Dressed impurities in an ideal Fermi gas: an (N+1)-body problem, with N>>1

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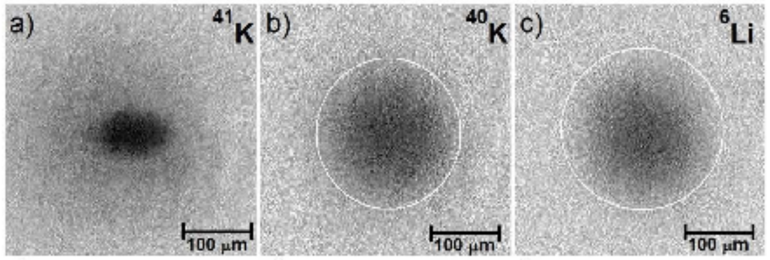
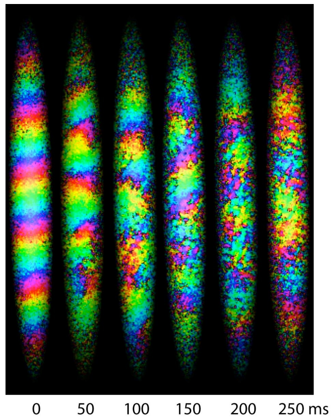
Introduction
Quantum Mixtures in CondMat



- Mixtures of fermionic/bosonic atoms
  (^{3}He-^{4}He, ultracold gases, neutron stars, Quark-Gluon Plasma, ...)



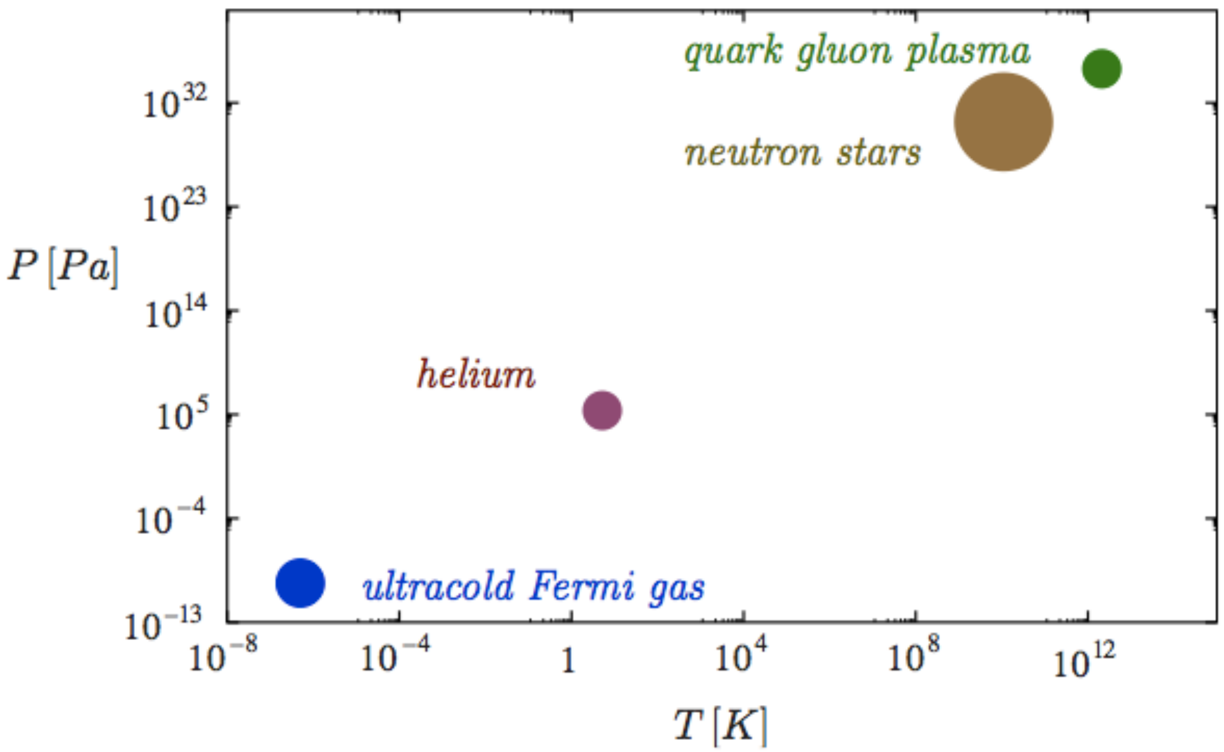
- Spinor gases, SU(N) invariant systems, ...

- Quantum magnets, quantum Hall systems, and spin-liquids

- Unconventional and multi-band superconductors

Despite different microscopic origins, at low energies these systems can be described by emergent many-body theories exhibiting a significant degree of universality.

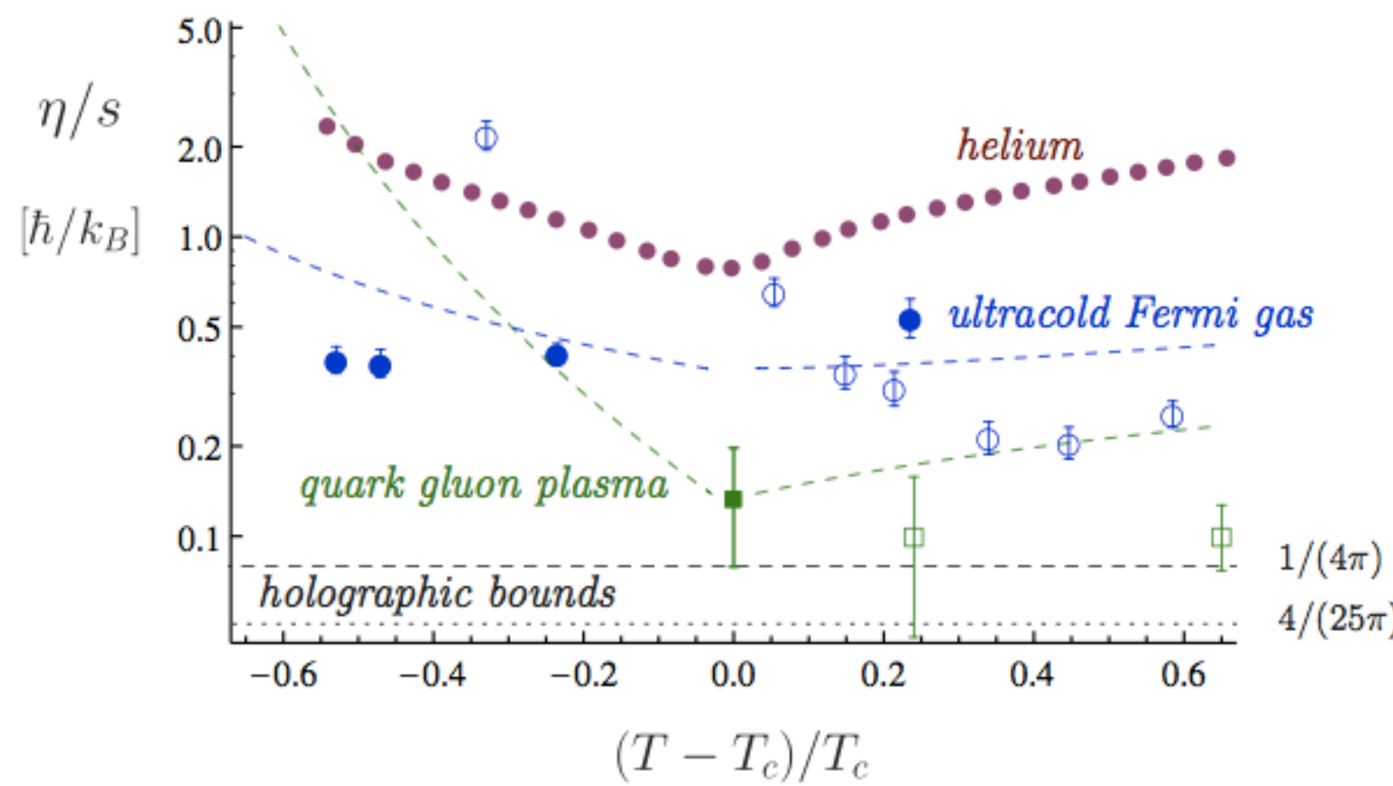
Universality in Quantum Mixtures



20 orders of magnitude difference in temperature

but similar transport properties!

example:
(shear viscosity/entropy density) close to Tc:



Quantum simulation with ultracold atoms

- chemical composition

- temperature
- interaction strength

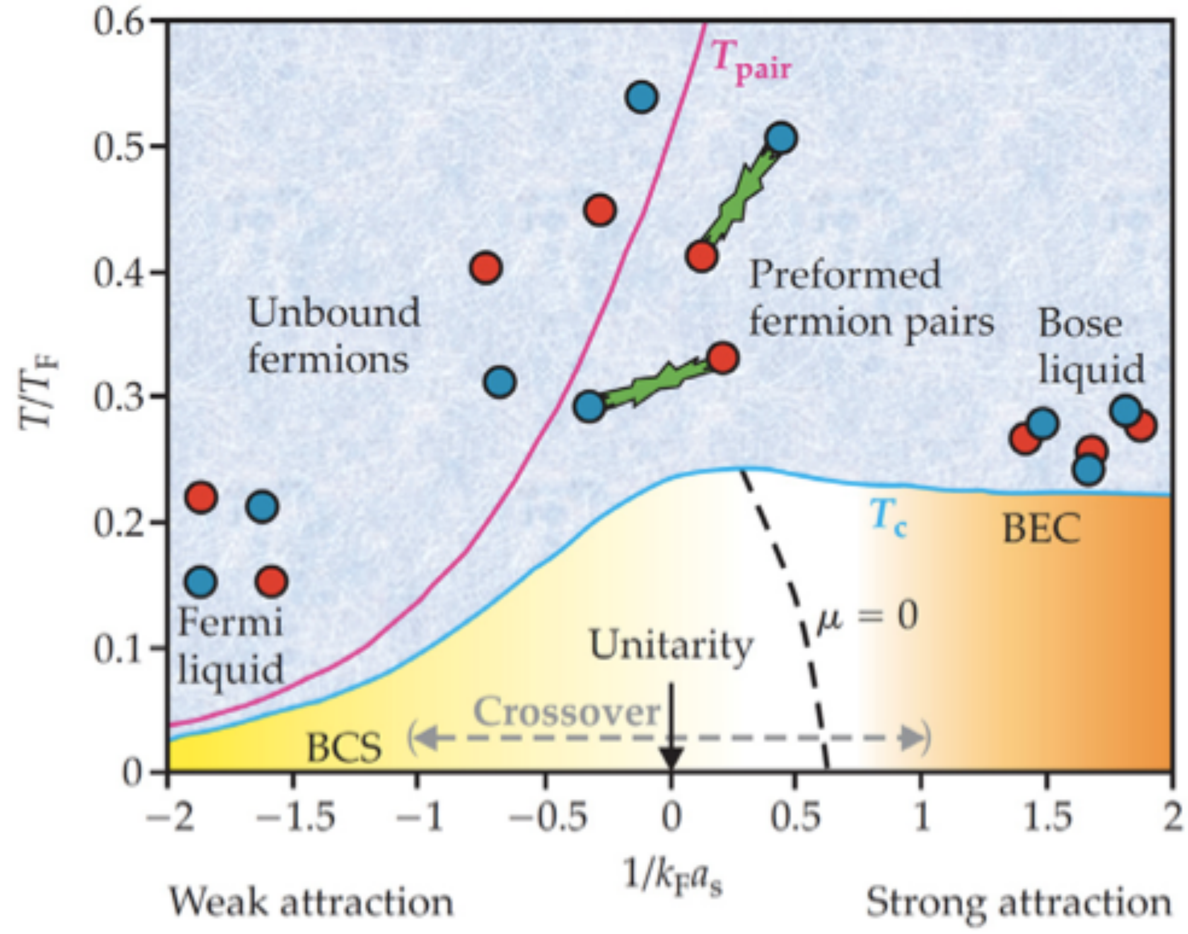
- periodic potentials
- physical dimension
- atom-light coupling

- exotic couplings
  (x-wave, spin-orbit, ...)
- dynamics

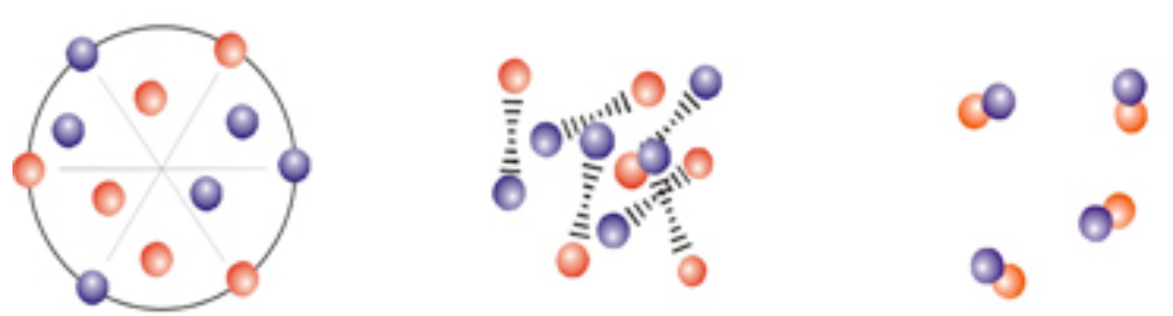
- disorder

- periodic driving
  (shaken optical lattices, ...)

