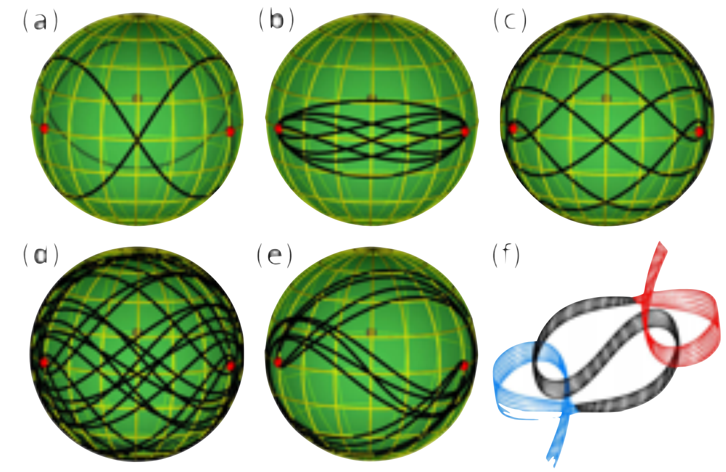
Super Efimov effect

together with Yusuke Nishida and Dam Thanh Son

Sergej Moroz
University of Washington

Few-body problems

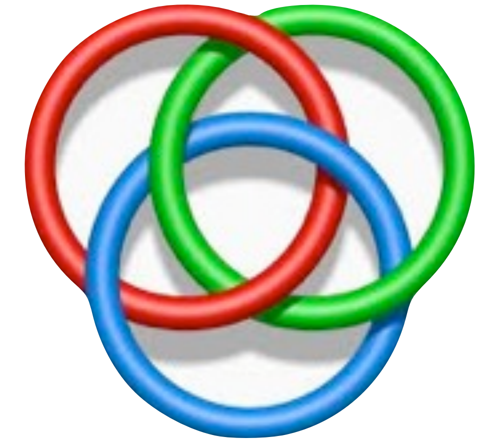


They are challenging but useful:

- Newton gravity $\longrightarrow$ perturbation theory, chaos
- Quantum atoms $\longrightarrow$ variational Hartree-Fock
- Quantum molecules $\longrightarrow$ Born-Oppenheimer

Efimov effect is "new" entry

Few-body universality



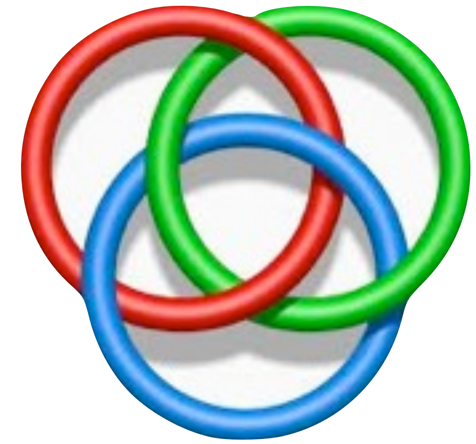
- Low energies, short-range interactions

$$\downarrow$$

in 3d: scattering length a, ...

- Universal regime: a $\gg$ other length scales
- Two-body bound state near resonance

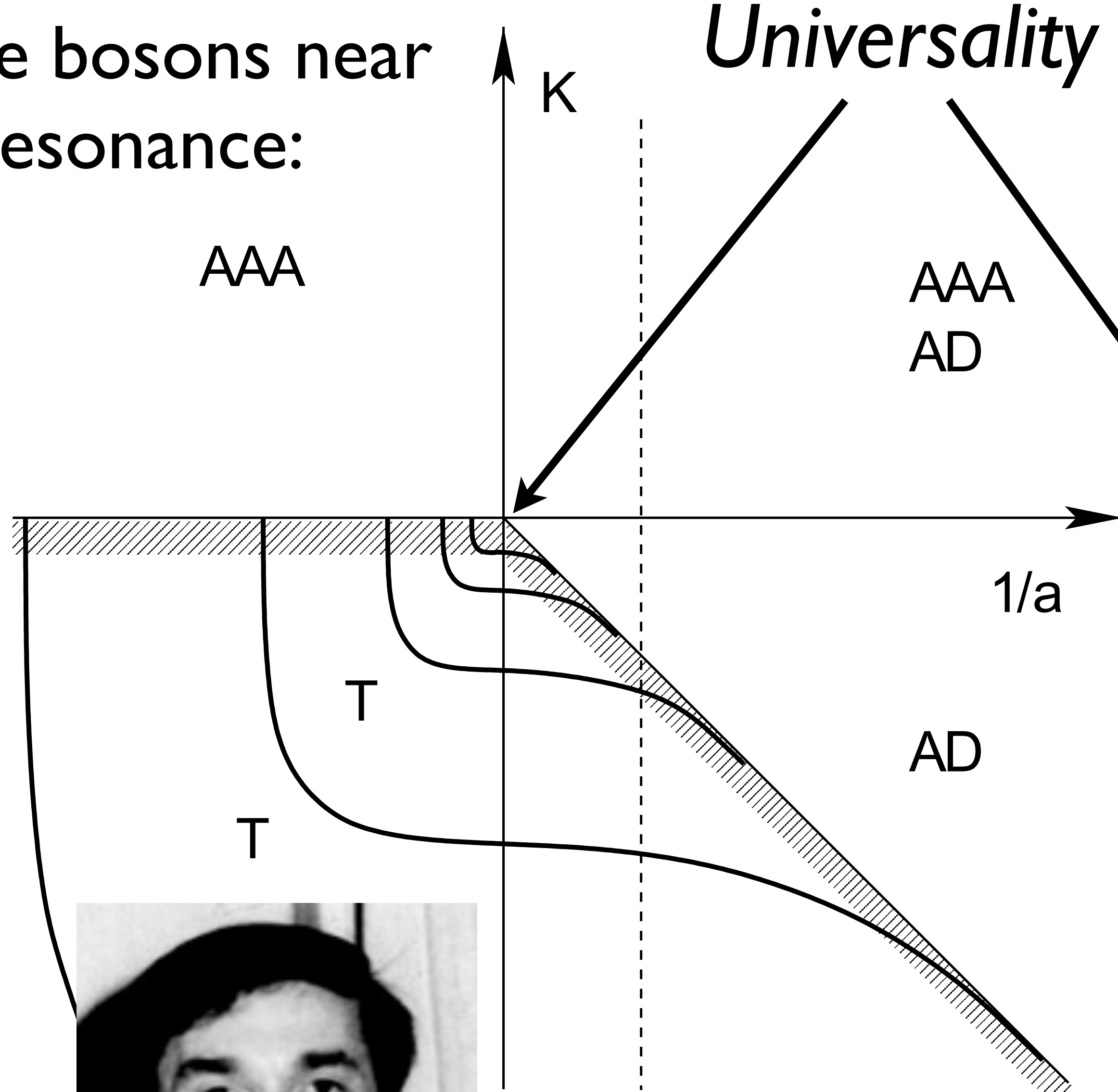
$$E_{\mathsf{D}} = \frac{\hbar^2}{ma^2} \quad \text{for} \quad a > 0$$



Efimov problem

Three bosons near resonance:

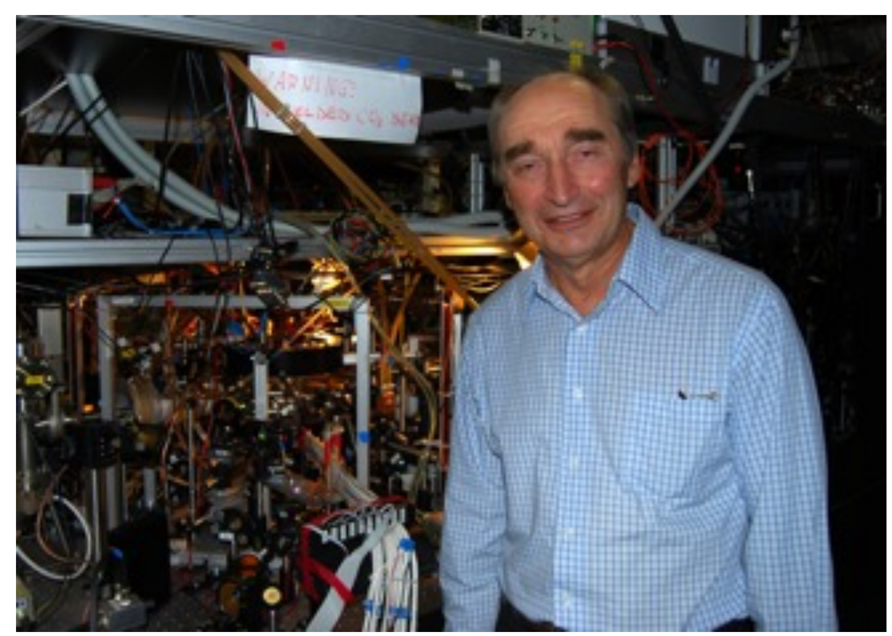
Universality



At resonance as $n \to \infty$

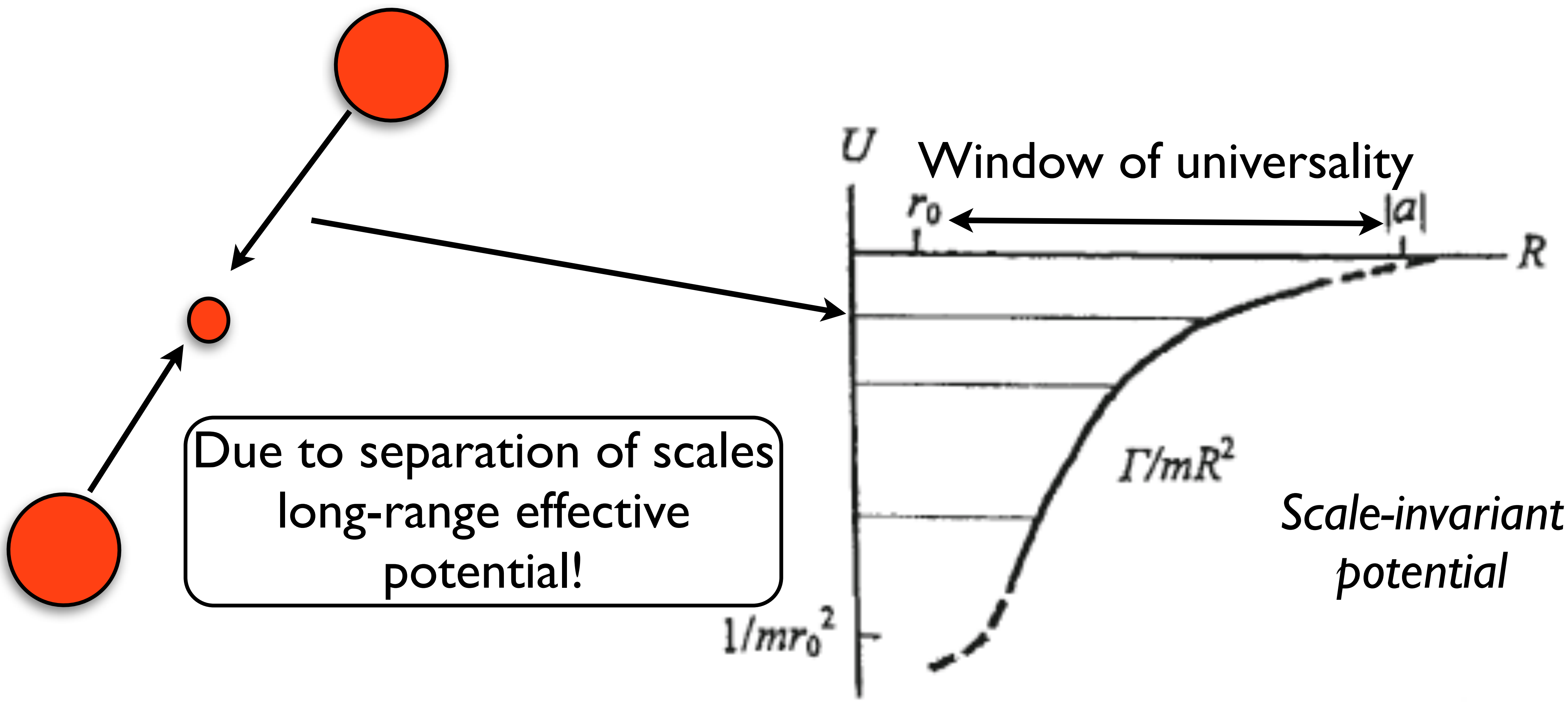
$$\frac{E_T^{(n+1)}}{E_T^{(n)}} \to e^{-2\pi/s_0}$$

