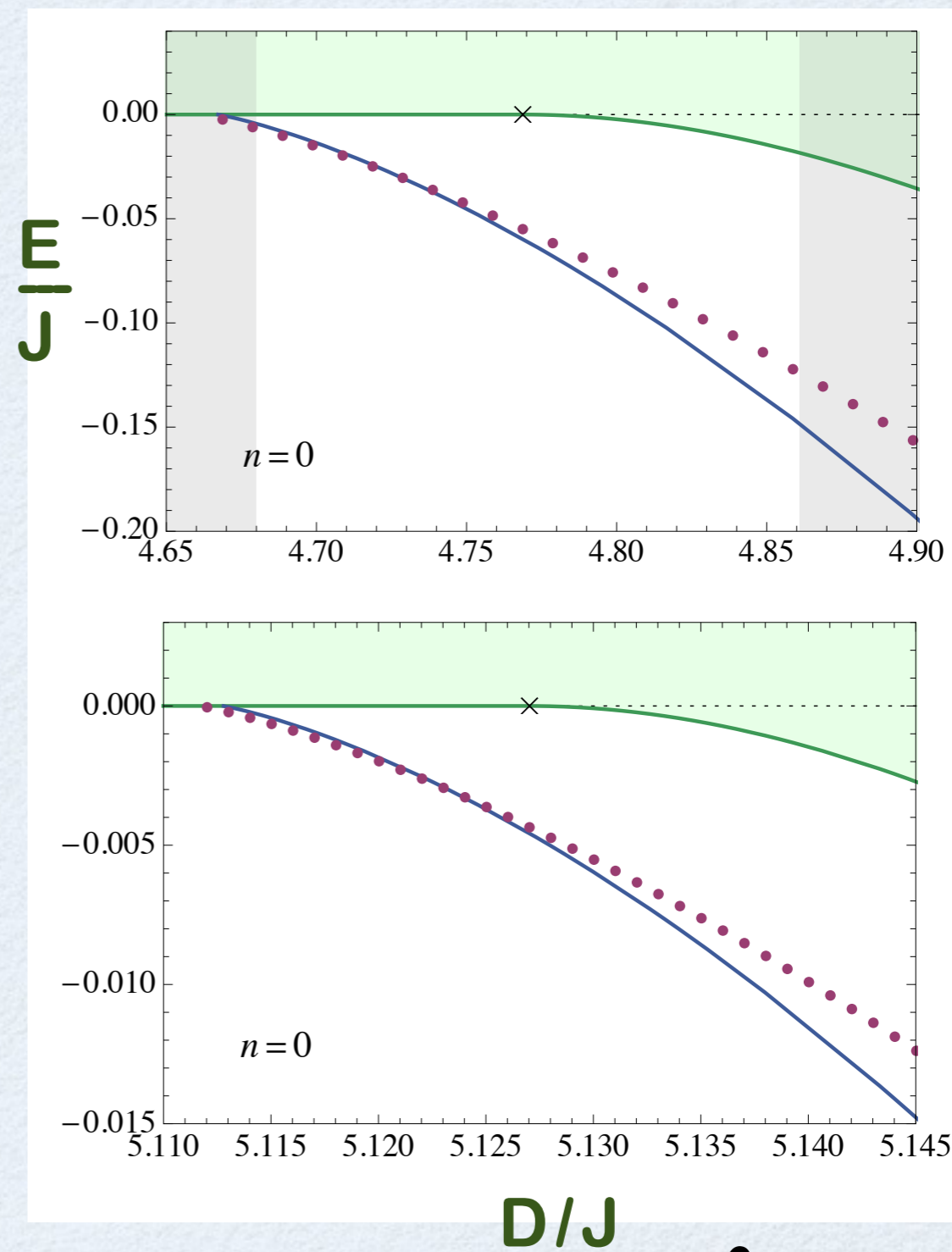
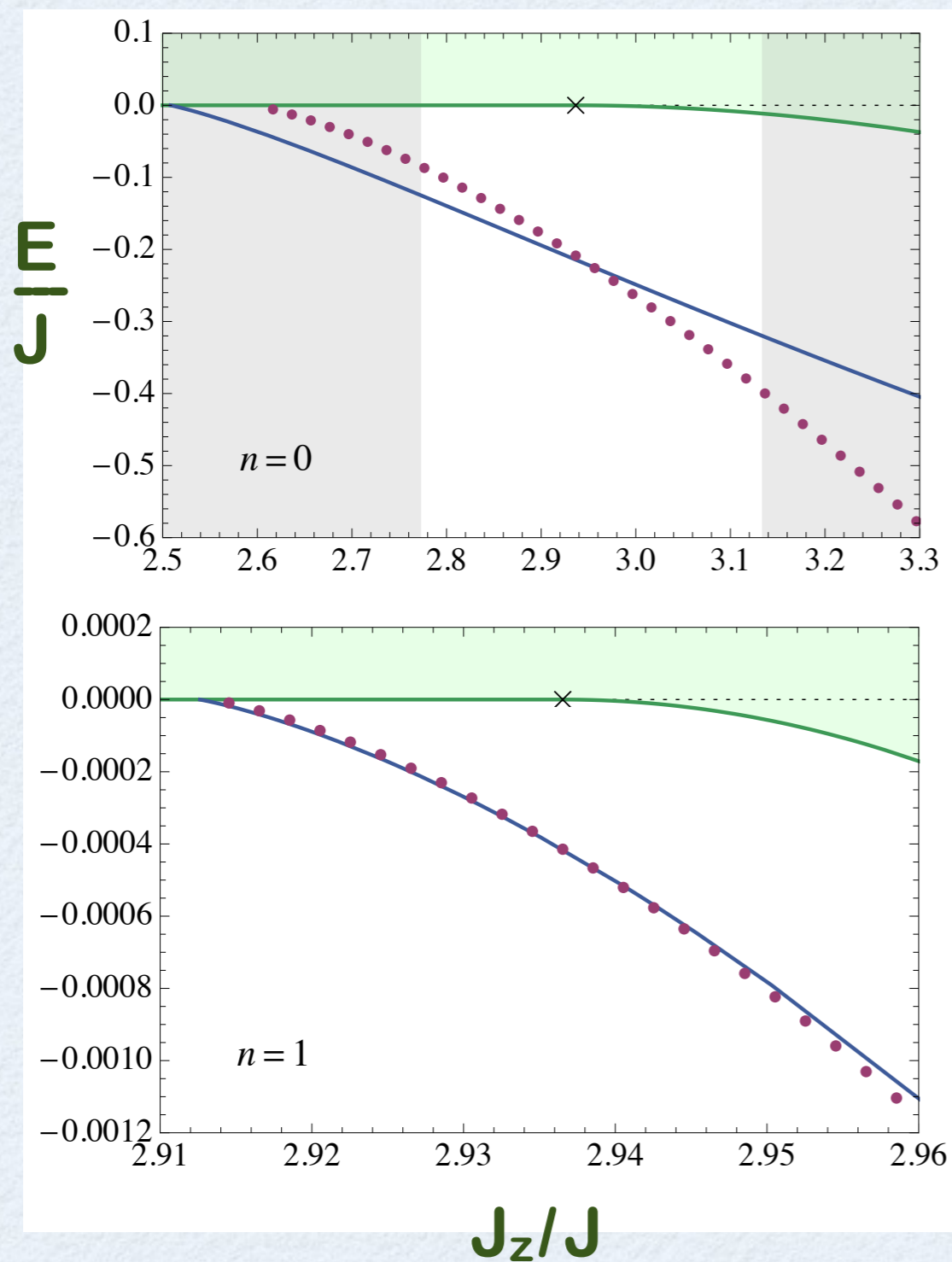


Universal scaling law by ~ 22.7

confirms they are **Efimov states**!