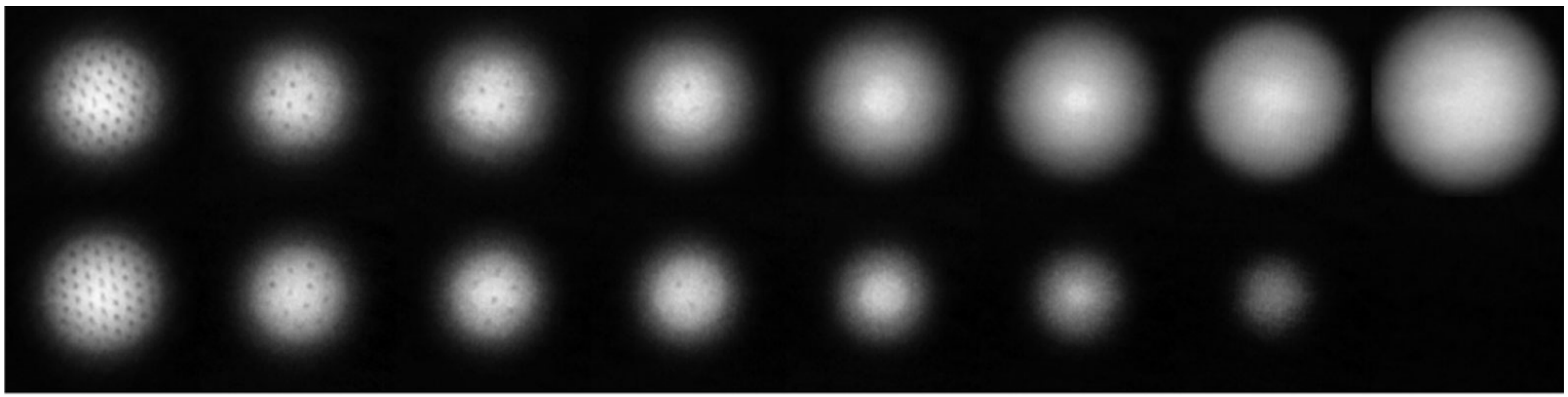
Attractive Fermi Mixtures



N=N : BCS-BEC crossover



Population-imbalanced attractive Fermi Mixtures



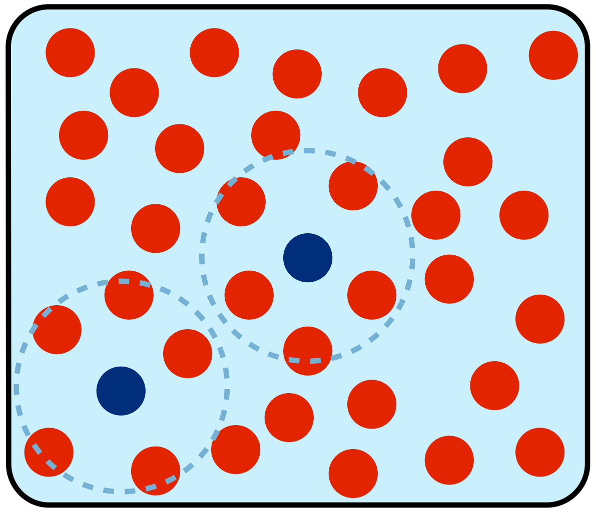
N=N N>N

SF-normal transition

Zwierlein et al., Nature 2005

Very imbalanced attractive Fermi mixtures

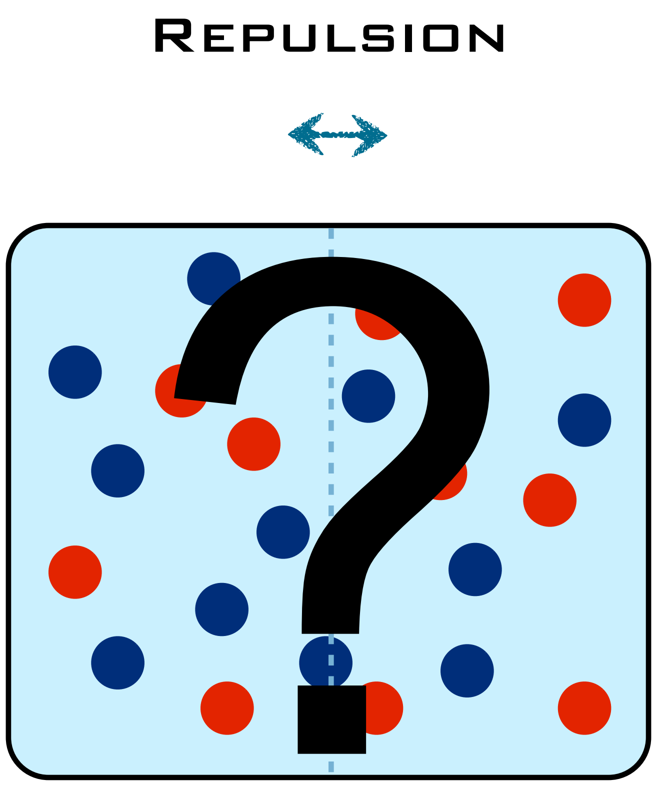
N>>N



polarons

Schirotzek et al., PRL 2009

Repulsive Fermi Mixtures

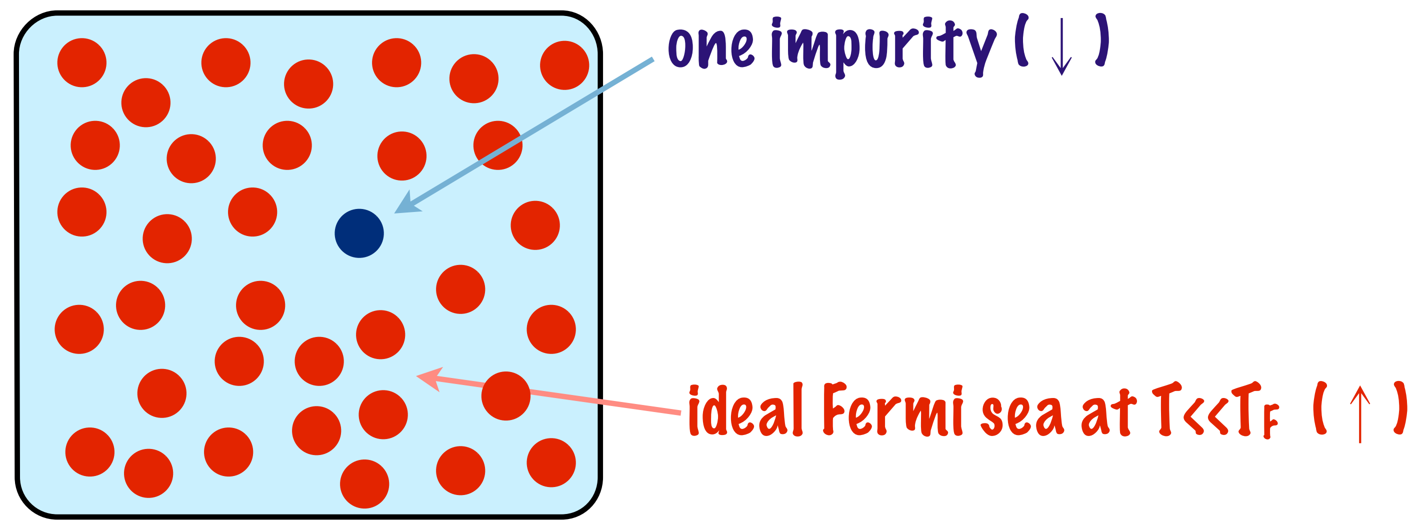


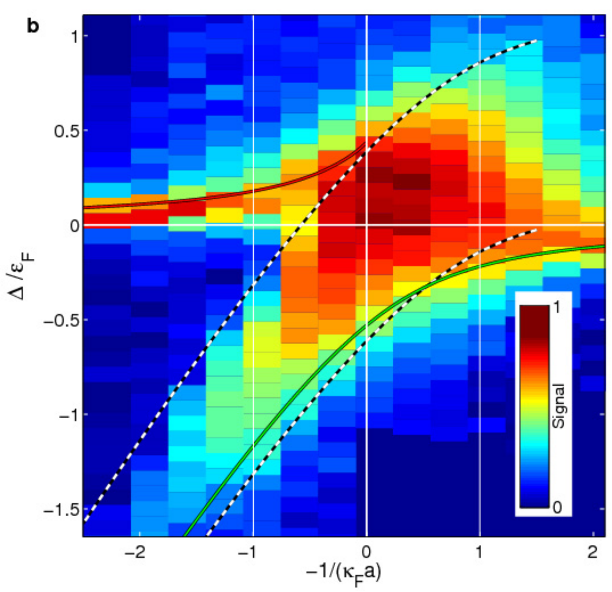
repulsion vs. Fermi pressure

Stoner's Itinerant Ferromagnetism

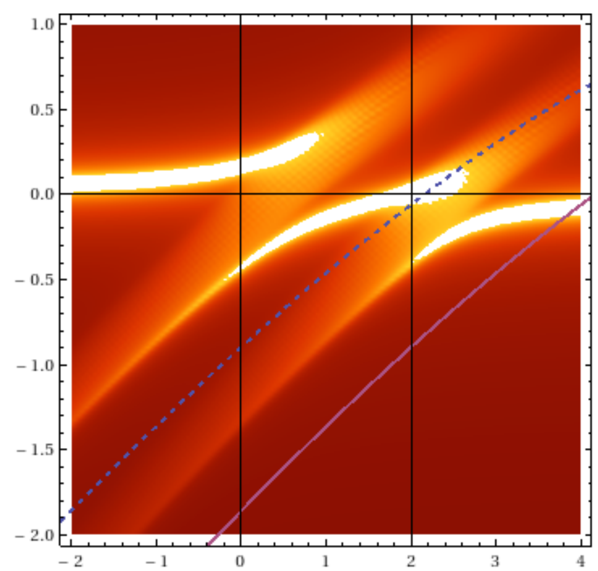
predicted in 1933, not yet realized..

Outline of this talk

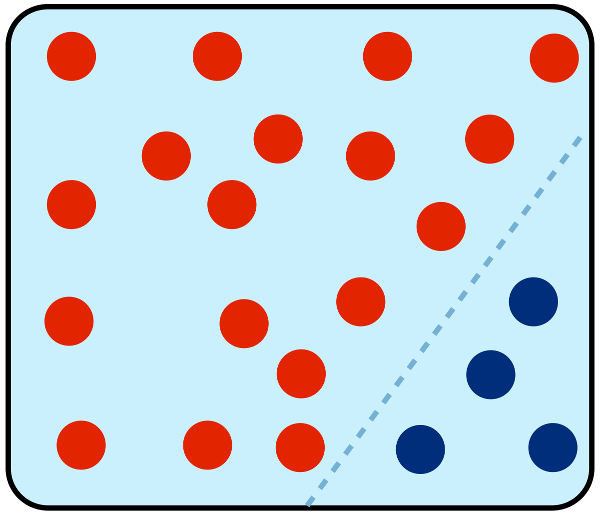




s-wave



p-wave



IFM

Motivation

Understanding the properties of a single impurity in a Fermi gas provides insight on:

- phase diagram of imbalanced Fermi gases
- coherence properties of fundamental quasiparticles
- their decay mechanisms

With p-wave interactions, superfluids may be polar, chiral, topological, ...

Routes towards Itinerant Ferromagnetism?

IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **77** (2014) 034401 (26pp)

doi:10.1088/0034-4885/77/3/034401

Report on Progress

Polarons, dressed molecules and itinerant ferromagnetism in ultracold Fermi gases

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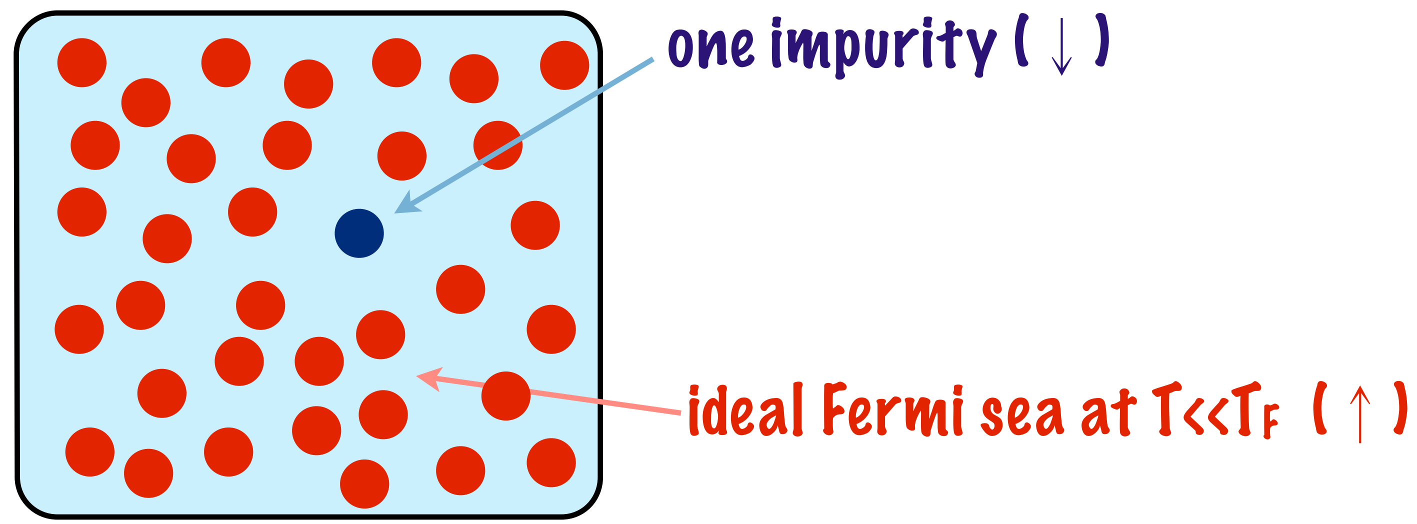
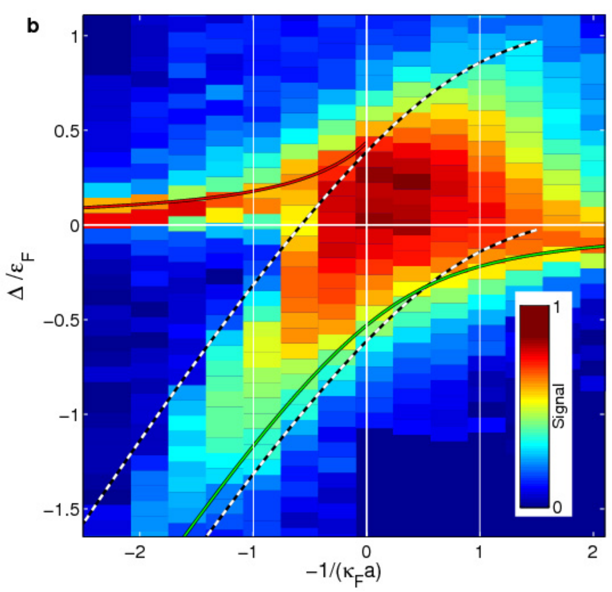
Received 20 December 2011, revised 19 December 2013
Accepted for publication 19 December 2013
Published 19 February 2014

a detailed review on:

- theoretical methods
- experimental probes and results
- mass imbalance
- reduced dimensionality
- decay processes





s-wave

p-wave

IFM

Many-body systems

(from Richard Mattuck's book)

