

Isospin-mixing corrections for super allowed Fermi-beta decay from ab-initio calculations

Universality in Few-Body Systems: Theoretical Challenges and New Directions.

March 12 2014, INT (Seattle, WA)

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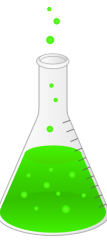
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Required ingredients to attain fundamental physics from the NP side



Understanding of nuclear forces or more correctly what does QCD look like at low energies (MeV).
- Chiral effective field theory.



Many-body techniques to turn the crank.
- No Core Shell Model and others.



Extrapolation techniques when the crank is too small.
- UV/ir extrapolation methods.



Clean signatures of new physics
- next slide.



What can nuclear physics give insights to in terms of fundamental physics?

Unitarity of the CKM matrix: (this presentation)

Determined by corrections to fit values from isospin-mixing corrections (+ QED corrections).

If CKM matrix is not unitary it could signal beyond standard model physics

Towner & Hardy, Ormand & Brown, Navratil & Ormand, Wiringa et al, Kruse & Ormand...

Permanent electric dipole moment (EDM) of light nuclei (He-3, Li-6):

Parity and time-reversal would be violated.

Meregethi, de Vries, van Kolck, Stetcu + others

Neutrinoless double-beta decay¹ (e.g. Ge-76).

If observed implies that the neutrino is its own anti-particle, i.e. Majorana.

Furthermore one could say something about the actual masses of the neutrinos directly, not just the differences. Slide at the end for the INT audience...

¹See Avignone III, Elliott, Engel in RMP **80** (2008) for a review

Background nuclear physics...

In order to make the talk more accessible let's bring everyone to the same page.

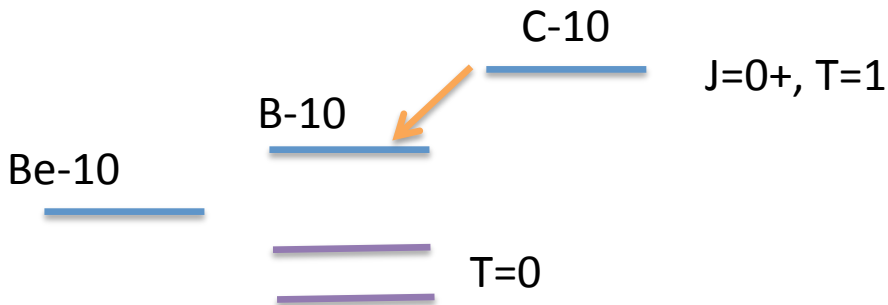
No Coulomb + nuclear force components equal

_____	_____	_____	$J=0+, T=1$
Be-10	B-10	C-10	
4p, 6n	5p, 5n	6p, 4n	
$t_z=-1$	$t_z=0$	$t_z=+1$	

Isobaric analogue states are nuclear states that appear in mirror-nuclei when p and n's are interchanged. They can be labeled by the isospin "T".

Isospin is much like spin (SU(2)). The third component $t_z=1/2(Z-N)$

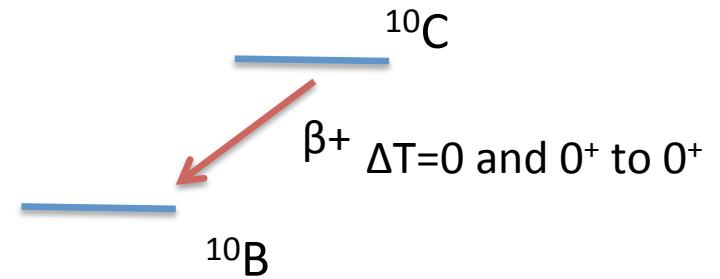
Coulomb + nuclear interaction



Superallowed Fermi transitions are beta-decays between $J=0+ T=1$ isobaric analog states.

Motivation

Superaligned Fermi β -decay transitions provide excellent tests of electroweak theory.



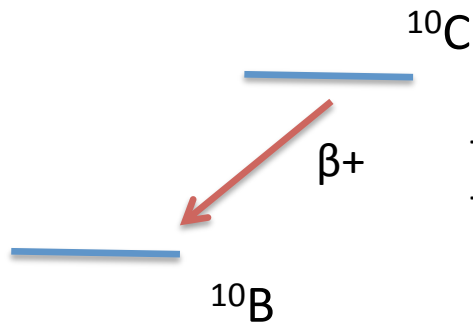
These transitions involve only the vector part of the weak interaction.

If the conserved-vector-current hypothesis is true then for pure Fermi transitions “ft” should be independent of nucleus (i.e. G_V is not renormalized in the nuclear medium).

$$ft = \frac{K}{G_V^2 |M_F|^2}$$

K fundamental constants and G_V is the vector coupling const.

Explanation of terms



t = partial lifetime of state (for branching)
f = statistical phase-space factor (Fermi function integral)

$$M_F = \langle \psi_f | T_{\pm} | \psi_i \rangle$$

T is the Isospin raising or lowering operator (changes neutron to proton or Vice versa.)

Corrections to measured “ft” values

But experimentally measured “ft’s” are not nucleus-independent.

For std. model tests one needs to apply two nucleus-dependent corrections to ft.

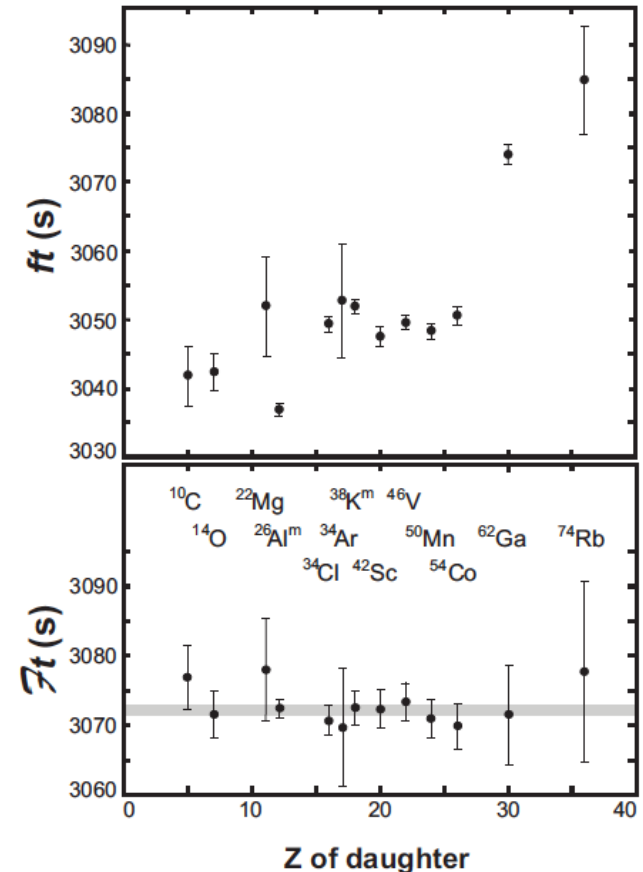
$$\mathcal{F}t = ft(1 + \delta_R + \Delta_R)(1 - \delta_C)$$

QED radiative corrections

Fermi-matrix element correction arising from nuclear structure due to isospin-breaking effects.

The nucleus-independent Ft values are then used to determine the CKM mixing matrix element between up and down quarks.

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t} \frac{\hbar^7}{G_F^2 m_e^5 c^4}$$



See Fig 3. of Hardy & Towner PRC 79, 055502 (2009) and references in caption.

The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} v_{ud} & v_{us} & v_{ub} \\ v_{cd} & v_{cs} & v_{cb} \\ v_{td} & v_{ts} & v_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Gives the relative probability that a down-type quark will weak decay into an up-type quark. d' is a superposition of down, strange and bottom. Or in HEP lingo: Transforms mass eigenstates into weak eigenstates.

The CKM matrix assumes three generations of quarks and is a fundamental part of the electroweak theory. It is believed to be unitary.

$$\sum_k |V_{ik}|^2 = \sum_i |V_{ik}|^2 = 1$$

Numerically, the values of the CKM matrix are below. V_{ud} is the largest component and thus you should spend most of your effort there (95% of the unitary check).

$$V_{\text{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

The sum of the squares of the top row give 0.9999 ± 0.0006

These numbers come from the PDG 2010 revised by A. Ceccucci, Z.Ligeti and Y.Sakai, in Section 11.

Determining V_{ud} in the CKM matrix

$$V_{ud} = G_V / G_F \quad \text{Fundamentally, ratio of vector-to-Fermi coupling constants.}$$

Corrected “ft” values from nuclear structure:

$$\mathcal{F}t = ft(1 + \delta_R + \Delta_R)(1 - \delta_C) \quad ft = \frac{K}{G_V^2 |M_F|^2}$$

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t} \frac{\hbar^7}{G_F^2 m_e^5 c^4} \quad \begin{array}{l} \text{Provided you know } G_F \\ \text{(from pure leptonic decays such as muon decay)} \end{array}$$

$$|V_{ud}| = 0.97425 \pm 0.00022 \quad \begin{array}{l} \text{Nuclear structure value taking into account uncertainties} \\ \text{in the radiative corrections as well as structure part.} \\ \text{Avg. of 13 superallowed Fermi transitions (20 measured)}^1. \end{array}$$

One can also determine V_{ud} from neutron beta decay or pion decay².

$$V_{ud} = 0.9746(18) \quad \text{Neutrons: have to consider axial-vector part too.}$$

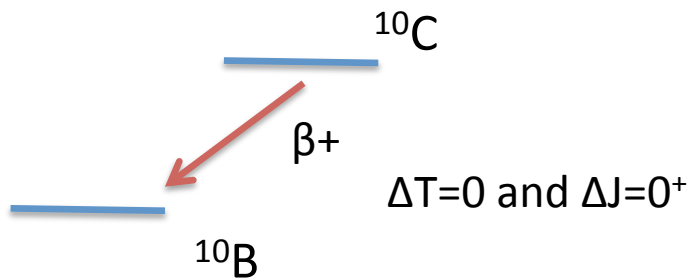
$$V_{ud} = 0.9749(26) \quad \text{Pions: branching ratio of } 10^{-8} \text{ that must be considered.}$$

1) Hardy and Towner, PRC 70, 055502 (2009)

2) A. Ceccucci, Z.Ligeti and Y.Sakai, PDG Feb 2010 Section 11

The isospin-mixing correction δ_c

Quantify isospin-symmetry breaking in a “heavier” nucleus by M_F .



