Fano Resonances in Atomic, Nuclear, and Hadronic Systems

Particle Dynamics at the Threshold

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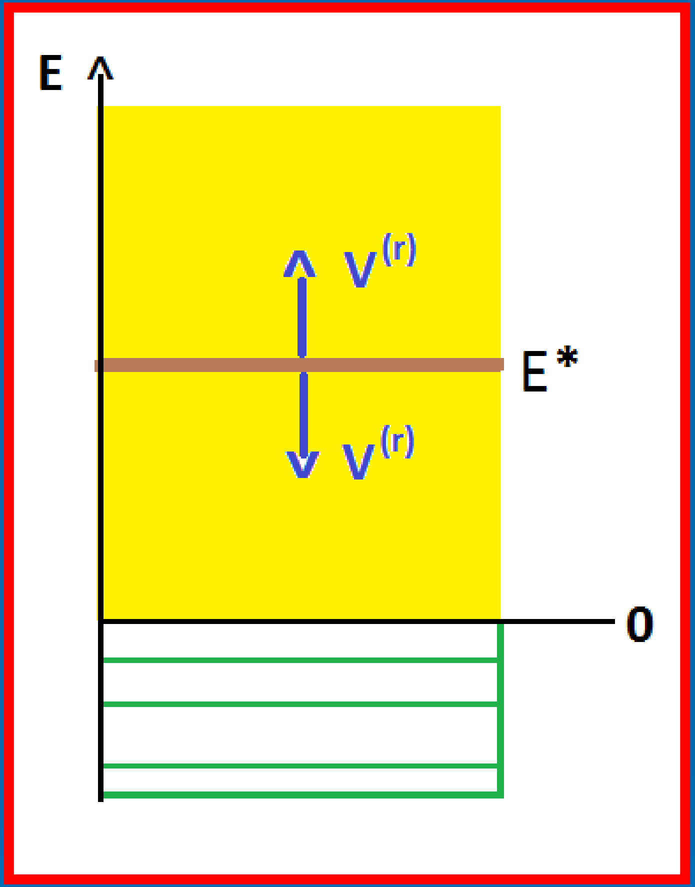
Agenda:

- Quantum Interference and Fano Resonances
- Fano resonances in atoms
- Continuum spectroscopy in nuclei
- Quantum Interference in hadron spectroscopy
- Summary

The Spectral Situation



- A closed channel E* is embedded into a continuum of open channels
- E* interacts via $V^{(r)}$ with open channels given by scattering states
- E* Interacts via $V^{(r)}$ with closed channels, e.g. of (simple) bound states

→ **Bound State Embedded into the Continuum - BSEC**

Examples:

- **Atoms: self-ionizing states of multi-electron configuration**
- **Nuclei: Multi-particle-hole states above threshold**
- **Mesons: Confined $q\bar{q}$-configurations embedded into the continuum of meson-meson scattering states, e.g. Δ(1232), ρ(770), f_0(980), X(3770)…**
- **Baryons: Confined qqq-configurations embedded into the continuum of meson-nucleon scattering states, e.g. Δ(1232), N*(1440), Λ(1405)…**

Fano Resonances in Atoms: Theory of Self-Ionizing States

The simplest case: One open and one closed channel

$$\Psi_E = a\varphi + \int dE'\, b_{E'}\psi_{E'}.$$

$$(\varphi|H|\varphi) = E_\varphi,$$

$$(\psi_{E'}|H|\varphi) = V_{E'},$$

$$(\psi_{E''}|H|\psi_{E'}) = E'\delta(E''-E').$$

$$E_\varphi a + \int dE'\, V_{E'}{}^* b_{E'} = Ea,$$

$$V_{E'}a + E'b_{E'} = Eb_{E'}.$$

Spectral Amplitude in the Open Channel

$$b_{E'} = \left[\frac{P}{E - E'} + z(E)\delta(E - E') \right] V_{E'} a,$$

$$E_\varphi + F(E) + z(E) |V_E|^2 = E,$$

$$F(E) = P \int dE' \frac{|V_{E'}|^2}{E - E'},$$

$$z(E) = \frac{E - E_\varphi - F(E)}{|V_E|^2}.$$

Spectral Amplitude in the Closed Channel

$$|a(E)|^2 = \frac{1}{|V_E|^2[\pi^2 + z^2(E)]} = \frac{|V_E|^2}{[E - E_\varphi - F(E)]^2 + \pi^2 |V_E|^4}.$$

- The discrete state obtains a spectral distribution because of the bound-continuum interaction;
- Line shape is of Lorentz form with a „mass shift" F(E) and a width $\Gamma=2\pi|V_E|^2$ (FWHM).
- If the system were prepared initially in the discrete state φ, it „auto-ionizes" through the coupling to the continuum with a mean life-time

$$\tau \sim 1/\Gamma \sim 1/|V_E|^2$$

The Open Channel Wave Function

Switching on an interaction → leads aymptotically to a change in the scattering phase shift: $\delta \rightarrow \delta^* = \delta + \Delta$

$r \rightarrow \infty$:

$$\int dE' \, b_{E'} \psi_{E'} \propto \sin[k(E)r + \delta + \Delta],$$

Interaction phase shift:

$$\Delta = -\arctan[\pi / z(E)]$$

$$\Delta = -\arctan\left[\frac{\pi |V_E|^2}{E - E_\varphi - F(E)}\right]$$

Explicit evaluation of the spectral amplitudes

…using:

$$b_{E'} = \left[\frac{1}{E-E'} + z(E)\delta(E-E')\right] V_{E'} a,$$

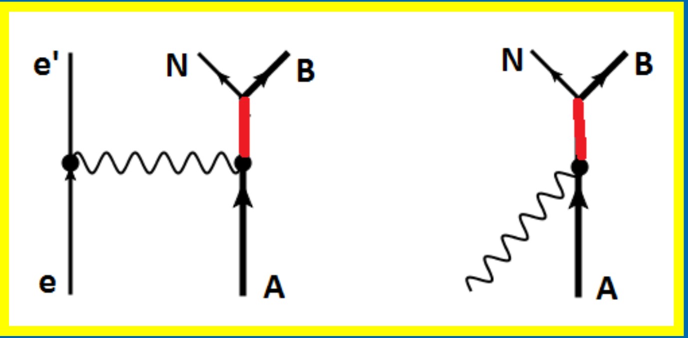
…and leading to:

$$a = \sin\Delta / \pi V_E,$$

$$b_{E'} = \frac{V_{E'}}{\pi V_E} \frac{\sin\Delta}{E-E'} - \cos\Delta\, \delta(E-E'),$$

Question:
What happens when a Fano State is excited by an external probe…

…e.g. in a (e,e') reaction or by a photon?



…resulting in the matrix element:

$$(\Psi_E|T|i) = \frac{1}{\pi V_E{}^*}(\Phi|T|i)\,\sin\Delta - (\psi_E|T|i)\,\cos\Delta,$$

$$\Phi = \varphi + P\int dE' \frac{V_{E'}\psi_{E'}}{E-E'}$$

Reaction Matrix Elements and Formation Cross Section

$$M_{\alpha\beta} = \left(\Psi_{E,\beta} \,|\, T \,|\, \chi_\alpha\right) = \frac{1}{\pi V_E^*}\left(\psi_{E,\beta} \,|\, T \,|\, \chi_\alpha\right)\sin\Delta\left(\frac{\left(\phi \,|\, T \,|\, \chi_\alpha\right)}{\left(\psi_{E,\beta} \,|\, T \,|\, \chi_\alpha\right)} - \cot\Delta\right)$$

The Fano-Formula:

$$\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{s} \frac{|q + \cot\Delta|^2}{1 + \cot^2\Delta} \sim \left|M_{\alpha\beta}\right|^2$$

$$\mathrm{q} = \frac{\left(\phi \,|\, T \,|\, \chi_\alpha\right)}{\left(\psi_{E,\beta} \,|\, T \,|\, \chi_\alpha\right)}$$

$$\sigma_{\alpha\beta}^{s} \sim \left|\frac{1}{\pi V_E^*}\left(\psi_{E,\beta} \,|\, T \,|\, \chi_\alpha\right)\right|^2$$