40 years later →



Basics intuition

- How can short-range forces create infinite number of bounds states?
- Born-Oppenheimer approximation:

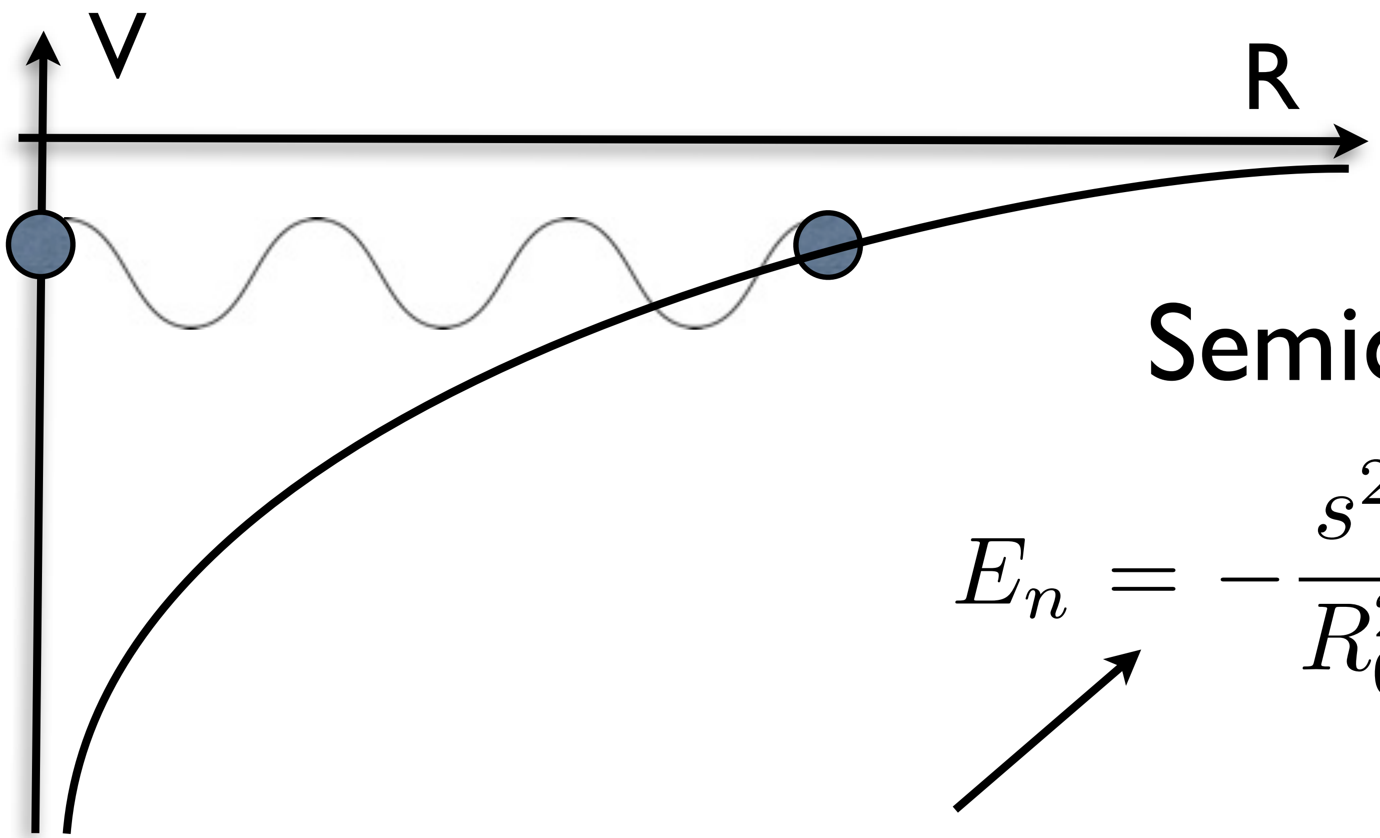


Energy spectrum

$$V(R) = -\frac{1/4 + s_0^2}{R^2}$$

Landau&Lifshitz:
Fall to center for strong attraction

$$s_0 > 0$$



Semiclassical solution

$$E_n = -\frac{s^2}{R_0^2} \exp\left(-\frac{2\pi n}{s} + \theta\right)$$

Efimov geometric spectrum

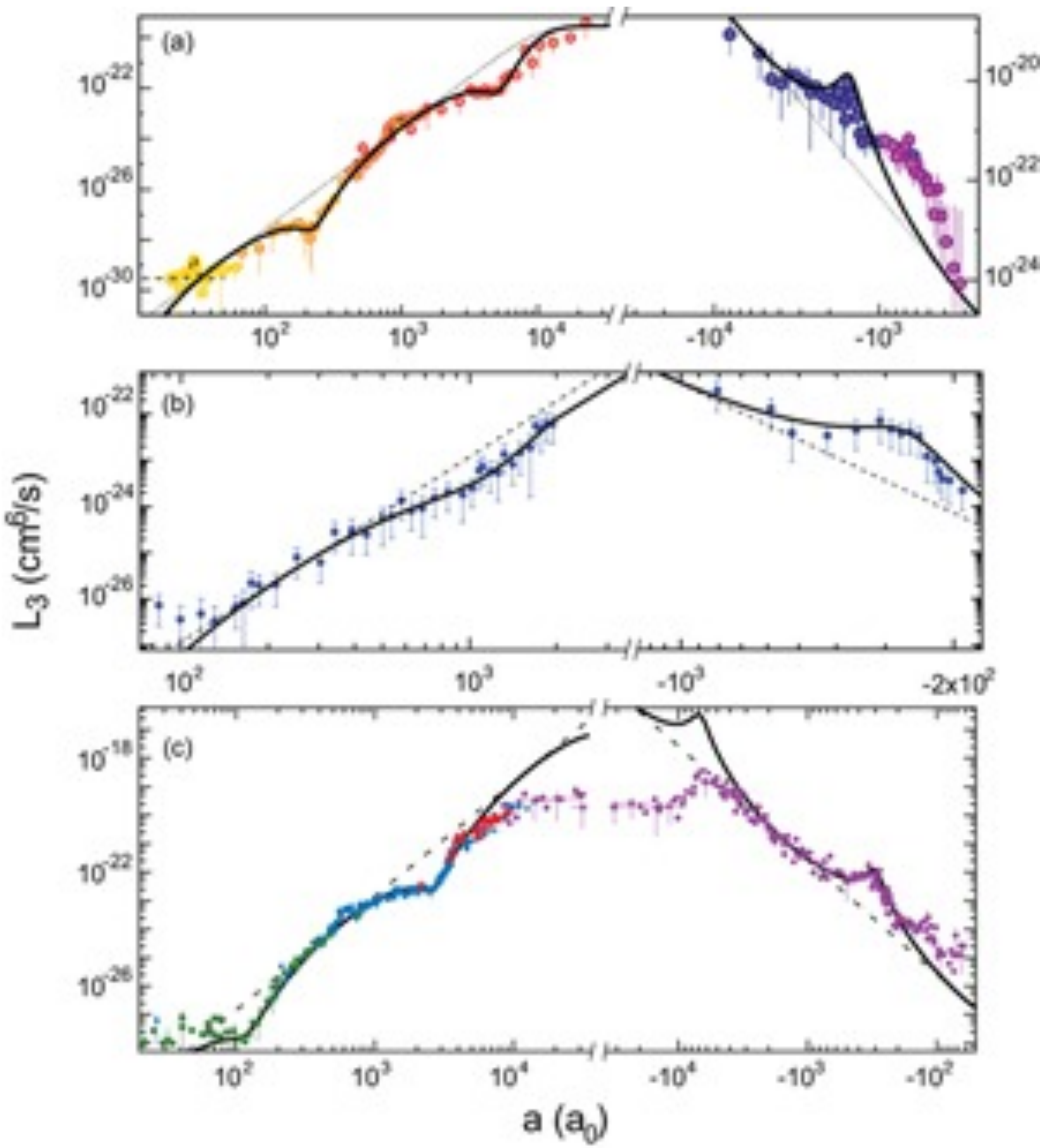
Experimental signatures

Three-body loss:

$$\dot{n} = -L_3 n^3$$

enhanced when trimers merge with atom threshold

First experiment Innsbruck 2006

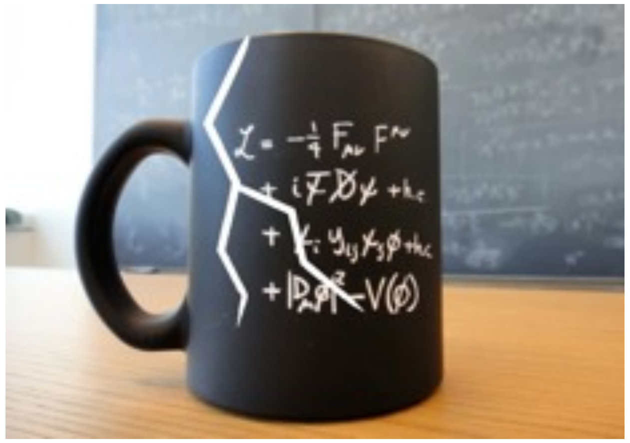


Florence *^{39}K*

Bar Ilan *^{7}Li*

Rice *^{7}Li*

Beyond standard model?

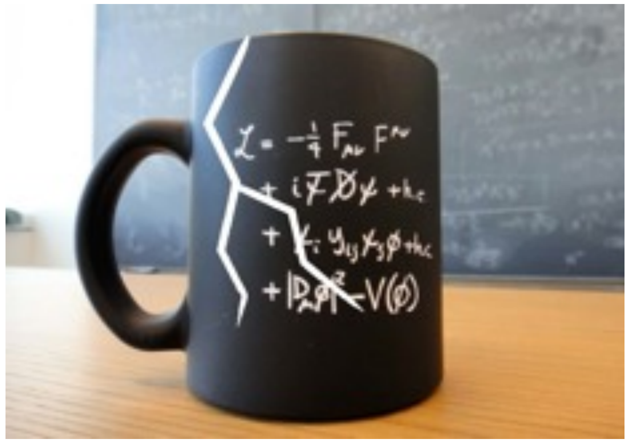


	$d=3$	$d=2$	$d=1$
s-wave	✓	X	X
p-wave	X	X	X
d-wave	X	X	X

No scale invariant two-body attraction away from 3d s-wave!

Efimov effect was liberated from 3d *Nishida, Tan*

Beyond standard model?



