Mass dependent energies and scattering lengths for three and four-particle two-component systems under 1D confinement

Nirav P. Mehta

Trinity University

Department of Physics and Astronomy

Outline of this talk

I. Current experiments with cold atoms

II. Two-Body problem

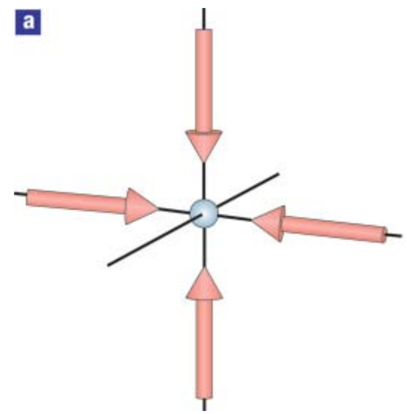
III. Three-body HHL Problem
   a) The Born-Oppenheimer potential curve
   b) Numerical results and comparison with more accurate methods

IV. Four-body HHHL Problem
   a) The coordinates
   b) The Born-Oppenheimer surface
   c) Numerical results

V. Current & future work

Atoms in waveguides

- Tune the laser frequency a little to the red of an atomic transition
- The "AC Stark shift" results in an effective potential energy well .



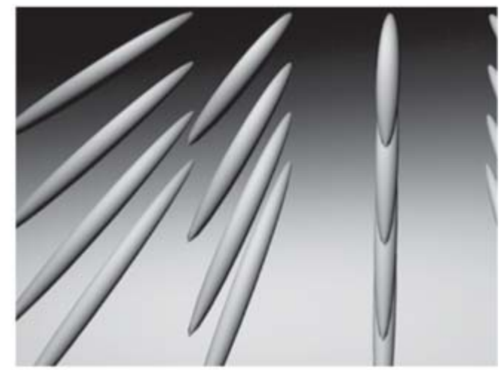
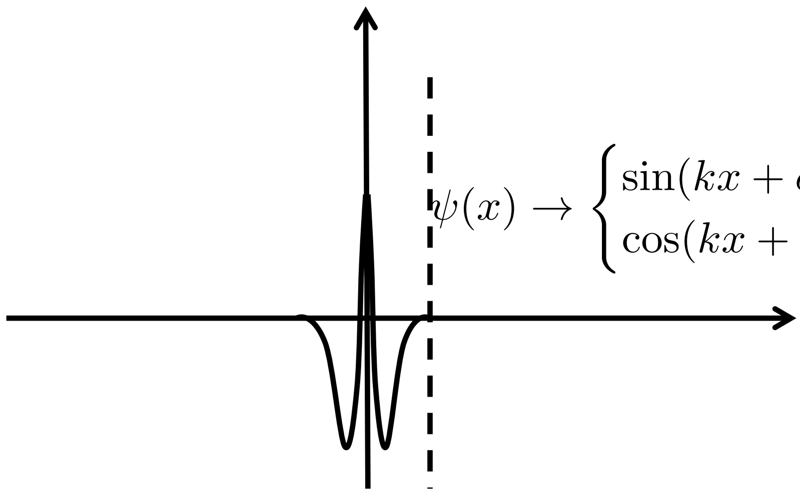


Image from: Immanuel Bloch
Nature Physics **1**, 23 - 30 (2005)

Assumption for this work: Atoms remain confined to the lowest transverse mode, and the Olshanii formula is meaningful.

$$a = -\frac{a_\perp^2}{2a_{3D}}\left(1 - C\frac{a_{3D}}{a_\perp}\right) \qquad C \approx 1.4603$$

Low-energy (2-body) scattering in 1D



$$\psi(x) \to \begin{cases} \sin(kx+\delta) \text{ odd} \\ \cos(kx+\delta) \text{ even} \end{cases}$$

$$\frac{1}{a} = \lim_{k\to 0} \begin{cases} -k\cot\delta & \text{(odd)} \\ k\tan\delta & \text{(even)} \end{cases}$$

For $V(x) = g\delta(x)$,

$$a = -1/(\mu g)$$

$$V(x) = \frac{-1}{\mu\, a}\,\delta(x)$$

For one heavy (H) and one light (L):

$$\beta = \frac{m_L}{m_H}$$

$$\mu_{HL} = m_H \frac{\beta}{1+\beta}$$

$$B_2 = \frac{1}{m_H a^2}\frac{\beta+1}{2\beta}$$

Some questions I want to answer:

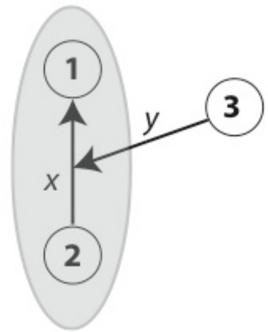
- What are the energy levels for the HHL and HHHL system?
- What is the atom-dimer a_{AD} scattering length for H+HL → H+HL ?
- What is the atom-trimer a_{AT} scattering length for H+HHL→H+HHL ?
- What are the specific mass ratios at which a new bound state appears (and a_{AD}, a_{AT} diverge)?
- What are the specific mass ratios at which a_{AD} or a_{AT} is zero?

Born-Oppenheimer approxiamtion for the 3-body problem

In units of the HL binding energy:

$$
\left.
\begin{aligned}
&[\hat{T}_{\mathrm{H}} + \hat{V}_{\mathrm{HH}}]\Psi \quad \left\{ \; -\frac{1}{2\mu_3}\frac{\partial^2\Psi}{\partial x^2} + g_3\lambda\delta(2x_0)\Psi \right. \\
&\hat{H}_{\mathrm{ad}}\Psi \quad \left\{ \; -\frac{1}{2\mu_3}\frac{\partial^2\Psi}{\partial y^2} + g_3\left[\delta(y+x_0)+\delta(y-x_0)\right]\Psi \right. \\
&= E\Psi
\end{aligned}
\right.
$$

Jacobi Coordinates:



$$\mu_3 = \frac{1+\beta}{2\sqrt{\beta(2+\beta)}}$$

$$g_3 = -2\sqrt{2}\left(\frac{\beta}{2+\beta}\right)^{1/4}$$

$$x_0 = x\sqrt{\frac{\beta}{2+\beta}}$$

Born-Oppenheimer factorization:

