Selected Topics in 3N and 4N Systems

A. Kievsky

INFN, Sezione di Pisa (Italy)

INT Workshop - Seattle 12-16 May 2014
Few-body Universality in Atomic and Nuclear Physics:
Recent Experimental and Theoretical Advances

Collaborators

- M. Viviani, L.E. Marcucci - *INFN & University of Pisa*
- L. Girlanda - *University of Lecce*
- M. Gattobigio - *Institute Non Lineaire de Nice (France)*
- E. Garrido, *CSIC - Madrid (Spain)*

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Outline of the talk

Topic 1: Critical analysis of the NN force

- quality of the description in the 2N system: χ^2 per datum ≈ 1

Potentials with charge dependence (1995):

- AV18 [0-350 MeV], $\chi^2/dat. \approx 1$ ($\approx$ 40 parameters)
- Nijmegen[0-350 MeV], $\chi^2/dat. \approx 1$
- CD Bonn [0-350 MeV], $\chi^2/dat. \approx 1$

Chiral potentials (2004)

- N2LO-BO [0-125 MeV], $\chi^2/dat. \approx 10$
- N3LO-EM [0-290 MeV], $\chi^2/dat. \approx 1 \rightarrow$ 29 parameters
- N2LO-opt [0-125 MeV], $\chi^2/dat. \approx 1 \rightarrow$ 15 parameters (2013)

$A = 3, 4$ Systems

The $A = 3, 4$ systems with NN interactions

- 2N system: χ^2 per datum ≈ 1
- 3N and 4N systems using a NN interaction: χ^2 per datum $>> 1$

The $A = 3, 4$ systems with NN and NNN forces

- Urbana IX $\rightarrow$ 2 parameters
- Tucson-Melbourne $\rightarrow$ 1 parameter
- 3N-N2LO $\rightarrow$ 2 parameters

The parameters are fixed to reproduce:

- The ^{3}H and ^{4}He binding energies
- The $^{(2)}a_{nd}$ scattering length or the triton half life

- 3N and 4N systems using a NN+NNN interaction: χ^2 per datum $>> 1$

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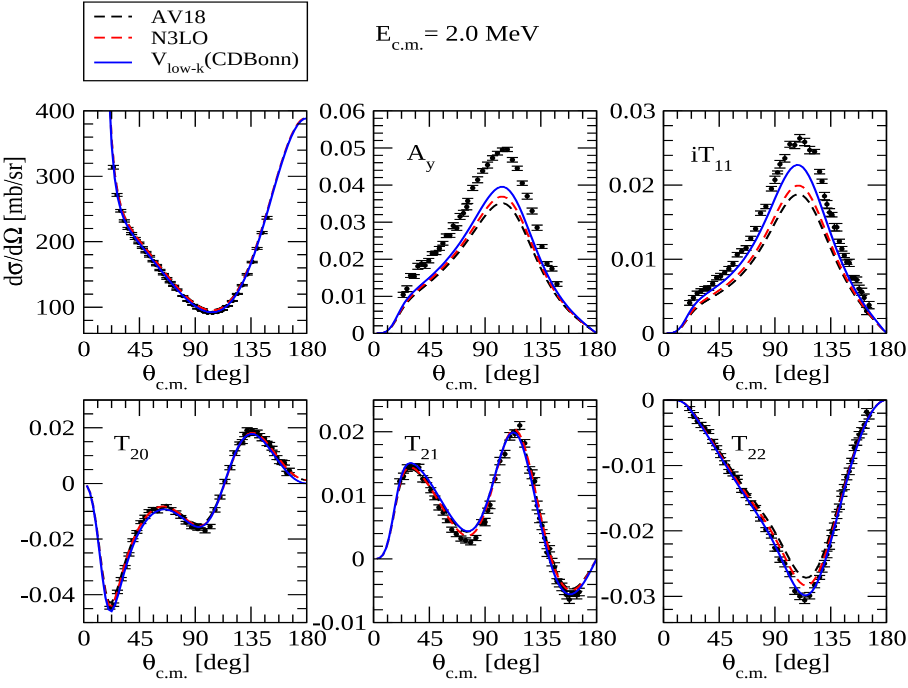
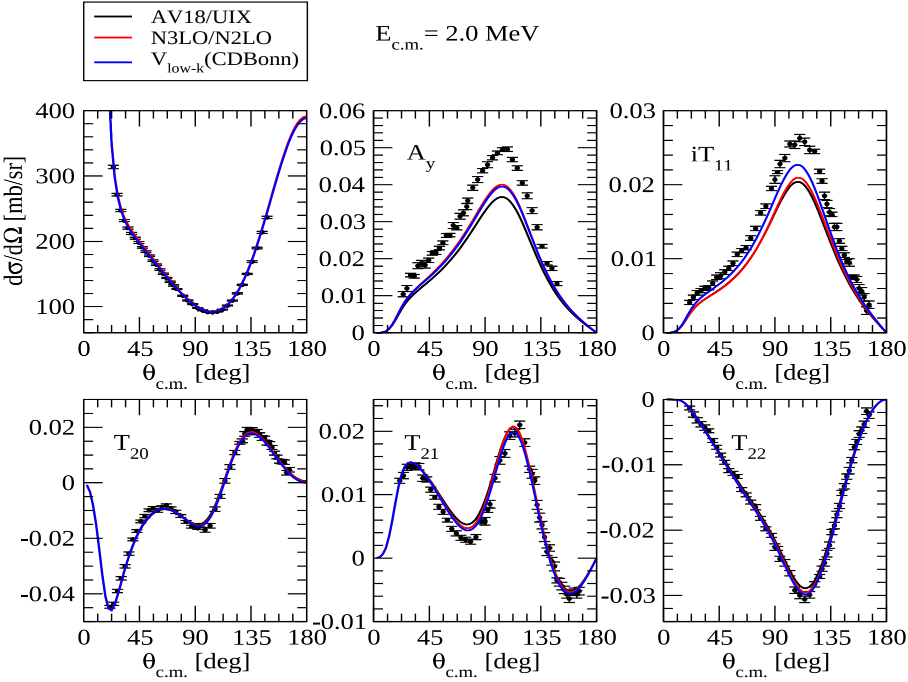
^{3}H and ^{4}He Bound States and $n-d$ scattering length

