Proton–deuteron scattering lengths in pionless effective field theory

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Current status of p–d scattering lengths

Proton

- spin $^1/_2$
- isospin $^1/_2$

Deuteron

- spin 1
- isospin 0

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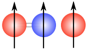
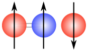
$\hookrightarrow$ **two S-wave channels:**

$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}}=\frac{\mathbf{3}}{\mathbf{2}}\Big(\sim$  $\Big)\oplus\frac{\mathbf{1}}{\mathbf{2}}\Big(\sim$  $+\cdots\Big)$

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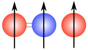
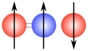
$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}}=\frac{\mathbf{3}}{\mathbf{2}}\Big(\sim$  $\Big)\oplus\frac{\mathbf{1}}{\mathbf{2}}\Big(\sim$  $+\cdots\Big)$

Quartet channel

Ref.	$^4a_{p\text{–}d}$ (fm)
van Oers, Brockman (1967)	$11.4^{+1.8}_{-1.2}$
Arvieux (1973)	11.88 ± 0.4
Huttel *et al.* (1983)	≈ 11.1
Kievsky *et al.* (1997)	13.8
Black *et al.* (1999)	14.7 ± 2.3

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$\hookrightarrow$ **two S-wave channels:**

$$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}}\left(\sim \uparrow\uparrow\uparrow\right)\oplus\frac{\mathbf{1}}{\mathbf{2}}\left(\sim \uparrow\uparrow\downarrow+\cdots\right)$$

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Doublet channel

Ref.	$^4a_{p\text{–}d}$ (fm)
van Oers, Brockman (1967)	1.2 ± 0.2
Arvieux (1973)	2.73 ± 0.10
Huttel *et al.* (1983)	≈ 4.0
Black *et al.* (1999)	-0.13 ± 0.04
Orlov, Orevkov (2006)	≈ 0.024

Motivation

Goal

Precise and controlled extraction from EFT calculation!

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Scope of method

- **Nuclear astrophysics**
  - Low-energy nuclear reactions in Halo-EFT
  - $\rightarrow$ one-neutron halo states in $^{11}\mathrm{Be}$
  - $\rightarrow$ one-proton halo state in $^{8}\mathrm{B}$?
- **Cold-atom systems**
  - EFT with van-der-Waals tails?

Outline

1. **Pionless effective field theory**
2. **Coulomb-modified effective range expansion**
3. **Quartet-channel scattering length**
4. **Doublet-channel scattering length**
5. **Summary and outlook**

SK, H.-W. Hammer, arXiv:1312.2573

SK, Ph.D. thesis (Bonn U, 2013)

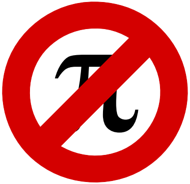
SK, H.-W. Hammer, PRC **83** (2011) 064001

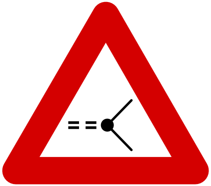
Part I

Pionless effective field theory

- **Effective Lagrangian**
- **Power counting**
- **Integral equations**

Foundation and basic features





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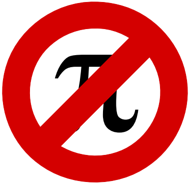
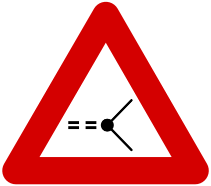
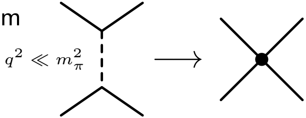


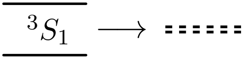


- at very low energies even pions can be integrated out
  $\hookrightarrow$ only nucleons left as effective degrees of freedom
- non-relativistic framework
- large scattering lengths in N-N scattering
  $\hookrightarrow$ additional low-energy scale



$$\gamma_d = \frac{1}{a_d}\big(1 + \mathcal{O}(a_0/r_d)\big)$$

Kaplan, Savage, Wise 1998; van Kolck 1997/98



- convenient description of three-body sector with dibaryon fields

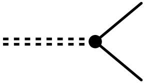
Bedaque, Hammer, van Kolck 1998

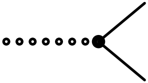
Effective Lagrangian

$$\mathcal{L} = \underline{N^\dagger \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{2M_N} \right) N} + \mathcal{L}_{\text{photon}} + \mathcal{L}_3$$

$$- d^{i\dagger} \left[\sigma_d + \ldots \right] d^i \quad - t^{A\dagger} \left[\sigma_t + \ldots \right] t^A$$

$$- y_d \left[d^{i\dagger} \left(N^T P_d^i N \right) + \text{h.c.} \right] - y_t \left[t^{A\dagger} \left(N^T P_t^A N \right) + \text{h.c.} \right]$$





