Microscopic origin and universality classes of the Efimov three-body parameter

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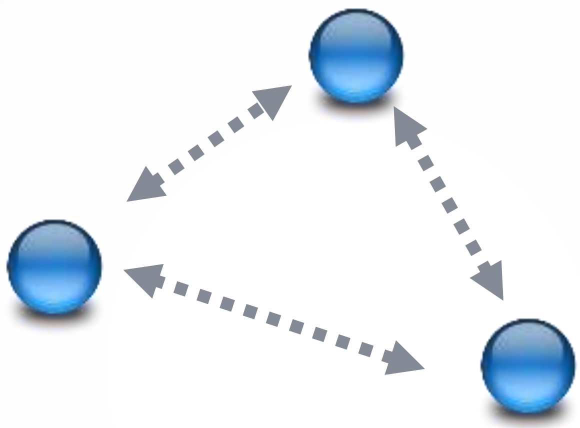
3 particles (bosons) with resonant two-body interactions



- Single-channel two-body interactions
- No three-body interaction

Summary

The **3-body parameter** is (mostly) determined by the **2-body correlation**.

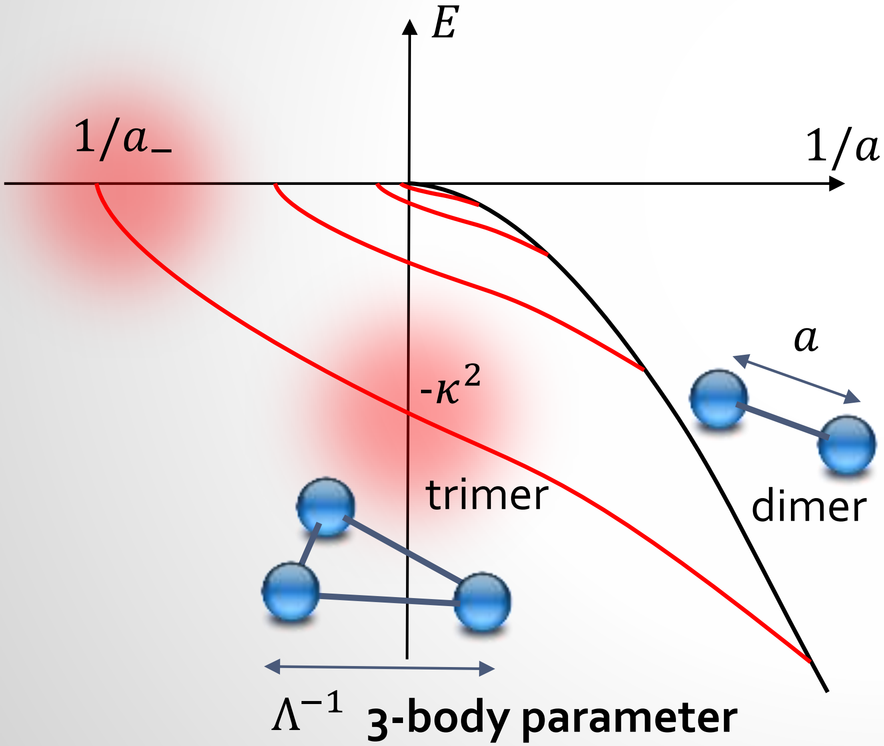
Reason:
2-body correlation induces a **deformation** of the 3-body system.

Consequences: the 3-body parameter
- is on the order of the **effective range**.
- has different universal values for **distinct classes** of interaction

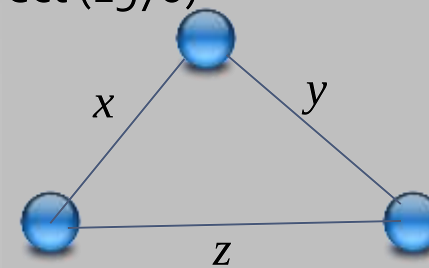
The Efimov 3-body parameter

Parameters describing particles at low energy

Scattering length *a* (2-body parameter)

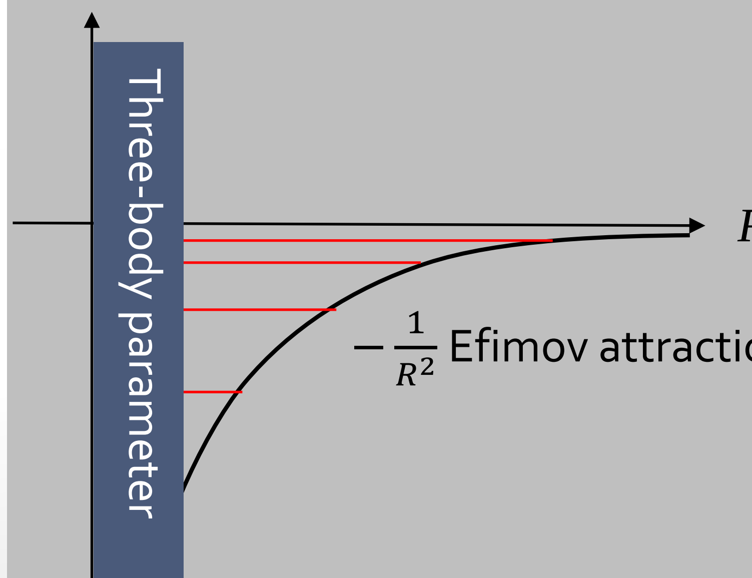


The Efimov effect (1970)



Hyperradius $R^2 = \frac{2}{3}(x^2 + y^2 + z^2)$

Zero-range condition with $a \to \infty$



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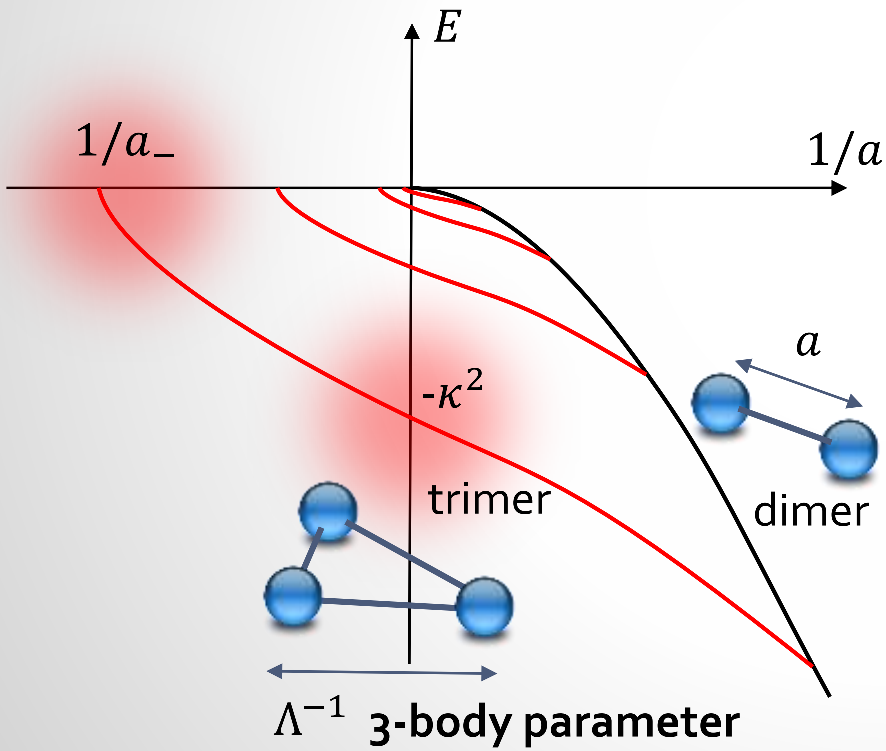
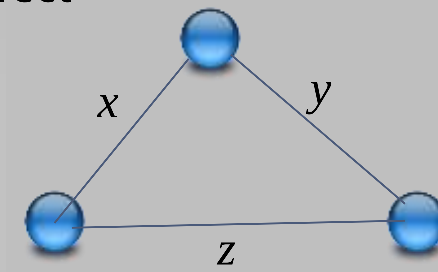