2 A GUIDE TO FEYNMAN DIAGRAMS [0.0

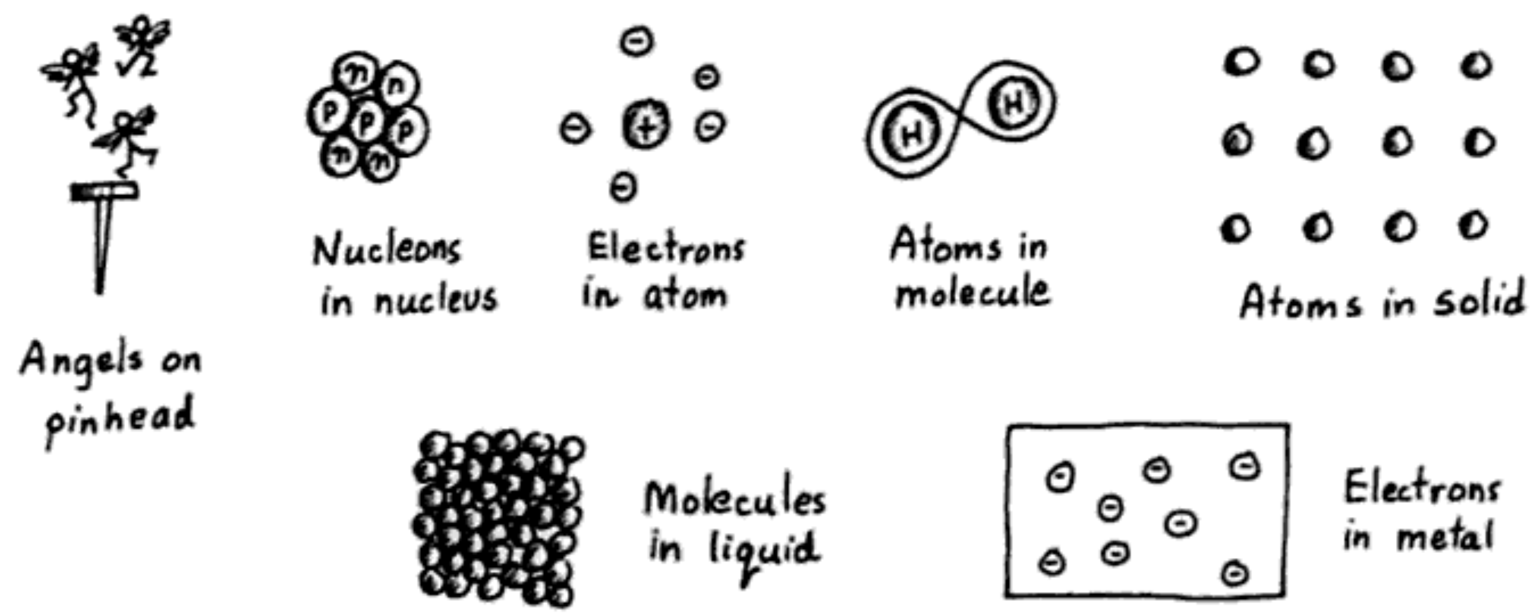
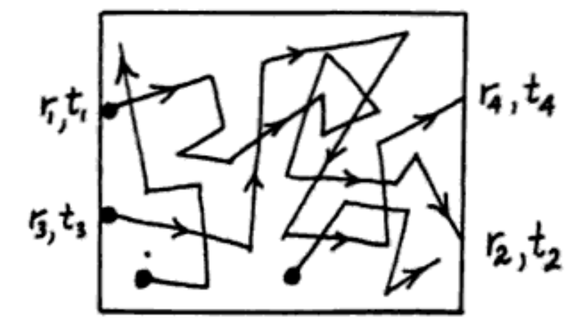
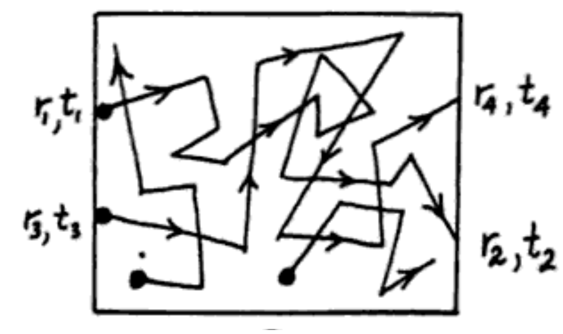


Fig. 0.1 *Some Many-body Systems*

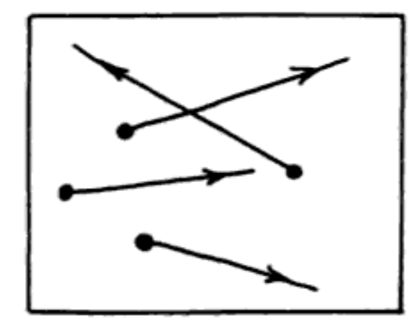


Quasi-Particles

Landau's idea:
why care about real particles?



Of importance are the excitations, which behave as quasi-particles!



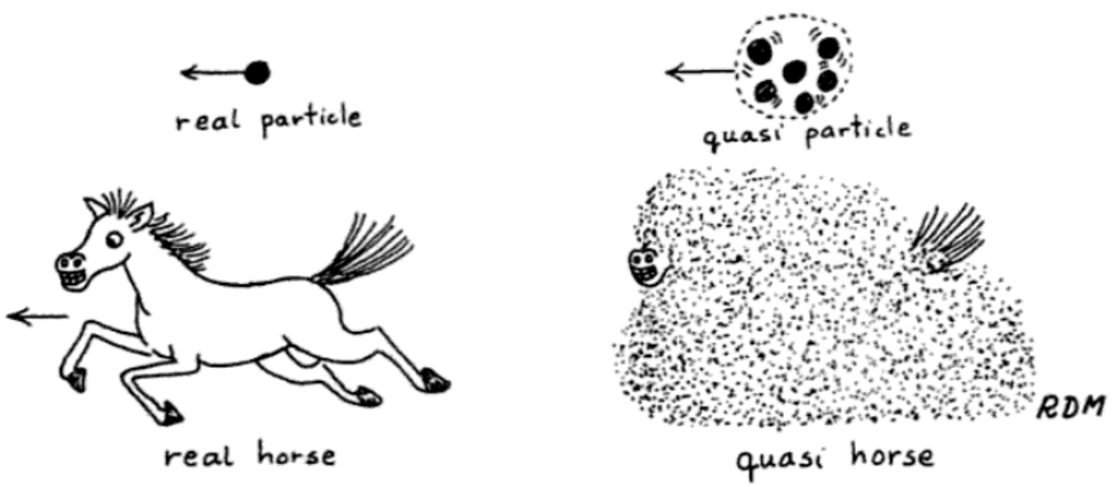


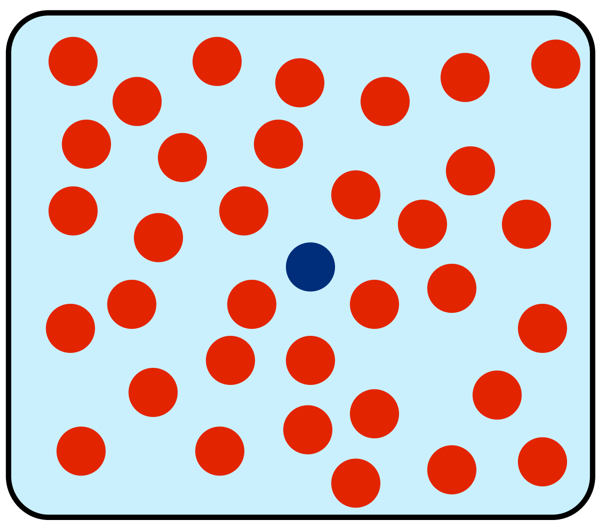
Fig. 0.4 *Quasi Particle Concept*

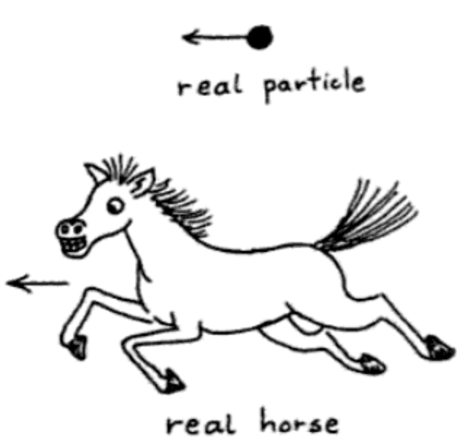
a QP is a "free particle" with:

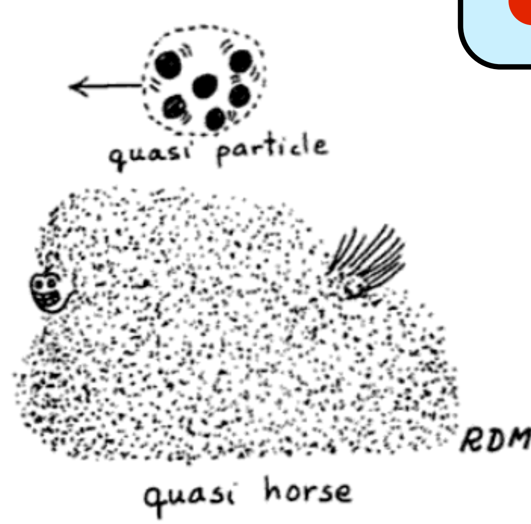
@ q. numbers (charge, spin, ...)
@ renormalized mass
@ chemical potential
@ shielded interactions
@ lifetime

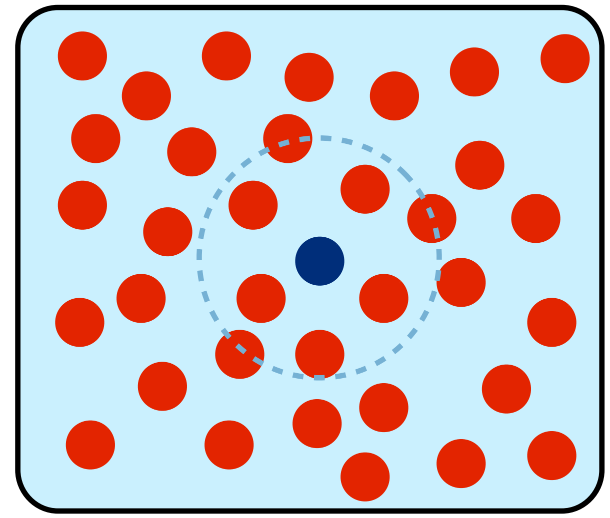
The impurity problem

Switch on interactions

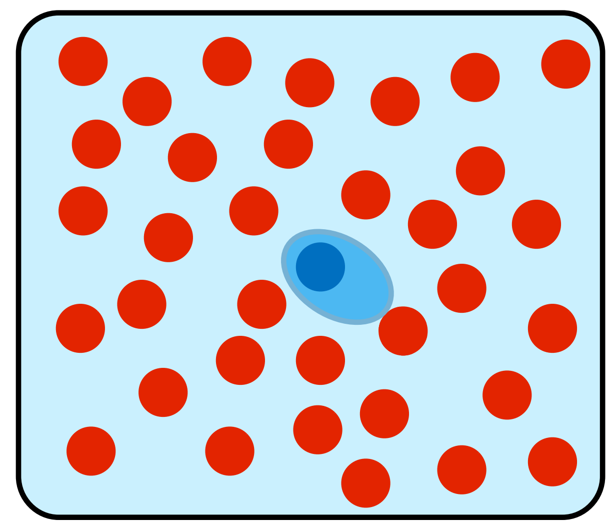




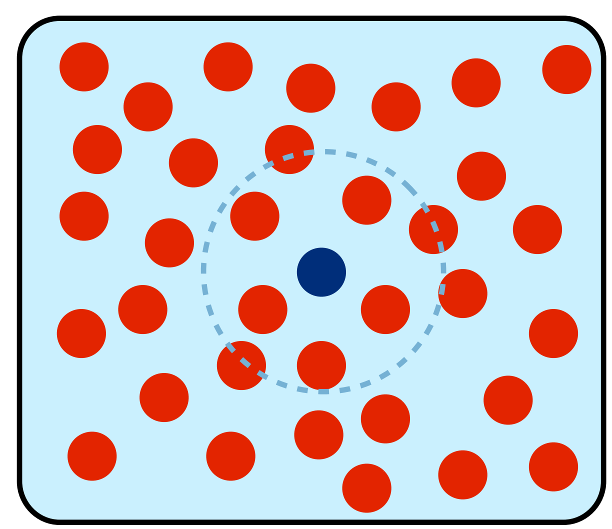




REPULSIVE POLARON



MOLECULE-HOLE



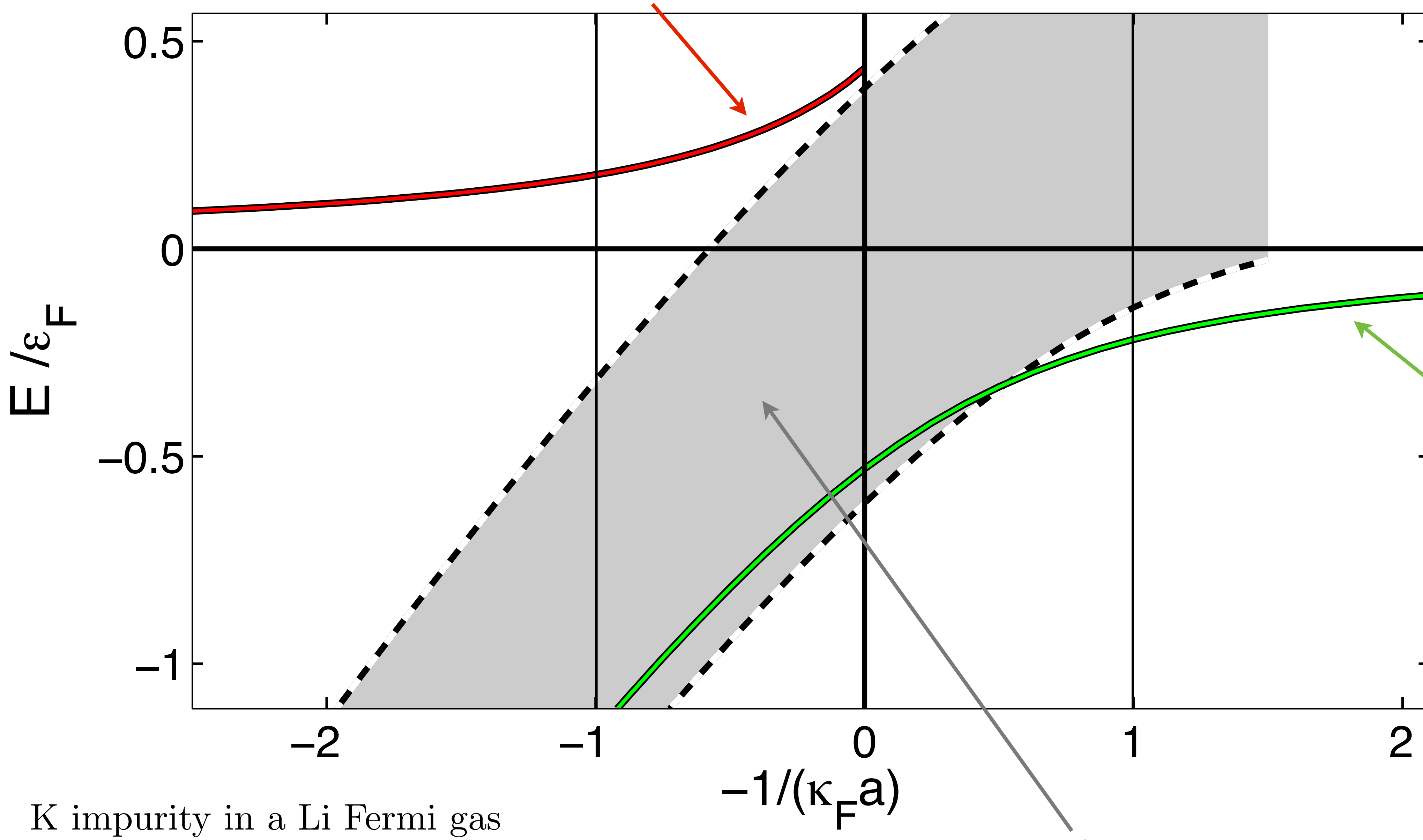
ATTRACTIVE POLARON

The impurity problem

new quantum toy!
a gas with strong repulsive interactions

(intrinsically metastable, due to the existence of weakly-bound lower-lying states)