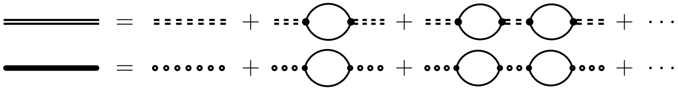
- **nucleon** field N, doublet in spin and isospin space
- auxiliary **dibaryon** fields d^i $({}^3S_1,\, I = 0)$ and t^A $({}^1S_0,\, I = 1)$
  $\leftrightarrow$ channels in N–N scattering
- **coupling constants** $y_{d,t}$ and $\sigma_{d,t}$
- dibaryon propagators are just constants at leading order

Dibaryon propagators

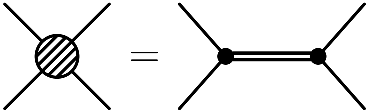
Bubble chains

$^3S_1: \quad \Delta_d =$

$^1S_0: \quad \Delta_t =$



Fix parameters from N-N scattering!



$$\mathrm{i}\mathcal{A}_{d,t}(k) = -y_{d,t}^2 \Delta_{d,t}\left(p_0 = \frac{\mathbf{k}^2}{2M_N}, \mathbf{p} = 0\right) = \frac{4\pi}{M_N} \frac{\mathrm{i}}{k\cot\delta_{d,t} - \mathrm{i}k}$$

- $k\cot\delta_d(k) = -\gamma_d + \frac{\rho_d}{2}(k^2 + \gamma_d^2) + \ldots \longrightarrow y_d, \sigma_d$
- $k\cot\delta_t(k) = -\gamma_t + \frac{r_{0t}}{2}k^2 + \ldots$ with $\gamma_t \equiv \frac{1}{a_t} \longrightarrow y_t, \sigma_t$

Range corrections

Dibaryon kinetic-energy terms

$$\sim \; \mathrm{i}\Delta_d^{\mathrm{LO}}(p) \times (-\mathrm{i}) \left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right) \times \mathrm{i}\Delta_d^{\mathrm{LO}}(p)$$

$\hookrightarrow$ effective-range corrections

$$\Delta_d(p) \sim \frac{1}{-\gamma_d + \sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon} - \frac{\rho_d}{2}\left(\frac{\mathbf{p}^2}{4} - M_N p_0 - \gamma_d^2\right)}$$

$\mathcal{O}(Q/\Lambda) \sim \mathcal{O}(\gamma_d \rho_d)$

expand in ρ_d, r_{0t} $\rightarrow$ NLO, N^{2}LO, ...

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expand in ρ_d, r_{0t} $\rightarrow$ NLO, N^{2}LO, ...

$$D_d(E;q) = D_d^{(0)}(E;q) + D_d^{(1)}(E;q) + \cdots$$

$$= -\frac{4\pi}{M_N y_d^2} \frac{1}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} \times \left[1 + \frac{\rho_d}{2} \frac{\left(3q^2/4 - M_N E - \gamma_d^2\right)}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} + \cdots \right]$$

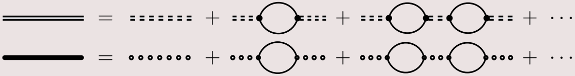
Resummations

Power counting $\hookrightarrow$ resum certain classes of diagrams!

Full dibaryon propagators

$^3S_1: \quad \Delta_d =$

$^1S_0: \quad \Delta_t =$



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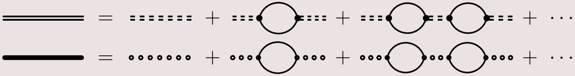
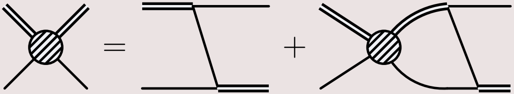


Scattering amplitude

$\sim \cdots$ all of same order $\rightarrow$ **Integral equation!**



Lippmann–Schwinger equation $\rightsquigarrow$ **solve numerically!**

What about Coulomb effects?

What about Coulomb effects?

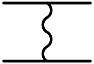
2-body sector

- p–p scattering — Kong, Ravndal 1999, 2000
- . . . at higher order — Ando, Shin, Hyun, Hong 2007

3-body sector

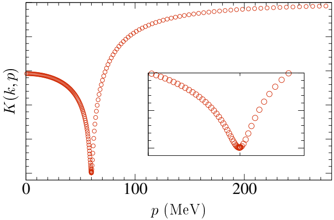
- p–d quartet-channel scattering — Rupak, Kong 2003
- ^{3}He binding energy (LO only) — Ando, Birse 2010
- p–d scattering (quartet + doublet) and ^{3}He — SK, Hammer, 2011

Coulomb contributions

Coulomb photons:  $\sim \; (\mathrm{ie}) \, \frac{\mathrm{i}}{\mathbf{q}^2} \, (\mathrm{ie}) \;\longrightarrow\; (\mathrm{ie}) \, \frac{\mathrm{i}}{\mathbf{q}^2 + \lambda^2} \, (\mathrm{ie})$

$\mathcal{O}(\alpha)$ diagrams

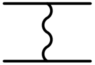
Coulomb peak



$\rightarrow$ re-shuffle mesh points!