The Efimov effect



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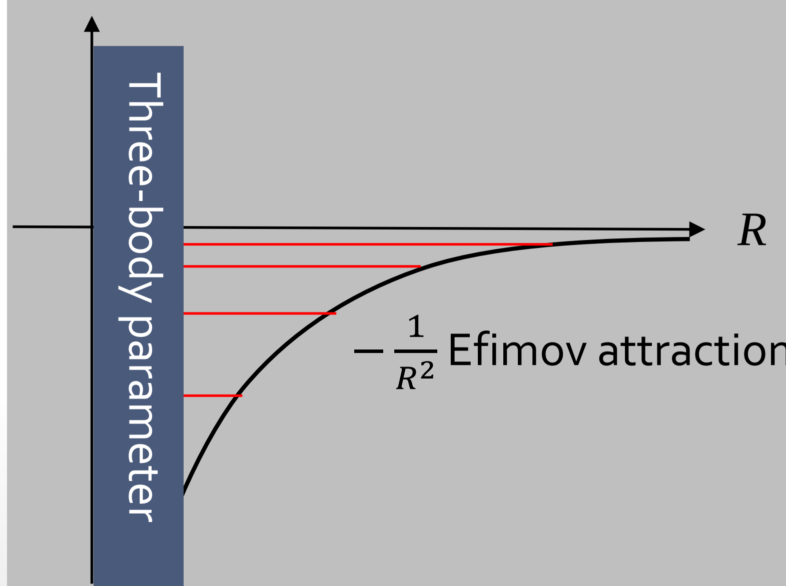
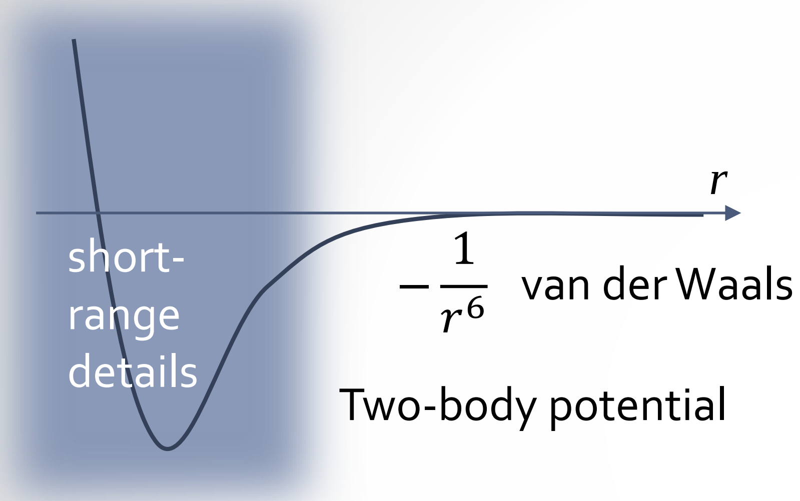
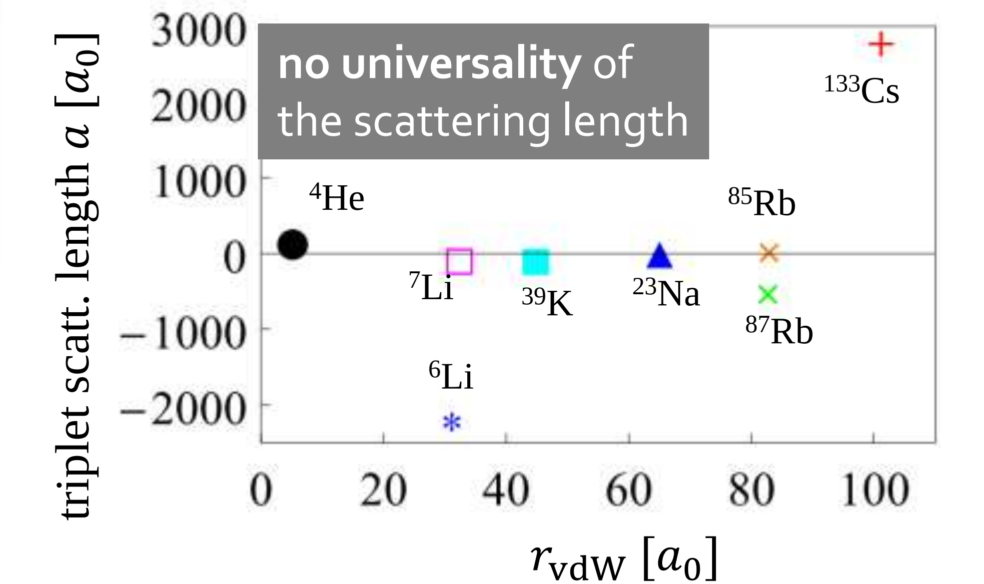
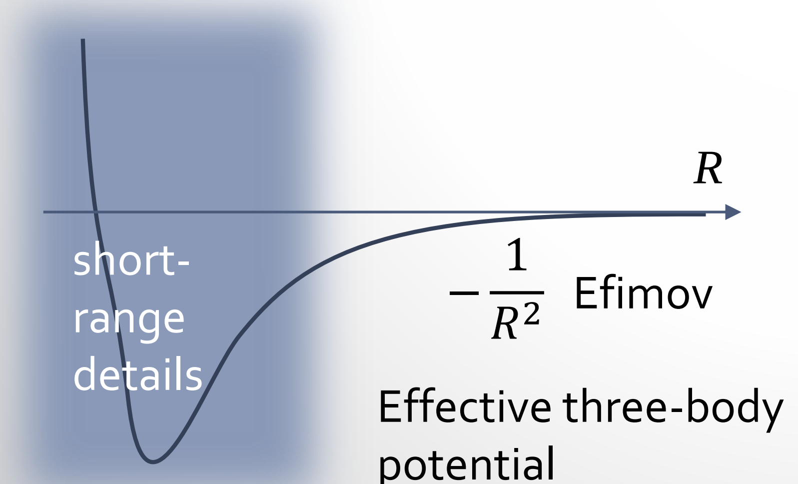


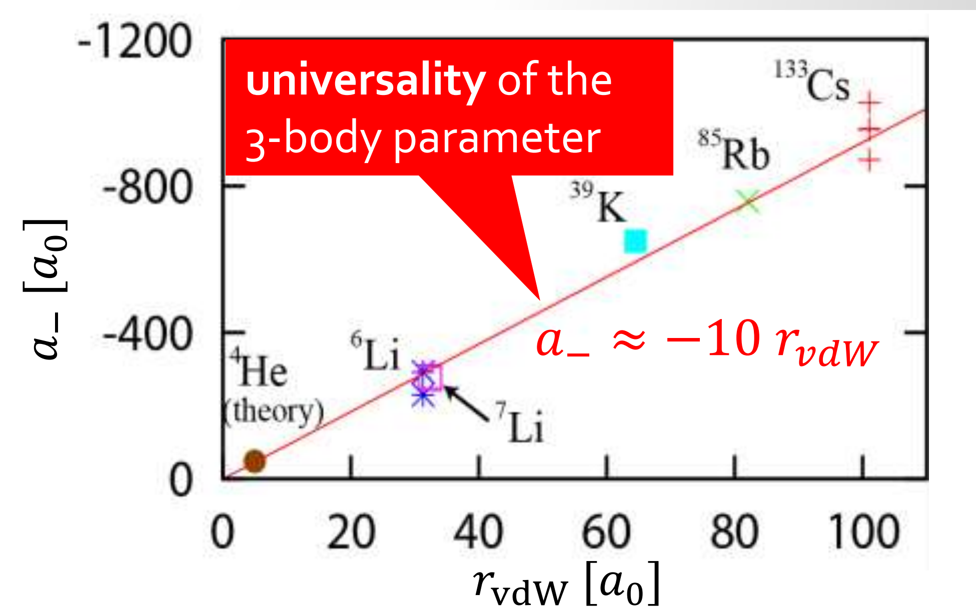
Universality for atoms

Microscopic determination?