Controlling the Line Shape…

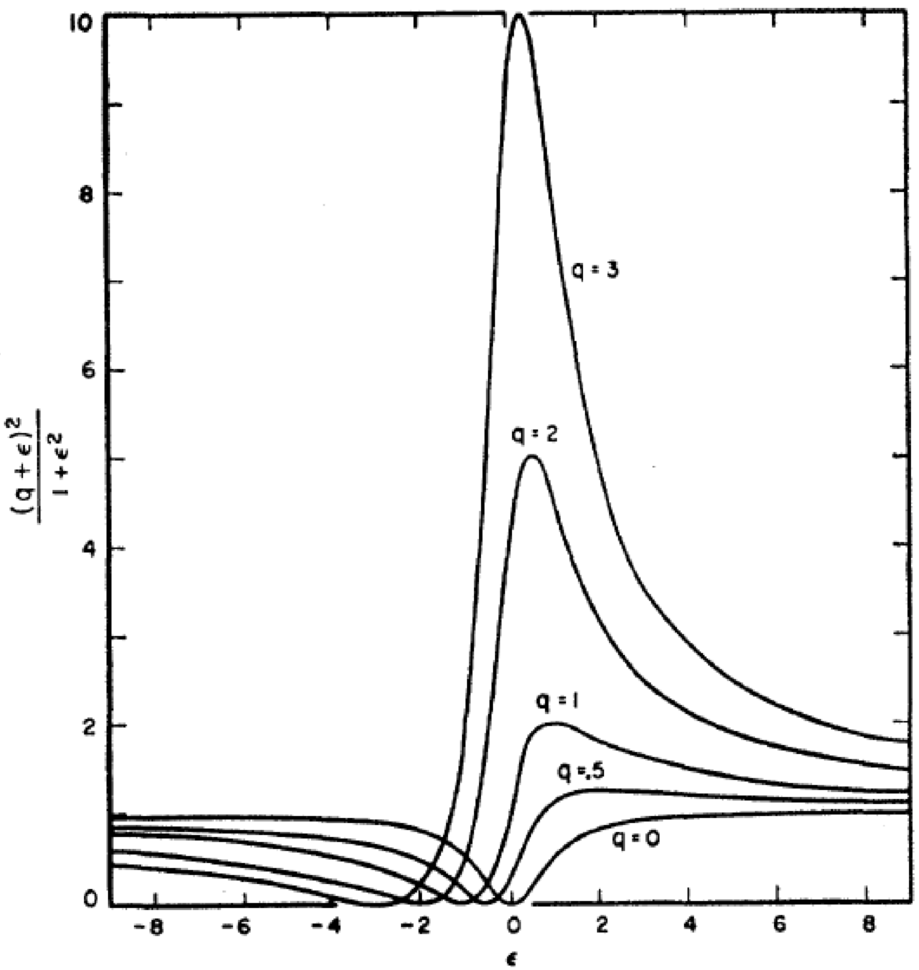
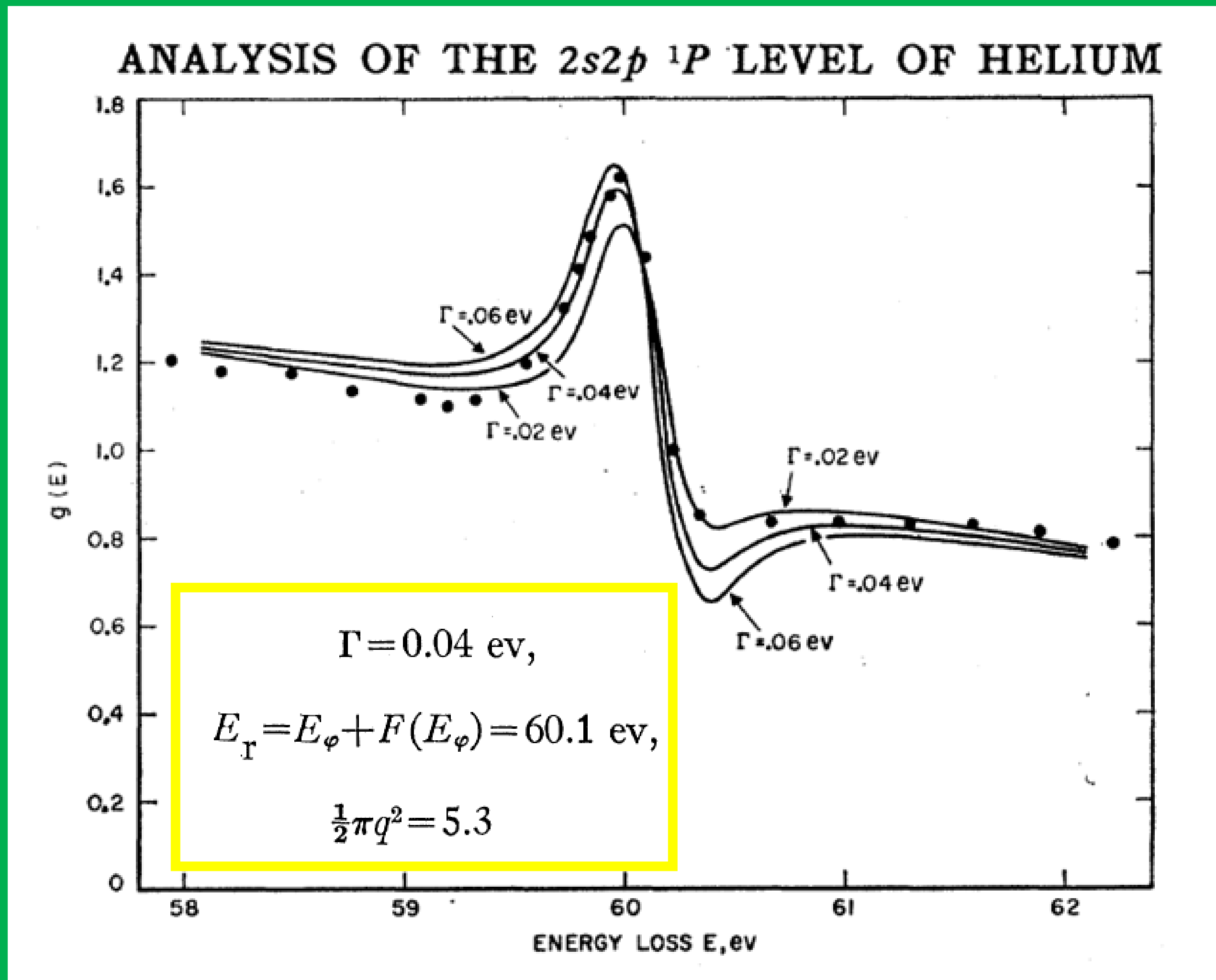


FIG. 1. Natural line shapes for different values of q. (Reverse the scale of abscissas for negative q.)

…taken from Fano's original work: Phys.Rev. 124,1866 (1961)

Historically:
The *famous* Silverman-Lassettre data
He(e,e')He*(^{1}P) @ 500eV



$\Gamma = 0.04$ ev,

$E_r = E_\varphi + F(E_\varphi) = 60.1$ ev,

$\frac{1}{2}\pi q^2 = 5.3$

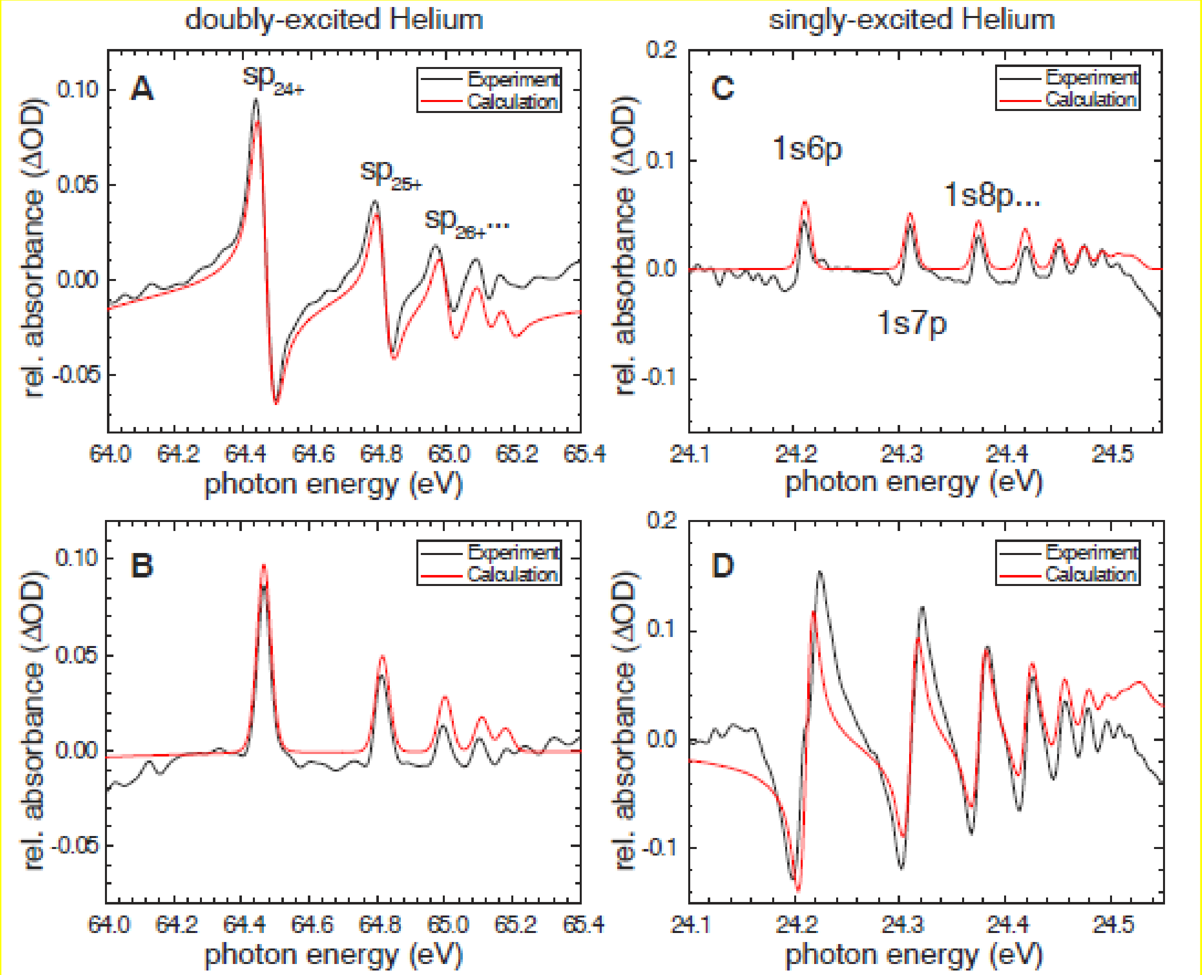
Note: q must be negative – q=-1.84

Manipulating Quantum Interference by Laser-Modelling of Fano-Line Shapes

Lorentz Meets Fano in Spectral Line Shapes: A Universal Phase and Its Laser Control

Christian Ott,[1] Andreas Kaldun,[1] Philipp Raith,[1] Kristina Meyer,[1] Martin Laux,[1] Jörg Evers,[1] Christoph H. Keitel,[1] Chris H. Greene,[3] Thomas Pfeifer[1,2*]