Why is isospin broken?

Coulomb force (protons \neq neutrons)
NN scattering lengths $pp \neq pn \neq nn$

Fundamentally due to quark interactions
These effects are encoded in the potential
at various orders.

If isospin were exact for $T=1$: $M_F^2 = 2$

We need to consider a small deviation

$$\delta_c = \frac{|M_F^2 - 2|}{2}$$

Typically $\delta_c \sim 0.1\%$ for light nuclei.

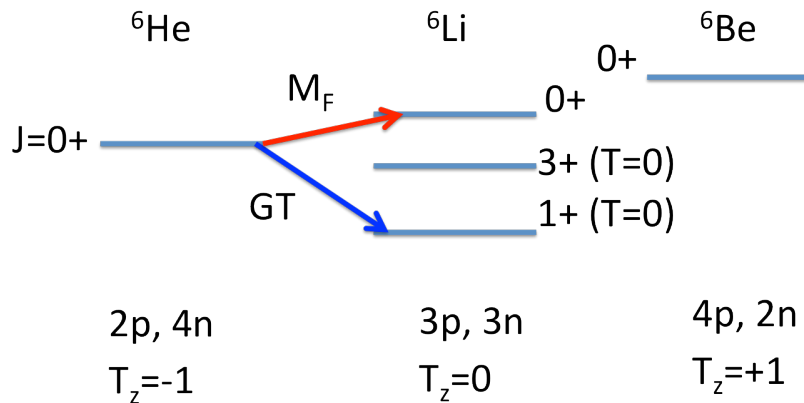
Corrections come from Coulomb as well as mixing of various isospin states and 1p-1h excitations.

Isobaric mass multiplet equation

- The IMME predicts parabolic energy dependence of similar isospin states in a mass multiplet.

- The IMME coefficients have a physical interpretation in terms of isovector (b) and isotensor (c) components.

$$E(A, T, T_z) = a + bT_z + cT_z^2$$



$$b = \frac{E({}^6\text{Be}) - E({}^6\text{He})}{2}$$

$$c = \frac{E({}^6\text{Be}) + E({}^6\text{He})}{2} + E({}^6\text{Li})$$

- “ b ” and “ c ” coefficients are experimentally measured. We set out to calculate these quantities too to judge convergence of various observables.

The No-Core Shell Model (NCSM)

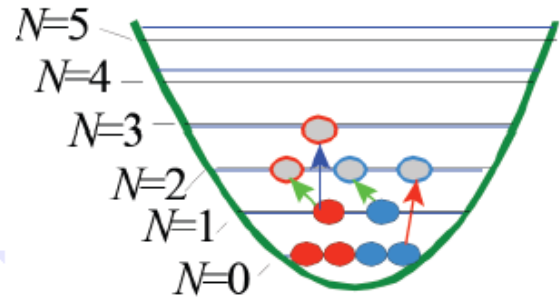
Starting Hamiltonian is translationally invariant.

$$H_A = \frac{1}{A} \sum_{i < j}^A \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i < j}^A V_{NN,ij}$$

NCSM has two parameters:
Nmax and Ω

Provided interaction is “soft” we don't need to do any renormalization of interaction,

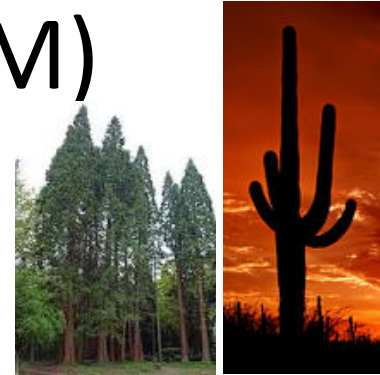
It's that “simple”.



If we now use a single-particle basis, we have to remove the spurious CM states.

Advantage in m-scheme: Antisymmetry is easy to implement.

Disadvantage in m-scheme: Number of basis states is much larger than JT basis



Chiral Effective Field theory

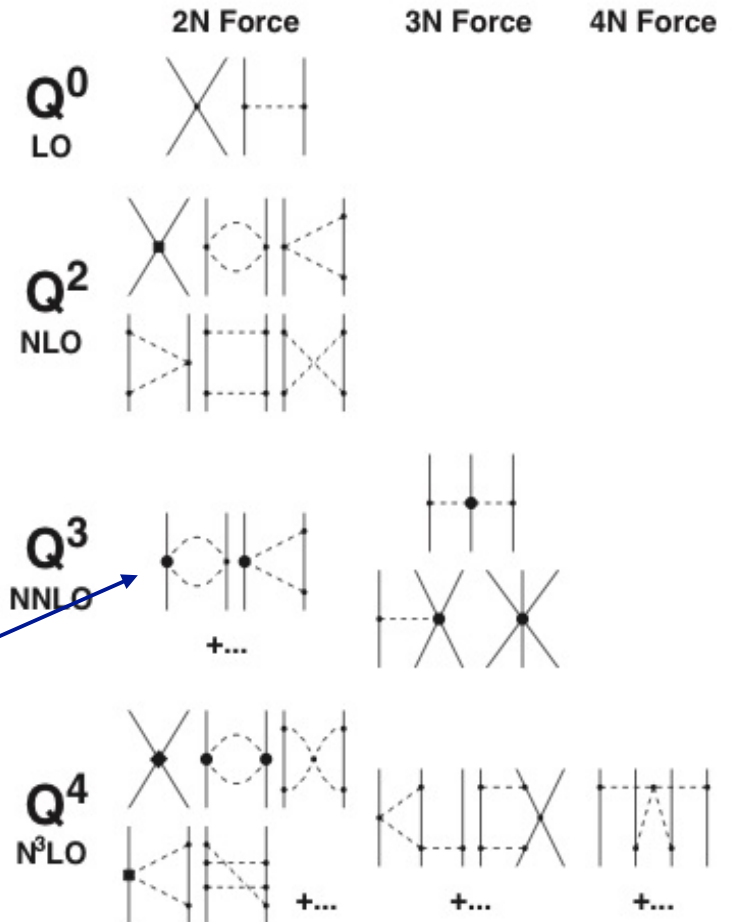
Low-energy theory of QCD in which the degrees of freedom are now nucleons and pions.

Therefore based on QCD symmetries.

Systematic power-expansion* (Weinberg) in powers of momentum over “QCD” scale.

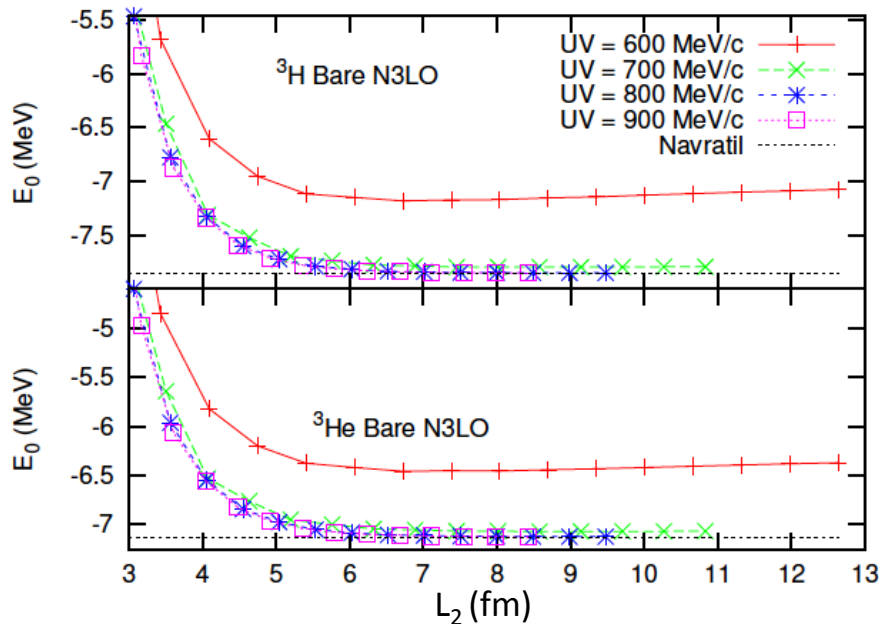
Short-range physics is integrated out, leading to Low-energy constants (LEC's) that need to be determined experimentally.

- But Weinberg counting is not renormalization-group invariant!¹



¹) Nogga, Timmermans, van Kolck PRC **72** 054006 (2006)