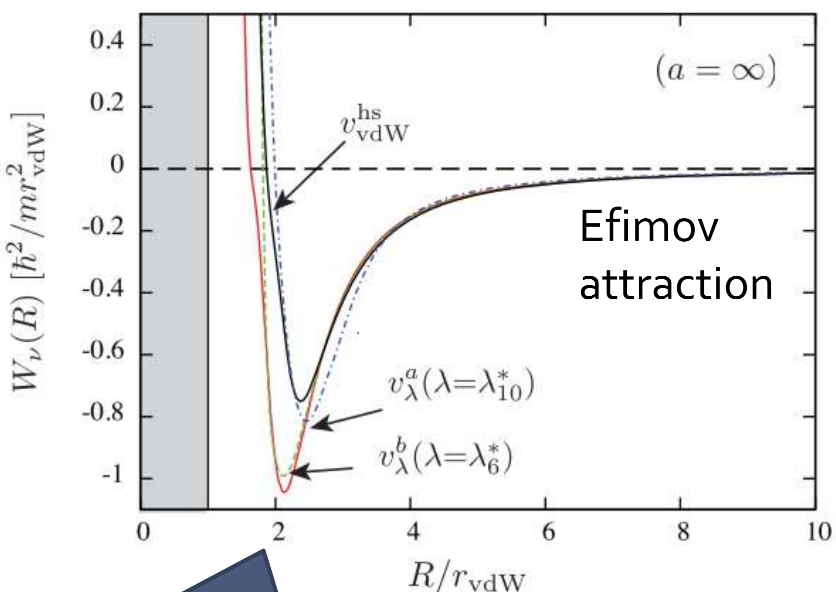


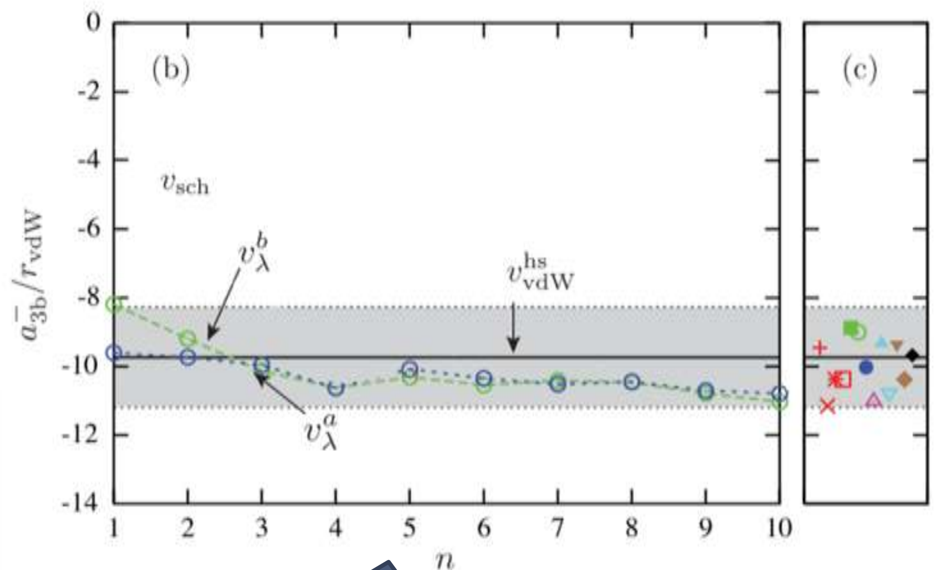


Three-body with van der Waals interactions

Phys. Rev. Lett. 108 263001 (2012)
J. Wang, J. D'Incao, B. Esry, C. Greene



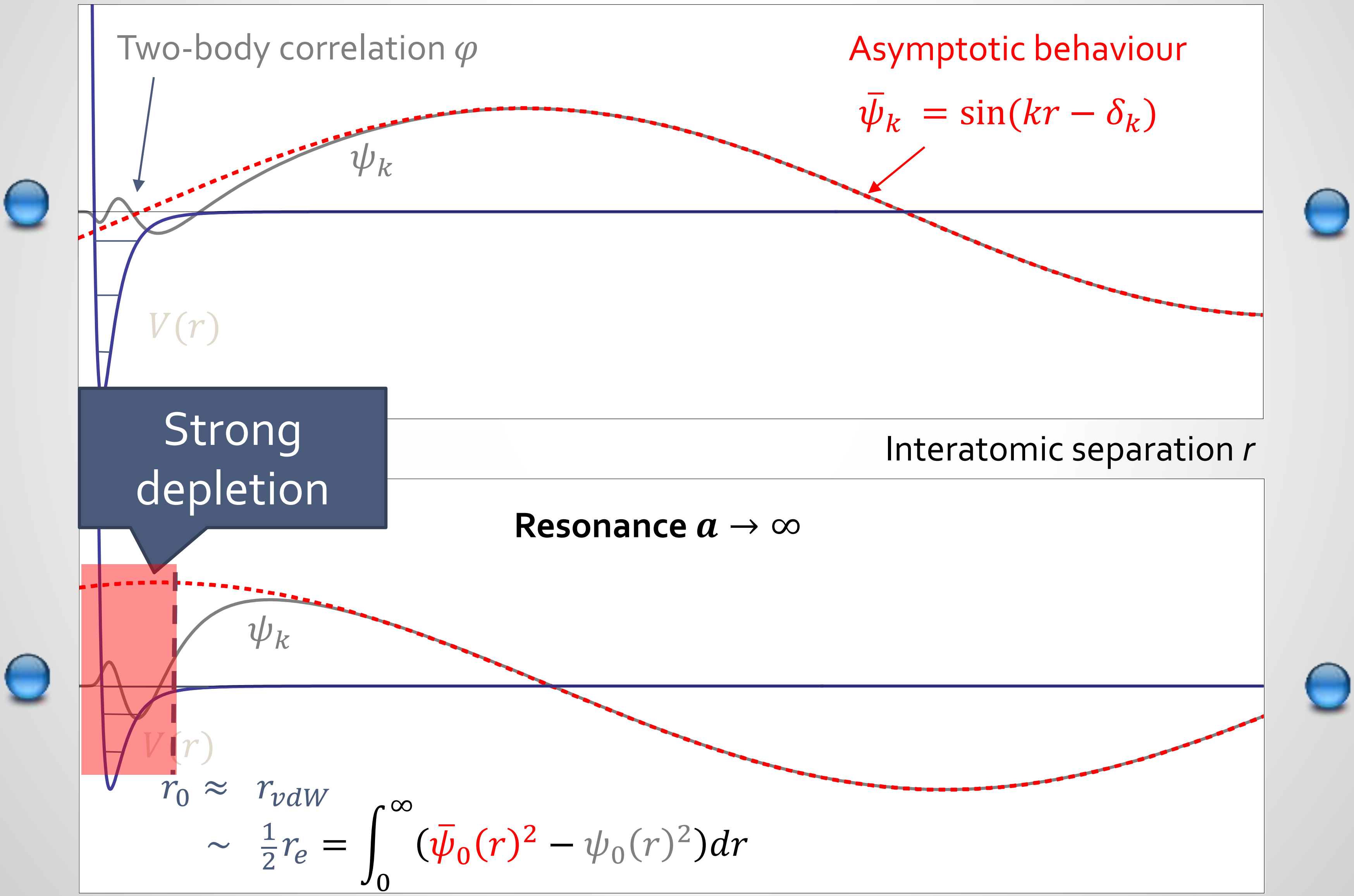


Three-body repulsion at $R \approx 2r_{vdW}$

→