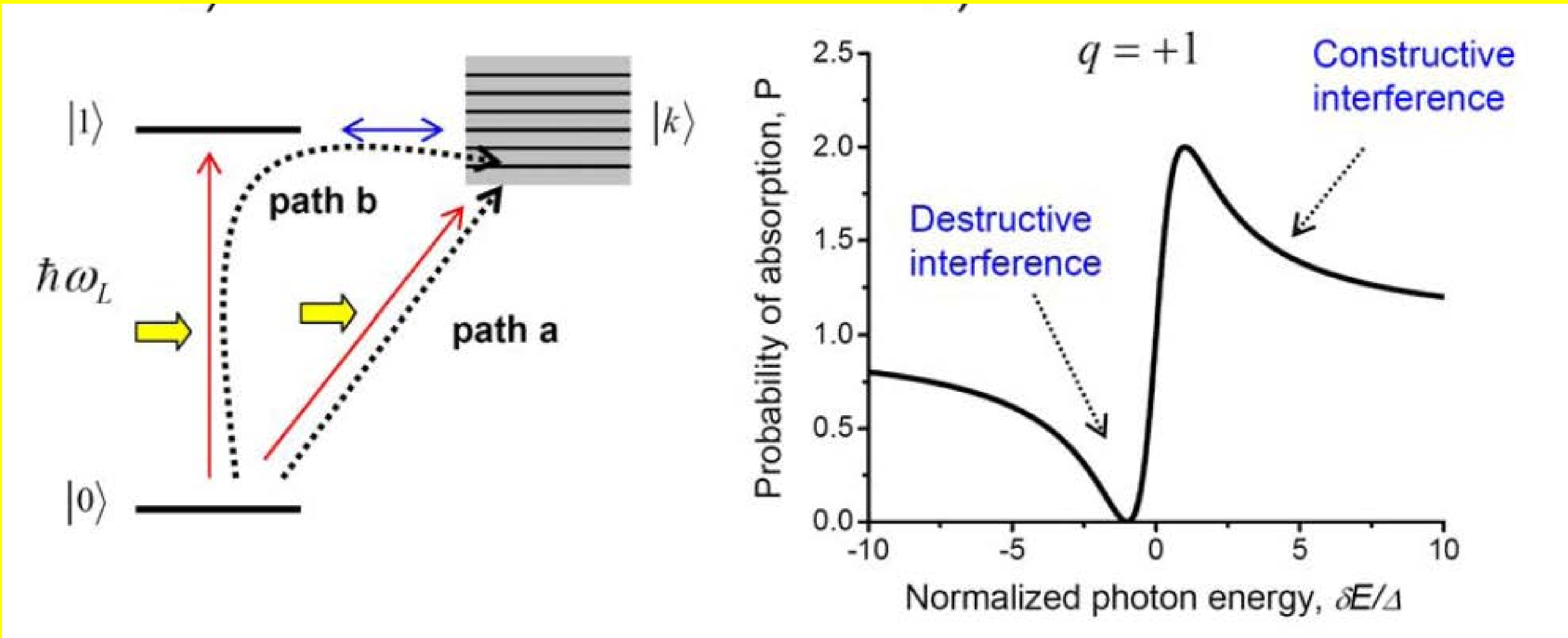
Fig. 3. Transforming asymmetric Fano spectral absorption lines into symmetric Lorentzian absorption peaks in doubly excited He and vice versa, from Lorentz to Fano, in singly excited He. (A) Field-free (static) absorption spectrum of doubly excited states of the *N* = 2 series in He. The well-known Fano absorption profiles are observed in the transmitted spectrum of a broad-band attosecond pulse. (B) When a 7-fs laser pulse immediately follows the attosecond-pulsed (deltalike) excitation (time delayed by ~5 fs) at an intensity of 2.0×10^{12} W/cm^2, the Fano absorption profiles are converted to Lorentzian profiles. (C) Field-free (static) absorption spectrum of singly excited He states below the first ionization threshold (24.6 eV): Lorentzian line shapes are visible in the attosecond-pulse absorption spectrum. (D) Absorption spectrum of the states in (C), when the attosecond pulse is again followed by the 7-fs laser pulse, at an intensity of 2.1×10^{12} W/cm^2. The initially Lorentzian absorption profile has been laser-transformed into an asymmetric Fano profile. The solid black lines are the measurement results; the red lines are generated by using tabulated values in (A) from (*6*) and (C) from (*30*), whereas the red line in (B) represents Lorentzians at the resonance positions of the original Fano lines. The red line in (D) shows Fano profiles with expected laser-induced $q = 1.49$ (Fig. 2) at the resonance positions of the original Lorentzian resonances.



Science **340**,716 (2013)

Visualizing Quantum Interference in Microscopic Systems: Asymmetric Fano-Line Shapes of Resonances

$$\Psi_E = \sum_i a_i \varphi_i^{(d)} + \sum_k \int d\varepsilon b_k \psi_k^{(c)}$$



Fano-Resonances in Nuclei

Hamiltonian and Wave Function

$$H = \begin{pmatrix} H_{11}^{b} & V_{12} & V_{13} \\ V_{21} & H_{22}^{s} & V_{23} \\ V_{31} & V_{32} & H_{33}^{x} \end{pmatrix}$$

The Fano wave function:

$$|\phi_J\rangle = \sum_n z_n(E)|n_J\rangle + \int d\varepsilon\, z_\varepsilon(E)|\varepsilon_J\rangle + \sum_{j' J_C} z_{j' J_C}(E)\big|(j' J_C)_J\big\rangle$$

The coupled equations (core nucleus integrated out):

$$\left(H_{11}^{b} - \varepsilon\right)\left|n_J\right\rangle = 0$$

$$\left(H_{22}^{s} - \varepsilon\right)\left|\varepsilon_J\right\rangle = 0$$

$$\left(H_{33}^{c} - (\varepsilon - \mathrm{E}_{J_C})\right)\left|j'\right\rangle = 0$$

Reduced Hamiltonian and Wave function

$$H = \begin{pmatrix} H_{11}^{b} & 0 & V_{13} \\ 0 & H_{22}^{s} & V_{23} \\ V_{31} & V_{32} & H_{33}^{x} \end{pmatrix}$$

The coupled equations (core nucleus integrated out):

$$\left.\begin{aligned} \left(\varepsilon_n^b - \varepsilon\right) z_n + \sum_c \langle n_J | V_{13} | j'c \rangle z_{j'c} = 0 \\ \left(\varepsilon_j^s - \varepsilon\right) z_\varepsilon + \sum_c \langle n_J | V_{23} | j'c \rangle z_{j'c} = 0 \end{aligned}\right\} \rightarrow \text{s.p. motion w.r.t. the g.s.}$$

$$\left(\varepsilon_{j'} - (\varepsilon - \mathrm{E}_{J_C})\right) z_{j'c} + \sum_n \langle j'c | V_{31} | n \rangle z_n + \int d\varepsilon' \langle j'c | V_{32} | \varepsilon' \rangle z_{\varepsilon'} = 0$$

Multi-channel Fano wave function:

$$|\phi_J\rangle = \sum_n z_n(E)|n_J\rangle + \int d\varepsilon\, z_\varepsilon(E)|\varepsilon_J\rangle + \sum_{j' J_C} z_{j' J_C}(E)\big|(j' J_C)_J\big\rangle$$

Extension to the Case of Several Open Channels

$$\Psi_{hE} = a_h \varphi + \int dE' [b_{hE'} \psi_{E'} + c_{hE'} \chi_{E'}]$$

- n=2 open channels
- n=2 energetically degenerate solutions with outgoing flux

$$(\varphi | H | \varphi) = E_\varphi,$$

$$(\psi_{E'} | H | \varphi) = V_{E'}, \quad (\chi_{E'} | H | \varphi) = W_{E'},$$

$$\Gamma_1 = \pi |V_E|^2 \;\; ; \;\; \Gamma_2 = \pi |W_E|^2 \Rightarrow \Gamma = \sum_i \Gamma_i$$

$$\Delta = -\arctan \frac{\Gamma(E)}{E - E_\varphi - G(E)}$$

$$G(E) = -\frac{P}{\pi} \int dE' \frac{\Gamma(E')}{E' - E}$$

Solution 1: fully mixed

$$z(E,E') = \frac{1}{\pi}\frac{P}{E-E'}\sin\Delta - \delta(E-E')\cos\Delta$$

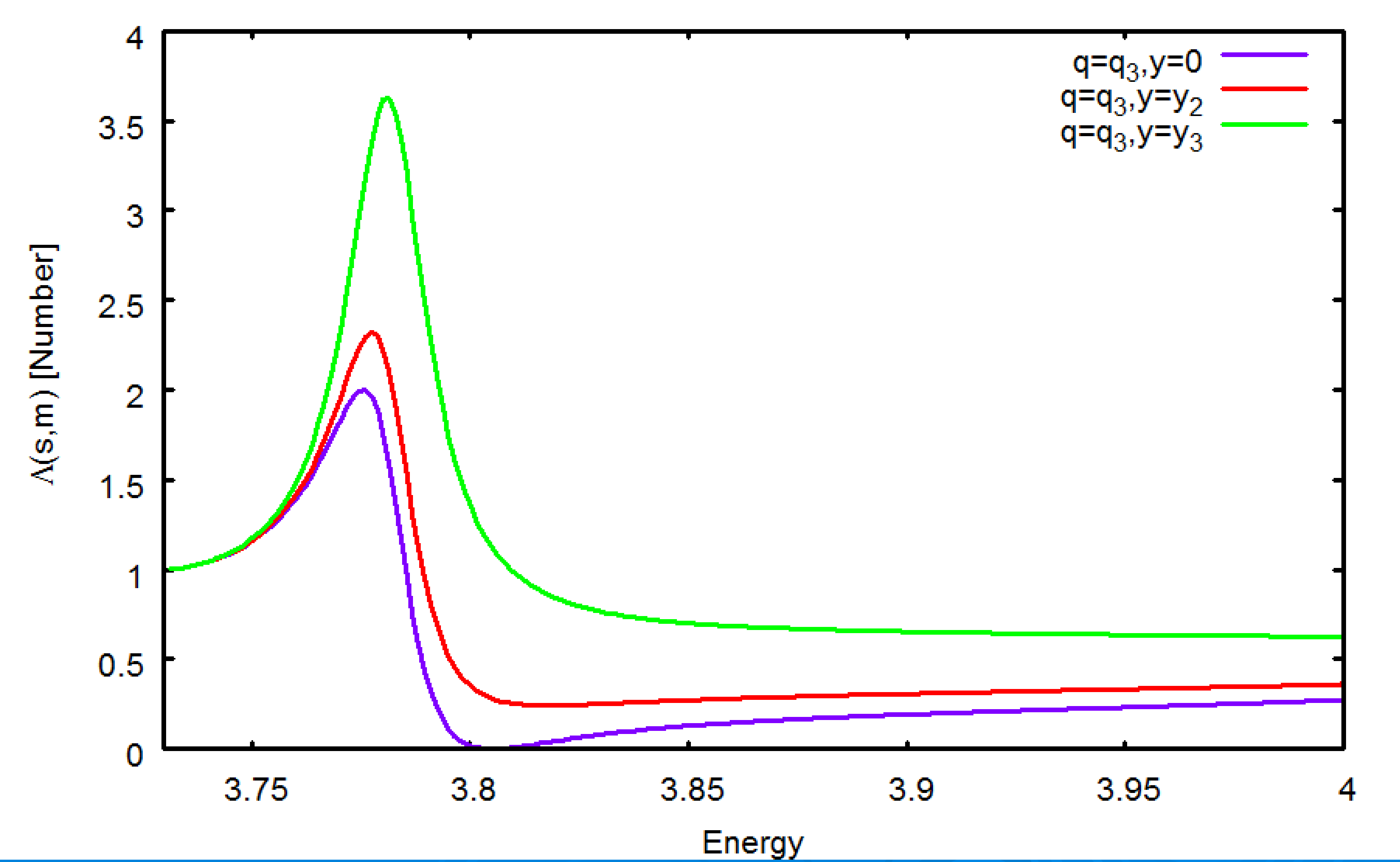
$$a_1 = \frac{\sin\Delta}{\Gamma} \;\; ; \;\; b_{1E'} = \sqrt{\frac{\Gamma_1}{\Gamma}}z(E,E') \;\; ; \;\; c_{1E'} = \sqrt{\frac{\Gamma_2}{\Gamma}}z(E,E')$$