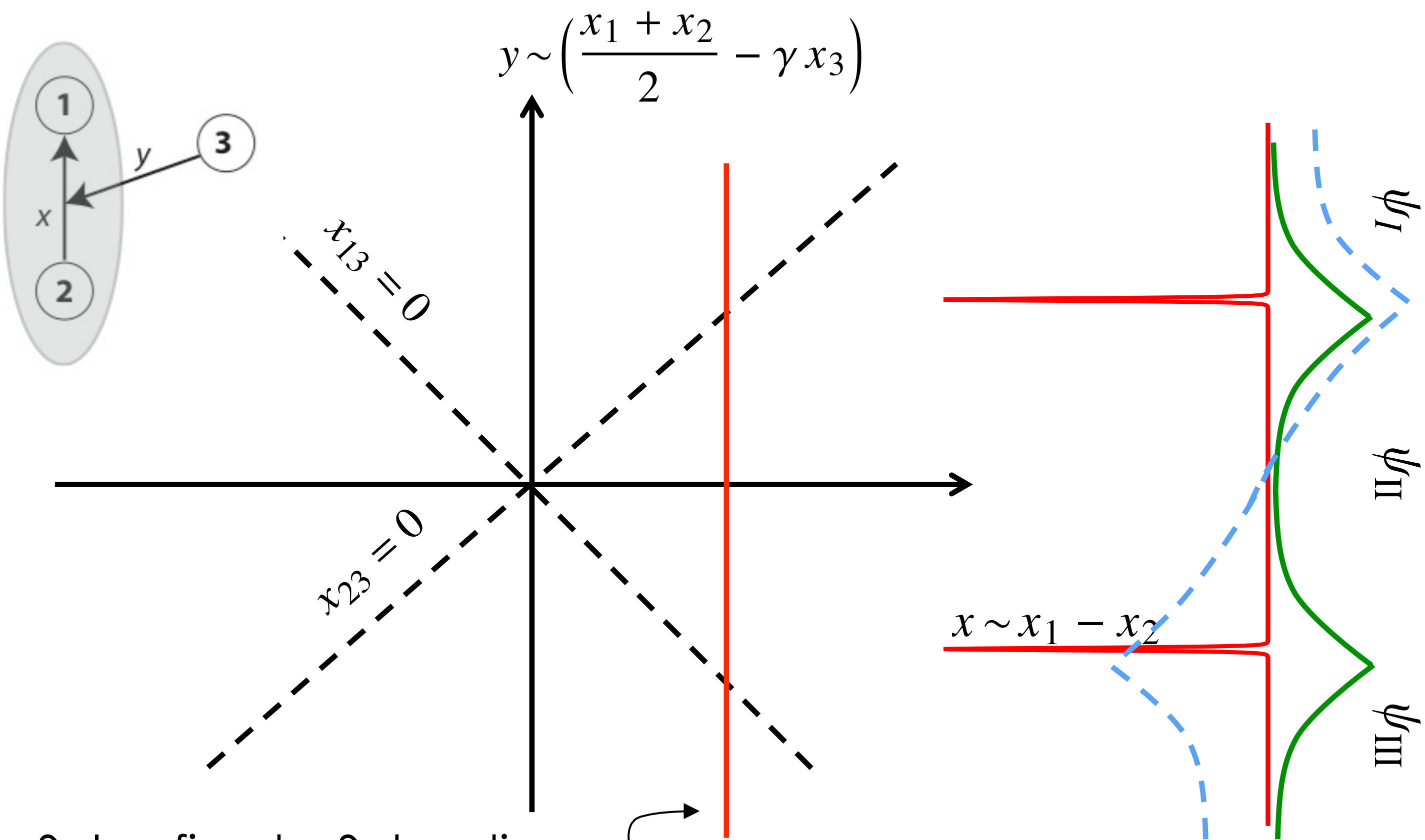
$$\Psi(x,y) = \Phi(x;y)\psi(x)$$

$$\left[\frac{-1}{2\mu_3}\frac{\partial^2}{\partial y^2} + g_3\left(\delta(y+x_0)+\delta(y-x_0)\right)\right]\Phi(x;y) = u(x)\Phi(x;y),$$

$$\left(\frac{-1}{2\mu_3}\frac{\partial^2}{\partial x^2} + g_3\lambda\delta(2x_0) + u(x) + \frac{\tilde{Q}(x)}{2\mu_3}\right)\psi(x) = E\psi(x)$$

$$\tilde{Q}(x) = \left\langle \frac{\partial\Phi}{\partial x} \middle| \frac{\partial\Phi}{\partial x} \right\rangle_y$$

3-Body HHL problem



Solve fixed-x Schrodinger equation along this line…

… with this potential for x-*dependent* eigenvalues *u(x)*.

Obtain a transcendental equation for the eigenvalue:

$$\frac{\kappa}{g_3 \mu_3} + 1 = (-1)^{P+1} e^{-2\kappa x_0}.$$

The nonadiabatic correction

$$\frac{\kappa}{g_3\mu_3} + 1 = (-1)^{P+1} e^{-2\kappa x_0}.$$

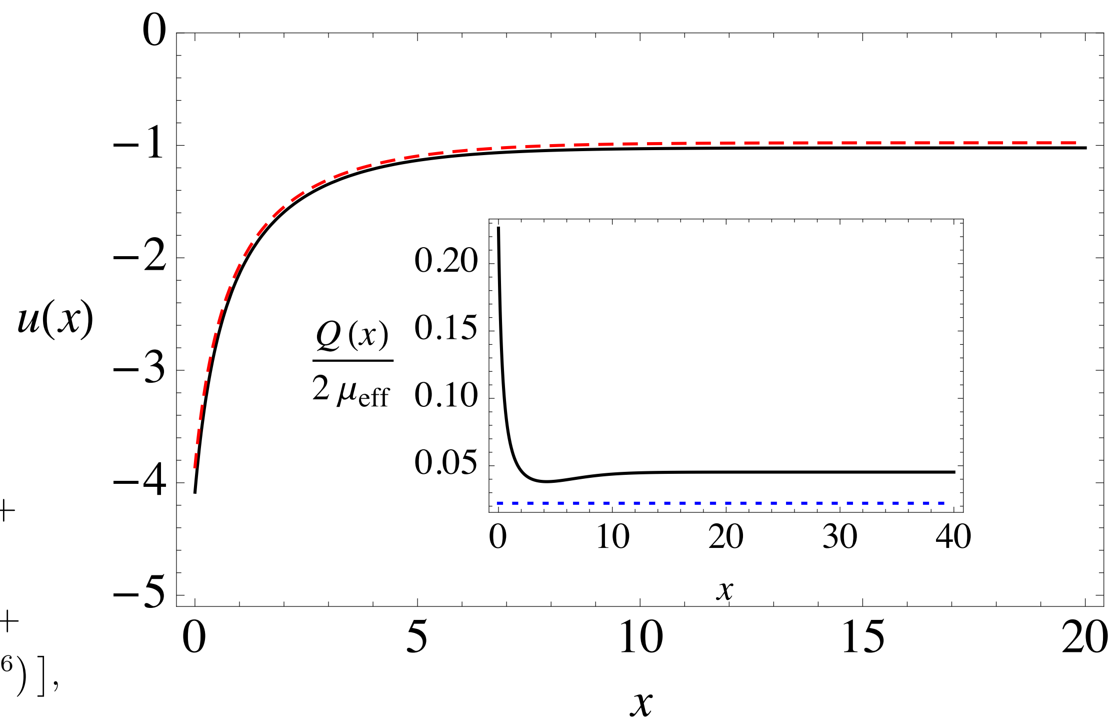
$$\kappa(x) = \sqrt{-2\mu_3 u(x)}$$

$$\tilde{Q}(x) = \left\langle \frac{\partial \Phi}{\partial x} \middle| \frac{\partial \Phi}{\partial x} \right\rangle_y$$