Potential(NN)	Method	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	HH	7.624	24.22	1.258
	FE/FY Bochum	7.621	24.23	1.248
	FE/FY Lisbon	7.621	24.24	
CDBonn	HH	7.998	26.13	
	FE/FY Bochum	8.005	26.16	0.925
	FE/FY Lisbon	7.998	26.11	
	NCSM	7.99(1)		
N3LO-EM	HH	7.854	25.38	1.100
	FE/FY Bochum	7.854	25.37	
	FE/FY Lisbon	7.854	25.38	
	NCSM	7.852(5)	25.39(1)	

Potential(NN+NNN)				
AV18/UIX	HH	8.479	28.47	0.590
	FE/FY Bochum	8.476	28.53	0.578
CDBonn/TM	HH	8.474	29.00	
	FE/FY Bochum	8.482	29.09	0.570
N3LO-EM/3N-N2LO	HH	8.474	28.37	0.675
	NCSM	8.473(5)	28.34(2)	
N2LO-opt/3N-N2LO		8.469	28.42	
Exp.		8.48	28.30	0.645±0.010


AV18
N3LO
V_{low-k}(CDBonn)
$E_{c.m.}$ = 2.0 MeV
dσ/dΩ [mb/sr]
A_y
iT_{11}
T_{20}
T_{21}
T_{22}
$\theta_{c.m.}$ [deg]

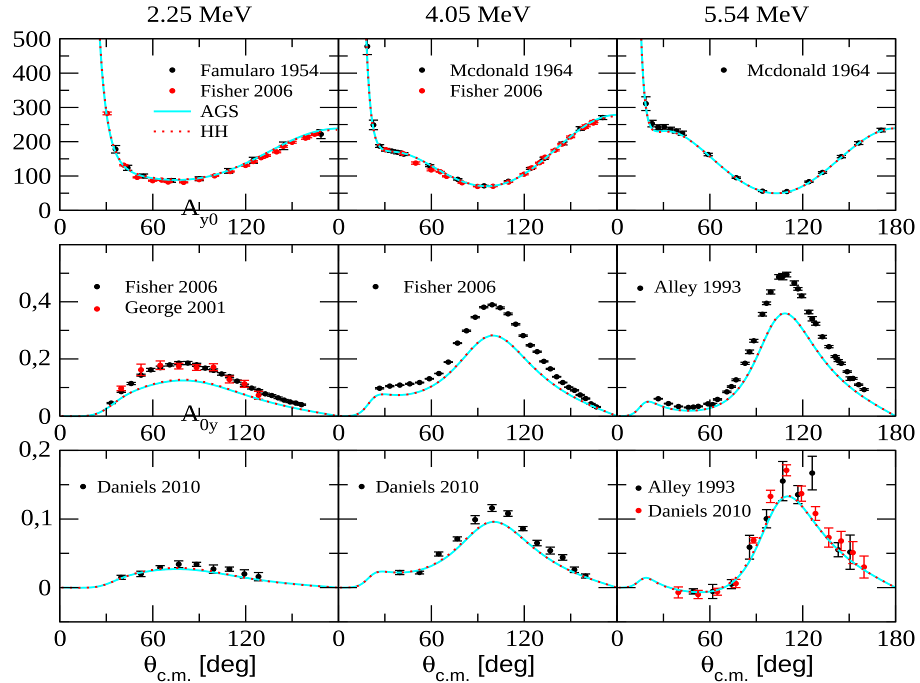

AV18/UIX
N3LO/N2LO
V_{low-k}(CDBonn)
$E_{c.m.}$ = 2.0 MeV
dσ/dΩ [mb/sr]
A_y
iT_{11}
T_{20}
T_{21}
T_{22}
$\theta_{c.m.}$ [deg]

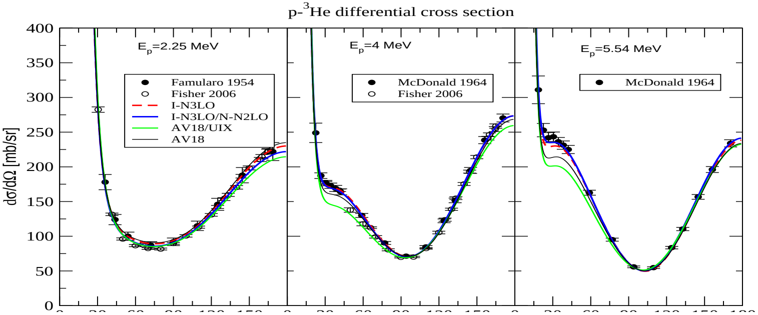
Benchmark calculations for $3+1$ scattering states