	$d=3$	$d=2$	$d=1$
s-wave	✓	X	X
p-wave	X	?	X
d-wave	X	X	X

No Efmov effect, but ...

p-wave in 2d

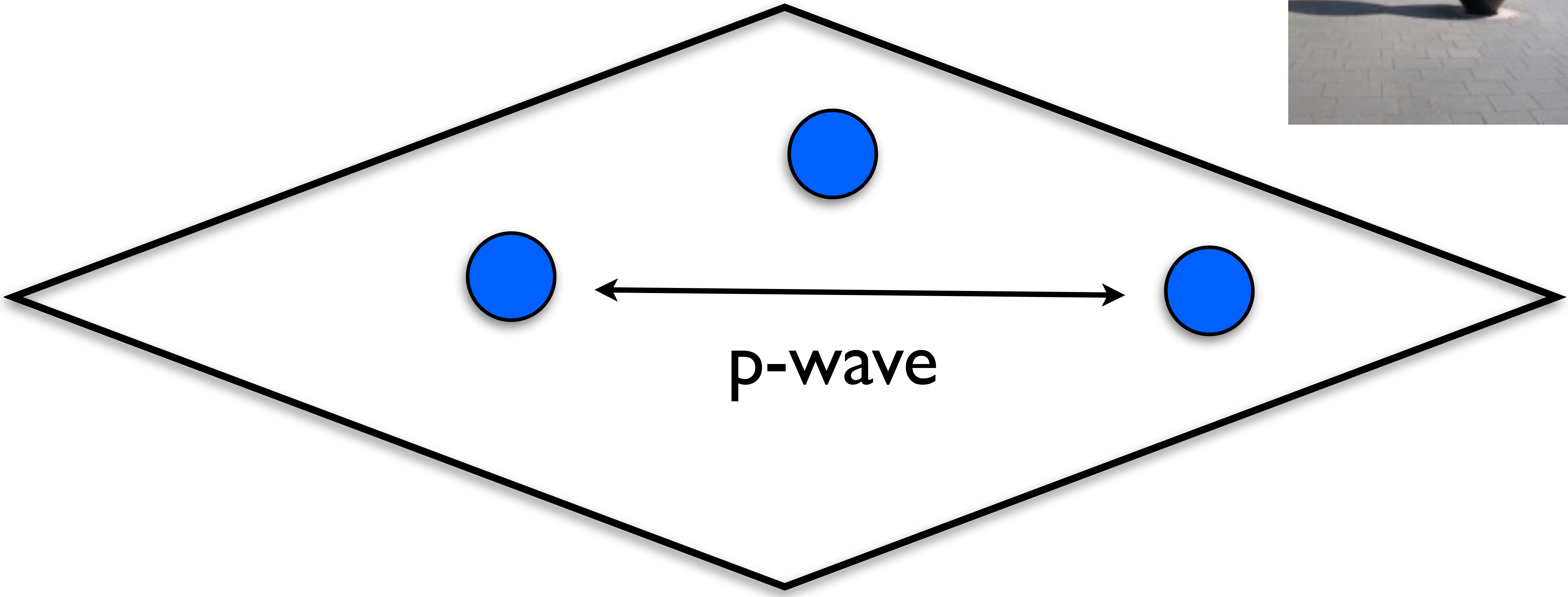
Square well solution
$$\frac{dJ_l(kr)/dr}{J_l(kr)} = \frac{dK_l(\kappa r)/dr}{K_l(\kappa r)}$$

In p-wave critical attraction needed $\quad V_0 r_0^2 = 5.784$

Normalized wave-function
$$\psi(r) = \frac{\kappa}{\sqrt{2\pi}} \frac{K_1(\kappa r)}{\sqrt{\ln(\kappa r_0)}}$$

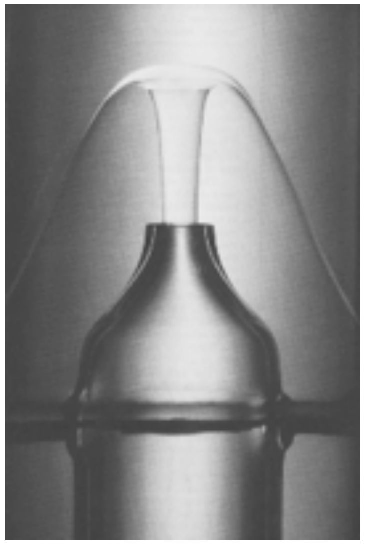
- No scale invariance
- Point-like boson as $r_0 \to 0$

This talk



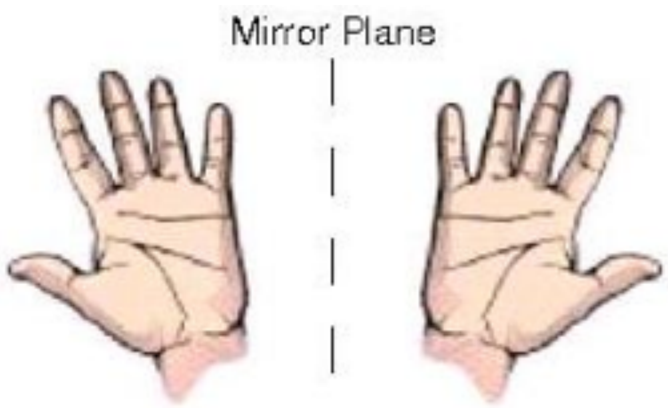
Few-body quantum physics of
resonantly interacting fermions in flatland

Superfluids



- T=0 state of a neutral many-body system
- No dissipation, quantum vortices, ...
- Old: ^4He and ^3He
- New: Bose and Fermi ultracold atoms

P-wave superfluids



From mean-field: *Volovik, Read, Green,...*

- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm i p_y)\hat{\Delta}$ preferred
- Topological phase transition at $\mu = 0$
- Chiral Majorana modes on boundaries
- Toy model for a film of ^{3}He

Sometimes mean-field is not good enough near resonance!

Super Efimov effect

<u>At resonance near threshold:</u>

- Infinite tower of $l = \pm 1$ trimer bound states

$$E_3^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta}\right)$$

- Infinite set of $l = \pm 2$ tetramer resonances

$$E_4^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta-0.188}\right)$$

Super exponential scaling!

P-wave model in d=2

TAKE ACTION!

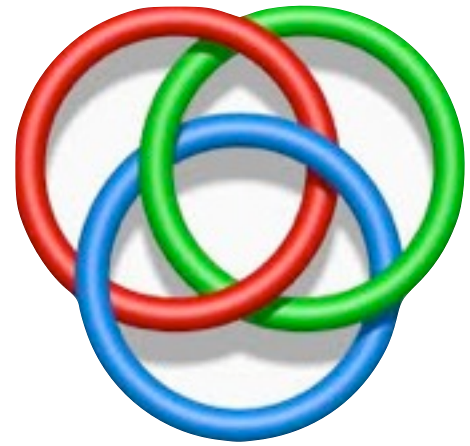
$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a$$
$$+g\,\phi_a^\dagger\psi(-i\nabla_a)\psi + g\,\psi^\dagger(-i\nabla_{-a})\psi^\dagger\phi_a$$
$$+v_3\,\psi^\dagger\phi_a^\dagger\phi_a\psi + v_4\,\phi_a^\dagger\phi_{-a}^\dagger\phi_{-a}\phi_a + v_4'\,\phi_a^\dagger\phi_a^\dagger\phi_a\phi_a$$

↑ spinless fermion

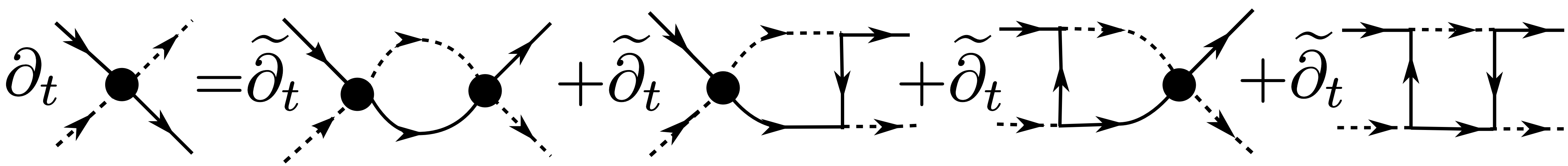
↑ composite $l = \pm 1$ boson

- P-wave resonance ⟷ zero energy bound state
- All dimensionless couplings are included

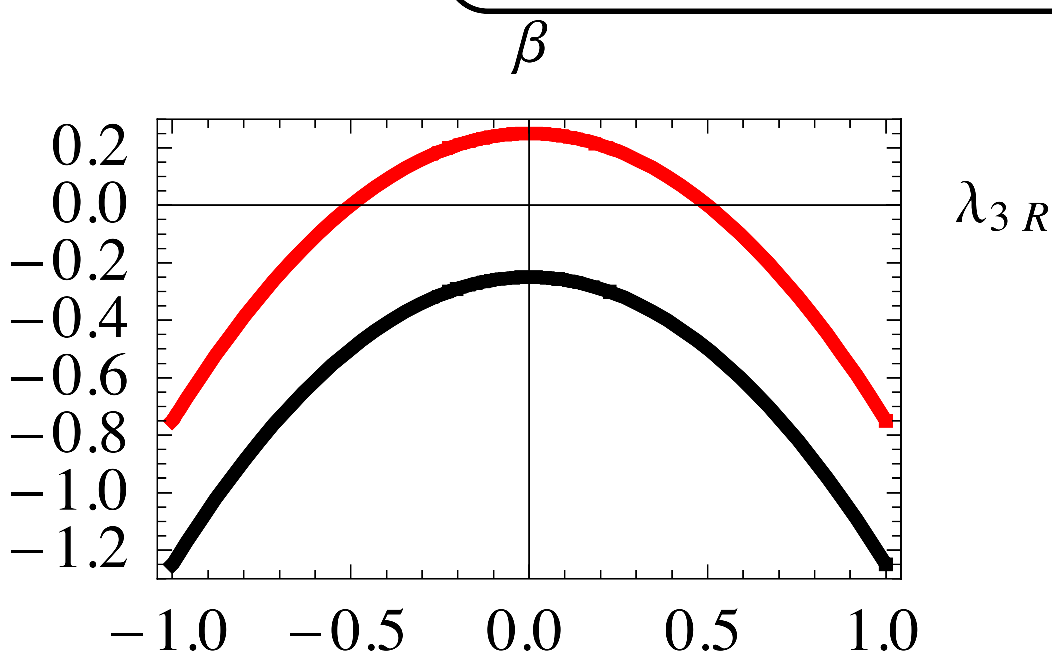
Efimov effect from RG



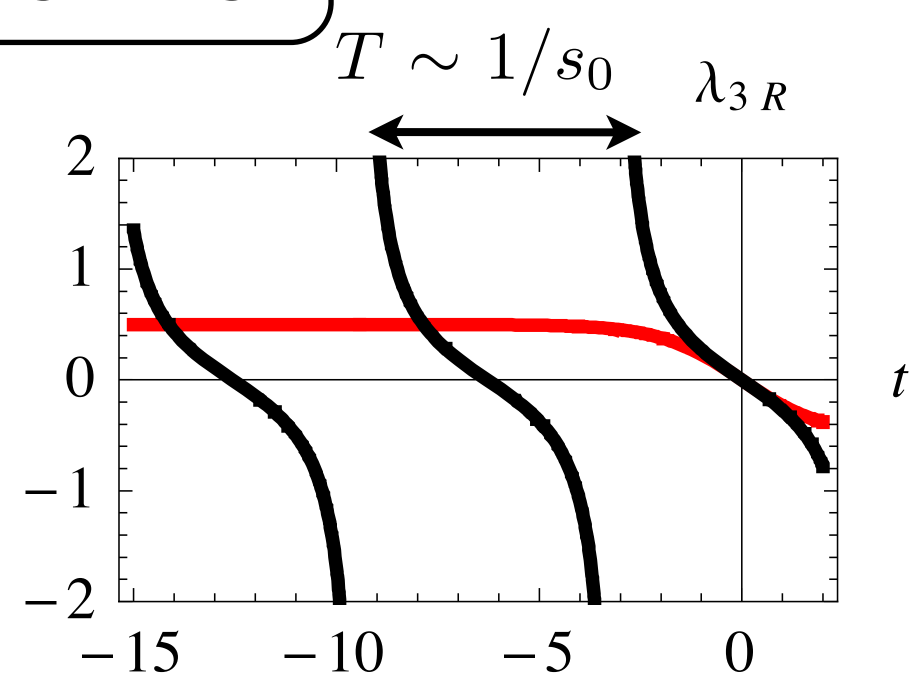
Flow of atom-dimer vertex: RG=one-loop diagrams