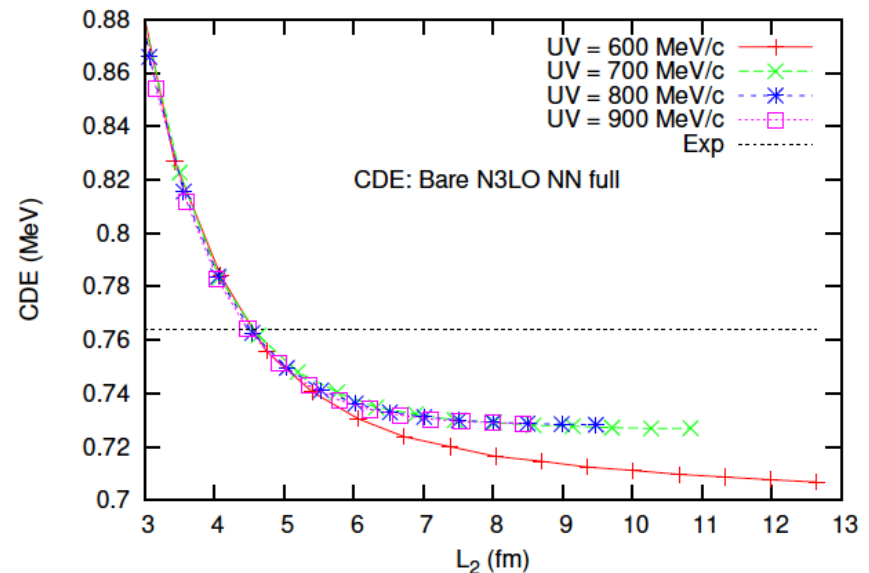
Coulomb-displacement energy A=3



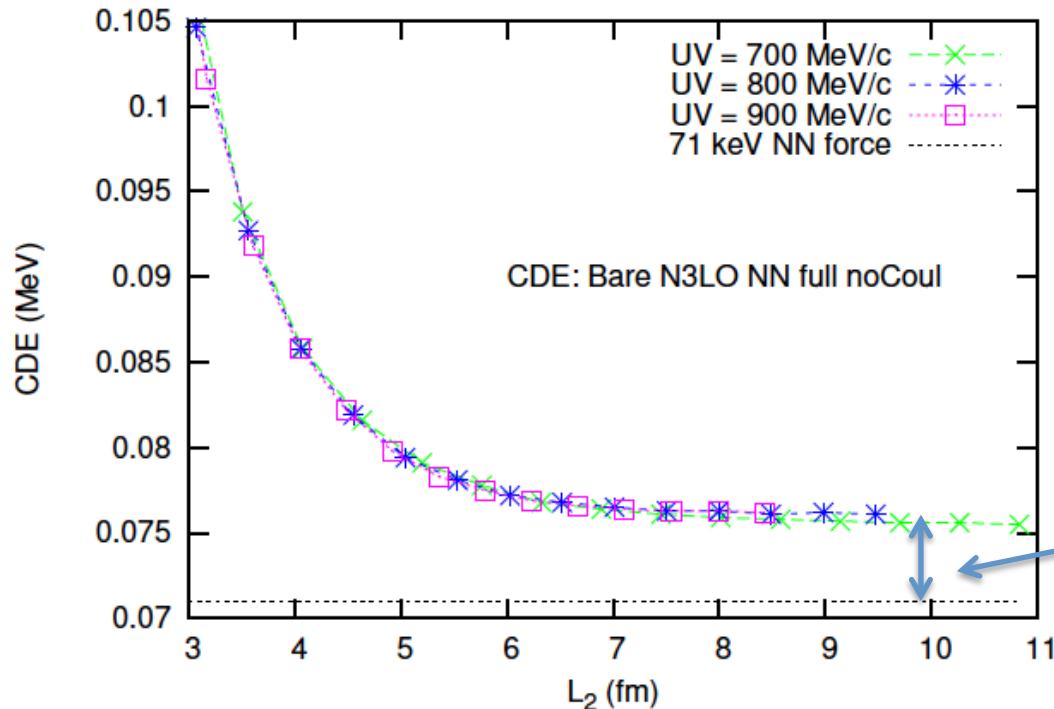
Ground-state energies for ${}^3\text{H}$ and ${}^3\text{He}$ are shown on the left for the NN N3LO int. UV convergence occurs for 800+ MeV/c. See later slides for def. of UV and ir (L_2) in terms of Nmax and $h\Omega$. Large $L_2 \Rightarrow$ large Nmax

$$\Delta E_c = \text{BE}({}^3\text{H}) - \text{BE}({}^3\text{He})$$

But for Coulomb-displacement energies UV convergence happens at ~ 700 MeV/c. This can be understood in terms of the repulsive core being subtracted out in the difference of the energies. Note we do not match the experimental CDE of 764 keV.



“Coulomb-displacement” for $A=3$ with no Coulomb (strong NN only)



Repeat the calculation of CDE with No coulomb interaction present. The point is to look at how much Charge-symmetry breaking is coming from the strong force.

For NN N3LO we determine the strong component to be about 76 keV, which is only 5 keV different from another accepted model¹ of 71 keV.

The conclusion seems to be that the charge-symmetry breaking terms in NN N3LO are ok even though these effects are included only up to NLO order. Recent work by Pastore and Epelbaum have added higher order terms.

¹Miller, Opper, Stephenson Annu. Rev. Nucl. Part. Sci 56, 253 (2006)

A=6 : Bare N3LO NN interaction

Begin by calculating M_F for A=6:

Computationally much easier than A=10.

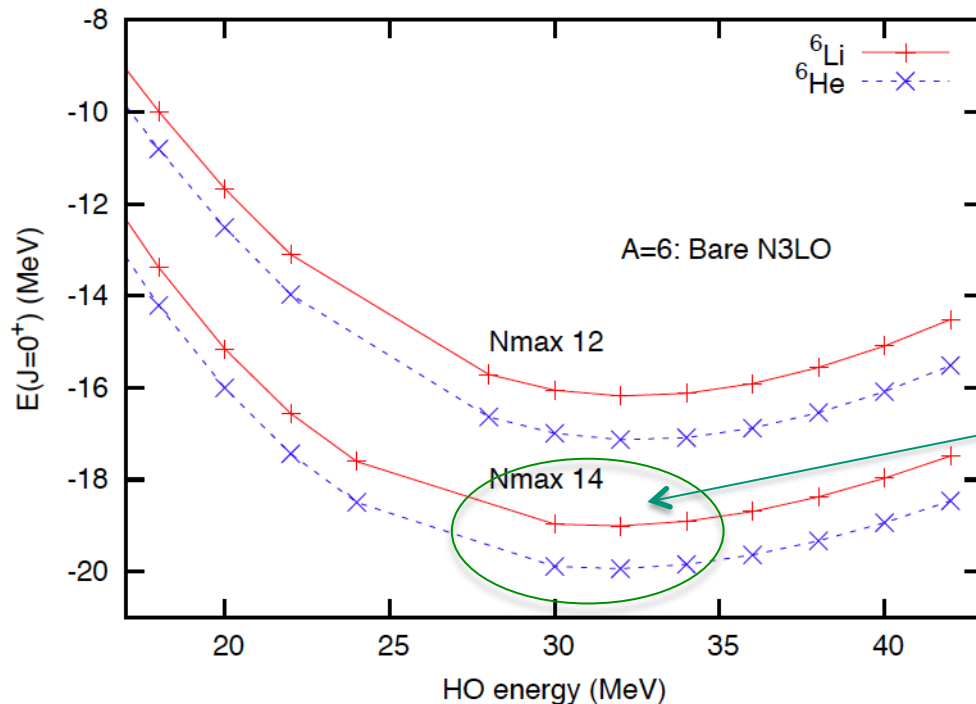
Extrapolation methods can be tested.

Guides us for A=10 calculation.

Bare interaction (are you crazy)?

No. A=6 can be calculated in the NCSM with the bare interaction by extrapolating.

What does SRG do to the isospin parts of the interaction (operator renormalization)?



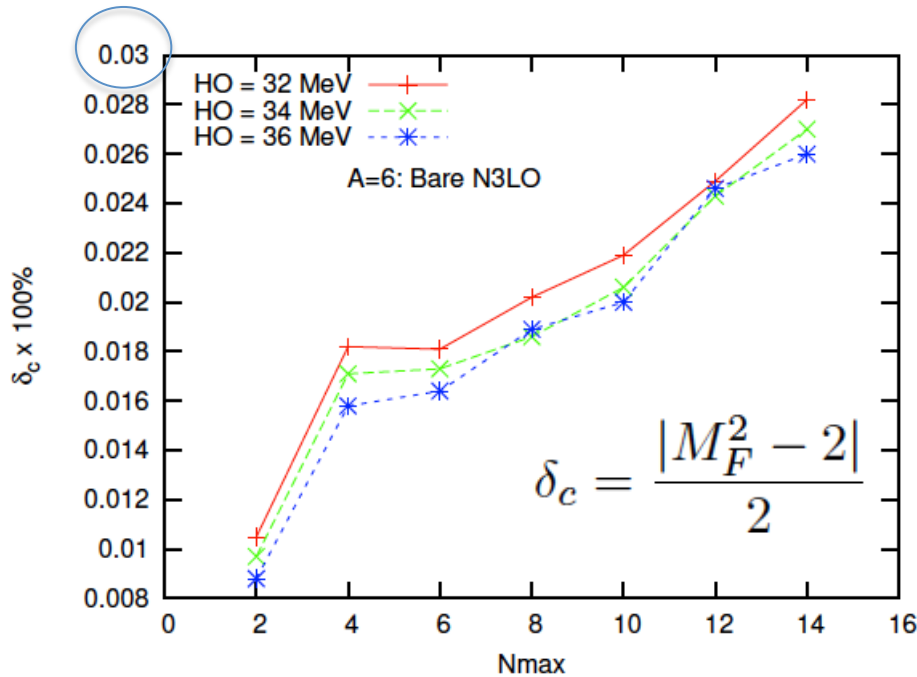
J=0+ energies of He6 (blue)/Li6(red).

Conventional wisdom indicates that one should calculate observables in this region here (variational min.)

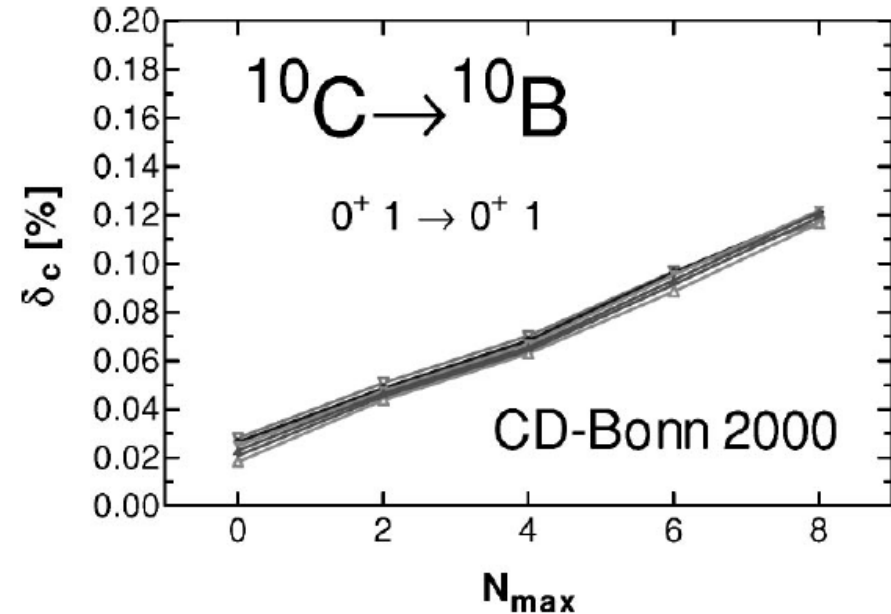
But...is that really true?