N3LO ($n-{}^3$H and $p-{}^3$He)

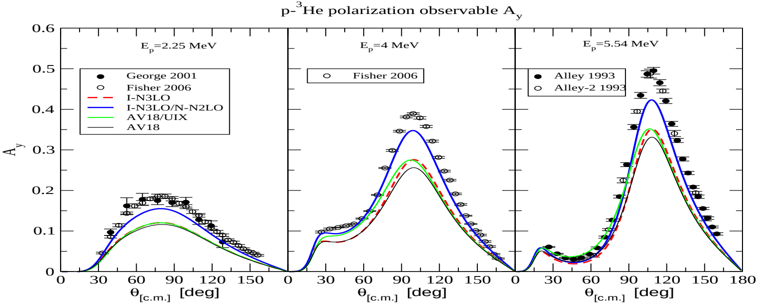
E_n(MeV)	$^1S_0$	$^3P_0$	$^3S_1$	$^3D_1$	ϵ_{1+}	$^1P_1$	$^3P_1$	ϵ_{1-}	
1.0	-38.10	4.15	-33.32	-0.09	-0.23	5.99	9.63	9.44	AGS
	-38.02	4.10	-33.25	-0.08	-0.20	5.59	9.44	8.85	HH
	-38.31	4.00	-33.56	-0.11	-0.24	6.13	10.13		FY
2.0	-51.93	10.54	-45.66	-0.36	-0.44	13.13	24.18	9.15	AGS
	-51.98	10.50	-45.71	-0.33	-0.39	12.87	23.90	9.09	HH
	-52.34	10.54	-45.99	-0.39	-0.50	13.55	25.15		FY
3.5	-65.54	20.31	-57.99	-0.91	-0.72	20.74	40.94	9.45	AGS
	-65.67	20.21	-58.20	-0.92	-0.67	20.90	40.98	9.57	HH
	-66.15	20.62	-58.40	-0.91	-0.79	21.17	41.50		FY

E_p(MeV)	$^1S_0$	$^3P_0$	$^3S_1$	$^3D_1$	ϵ_{1+}	$^1P_1$	$^3P_1$	ϵ_{1-}	
2.25	-40.64	8.04	-35.00	-0.24	-0.53	10.64	17.29	8.61	AGS
	-40.87	7.67	-35.31	-0.30	-0.43	10.08	16.88	8.47	HH
	-41.57	7.74	-35.49	-0.28	-0.58	10.84	17.75		FY
4.05	-58.23	17.94	-50.79	-0.94	-0.84	18.90	35.50	8.73	AGS
	-58.65	17.84	-51.25	-0.97	-0.74	19.05	35.54	8.84	HH
	-59.12	18.12	-51.15	-0.96	-0.94	19.26	35.78		FY
5.54	-68.28	25.41	-60.02	-1.45	-1.08	23.05	44.54	9.28	AGS
	-68.50	25.08	-60.15	-1.53	-1.08	23.07	44.46	9.30	HH
5.51	-69.00	25.81	-60.03	-1.40	-1.18	23.16	44.13		FY

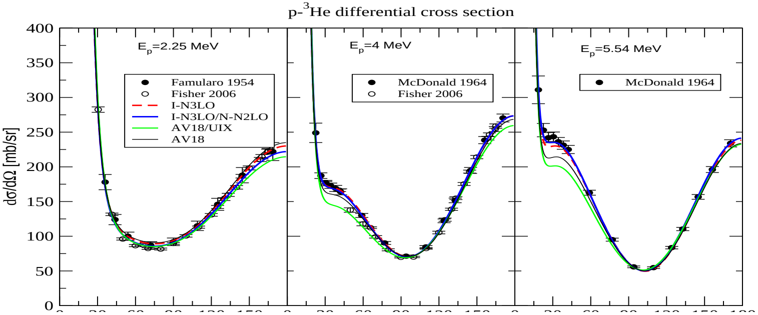

2.25 MeV
4.05 MeV
5.54 MeV
Famularo 1954
Fisher 2006
AGS
HH
Mcdonald 1964
Fisher 2006
Mcdonald 1964
A_{y0}
Fisher 2006
George 2001
Fisher 2006
Alley 1993
A_{0y}
Daniels 2010
Daniels 2010
Alley 1993
Daniels 2010
$\theta_{c.m.}$ [deg]
$\theta_{c.m.}$ [deg]
$\theta_{c.m.}$ [deg]



p-3He differential cross section
Ep=2.25 MeV
Ep=4 MeV
Ep=5.54 MeV
Famularo 1954
Fisher 2006
I-N3LO
I-N3LO/N-N2LO
AV18/UIX
AV18
McDonald 1964
Fisher 2006
McDonald 1964
dσ/dΩ [mb/sr]



