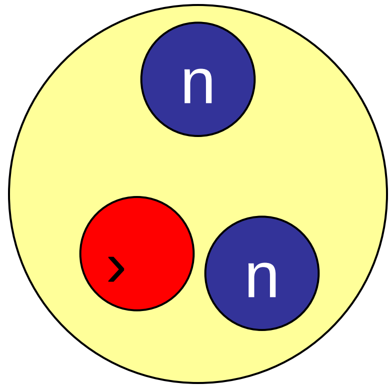
Four-body calculations of ^{4}He tetramer and light hypernuclei using realistic two-body potentials

E. Hiyama (RIKEN)

Outline of my talk

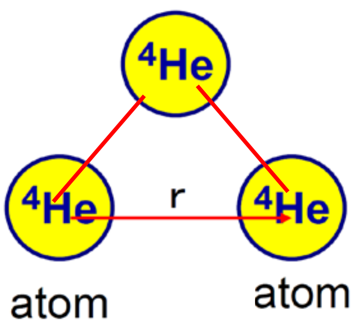
(1) Introduction

(2)

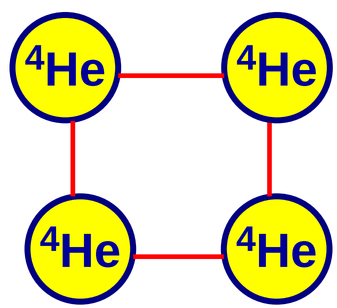


› ^{3}n

(2)



In 3-dimension and 2-dimension world



LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential

Introduction

Many of important subjects in physics come finally to solving few-body (mainly 3- and 4-body) Schrödinger equations accurately.

By solving the equations, we can predict various observables before measurements and can obtain new understandings by analyzing the experimental data.

For this purpose, it is necessary to develop any accurate calculation-method for few-body problems and apply it to many subjects in various fields such as nuclear physics and atomic physics.

Our few-body caluclational method

Gaussian Expansion Method (GEM) , since 1987

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

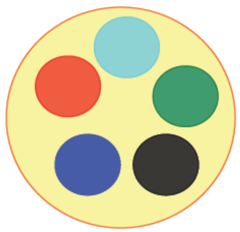
Exotic atoms / molecules ,	Light hypernuclei,
3- and 4-nucleon systems,	3-quark systems,
multi-cluster structure of light nuclei,	

Advantage of this method: We can treat

1) any kinds of particles (nucleon, electron, quark, ….)

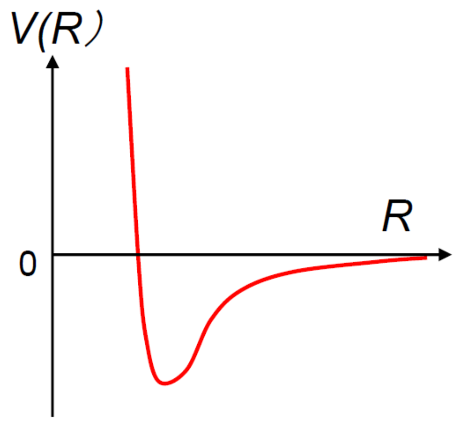
3-body problem

4-body problem



Currently, 5-body problem

2) any types of interactions (even if they have strong short-range correlations, spin-dependence or momentum dependence, ……).



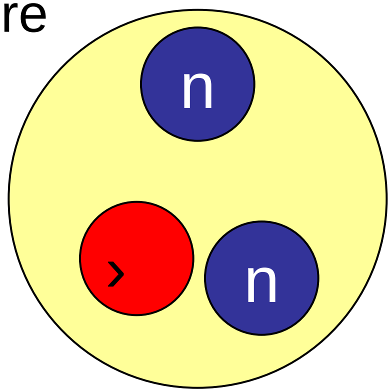
Few-nucleon systems and hypernuclear physics has been encouraging my method to develop to the above treatments.

Especially, to treat potential to have high repulsive core and long range tail is interesting subject for me.

For this purpose, hypernuclear physics provide us many challenging subjects.