bosons vs fermions in 3d



Tetramers can be found from RG

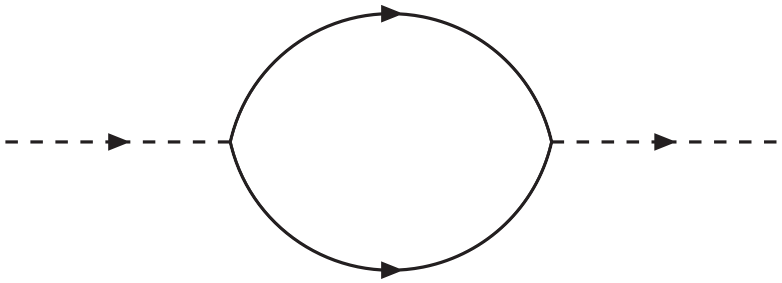


limit cycle

Super Efimov from RG

Two-body:

Perturbative counting is reliable!



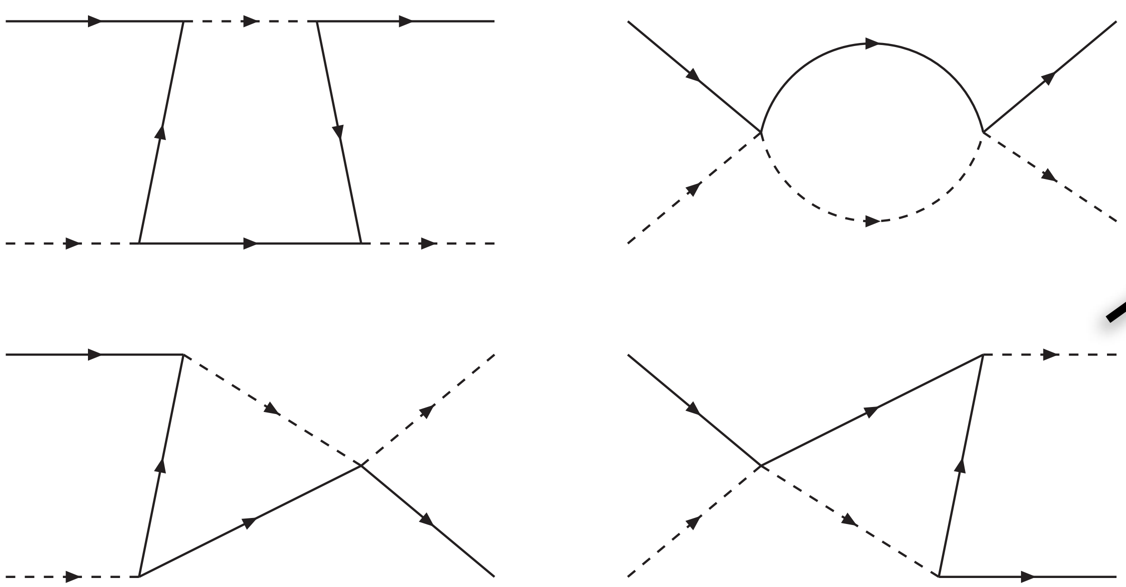
$$s = \ln \Lambda / k$$

irrelevant in IR

like QED

$$g^2(s) = \frac{1}{\frac{s}{\pi} + \frac{1}{g^2(0)}}$$

Three-body:

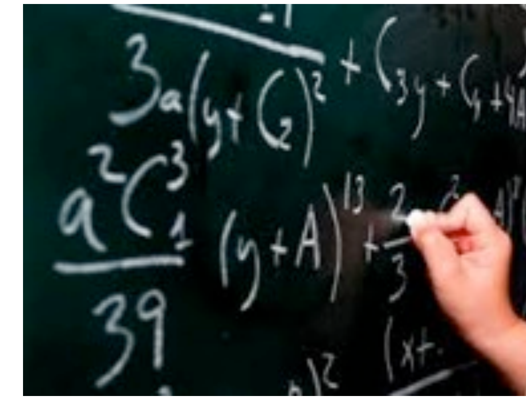


Double log periodic solution:

$$v_3(s) \to \frac{2\pi}{s}\left[1 - \cot\left(\frac{4}{3}(\ln s - \theta)\right)\right]$$

Divergences= trimer bound states

T-matrix solution



One-channel model:

$$H = \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{\mathbf{k}^2}{2} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} - v_0 \sum_{a=\pm} \int \frac{d\mathbf{k} d\mathbf{p} d\mathbf{q}}{(2\pi)^6}$$
$$\times \chi_a(\mathbf{p}) \chi_{-a}(\mathbf{q})\, \psi^\dagger_{\frac{\mathbf{k}}{2}+\mathbf{p}} \psi^\dagger_{\frac{\mathbf{k}}{2}-\mathbf{p}} \psi_{\frac{\mathbf{k}}{2}-\mathbf{q}} \psi_{\frac{\mathbf{k}}{2}+\mathbf{q}}$$

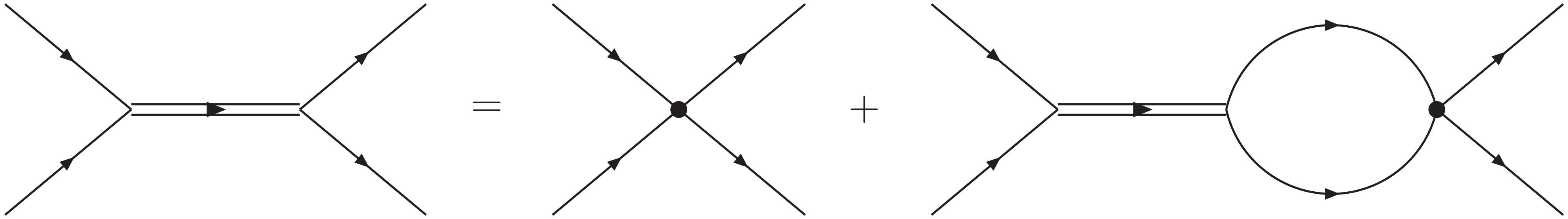
Separable interaction

$$\chi_a(\mathbf{p}) = p_a\, e^{-\mathbf{p}^2/(2\Lambda^2)}$$

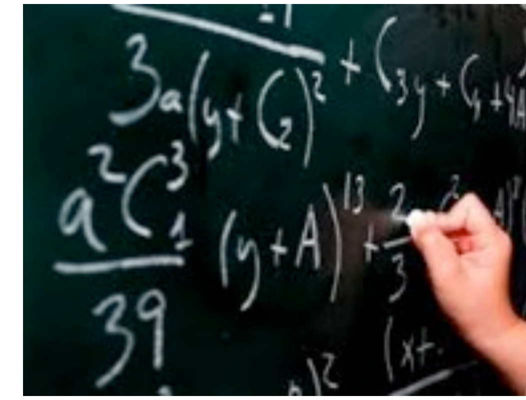
T-matrix solution

Two-fermion scattering T-matrix:

$$T(E;\mathbf{p},\mathbf{q}) = \frac{16\pi|\mathbf{p}||\mathbf{q}|\cos(\varphi_{\mathbf{p}}-\varphi_{\mathbf{q}})\,e^{-(\mathbf{p}^2+\mathbf{q}^2)/(2\Lambda^2)}}{\frac{2\pi}{v_0}-\Lambda^2-E\,e^{-E/\Lambda^2}\,\mathrm{E}_1(-E/\Lambda^2)}$$



T-matrix solution

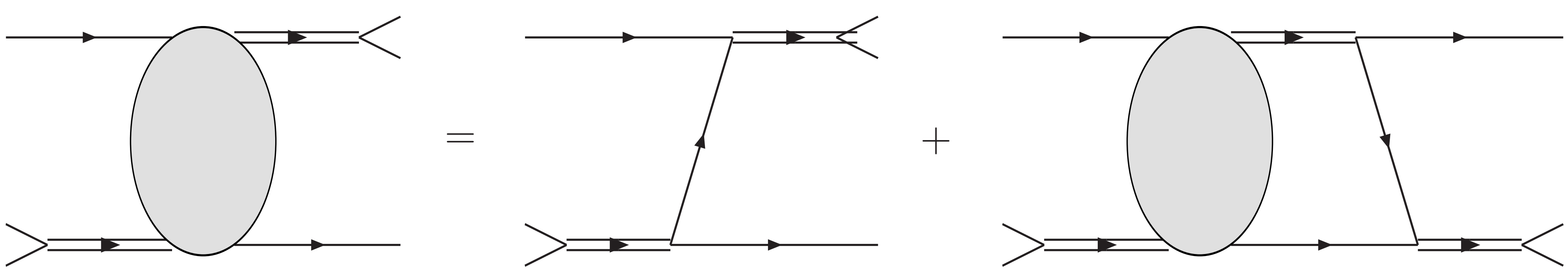


Three-fermion scattering T-matrix:

Near binding energy $T_{ab}(E;\vec{p},\vec{q}) \to Z_a(\vec{p})Z_b^*(\vec{q})/(E+\kappa^2)$

$$Z_a(\mathbf{p}) \quad = -\int \frac{d\mathbf{q}}{2\pi} \; \frac{(\mathbf{p}+2\mathbf{q})_{-a}\; e^{-(5\mathbf{p}^2+5\mathbf{q}^2+8\mathbf{p}\cdot\mathbf{q})/(8\Lambda^2)}}{\mathbf{p}^2+\mathbf{q}^2+\mathbf{p}\cdot\mathbf{q}+\kappa^2}$$

$$\times \frac{\sum_{b=\pm}(2\mathbf{p}+\mathbf{q})_b\; Z_b(\mathbf{q})}{(\frac{3}{4}\mathbf{q}^2+\kappa^2)\; e^{(\frac{3}{4}\mathbf{q}^2+\kappa^2)/\Lambda^2}\; \mathrm{E}_1((\frac{3}{4}\mathbf{q}^2+\kappa^2)/\Lambda^2)}$$



T-matrix solution

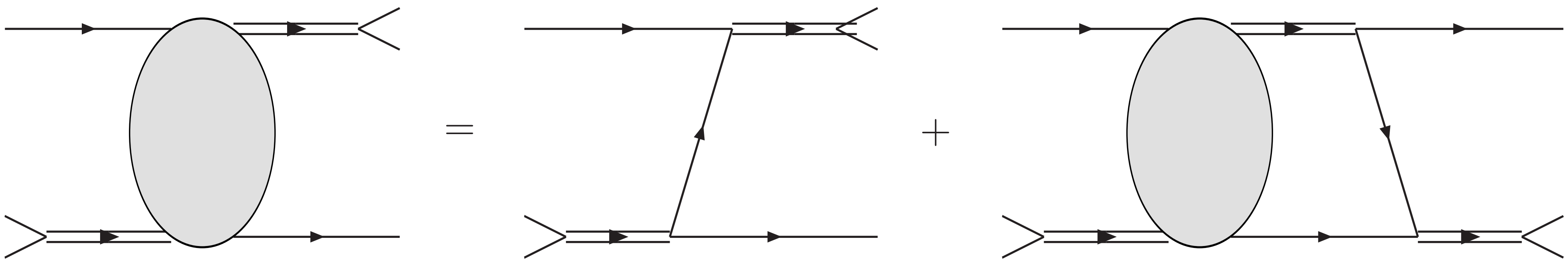
Partial wave decomposition:

$$Z_a(\mathbf{p}) = e^{i\ell\varphi_{\mathbf{p}}} z_a(p)$$

s and d waves are <u>coupled</u>!