At $N_{\text{max}}=14$ the bare calculation is about 5 MeV away from extrapolated g_s^1 .

Isospin-mixing correction (bare int)



The Fermi-matrix element at the variational minima does not seem to converge!
Can't extrapolate reliably.
What to do?



Linear scaling with N_{\max} is known from older calculations.
E. Caurier, P. Navratil, W.E. Ormand, J.P. Vary
PRC **66**, 024314 (2002)

Approach convergence by new means.

What can δ_c depend on?

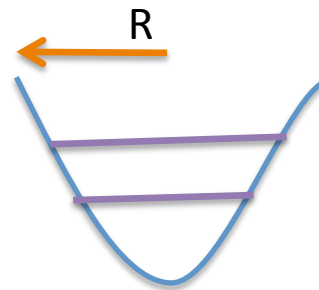


Coulomb force obviously!

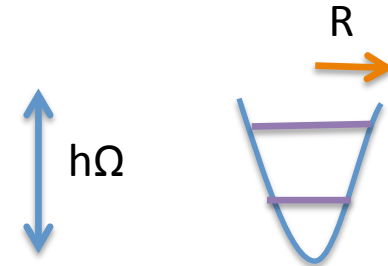


Dependence quantified through Coulomb-energy (i.e. charge radii) or “b” coeff. (1st order).

But if δ_c depends on radii there should be dependence on $h\Omega$.



HO well

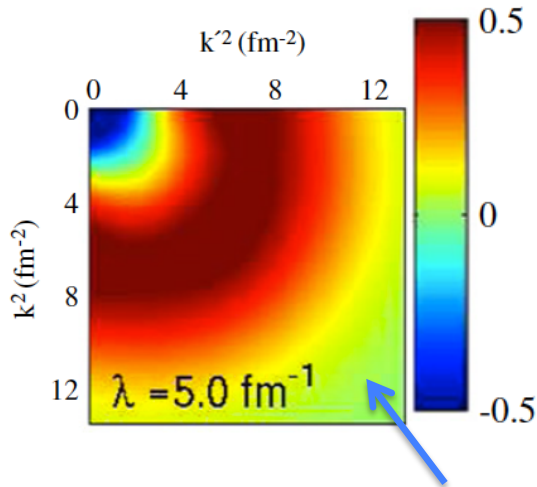


“Squeezed” nucleus

δ_c also has contributions from states mixing – subtle $h\Omega$ dependence.

And then there are the various contributions from isovector (b) and isotensor (c) components of the NN force.

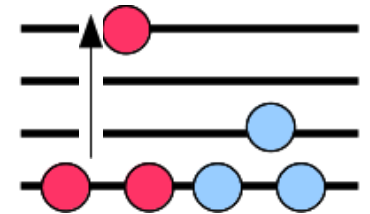
Unified Extrapolations (UV+ir)



Nuclear interaction is expressed in terms of matrix elements:
In order to correctly capture the physics in the interaction you need both UV and ir convergence.

Harmonic oscillator basis regulators

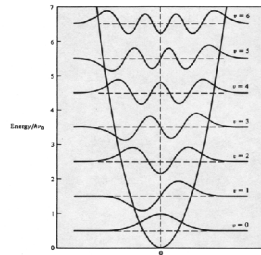
$$\Lambda = \sqrt{m_N (N + 3/2) \hbar \Omega}$$



NN interaction at high momentum is (super)-exponentially regulated.
For Chiral N3LO: UV \sim 800 MeV/c

NN int. is also fit to low-energy scattering data, e.g. scattering lengths or the Deuteron.
All NN int's should have ir \sim 20-45 MeV/c

$$\lambda_{ir} = \lambda_{sc} = \sqrt{\frac{m_N \hbar \Omega}{N + 3/2}}$$

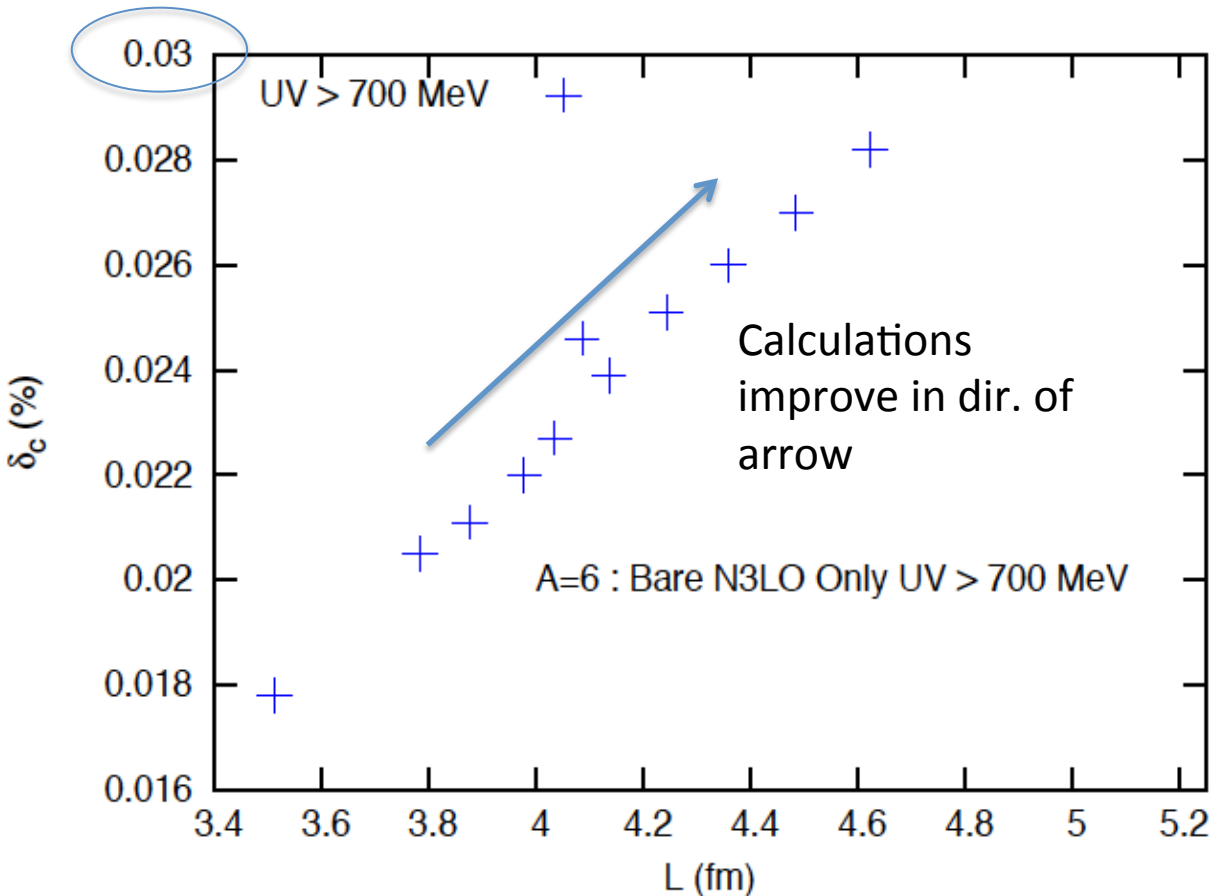


Coon, Avetian, Kruse, van Kolck, Maris, Vary, PRC 86 054002 (2012)

Furnstahl, Hagen, Papenbrock, PRC 86, 031301 (2012)

More, Ekström, Furnstahl, Hagen, Papenbrock, PRC 87, 044326 (2013)

UV converged δ_c extrapolation



Arizona (i.e. Sid, MKGK prefer)

$$\lambda_{ir} = \lambda_{sc} = \sqrt{\frac{m_N \hbar \Omega}{N + 3/2}}$$

But of course you could use

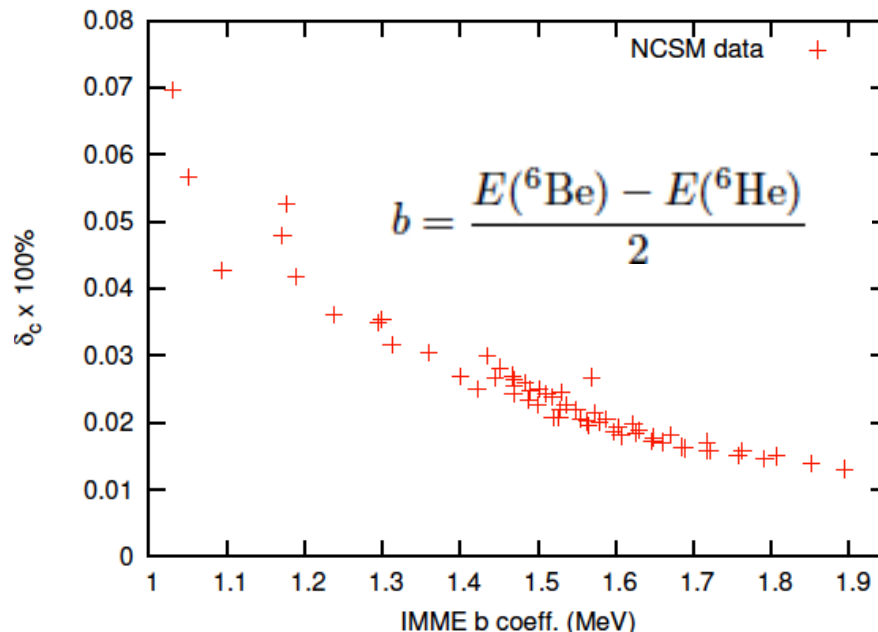
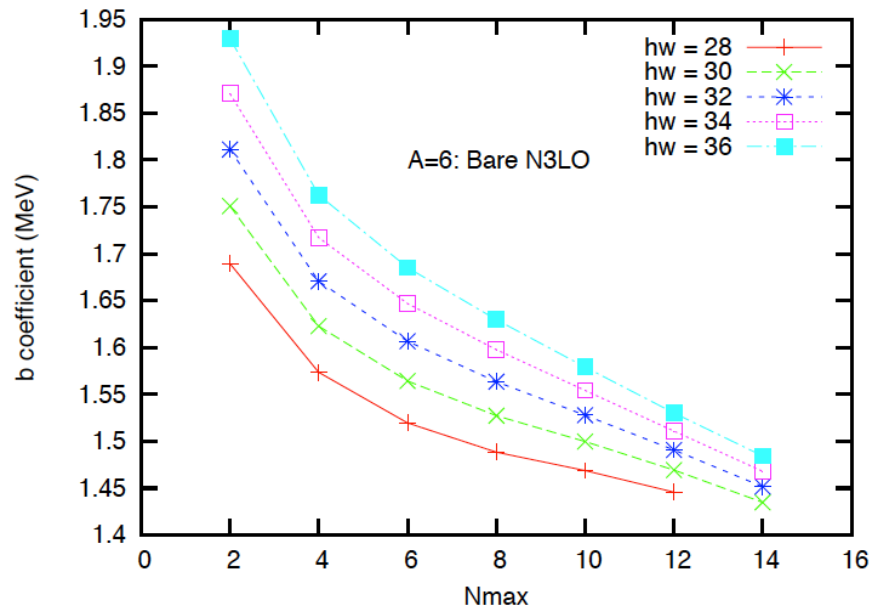
$$L_i = \sqrt{N_{\max} + 3/2 + ib}$$