In hypernuclear physics, we have realistic interactions such as Nijmegen model (Nijmegen soft core 97, Extended soft core 08, etc)

・To have high repulsive core

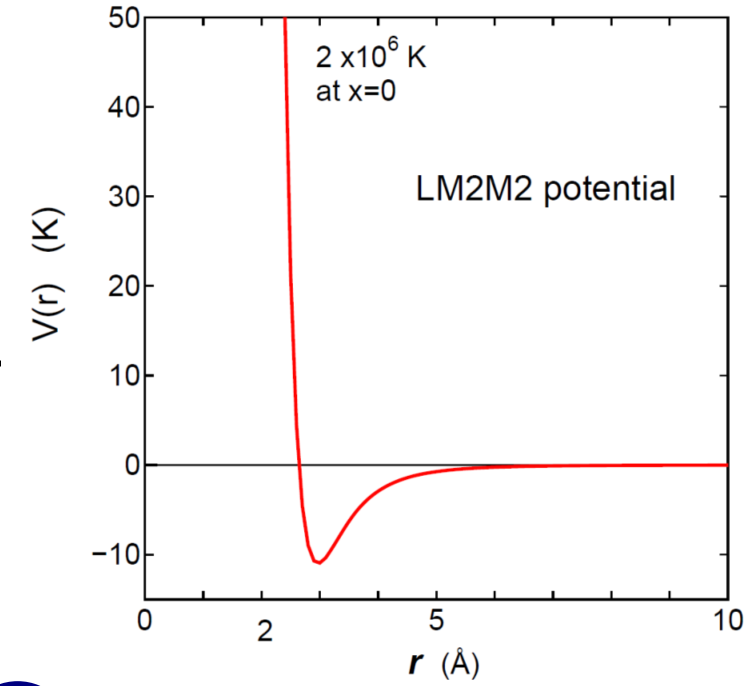


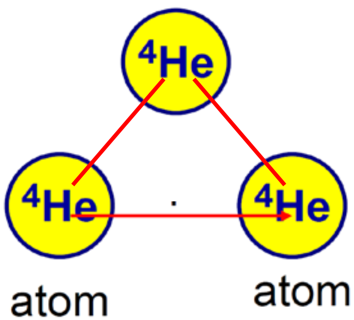
$^{3}_{›}\mathrm{n}$

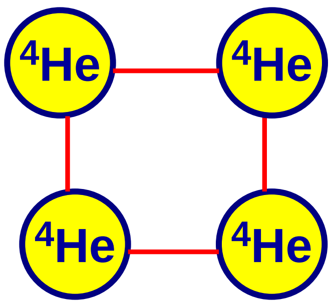
Another interesting subject is
to solve bound states in ^{4}He (^{3}He) trimer and tetramer systems.

The potential between two ^{4}He has high repulsive core and long-ranged tail.

To solve these systems encourages us to develop our method, Gaussian Expansion Method.







Outline of my talk

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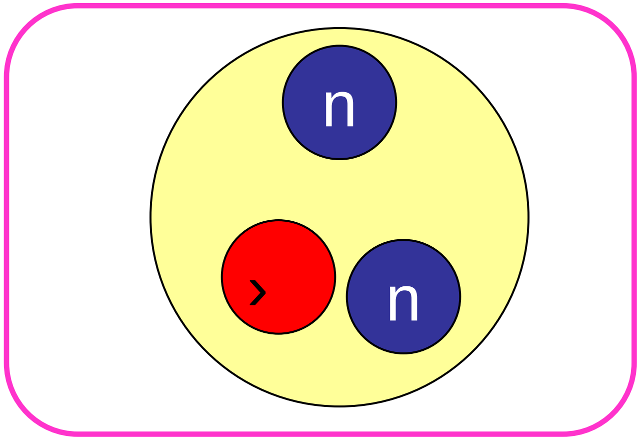
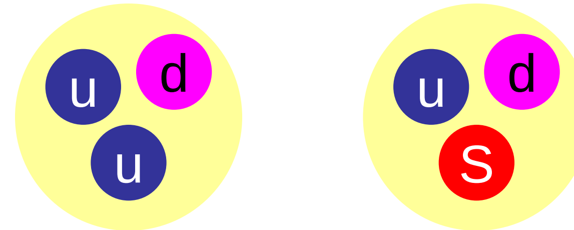
(2)



