

# ~~From Cold Atoms to Halo Nuclei~~

*An EFT description of three-body physics*

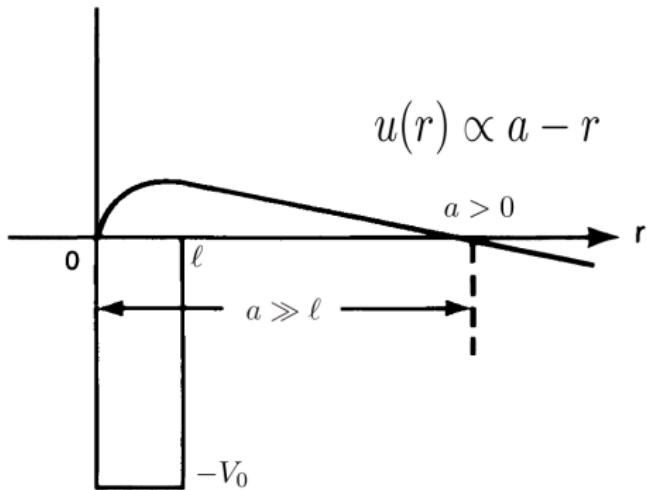
Chen Ji || TRIUMF

In collaboration with  
B. Acharya, D.R. Phillips & Ch. Elster (Ohio Univ.)

INT workshop, May 12-16, 2014



- Separation of scales:  
 $a \gg \ell$
- 2-body universality:  
 $B_2 = 1/ma^2$



- **Separation of scales:**

$$a \gg \ell$$

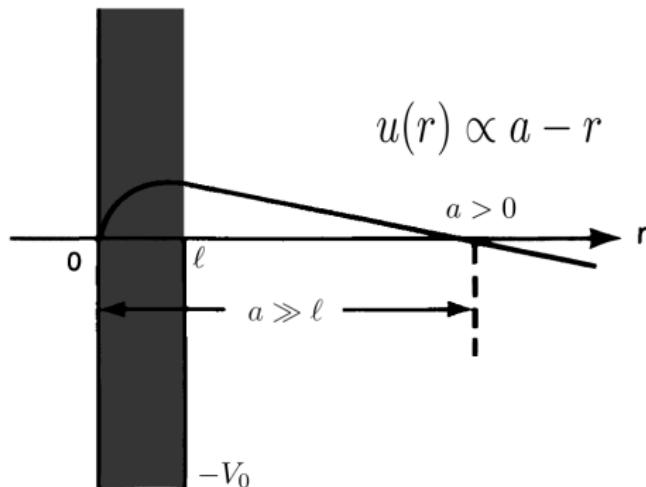
- **2-body universality:**

$$B_2 = 1/m a^2$$

- physics at  $r \sim a$  is insensitive to physics at  $r \sim \ell$

- large- $a$  physics is studied in  $\ell/a$  expansion

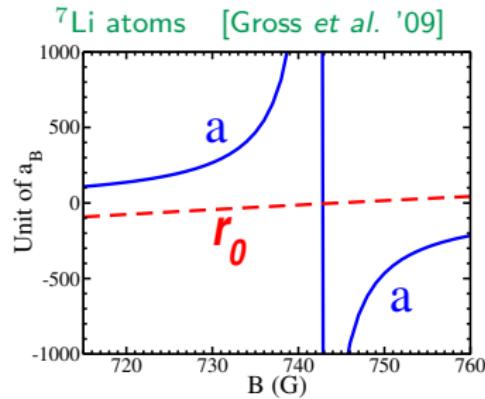
- effects from SR-dynamics can be included as perturbation



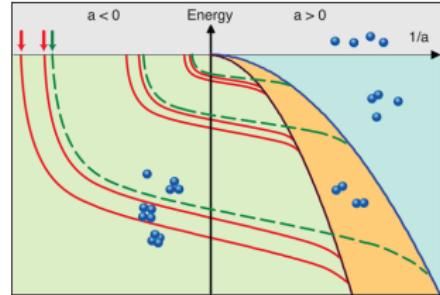
# Large- $a$ physics

universal physics exists in systems with  $\ell \ll a$

- atomic physics
  - cold atomic gases ( $^{133}\text{Cs}$ ,  $^7\text{Li}$ ,  $^{39}\text{K}$ ):  
 $\ell \ll a$  varies near Feshbach resonance



Efimov physics [pic:Esry '09]



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  - $^4\text{He}$  atomic clusters:  
 $\ell_{vdw} \sim 7\text{\AA}$ ,  $a \sim 100\text{\AA}$

$^4\text{He}$  trimer

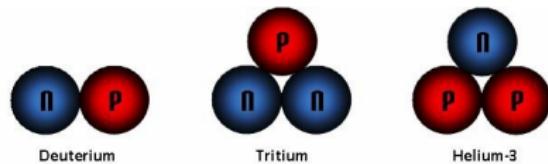
[pic: Blume@physics.wsu.edu]



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  - few-nucleon systems ( $^3\text{H}$ ,  $^3\text{He}$ ,  $^4\text{He}$ ):  
 $\ell_{np}^t \sim 1.7 \text{ fm}$ ,  $a_{np}^t \sim 5.4 \text{ fm}$



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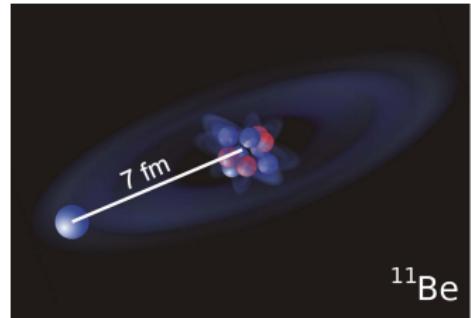
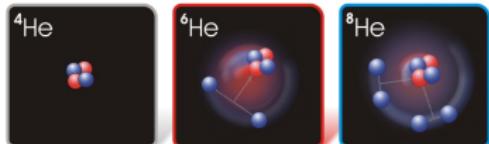
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- halo nuclei** (core + valence  $N$ )

$$^6\text{He}: S_{2n} \approx 1 \text{ MeV} \quad E_c^* \approx 20 \text{ MeV}$$

$$Q \sim \sqrt{m_N S_{2n}} \approx 30 \text{ MeV} \quad \Lambda \sim \sqrt{m_N E_c^*} \approx 140 \text{ MeV}$$



- EFT is an approach to systems with a separation of scales
  - atomic systems → zero-range EFT
  - few-nucleon systems → pionless EFT
  - halo nuclei → halo EFT
  - study physics in expansion of  $\ell/a$  or  $Q/\Lambda$