SK, Hammer, 2011

Coulomb contributions

Coulomb photons:  $\sim (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie}) \longrightarrow (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$

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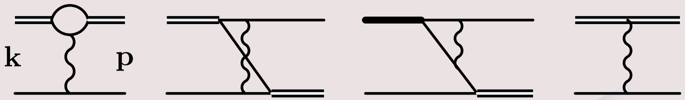

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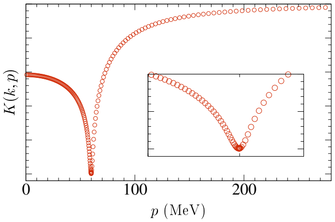

→ re-shuffle mesh points!

SK, Hammer, 2011

generated by dibaryon kinetic term!

$$\mathcal{L} \supset d^{i\dagger}\left[\sigma_d + \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{4M_N}\right)\right]d^i$$

$\hookrightarrow$ **range correction!**

Part II

Coulomb-modified effective range expansion

- **Coulomb-subtracted phase shifts**
- **Modified effective range expansion**
- **The Gamow factor**

Coulomb-subtracted phase shifts

Coulomb force

- long (infinite) range $\rightarrow$ very strong at small momentum transfer
- pure Coulomb scattering can be solved analytically

$\hookrightarrow$ **use Coulomb wave functions as reference states!**

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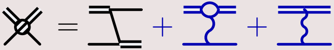
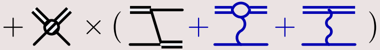
Bottom line

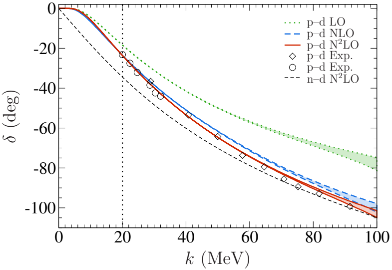




→ full amplitude $\mathcal{T}_{\text{full}}$

→ Coulomb amplitude $\mathcal{T}_{\text{c}}$

$\tilde{\delta}(k) \approx \delta_{\text{diff}}(k) \equiv \delta_{\text{full}}(k) - \delta_{\text{c}}(k)$



Rupak, Kong (2001); SK, Hammer (2011)

Modified effective range expansion

Ordinary effective range expansion

$$k \cot \delta(k) = -\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots$$

$a =$ scattering length
$r =$ effective range

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Modified effective range expansion

$$C^2_{\eta,0}\,k\cot\delta_{\text{diff}}(k) + \alpha\mu\,h_0(\eta) = -\frac{1}{a^C_0} + \cdots$$

Gamow factor

$$C^2_{\eta,0} = \frac{2\pi\eta}{e^{2\pi\eta} - 1}$$

$$\eta = \alpha\mu/k$$

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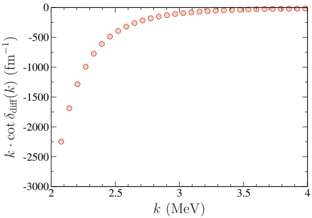

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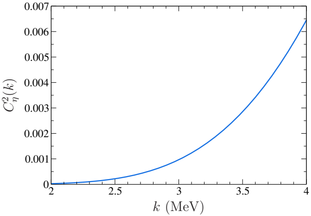

= finite value

The Gamow factor

$$C_{\eta,0}^2 = \frac{2\pi\eta}{\mathrm{e}^{2\pi\eta} - 1}$$

But we have a screened Coulomb potential!

$$\frac{1}{q^2 + \lambda^2} \leftrightarrow \frac{\mathrm{e}^{-\lambda r}}{r}$$

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- furthermore: $\psi^{(+)}_{\mathbf{k},0}(p) = \frac{2\pi^2}{k^2}\delta(k-p) - \frac{2\mu\, Z_0 \mathcal{T}(E;p,k)}{k^2 - p^2 + \mathrm{i}\varepsilon}$, $E = E(k)$

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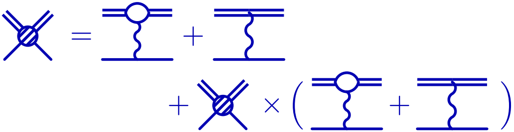