$$\tilde{Q}(x) = \frac{\beta}{3(2+\beta)\left(-2hx_0 + 2\kappa x_0 + 1\right)^4}$$
$$\times \left[3h^2 + x_0\left(-12h^3 + 24h^2\kappa - 36h\kappa^2 + 24\kappa^3\right) + x_0^3\left(-16h^3\kappa^2 + 48h^2\kappa^3 - 48h\kappa^4 + 16\kappa^5\right) + x_0^2\left(12h^4 - 48h^3\kappa + 108h^2\kappa^2 - 120h\kappa^3 + 48\kappa^4\right) + x_0^4\left(-16h^4\kappa^2 + 64h^3\kappa^3 - 96h^2\kappa^4 + 64h\kappa^5 - 16\kappa^6\right)\right],$$

$$h = \mu_3 g_3$$

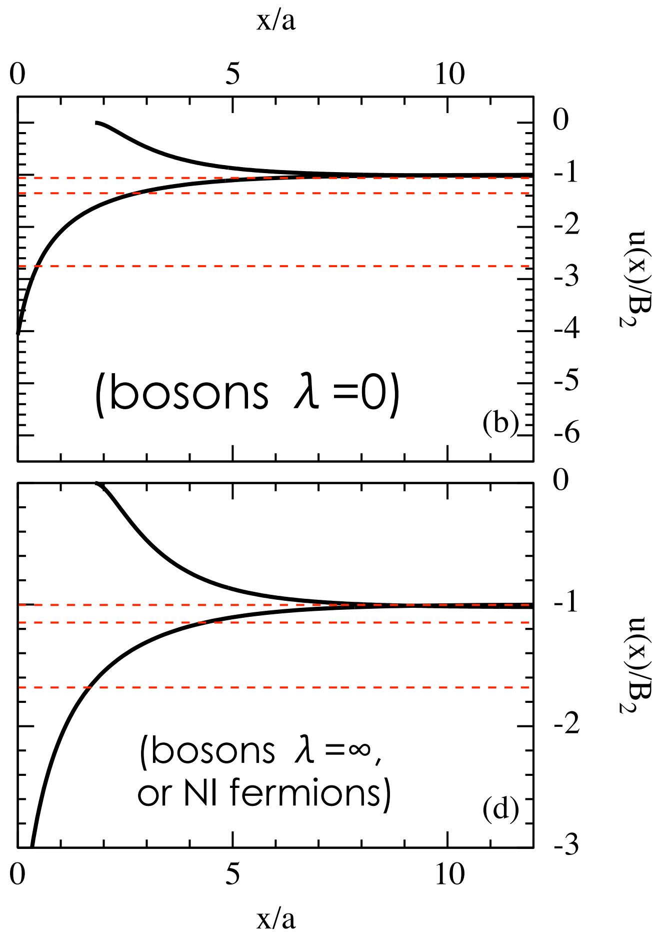
$$\lim_{|x|\to\infty} u(x) = -1 - \frac{\beta}{2+\beta}$$

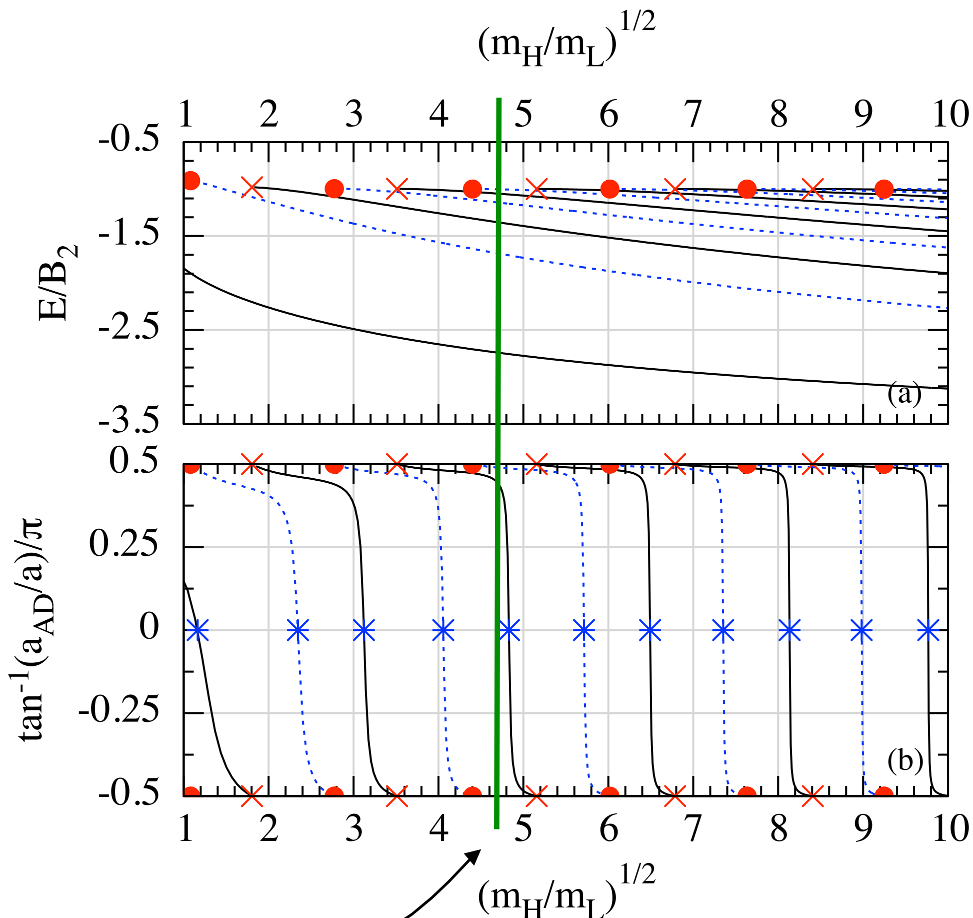


$$\lim_{|x|\to\infty}\left[u(x) + \frac{\tilde{Q}}{2\mu_3}\right] = -1 + \left(\frac{\beta}{2+\beta}\right)^2.$$