› ^{3}n

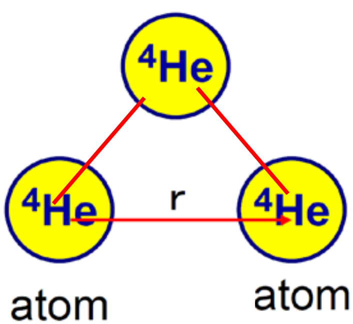


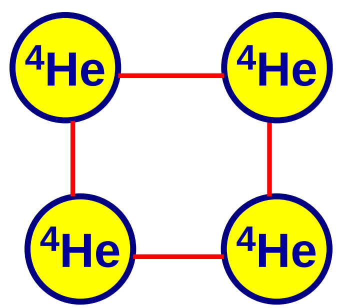
neutron

›

In 3-dimension and 2-dimension world

(2)





LM2M2 potential
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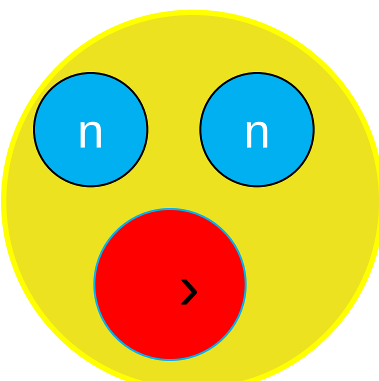
Section 2

three-body calculation of ${}^{3}_{\Lambda}n$



E. Hiyama, S. Ohnishi,
B.F. Gibson, and T. A. Rijken,
The paper will be publised in
PRC as a Rapid communication soon.

Major goals of hypernuclear physics

1) To understand baryon-baryon interactions

2) To study the structure of multi-strangeness systems

In order to understand the baryon-baryon interaction, two-body scattering experiment is most useful.

Total number of
Nucleon (N) -Nucleon (N) data: 4,000

→ Study of NN intereaction has been developed.

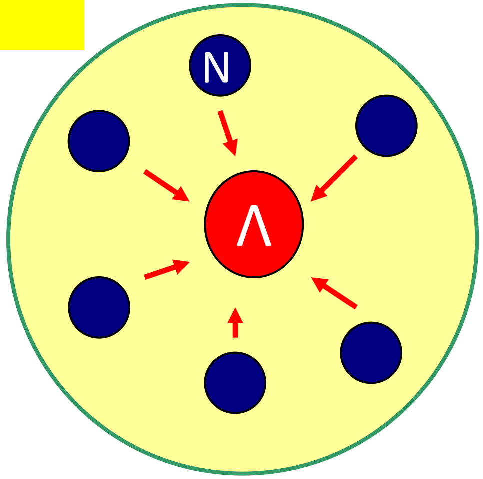
↕

- Total number of differential cross section
  Hyperon (Y) -Nucleon (N) data: 40
- NO YY scattering data

→ YN and YY potential models so far proposed (ex. Nijmegen, Julich, Kyoto-Niigata) have large ambiguity.

since it is difficult to perform YN scattering experiment even at J-PARC.