Three-magnon spectrum

• Spin-1/2



• $S=1, J_z=J>0$

• $S=1, J_z=J<0$

Agree with universal prediction : $E_n = -\lambda^{-2n} \frac{\kappa_*^2}{m} F\left(\frac{\lambda^n}{\kappa_* a_s}\right)$

Toward experimental realization 16/38

1. Find a good compound

whose anisotropy is close to the critical value

E.g. Ni-based organic ferromagnet with $D/J \sim 3$ (critical 4.8)

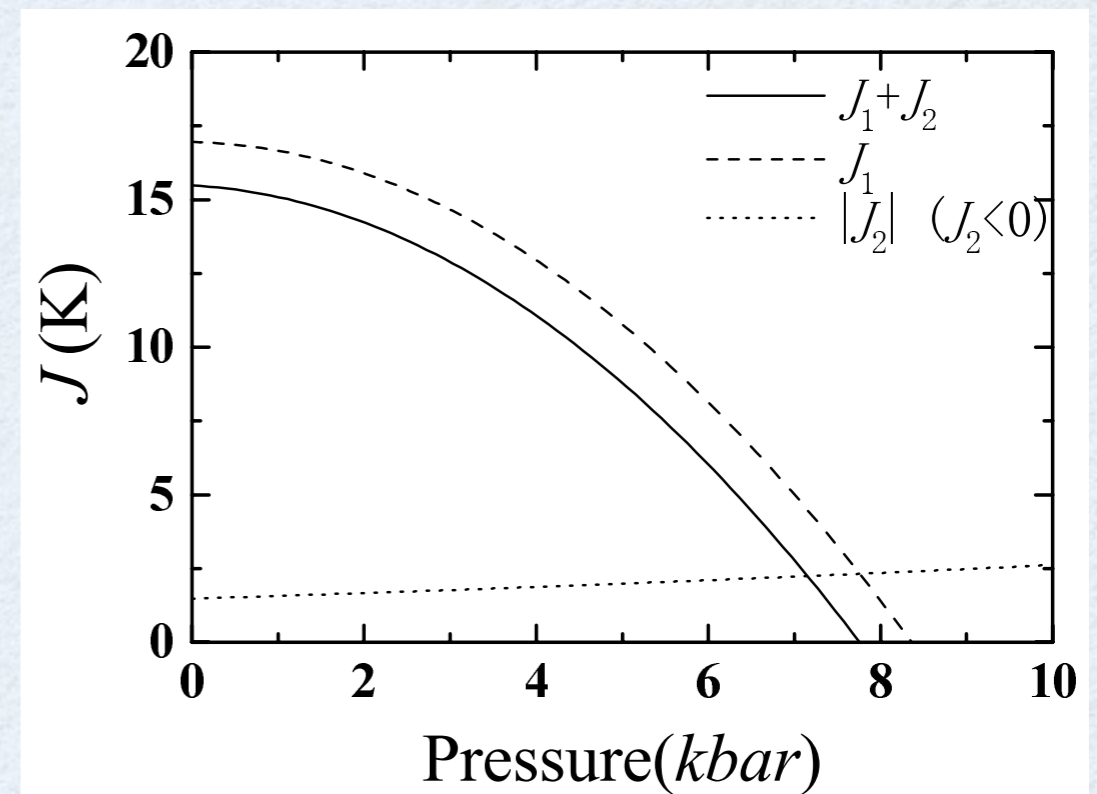
R. Koch et al., Phys. Rev. B 67, 094407 (2003)

C.f. TDAE-C₆₀

2. Tune the exchange coupling with pressure to induce the two-magnon resonance

3. Observe the Efimov states of three magnons with

- absorption spectroscopy
 - inelastic neutron scattering
 - electron spin resonance
- [see Y.N., PRB88, 224402 (2013)]



T. Kawamoto et al, JPSJ (2001)

Find interested experimentalists !

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic
physics

nuclear
physics

condensed
matter

Efimov effect in quantum magnets induced by

- exchange anisotropy
- spatial anisotropy
- single-ion anisotropy
- frustration

nature
physics

ARTICLES

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Efimov effect in quantum magnets

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Atomic vs magnetic systems

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

Atomic BEC (1995 )

Magnon BEC (1999 )



Efimov effect (2006 )

Efimov effect (201? )

Atomic vs magnetic systems

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic
physics



condensed
matter

Atomic BEC (1995 )

Magnon BEC (1999 )

Efimov effect (2006 )

Efimov effect (201? )

New link between atomic and magnetic systems

Unitary magnon gas ?!

Novel universality: Super Efimov effect

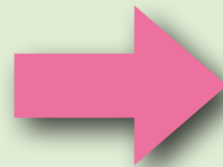


Few-body universality



Efimov effect (1970)

- 3 bosons
- **3 dimensions**
- **s-wave** resonance



Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Efimov effect in other systems ?

No, only in 3D with s-wave resonance

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x	x
1D	x	x	

Y.N. & S.Tan,
Few-Body Syst

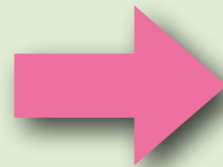
Y.N. & D.Lee
Phys Rev A

Few-body universality



Efimov effect (1970)

- 3 bosons
- **3 dimensions**
- **s-wave** resonance



Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Different universality in other systems ?

Yes, super Efimov effect in 2D with p-wave !

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	!!x!	x
1D	x	x	

Y.N. & S.Tan,
Few-Body Syst

Y.N. & D.Lee
Phys Rev A

Few-body universality

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance



exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance



“doubly” exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

PRL 110, 235301 (2013)

PHYSICAL REVIEW LETTERS

week ending
7 JUNE 2013



Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

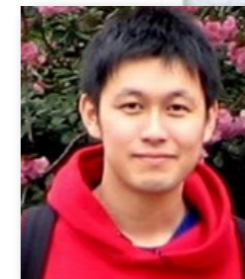
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²Department of Physics, University of Washington, Seattle, Washington 98195, USA