At zero momentum of the impurity:

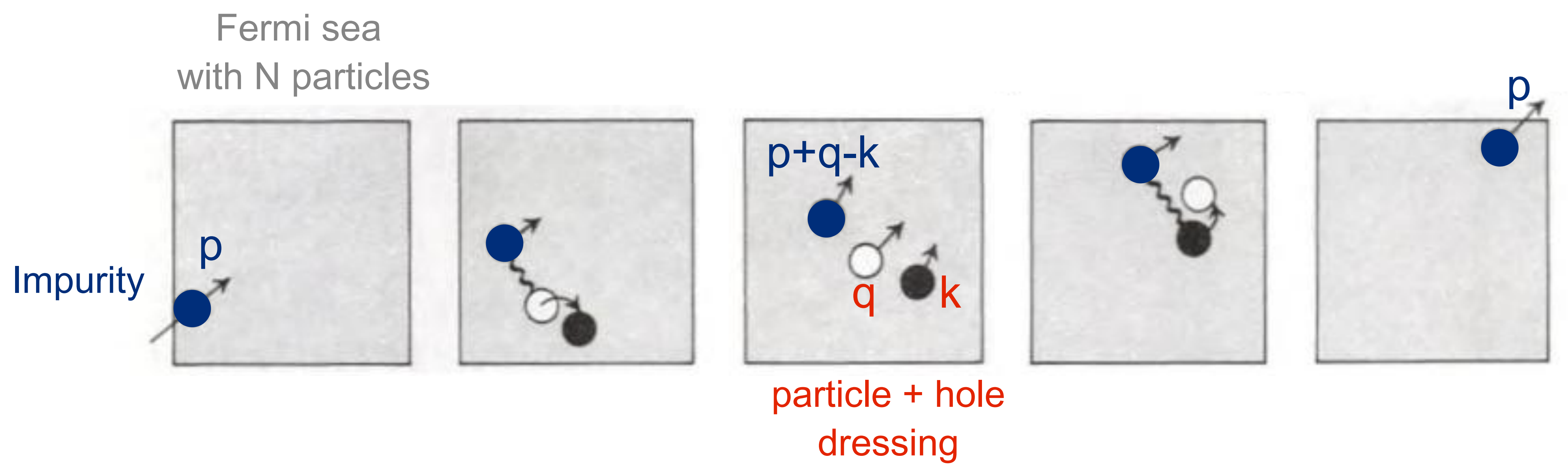


K impurity in a Li Fermi gas

$m_\downarrow/m_\uparrow = 40/6$

$k_F R^* \sim 1$

The polaron: a dressed impurity



$$|\psi_{\mathbf{p}}\rangle = \phi_{\mathbf{p}} c^{\dagger}_{\mathbf{p}_{\downarrow}} |FS_N\rangle + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{pqk}} \; c^{\dagger}_{\mathbf{p+q-k}_{\downarrow}} \; c^{\dagger}_{\mathbf{k}_{\uparrow}} \; c_{\mathbf{q}_{\uparrow}} \; |FS_N\rangle$$

(and a similar variational w.f. may be written for the molecule)

Quasiparticle properties

self-energy of the impurity: $\Sigma_{\mathrm{P}}(\mathbf{p},E) = \sum_{q<k_F} T(\mathbf{p}+\mathbf{q},E+\xi_{q\uparrow})$

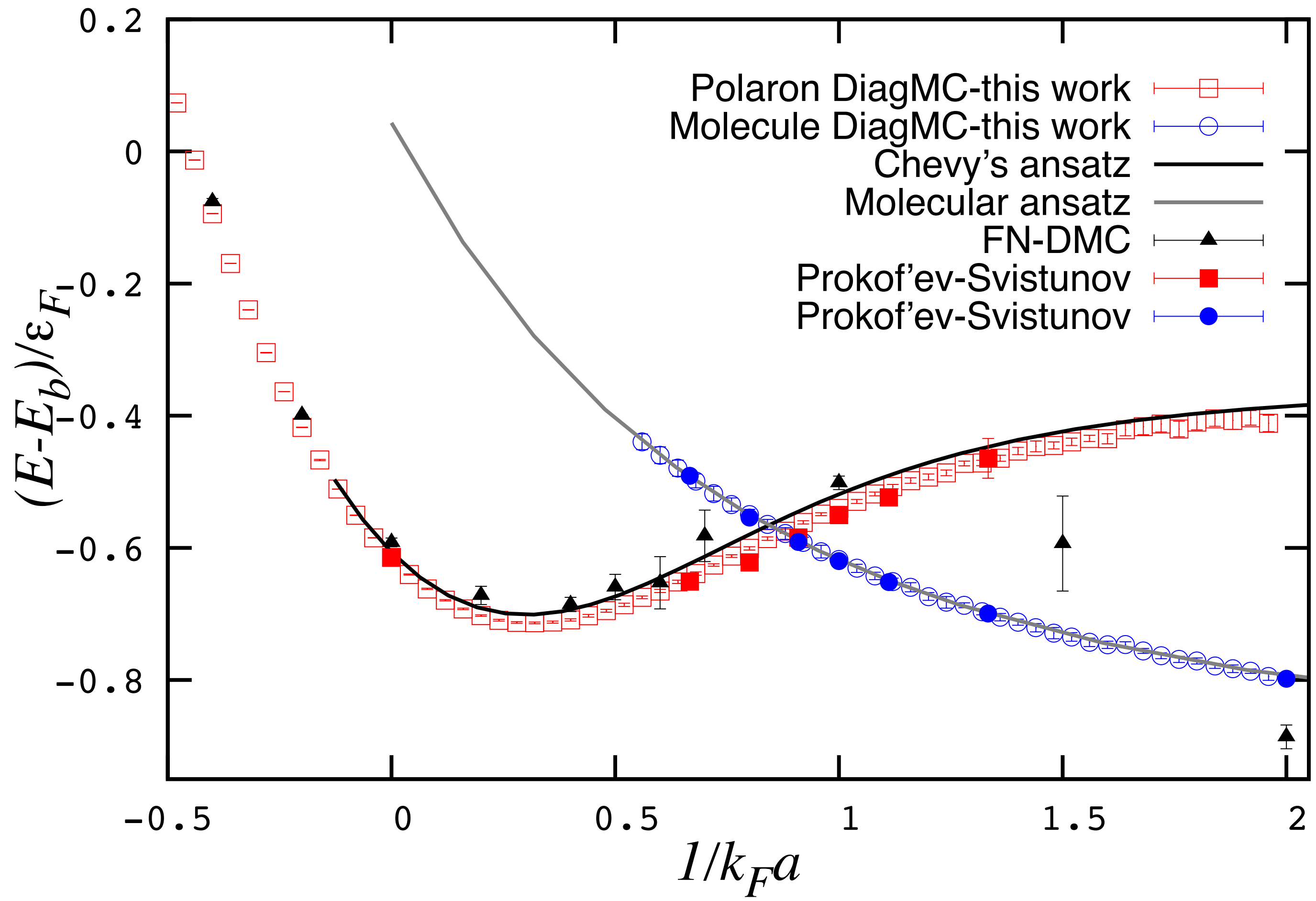
energies of the two polarons: $E_{\pm} = \Re[\Sigma_{\mathrm{P}}(\mathbf{p},E_{\pm}+i0^{+})]$

residues: $Z_{\pm} = \dfrac{1}{1-\partial_{\omega}\Re(\Sigma_{\mathrm{P}})}$

effective masses: $\dfrac{m^*}{m_{\downarrow}} = \dfrac{1}{Z_{\pm}}\left[1+\dfrac{\partial\Re(\Sigma_{\mathrm{P}})}{\partial(p^2/2m_{\downarrow})}\right]^{-1}$

self-consistent equation for the molecule energy: $\sum_{\mathbf{k}'}\dfrac{K_{\mathbf{k}'\mathbf{p}\mathbf{q}}}{E^{(2)}_{\mathbf{k}\mathbf{k}'\mathbf{p}\mathbf{q}}} - \sum_{\mathbf{q}'}\dfrac{K_{\mathbf{k}\mathbf{p}\mathbf{q}'}}{E^{(1)}_{\mathbf{k}\mathbf{p}}} - \dfrac{T(\mathbf{p},0)}{E^{(1)}_{\mathbf{k}\mathbf{p}}}\sum_{\mathbf{k}'\mathbf{q}'}\dfrac{K_{\mathbf{k}'\mathbf{p}\mathbf{q}'}}{E^{(1)}_{\mathbf{k}'\mathbf{p}}} + \dfrac{K_{\mathbf{k}\mathbf{p}\mathbf{q}}}{T(\mathbf{q}+\mathbf{p}-\mathbf{k},\xi_{q\uparrow}-\xi_{k\uparrow})} = -\dfrac{T(\mathbf{p},0)}{E^{(1)}_{\mathbf{k}\mathbf{p}}}$

Comparison with Diagrammatic QMC



Vlietinck et al., PRB (2013)

Narrow Feshbach Resonances

Scattering amplitude: $f = -\left[a^{-1} + ik + R^* k^2 + \ldots\right]^{-1}$

close to resonance: $R^* = -\frac{r_s}{2} = \frac{1}{2m_r a_{\rm bg} \Delta B \delta\mu} > 0$

(Petrov, PRL 2004)

a FR is broad if $R^* \ll R_{VdW}$ or $k_F R^* \ll 1$

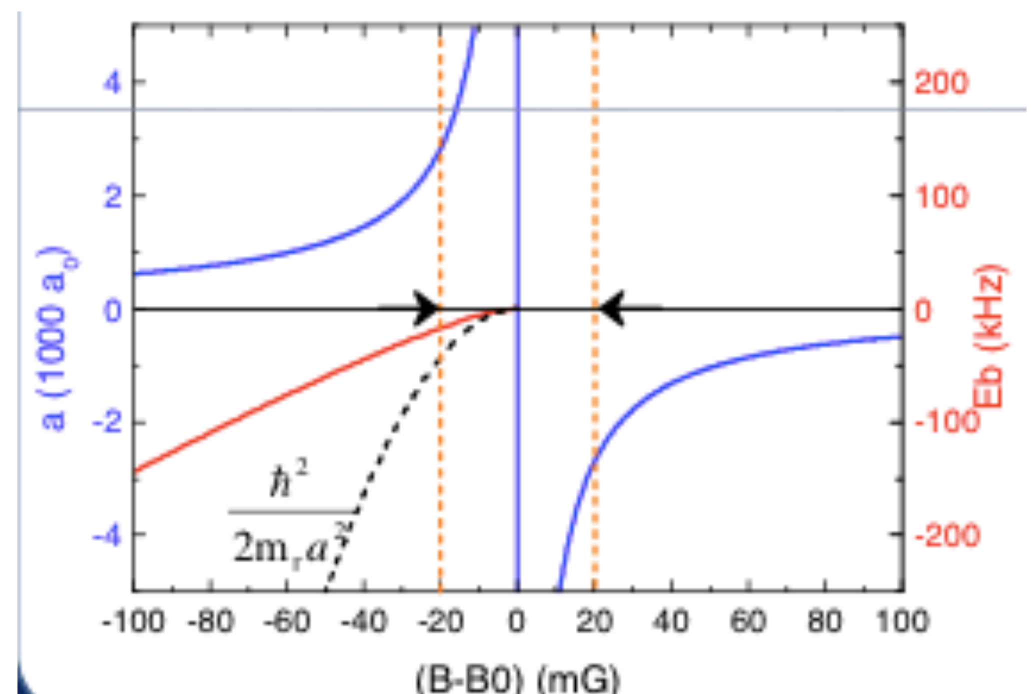
most heteronuclear FR are narrow

Molecule energy:

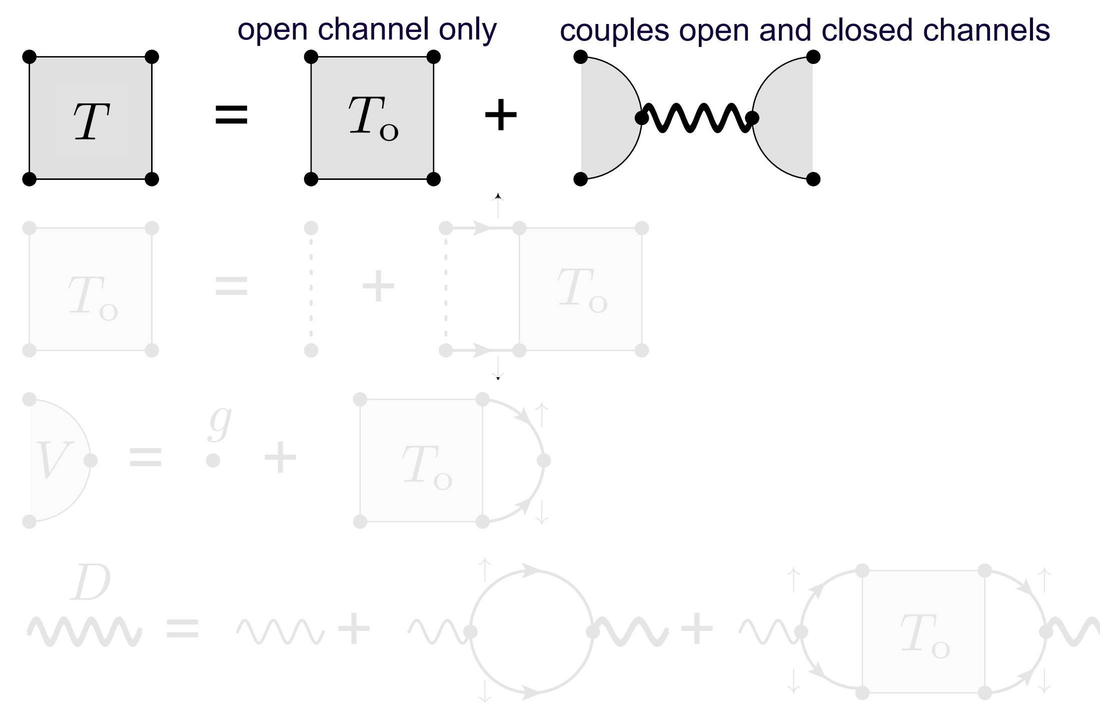
$$E_M = -\frac{\hbar^2}{2m_r (a_*)^2} \quad \text{with} \quad a^* = \frac{2R^*}{\sqrt{1+4R^*/a} - 1}$$

$a \gg R^*: \quad a^* \sim a$

$a \ll R^*: \quad a^* \sim \sqrt{aR^*}$



many-body + narrow FR + bg scatt.



$$T(\mathbf{P},\omega) = T_{\mathrm{open}}(\mathbf{P},\omega) + T_{\mathrm{closed}}(\mathbf{P},\omega)$$

$$T_{\mathrm{open}}(\mathbf{P},\omega) = \frac{1}{T_{\mathrm{bg}}^{-1} - \Pi(\mathbf{P},\omega)}$$

$$T_{\mathrm{bg}} = 2\pi a_{\mathrm{bg}}/m_r$$

$$\Pi(\mathbf{P},\omega) = \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\left[\frac{1 - f_\uparrow(\mathbf{k}) - f_\downarrow(\mathbf{P}+\mathbf{k})}{\omega + i0^+ - \xi_{\uparrow\mathbf{k}} - \xi_{\downarrow\mathbf{P}+\mathbf{k}}} + \frac{2m_r}{k^2}\right]$$

renormalized (↑↓)-dimer coupling: $g = \sqrt{T_{\mathrm{bg}}\Delta B \delta\mu}$

$$T_{\mathrm{closed}}(\mathbf{P},\omega) = V(\mathbf{P},\omega)D(\mathbf{P},\omega)V(\mathbf{P},\omega)$$

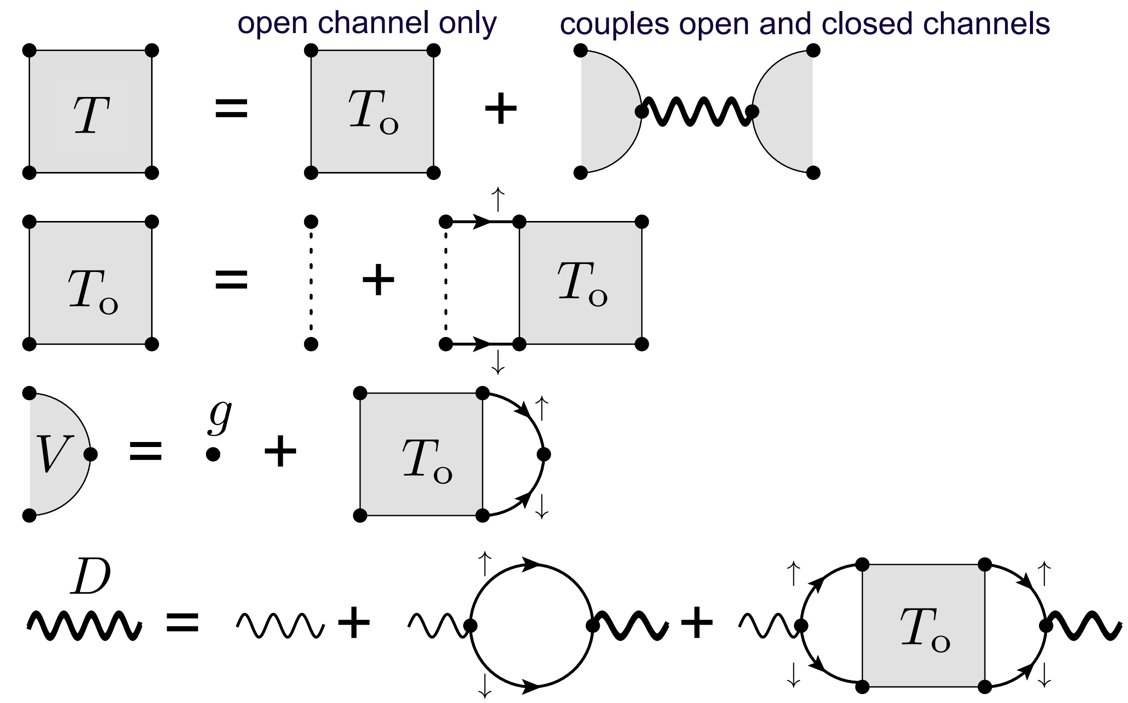
$$V(\mathbf{P},\omega) = g[1 - T_{\mathrm{bg}}\Pi(\mathbf{P},\omega)]^{-1}$$

$$D(\mathbf{P},\omega) = [E_{\mathrm{CM}} - \delta\mu(B - B_0) - \Sigma_{\mathrm{mol}}(\mathbf{P},\omega)]^{-1}$$

$$\Sigma_{\mathrm{mol}}(\mathbf{P},\omega) = g\,\Pi(\mathbf{P},\omega)\,V(\mathbf{P},\omega)$$

Bruun, Jackson & Kolomeitsev, PRA 2005
PM & Stoof, PRA 2008

many-body + narrow FR + bg scatt.



$$T(\mathbf{P},\omega) = T_{\text{open}}(\mathbf{P},\omega) + T_{\text{closed}}(\mathbf{P},\omega)$$

$$T(\mathbf{P},\omega) = \frac{1}{\frac{m_r}{2\pi\tilde{a}(E_{CM})} - \Pi(\mathbf{P},\omega)}$$

energy-dependent "scattering length": $\tilde{a}(E_{\text{CM}}) \equiv a_{\text{bg}}\left(1 - \frac{\Delta B}{B - B_0 - E_{\text{CM}}/\delta\mu}\right)$

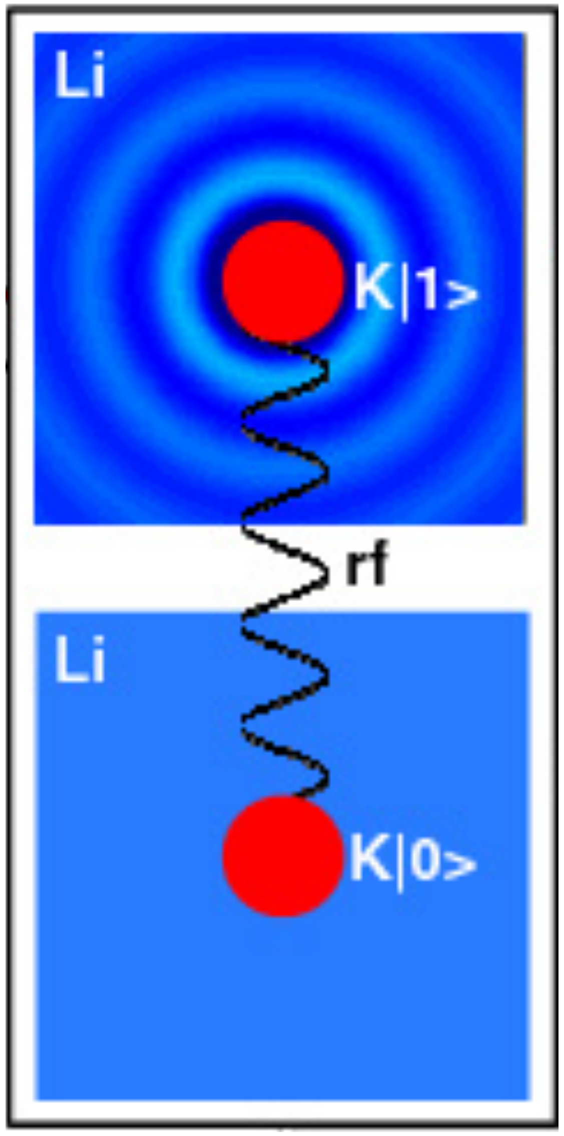
low energy expansion in vacuum: $-\frac{1}{f_{\text{vac}}} = a^{-1} + ik + \tilde{R}^* + \ldots$

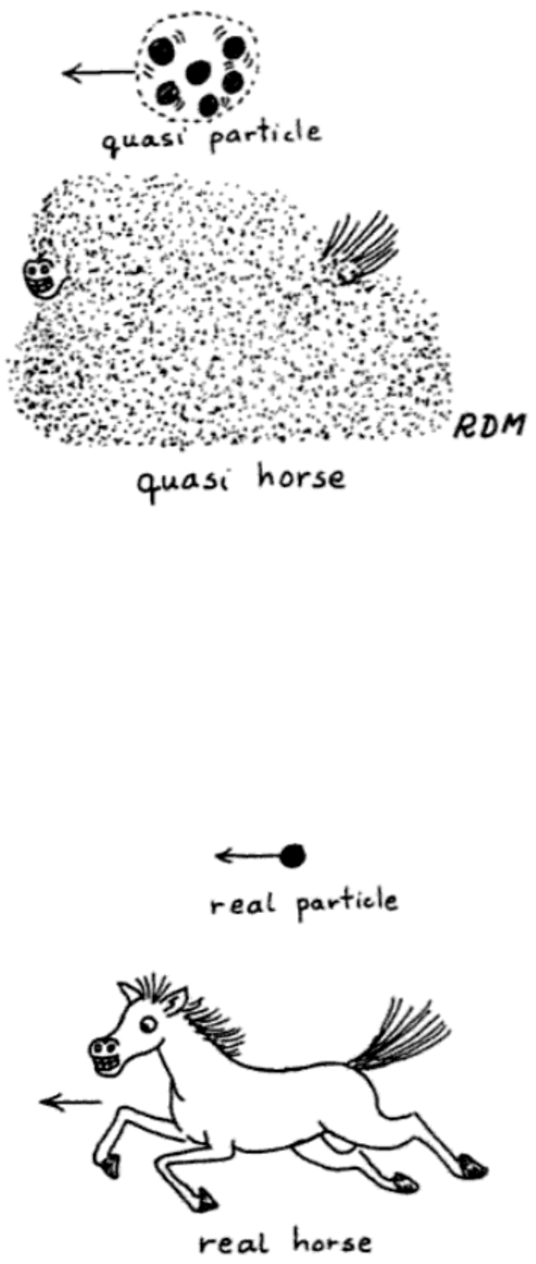
$$a = a_{\text{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

$$\tilde{R}^* = R^*\left(1 - \frac{a_{\text{bg}}}{a}\right)^2$$

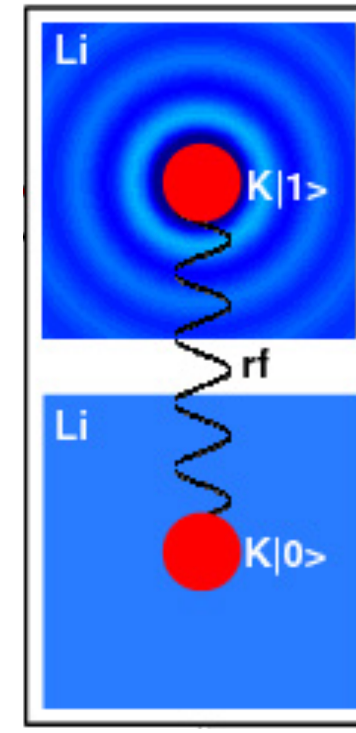
Bruun, Jackson & Kolomeitsev, PRA 2005
PM & Stoof, PRA 2008