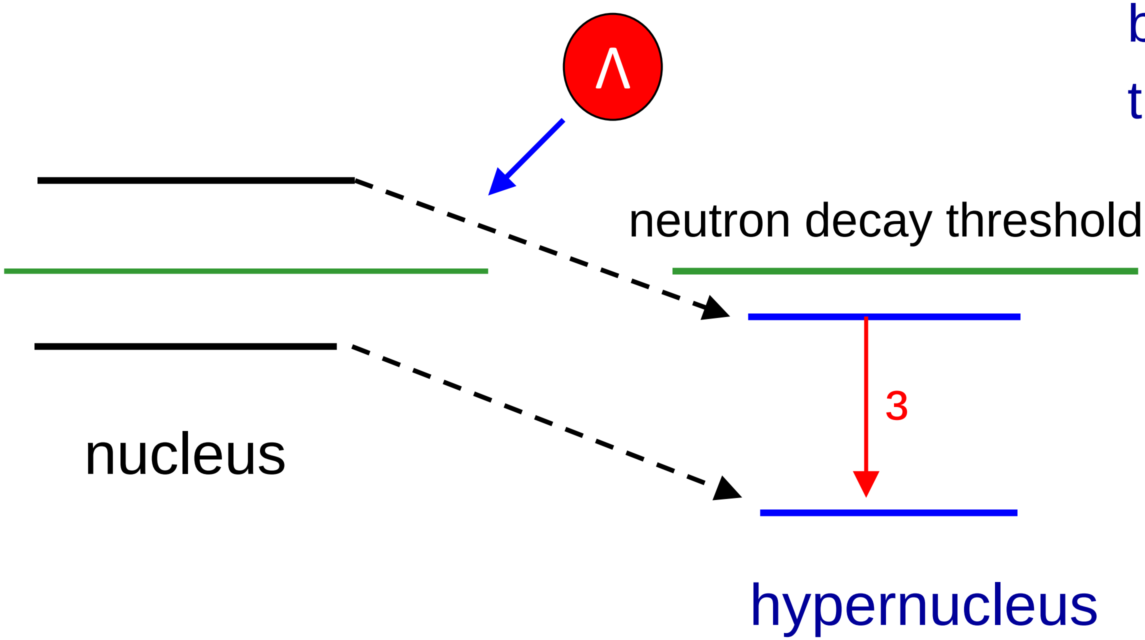
No Pauli principle
Between N and Λ



Hypernucleus

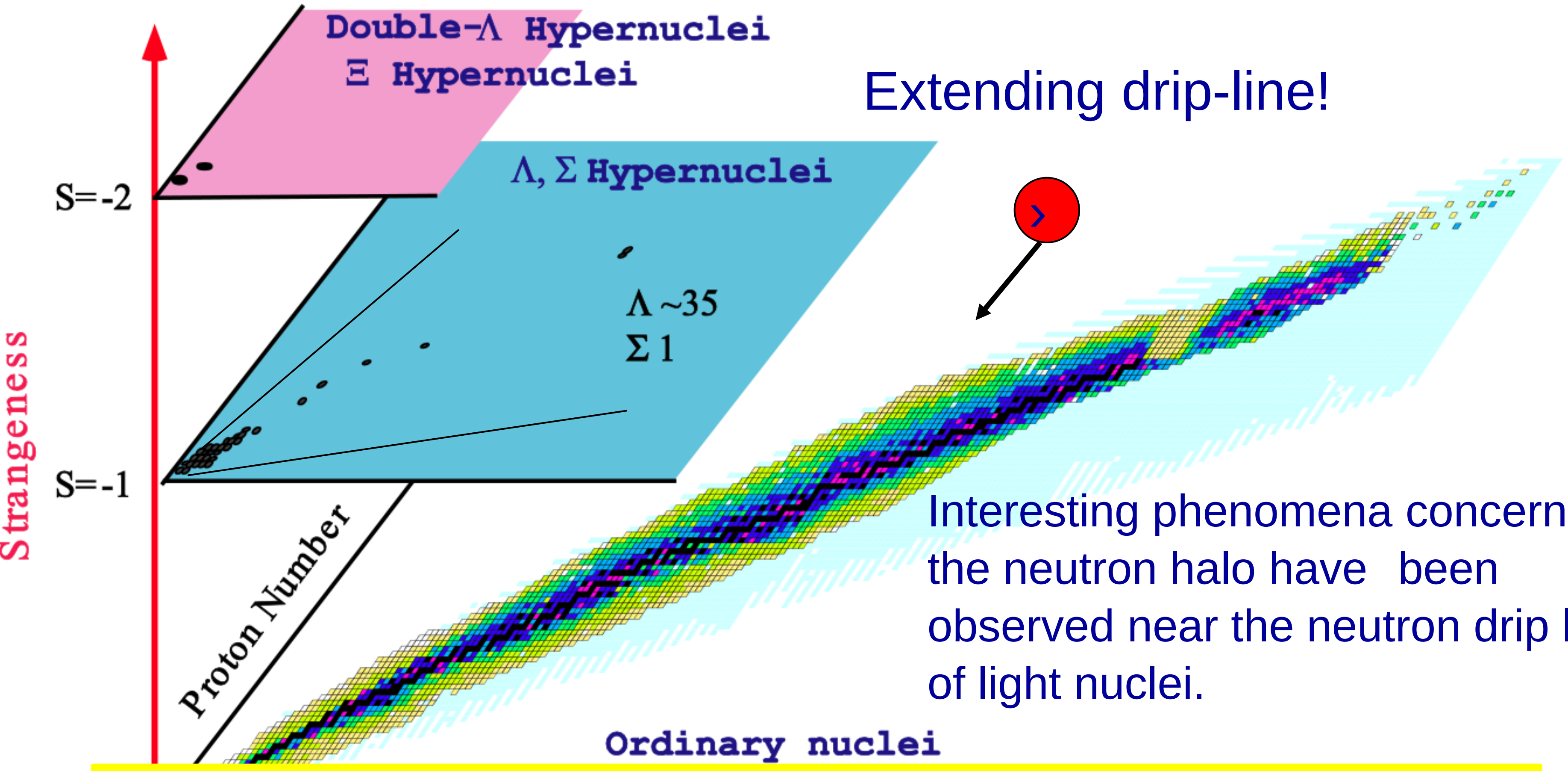
Λ particle can reach deep inside, and attract the surrounding nucleons towards the interior of the nucleus.