- EFT with contact interactions

- 2-body contact (LO)


$$= -iC_0$$

$C_0$  determined by a 2-body observable

- 3-body contact (LO)


$$= -iD_0$$

$D_0$  determined by a 3-body observable

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- EFT with contact interactions

- 2-body contact (LO) introduce a dimer field



$$= -iC_0$$

$$\xrightarrow{C_0=g^2/\Delta}$$



$$= -i\sqrt{2}g$$

$C_0$  determined by a 2-body observable

- 3-body contact (LO)



$$= -iD_0$$

$$\xrightarrow{D_0=-3hg^2/\Delta^2}$$



$$= ih$$

$D_0$  determined by a 3-body observable

Bedaque, Hammer, van Kolck '99

- **EFT Lagrangian for 3 identical bosons at LO**

$$\mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - d^\dagger \left( i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d - \frac{g}{\sqrt{2}} \left( d^\dagger \psi \psi + \text{h.c.} \right) + h d^\dagger d \psi^\dagger \psi + \dots$$

- terms with more derivatives are at higher orders

- **Non-perturbative features at LO**

- particle-particle scattering (tune  $g$ )

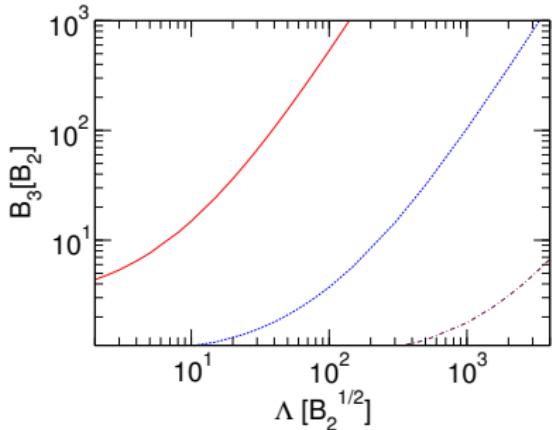


- particle-dimer scattering (tune  $h$ )



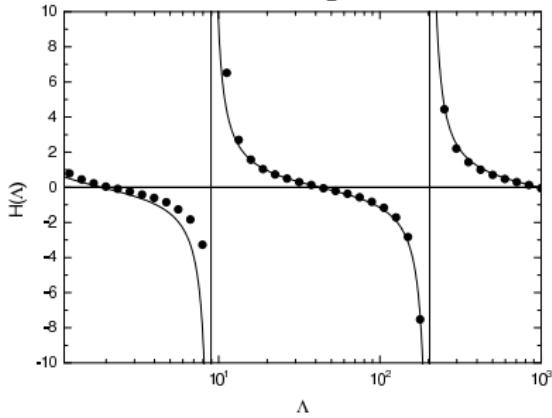
- Without 3BF:

- 3-body spectrum:  
cutoff dependent ( $\Lambda \sim 1/\ell$ )  
Platter '09



- LO 3BF  $h$ :

- tune  $H(\Lambda) = \Lambda^2 h / 2mg^2$ :  
fix one 3-body observable
- limit cycle:  
 $H(\Lambda)$  periodic for  $\Lambda \rightarrow \Lambda(22.7)^n$   
Bedaque *et al.* '00
- Efimov physics



- with perturbation theory:
  - study effective-range corrections ( $r_0/a$ ) to 3-atom systems
  - NLO ( $\sim r_0/a$ ) range effects in 3body Efimov spectrum  
Platter, C.J., Phillips, PRA 79, 022702 (2009)
  - NLO ( $\sim r_0/a$ ) range effects in cold-atom recombination  
C.J., Platter, Phillips, EPL. 92, 13003 (2010)  
C.J., Phillips, Platter, Ann. Phys. 327, 1803 (2012)
  - N<sup>2</sup>LO ( $\sim r_0^2/a^2$ ) range effects on Helium-4 trimer  
C.J., Phillips, Few Body Syst. 54, 2317 (2013)

- include core + valence nucleon d.o.f. e.g.,  ${}^6\text{He} \rightarrow n + n + {}^4\text{He}$ ;  ${}^{11}\text{Be} \rightarrow n + {}^{10}\text{Be}$
- include spin & isospin d.o.f.

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- 2n-halo wave functions:

$$\Psi_x(p, q) = \begin{array}{c} \text{---} \\ | \\ | \\ \text{---} \end{array} + 2 \times \begin{array}{c} \text{---} \\ | \\ | \\ \text{---} \\ | \\ | \\ \text{---} \\ | \\ | \\ \text{---} \end{array}$$

$\mathbf{F}_c$       +       $\mathbf{F}_n$

## EFT in halo nuclei

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$$\Psi_x(p, q) = \text{Diagram A} + 2 \times \text{Diagram B}$$

- Faddeev equation

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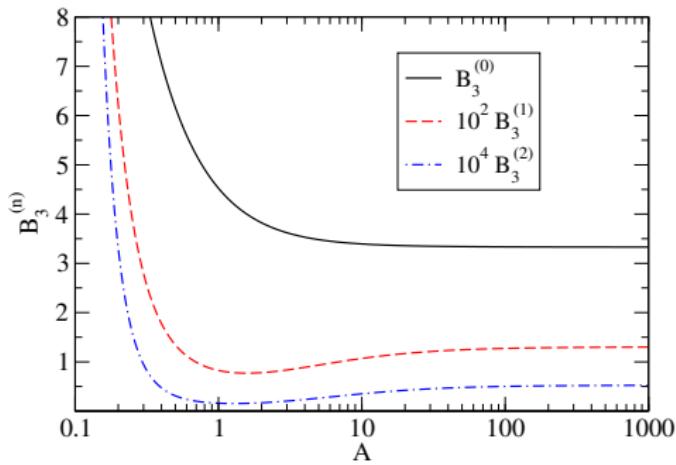
- Faddeev equation + 3-body force

$$n - n \begin{array}{c} \text{---} \\ | \\ \text{F}_c \\ | \\ \text{---} \end{array} = 2 \times \begin{array}{c} \text{---} \\ | \\ \text{F}_n \\ | \\ \text{---} \end{array}$$