RF spectroscopy





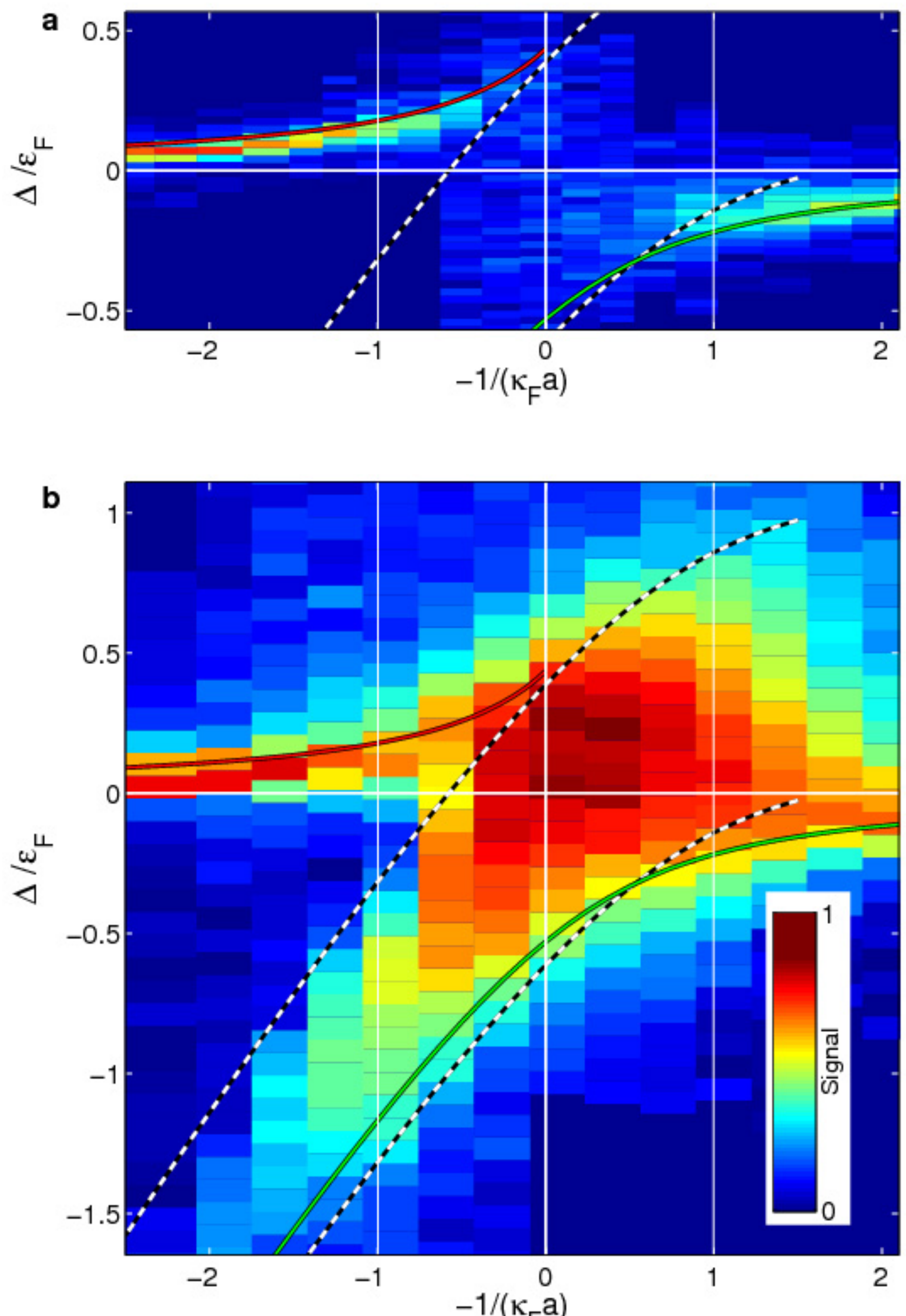
RF spectroscopy



low power RF:

high power RF:

high power is needed to
couple to the MH continuum,
due to a small FC overlap

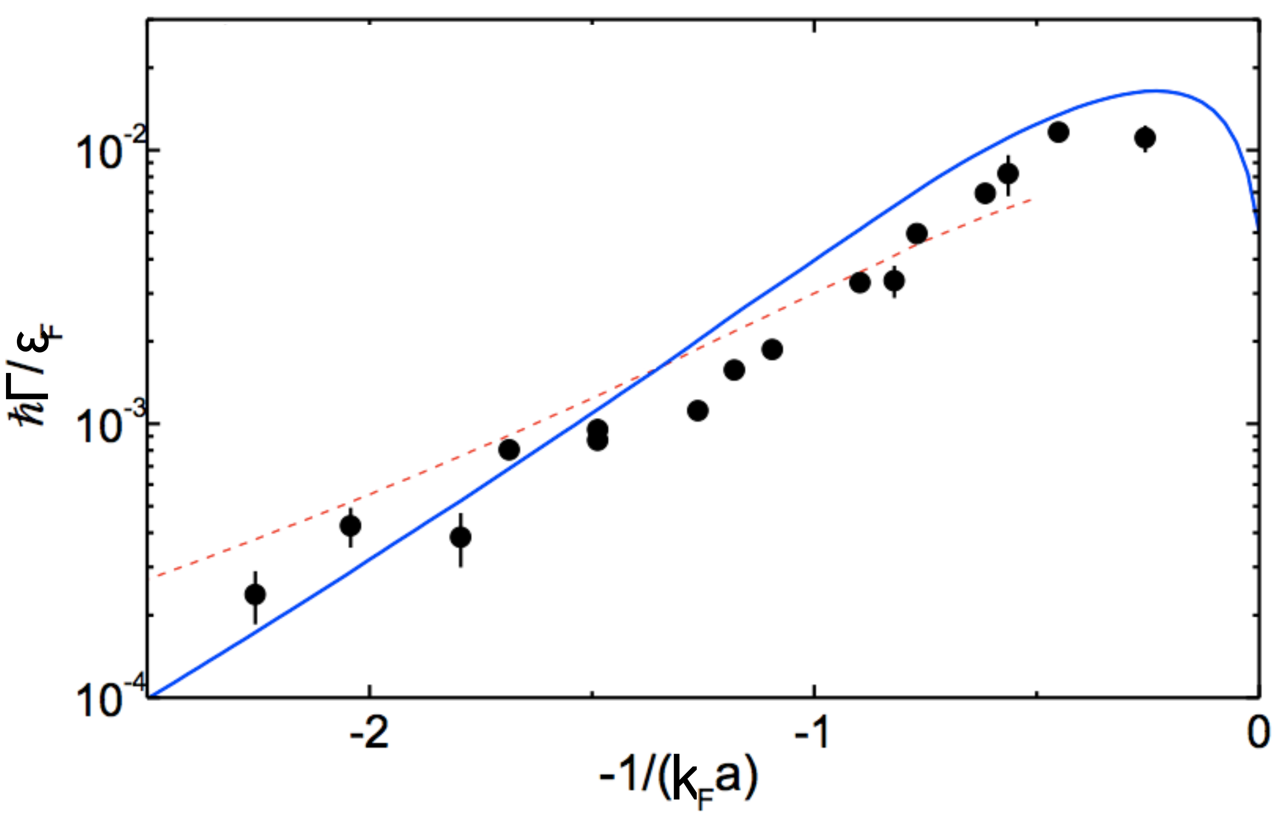


repulsive polarons exist
as well-defined quasiparticles
even in the strongly-interacting regime

repulsive pol.
attractive pol.
molecule+hole continuum

- Kohstall, Zaccanti, Jag, Trenkwalder, PM, Bruun, Schreck & Grimm, Nature (2012)
- P. Massignan, EPL (2012)

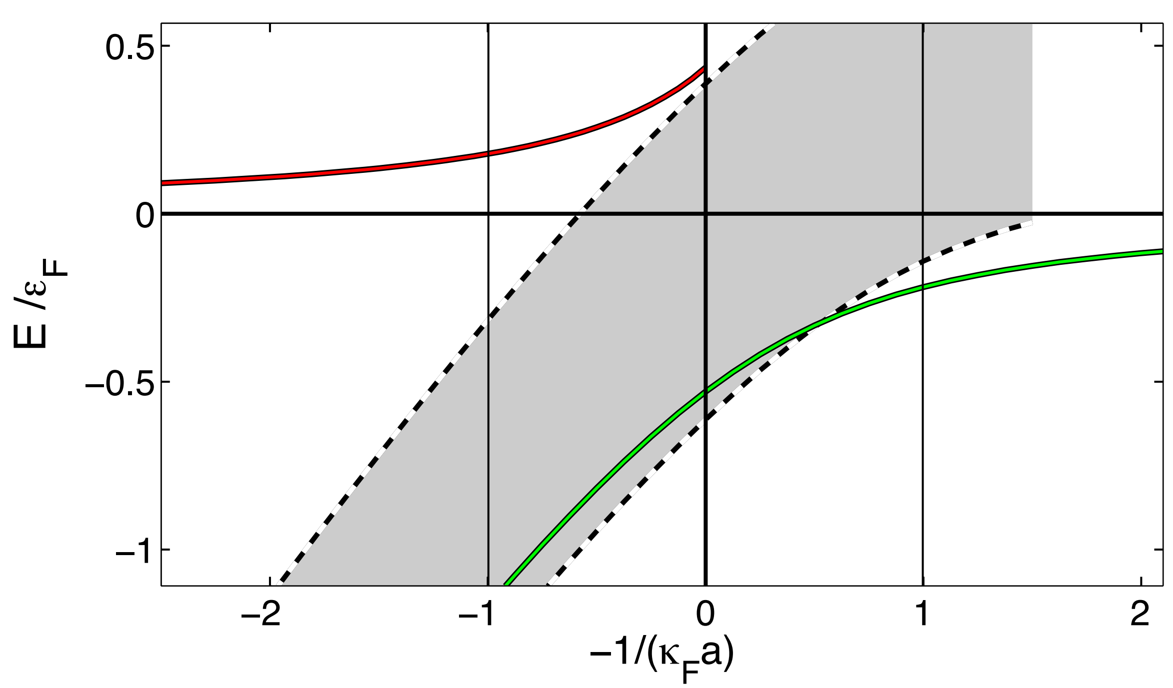
Decay of repulsive polarons



exp. data

vs. theory for

Pol+→Pol- and Pol+→Mol



long lifetimes!

10 times more than in the MIT expmt. (Science 2009)

Rabi oscillations

$$\hat{R} \propto \Omega_0 \sum_{\mathbf{q}} (\hat{a}^\dagger_{1\mathbf{q}} \hat{a}_{0\mathbf{q}} + h.c.)$$

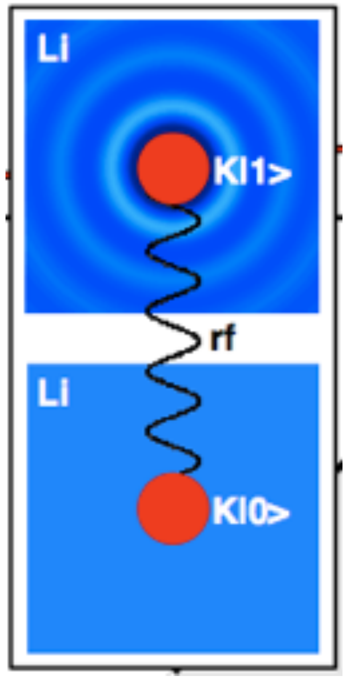
$$|I\rangle = \hat{a}^\dagger_{0\mathbf{q}=0} |\mathrm{FS}\rangle$$

$$|F\rangle = \sqrt{Z} \hat{a}^\dagger_{1\mathbf{q}=0} |\mathrm{FS}\rangle + \sum_{p<\hbar\kappa_F<q} \phi_{\mathbf{q},\mathbf{p}} \hat{a}^\dagger_{1\mathbf{p}-\mathbf{q}} \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{p}} |\mathrm{FS}\rangle + \dots$$

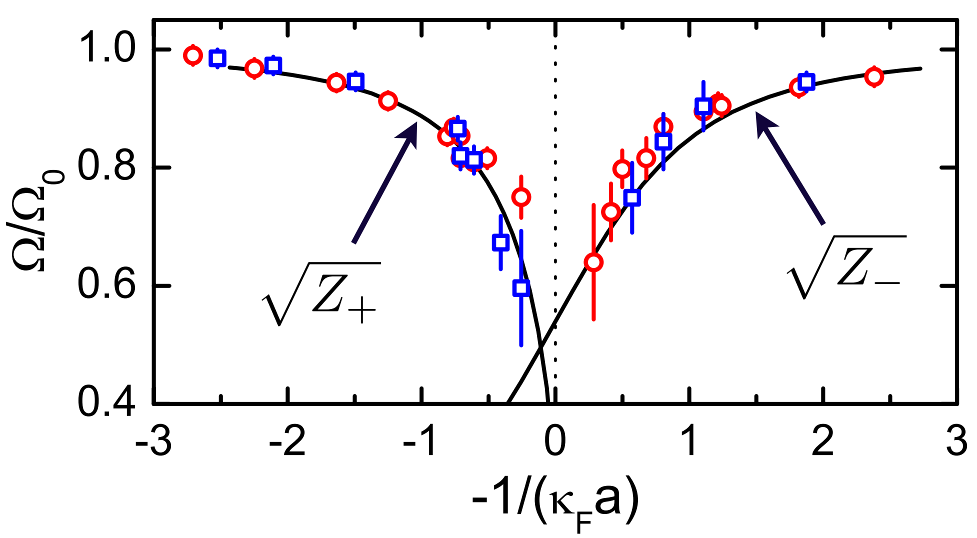
$$\langle F|\hat{R}|I\rangle = \sqrt{Z}\Omega_0$$

regime of very high RF power,
well beyond linear response regime:
fast oscillations, and quasiparticle decay
may be ignored

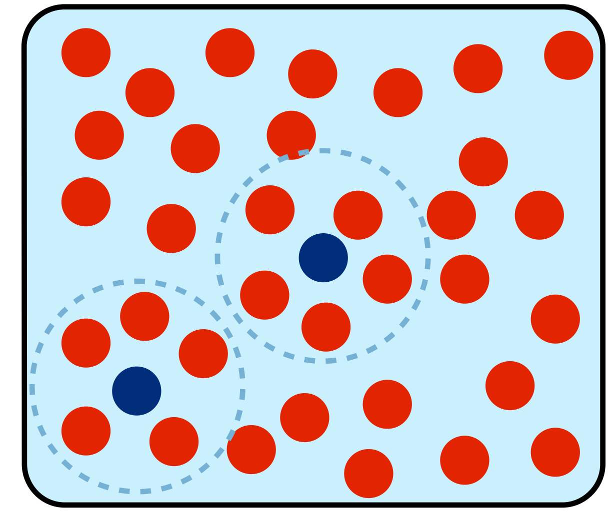
collision-induced decoherence
is the main damping mechanism



Rabi frequency as a measure of polaron residues



Equation of state



A strongly-interacting system, described as
an ensemble of weakly-interacting quasi-particles
(a Fermi liquid)

$$E = \frac{3}{5}\epsilon_F N_\uparrow \left[1 + \frac{m}{m^*}\left(\frac{N_\downarrow}{N_\uparrow}\right)^{5/3}\right] + N_\downarrow E_p + \ldots,$$

kinetic energy
of the Fermi sea

kinetic energy
of the polarons
(m* is their effective mass)