Solution

$$\rightsquigarrow C^2_{\eta,\lambda} = \left|1 + \frac{2M_N}{3\pi^2}\int_0^\Lambda \frac{\mathrm{d}p\, p^2}{p^2 - k^2 - \mathrm{i}\varepsilon} Z_0 \mathcal{T}_c(E;p,k)\right|^2$$

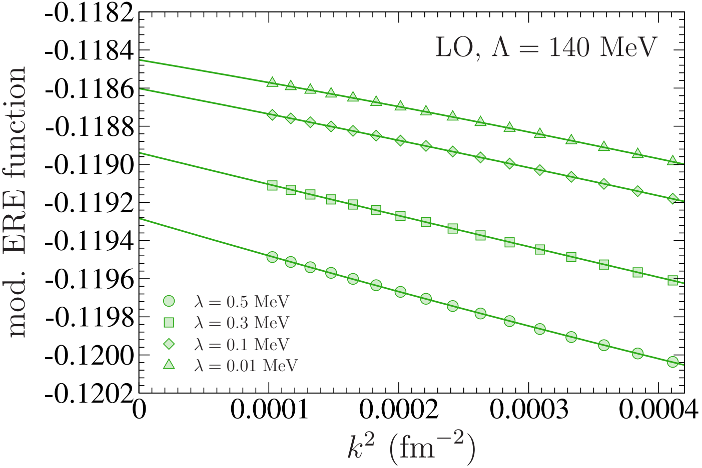
$\hookrightarrow$ **consistent extraction from numerical calculation!**

Part III

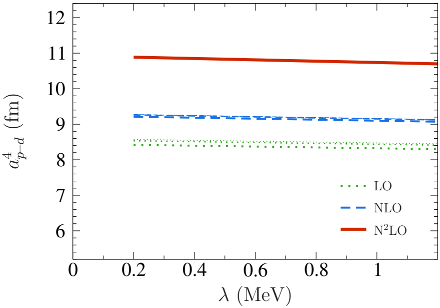
Quartet channel

- **Convergence pattern**
- **Fully perturbative calculation**
- **Results**

Quartet-channel scattering length

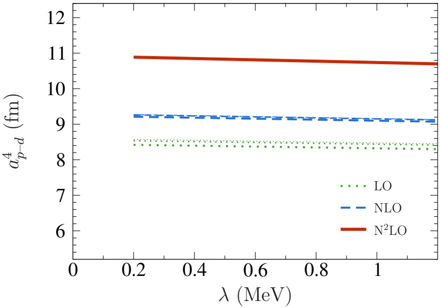


Convergence pattern



- right order of magnitude ✓
- nice (weak) photon-mass dependence ✓

Convergence pattern



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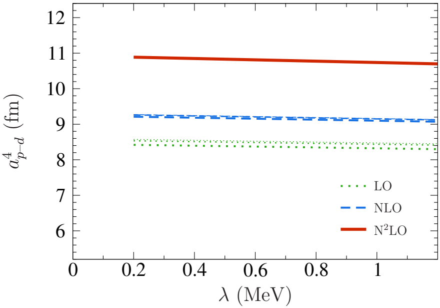
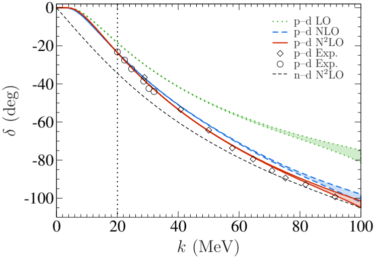
- **but:** strange convergence pattern!

Convergence pattern



$\overset{?}{\Longleftrightarrow}$



- right order of magnitude ✓
- nice (weak) photon-mass dependence ✓
- **but:** strange convergence pattern!

Fully perturbative calculation (I)

So far...

Partial-resummation approach Bedaque, Grießhammer, Hammer, Rupak

- $\mathcal{T}_{\text{LO}} = K_{\text{LO}} + \mathcal{T}_{\text{LO}} \otimes (D_{\text{LO}} K_{\text{LO}})$
- $\mathcal{T}_{\text{NLO}} = K_{\text{NLO}} + \mathcal{T}_{\text{NLO}} \otimes (D_{\text{NLO}} K_{\text{NLO}})$
- etc. $\hookrightarrow$ resums certain higher-order corrections!

Better (cleaner) approach

Fully perturbative calculation see, *e.g.*, Ji, Phillips 2012

- $\mathcal{T}_{\text{NLO}} = \mathcal{T}_{\text{LO}} + \Delta\mathcal{T}_{\text{NLO}}$
- $\Delta\mathcal{T}_{\text{NLO}} = \mathcal{T}_{\text{LO}} \otimes (D^{(1)} K_{\text{LO}}) \otimes \mathcal{T}_{\text{LO}} + \cdots$
- $\delta(k) = \delta^{(0)} + \delta^{(1)} + \cdots$
- complicated at N^{2}LO!