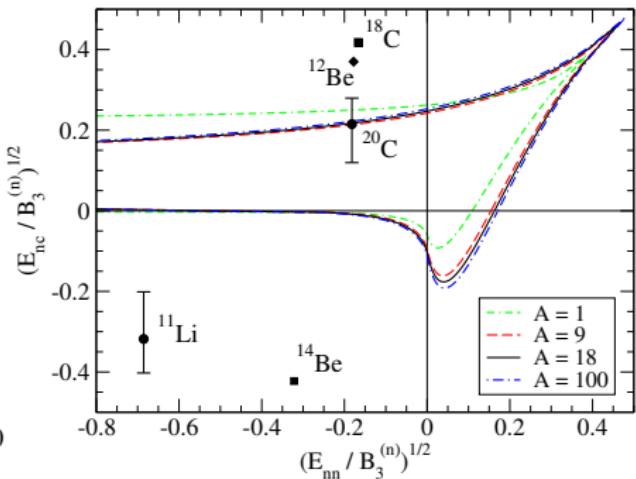
$$c - n = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3}$$

# Universality in $2n$ s-wave halo

- Efimov states in  $a_{nn}, a_{nc} \rightarrow \infty$



- Implication of excited Efimov halo



Canham, Hammer EPJA 2008

- **1n halo and resonant state**

- p-wave resonance:

$^5\text{He}$  [Bertulani *et al.* '02, Bedaque *et al.* '02]

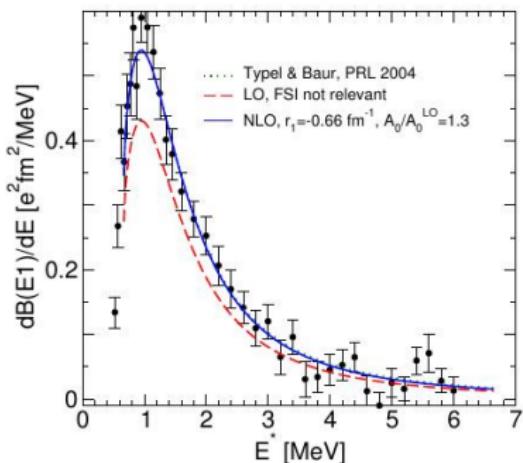
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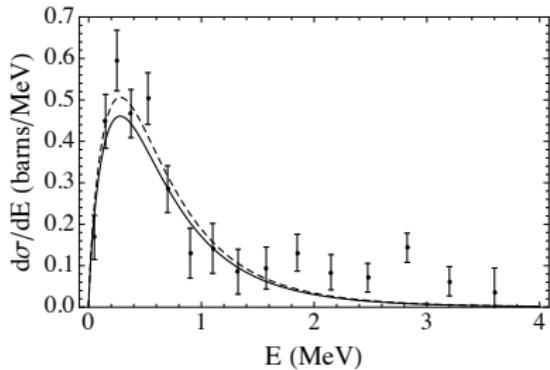
- E1 transition:

$^{11}\text{Be}$  photo-dissociation



[Hammer, Phillips '11]

$^{19}\text{C}$  photo-dissociation



Data: Nakamura *et al.*, RIKEN ('99, '03);  
Calculation: Acharya, Phillips ('13)

- **1p halo**

- $^{17}\text{F}$  [Ryberg, Forssén, Platter '13]

- $p\text{-}\alpha$  scattering [Higa, Bertulani, van Kolck '14]

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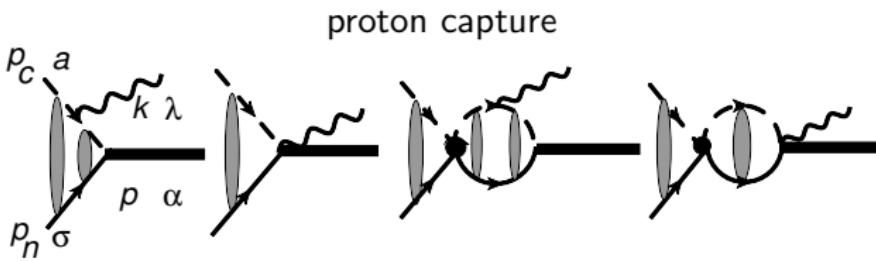
$p\text{-}\alpha$  scattering [Higa, Bertulani, van Kolck '14]

- **radiative nucleon captures**

$^7\text{Li} + n \rightarrow ^8\text{Li} + \gamma$  [Rupak, Higga '11, Zhang, Nollett, Phillips '13]

$^{14}\text{C} + n \rightarrow ^{15}\text{C} + \gamma$  [Rupak, Fernando, Vaghani '12]

$^7\text{Be} + p \rightarrow ^8\text{B} + \gamma$  [Zhang, Nollett, Phillips '14]



- **2-neutron halo:**

- $n$ -core in s-wave virtual/real bound state:  
 $^{11}\text{Li}$ ,  $^{12}\text{Be}$ ,  $^{20}\text{C}$  [Canham, Hammer '08, '10]  
 $^{22}\text{C}$  [Yamashita, Carvalho, Frederico, Tomio '11]  
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 $^{62}\text{Ca}$  [Hagen, Hagen, Hammer, Platter '13]

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- $^6\text{He}$ :  $n$ - $\alpha$  in p-wave resonance
  - EFT + Gamow shell model [Rotureau, van Kolck '12]
  - EFT + Faddeev Equations C.J., Elster, Phillips arXiv:1405.2394 (2014)

	$^{20}\text{C}$	$^{21}\text{C}$	$^{22}\text{C}$
bound/unbound	bound		
ground state	$0^+$		
binding/virtual energy	$S_{2n} = 4.76 \text{ MeV}$ <a href="#">Ozawa et al. '11</a>		
matter radius $r_m$	2.97(5) fm <a href="#">Ozawa et al. '01</a>		