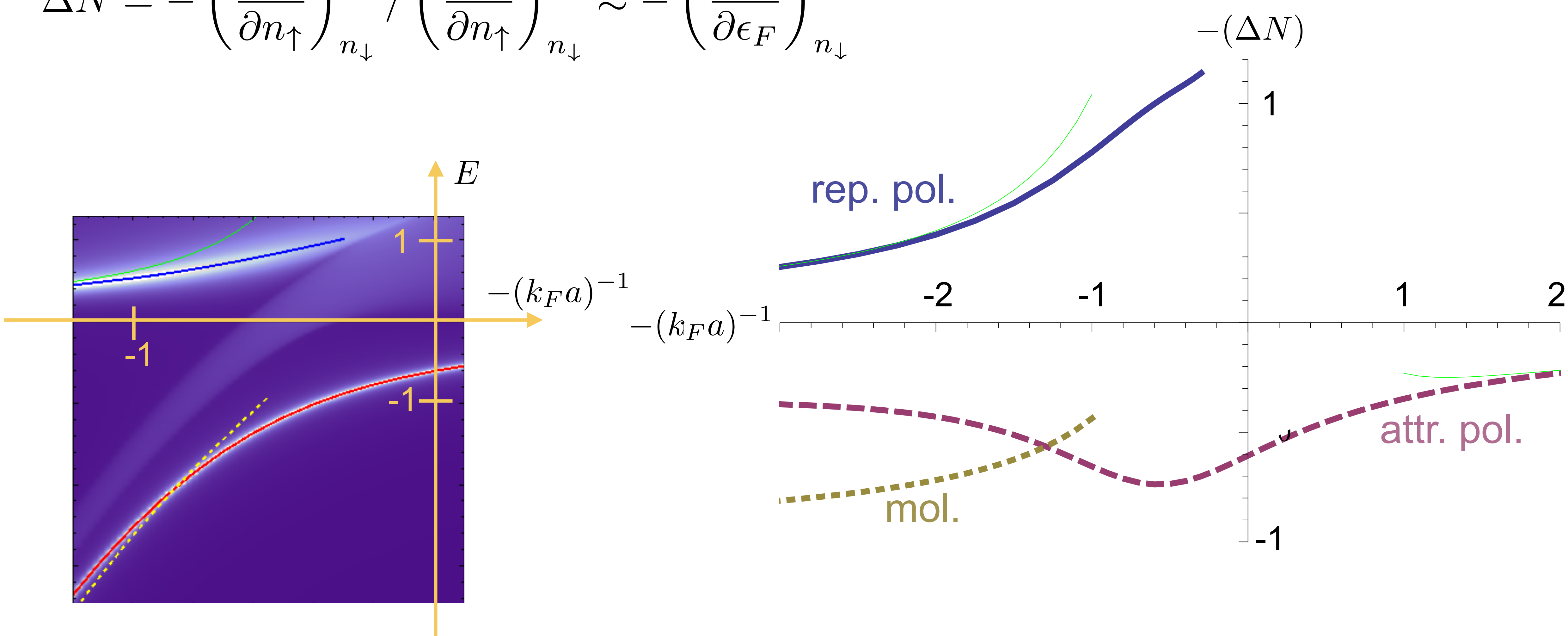
chemical potential
of one polaron

How many $\uparrow$ in the dressing cloud?

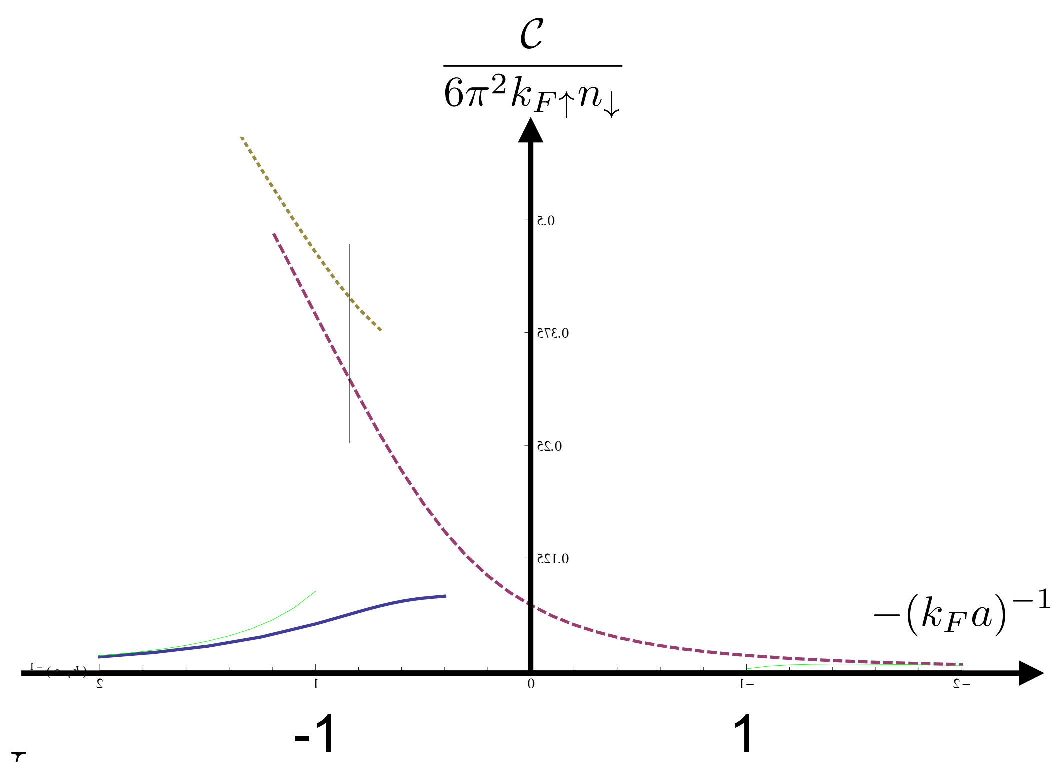
The density of the majority atoms far away from the impurity should remain unchanged when adding one impurity:

$$\delta\mu_\uparrow = \frac{\partial^2 \varepsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2 \varepsilon}{(\partial n_\uparrow)^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial \mu_\downarrow}{\partial n_\uparrow}\right)_{n_\downarrow} / \left(\frac{\partial \mu_\uparrow}{\partial n_\uparrow}\right)_{n_\downarrow} \approx -\left(\frac{\partial \mu_\downarrow}{\partial \epsilon_F}\right)_{n_\downarrow}$$



Contact density



Punk et al., PRA 2009
PM and G. Bruun, EPJD 2011

$$\frac{\mathcal{C}}{8\pi m_r} = -\frac{\mathrm{d}\epsilon}{\mathrm{d}(1/a)} = -\frac{\mathrm{d}E_\downarrow}{\mathrm{d}(1/a)}$$

link with the cloud “weight”: $\frac{\mathcal{C}}{16\pi m_r a} = -n_\downarrow E_\downarrow - n_\downarrow \Delta N \epsilon_F$

at resonance: $\Delta N = -E_\downarrow/\epsilon_F$

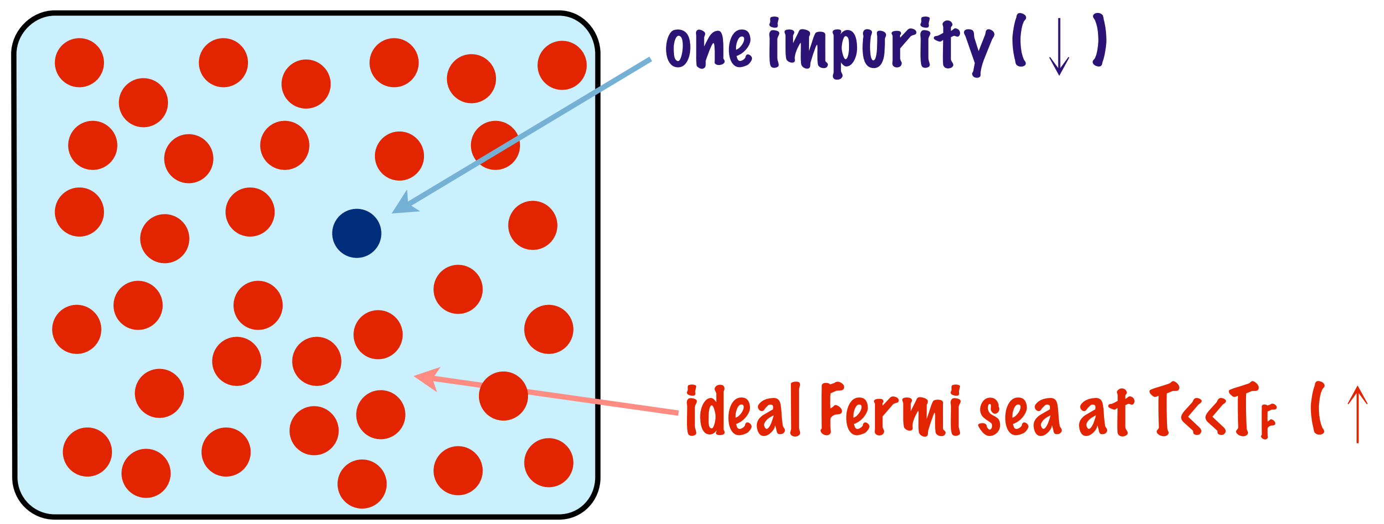
link with Tan’s pressure relation: $\mathcal{P}_0 = 2\epsilon_F n_\uparrow/5$

$$\Delta\mathcal{P} = \mathcal{P} - \mathcal{P}_0 = 2\epsilon_F(\Delta n_\uparrow)/3$$

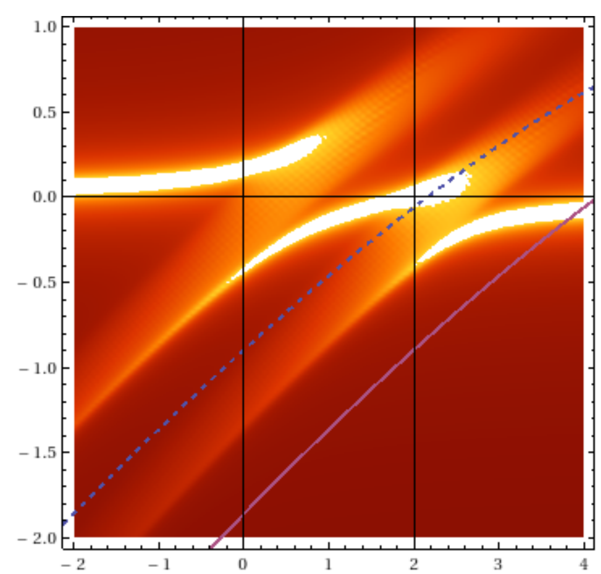
$$\Delta n_\uparrow = -n_\downarrow \Delta N$$

$$\frac{\mathcal{C}}{16\pi m_r a} = -\Delta\epsilon + \epsilon_F(\Delta n_\uparrow) = -\Delta\epsilon + \frac{3}{2}(\mathcal{P} - \mathcal{P}_0)$$

adding the contribution of the non-int. Fermi sea: $\frac{\mathcal{C}}{16\pi m_r a} = \frac{3}{2}\mathcal{P} - \epsilon$



s-wave



p-wave

IFM

p-wave scattering

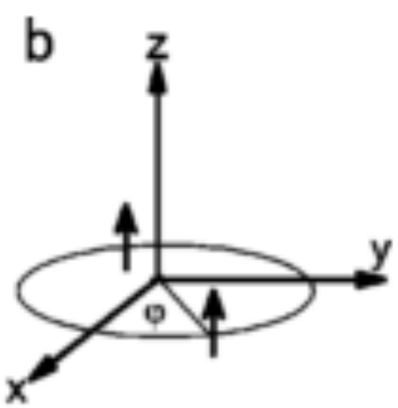
$$f_\mu(k) = \frac{k^2}{-v_\mu^{-1} + \frac{1}{2}k_0 k^2 - ik^3}.$$

scattering volume

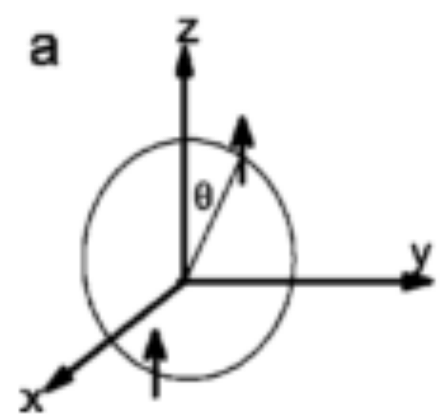
effective range

p-wave molecules with m=+1,0,-1 in an external magnetic field have different energies due to dipole-dipole interactions:

m=±1



m=0



$E_{\pm 1} > E_0$

(Ticknor, Regal, Jin, and Bohn, PRA 2004)

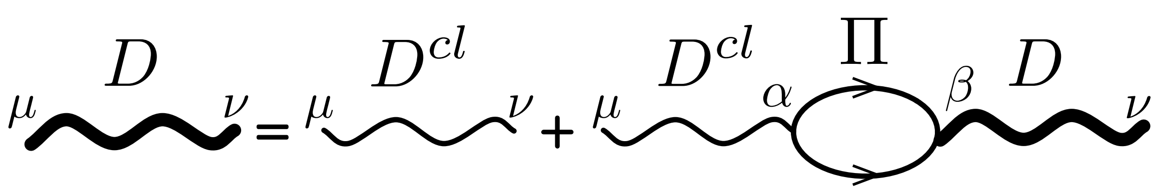
p-wave polarons

cut-off: $g(p) = g\Theta(\Lambda - p)$

two-channel Hamiltonian

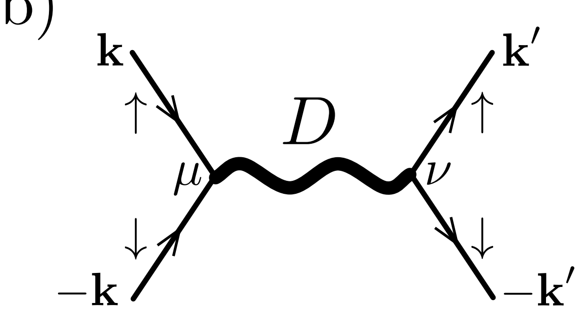
$$H = \sum_{\mathbf{p},\sigma} \frac{p^2}{2m} a^\dagger_{\sigma\mathbf{p}} a_{\sigma\mathbf{p}} + \sum_{\mathbf{q},\mu} \left(\epsilon_\mu + \frac{q^2}{4m}\right) b^\dagger_{\mu\mathbf{q}} b_{\mu\mathbf{q}} + \sum_{\mathbf{p},\mathbf{q},\mu} \frac{g(p)}{\sqrt{V}} \mathbf{p}_\mu \left(b^\dagger_{\mathbf{q}\mu} a_{\uparrow\frac{\mathbf{q}}{2}+\mathbf{p}} a_{\downarrow\frac{\mathbf{q}}{2}-\mathbf{p}} + a^\dagger_{\downarrow\frac{\mathbf{q}}{2}-\mathbf{p}} a^\dagger_{\uparrow\frac{\mathbf{q}}{2}+\mathbf{p}} b_{\mathbf{q}\mu} \right)$$