Similar to deuteron due to tensor one-pion force

T-matrix solution



Analytic solution

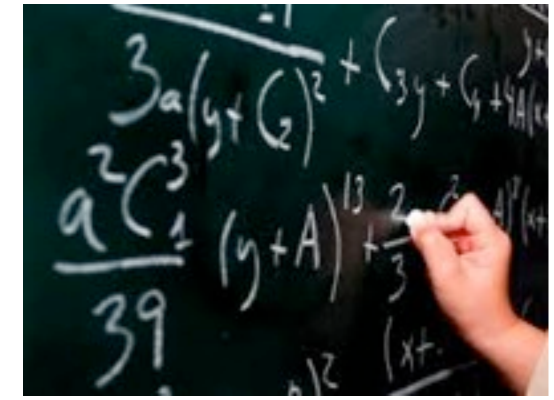
leading log approximation

$$\lambda_n \equiv \ln \ln \Lambda/\kappa_n$$

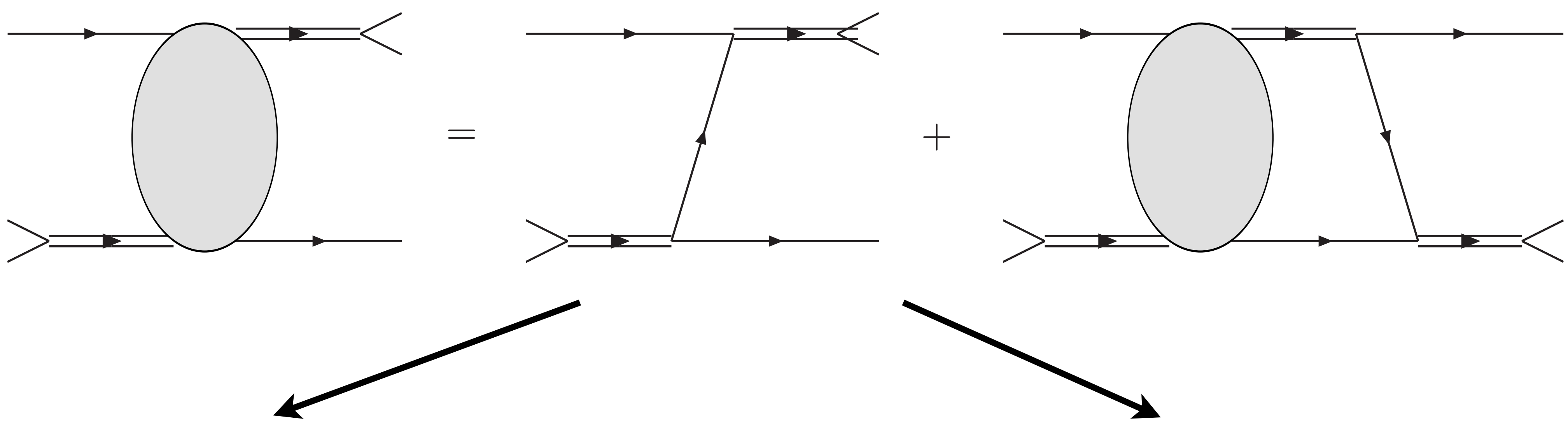
Numeric solution

n	λ_n	$\lambda_n - \lambda_{n-1}$
0	0.5632	---
1	2.770	2.207
2	5.078	2.308
3	7.430	2.352
4	9.785	2.355
∞	---	2.35619

T-matrix solution



Near binding energy $T_{ab}(E;\vec{p},\vec{q}) \to Z_a(\vec{p})Z_b^*(\vec{q})/(E+\kappa^2)$



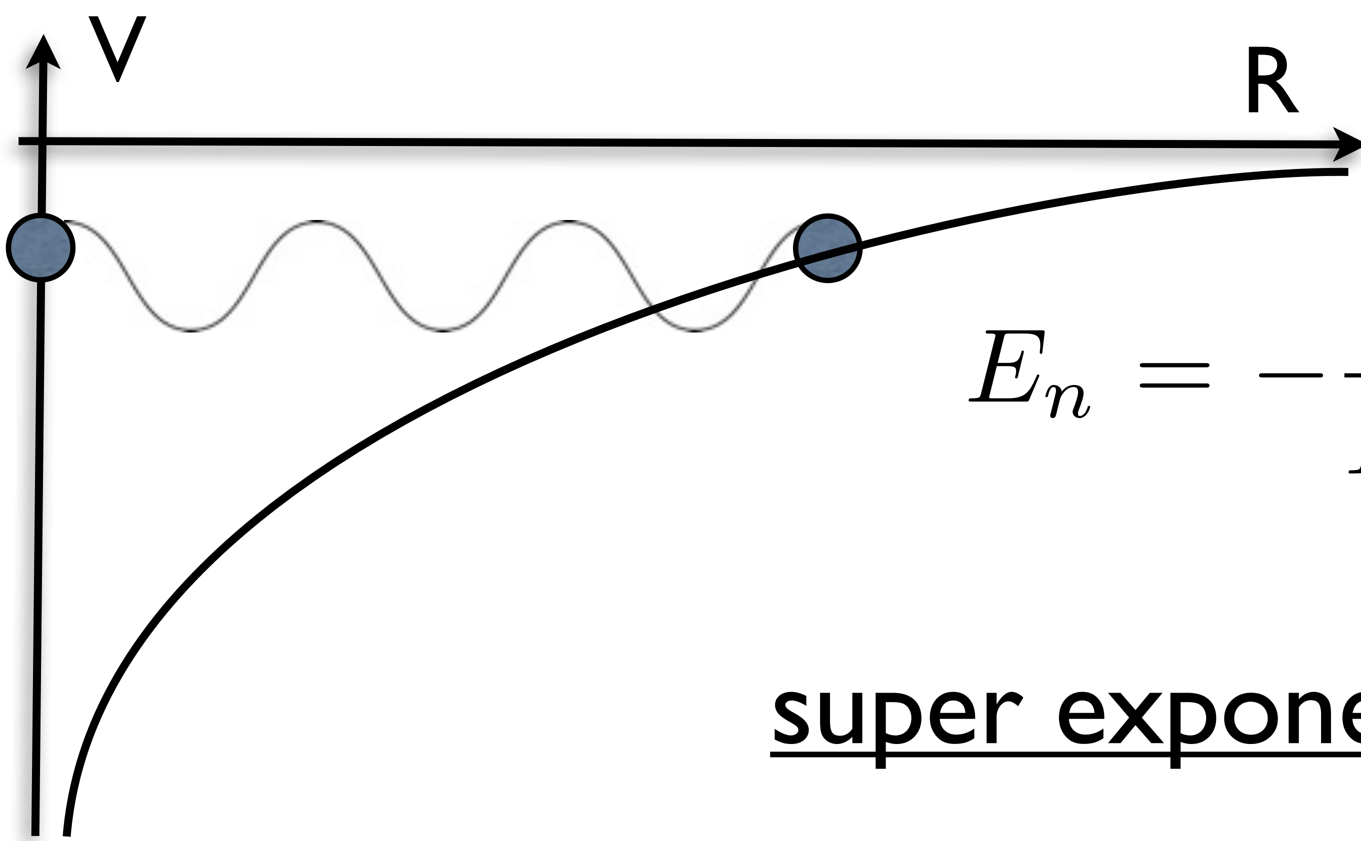
$$E_3^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta}\right)$$

Agreement with RG result

Effective potential

$$V(R) = -\frac{1}{4R^2} - \frac{1/4 + r^2}{(R \ln \frac{R}{R_0})^2}$$

Semiclassical solution with double Langer correction



$$E_n = -\frac{r^2}{R_0^2}\frac{\exp\left(-2e^{\frac{\pi}{r}n+\theta}\right)}{e^{2\left(\frac{\pi}{r}n+\theta\right)}}$$

<u>super exponential</u> scaling

Hyperspherical calculation

Volosniev et al.; Gao&Yu

- Adiabatic approximation

$$\Psi_{l=1} = R^{-3/2} f_{l=1}(R)\Phi_{l=1}(\Omega; R)$$

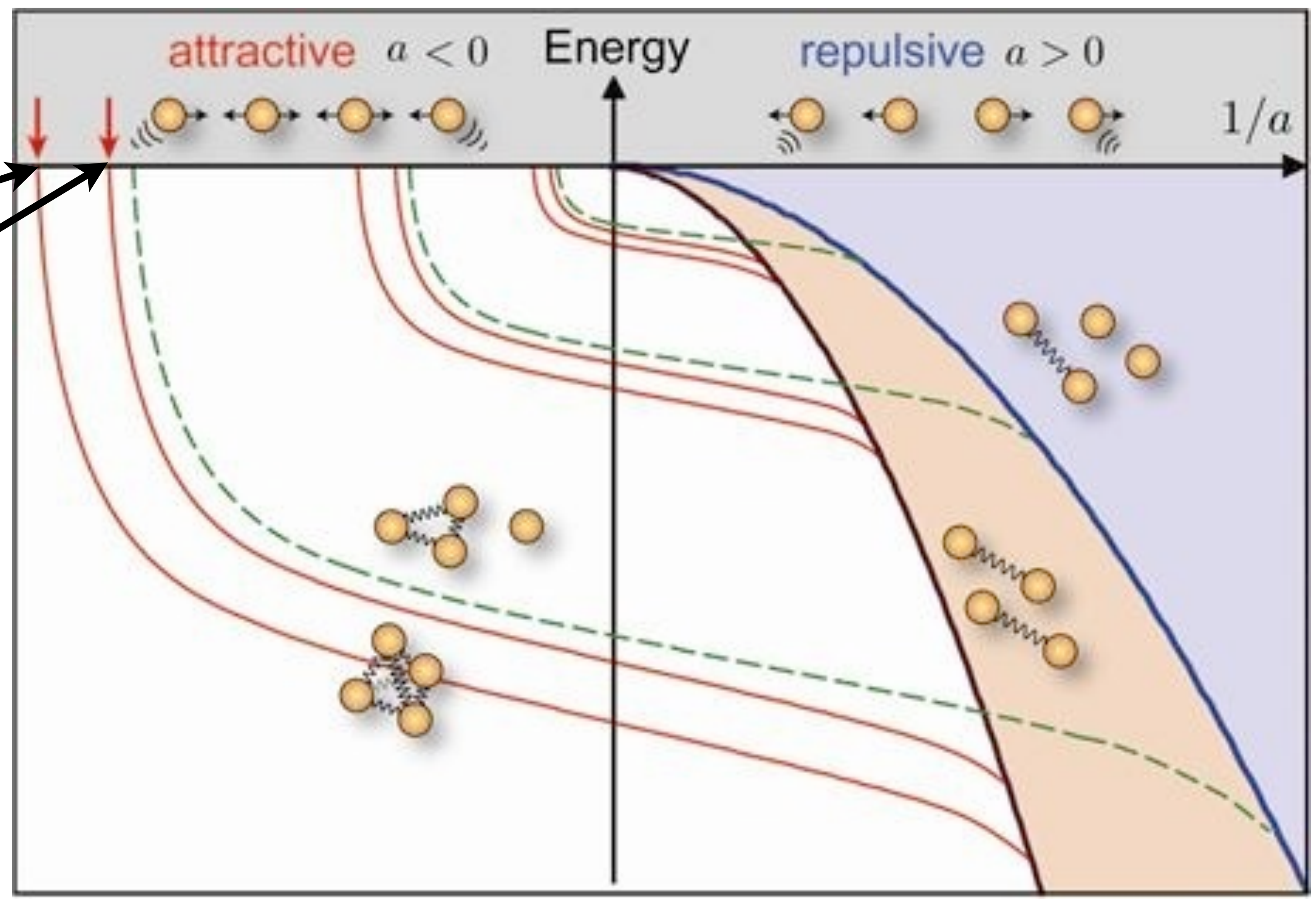
- s-d wave mixing is well captured
- Diagonal corrections <u>important</u> for super Efimov effect
- Is adiabatic approximation reliable?