$a_- \approx -11\, r_{vdW}$

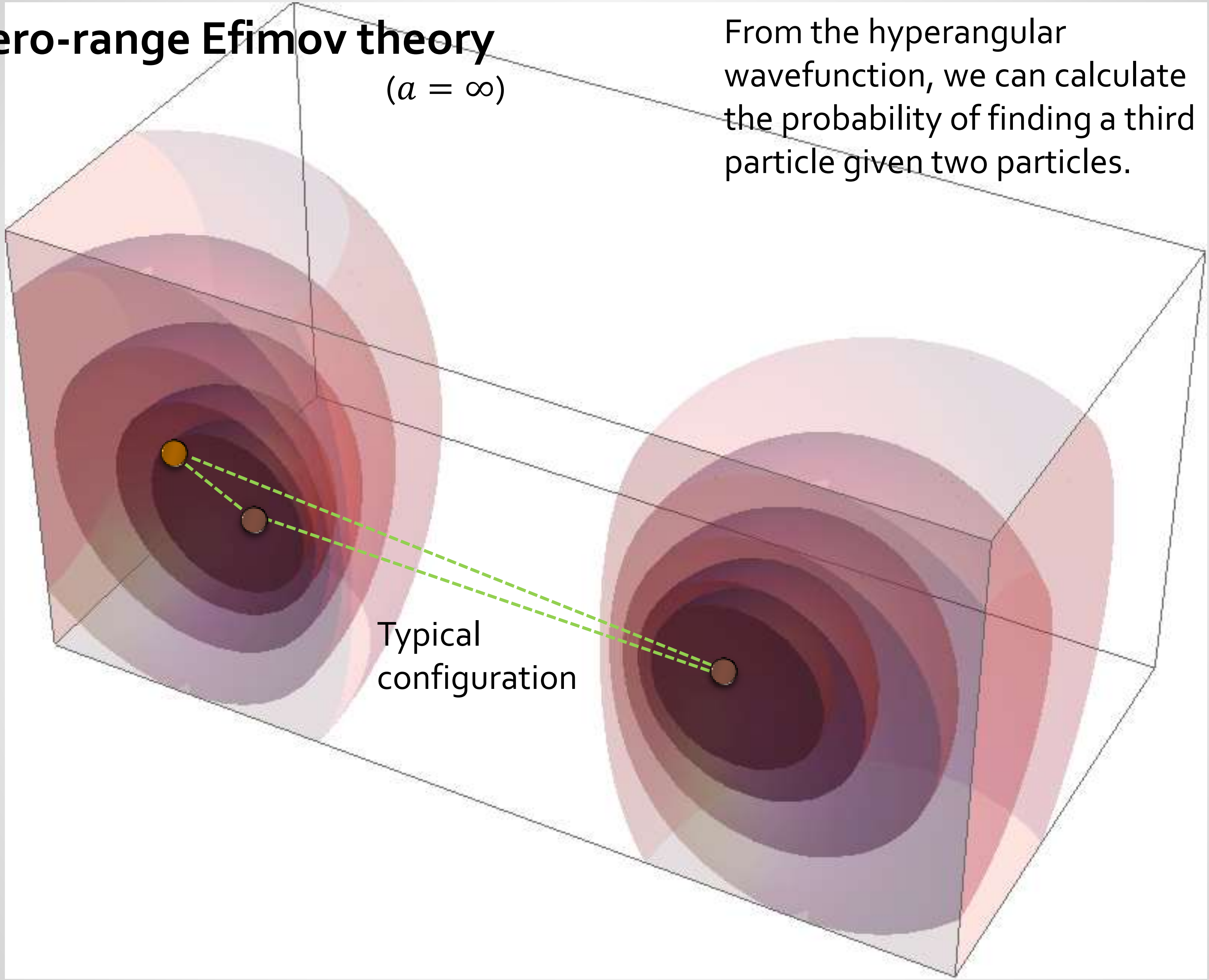
Interpretation: two-body correlation



Zero-range Efimov theory

$(a = \infty)$

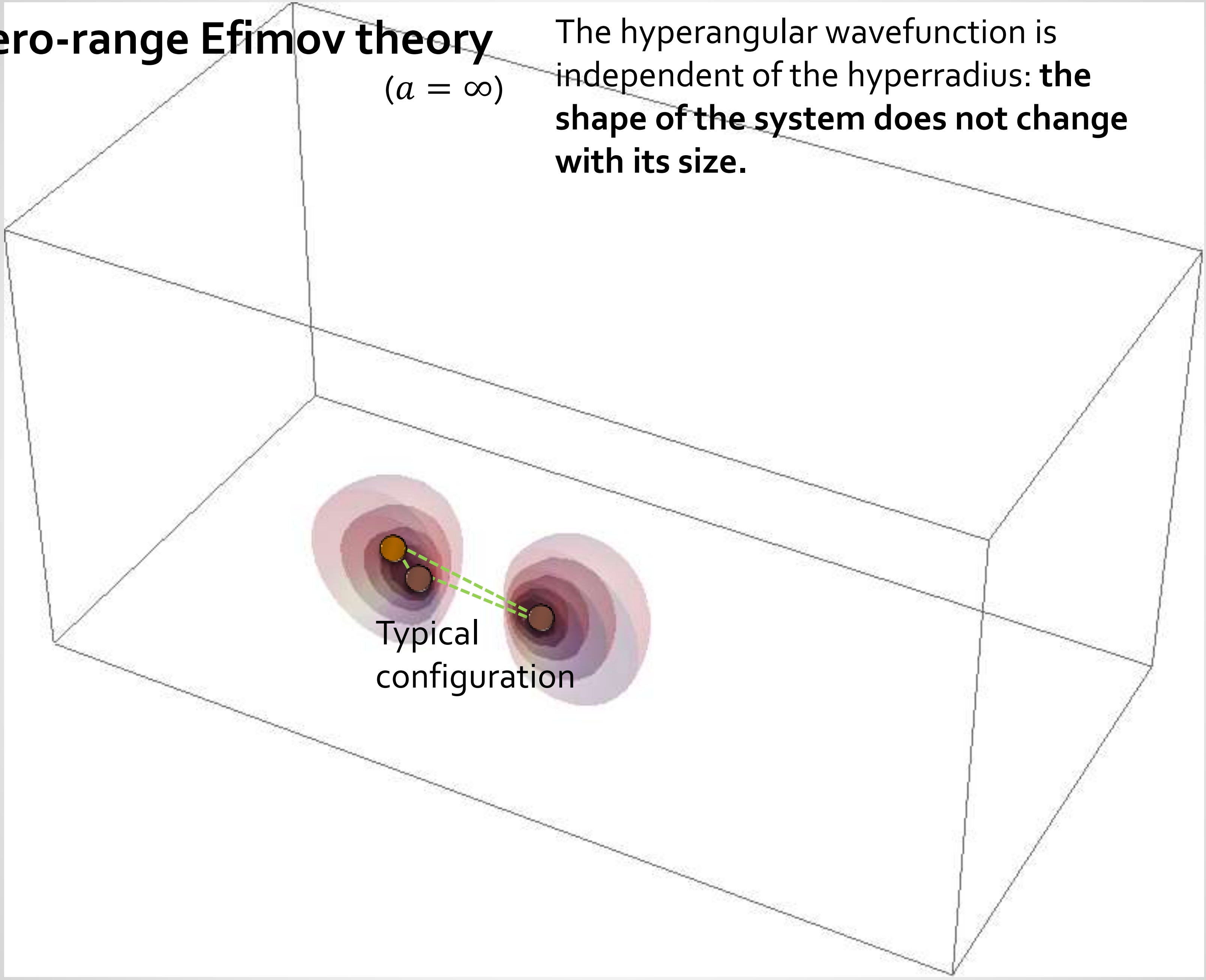
From the hyperangular wavefunction, we can calculate the probability of finding a third particle given two particles.