(a) **dressed molecule**

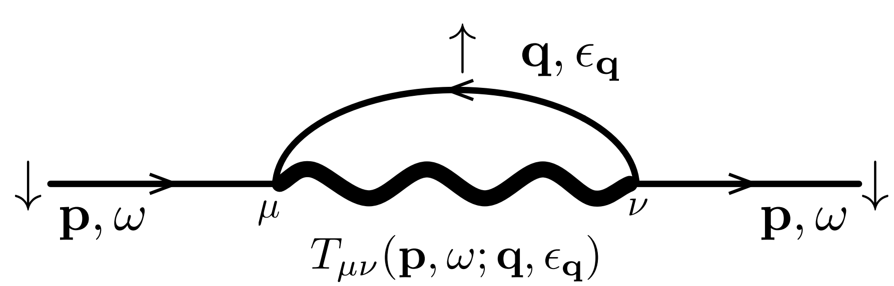


T-matrix

(b)

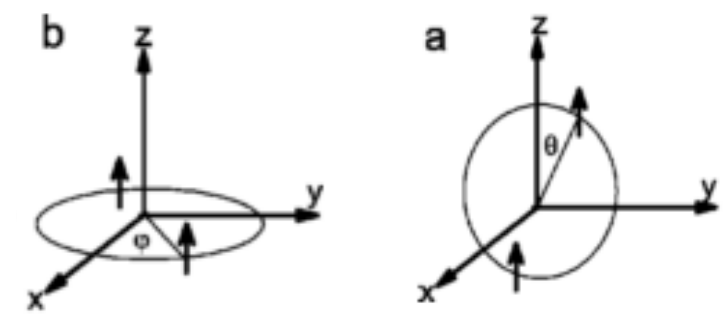


self-energy:



upon proper renormalization, the theory depends on: scattering volumes v_μ, and effective range $k0 \sim -1/R_{vdW}$

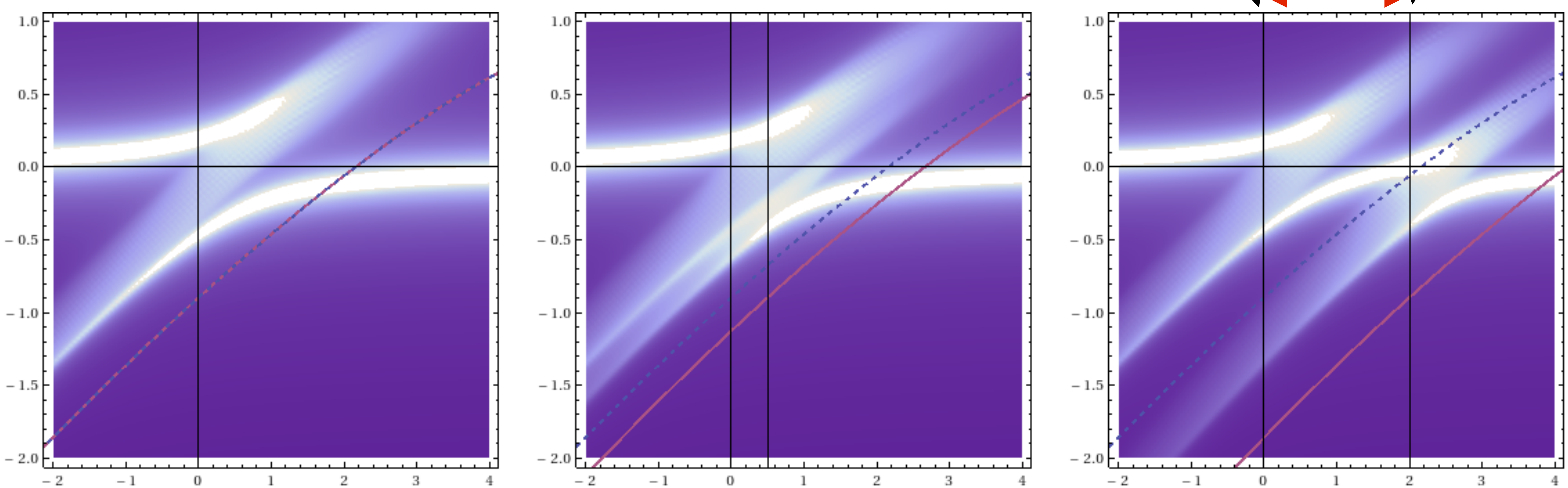
p-wave polarons



m=±1   m=0

p=0 polaron spectra for various resonance splittings δ:

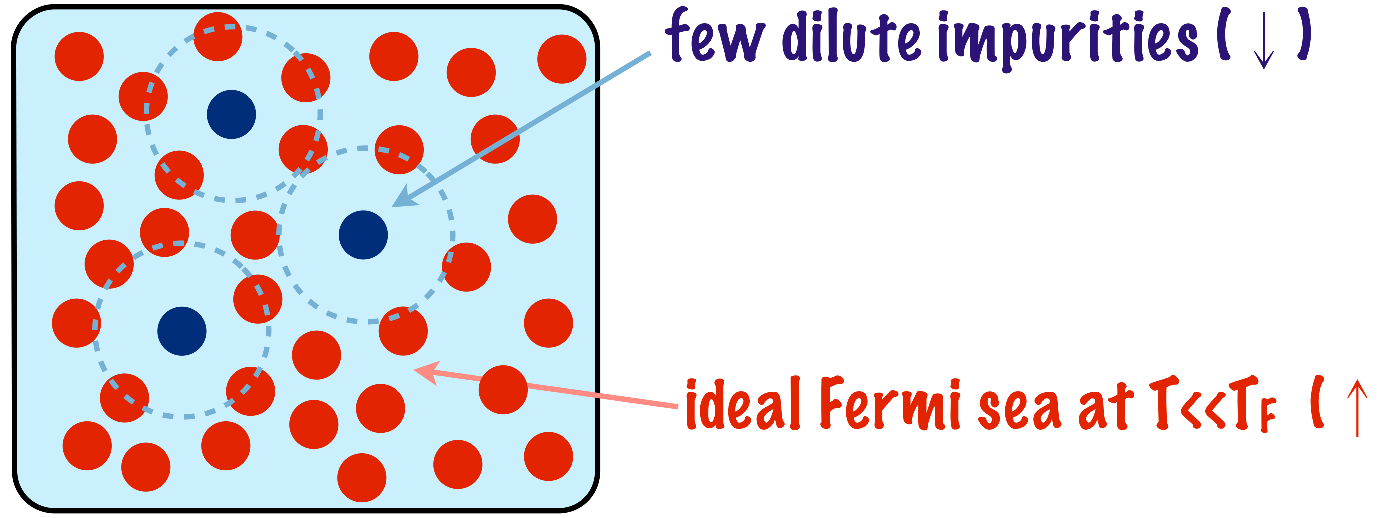
δ=0   δ



$Z_{\pm 1}=2Z_0$

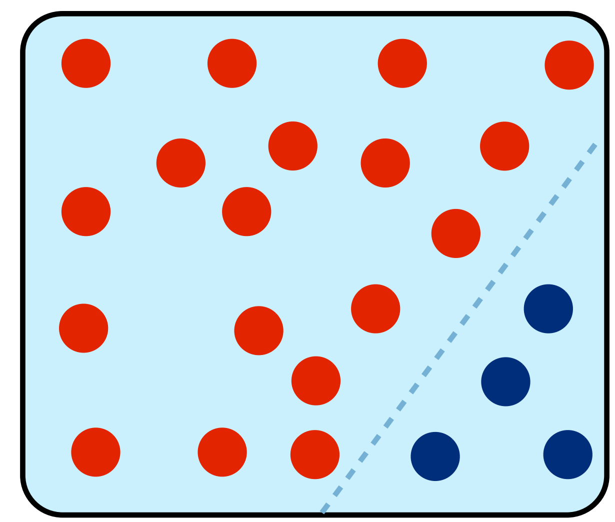
J. Levinsen, PM, F. Chevy, and C. Lobo, PRL 2012

$k_0=-10k_F$



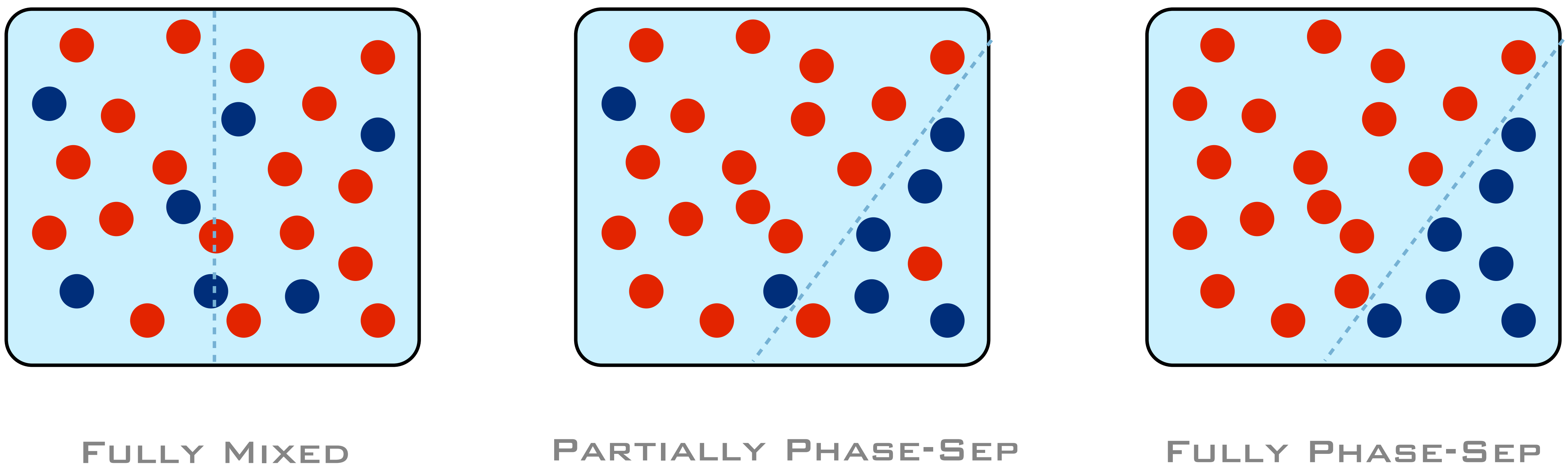
s-wave

p-wave



IFM

Itinerant FerroMagnetism



Thermodynamic analysis at T≥0:
Maxwell construction on the free energy of the mixed phase

Itinerant FerroMagnetism

broad res

narrow res ($k_F R^*=1$)

$$\text{Polarization}: P = \frac{N_1 - N_2}{N_1 + N_2}$$

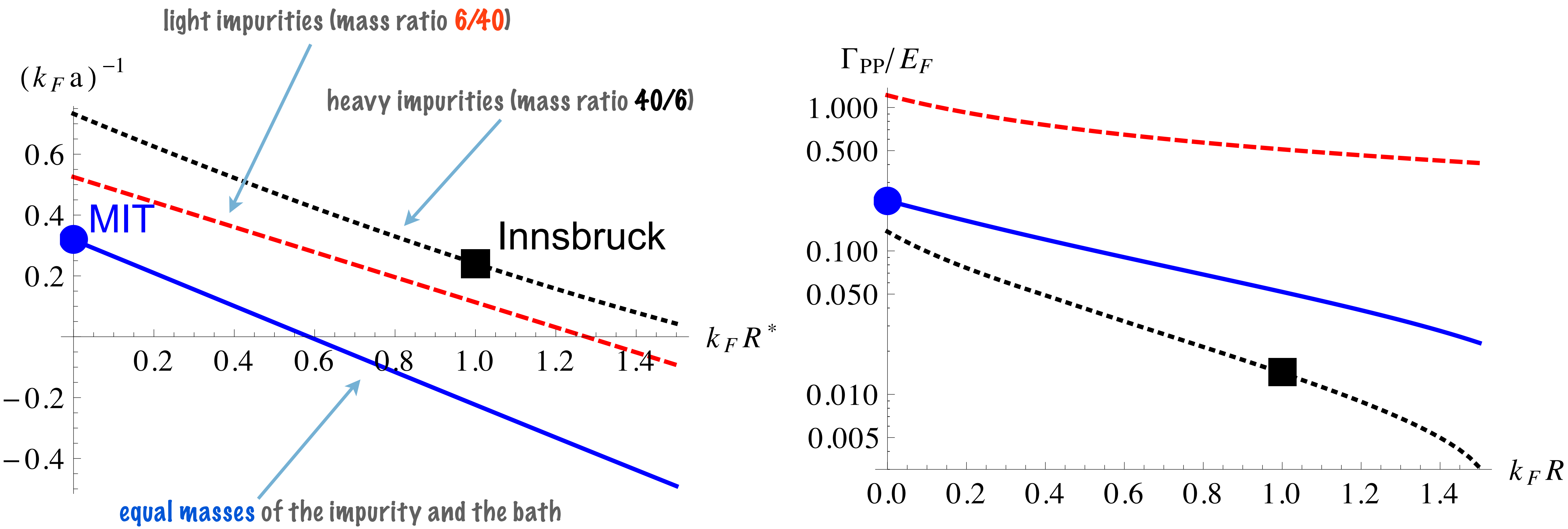
earlier
predictions
at T=0

2^{nd} order

The gas is mixed above the lines, and phase separated below.

PM, Z. Yu, and G. Bruun, PRL 2013

Critical interaction for IFM and decay rates



long lifetimes for K impurities
in a bath of Li atoms
at a narrow Feshbach resonance!
(as in the Innsbruck FeLiKx expmt.)

PM, Z. Yu, and G. Bruun, PRL 2013

in collaboration with:

Theory:

- Georg Bruun
- Frederic Chevy
- Carlos Lobo
- Jesper Levinsen
- Zhenhua Yu

Experiment:

- Christoph Kohstall
- Matteo Zaccanti
- Michael Jag
- Andreas Trenkwalder
- Florian Schreck
- Rudi Grimm

Conclusions

- The properties of one dressed impurity give important insights in the many-body behavior of a complex system
- A new strongly interacting quantum state: the repulsive polaron (meta-stable, but very long-lived)
- Rabi oscillations confirm the coherence of the quasi-particles
- new polaron/molecule branches appear in the p-wave case
- Smaller losses at narrow resonances may open the way to IFM