Results for the HHL system

For Li-Cs mixtures ($m_H/m_L \approx 22$)





These potential curves give the spectrum & a_{AD} for one mass ratio

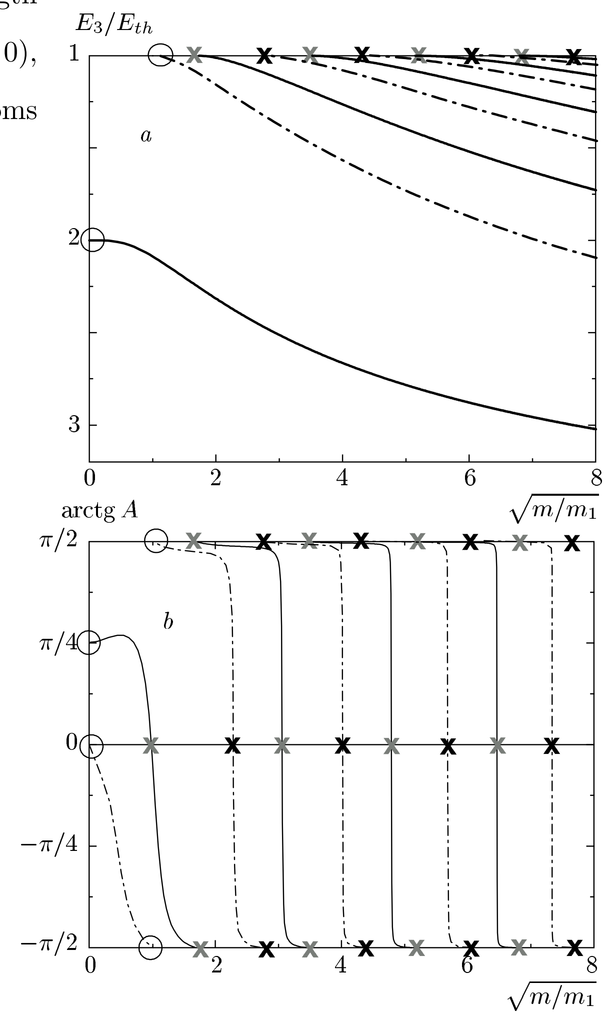
Comparison with Kartavtsev, Malykh and Sofianos

TABLE I. The values of the mass ratio $\beta^{-1} = m_H/m_L$ for which the atom-dimer scattering length is infinite ($a_{AD} \to \infty$, corresponding to the appearance of the n^{th} trimer state), or zero ($a_{AD} \to 0$), are tabulated both the case of noninteracting bosonic H atoms ($\lambda \to 0$) and fermionic H atoms ($\lambda \to \infty$). Results are compared to Ref. [23]. An asterisk (*) denotes an exact result.

n	$\lambda = 0$ $\beta^{-1}\|_{a_{AD}\to 0}$		$\lambda = 0$ $\beta^{-1}\|_{a_{AD}\to\infty}$	
	This work	Ref. [23]	This work	Ref. [23]
1	–	–	–	–
2	1.357	0.971	3.255	2.86954
3	9.747	9.365	12.336	11.9510
4	23.333	22.951	26.602	26.218
5	42.142	41.762	46.055	45.673
6	66.168	65.791	70.695	70.317
7	95.404	95.032	100.523	100.151
8	129.845	129.477	135.539	135.170
9	169.488	169.120	175.742	175.374
10	214.331	213.964	221.133	220.765

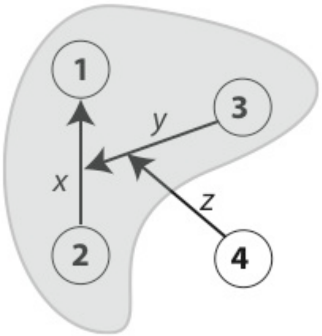
n	$\lambda = \infty$ $\beta^{-1}\|_{a_{AD}\to 0}$		$\lambda = \infty$ $\beta^{-1}\|_{a_{AD}\to\infty}$	
	This work	Ref. [23]	This work	Ref. [23]
1	–	0*	1.170	1*
2	5.499	5.2107	7.694	7.3791
3	16.456	16.1197	19.373	19.0289
4	32.650	32.298	36.235	35.879
5	54.067	53.709	58.283	57.923
6	80.697	80.339	85.518	85.159
7	112.535	112.179	117.940	117.583
8	149.577	149.222	155.550	155.193
9	191.820	191.463	198.347	197.989
10	239.262	238.904	246.331	245.973