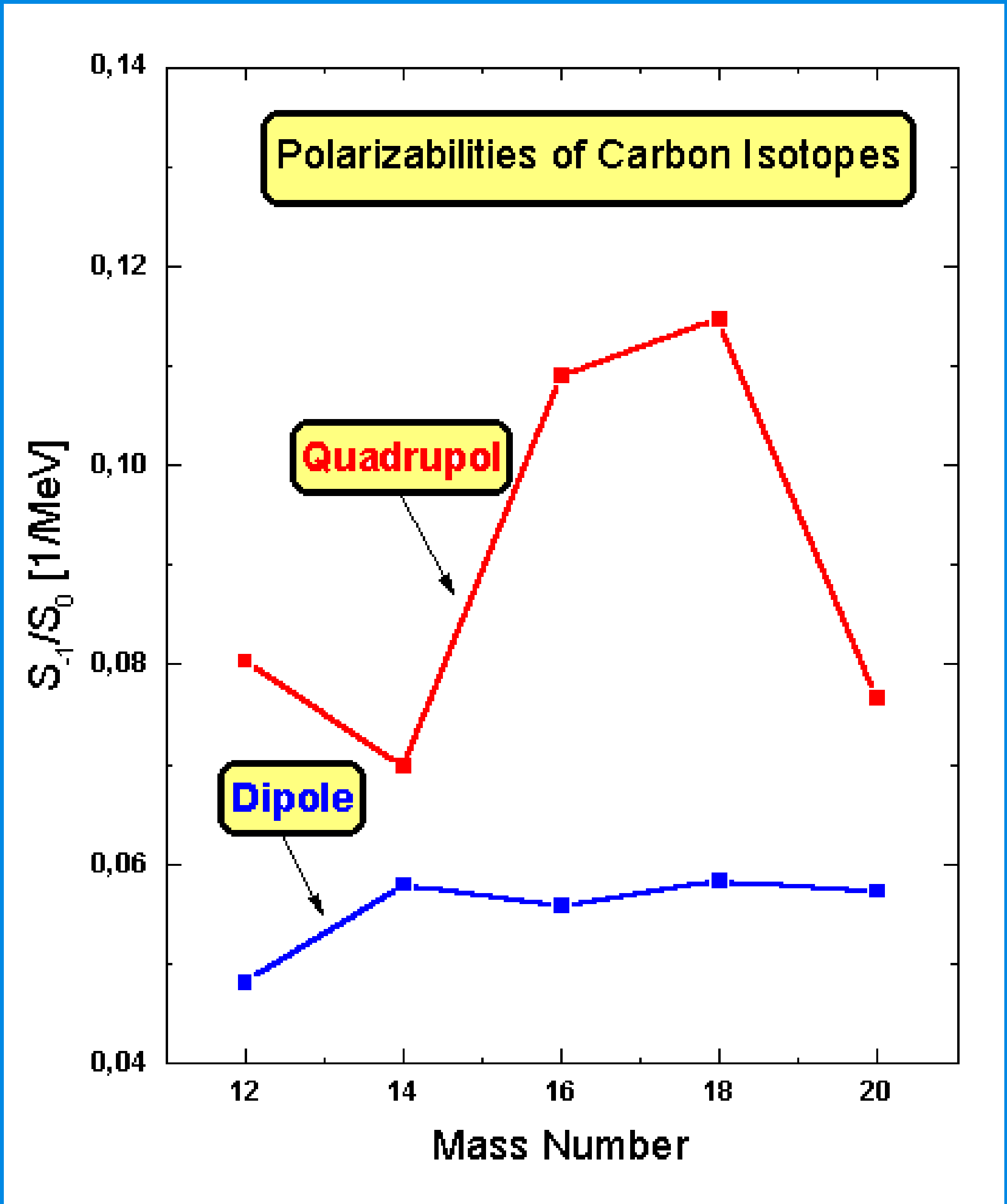
Solution 2: continuum mixed

$$a_2 = 0 \;\; ; \;\; b_{2E'} = \sqrt{\frac{\Gamma_1}{\Gamma}}\delta(E-E') \;\; ; \;\; c_{2E'} = -\sqrt{\frac{\Gamma_2}{\Gamma}}\delta(E-E')$$

→ Resonance superimposed on a smoothly varying background!

Multi-channel Coupling



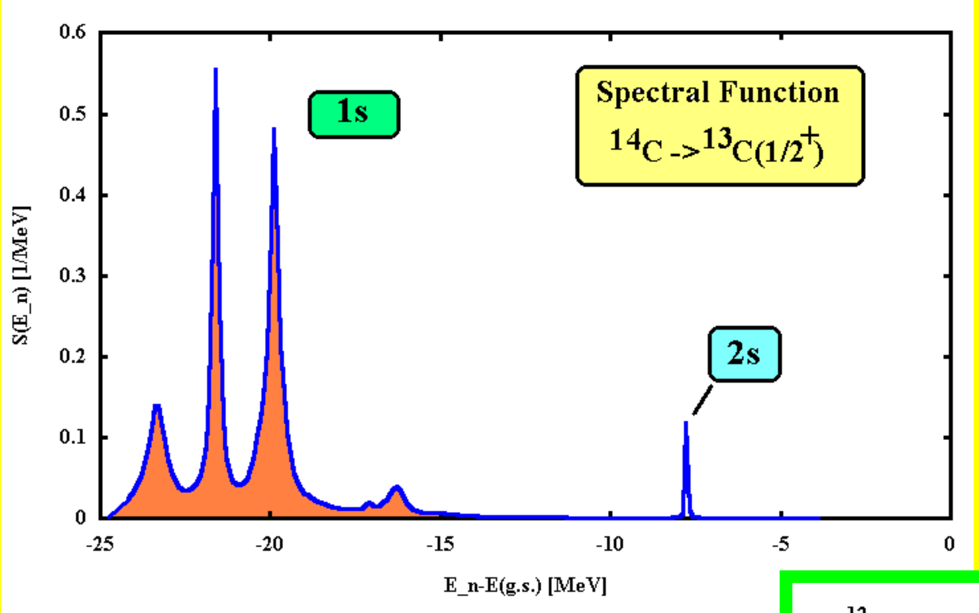


Polarizability of Even-Mass Carbon Isotopes: HFB+QRPA results

$$S_n = \sum E_a^n \left| \langle a \| T_\lambda \| 0 \rangle \right|^2$$

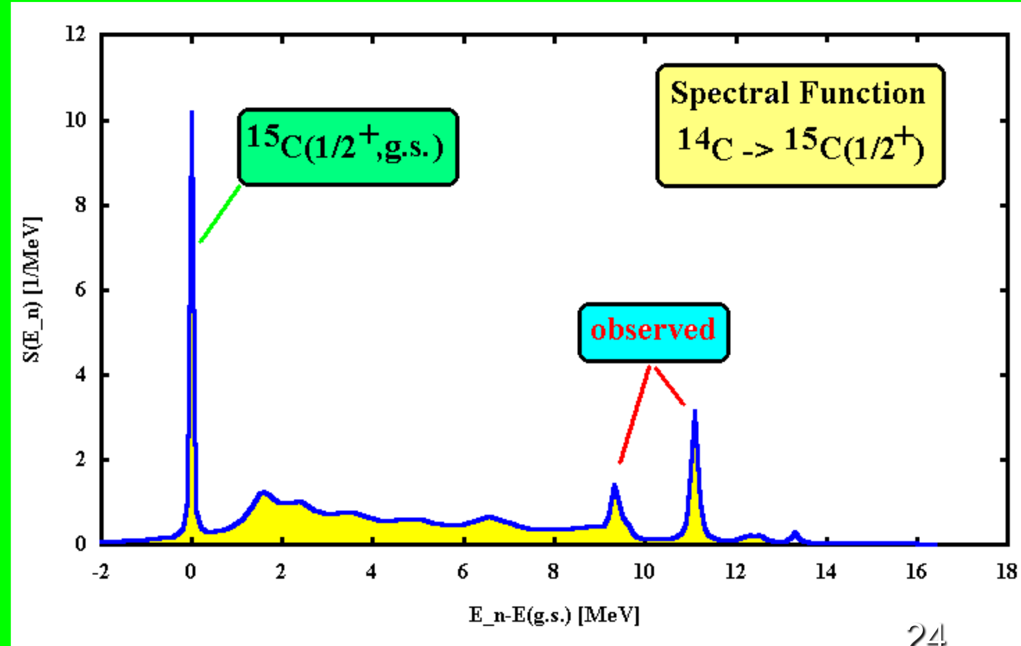
$$P = \frac{S_{-1}}{S_0}$$

1/2⁺ Particle and Hole Spectral Functions in ^{14}C



Hole strength function:

$^{13}C \sim {}^{14}C\text{-}n$



Particle strength function

$^{15}C \sim {}^{14}C+n$

Fano-Resonances in the Nuclear Continuum

$$\left(\mathrm{h}_{\mathrm{j}}^{(1)}-\varepsilon_1\right)\phi_j+\sum_{\mathrm{j}'\ \mathrm{J_C}}\left\langle 0\left|\mathrm{V}_{13}\right|\ \mathrm{J_C}\right\rangle\phi_{j'J_c}=0$$

g.s., elastic

$$\left(\mathrm{h}_{\mathrm{j'J_C}}^{(\mathrm{i})}-\varepsilon_{\mathrm{i}}\right)\phi_{j'J_c}+\sum_{\mathrm{n}}\left\langle \mathrm{J_C}\left|\mathrm{V}_{13}\right|0\right\rangle\phi_j=0$$