p-3He polarization observable Ay
Ep=2.25 MeV
Ep=4 MeV
Ep=5.54 MeV
George 2001
Fisher 2006
I-N3LO
I-N3LO/N-N2LO
AV18/UIX
AV18
Fisher 2006
Alley 1993
Alley-2 1993
Ay
θ[c.m.] [deg]
θ[c.m.] [deg]
θ[c.m.] [deg]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I-N3LO/N-N2LO
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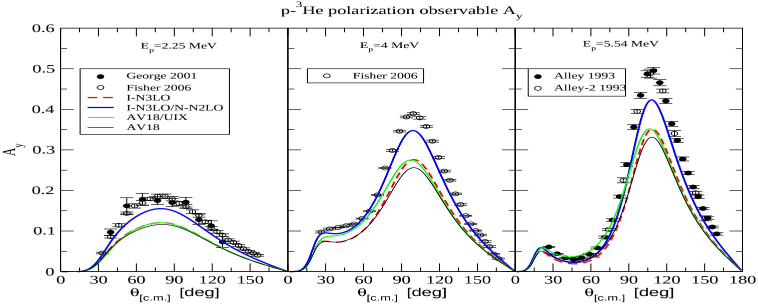
AV18
McDonald 1964
Fisher 2006
McDonald 1964
dσ/dΩ [mb/sr]
400
350
300
250
200
150
100
50
0


p-3He polarization observable A_y
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George 2001
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I-N3LO
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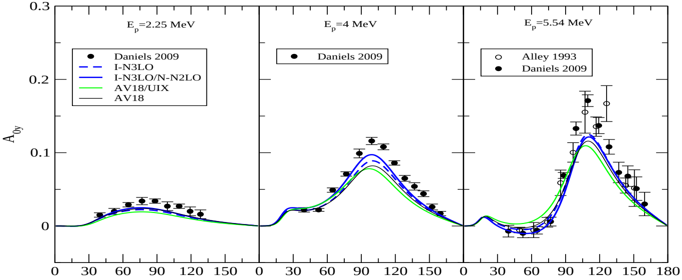
Alley-2 1993
A_y
0.6
0.5
0.4
0.3
0.2
0.1
0
θ[c.m.] [deg]
θ[c.m.] [deg]
θ[c.m.] [deg]



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Daniels 2009
I-N3LO
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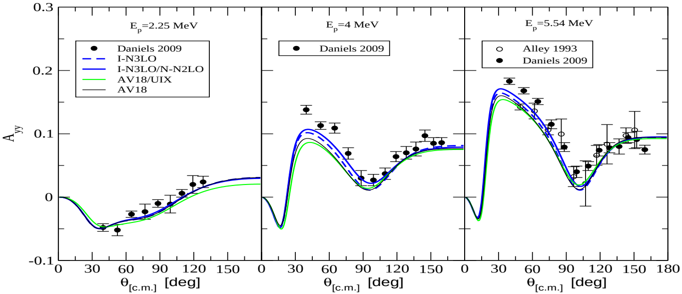
A_{0y}
0.3
0.2
0.1
0
0 30 60 90 120 150 180



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0.2
0.1
0
-0.1
0 30 60 90 120 150 180
$\theta_{[c.m.]}$ [deg]

Some Remarks

- In the $3N$ system $\chi^2 \approx 100$ using NN+NNN forces
- High quality NN potentials are unable to describe polarization observables with high acuracy
- Including NNN forces the description slightly improves (in some cases is worst)

But

- The singlet and triplet scattering lenghts are: $a_0 \approx -23$ fm and $a_1 \approx 5$ fm
- The NN force range $r_0 \approx 1.5$ fm, therefore $r_0/a_0 < 1$ and $r_0/a_1 < 1$ and also $E_d \approx -\hbar^2/ma_1^2$
- To which extend a_0 and a_1 are control parameters of the few-nucleon dynamics? $\rightarrow$ Efimov physics
- In this contex the ^{3}H bining energy appears as a scale parameter (the three-body parameter)

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LO (pionless) description

The 2N sector: Low energy data

$E_d = -2.2245$ MeV

$a_1 = 5.424 \pm 0.003$ fm $r_1^{eff} = 1.760 \pm 0.005$ fm

$a_0 = -23.740 \pm 0.020$ fm $r_0^{eff} = 2.77 \pm 0.05$ fm

Constructing the LO 2N potential

Two parameters corresponding to the $\ell = 0$ partial waves with $S = 0, 1$:

$V_0(r) = -V_0 \mathrm{e}^{-r^2/r_0^2}$, $V_1(r) = -V_1 \mathrm{e}^{-r^2/r_1^2}$

V_0[MeV]	r_0[fm]	a_0[fm]	r_0^{eff}[fm]	V_1[MeV]	r_1[fm]	a_1[fm]	r_1^{eff}[fm]
53.255	1.40	−23.741	2.094	79.600	1.40	5.309	1.622
42.028	1.57	−23.745	2.360	65.750	1.57	5.423	1.776
40.413	1.60	−23.745	2.407	63.712	1.60	5.447	1.802
37.900	1.65	−23.601	2.487	60.575	1.65	5.482	1.846
33.559	1.75	−23.745	2.644	55.036	1.75	5.548	1.930
30.932	1.82	−23.746	2.756				

The 3N sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	$^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	−12.40	−2.191	−2.175
42.028	1.57	65.750	1.57	−10.83	−2.199	−1.236
40.413	1.60	63.712	1.60	−10.59	−2.197	−1.097
37.900	1.65	60.575	1.65	−10.22	−2.199	−0.860
33.559	1.75	55.036	1.75	−9.584	−2.201	
30.932	1.82	65.750	1.57	−9.715	−2.200	−0.285
Exp.				−8.482		0.645 ± 0.010