Tetramer states in 3d

- <u>Universal tetramer resonances</u>

Hammer&Platter
von Stecher et al,...

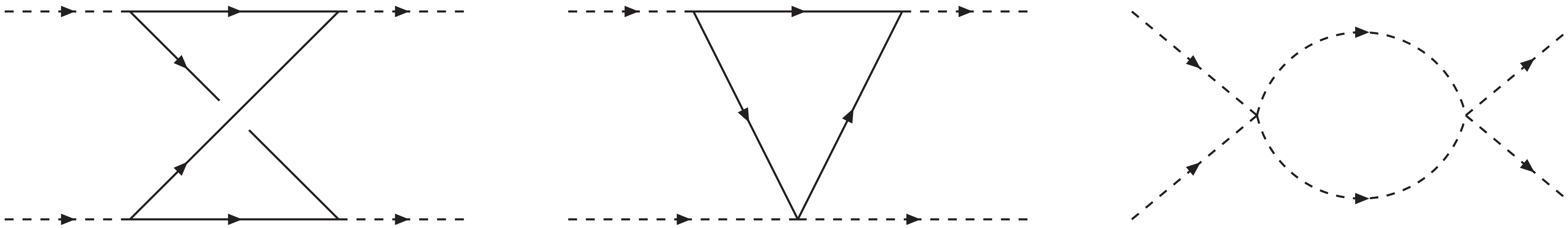
Innsbruck



- Universality and # tetramers not settled

Hadizadeh et al

Tetramers from RG



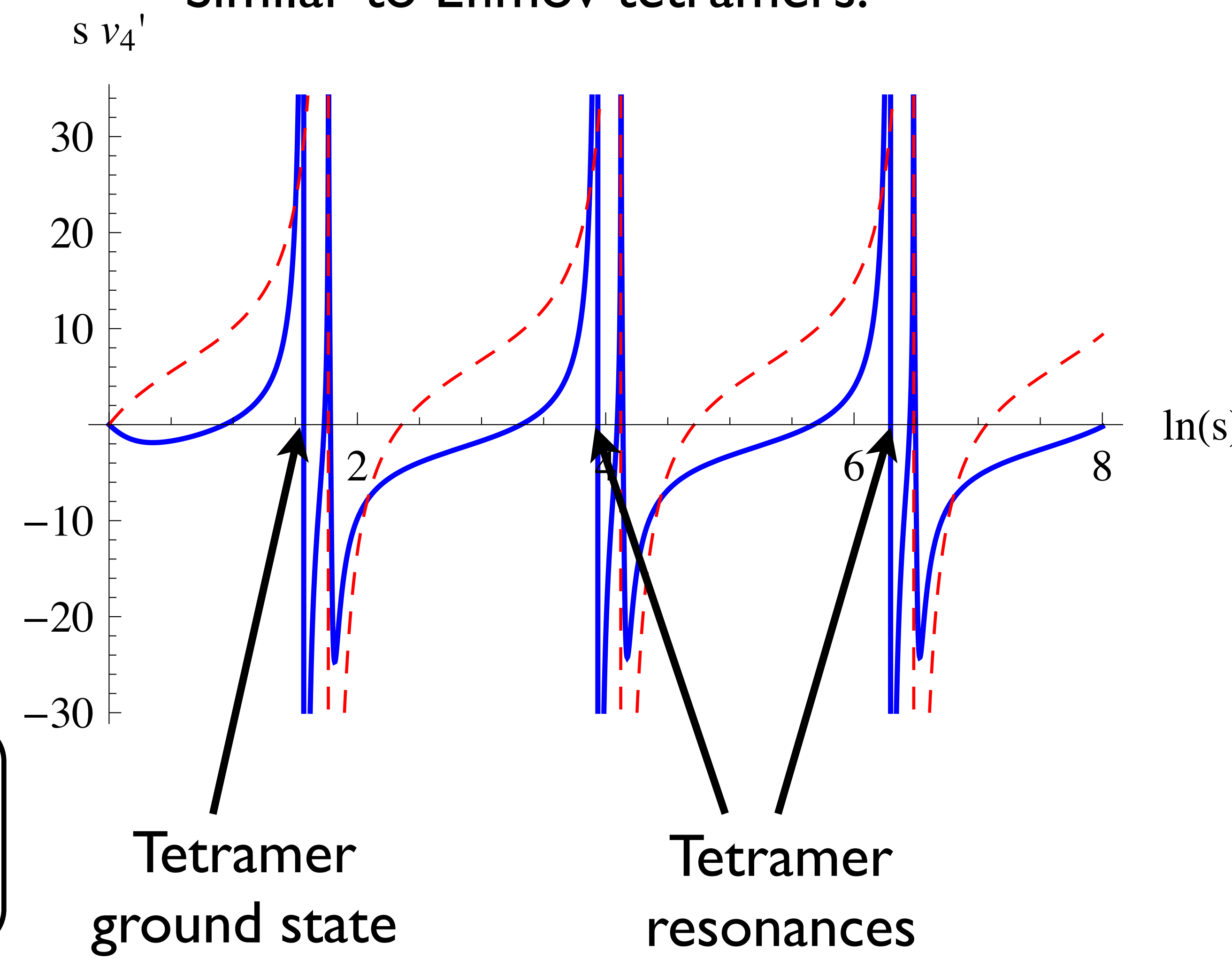
l=2 sector:

$$\frac{dv'_4}{ds} = -\frac{4g^4}{\pi} + \frac{2g^2 v_3}{\pi} - \frac{2g^2 v'_4}{\pi} + \frac{2v'^2_4}{\pi}$$

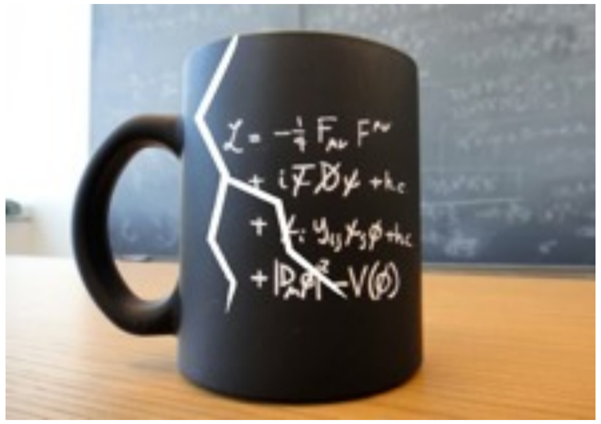
- Numerical solution necessary
- Singularities understood analytically

$$E_4^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta-0.188}\right)$$

Similar to Efimov tetramers:



Super Efimov in 3d?



- Recent RG calculation includes trimer degrees of freedom

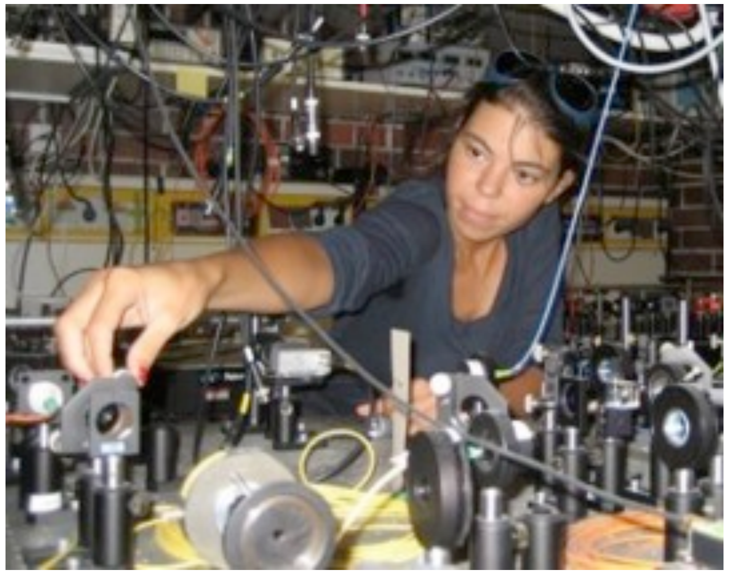
Jaramillo Avila&Birse

- Suggests super Efimov tower of tetramers for every Efimov trimer in 3d!

$$k_4^{(n)} = k_3 \exp(\alpha e^{-\beta n})$$

- Hand-waving RG argument: appears due to <u>logarithmic</u> trimer divergences that feed into the four-body solution

Experiments



- Great success in three dimensions
- Quasi 2d fermions near p-wave resonance *ETH 2005*
- Trimers sizes:

many-body physics

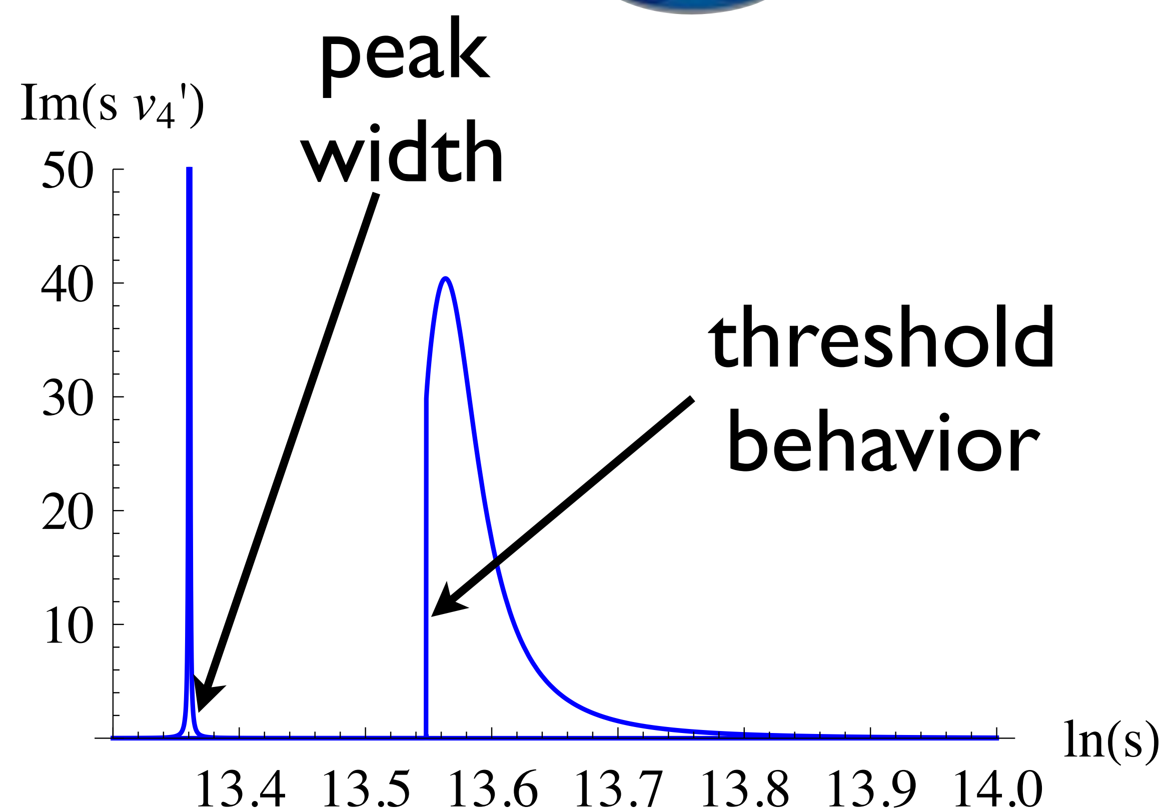
n	GS	1	2	3
size	Å	μ	10^{38}m	10^{499}m

but quasi 2d!