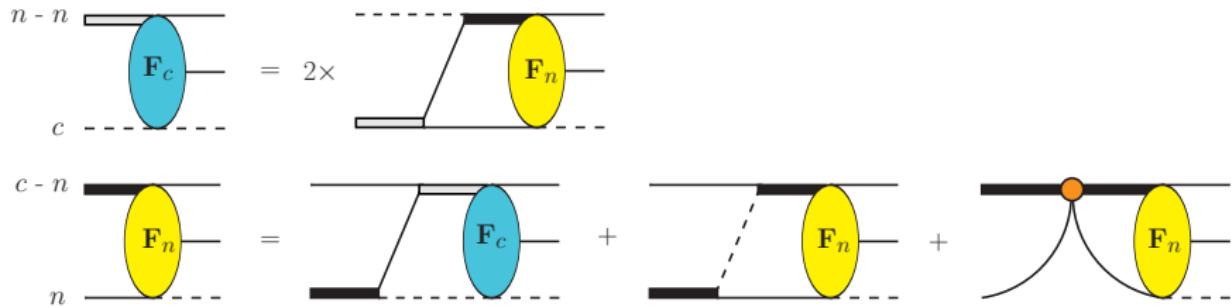
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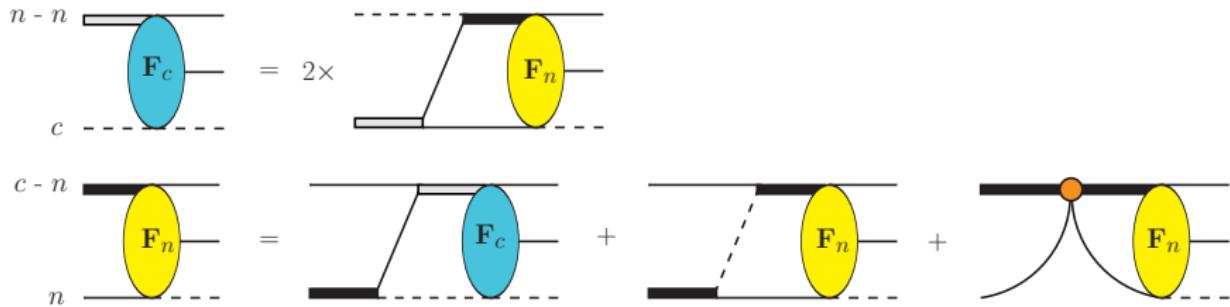
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- Halo EFT: use measured  $^{22}\text{C}$   $r_m$  to put constraints on:
  - $E_{nc}$  of  $^{21}\text{C}$  ( $a < 0$ )
  - $S_{2n}$  of  $^{22}\text{C}$

# $n - n - ^{20}\text{C}$ Faddeev equations

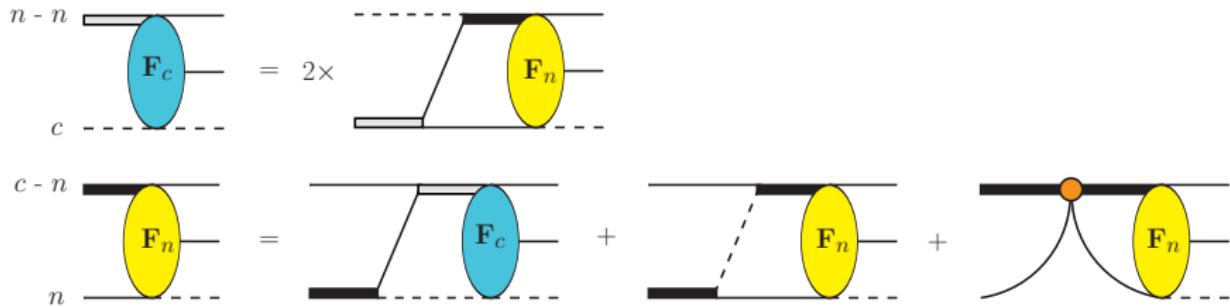


# $n - n - ^{20}\text{C}$ Faddeev equations



- parameterize  $n - n$  and  $n - ^{20}\text{C}$  interactions:
  - 2body virtual energies:  $E_{nn} = 1/(M_n a_{nn}^2)$ ,  $E_{nc}$

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- parameterize  $n - n$  and  $n - ^{20}\text{C}$  interactions:
  - 2body virtual energies:  $E_{nn} = 1/(M_n a_{nn}^2)$ ,  $E_{nc}$
- parameterize  $n - ^{21}\text{C}$  3body contact term
  - fix  $r_m[^{22}\text{C}]$

- two-body form factor

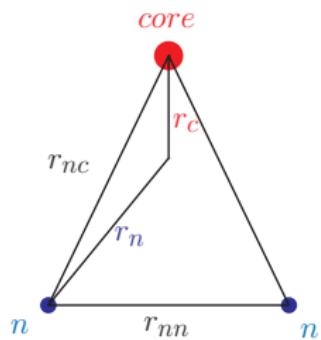
$$\mathcal{F}(k^2) = \int d\mathbf{p} \int d\mathbf{q} \Psi^\dagger(\mathbf{p}, \mathbf{q}) \Psi(\mathbf{p} - \mathbf{k}, \mathbf{q}) \approx 1 - \frac{1}{6} r_{2b}^2 k^2$$

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- matter radius (with point-like core)

$$R = \sqrt{A r_c^2 + 2 r_n^2} / (A + 2)$$



- Define a dimensionless quantity:

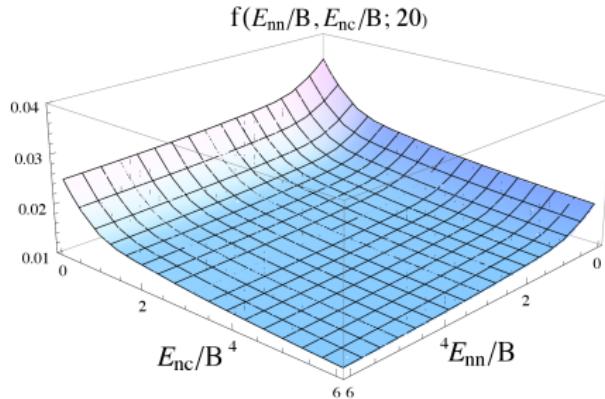
$$M_n B_3 \cdot R^2 \equiv f \left( \frac{E_{nn}}{B_3}, \frac{E_{nc}}{B_3}; A \right)$$

# Universality and matter radii

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- At  $A = 20$ :
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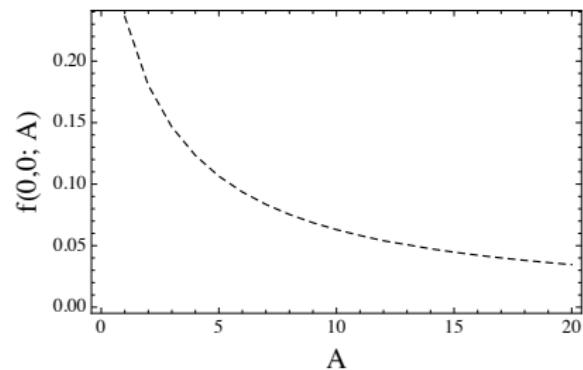
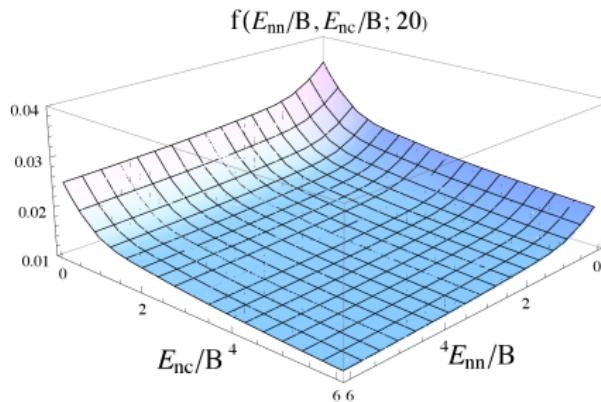
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- At  $A = 20$ :