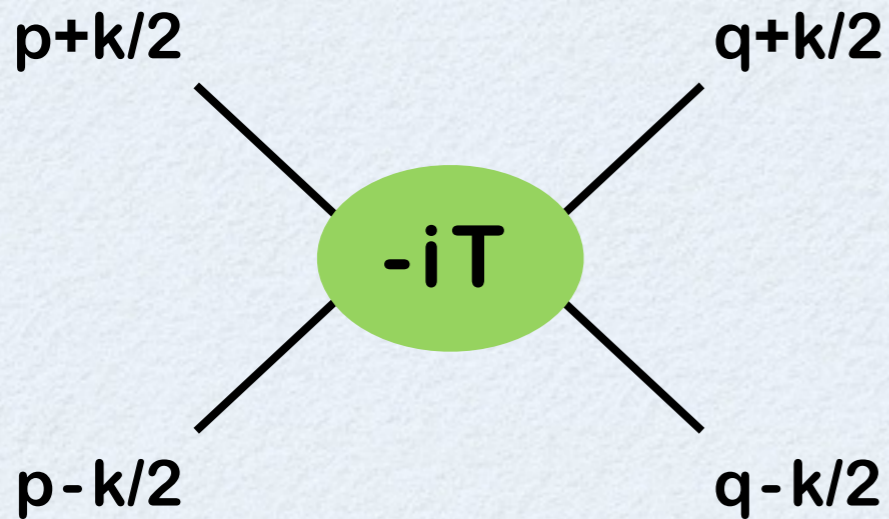
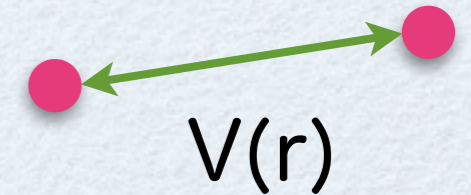
³Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA

(Received 18 January 2013; published 4 June 2013)



P-wave scattering in 2D

Two fermions with short-range potential



⇒ Effective range expansion

Cf. H.-W. Hammer & D. Lee
Ann. Phys. 325, 2212 (2010)

$$-iT = \frac{2i}{m} \frac{\vec{p} \cdot \vec{q}}{-\frac{1}{a} - \frac{m\varepsilon}{\pi} \ln\left(-\frac{\Lambda^2}{m\varepsilon}\right) + \sum_{n=2}^{\infty} c_n (m\varepsilon)^n}$$

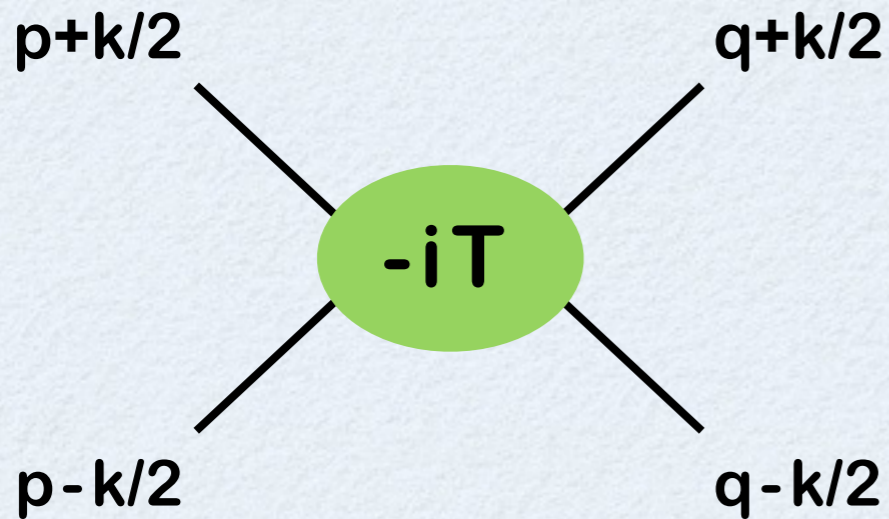
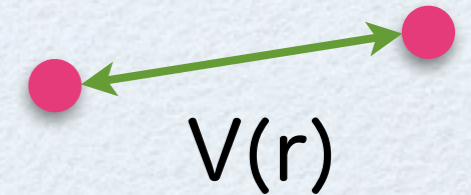
↑ scattering “length”

↑ effective “range”

collision energy $\varepsilon = E - \frac{k^2}{4m} + i0^+$

P-wave scattering in 2D

Two fermions with short-range potential



⇒ Effective range expansion

Cf. H.-W. Hammer & D. Lee
Ann. Phys. 325, 2212 (2010)

$$-iT = \frac{2i}{m} \frac{\vec{p} \cdot \vec{q}}{-\frac{1}{a} - \frac{m\varepsilon}{\pi} \ln\left(-\frac{\Lambda^2}{m\varepsilon}\right) + \sum_{n=2}^{\infty} c_n (m\varepsilon)^n}$$

resonance

($a \rightarrow \infty$)

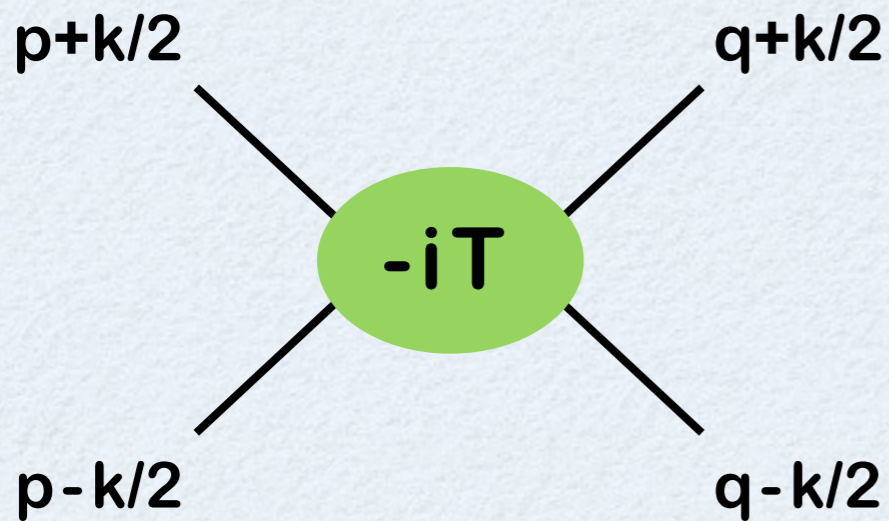
low-energy

($\varepsilon \rightarrow 0$)

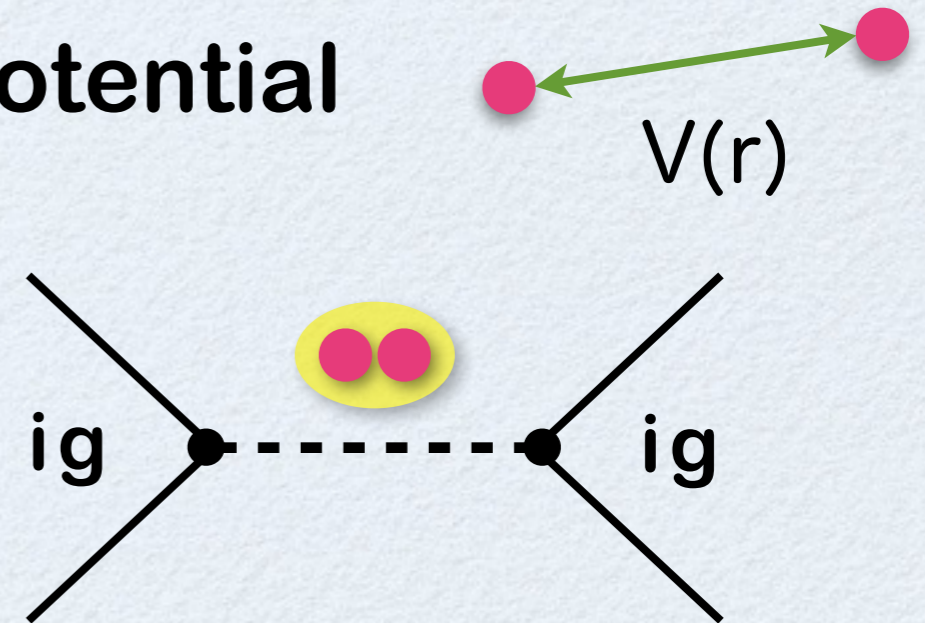
collision energy $\varepsilon = E - \frac{k^2}{4m} + i0^+$