Due to the attraction of › N interaction, the resultant hypernucleus will become more stable against the neutron decay.



Nuclear chart with strangeness

Multi-strangeness system
such as Neutron star



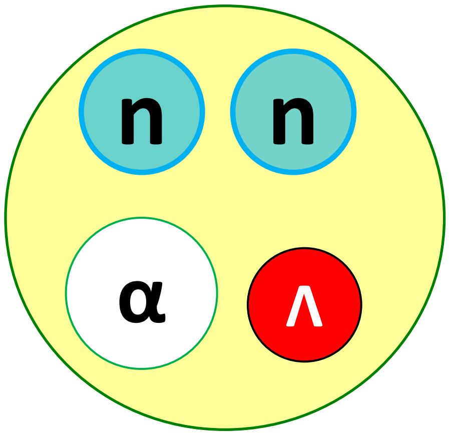
Extending drip-line!

Interesting phenomena concerning the neutron halo have been observed near the neutron drip line of light nuclei.

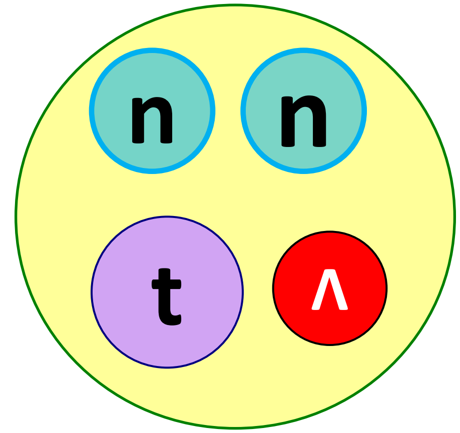
How is structure change when a › particle is injected into neutron-rich nuclei ?

Question : How is structure change when a Λ particle is injected into neutron-rich nuclei?

$^{7}_{\Lambda}$He

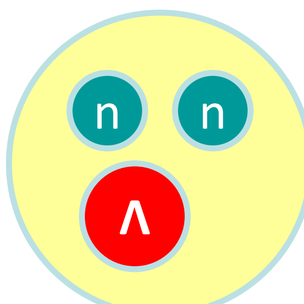


$^{6}_{\Lambda}$H



Observed at JLAB, Phys. Rev. Lett. **110**, 12502 (2013).

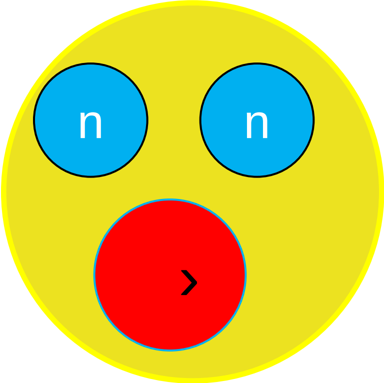
Observed by FINUDA group, Phys. Rev. Lett. **108**, 042051 (2012).



$^{3}_{\Lambda}$n

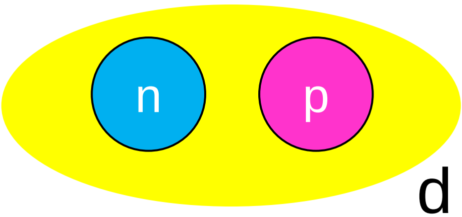
C. Rappold et al., HypHI collaboration
Phys. Rev. C 88, 041001 (R) (2013)

What is interesting to study nn› system?