Introducing a Three-Body Force

We choose a simple (two-parameter) form:

$$W(\rho) = W_0 e^{-\rho^2/\rho_0^2}$$

with $\rho^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$

- W_0 fixed to describe $E(^3\mathrm{H})$
- The dependence in ρ_0 is analyzed using $^2a_{nd}$

The 3N sector

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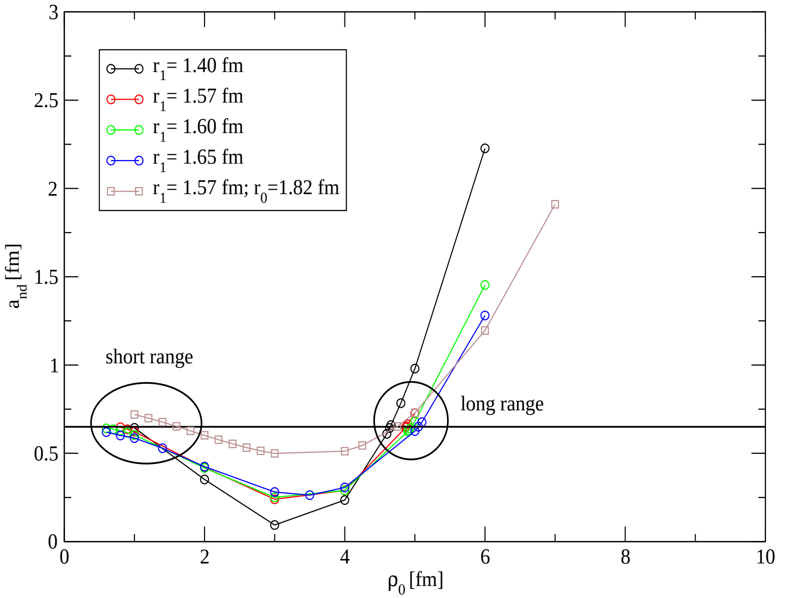
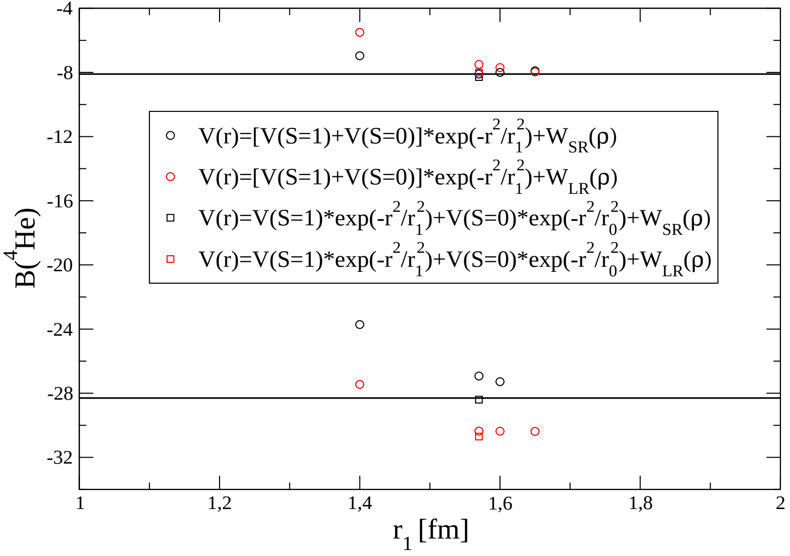
- The dependence in ρ_0 is analyzed using $^2a_{nd}$

$$V(r)=[V(S=1)+V(S=0)]*\exp(-r^2/r_1^2)+W_0*\exp(-\rho^2/\rho_0^2)$$

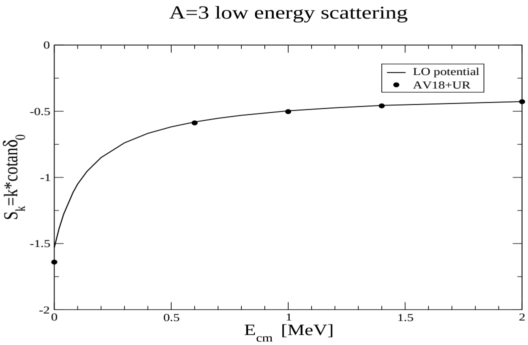


The N=4 ground and excited states



Summary of the (optimized) LO potential

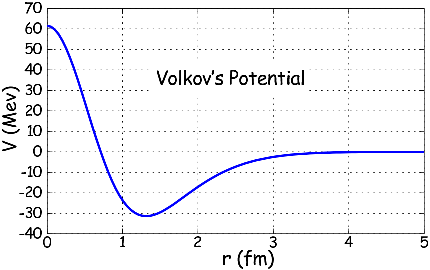
LO	E_d	$B(^3\mathrm{H})$	$B(^4\mathrm{He})$	$B(^4\mathrm{He}^*)$	$^2a_{nd}$
	-2.225	-8.480	-28.41	-8.29	0.652
Exp.	-2.225	-8.482	-28.296	-8.10	0.645



No bad for a 4-parameter 2*N* potential + 2-parameter 3*N* potential!
next step (in progress)→ ^{6}He and ^{6}Li ground states