P-wave scattering in 2D

Two fermions with short-range potential



resonance
 \rightarrow
 low-energy



\Rightarrow Effective range expansion

$$-iT \rightarrow \underbrace{-\frac{2\pi \vec{p} \cdot \vec{q}}{m^2 \ln\left(-\frac{\Lambda^2}{m\varepsilon}\right)}}_{(ig)^2 p \cdot q} \times \underbrace{\frac{i}{E - \frac{k^2}{4m} + i0^+}}_{\text{propagator of dimer}}$$

$= (ig)^2 p \cdot q$

propagator of dimer

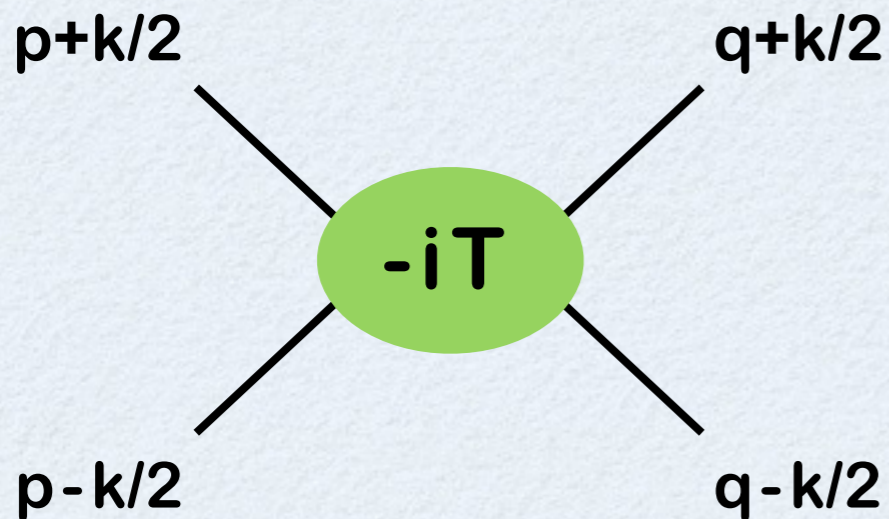


“running” coupling

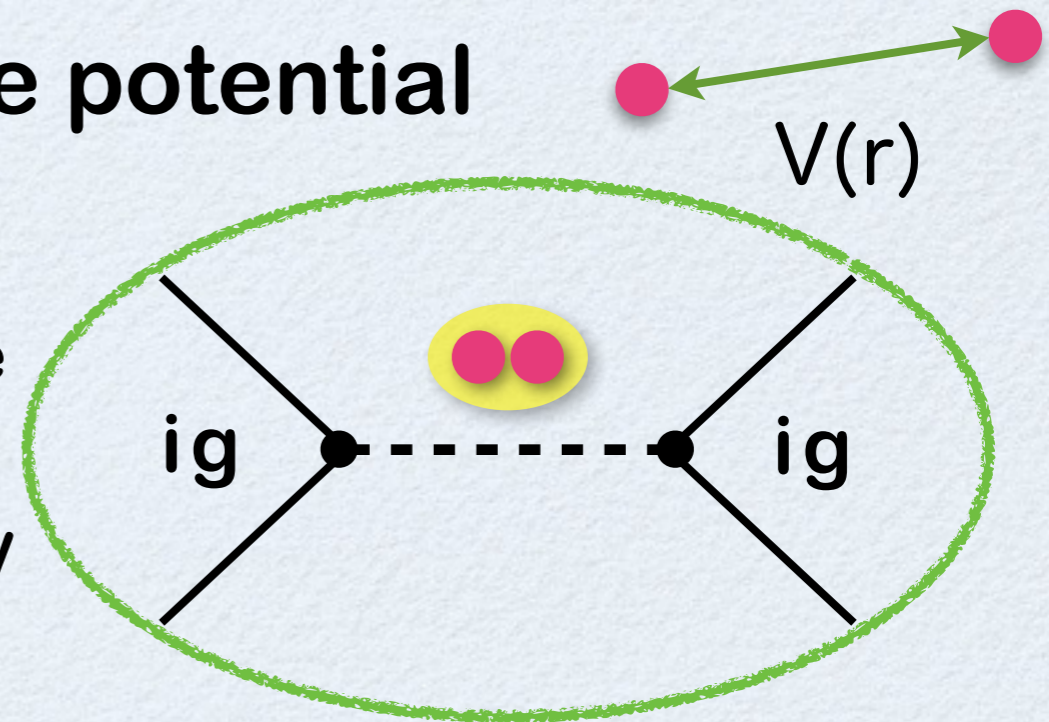
(logarithmic decrease toward low-energy $p/\Lambda \rightarrow 0$)

P-wave scattering in 2D

Two fermions with short-range potential



resonance
→
low-energy



⇒ Low-energy effective field theory

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + \sum_{\pm} \left[\phi_{\pm}^\dagger \left(i\partial_t + \frac{\nabla^2}{4m} \right) \phi_{\pm} + g \phi_{\pm}^\dagger \psi (-i) (\nabla_x \pm i\nabla_y) \psi + \text{h. c.} \right]$$