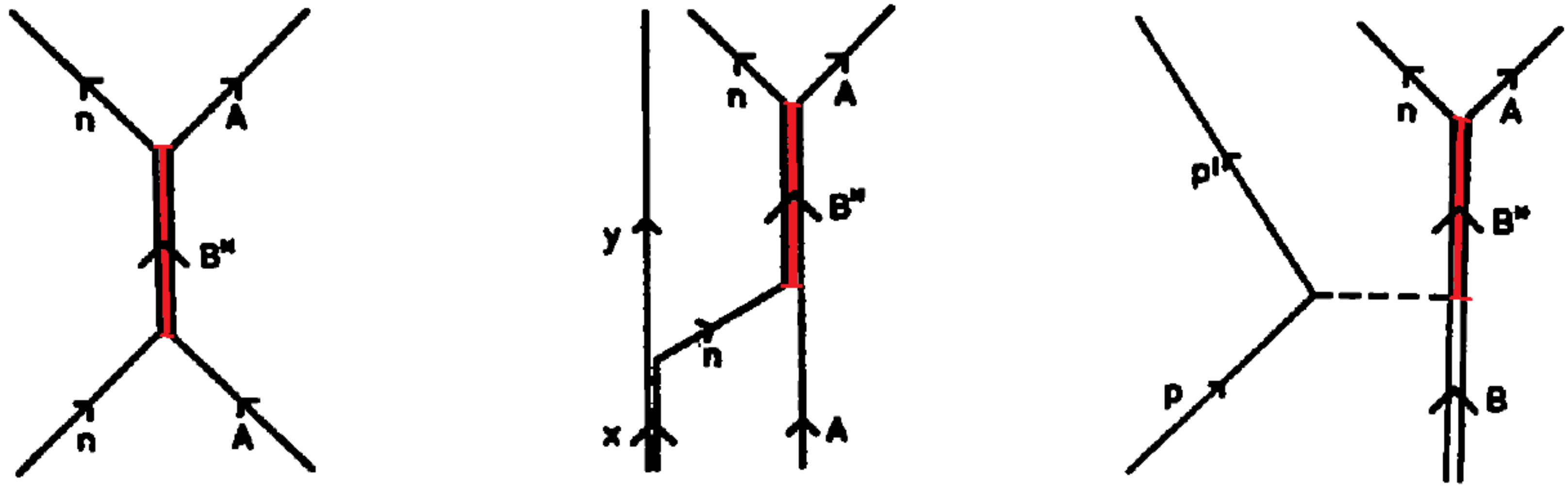
inelastic

Important core excitations ^{14}C

$$E_C(J^\pi)=6.094\ (1^-),\ 6.728\ (3^-),\ 7.012\ (2^+),\ 8.317\ (2^+)\ \mathrm{MeV}.$$

Phys.Lett. B633 (2006) 469

Resonance Scenarios in Nuclear Physics



$A + n \rightarrow B^{*} \rightarrow A + n$

Resonance Formation

$A + x \rightarrow B^{*} + y \rightarrow A + n + y$

Population by Transfer

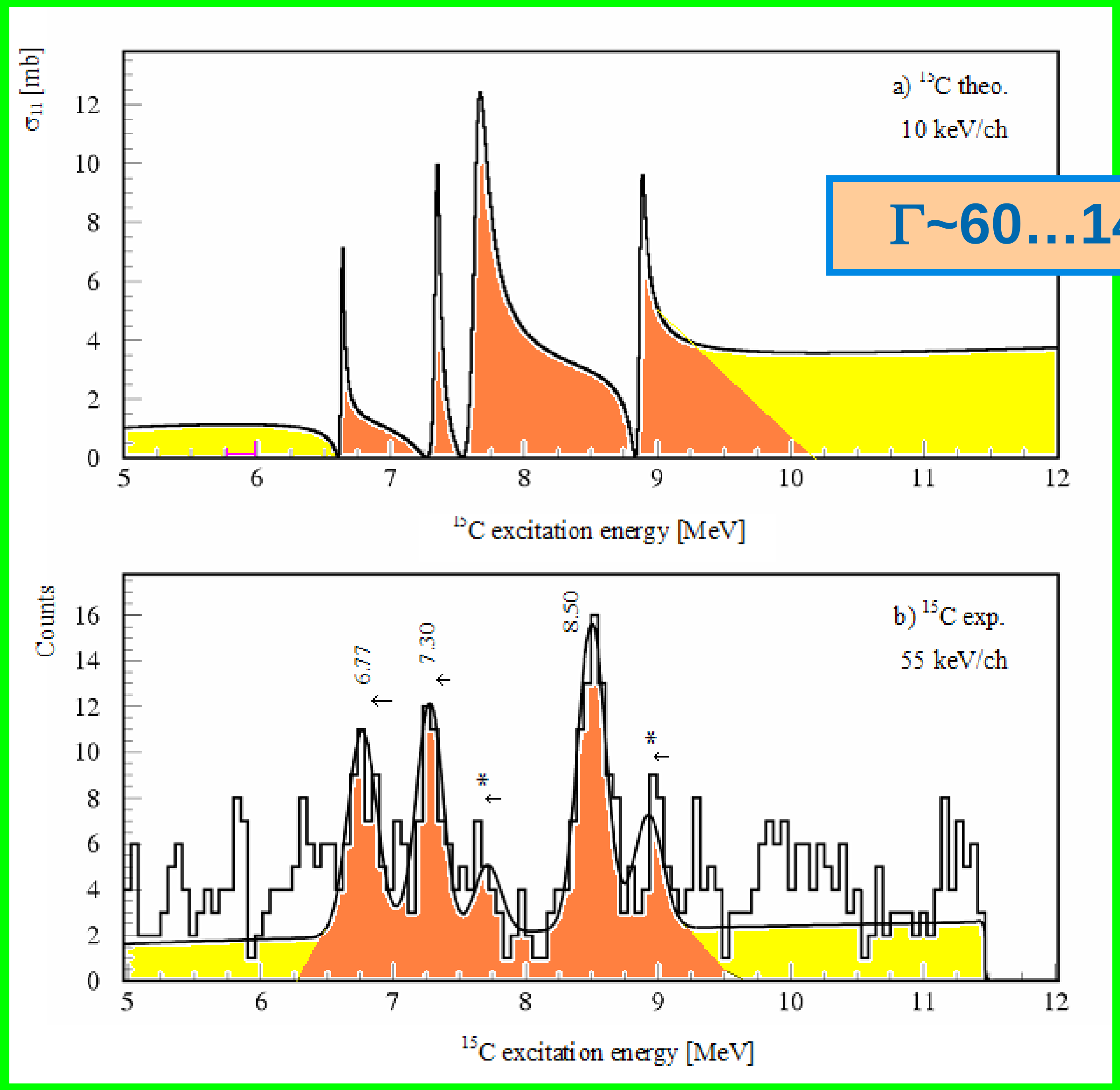
$B + p \rightarrow B^{*} + p' \rightarrow A + n + p'$

Inelastic Excitation

The Fano-Wave Function:

$$\psi_E^{JM} = e^{i\Delta}\{\cos\Delta\,[\phi_A \chi_n^0]^{JM} + \sin\Delta\,\frac{1}{\pi V_{E*}}\,[\phi_b^{JM} + [\phi_A \tilde{\chi}_n^0]^{JM}].$$

Correlation Dynamics in an Open Quantum System: d-wave Fano-Resonances in ^{15}C



Γ~60…140keV

$^{15}N(^{7}Li,\,^{7}Be)^{15}C$

$E_{inc} = 55$ MeV

Sonja Orrigo, H.L., Phys.Lett. B633 (2006)

Pairing in the Continuum: Quasiparticle Resonances

Pairing Theory as Coupled Channels Problem: The Gorkov-Equations

$$\begin{pmatrix} H-\lambda & -\Delta \\ -\Delta^{+} & -(H-\lambda) \end{pmatrix}\begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = E\begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}$$

$$\phi_+ \sim u_{\ell j}^{(q)}(r)\,|(\ell s)\,jm\rangle;\ \phi_- \sim v_{\ell j}^{(q)}(r)\,|(\ell s)\,jm\rangle$$

Mean-Field Hamiltonian $(q = p,n)$:

$$H = -\frac{\hbar^2}{2m}\vec{\nabla}^2 + U(\rho)$$

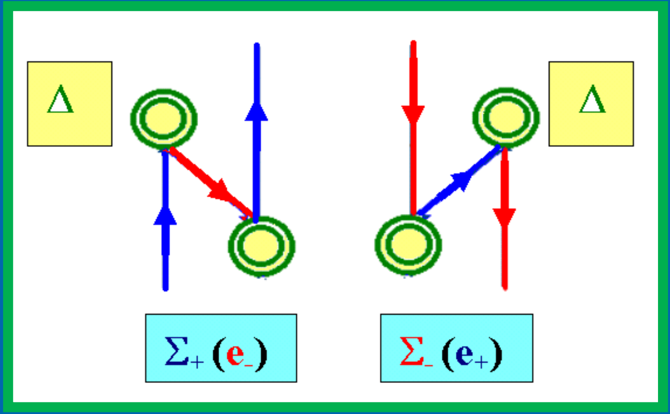
$$\rho_q(r) = \sum_{n\ell j}\frac{2j+1}{4\pi}\,|\,v_{n\ell j}^{(q)}(r)\,|^2$$

Pairing-Field & Density $(q = p,n)$:

$$\Delta_q = \tfrac{1}{2}V_{SE}(\rho)\,\kappa_q$$

$$\kappa_q(r) = \sum_{n\ell j}\frac{2j+1}{4\pi}u_{n\ell j}^{(q)}(r)\,v_{n\ell j}^{(q)*}(r)$$

Pairing in the Continuum



$$u_\alpha(r) \to \cos\left(\delta_\alpha^{(c)}\right) f_\alpha(r) + \sin\left(\delta_\alpha^{(c)}\right) g_\alpha(r),$$