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- $\delta(k) = \delta^{(0)} + \delta^{(1)} + \cdots$
- complicated at N^2LO!

Much more efficient calculation with re-shuffling of terms!

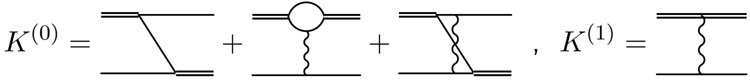
Vanasse 2013

Fully perturbative calculation (II)

$$\mathcal{T}_{\text{full}}^{(0)} = K^{(0)} + \mathcal{T}_{\text{full}}^{(0)} \otimes D_d^{(0)}\, K^{(0)}$$

$$\mathcal{T}_{\text{full}}^{(1)} = K^{(1)} + \mathcal{T}_{\text{full}}^{(0)} \otimes \left[D_d^{(0)}\, K^{(1)} + D_d^{(1)}\, K^{(0)}\right] + \mathcal{T}_{\text{full}}^{(1)} \otimes D_d^{(0)}\, K^{(0)}$$

$$\mathcal{T}_{\text{full}}^{(2)} = \mathcal{T}_{\text{full}}^{(0)} \otimes \left[D_d^{(1)}\, K^{(1)} + D_d^{(2)}\, K^{(0)}\right] + \mathcal{T}_{\text{full}}^{(1)} \otimes \left[D_d^{(0)}\, K^{(1)} + D_d^{(1)}\, K^{(0)}\right] + \mathcal{T}_{\text{full}}^{(2)} \otimes D_d^{(0)}\, K^{(0)}$$



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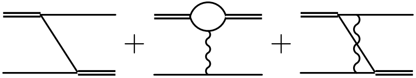
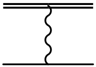
$K^{(0)} =$  , $K^{(1)} =$ 

$$[k\cot\delta_{\text{diff}}]^{(0)} = \frac{2\pi}{\mu}\frac{\mathrm{e}^{2\mathrm{i}\delta_c^{(0)}}}{T_{\text{diff}}^{(0)}} + \mathrm{i}k$$

$$[k\cot\delta_{\text{diff}}]^{(1)} = \frac{2\pi}{\mu}\mathrm{e}^{2\mathrm{i}\delta_c^{(0)}} \times \left[\frac{2\mathrm{i}\delta_c^{(1)}}{T_{\text{diff}}^{(0)}} - \frac{T_{\text{diff}}^{(1)}}{(T_{\text{diff}}^{(0)})^2}\right]$$

$$[k\cot\delta_{\text{diff}}]^{(2)} = -\frac{2\pi}{\mu}\mathrm{e}^{2\mathrm{i}\delta_c^{(0)}} \times \left[\frac{2(\delta_c^{(1)})^2 - 2\mathrm{i}\delta_c^{(2)}}{T_{\text{diff}}^{(0)}} + \frac{2\mathrm{i}\delta_c^{(1)}T_{\text{diff}}^{(1)} + T_{\text{diff}}^{(2)}}{(T_{\text{diff}}^{(0)})^2} - \frac{(T_{\text{diff}}^{(1)})^2}{(T_{\text{diff}}^{(0)})^3}\right]$$

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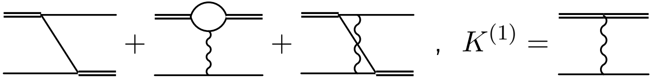
$K^{(0)} =$  $, \quad K^{(1)} =$

$$[k\cot\delta_{\rm diff}]^{(0)} = \frac{2\pi}{\mu}\frac{e^{2i\delta_c^{(0)}}}{T_{\rm diff}^{(0)}} + ik$$

$$[k\cot\delta_{\rm diff}]^{(1)} = \frac{2\pi}{\mu}e^{2i\delta_c^{(0)}} \times \left[\frac{2i\delta_c^{(1)}}{T_{\rm diff}^{(0)}} - \frac{T_{\rm diff}^{(1)}}{(T_{\rm diff}^{(0)})^2}\right]$$