dimer field ϕ_{\pm} couples to two fermions ψ

with orbital angular momentum $L=\pm 1$

RG in 2-body sector

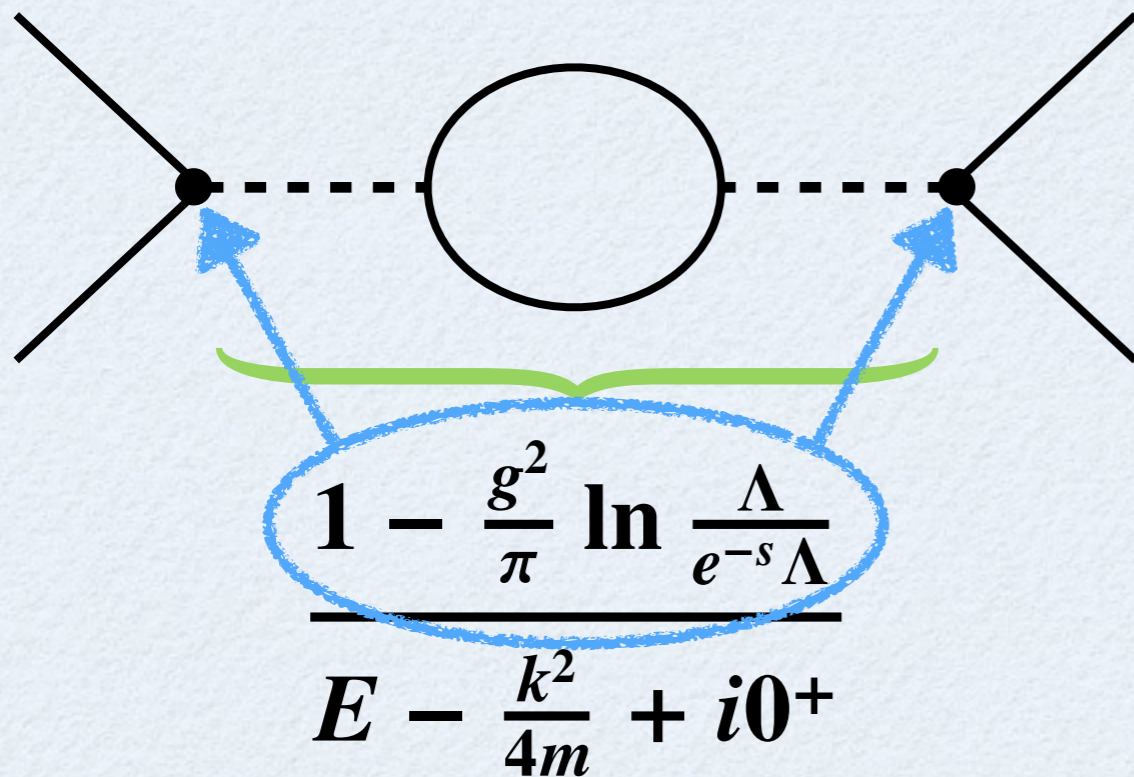
Low-energy effective field theory

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + \sum_{\pm} \left[\phi_{\pm}^\dagger \left(i\partial_t + \frac{\nabla^2}{4m} \right) \phi_{\pm} \right.$$

$$\left. + g \phi_{\pm}^\dagger \psi (-i) (\nabla_x \pm i\nabla_y) \psi + \text{h. c.} \right] + \dots$$

marginal coupling

irrelevant



($e^{-s}\Lambda < p < \Lambda$ integrated out)

RG equation $\frac{dg}{ds} = -\frac{g^3}{2\pi}$

$$\Rightarrow g^2(s) = \frac{1}{\frac{1}{g^2(0)} + \frac{s}{\pi}} \rightarrow \frac{\pi}{s}$$

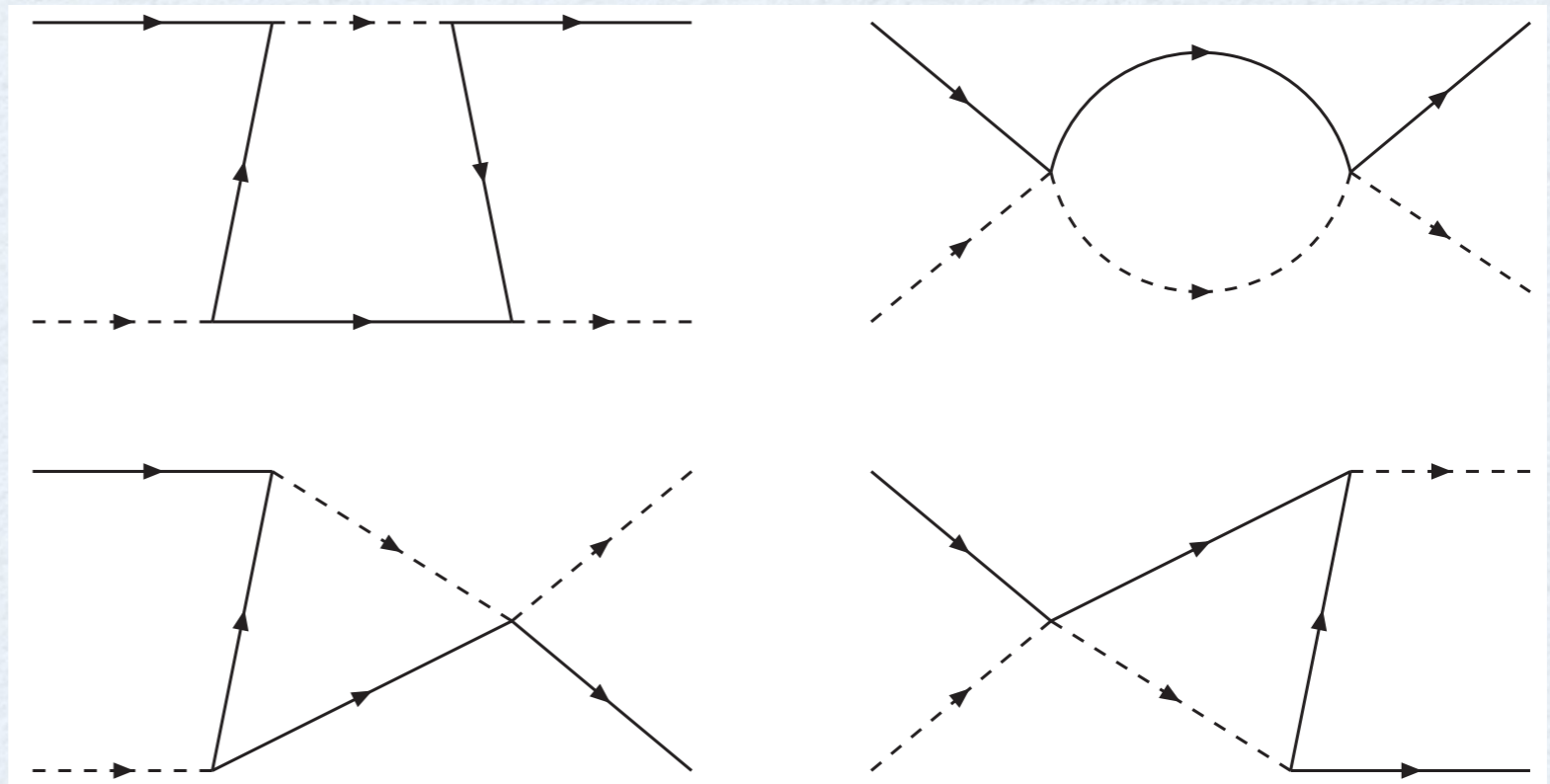
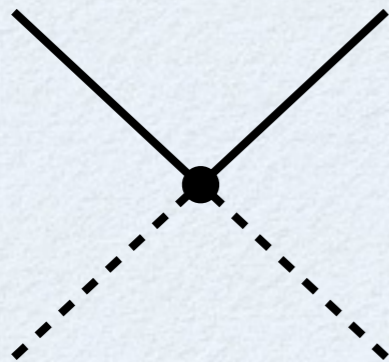
logarithmical decrease
toward low-energy $s \rightarrow \infty$

RG in 3-body sector

3-body problem \Leftrightarrow fermion+dimer scattering

$$\mathcal{L}_{3\text{-body}} = \underbrace{v_3}_{\text{marginal coupling}} \sum_{a=\pm} \psi^\dagger \phi_a^\dagger \phi_a \psi + \underbrace{\dots}_{\text{irrelevant}}$$

marginal coupling renormalized by



\Rightarrow RG equation

$$\frac{dv_3}{ds} = \frac{16}{3\pi} g^4 - \frac{11}{3\pi} g^2 v_3 + \frac{2}{3\pi} v_3^2$$