S=0

The lightest nucleus to have a bound state is deuteron.



d

n+p threshold

J=1⁺

-2.22 MeV

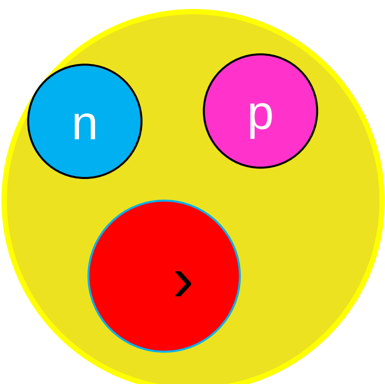
Exp.

S=-1 (›   hypernucear sector)

n+p+›

Lightest hypernucleus to have a bound state

d+›



$_{›}^{3}$H (hyper-triton)

0.13 MeV

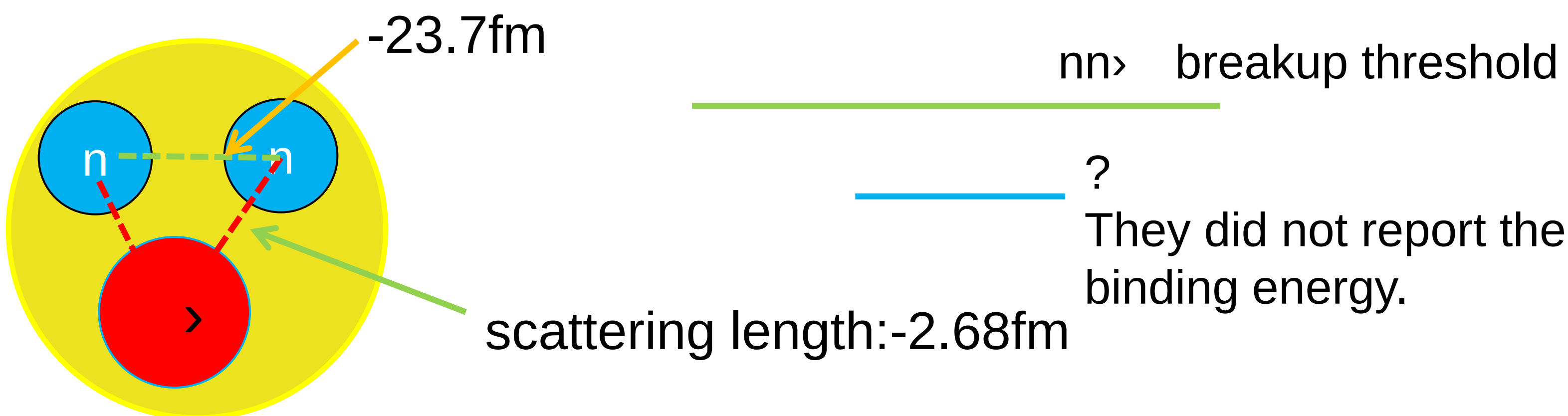
J=1/2⁺

Exp.

PHYSICAL REVIEW C 88, 041001(R) (2013)

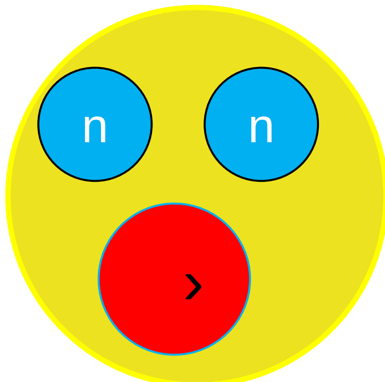
Search for evidence of $^{3}_{\Lambda}n$ by observing $d + \pi^-$ and $t + \pi^-$ final states in the reaction of ^{6}Li + ^{12}C at 2*A* GeV