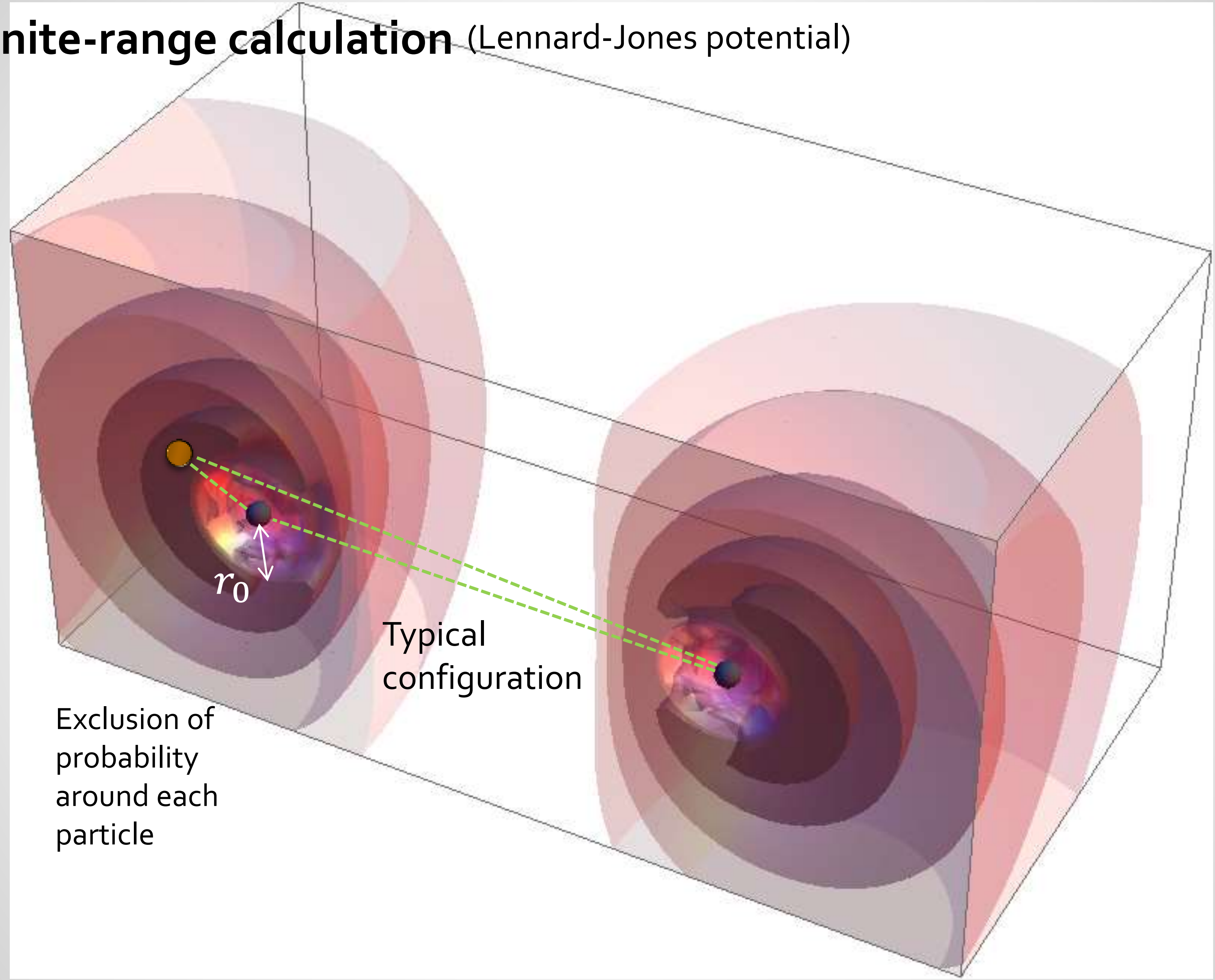
Zero-range Efimov theory

($a = \infty$)

The hyperangular wavefunction is independent of the hyperradius: **the shape of the system does not change with its size.**



Finite-range calculation (Lennard-Jones potential)



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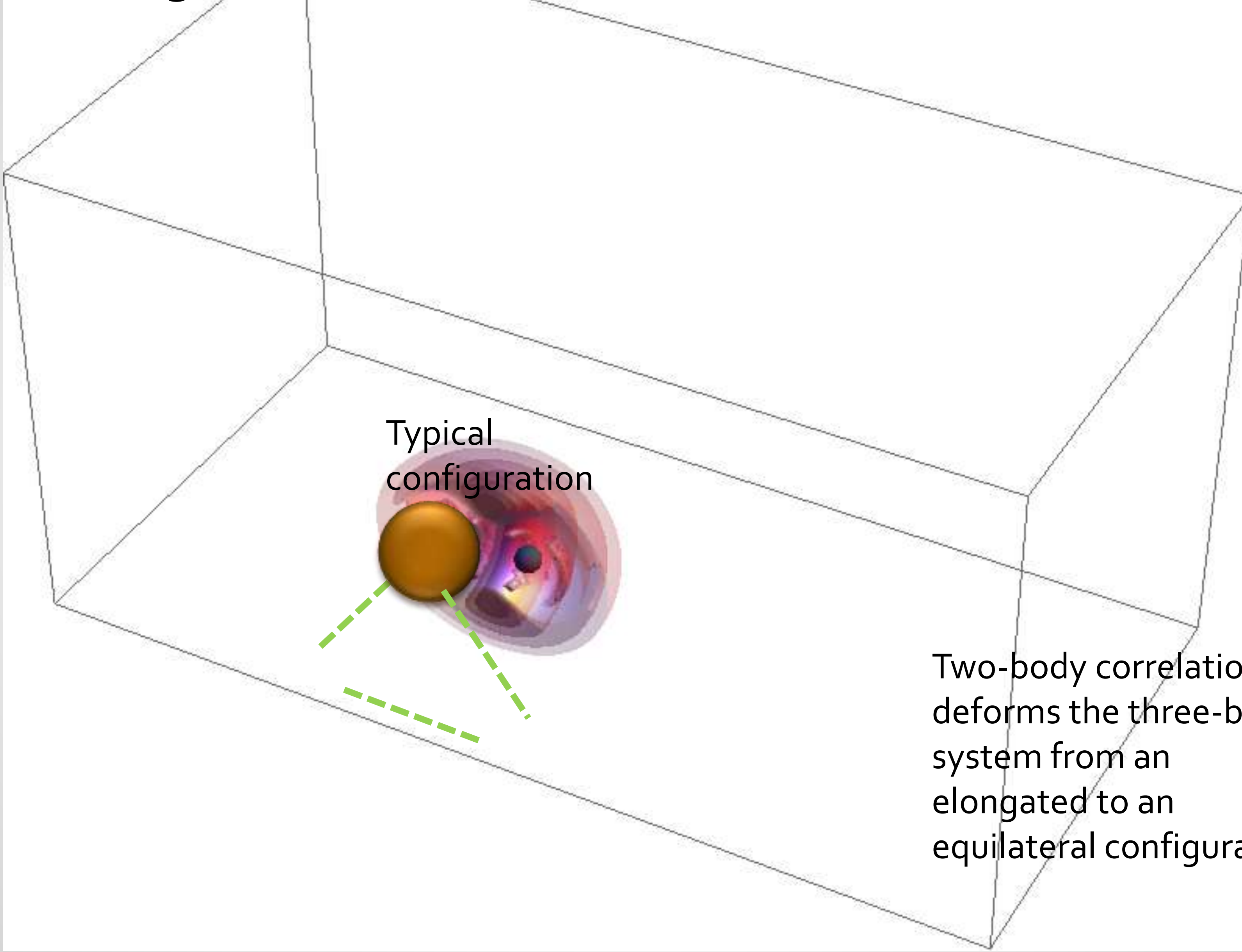
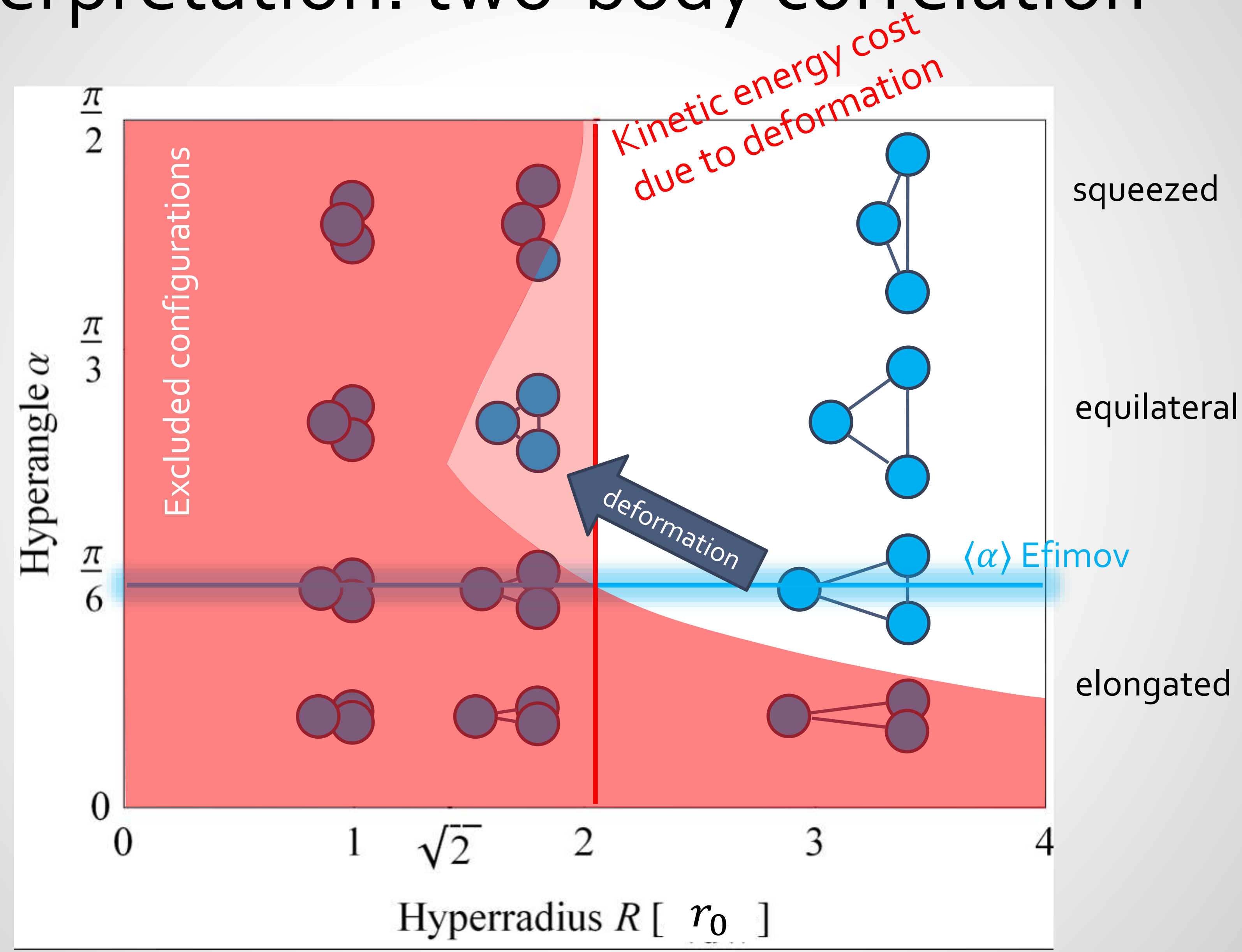