- a function of  $E_{nn}/B_3$  and  $E_{nc}/B_3$

- Unitary limit  $E_{nn} = E_{nc} = 0$ :
  - a function of  $A$

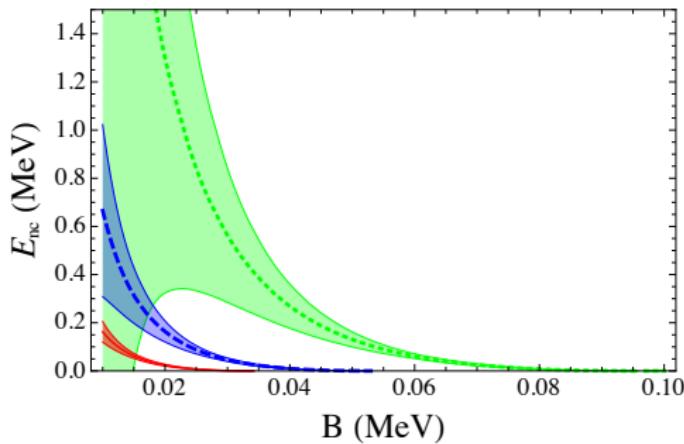


Acharya, C.J., Phillips PLB 2013

# Constraints on $^{21}\text{C}$ and $^{22}\text{C}$

- Include finite size of  $^{20}\text{C}$

$$R^2 = r_m^2[{}^{22}\text{C}] - \frac{20}{22} r_m^2[{}^{20}\text{C}] = (5.4_{-0.9}^{+0.9})^2 - \frac{20}{22} (2.97 \pm 0.05)^2 \text{ fm}^2$$

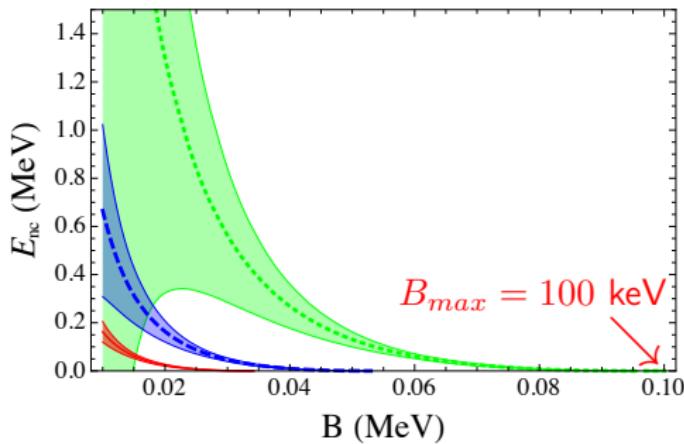


bands: EFT power counting

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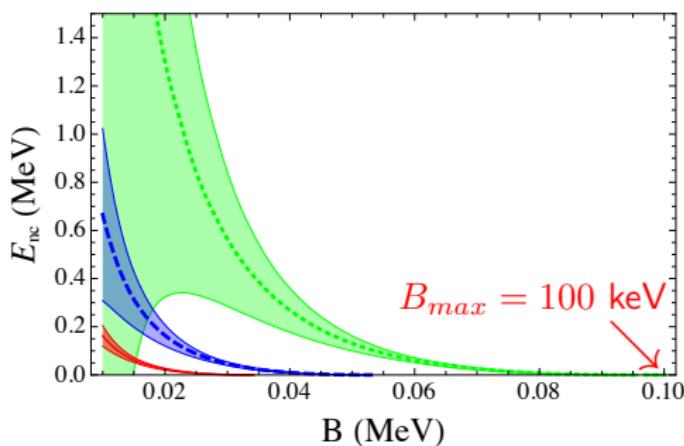


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c.f. Yamashita et al. '11

$\rightarrow B_{max} \sim 120 \text{ keV}$

Fortune & Sherr '12

$\rightarrow B_{max} \sim 220 \text{ keV}$

Gaudefroy et al. '12

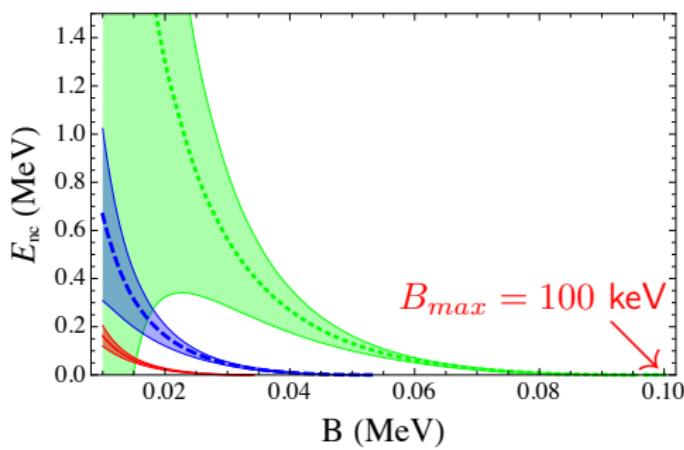
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Gaudefroy et al. '12  
 $\rightarrow B_{max} \sim 320 \text{ keV}$

Mosby et al. '13  
 $a_{nc} < 2.8 \text{ fm} \iff E_{nc} > 2.9 \text{ MeV}$   
 $\rightarrow B_{max} < 20 \text{ keV}$   
inconsistency in measurements?