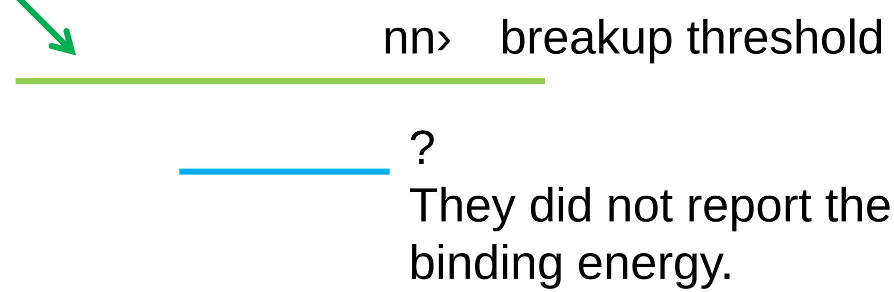
C. Rappold,[1,2,*] E. Kim,[1,3] T. R. Saito,[1,4,5,†] O. Bertini,[1,4] S. Bianchin,[1] V. Bozkurt,[1,6] M. Kavatsyuk,[7] Y. Ma,[1,4] F. Maas,[1,4,5] S. Minami,[1] D. Nakajima,[1,8] B. Özel-Tashenov,[1] K. Yoshida,[1,5,9] P. Achenbach,[4] S. Ajimura,[10] T. Aumann,[1,11] C. Ayerbe Gayoso,[4] H. C. Bhang,[3] C. Caesar,[1,11] S. Erturk,[6] T. Fukuda,[12] B. Göküzüm,[1,6] E. Guliev,[7] J. Hoffmann,[1] G. Ickert,[1] Z. S. Ketenci,[6] D. Khaneft,[1,4] M. Kim,[3] S. Kim,[3] K. Koch,[1] N. Kurz,[1] A. Le Fèvre,[1,13] Y. Mizoi,[12] L. Nungesser,[4] W. Ott,[1] J. Pochodzalla,[4] A. Sakaguchi,[9] C. J. Schmidt,[1] M. Sekimoto,[14] H. Simon,[1] T. Takahashi,[14] G. J. Tambave,[7] H. Tamura,[15] W. Trautmann,[1] S. Voltz,[1] and C. J. Yoon[3]
(HypHI Collaboration)



Observation of nn› system (2013)
Lightest hypernucleus to have a bound state
Any two-body systems are unbound.=>nn› system is bound.
Lightest Borromean system.

Theoretical important issue:
Do we have bound state for nn› system?
If we have a bound state for this system, how much is binding energy?



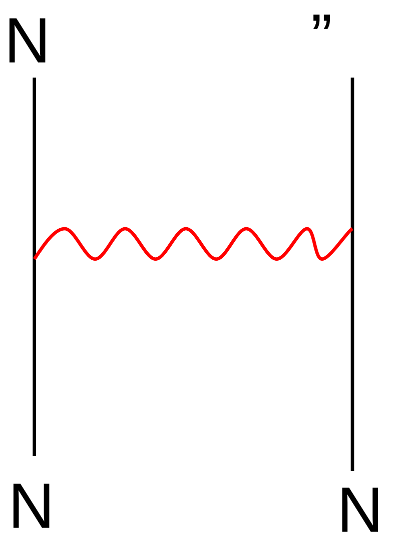


NN interaction : to reproduce the observed binding energies of ^3H and ^3He

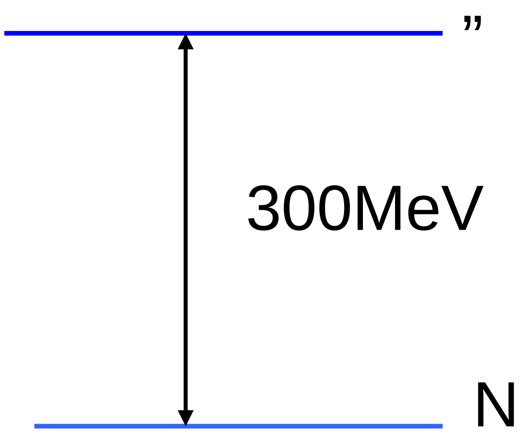
NN: AV8 potential
We do not include 3-body force for nuclear sector.

How about YN interaction?

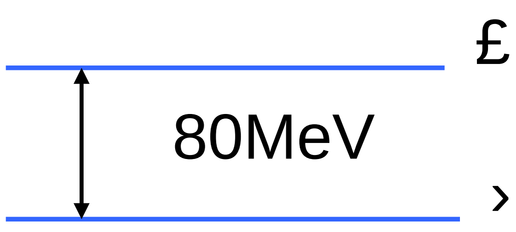
Non-strangeness nuclei

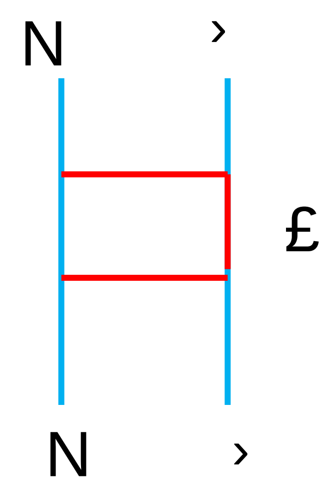


Nucleon can be converted into ” .
However, since mass difference between nucleon and ” is large, then probability of ” in nucleus is not large.