Two-body correlation deforms the three-body system from an elongated to an equilateral configuration

Interpretation: two-body correlation

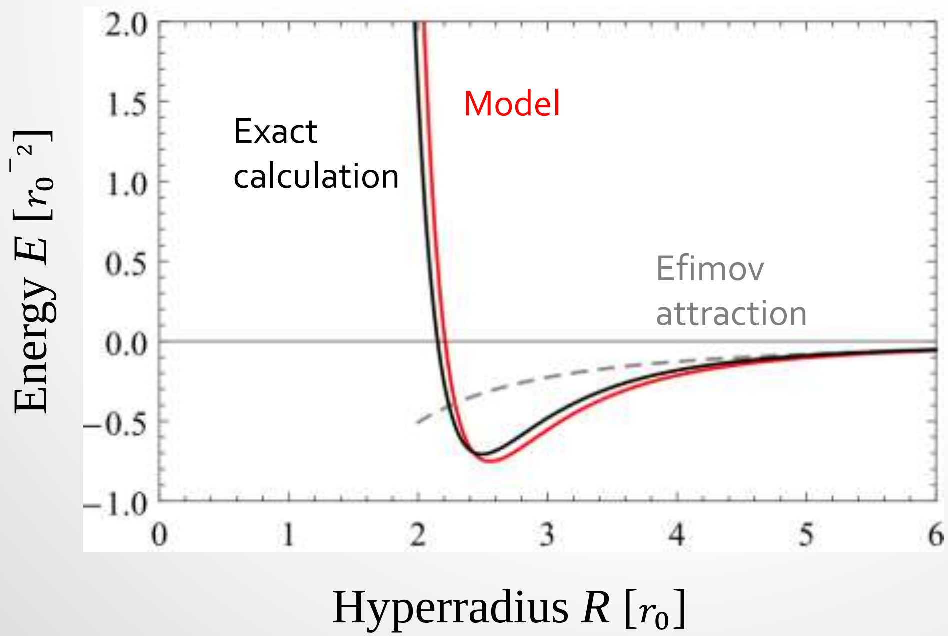


Confirmation 1: pair correlation model

$$\Phi_{\text{Model}} = \Phi_{\text{Efimov}} \times \varphi(r_{12})\,\varphi(r_{23})\,\varphi(r_{31})$$

(hyperangular wave function) (product of pair correlations)

3-body potential $U(R) = \frac{\lambda}{R^2} + \int d\cos\theta\, d\alpha \left|\frac{\partial \Phi}{\partial R}\right|^2$



(for a Lennard-Jones interaction with 5 bound states)

Confirmation 2: separable model

$$V = \xi|\chi\rangle\langle\chi|$$

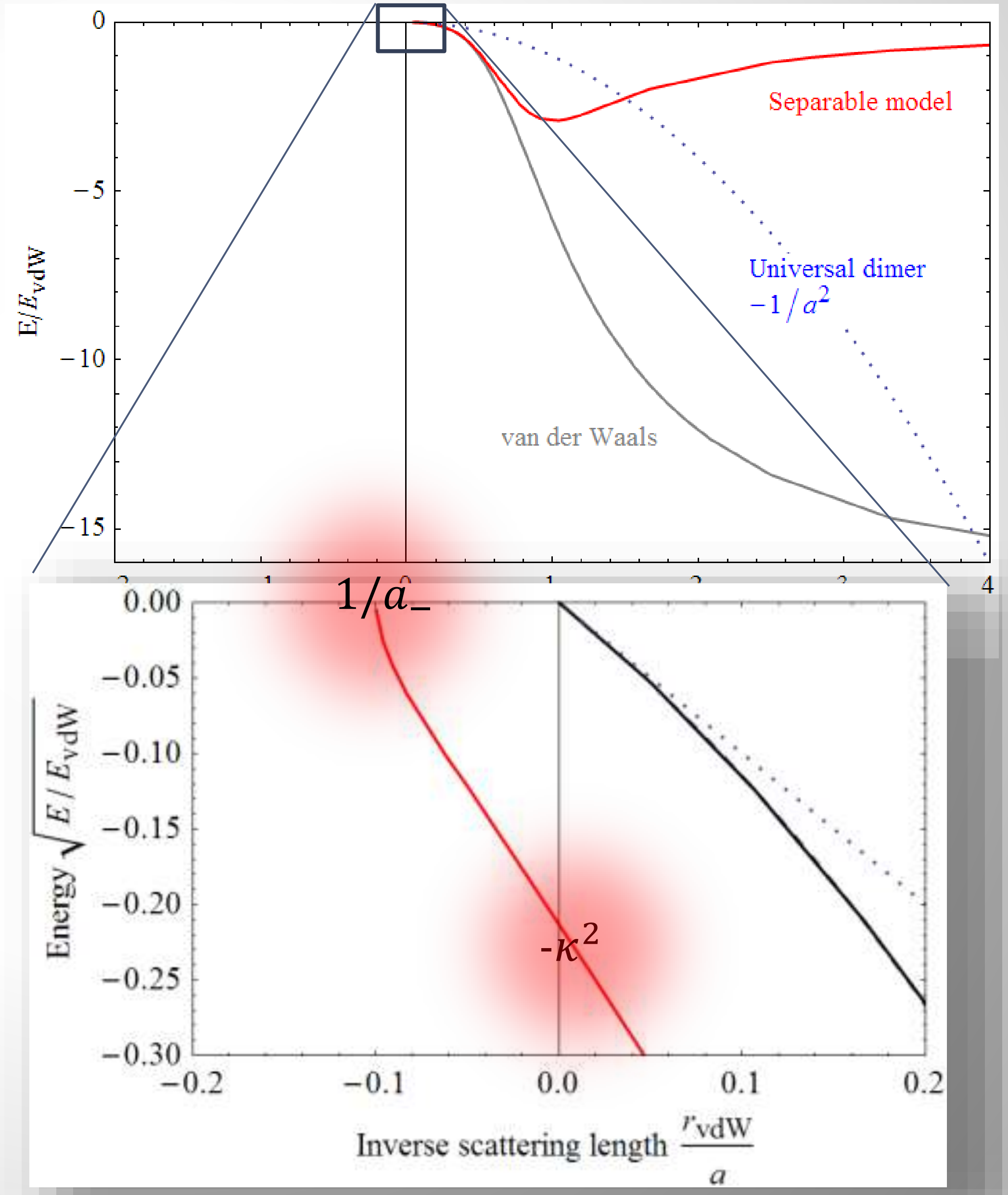
Parameterised to reproduce **exactly** the two-body correlation at zero energy.

$$\chi(q) = 1 - q\int_0^\infty (\bar{\psi}_0(r) - \psi_0(r)) \sin qr\, dr$$

$$\xi = 4\pi\left(\frac{1}{a} - \frac{2}{\pi}\int_0^\infty |\chi(q)|^2 dq\right)^{-1}$$

Reproduces the low-energy 2-body physics very well

- Scattering length
- Effective range
- Last bound state
- ….