Prelimar study up to $A = 6$ using a central potential

- The Volkov Potential: $V(r) = E_1\, e^{-r^2/R_1^2} + E_2\, e^{-r^2/R_2^2}$ with $E_1 = 144.86$ Mev, $R_1 = 0.82$ fm, $E_2 = -83.34$ Mev, $R_2 = 1.6$ fm
- No three-body force
- $A = 2$: $E_d = -0.546$ MeV, $a = 10.08$ fm
- $A = 3$: $E(^3\mathrm{H}) = -8.43$ MeV, $E(^3\mathrm{He}) = -7.72$ MeV



- S-wave potential – only acts when $l_{ij} = 0$

M. Gattobigio, A.K. and M. Viviani, PRC83, 024001 (2011)

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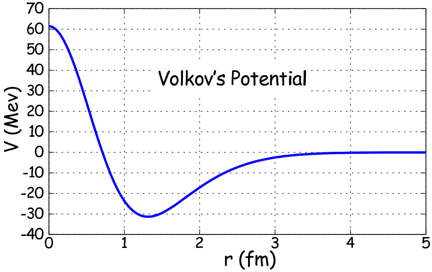

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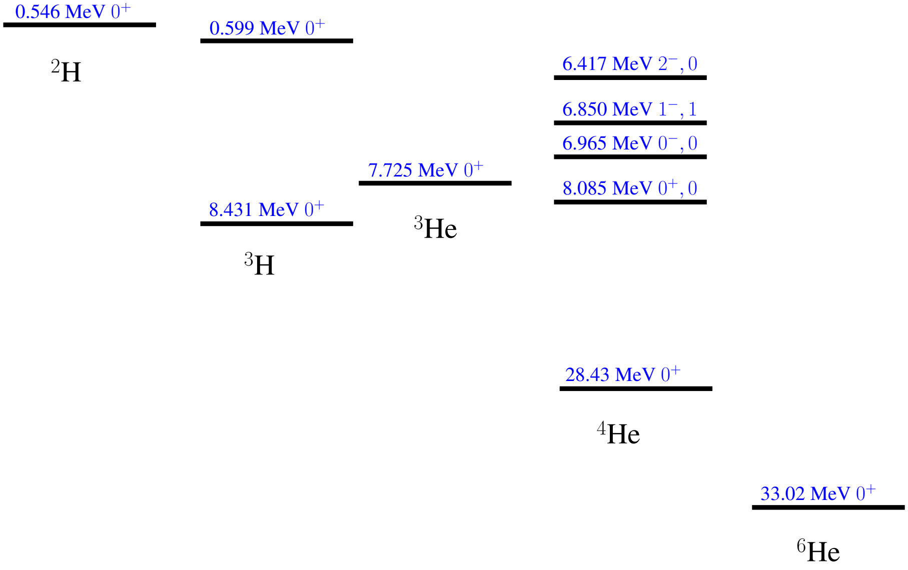
All-waves Volkov for $A = 6$, $L^\pi = 0^+$

K_{max}	N_{HH}	E_0 (MeV) [6]	E_1 (MeV) [6]	E_2 (MeV) [5 1]	E_3 (MeV) [4 2]
2	15	117.205	64.701	62.513	61.142
4	120	118.861	69.450	64.277	62.015
6	680	120.345	70.544	66.268	63.377
8	3045	121.738	71.443	67.280	64.437
10	11427	122.317	71.923	68.371	65.354
12	37310	122.597	72.477	69.029	65.886
14	108810	122.711	72.822	69.531	66.201
16	288990	122.752	73.101	69.842	66.360
18	709410	122.768	73.284	70.051	66.437
20	1628328	122.774	73.407	70.189	66.474
22	3527160	122.776	73.485	70.283	66.491
SVM*					66.25

* K. Varga and Y. Suzuki, Phys. Rev. C **52**, 2885 (1995)

S-wave Volkov



Topic 2: Use the Subleading contributions to the 3N contact interaction to improve the 3N continuum description

At LO there is one 3N contact interaction. The local form used is

$$V_{LO}(3N) = \frac{1}{2}\sum_{j\neq k} E\,\boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k \rightarrow c_E \sum_{j\neq k} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k Z_0(r_{ji})Z_0(r_{ki})$$

with

$$Z_0(r) = \int \frac{d\boldsymbol{p}}{(2\pi)^3} e^{i\boldsymbol{p}\cdot\boldsymbol{r}} F_\Lambda(p)$$

In Topic 1 we have used: $W_0 e^{-\rho^2/\rho_0^2}$ and we have identified the cutoff Λ with ρ_0

The NLO contact interaction

The NLO $3N$ contact Lagrangian contain two spatial derivatives. The space-structures are of the form

$$
\begin{aligned}
X^{+}_{A,ij} &= (N^\dagger \overleftrightarrow{\nabla}_i N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\
X^{+}_{B,ij} &= \nabla_i(N^\dagger N)\nabla_j(N^\dagger N)(N^\dagger N) \\
X^{-}_{C,ij} &= i\nabla_i(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\
X^{+}_{D,ij} &= (N^\dagger \overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j N)(N^\dagger N)(N^\dagger N),
\end{aligned}
$$

The relevant isospin structures are

$$
T^{+} = \mathbf{1}, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3, \quad \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3, \quad T^{-} = \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3,
$$

Even (odd) combinations of $X \otimes T$ structures under time-reversal have to be associated with spin structures containing even (odd) numbers of σ matrices.