- No tuning possible in this theory!

Open questions

- Decay of tetramers from RG

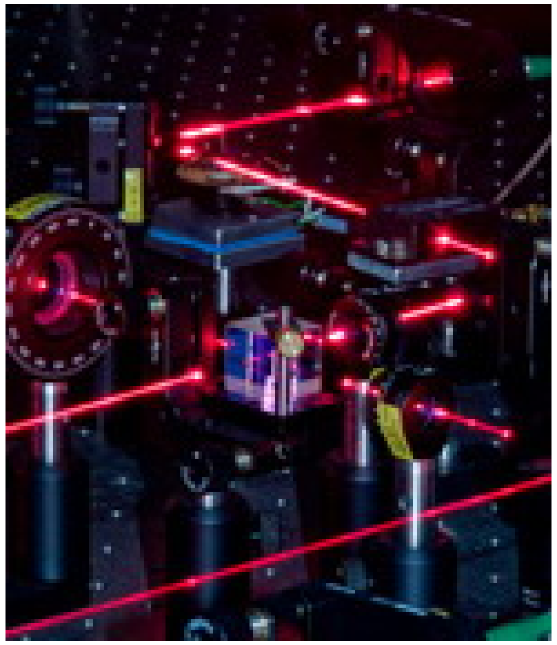


- Tetramers and higher-body from T-matrix?
- Superfluid near resonance

Conclusion

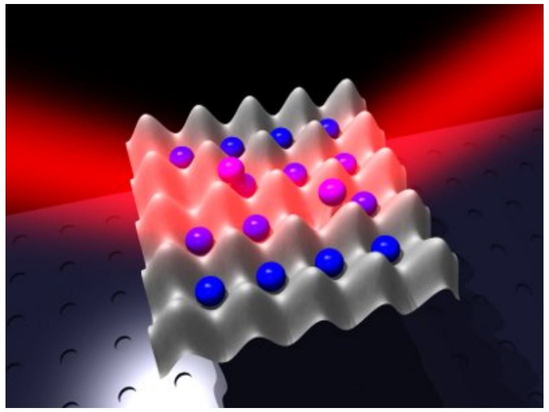
- Super Efimov -- double exponential scaling
- Many question to be asked and answered...

Ultracold atoms



- Ensembles of neutral alkali atoms
- Low densities $n \sim 10^{14}\text{cm}^{-3} \longrightarrow$ gases
- Laser cooling $T \sim 10^{-9}\text{K} \longrightarrow$ quantum
- Tunable interactions and geometry
- Harmonic trap keeps atoms together

Quantum simulator

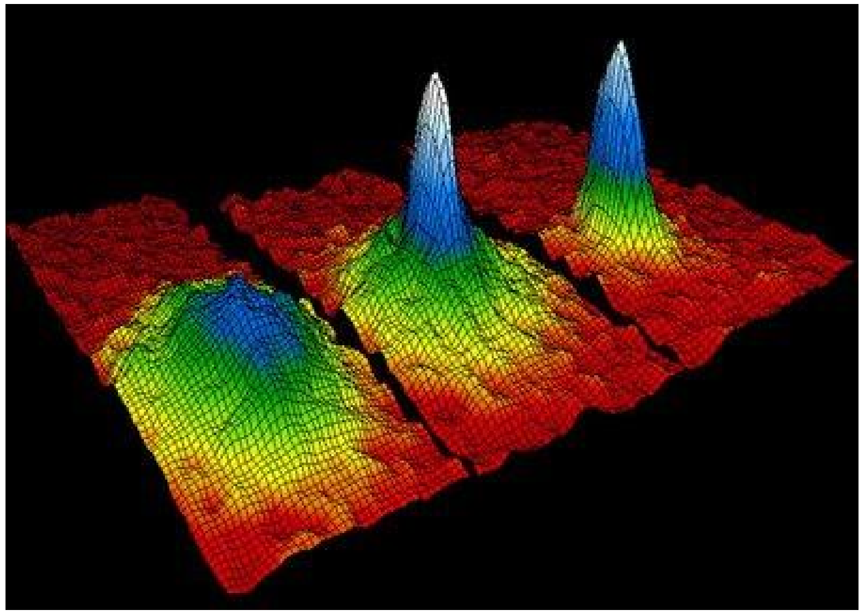


- *Quantum*
- *High degree of tuning*
- *Table-top size*

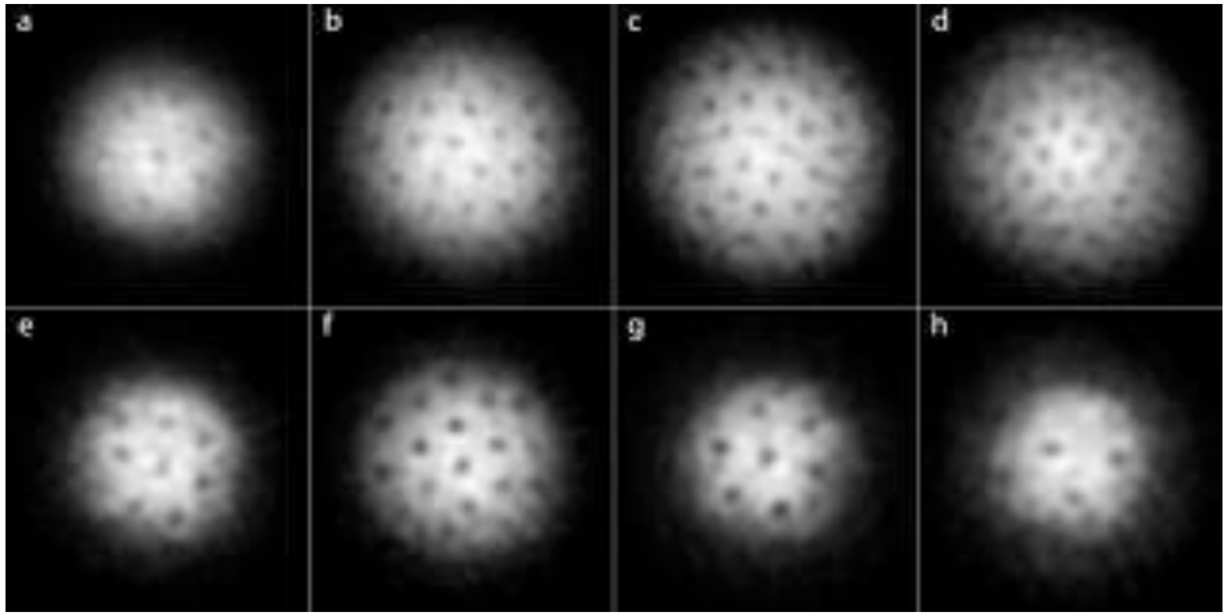
"Let nature do the calculation"

- Lattice models → high Tc superconductors
- Artificial gauge fields → topological states of matter
- Precision measurements → equation of state of neutrons
- Few-body physics → quantum chemistry
- Single atom manipulation → quantum computer