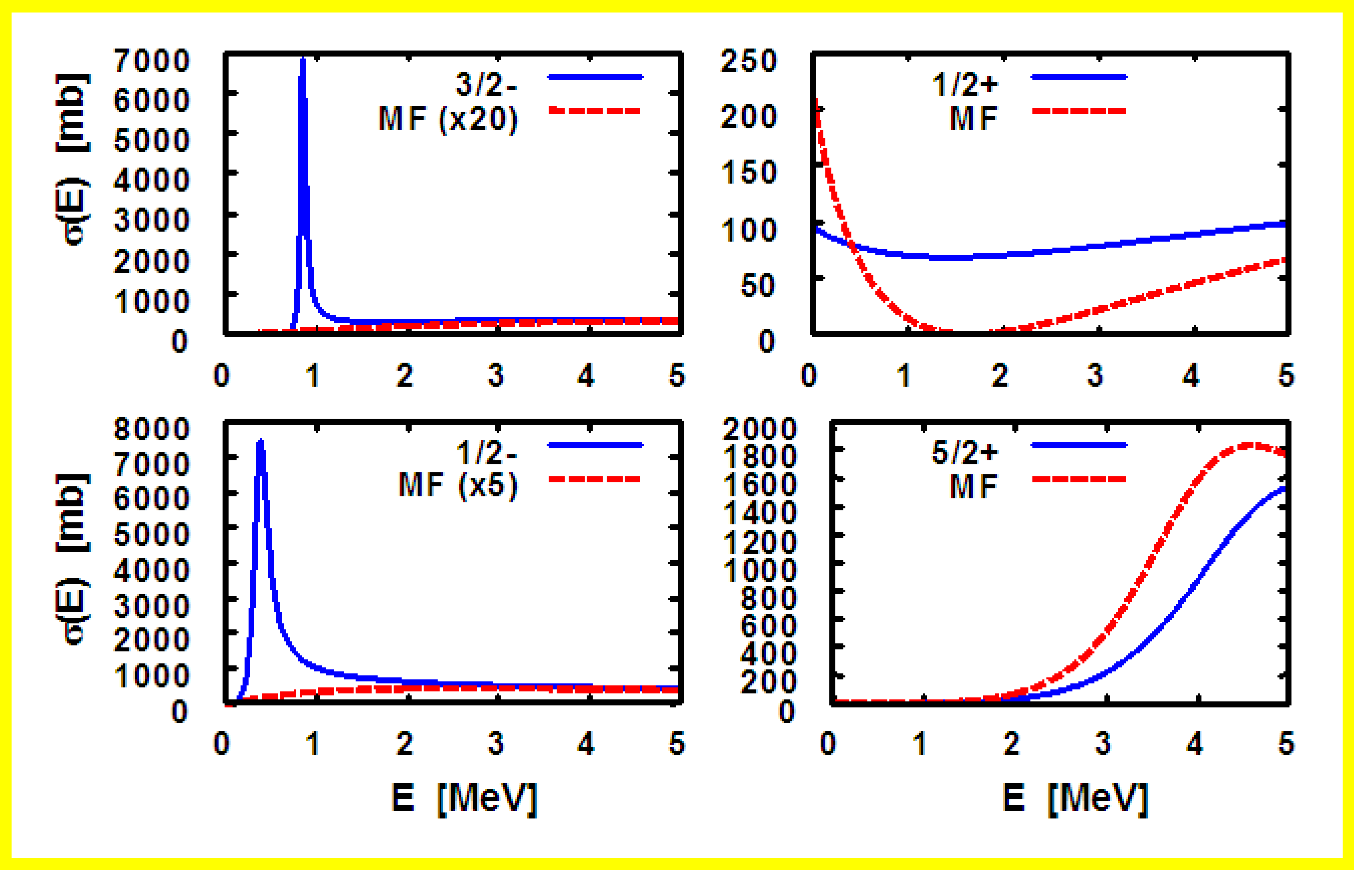
$$\tan\left(\delta_\alpha^{(c)}\right) = -\frac{2\tilde{m}k_\alpha}{4\pi\hbar^2}\langle f_\alpha | \Sigma_q^{(c)} | u_\alpha\rangle \sim -\frac{2\tilde{m}k_\alpha}{4\pi\hbar^2}\langle f_\alpha | \Sigma_q^{(c)} | f_\alpha\rangle$$

$$\Gamma_\alpha^{(c)} \sim N(k_\alpha) |\langle f_\alpha | \Delta_\alpha | v_b\rangle|^2$$

S. Orrigo, H.L., PLB 677 (2009)

Pairing Resonances in Dripline Nuclei

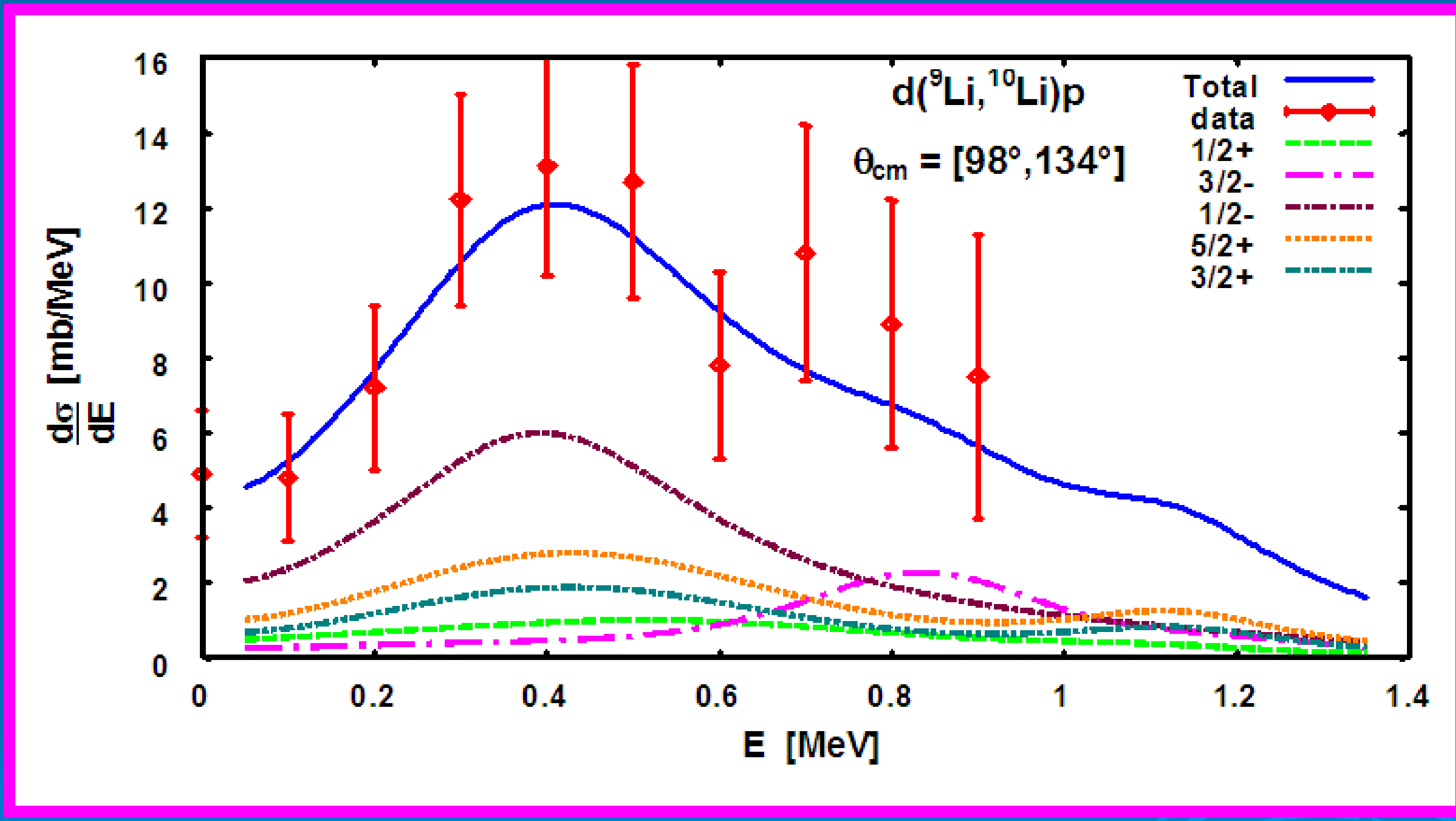
$^{9}Li+n \longleftrightarrow {}^{10}Li$



$$\begin{pmatrix} T_q + U_q - 2\lambda_q + e_\alpha & \Delta_q(\vec{r}) \\ -\Delta_q^\dagger(\vec{r}) & -(T_q + U_q - e_\alpha) \end{pmatrix} \begin{pmatrix} u_{\alpha q}(\vec{r}) \\ v_{\alpha q}(\vec{r}) \end{pmatrix} = 0$$

S. Orrigo, H.L., PLB 677 (2009) & ISOLDE newsletter Spring 2010, p.5

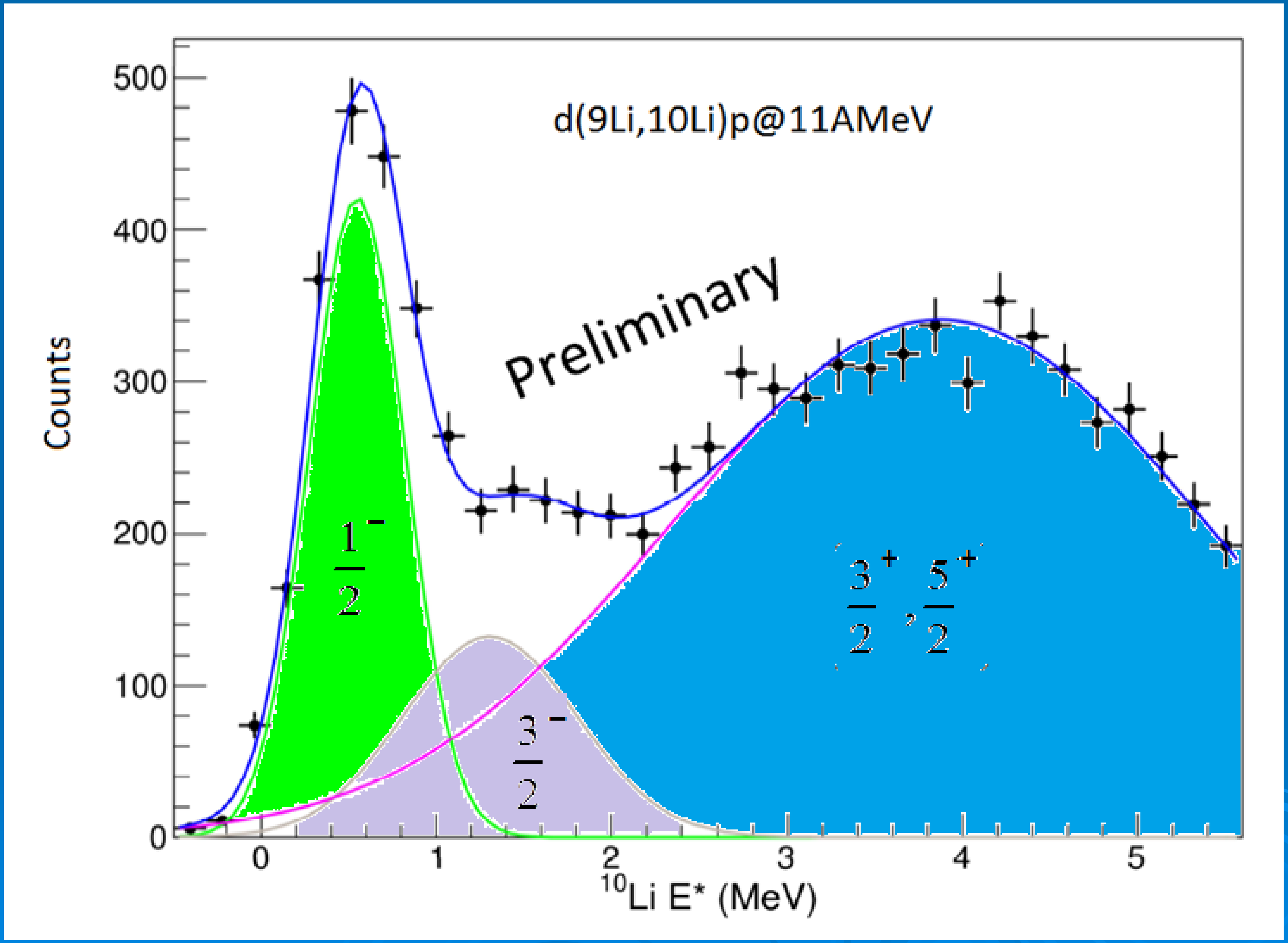
Continuum Spectroscopy at REX-ISOLDE: $^{10}Li = ^{9}Li + n$
$d(^{9}Li, ^{10}Li)p$@2.36AMeV