A local form of the NLO 3N potential

The NLO 3N contact Lagrangian contain two spatial derivatives. Its local form has been derived in L. Girlanda, A. K. and M. Viviani, PRC84, 014001 (2011).

$$
\begin{aligned}
V_{NLO}(3N) = & \sum_{i\neq j\neq k} (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) \\
& \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k) \boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}
$$

It contains 10 LEC

$A = 3$ calculations

The $A = 3$ Hamiltonian is

$$H = T + V(2N) + V_{LO}(3N) + V_{NLO}(3N)$$

For $V(2N)$ we use AV18

$V_{LO}(3N)$ has one LEC $\rightarrow c_E$

$V_{NLO}(3N)$ has 10 LEC $\rightarrow [E_1, \ldots, E_{10}]$

To determine the LECs we use $B(^3\mathrm{H})$, $^2a_{nd}$ and scattering observables at $E_{lab} = 3$ MeV

Using AV18 or AV18+UR, at $E_{lab} = 3$ MeV the $\chi^2/datum \approx 100$

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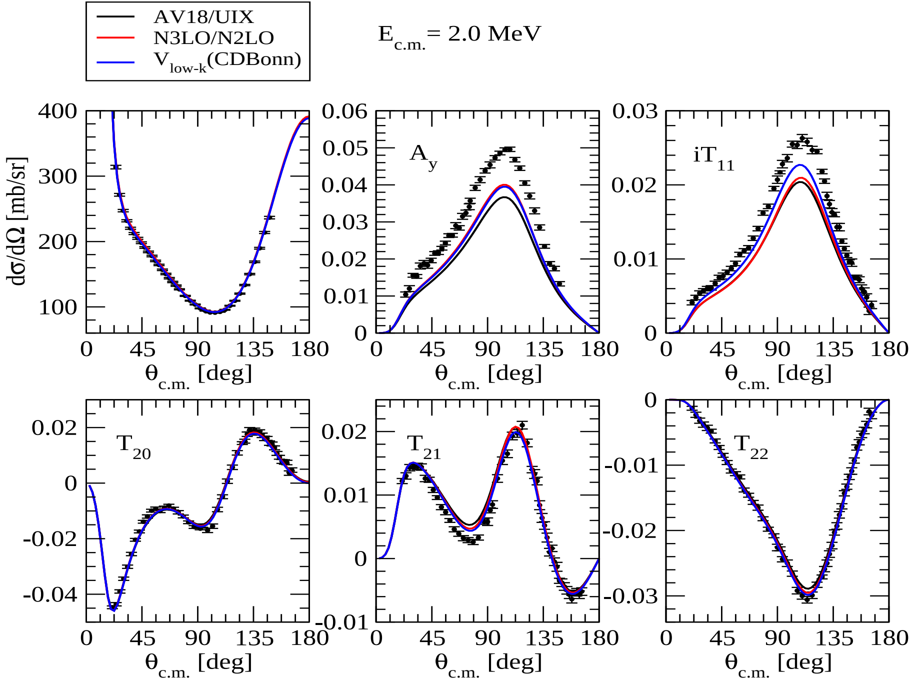
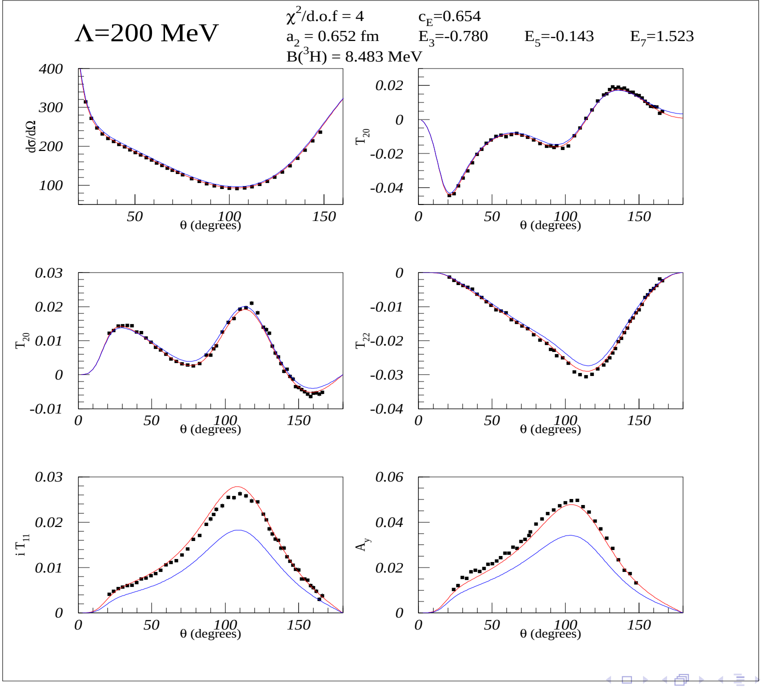
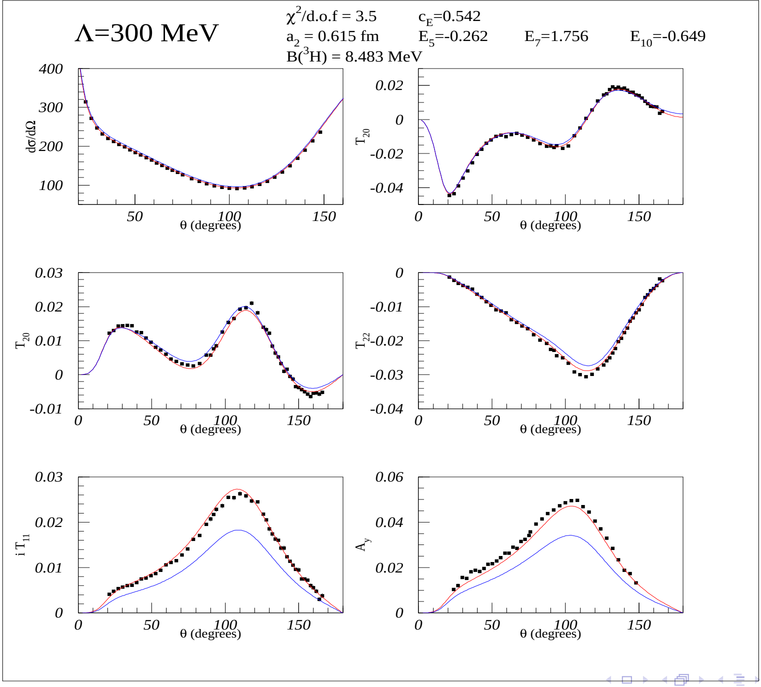