RG in 3-body sector

3-body problem \Leftrightarrow fermion+dimer scattering

$$\mathcal{L}_{3\text{-body}} = v_3 \sum_{a=\pm} \psi^\dagger \phi_a^\dagger \phi_a \psi + \dots$$

irrelevant

marginal coupling @ low-energy limit $s \rightarrow \infty$

$$v_3(s) \rightarrow \frac{2\pi}{s} \left\{ 1 - \cot \left[\frac{4}{3} (\ln s - \theta) \right] \right\}$$

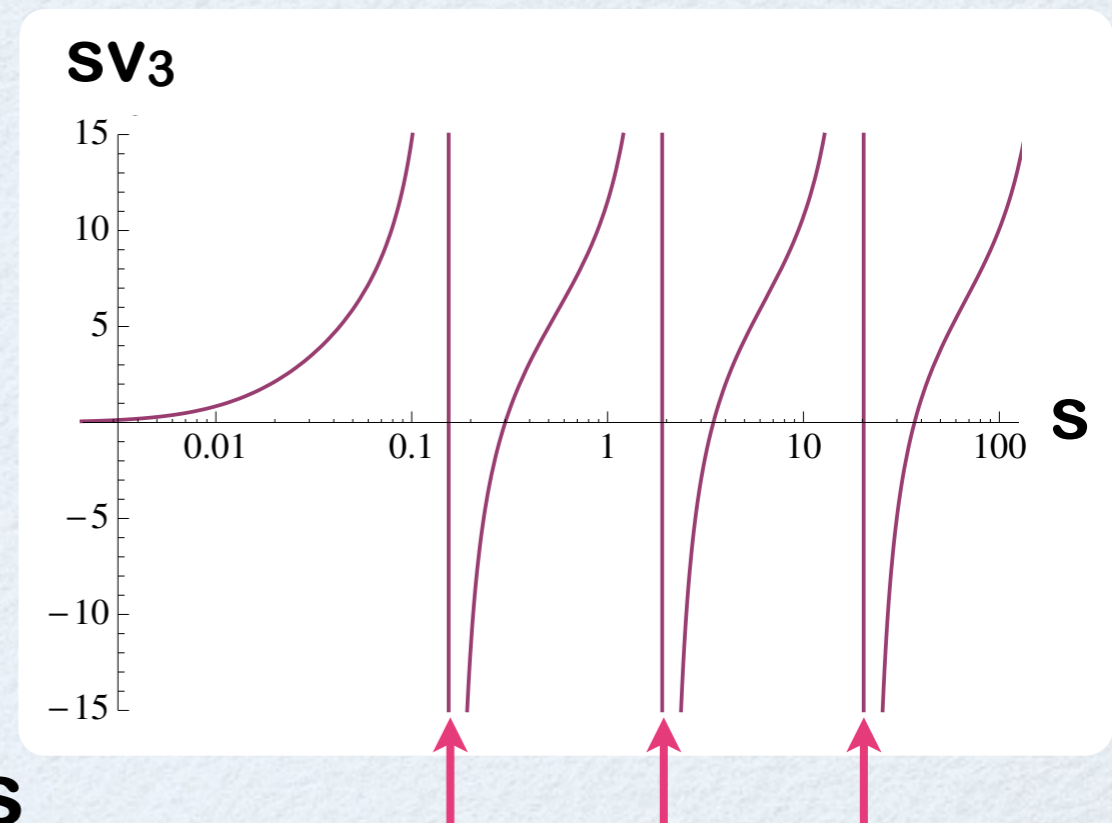
non-universal

diverges at $\ln s = \frac{3\pi n}{4} + \theta$

\Rightarrow characteristic energy scales

$$E_n \propto \frac{\Lambda^2}{m} e^{-2e^{3\pi n/4 + \theta}}$$

Super Efimov effect !



Confirmation by model

$$H = \int \frac{dk}{(2\pi)^2} \frac{k^2}{2m} \psi_k^\dagger \psi_k$$

Spinless fermions
with a separable potential

$$-v_0 \sum_{a=\pm} \int \frac{dk dp dq}{(2\pi)^6} \underbrace{\psi_{\frac{k}{2}+p}^\dagger \chi_a(p) \psi_{\frac{k}{2}-p}^\dagger}_{\chi_{\pm}(p)} \times \underbrace{\psi_{\frac{k}{2}-q} \chi_{-a}(q) \psi_{\frac{k}{2}+q}}_{\chi_{\pm}(q)}$$

↑
resonance ($a \rightarrow \infty$)

$$\chi_{\pm}(p) = (p_x \pm ip_y) e^{-p^2/(2\Lambda^2)}$$

3-body binding energies $\lambda_n = \ln \ln (mE_n/\Lambda^2)^{-1/2}$

\Rightarrow solve STM equation numerically

$$Z_a(p) = - \int \frac{dq}{2\pi} \frac{(p+2q)_{-a} e^{-(5p^2+5q^2+8p\cdot q)/(8\Lambda^2)}}{p^2+q^2+p\cdot q+\kappa^2} \times \frac{\sum_{b=\pm} (2p+q)_b Z_b(q)}{(\frac{3}{4}q^2+\kappa^2) e^{(\frac{3}{4}q^2+\kappa^2)/\Lambda^2} \mathbf{E}_1[(\frac{3}{4}q^2+\kappa^2)/\Lambda^2]}$$

Confirmation by model

$$H = \int \frac{dk}{(2\pi)^2} \frac{k^2}{2m} \psi_k^\dagger \psi_k$$

Spinless fermions
with a separable potential

$$-v_0 \sum_{a=\pm} \int \frac{dk dp dq}{(2\pi)^6} \underbrace{\psi_{\frac{k}{2}+p}^\dagger \chi_a(p) \psi_{\frac{k}{2}-p}^\dagger}_{\chi_\pm(p)} \times \underbrace{\psi_{\frac{k}{2}-q} \chi_{-a}(q) \psi_{\frac{k}{2}+q}}_{\chi_\pm(q)}$$

resonance ($a \rightarrow \infty$)

$$\chi_\pm(p) = (p_x \pm ip_y) e^{-p^2/(2\Lambda^2)}$$

3-body binding energies $\lambda_n = \ln \ln (mE_n / \Lambda^2)^{-1/2}$

n	λ_n	$\lambda_n - \lambda_{n-1}$			
			3	7.430	2.352
0	0.5632	—	4	9.785	2.355
1	2.770	2.207	5	12.141	2.356
2	5.078	2.308	∞	—	2.35619 $\leftarrow 3\pi/4$

Super Efimov effect!