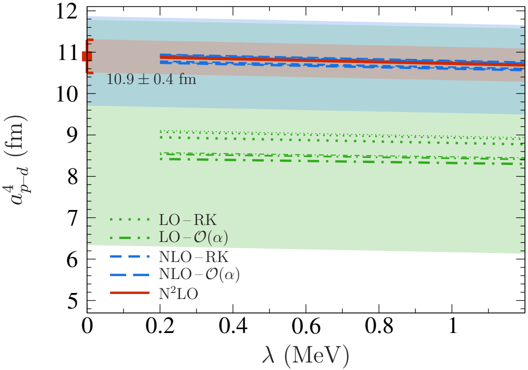
$$[k\cot\delta_{\rm diff}]^{(2)} = -\frac{2\pi}{\mu}e^{2i\delta_c^{(0)}} \times \left[\frac{2(\delta_c^{(1)})^2 - 2i\delta_c^{(2)}}{T_{\rm diff}^{(0)}} + \frac{2i\delta_c^{(1)}T_{\rm diff}^{(1)} + T_{\rm diff}^{(2)}}{(T_{\rm diff}^{(0)})^2} - \frac{(T_{\rm diff}^{(1)})^2}{(T_{\rm diff}^{(0)})^3}\right]$$

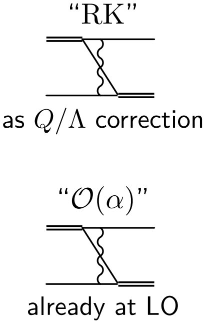
Scattering lenth

$$C_{\eta,\lambda}^2\,[k\cot\delta_{\rm diff}(k)] + \gamma\, h(\eta) = -\frac{1}{a_{p\text{–}d}} + \mathcal{O}(k^2)$$

Combine with $C_{\eta,\lambda}^2 = [C_{\eta,\lambda}^2]^{(0)} + [C_{\eta,\lambda}^2]^{(1)} + \cdots$

Fully perturbative result





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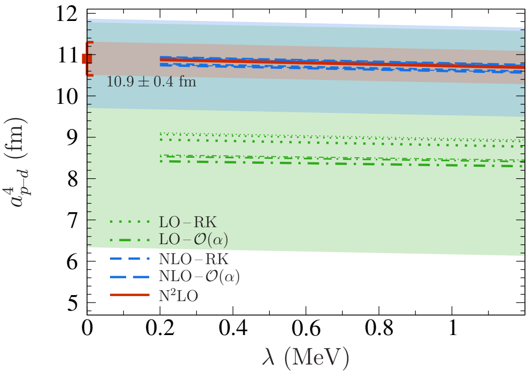
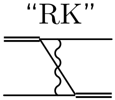
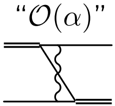




as Q/Λ correction



already at LO

Older experimental determinations

- $a^4_{p\text{–}d} = 11.88 \pm 0.4$ fm — Arvieux (1973)
- $a^4_{p\text{–}d} = 11.1$ fm — Huttel *et al.* (1983)

More recent result

$$a^4_{p\text{–}d} = 14.7 \pm 2.3 \text{ fm}$$

Black *et al.* (1999)

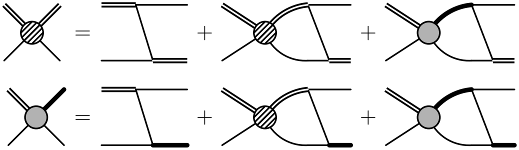
Part IV

Doublet channel

- **Coupled channels**
- **Three-nucleon forces**
- **Results (preliminary)**

Complications

1. coupled channels!



Complications

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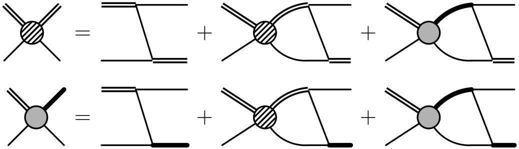
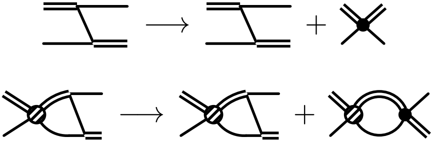


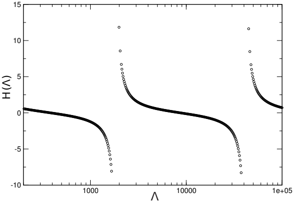
2. strong cutoff dependence!

$\hookrightarrow$ renormalize with leading order 3N-force force ($SU(4)$-symmetric)

Bedaque, Hammer, van Kolck 1999

$$\mathcal{L}_3 = -M_N \frac{H(\Lambda)}{\Lambda^2}\left(y_d^2 N^\dagger(\vec{d}\cdot\vec{\sigma})^\dagger(\vec{d}\cdot\vec{\sigma})N + \ldots\right)$$

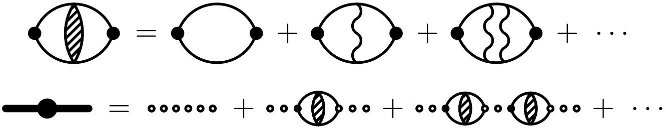




. . . fix $H(\Lambda)$ with three-body input $\rightarrow$ triton binding energy, ${}^2a_{n-d}$