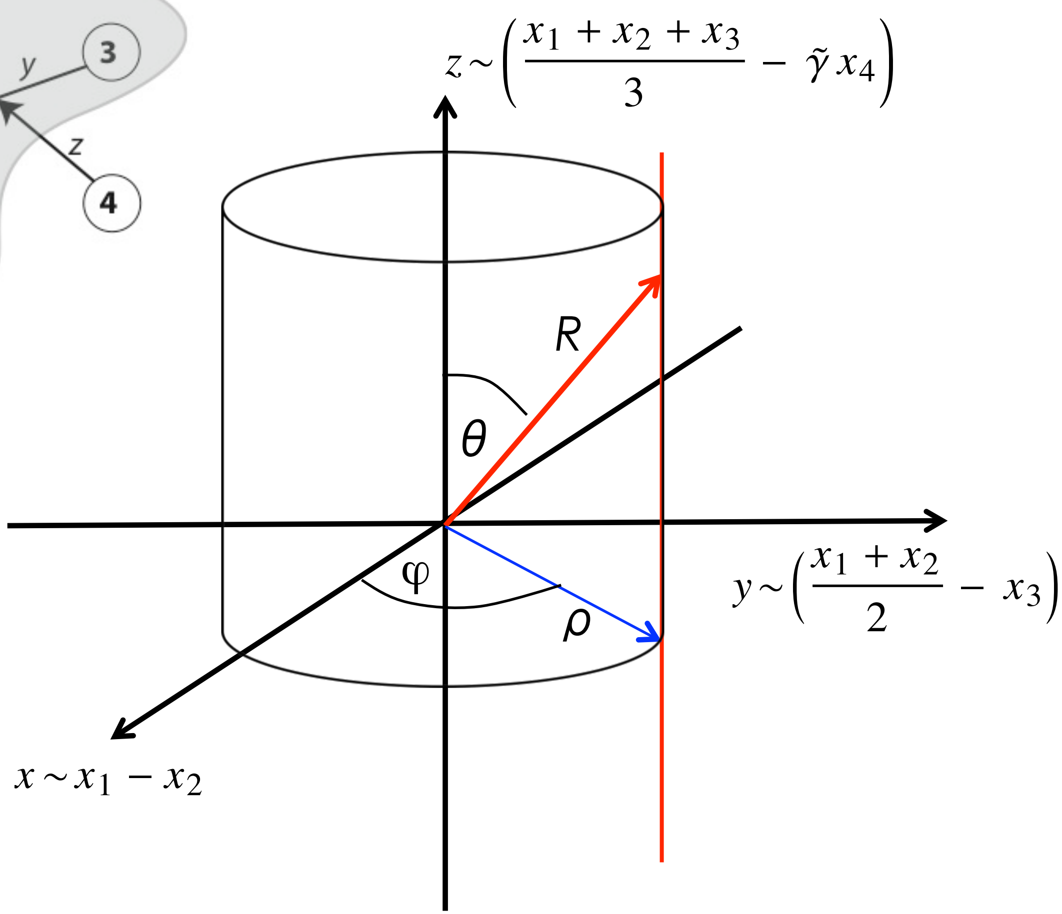
From KM&S



O. I. Kartavtsev, A. V. Malykh, and S. A. Sofianos, ZhETF 135, 419 (2009)

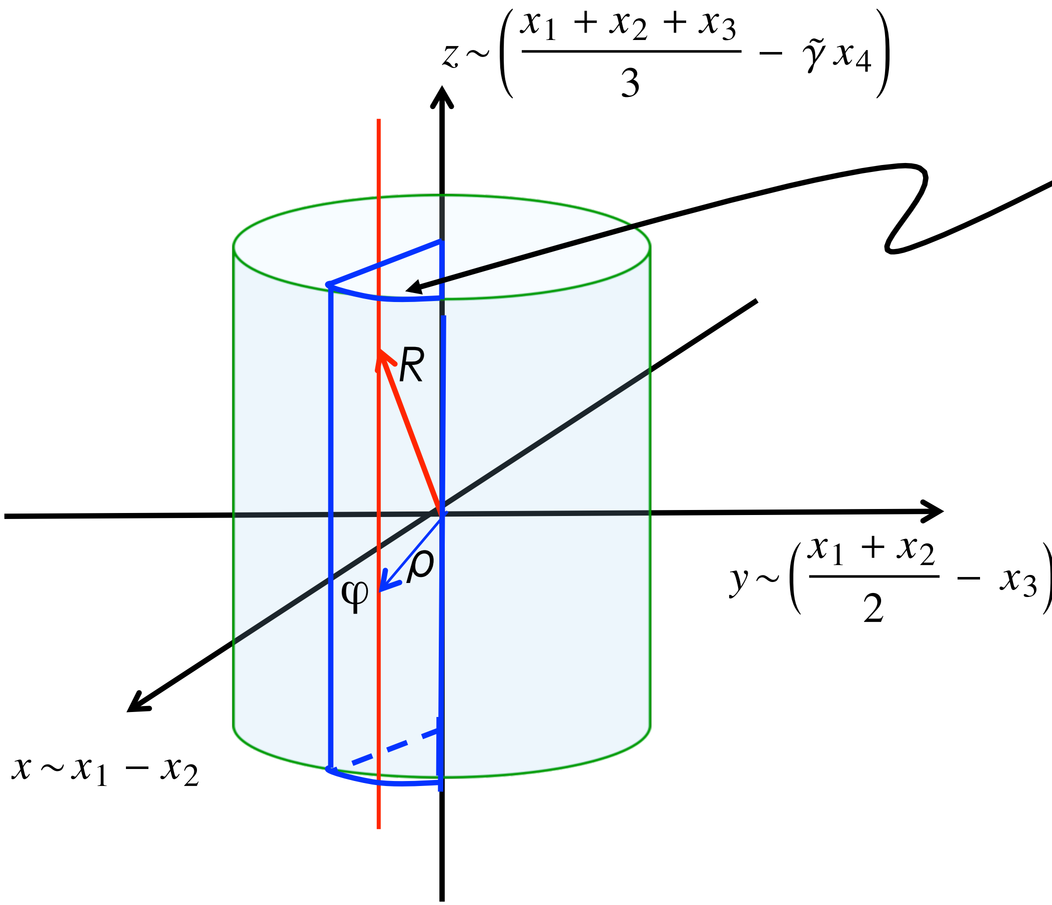
4-Body HHHL coordinates





- Heavy particle coordinates make the x-y plane
- Because H-particles are identical, only need $0 < \varphi < \pi/6$ (for a given parity)
- Solve the fixed (ρ, φ) Schrodinger equation (along the red line)
- Plot the eigenvalue $U(\rho, \varphi)$.

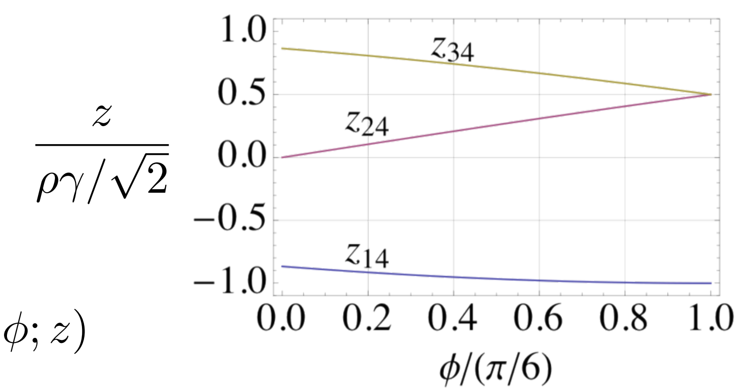
Where are the interactions?