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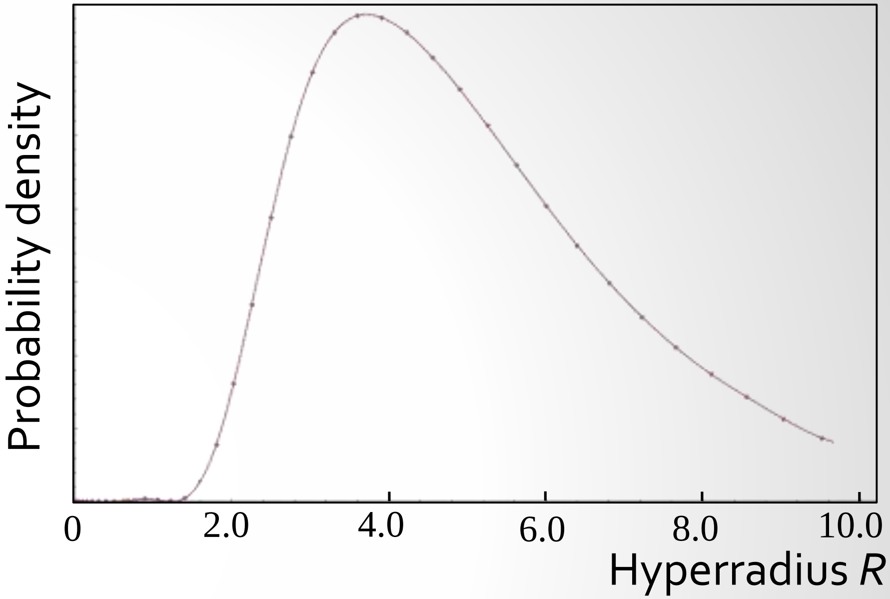
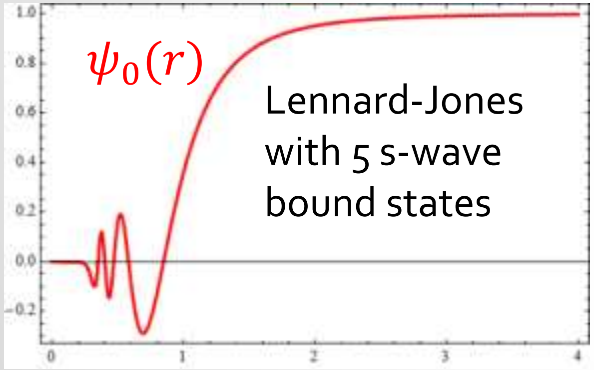


van der Waals two-body correlation



$$a_- = -10.86(1)\, r_{vdW}$$

Exact: $a_- = -11(1)\, r_{vdW}$

Experiments: $a_- = -10(1)\, r_{vdW}$

Confirmation 2: separable model

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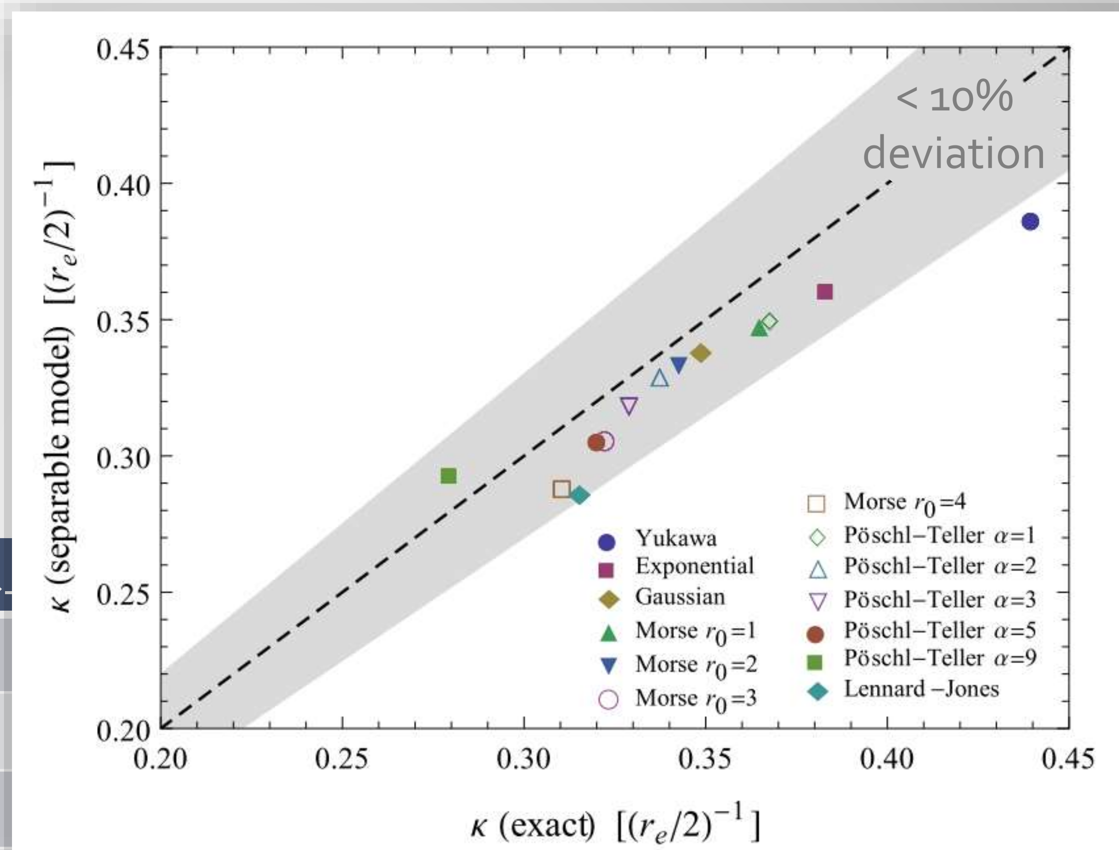
Parameterised to reproduce **exactly** the two-body correlation at zero energy.