Coulomb effects in the proton–proton channel

In doublet channel, the singlet dibaryon can be in a pure $p - p$ state



$$\Delta_{t,pp}(p) \sim \frac{1}{-1/a_C - 2\kappa H(\kappa/p')} \;\;,\;\; \kappa = \frac{\alpha M_N}{2} \;\;,\;\; p' = \mathrm{i}\sqrt{\mathbf{p}^2/4 - M_N p_0 - \mathrm{i}\varepsilon}$$

Kong, Ravndal 1999

$\rightarrow$ Coulomb-modified effective range expansion

Bethe 1949

cf. Ando, Birse 2010

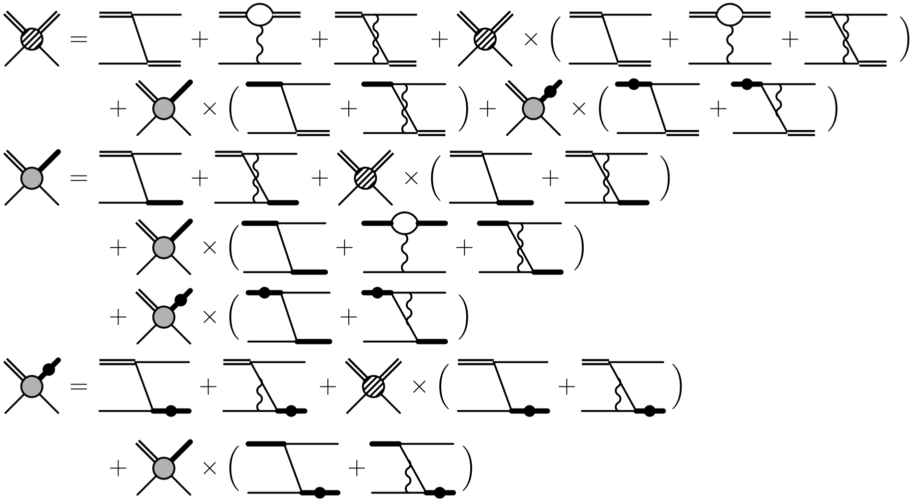
The third nucleon neccessarily has to be a neutron!

$\rightarrow$ no additional Coulomb-photon exchange!

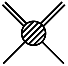
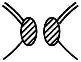
$\hookrightarrow$ 3-channel integral equation

Full doublet-channel integral equation

Include all $\mathcal{O}(\alpha)$ Coulomb diagrams. . .

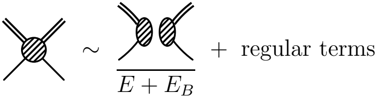


He-3 binding energy (LO)

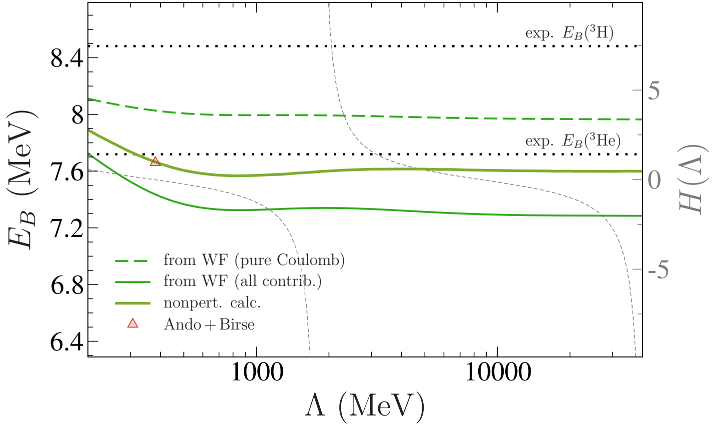
bound-sate regime:  $\sim$  $/ \, (E + E_B)$ $+$ regular terms

He-3 binding energy (LO)

bound-sate regime:

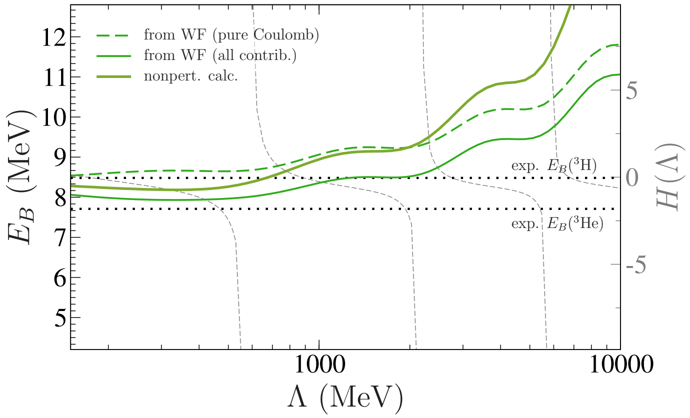


↪ predict ^{3}He binding energy!