As a first-attempt, we take approximately UV converged points for δ_c and then extrapolate into the ir region (large “L”). Unfortunately this procedure does not seem to converge either (for later: but might be ok).

δ_c as a function of the b coefficient

Strategy: Can we extrapolate other observables to indirectly determine δ_c ?

Perhaps we can correlate multiple observables to make a consistent prediction of δ_c .



b as a function of Nmax:
 $h\Omega$ dependence and no obvious form
of extrapolating function.
Now what?

NCSM parameters:

Nmax = 4 – 14

HO = 12-42 MeV (Nmax \leq 10)

HO = 28-42 MeV (Nmax 12-14)

Extrapolating the b coefficient

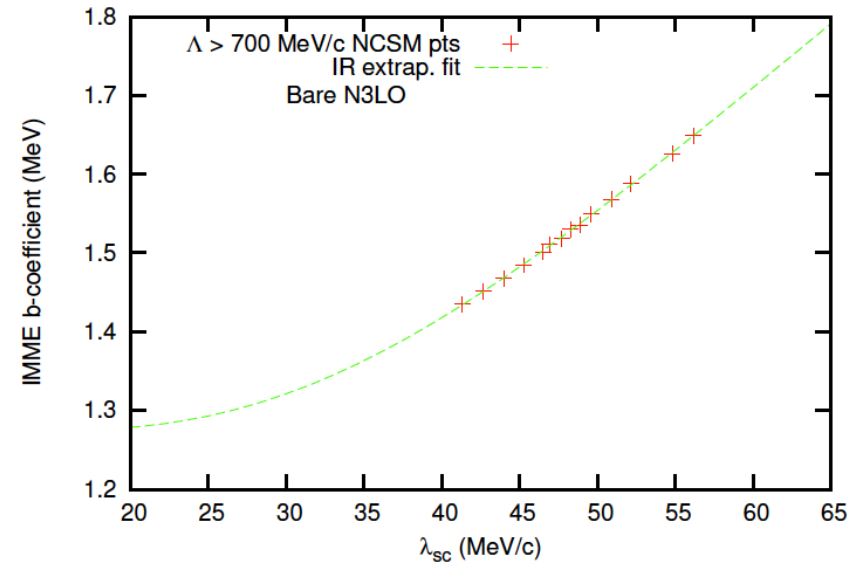
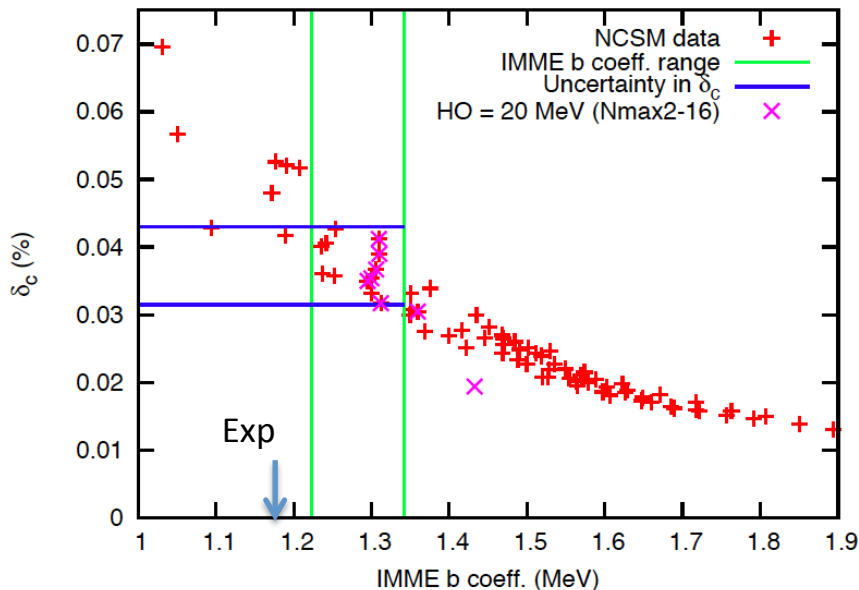
The b coef. is extrapolated for various UV values.

UV = 650,675,...,725 MeV/c.

UV convergence for b \sim 700 MeV/c¹

Note: "Blind" to Nmax and h Ω .

$$b(\lambda_{sc}) = A \exp(-B/\lambda_{sc}) + b(\lambda_{sc} = 0)$$



δ_c values consistent with extrapolated b's
 $\delta_c = 0.035\%$ with 10% error.
 Bracketed points are all $h\Omega \sim 20$ MeV (?).

1) Also known from He3-H3 studies by MKGK

Coulomb-energy and b: Consistent?

Argument: If you determine b then you have a range for δ_c that is acceptable.

Question: Did you determine b correctly?

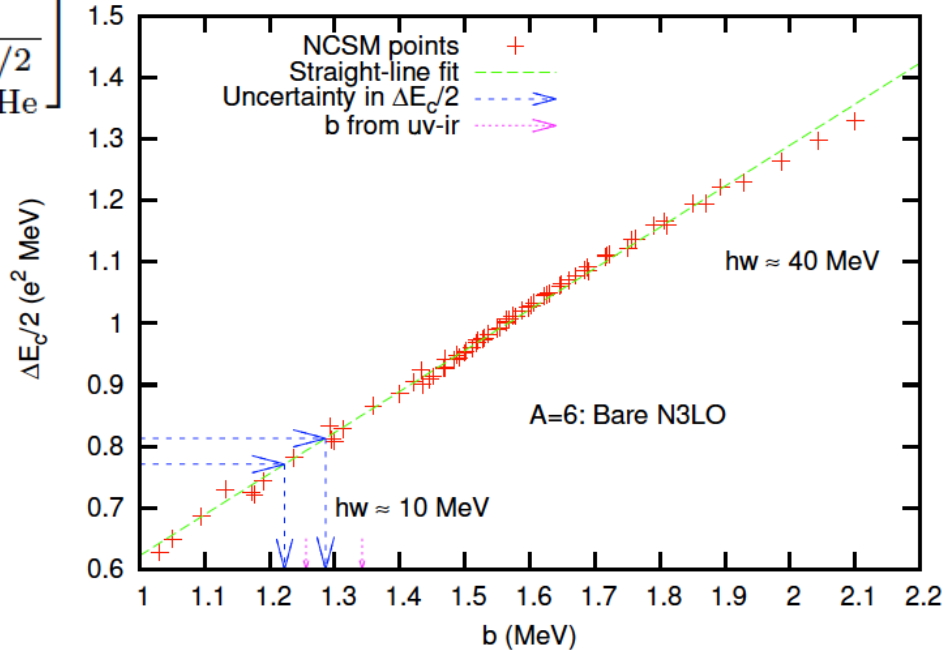
Extrapolate rms charge-radii of He-6 and Be-6 to determine Coulomb-energy diff. Then, by the plot below you could indirectly determine a different b coefficient.

$$\Delta E_c(^6\text{Be} - ^6\text{He}) = \left(\frac{3}{5}\right)^2 e^2 \left[\frac{16}{\langle r_p^2 \rangle_{^6\text{Be}}^{1/2}} - \frac{4}{\langle r_p^2 \rangle_{^6\text{He}}^{1/2}} \right]$$

The radii are extrapolated with UV/ir:¹

$$\langle r_p^2 \rangle_L = \langle r_p^2 \rangle_\infty [1 - c_0 \beta^3 e^{-\beta}]$$