S. Orrigo, H.L., PLB 677 (2009) & ISOLDE newsletter Spring 2010, p.5

Data: H. Jeppesen et al., REX-ISOLDE Collaboration, NPA 738 (2004) 511 & NPA 748 (2005) 374.

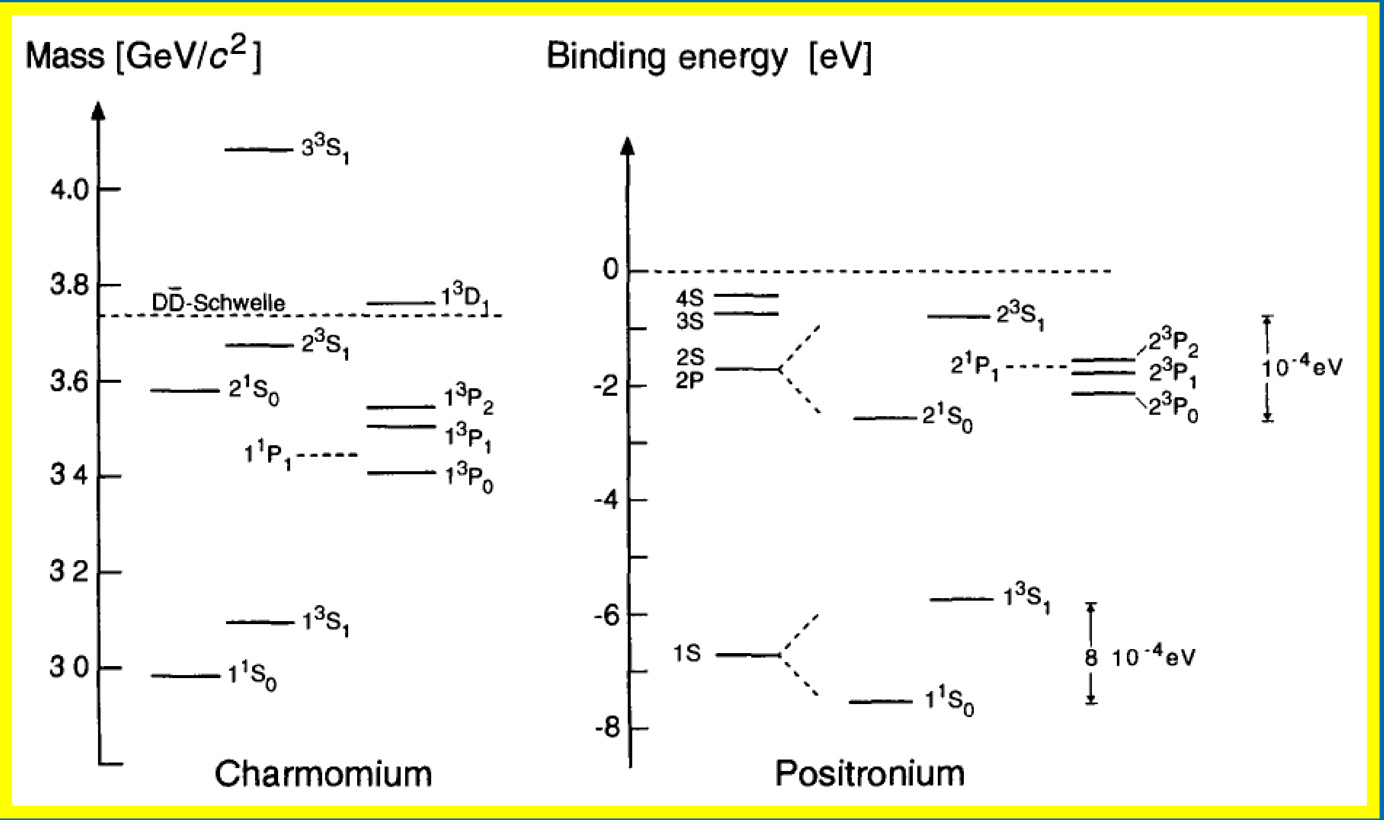
New experimental results (Dec. 2013): ^{10}Li continuum spectroscopy at TRIUMF



S. Orrigo, M. Cavallo, F. Capppuzzello et al.

Fano-Resonances in Hadron Physics: Charmonium Spectroscopy

Quarkonium: Confined Systems of a Light and a Heavy Quark



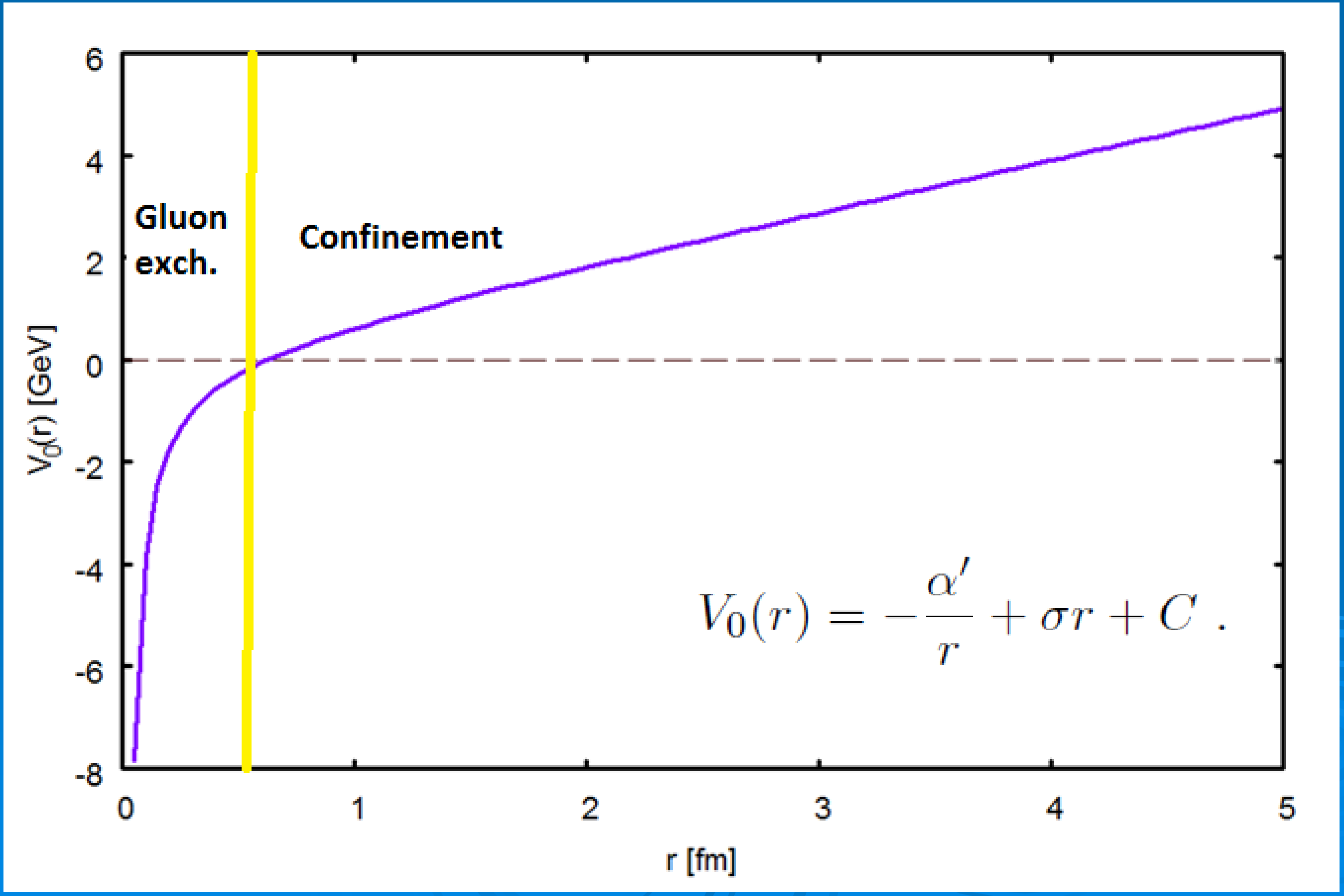
The Cornell Potential (Eichten, Gottfried ~1980):