- **experiment in  ${}^6\text{He}$**

- matter radius Tanihata *et al.* '92, Alkhazov *et al.* '97, Kislev *et al.* '05
- charge radius Wang *et al.* '04, Mueller *et al.* '07
- ${}^6\text{He}$  mass Brodeur *et al.* '12

- **cluster model**

- separable potential Ghovanlou, Lehman '74
- density-dependent  $nn$  contact interaction Esbensen *et al.* '97

- ***ab initio* calculation**

- no-core shell model Navrátil *et al.* '01
- hyperspherical harmonics Bacca *et al.* '12
- Green's function Monte Carlo Pieper *et al.* '01

- **halo EFT**

- explore **universal physics** in halo nuclei
- compare **predictions** with experiments and other theories

- $n\alpha$  interaction is dominated by the  $^2P_{\frac{3}{2}}$  state



A Feynman diagram showing the interaction between a neutron ( $n$ ) and an alpha particle ( $\alpha$ ). The neutron is represented by a solid line labeled  $n$ , and the alpha particle by a dashed line labeled  $\alpha$ . They interact via a central vertex connected to a horizontal black bar representing the exchange of a virtual particle.

$$= \frac{1}{4\pi^2\mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1}$  Arndt et al. '73

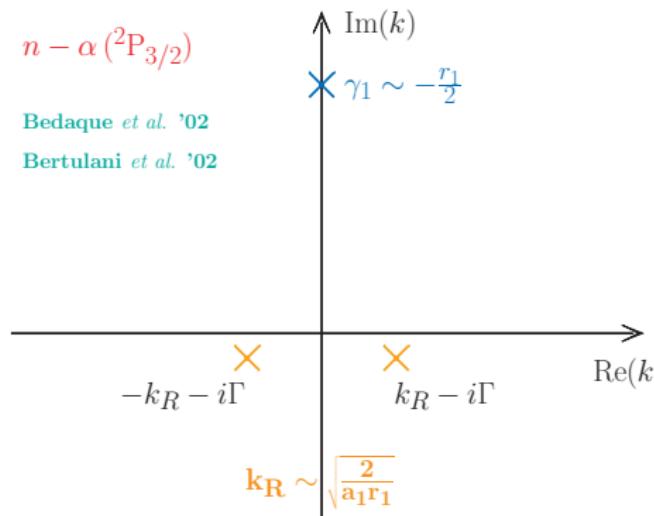
- $n\alpha$  interaction is dominated by the  $^2P_{\frac{3}{2}}$  state

$$\begin{array}{c} n \\ \backslash \quad / \\ \text{---} \quad \text{---} \\ / \quad \backslash \\ \alpha \end{array} = \frac{1}{4\pi^2\mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1} \text{ Arndt et al. '73}$$

- $r_1 \neq 0$  Nishida '12

- $n\alpha$  EFT power counting: Bedaque, Hammer, van Kolck '02
  - $a_1 \sim 1/(Q^2\Lambda)$   $r_1 \sim \Lambda$
  - $Q/\Lambda \sim 0.15$
- ${}^2P_{\frac{3}{2}}$  :
  - shallow resonance:  
 $k_R \sim Q, \quad \Gamma \sim Q^2/\Lambda$
  - deep bound state:  $\gamma_1 \sim \Lambda$



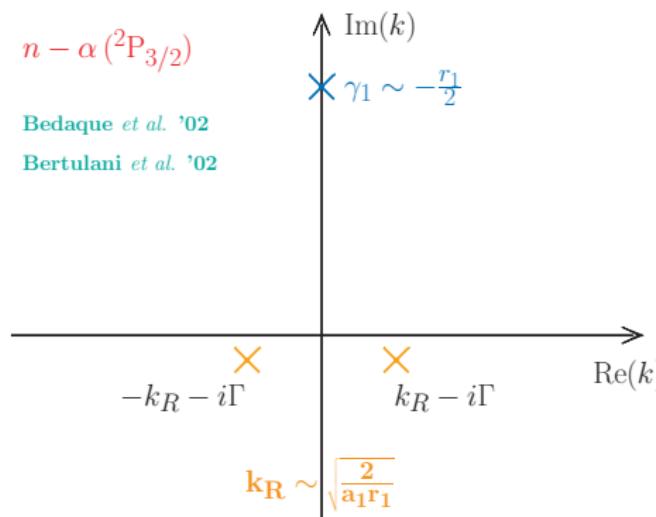
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- ${}^2P_{\frac{3}{2}}$  :
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- LO  $n\alpha$  t-matrix ( $ik^3 \rightarrow 0$ )

$$\begin{aligned} t_{n\alpha} &= \frac{1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2} \\ &= \frac{-1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{\gamma_1 (k^2 - k_R^2)} \end{aligned}$$



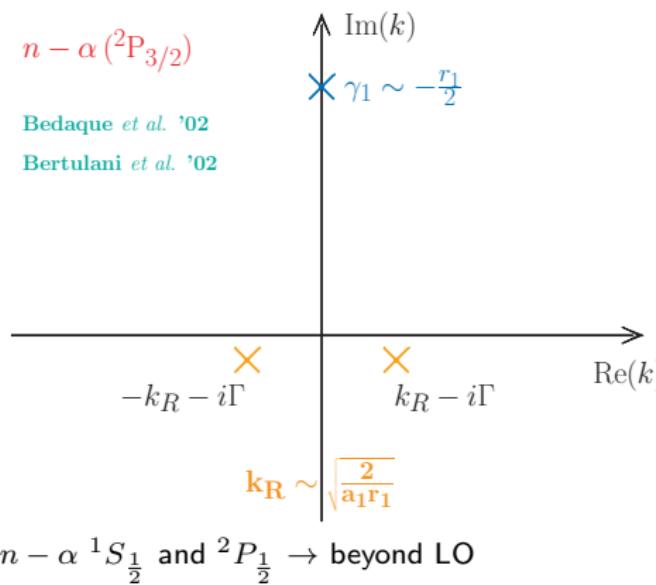
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$n - \alpha {}^1S_{\frac{1}{2}}$  and  ${}^2P_{\frac{1}{2}} \rightarrow$  beyond LO

# $^6\text{He}$ in Jacobi coordinates

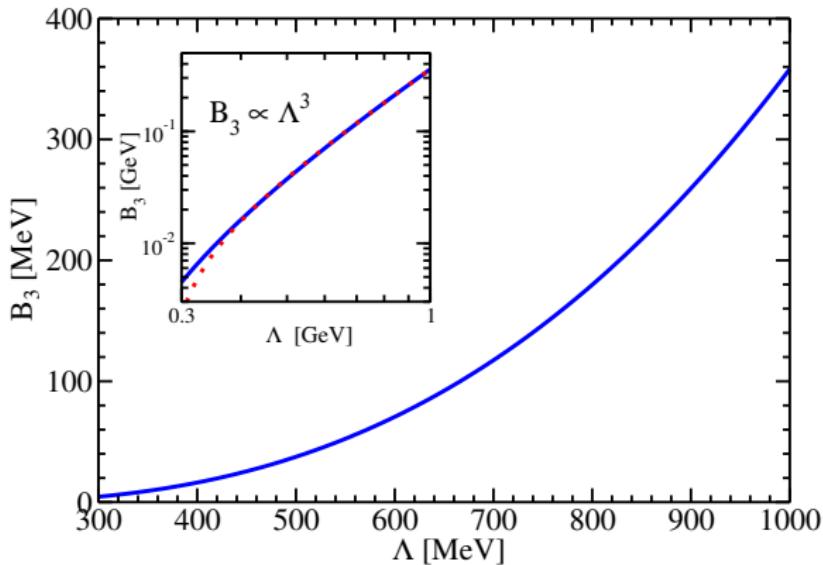
- Jacobi-momentum



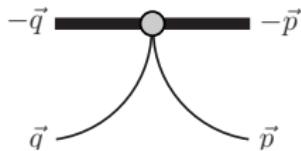
## spin-orbit coupling for $^6\text{He}$ ( $J = 0^+$ )