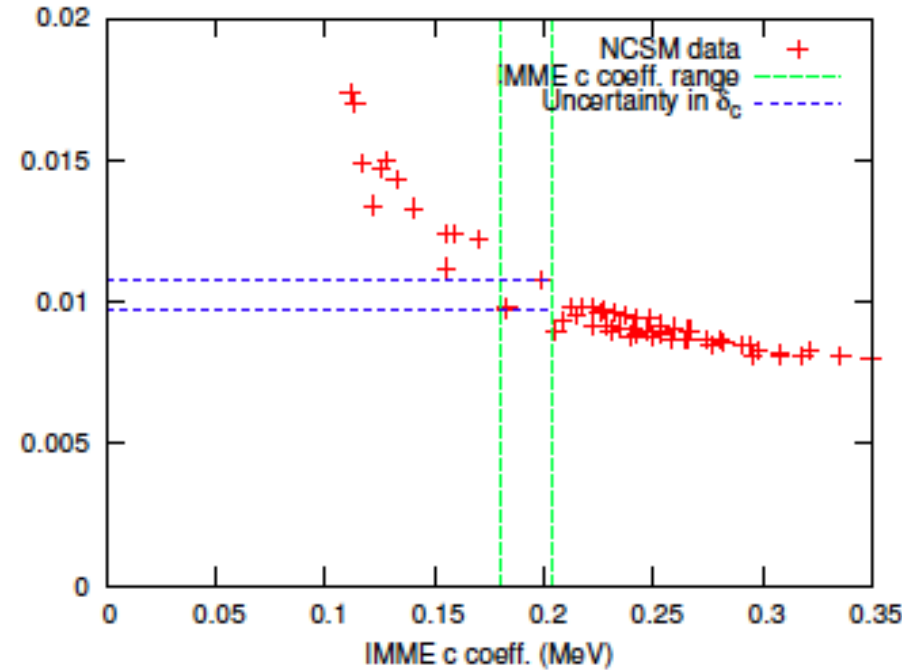
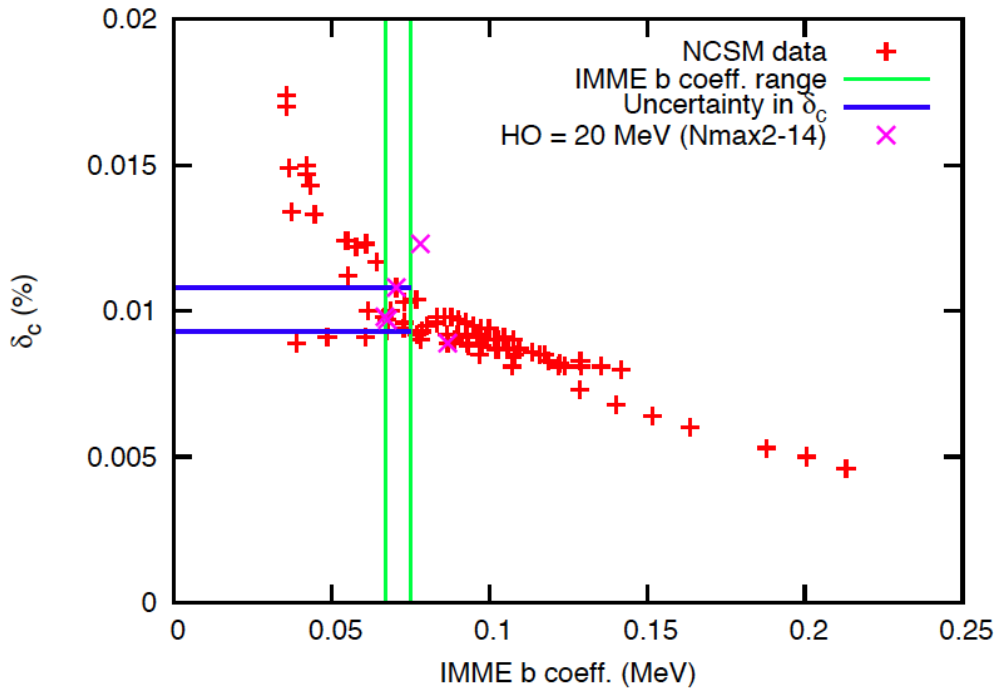
Short answer: Yes, consistent.



1) R. J. Furnstahl, G. Hagen, and T. Papenbrock Phys. Rev. C **86**, 031301(R)

No Coulomb (only NN parts)

How large are the contributions to δ_c from Coulomb/strong interaction?
As a first step, we neglect the Coulomb interaction and redo A=6 calculations.



Extrapolate b with UV/ir as before.

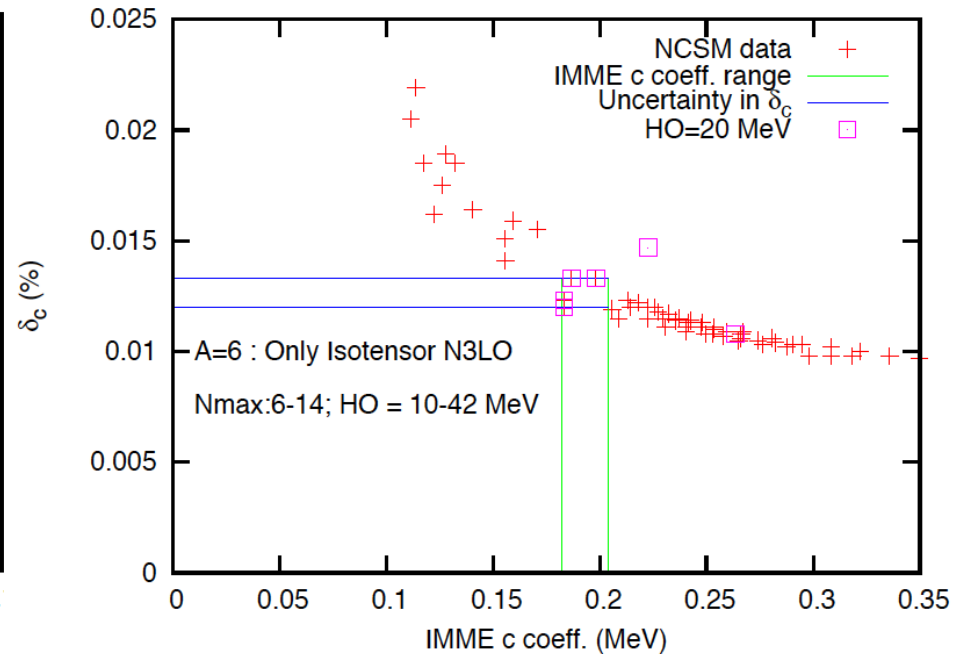
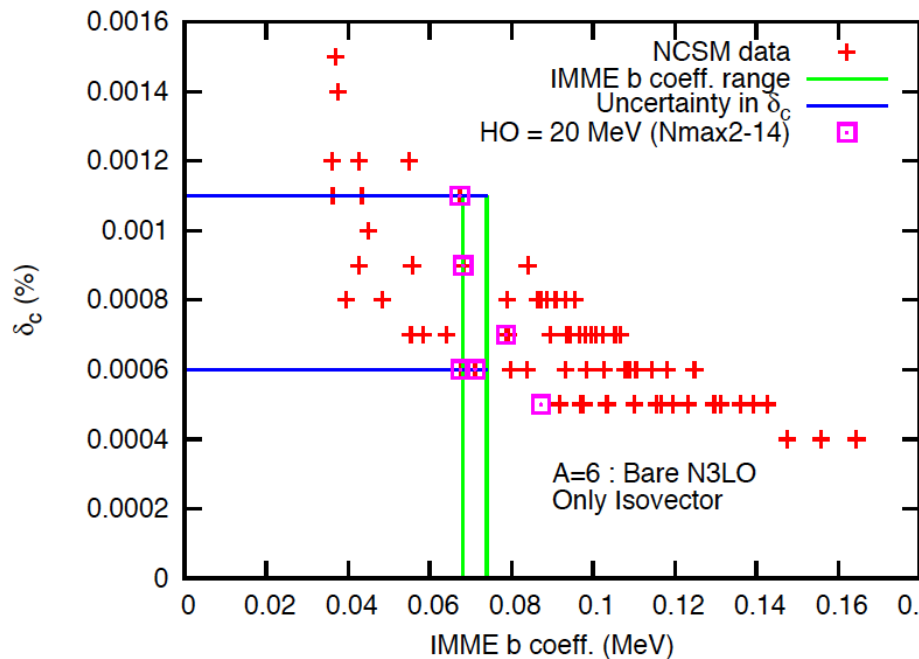
We can also extrapolate c.

b and c extrapolations give the same range for δ_c (consistent).

δ_c is about 1/3 the value of the full bare interaction – 10 % uncertainty on δ_c .

Isovector and isotensor components

Now turn off the isovector and isotensor parts separately to see which part of the interaction gives rise to isospin-breaking. There is no Coulomb interaction present.



Notice that δ_c is receiving only contributions from the isotensor part of the interaction. The value of δ_c is about 15-20% larger than when both components are present (prev. slide) but is still about a factor 1/3 smaller than the bare+coulomb interaction.

Giant isoscalar monopole modes

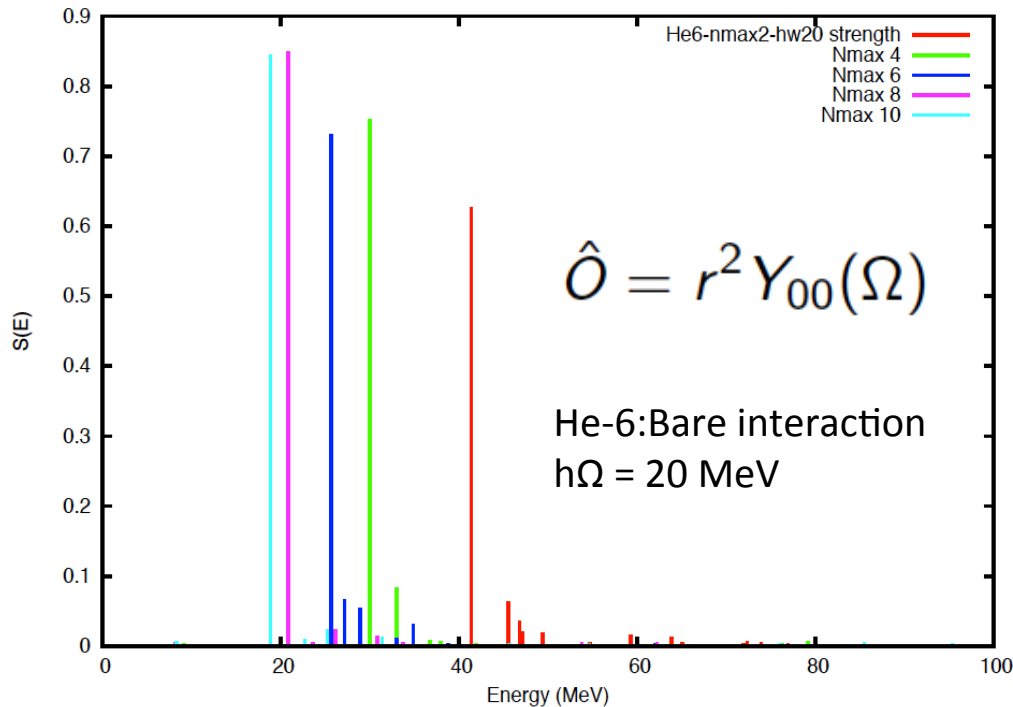
δ_c receives contributions from isospin-mixing.