\Rightarrow doubly exponential scaling $mE_n / \Lambda^2 \propto e^{-2e^{3\pi n/4 + \theta}}$

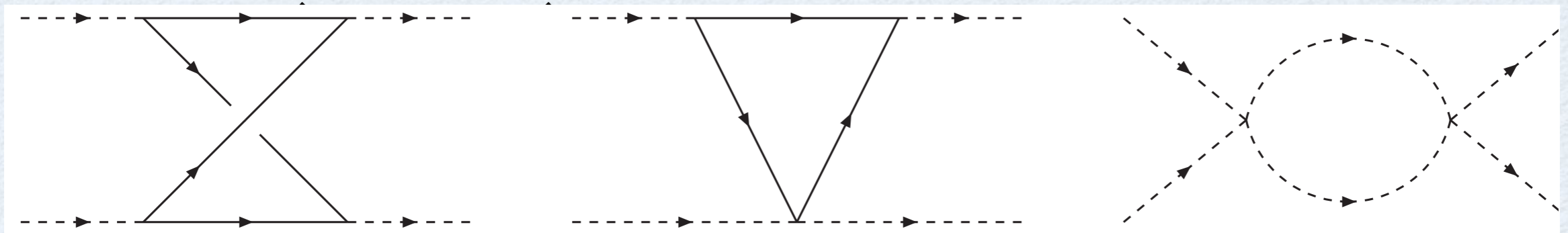
RG in 4-body sector

4-body problem \Leftrightarrow dimer+dimer scattering

$$\mathcal{L}_{4\text{-body}} = \sum_{a=\pm} \left[v_4 \phi_a^\dagger \phi_{-a}^\dagger \phi_{-a} \phi_a + v'_4 \phi_a^\dagger \phi_a^\dagger \phi_a \phi_a \right] + \dots$$

irrelevant

marginal couplings renormalized by



\Rightarrow RG equations

$$\frac{dv_4}{ds} = -\frac{8}{\pi} g^4 + \frac{2}{\pi} g^2 v_3 - \frac{2}{\pi} g^2 v_4 + \frac{2}{\pi} v_4^2$$

$$\frac{dv'_4}{ds} = -\frac{4}{\pi} g^4 + \frac{2}{\pi} g^2 v_3 - \frac{2}{\pi} g^2 v'_4 + \frac{2}{\pi} v_4'^2$$

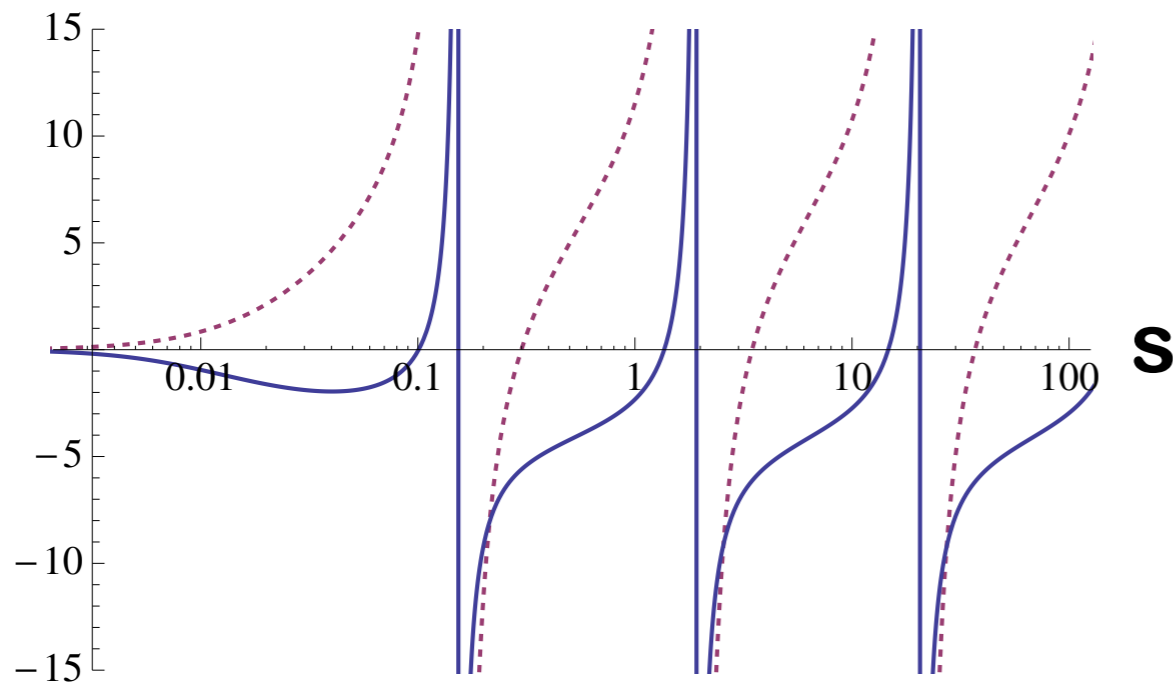
RG in 4-body sector

4-body problem \Leftrightarrow dimer+dimer scattering

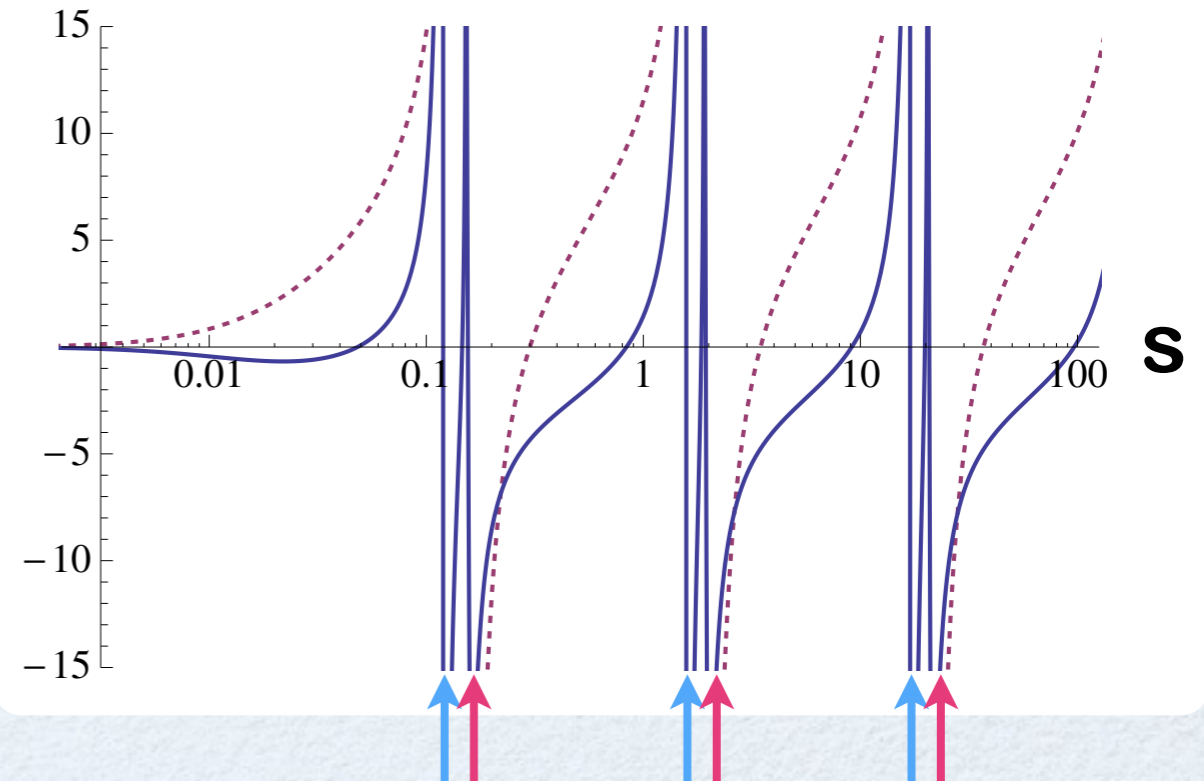
$$\mathcal{L}_{4\text{-body}} = \sum_{a=\pm} \left[v_4 \phi_a^\dagger \phi_{-a}^\dagger \phi_{-a} \phi_a + v'_4 \phi_a^\dagger \phi_a^\dagger \phi_a \phi_a \right] + \dots$$