He-3 binding energy (NLO)

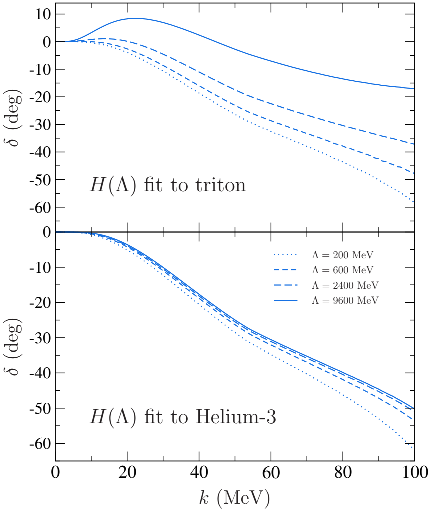
At NLO, things don't work so well. . .



↪ **incomplete renormalization!**

New "Coulomb" counterterm

Re-fit $H(\Lambda)$ to ^{3}He energy at NLO



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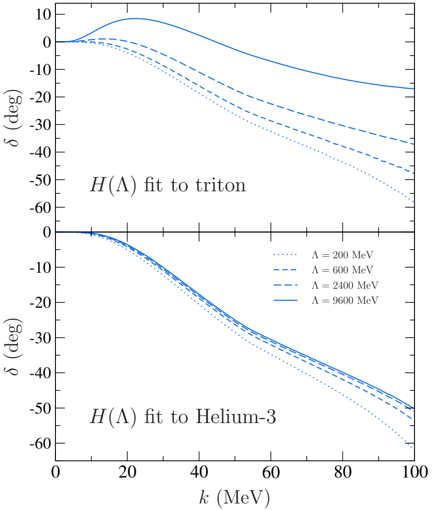
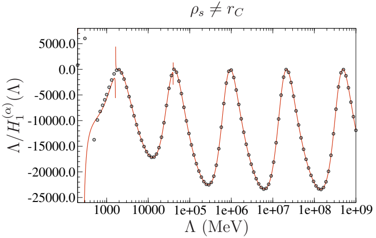


Can be shown analytically!

Vanasse, Egolf, Kerin, SK, Springer 2014

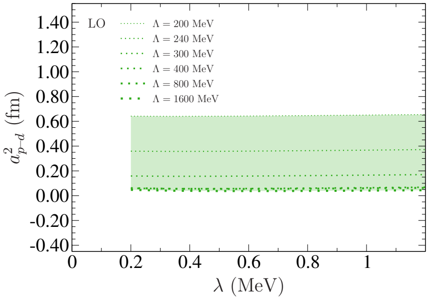
$$H(\Lambda) = H_0(\Lambda) + H_1(\Lambda) + H_1^{(\alpha)}(\Lambda)$$



Doublet-channel scattering length

Back to the fully perturbative approach...

- fit $H_1^{(\alpha)}(\Lambda)$ to ^{3}He binding energy
- predict doublet-channel p–d scattering length



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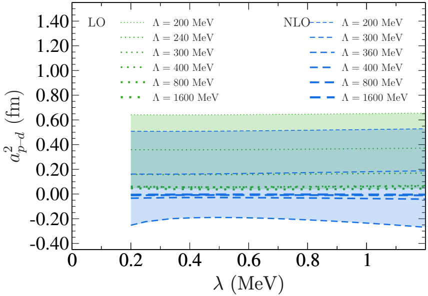

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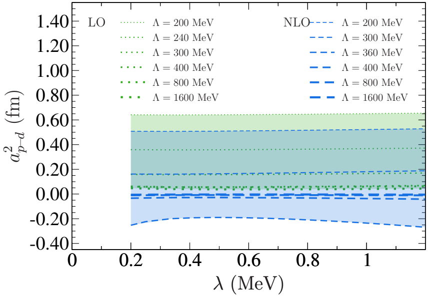

Other determinations

Ref.	$^4a_{p\text{–}d}$ (fm)
van Oers, Brockman (1967)	1.2 ± 0.2
Arvieux (1973)	2.73 ± 0.10
Huttel *et al.* (1983)	≈ 4.0
Black *et al.* (1999)	-0.13 ± 0.04
Orlov, Orevkov (2006)	≈ 0.024

Summary and outlook

- Coulomb effects are well under control
- Clean perturbative expansion very important at low energies
- Quartet-channel scattering length agrees well with older experimental determinations
- Need to go to higher orders to nail down doublet-channel scattering length

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***

Thanks for your attention!