pair, spec	pair	spectator	total $L, S$	total $J$
$nn, \alpha$	$\ell = 0, s_1 = 0$	$\lambda = 0, s_2 = 0$	$L = 0, S = 0$	$J = 0^+$
$n\alpha, n$	$\ell = 1, s_1 = \frac{1}{2}$	$\lambda = 1, s_2 = \frac{1}{2}$	$L = 0, S = 0$ $L = 1, S = 1$	

- without  $nna$  3-body force:
  - $S_{2n}$  is strongly cutoff dependent:  $S_{2n} \sim \Lambda^3$  ← need 3body force!

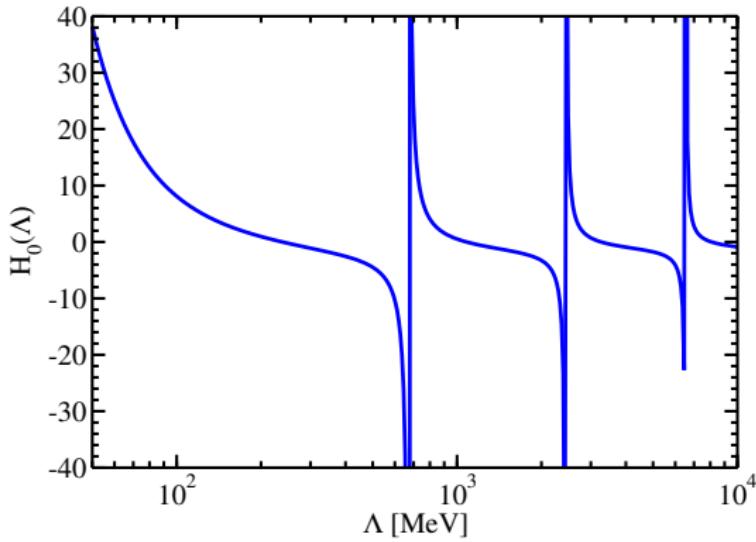


- p-wave 3BF:



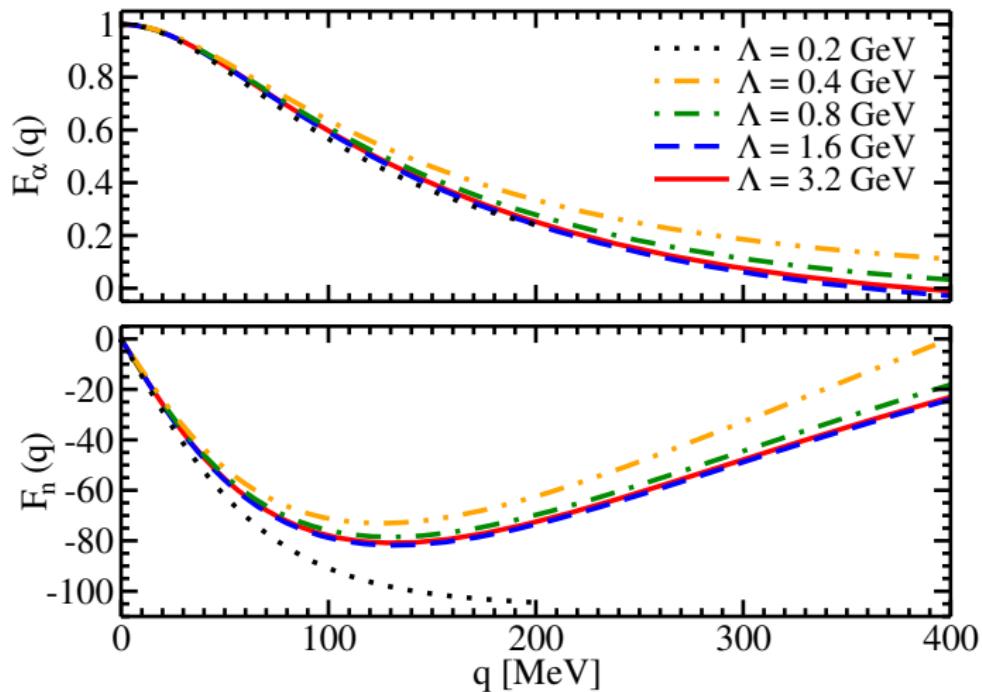
$$= M_n \textcolor{red}{q} \textcolor{red}{p} \frac{H(\Lambda)}{\Lambda^2}$$

- reproduce  
 $S_{2n} = 0.973 \text{ MeV}$
- log oscillation
- No limit cycle  
(c.f. 3-body in S-wave)



# Renormalized Faddeev Components

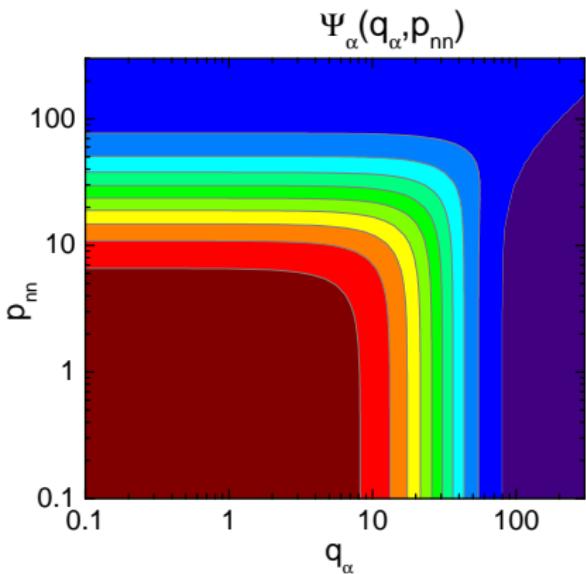
$F_\alpha(\alpha, nn)$  and  $F_n(n, \alpha n)$ :  $\Lambda$  independent