↑ }
marginal couplings irrelevant

SV₄



SV'₄



L=±2 tetramers attached to every **trimer**

with resonance energy $E_n \sim e^{-2e^{3\pi n/4+\theta-0.188}}$

Efimov vs super Efimov

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance



exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance



“doubly” exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

PRL 110, 235301 (2013)

PHYSICAL REVIEW LETTERS

week ending
7 JUNE 2013



Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

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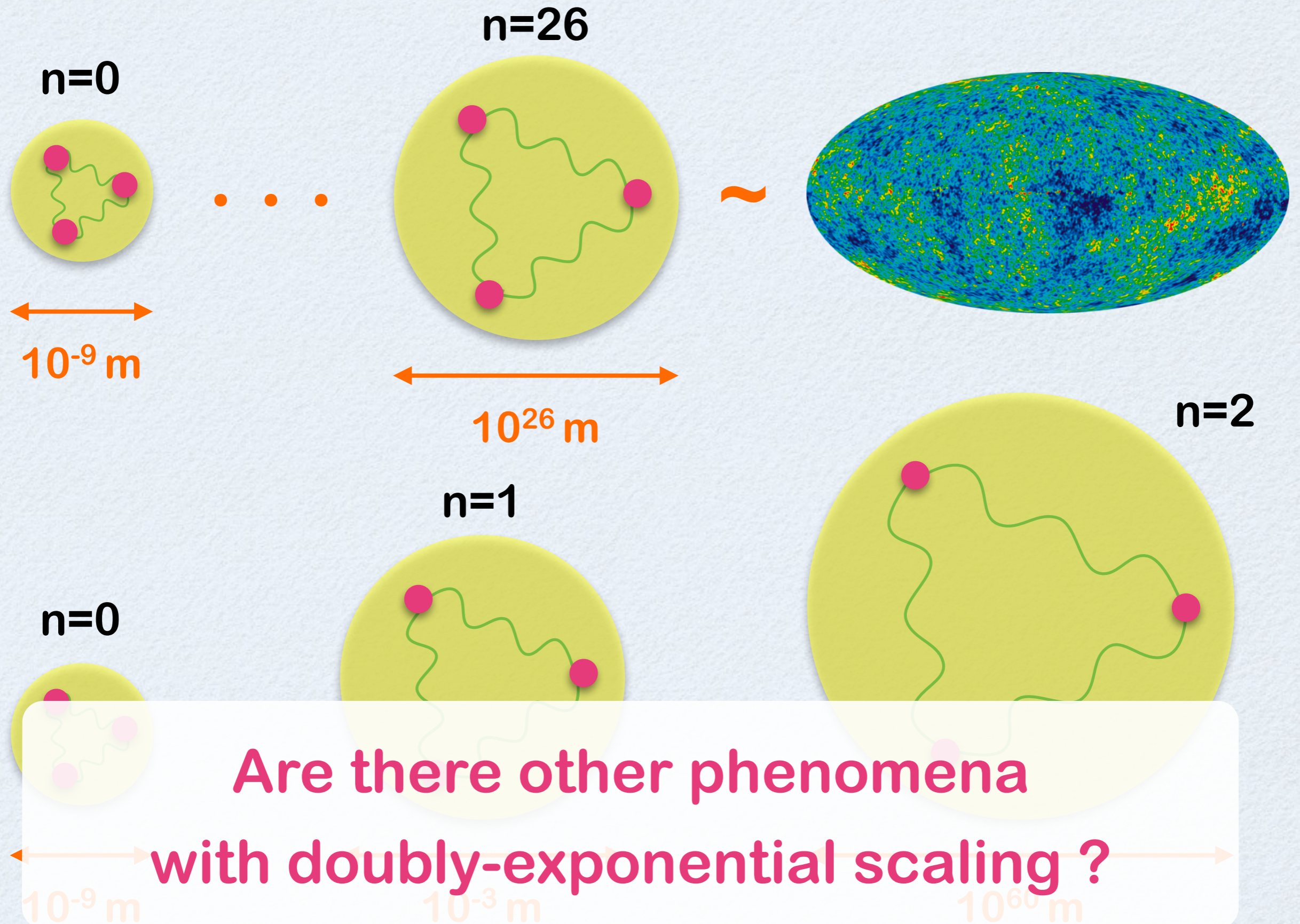
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Efimov vs super Efimov



The image shows two overlapping web pages. The top page is a Wikipedia article titled "Hyperinflation". It includes a globe icon with various characters, the Wikipedia logo, and a navigation menu. The article text discusses hyperinflation in economics, mentioning that it occurs when a country experiences very high and usually accelerating rates of monetary and price inflation. A black and white photograph shows a street scene with a man sweeping up a large amount of discarded banknotes. The bottom page is an arXiv preprint titled "The mechanism of double exponential growth in hyper-inflation" by Takayuki Mizuno, Misako Takayasu, and Hideki Takayasu. The title "double exponential growth" is circled in green. The preprint abstract discusses analyzing historical data of price indices and introducing the general coarse-graining technique in physics, the Monte Carlo renormalization group method, to the price dynamics. A large pink text overlay asks: "Are there other 'physics' phenomena with doubly-exponential scaling?".

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Hyperinflation

From Wikipedia, the free encyclopedia

For lungs filling with excessive air, see Hyperaeration.

Certain figures in this article use scientific notation for readability.

In economics, **hyperinflation** occurs when a country experiences very high and usually accelerating rates of monetary and price inflation, causing the population to minimize their holdings of money. Under such conditions, the general price level within an economy increases rapidly as the official currency quickly loses real value.^[1] Meanwhile, the real value of economic items generally stay the same with respect to one another, and remain

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The mechanism of double exponential growth in hyper-inflation

Takayuki Mizuno, Misako Takayasu, Hideki Takayasu

(Submitted on 24 Dec 2001)

Analyzing historical data of price indices we find an extraordinary growth phenomenon in several examples of hyper-inflation in which prices change as a super-exponential function. In order to explain such behavior we introduce the general coarse-graining technique in physics, the Monte Carlo renormalization group method, to the price dynamics. Starting from a microscopic stochastic equation describing dealer's actions in open markets we obtain a macroscopic noiseless equation of price consistent with the observed behavior. The emergence of the super-exponential growth is shown to be responsible for the double-exponential behavior.

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Are there other "physics" phenomena with doubly-exponential scaling?

Efimov effect: universality, discrete scale invariance, RG limit cycle

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✓ **Efimov effect in quantum magnets**

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ **Novel universality: Super Efimov effect**

Y.N, S.M, D.T.S, Phys Rev Lett 110, 235301 (2013)