$$V_0(r) = -\frac{\alpha'}{r} + \sigma r + C \ .$$

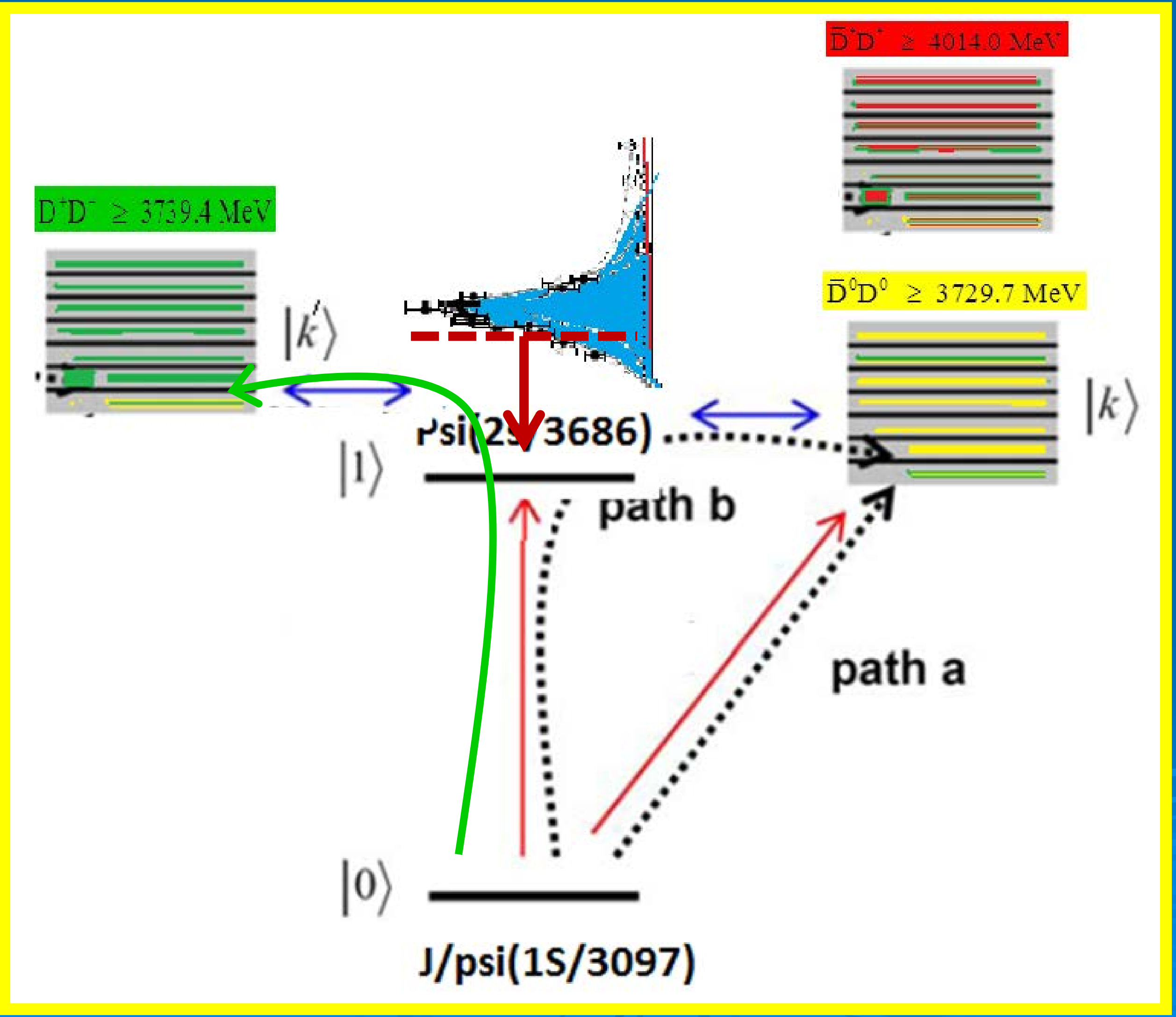
…enriched by relativistic corrections (Isgur, Godfrey ~1985):

Charmonium and Bottonium Spectroscopy

	m_q [GeV]	α_s	σ [GeV / fm]	energy interval [GeV]
Charmonium	1.2185	0.29	1.306	2.8 - 4.8
Bottonoum	4.7645	0.388	1.02	9.0 - 11.0



Spectroscopy at the Open Charm Threshold:
→ Interaction of a confined $c\bar{c}$-configuration with two $D\bar{D}$- continua

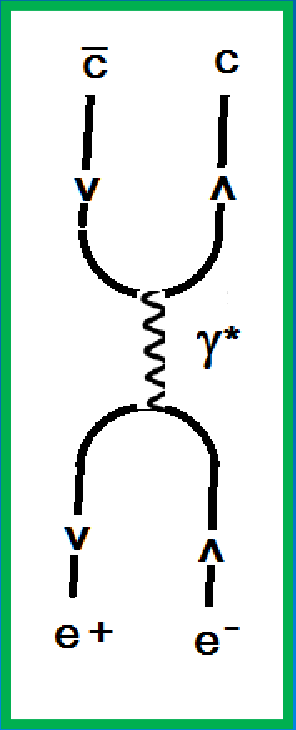


Xu Cao, H. L., PRL, to appear

Model Wave Function at the Open Charm Threshold

$$\left|\psi_b\right\rangle \sim x_b(\omega)\left|(c\overline{c})_{2S}\right\rangle + \int d^3k\; x_k(\omega)\left|k_{\bar{D}D}\right\rangle$$

$$\left|\psi_r\right\rangle \sim z_b(\omega)\left|(c\overline{c})_{2S}\right\rangle + \int d^3k\; z_k(\omega)\left|k_{\bar{D}D}\right\rangle$$



Charm Production by e+e- Annihilation at BES

$$e^+ + e^- \rightarrow (c\overline{c})_{1^-}$$

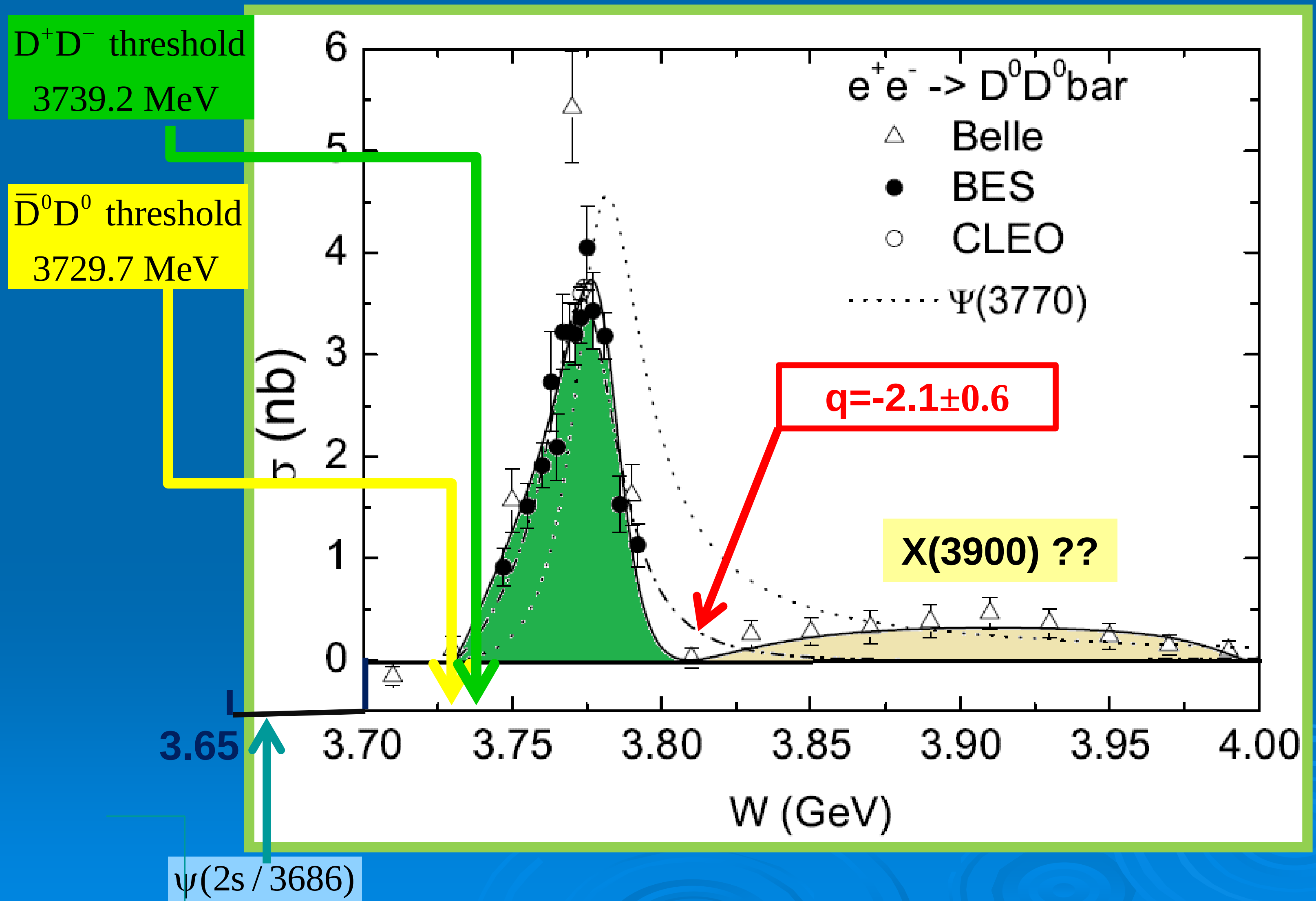
Fano-Cross Section@BES III

$$\cot\Delta_r = \frac{s^* - m_R^2}{\sqrt{s^*}\,\Gamma_R(s)} \quad ; \quad s^* = \left(\sqrt{s} + \Delta m(s)\right)^2$$

$$\sigma\left(e^+e^- \to D\bar{D}\right) \sim \sigma_{bg}\frac{\left(q + \cot\Delta_r\right)^2}{1 + \cot^2\Delta_r}$$

$$\sigma_{bg} \sim \left|\left\langle e^+e^- \middle| T \middle| k_{\bar{D}D}\right\rangle\right|^2 \quad ; \quad q = \frac{\left(\phi\,|\,T\,|\,\chi_\alpha\right)}{\left(k_{\bar{D}D}\,|\,T\,|\,\chi_\alpha\right)}$$

$D\bar{D}$-Dynamics at Threshold
Channel Coupling and the Line Shape of Ψ(3770)



Xu Cao, H. L., PRL, submitted

Summary

- Dynamics close to the particle threshold
- Fano resonances in atoms
- Fano resonances in atomic nuclei
- Fano resonances in hadrons
- Universality of quantum interference
- Universal and versatile tool for continuum spectroscopy

…with contributions of Sonja Orrigo and Xu Cao