What states in the excitation spectrum are responsible?

As a first step, consider the isoscalar monopole operator.

$$\hat{O} = r^2 Y_{00}(\Omega)$$

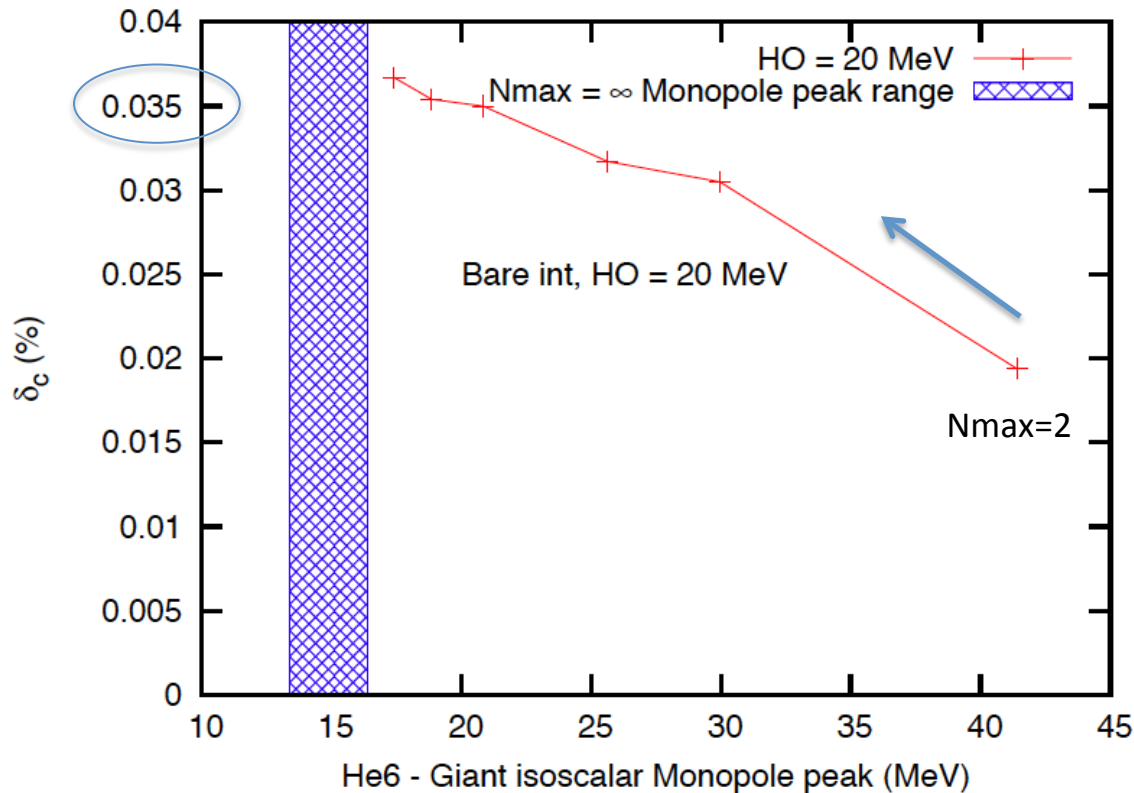
Act on the g.s. creating a pivot from which we calculate the strength function.



Note that 1 particular state is coming down in energy as Nmax increases. This state is responsible in part for isospin-mixing corrections. Does it converge?

Fine detail: the plot on the left has normalized strength.

δ_c is elusive, but consistent...



We had thought that by plotting the value of δ_c as a function of the peak of the collective state that we may be able to determine δ_c .

Not so easily, but at least the value of δ_c is consistent to what was determined before.

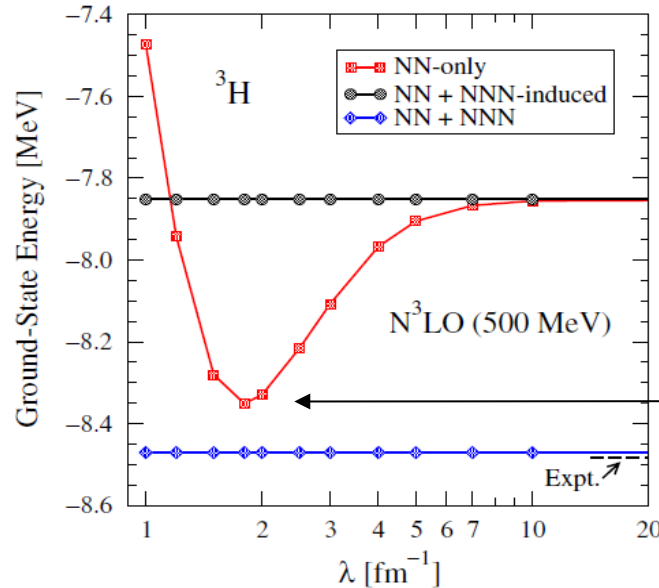
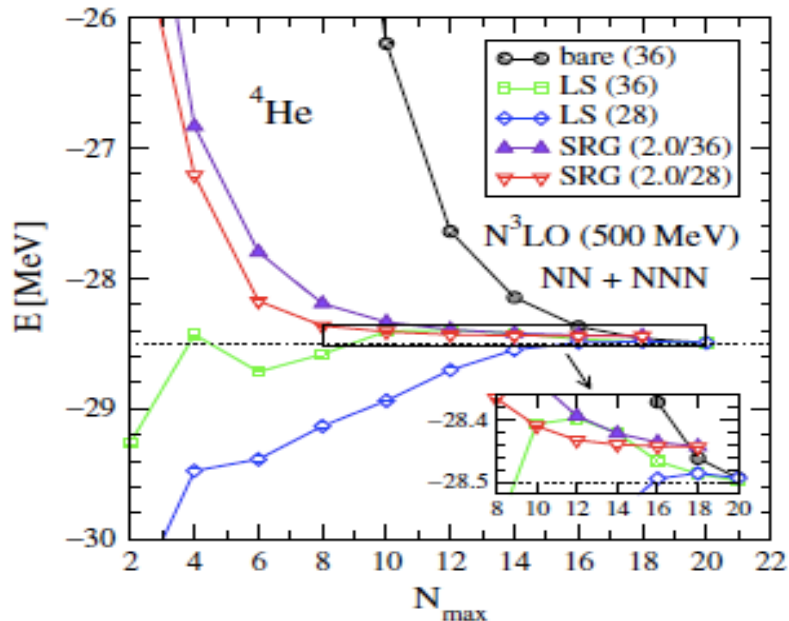
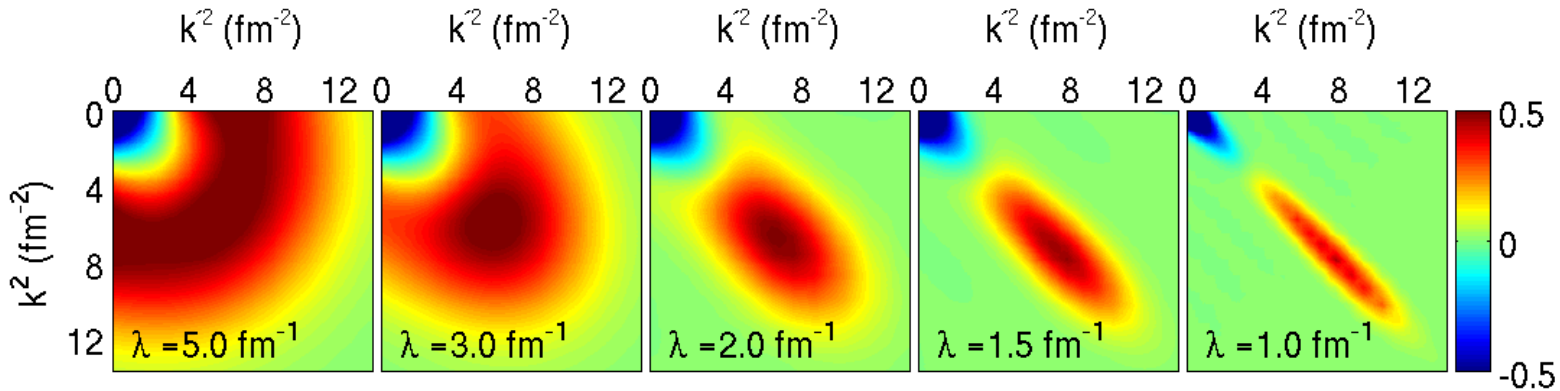
We think the problem is that many excited states are mixing (i.e. isospin-mixing) with the g.s. As N_{max} increases (blue arrow) these states come down in energy (thus more mixing) but newer states are introduced higher up in energy as well.