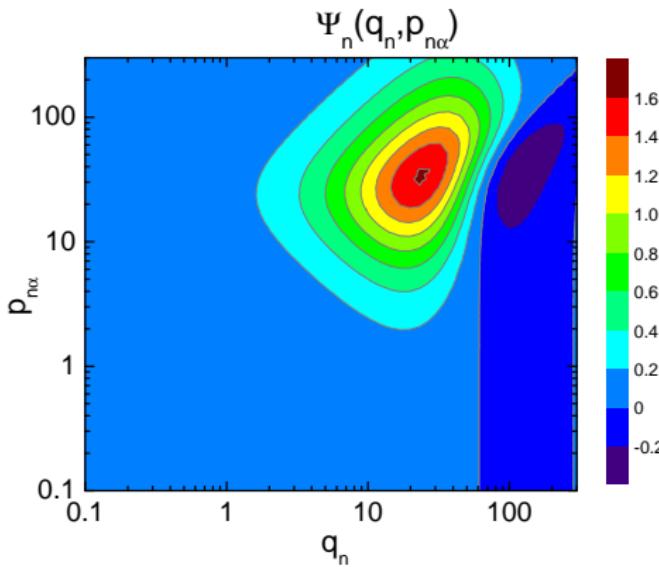
C.J., Elster, Phillips arXiv:1405.2394 (2014)

# $^6\text{He}$ ground-state wave function

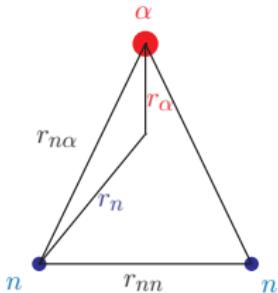
- $|\Psi\rangle$  in  $\alpha - nn$  basis



- $|\Psi\rangle$  in  $n - \alpha n$  basis



momenta in units of MeV



[ Preliminary ]

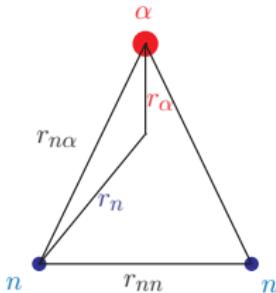
$$r_{nn} = 3.77 \text{ fm} \pm 20\%$$

$$r_{n\alpha} = 4.33 \text{ fm} \pm 20\%$$

- $\pm 20\%$  uncertainty is from higher-order EFT

	EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—
	$r_m[\alpha]$	1.455(1)

He-6	—
------	---



[ Preliminary ]

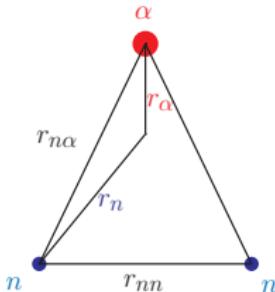
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	EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—
	$r_m[\alpha]$	1.455(1)
He-6	$r_\alpha$	1.30(26)
	$r_n$	3.21(64)

# Matter and point-proton radii



[ Preliminary ]

$$r_{nn} = 3.77 \text{ fm} \pm 20\%$$

$$r_{n\alpha} = 4.33 \text{ fm} \pm 20\%$$

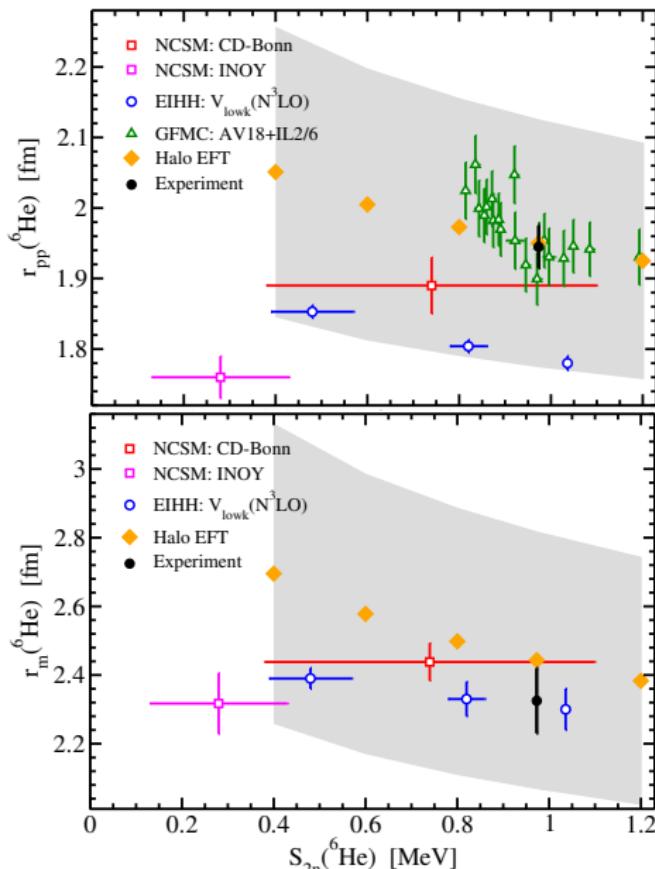
- $\pm 20\%$  uncertainty is from higher-order EFT

	EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—
	$r_m[\alpha]$	1.455(1)
He-6	$r_\alpha$	1.30(26)
	$r_n$	3.21(64)
	$r_{pp}[{}^6\text{He}] = \sqrt{r_\alpha^2 + r_{pp}^2[\alpha]}$	1.95(17)
	$r_m[{}^6\text{He}] = \sqrt{\frac{1}{3}(2r_\alpha^2 + r_n^2 + 2r_m^2[\alpha])}$	2.44(37)
		1.938(23), 1.953(22)
		2.33(4), 2.30(7), 2.37(5)

# compare with theory and experiment

[ Preliminary ]

● He-6 point-proton radius



● He-6 matter radius

c.f. Bacca, Barnea & Schwenk, '12

- Halo nuclei are ideal places to study universal physics
  - $^{22}\text{C}$ :  $n$ -core in s-wave resonance
  - $^6\text{He}$ :  $n$ -core in p-wave resonance
- with halo EFT analysis:
  - s- / p-wave 3body force is needed for proper renormalization
  - predict probability density distribution, matter radii, point-proton radii
  - use universality to constrain experiments & other theories
- further work
  - study  $^6\text{He}$   $2^+$  resonance state
  - extension to  $^{11}\text{Li}$
  - higher-order effects: range corrections, more partial waves, ...