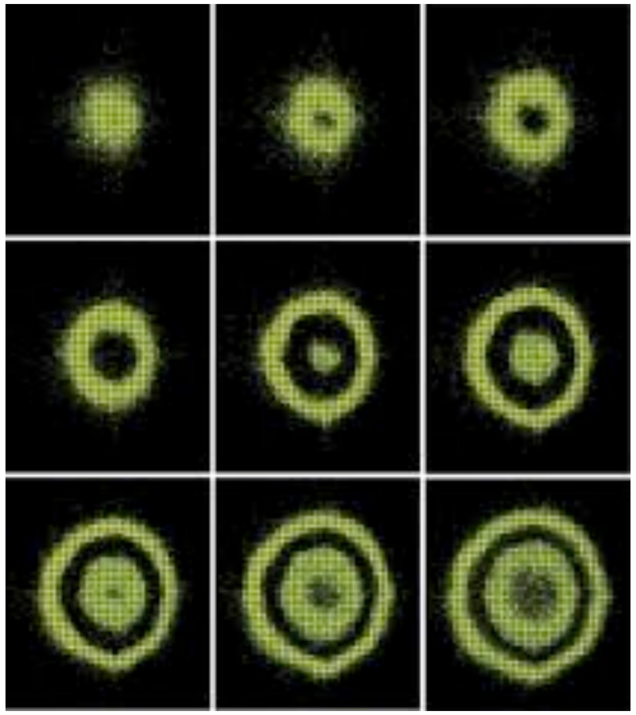
Experimental achievements



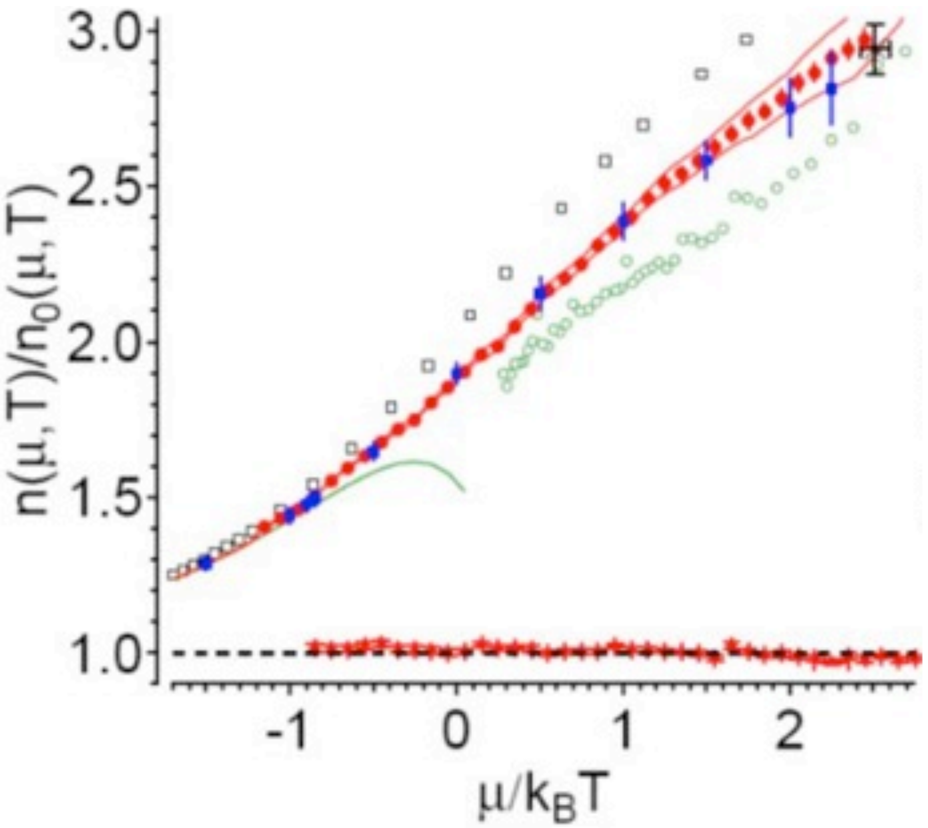
BEC@JILA&MIT 1995



Vortices@MIT 2005

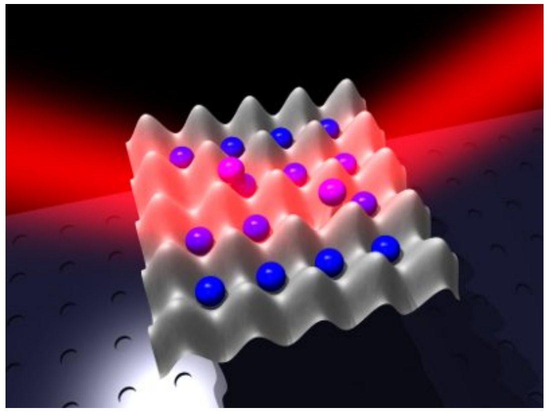


Mott shells@
MPI&Harvard 2010



EOS@ENS&MIT 2012

Quantum simulator



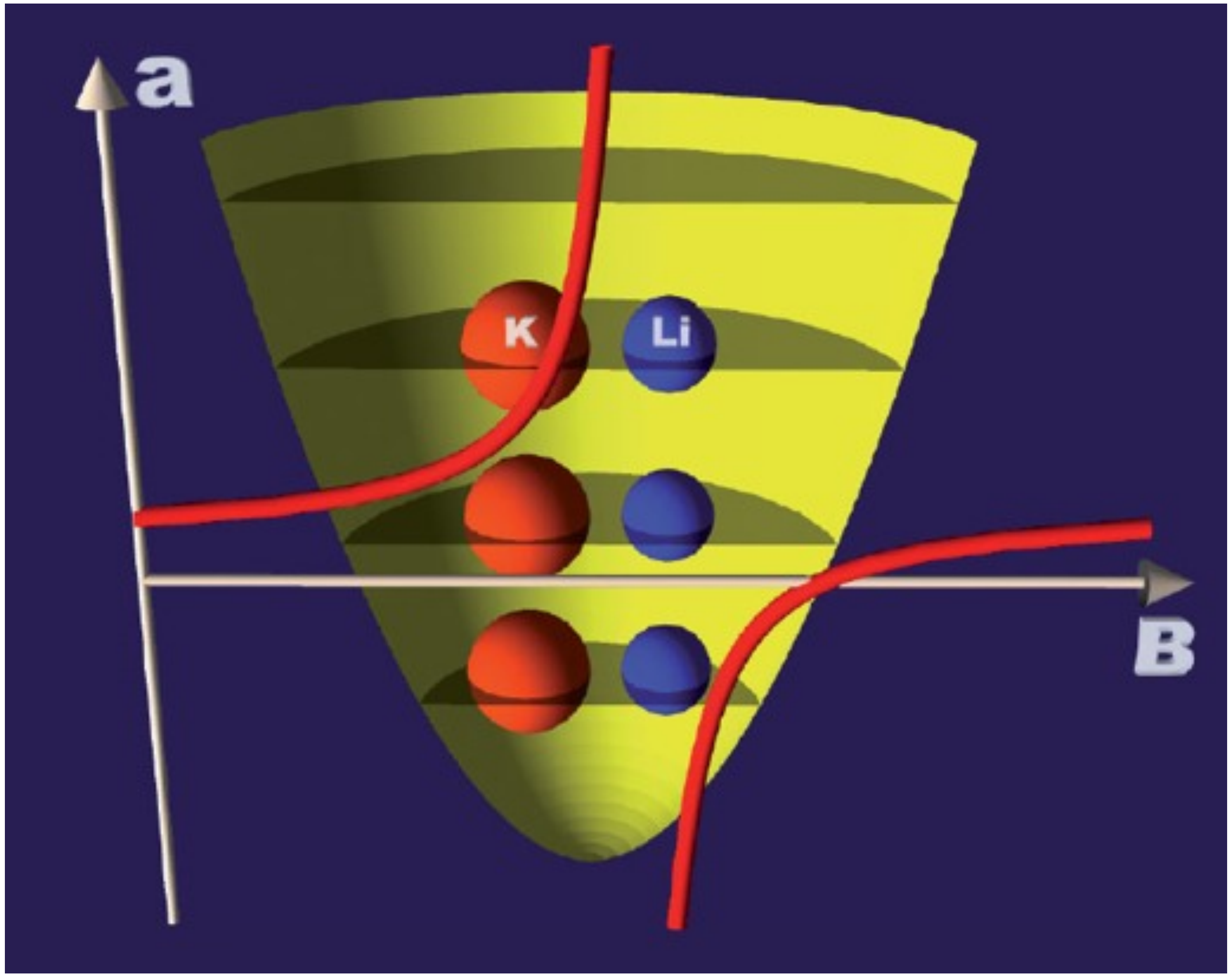
"Let Nature do the calculation"

- *Quantum*
- *High degree of tuning*
- *Table-top size*

- Lattice models → high Tc superconductors
- Artificial gauge fields → topological states of matter
- Precision measurements → equation of state of neutrons
- Few-body physics → quantum chemistry
- Single atom manipulation → quantum computer

Feshbach resonance

Tunable interactions in ultracold gases

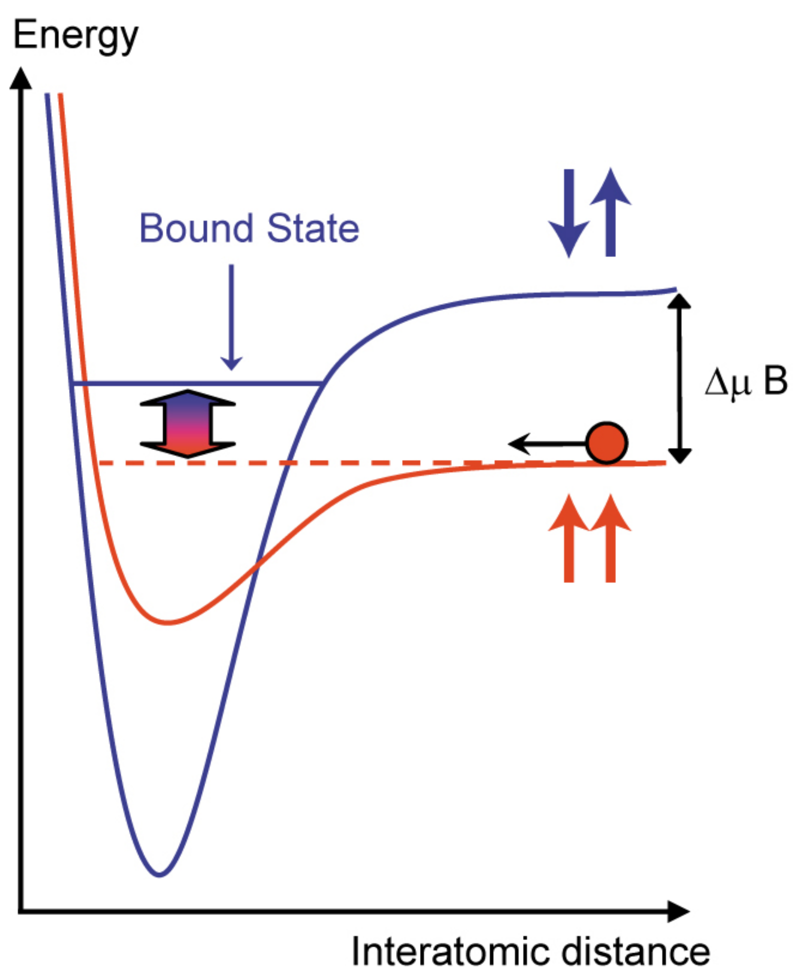


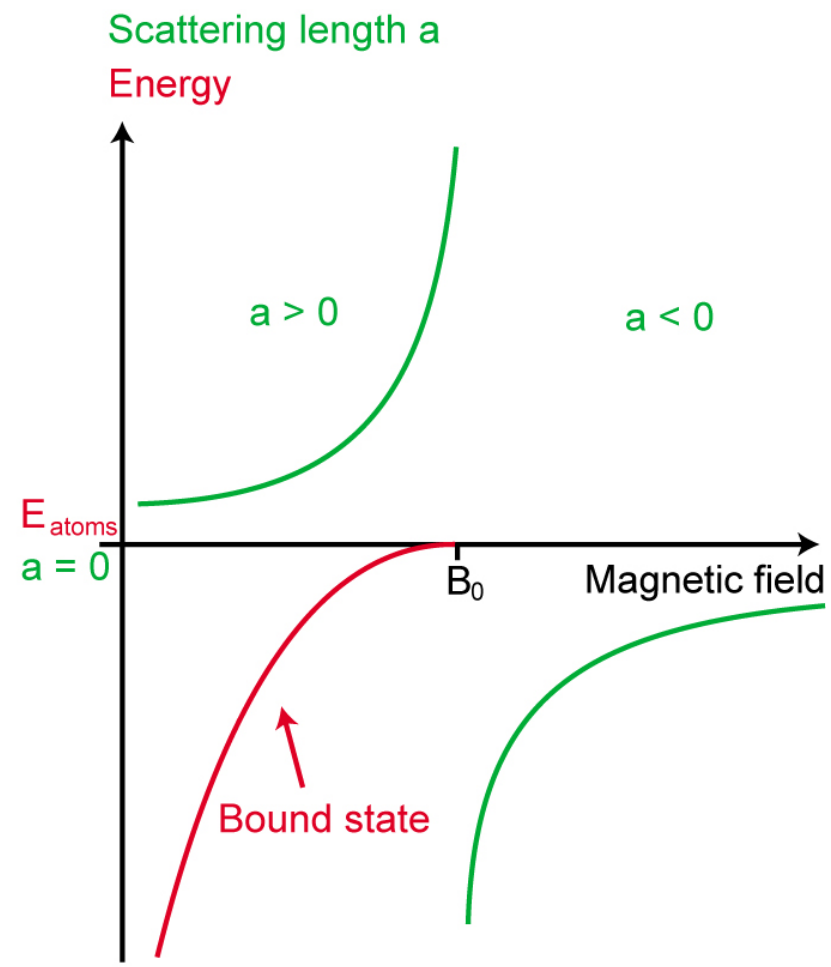
Innsbruck

Interaction strength tuned by magnetic field B

Feshbach resonance

Tunable interactions in ultracold gases





MIT

Resonance phenomenon: $a(B) \approx a_{\text{bg}}\left[1 + \frac{\Delta B}{B - B_0}\right]$