SRG evolved potentials

PRL 103, 082501 (2009)

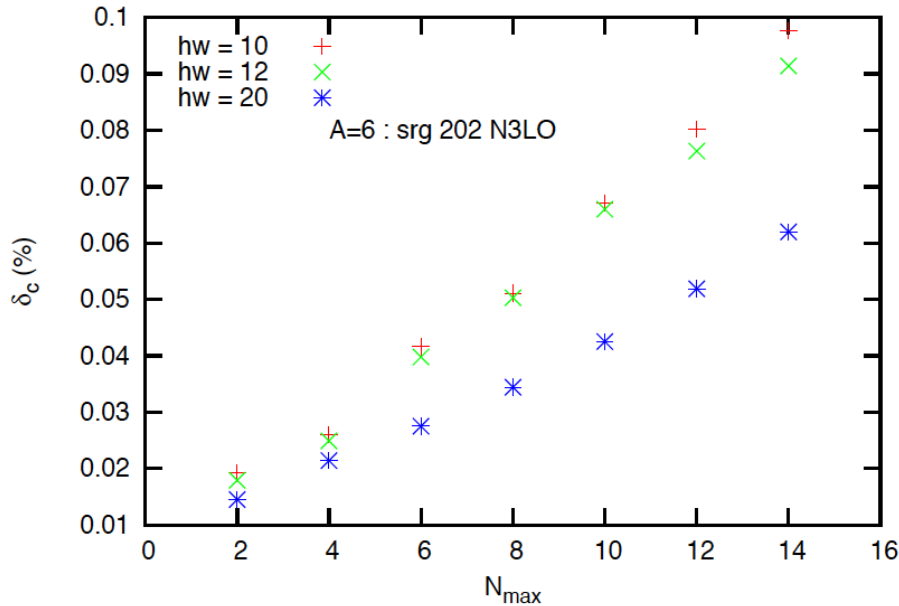
Jurgenson et al

$$H_\lambda = U_\lambda H_{\lambda=\infty} U_\lambda^\dagger; \quad \frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5} [[T, H_\lambda], H_\lambda]$$



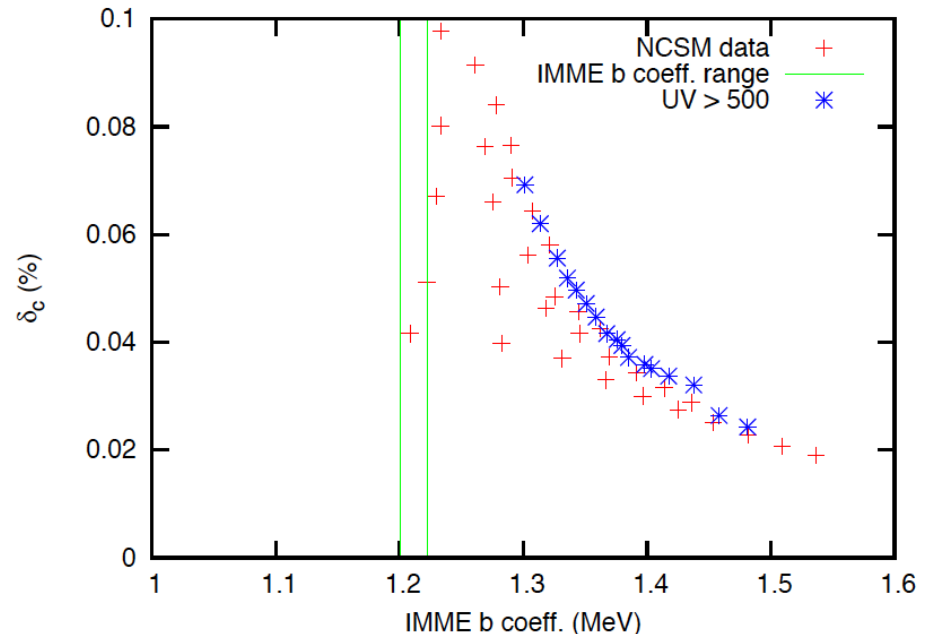
NN force „acts“
as though it is a
NNN force.
Closest to 3N at
about 1.5-2.0/
fm

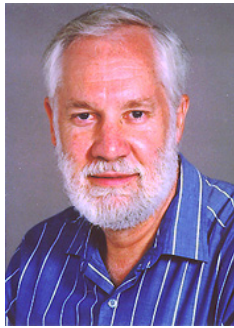
And what about SRG interactions?



δ_c increases with N_{\max} once more, but note that it is about 1/3 in magnitude than the bare interaction. Coulomb has also been srg'ed.

Unfortunately the correlation between extrapolated 'b' coefficient and δ_c is now completely inconsistent. 'b' has been extrapolated to within 40 keV of experimental value. δ_c does follow some sort of 'universal' curve for UV converged points. Convergence with ir has been tested but leads to inconsistent results.





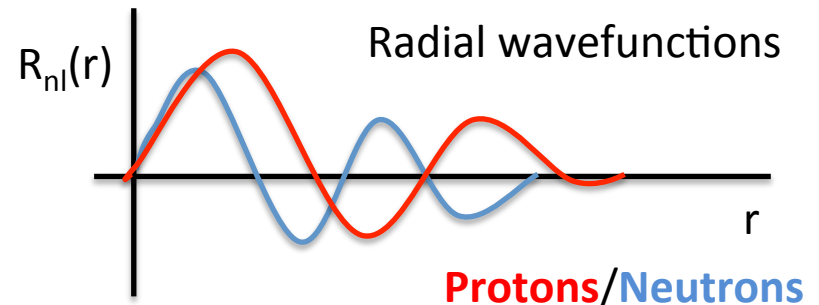
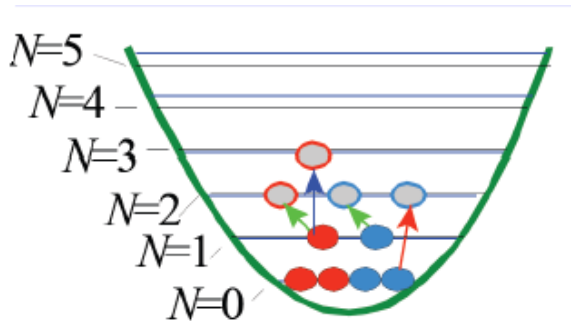
Towner and Hardy vs Ormand, Brown, MKGK

Towner and Hardy have dominated the calculations of the isospin-mixing correction for decades. Why bother competing? To provide another opinion on the matter.

What do Towner and Hardy “do”?

- Shell model calculations (restricted model-space i.e. only some nucleons “active”).
- Shell model implies phenomenological interactions (Cohen-Kurath, USD, ...).
- Taking care of isospin-mixing states outside their model space.

T&H introduce two corrections they need to take care of by doing shell-model: the largest correction is the radial mismatch but there is also the case of non-Hermiticity in their isospin-operator¹. In principle we should have this in NCSM.



¹) See Miller, Schwenk PRC 78, 035501 (2008) for a discussion

Conclusion

Really too early to say anything just yet – need to complete this work.

Unfortunately subtleties are present:

δ_c depends on a number of parameters/observables. Do them consistently!

SRG results are confusing but perhaps are due to the RG itself.

$\Delta T=1$ matrix elements in Argonne calculation (we don't consider them).

BUT we are figuring out how to present a truly ab-initio approach to isospin-mixing δ_c

Extrapolation techniques (UV/ir)

Correlation of observables to make consistent predictions.

Uncertainty quantification (theory errors).

Vital for determining δ_c in $A=10$ system which we have started.

Thanks to the following people:

Erich Ormand (LLNL)

Calvin Johnson (SDSU)

Sid Coon (UofA)

Coulomb-energy, b-coefficients and rms charge-radii...

- “b” coefficient is connected to the Coulomb-displacement energy.
- The Coulomb-energy in a uniformly charged sphere $E_c = \frac{3}{5} \frac{(Ze)^2}{R}$
- Writing the “b” coefficient in terms of Coulomb-displacement energies

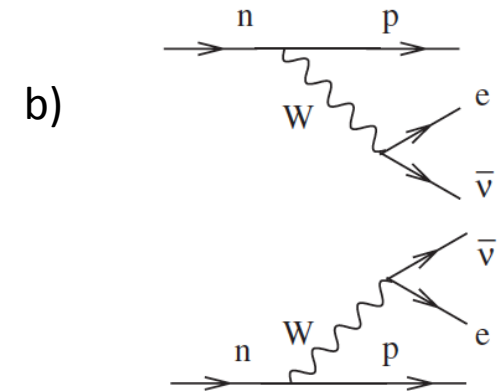
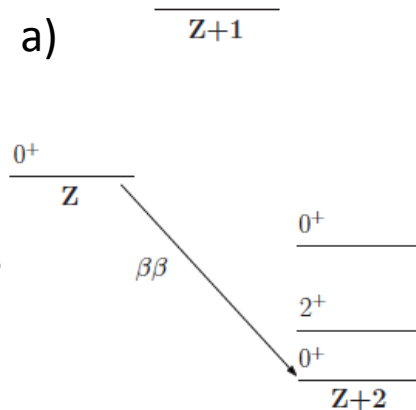
$$\Delta E_c(^6\text{Be} - ^6\text{He}) = \left(\frac{3}{5}\right)^2 e^2 \left[\frac{16}{\langle r_p^2 \rangle_{^6\text{Be}}^{1/2}} - \frac{4}{\langle r_p^2 \rangle_{^6\text{He}}^{1/2}} \right]$$

- Note that I replaced “R” by the rms charge radius since that is what we measure and calculate.
- Thus we need the rms radii as well!

Neutrinoless double beta-decay

Ordinary double beta decay (2ν) does indeed happen in nature. It is a “second-order” process meaning it is “rare” (i.e. long half-lives on the order of 10^{21} years).

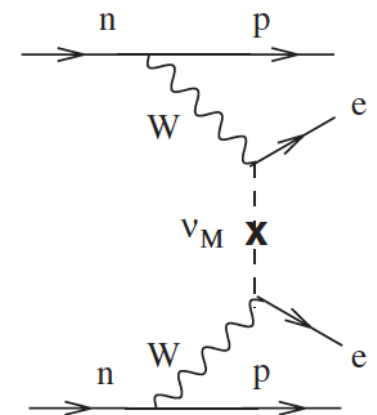
On the right in a) I show the typical energy level diagram of $\beta\beta$ -decay. The parent is an even-even nucleus which implies it is more tightly bound (by pairing) than the $Z+1$ nucleus but less-bound than the $Z+2$ nucleus.



$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}(Q_{\beta\beta}, Z) |M_{2\nu}|^2$$

Neutrinoless double beta decay or $\beta\beta$ (0ν) requires that neutrinos have mass (which they do) and that they are their own anti-particle (Majorana).

The minimal model simply requires that light-neutrinos are exchanged amongst the W bosons. Note the process is lepton-number violating and depends on the masses of the neutrinos.



The masses enter through: $\langle m_{\beta\beta} \rangle \equiv \left| \sum_k m_k U_{ek}^2 \right|$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z) |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$