AV18/UIX
N3LO/N2LO
V_{low-k}(CDBonn)
$E_{c.m.}$ = 2.0 MeV
dσ/dΩ [mb/sr]
A_y
iT_{11}
T_{20}
T_{21}
T_{22}
$\theta_{c.m.}$ [deg]


Λ=200 MeV
χ²/d.o.f = 4
c_E=0.654
a_2 = 0.652 fm
E_3=-0.780
E_5=-0.143
E_7=1.523
B(³H) = 8.483 MeV
dσ/dΩ
T_20
T_20
T_22
i T_11
A_y
θ (degrees)


Λ=300 MeV
χ²/d.o.f = 3.5
c_E=0.542
a_2 = 0.615 fm
E_5=-0.262
E_7=1.756
E_{10}=-0.649
B(³H) = 8.483 MeV
dσ/dΩ
T20
T20
T22
i T11
Ay
θ (degrees)

Some hints to perform the minimization

The $A = 3$ Hamiltonian can be written as

$$H = T + V(2N) + c_E * V_0(3N) + \sum_{i=1,10} E_i * V_i(3N)$$

Using the HH basis it can be transformed to matrices

$$H_{nn'} = [T + V(2N)]_{nn'} + \sum_{i=0,10} E_i * [V_i(3N)]_{nn'}$$

- The maximum size of the basis is fixed (accuracy below 0.5%)
- The $[T + V(2N)]_{nn'}$ matrix is constructed (no free parameters)
- For each operator $[V_i]$ the matrix is constructed (11 matrices)
- For each set $[E_i]$ the Hamiltonian is diagonalized (bound state)
- Different cutoffs are used

For the continuum

- The matrices are calculated for the different J^π states
- At low energies the 3N force is active up to $J^\pm = 5/2$
- To calculate the S matrix we use the KVP

$$\sum_{n'} [H_{nn'} - E\, N_{nn'}] A^i_{n'} = F^i_{n'}; \qquad \sum_{n'} [H_{nn'} - E\, N_{nn'}] B^i_{n'} = G^i_{n'}$$

  where $F^i_{n'} =< \phi_n | H - E | F_{LSJ} >$ and $G^i_{n'} =< \phi_n | H - E | G_{LSJ} >$ are the asymptotic driving terms. The S matrix is
  $[S_{ij}]^{2nd} = A^{-1} B$
  with $A_{ij} =< \Psi_i | H - E | F_i >$ and $B_{ij} =< \Psi_i | H - E | G_i >$
- The F^i_n are:
  $F^i_n = F^i_n(2N) + \sum_{k=0,10} E_k * F^i_n(k)$
- This coefficients are calculated for each term and for each energy
- The matrix operations are linear in the coefficients $[E_k]$ and the minimization can be implemented efficiently

The $A = 4$ system

The LO 4N potential

- The obtained 3N interaction has to be used in the 4N system
- We expect a good description of the polarization observables
- However the $E(^4\mathrm{He})$ has to be tuned
- At present this could be too difficult
- Alternatively, we can use the LO 4N potential:

$$V_{4N} = F \sum_{i \neq j \neq k \neq l} Z_0(r_{ij}) Z_0(r_{ik}) Z_0(r_{il})$$

- The LEC F can be fixed to reproduce the $E(^4\mathrm{He})$

Summary Topics 1 and 2

- We have studied the role of a_0 and a_1 as control parameters
- The few-nuceon dynamics is to a large extent universal and fits inside the Efimov Physics
- A (pionless) potential up to two derivatives can be constructed. It should have $\chi^2 \approx 1$ in the range $[0, 100]$ MeV (to be done).
- It has to be supplemented with a three-body force.
- This three-body force contains a leading term and subleading terms
- Probably with this NN+NNN interaction the χ^2 per datum in the low energy sector of the $3N$ and $4N$ systems can be reduced by two order of magnitude (from $100 \rightarrow 1$)
- Work in progress:
  low energy $N - {}^4\mathrm{He}$ scattering states
  ^{6}He and ^{6}Li binding energies
  determination of the NLO($3N$) potential

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