Heavy particles have a definite ordering within this wedge

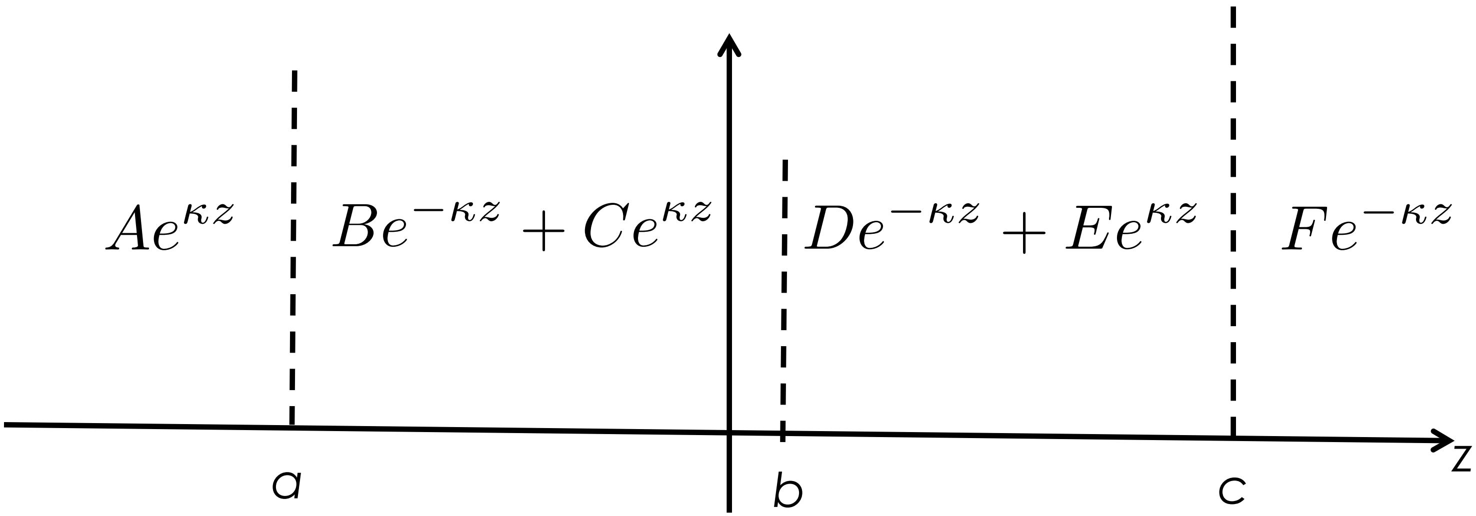
δ - function spikes when:

$$z = \begin{cases} -\frac{\rho\gamma}{\sqrt{2}} \sin\left(\phi + \pi/3\right) \\ -\frac{\rho\gamma}{\sqrt{2}} \sin\left(\phi - \pi/3\right) \\ \frac{\rho\gamma}{\sqrt{2}} \sin\left(\phi\right) \end{cases}$$



$$\left(\frac{-1}{2\mu_4}\frac{\partial^2}{\partial z^2} + g_4 \sum_{i=1}^{3} \delta(z - z_i)\right)\Phi(\rho, \phi; z) = U(\rho, \phi)\Phi(\rho, \phi; z)$$

The triple delta-function problem

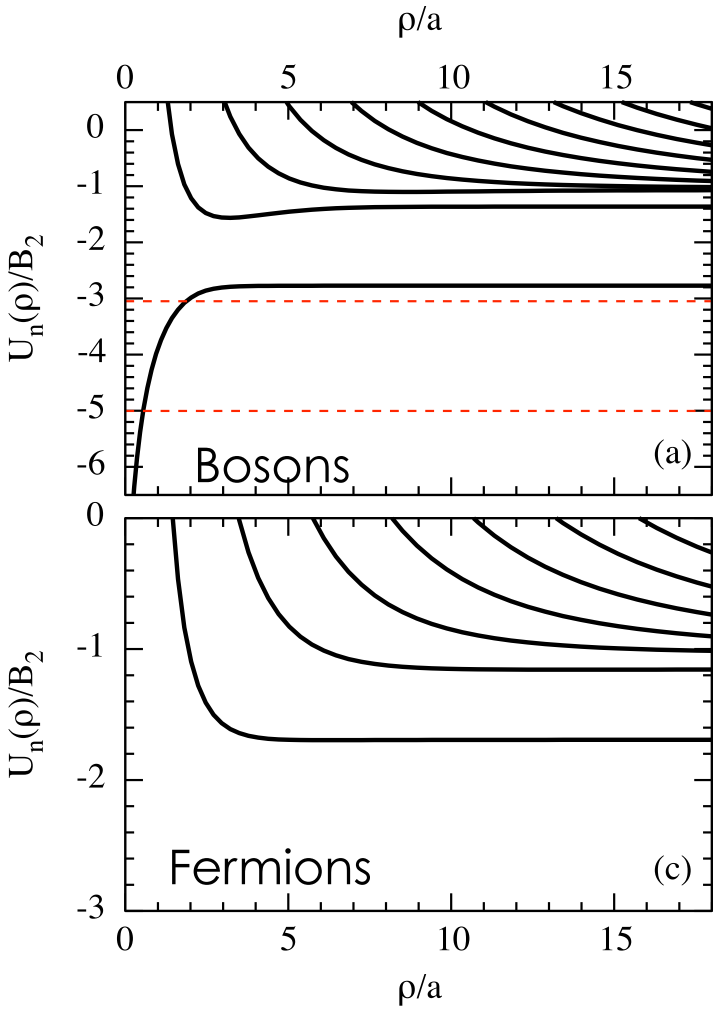


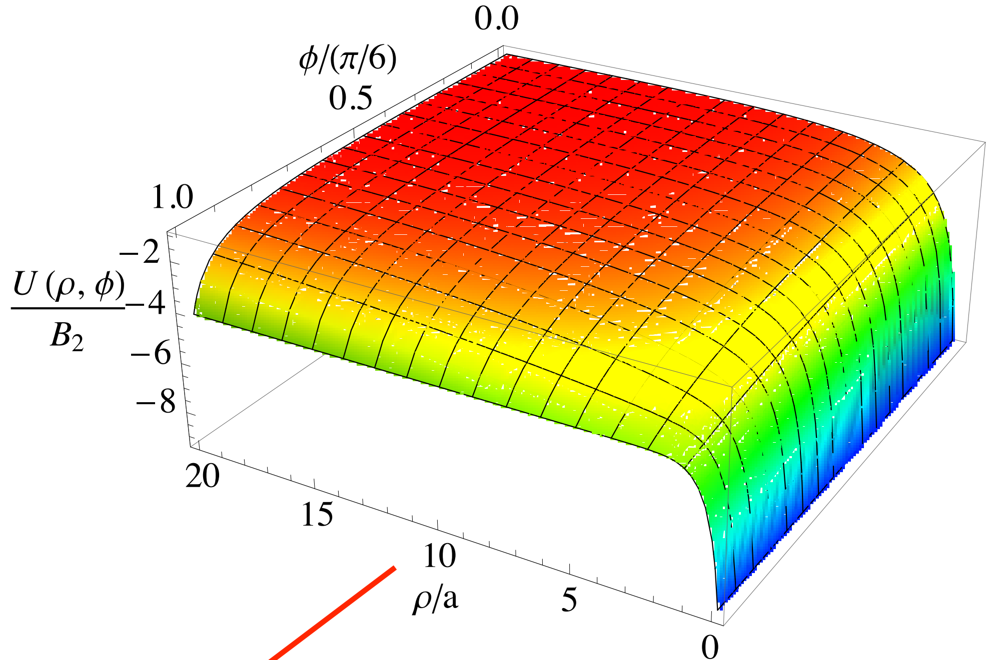
$$\left[-\frac{\partial^2}{\partial z^2} + g_a\delta(z-a) + g_b\delta(z-b) + g_c\delta(z-c)\right]\Phi(z) = -\kappa^2\Phi(z)$$

$$g_a g_c (g_b - 2\kappa)e^{2\kappa(a+b)} - g_a g_b (g_c + 2\kappa)e^{2\kappa(a+c)}$$
$$+ (g_a + 2\kappa)(g_b + 2\kappa)(g_c + 2\kappa)e^{2\kappa(b+c)}$$
$$- g_b g_c e^{4b\kappa}(g_a + 2\kappa) = 0$$

$$g_a = g_b = g_c = 2\mu_4 g_4,$$
$$\kappa^2 = -2\mu_4 U(\rho, \phi) > 0,$$
$$a = z_1,\ b = z_2 \text{ and } c = z_3.$$