Other potentials

Potential	a			
Yukawa	-5.73			
Exponential	-10.7			
Gaussian	-4.27			
Morse ($r_0 = 1$)	-12.3	-12.6	0.180	0.173
Morse ($r_0 = 2$)	-16.4	-16.3	0.131	0.128
Pöschl-Teller ($\alpha = 1$)	-6.02	-6.23	0.367	0.350

Separable model

Exact calculations

S. Moszkowski, S. Fleck, A. Krikeb, L. Theuÿl, J.-M.Richard, and K. Varga, Phys. Rev. A 62 , 032504 (2000).

Summary

universal two-body correlation $\sim r_e$ effective range

↓

three-body deformation

↓

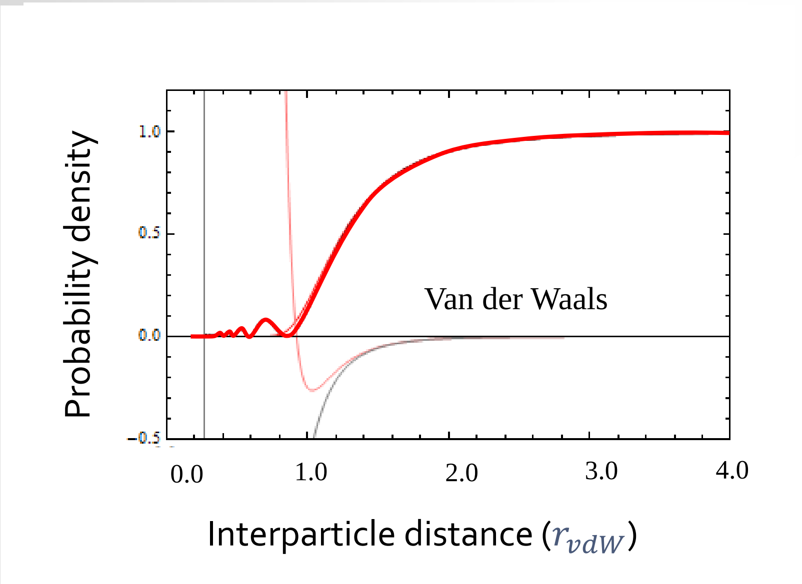
three-body repulsion

↓

universal three-body parameter $\sim r_e$ effective range

Two-body correlation universality classes

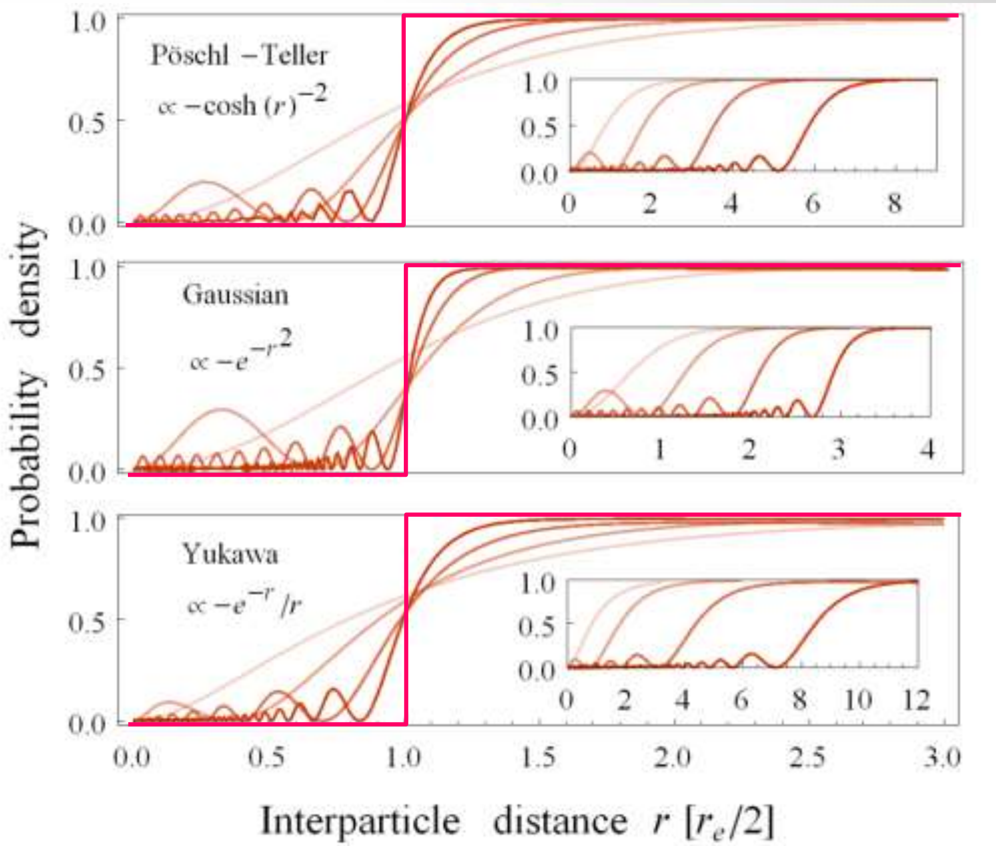
Power-law tails $\propto -\frac{1}{r^n} \quad (n > 3)$



Universal correlation

$$\psi_0(r) = \Gamma\left(\frac{n-1}{n-2}\right)(r/r_n)^{1/2} J_{1/(n-2)}\left(2(r/r_n)^{-(n-2)/2}\right)$$

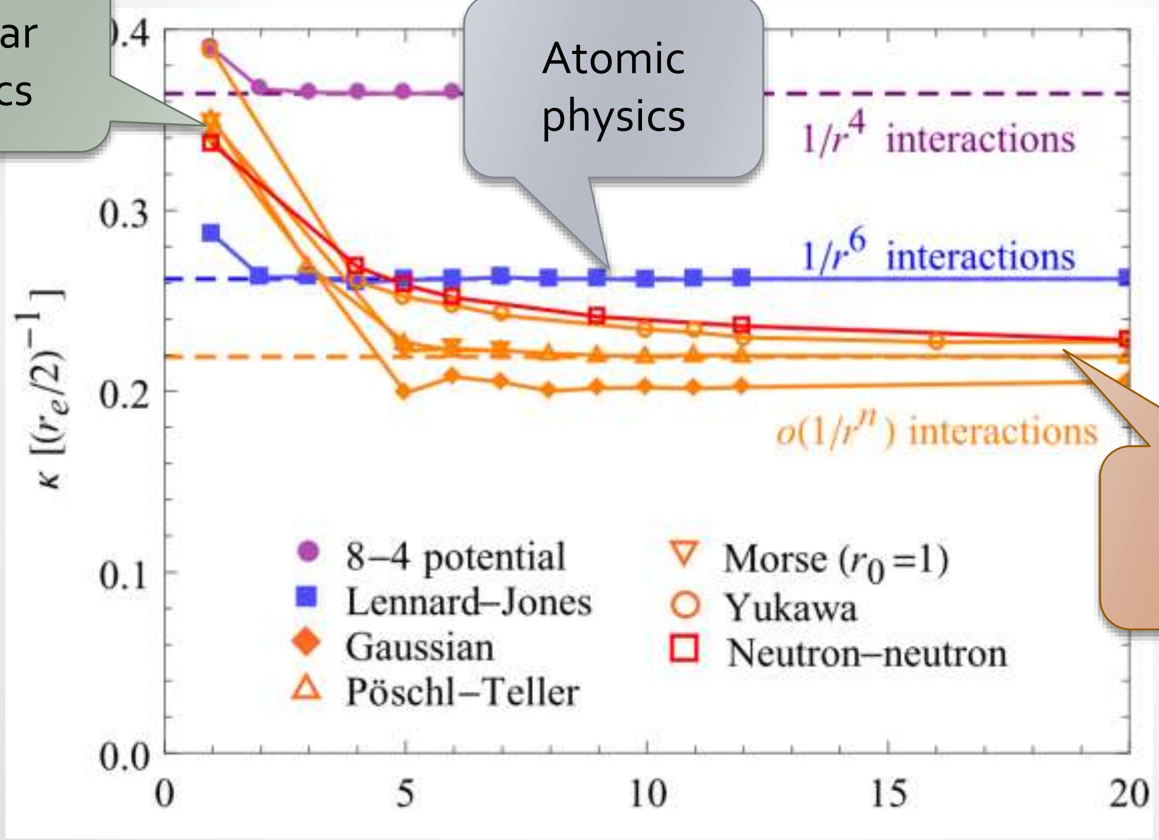
Faster than Power-law tails



Step function correlation limit

Separable model

3-body parameter in units of the *two-body effective range*
(= size of two-body correlation)



Nuclear physics

Atomic physics

?

Binding wave number at unitar

Number of two-body bound states

$$\kappa = -0.364(1)\left(\frac{r_e}{2}\right)^{-1}$$

$$\kappa = -0.261(1)\left(\frac{r_e}{2}\right)^{-1}$$

$$\kappa = -0.2190(1)\left(\frac{r_e}{2}\right)^{-1}$$

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P. Naidon, S. Endo, M. Ueda, arXiv:1208.3912

P. Naidon, S. Endo, M. Ueda, PRL 112, 105301 (2014)


Thank you