HHHL potential energy surface & curves (for $m_H/m_L \approx 22$) (Even parity)



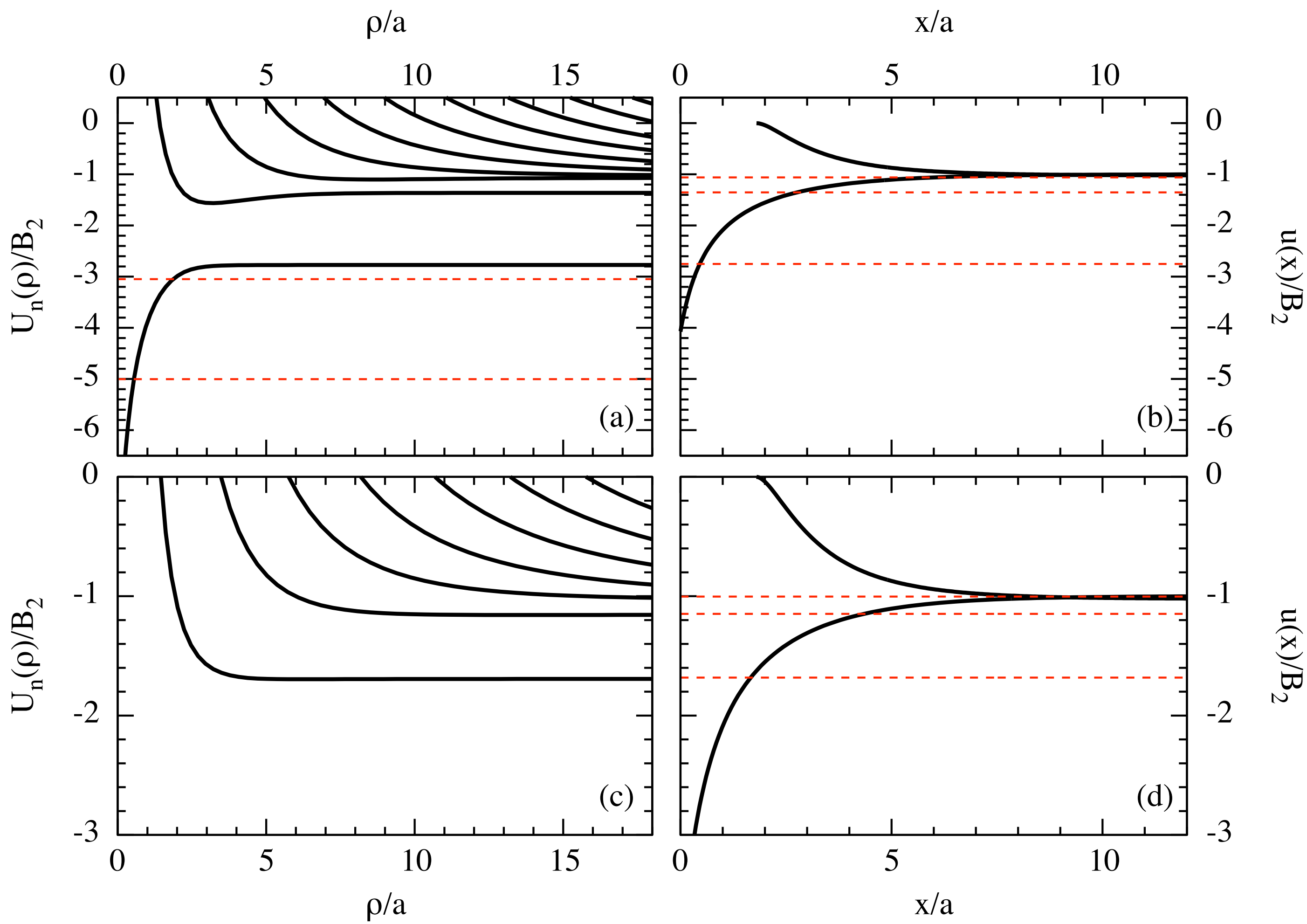


Solve the fixed-ρ equation to get "hyperspherical" curves

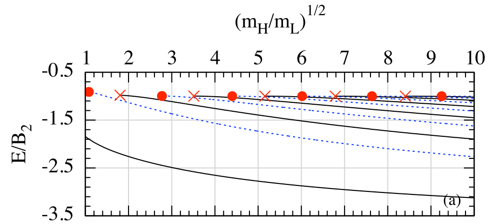
The energy landscape ($m_H/m_L \approx 22$)

Bosons

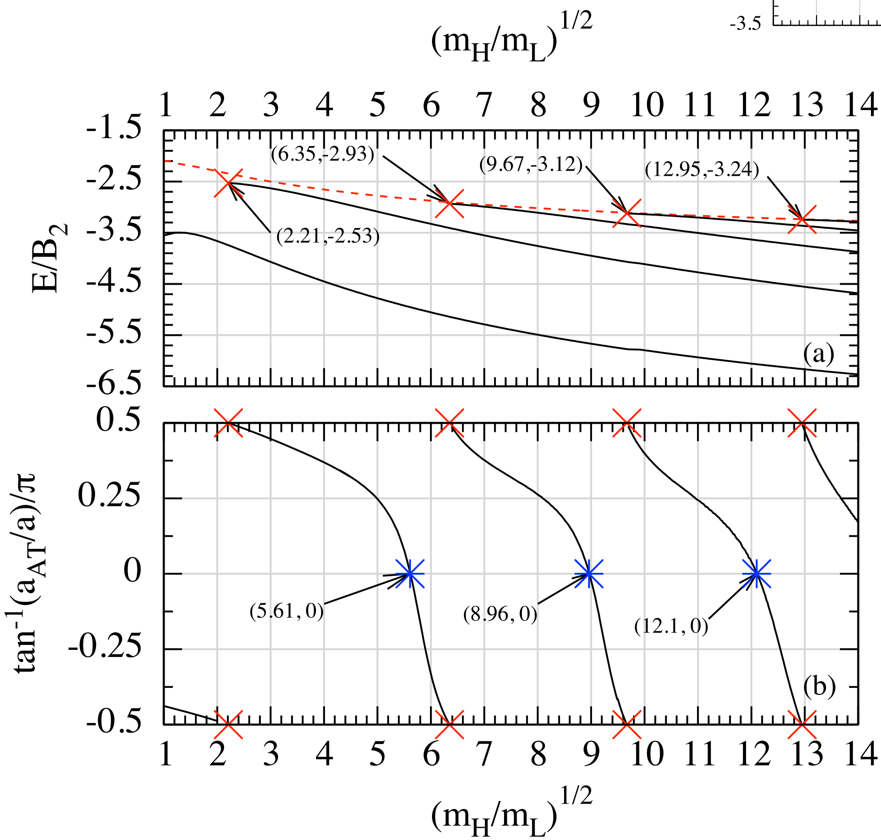
Fermions



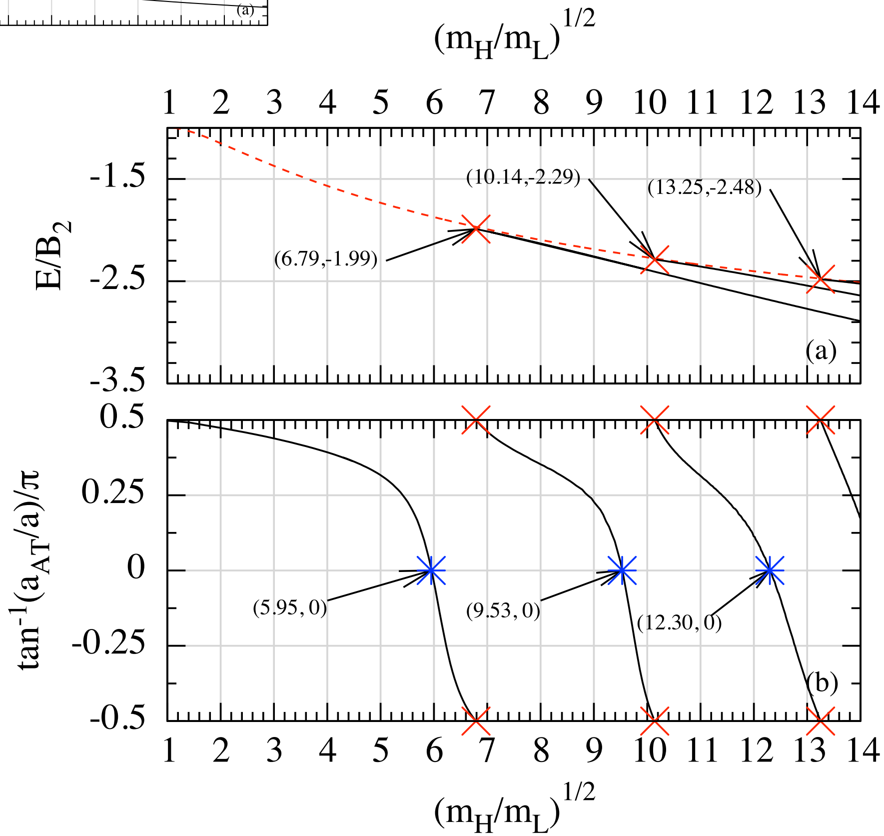
The HHL Spectrum and the atom-trimer scattering length a_{AT} *(Even Parity)*



Bosons



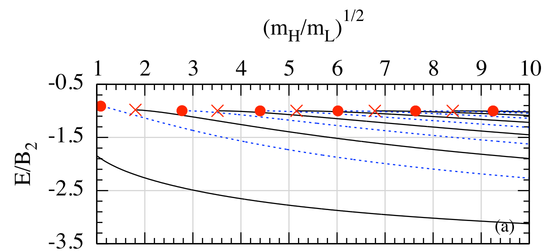
Fermions



The HHL Spectrum and the atom-trimer scattering length a_{AT} *(Odd Parity)*

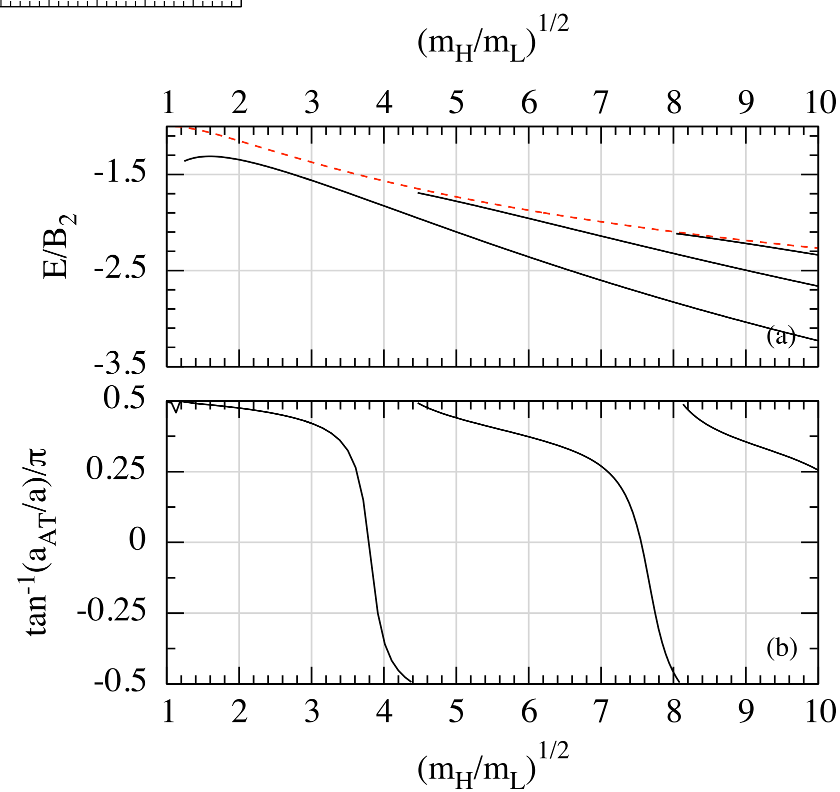
*preliminary results

Bosons



?

Fermions



Current and Future Work

- Treat arbitrary heavy particle interactions. (not just the noninteracting, or infinitely repulsive cases.)
- Construct an HHHL "phase diagram".
- Add harmonic confinement, compare with recent publications.
- Work towards a fully 3D 3-body problem *with* a cigar-trap?
- Other systems like HLL, HHLL, HLLL?
- Thanks to Jose D'Incao and Jesper Levinson for helpful discussions. Thanks also to C.H. Greene for early inspiration to start this problem.

THANK YOU!