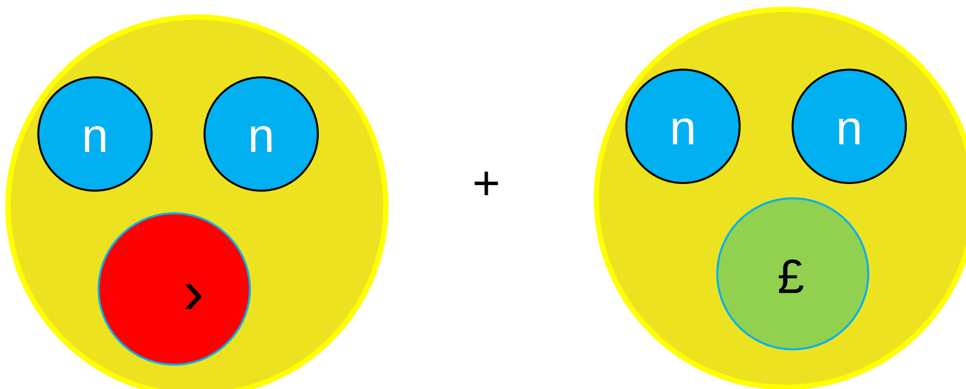


On the other hand, the mass difference between › and £ is much smaller, then › can be converted into £ particle easily.



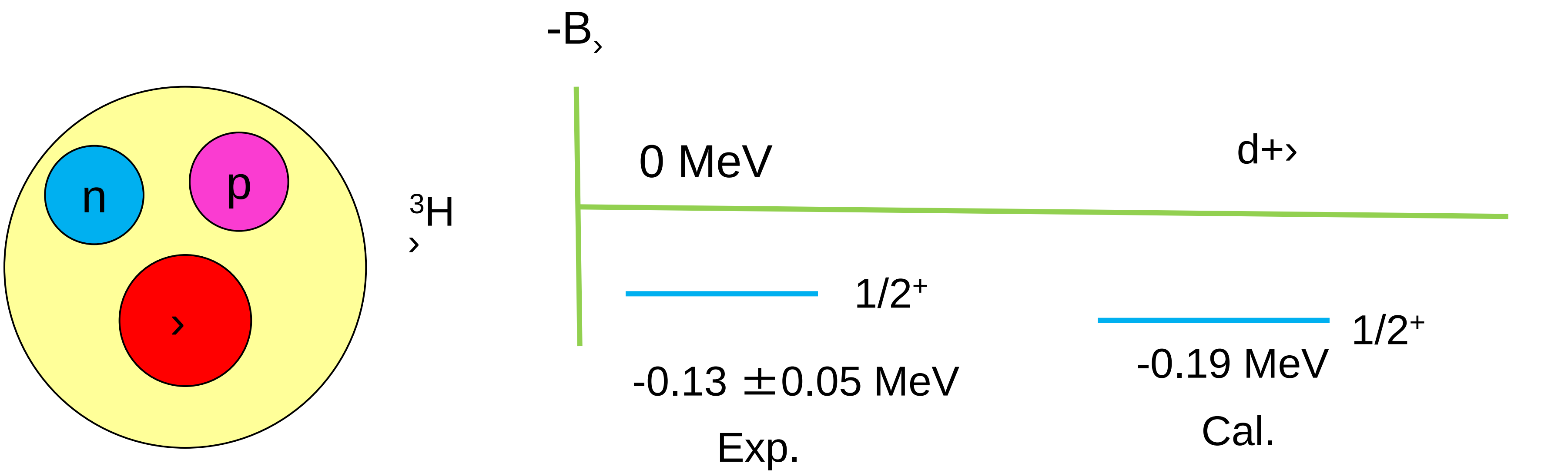


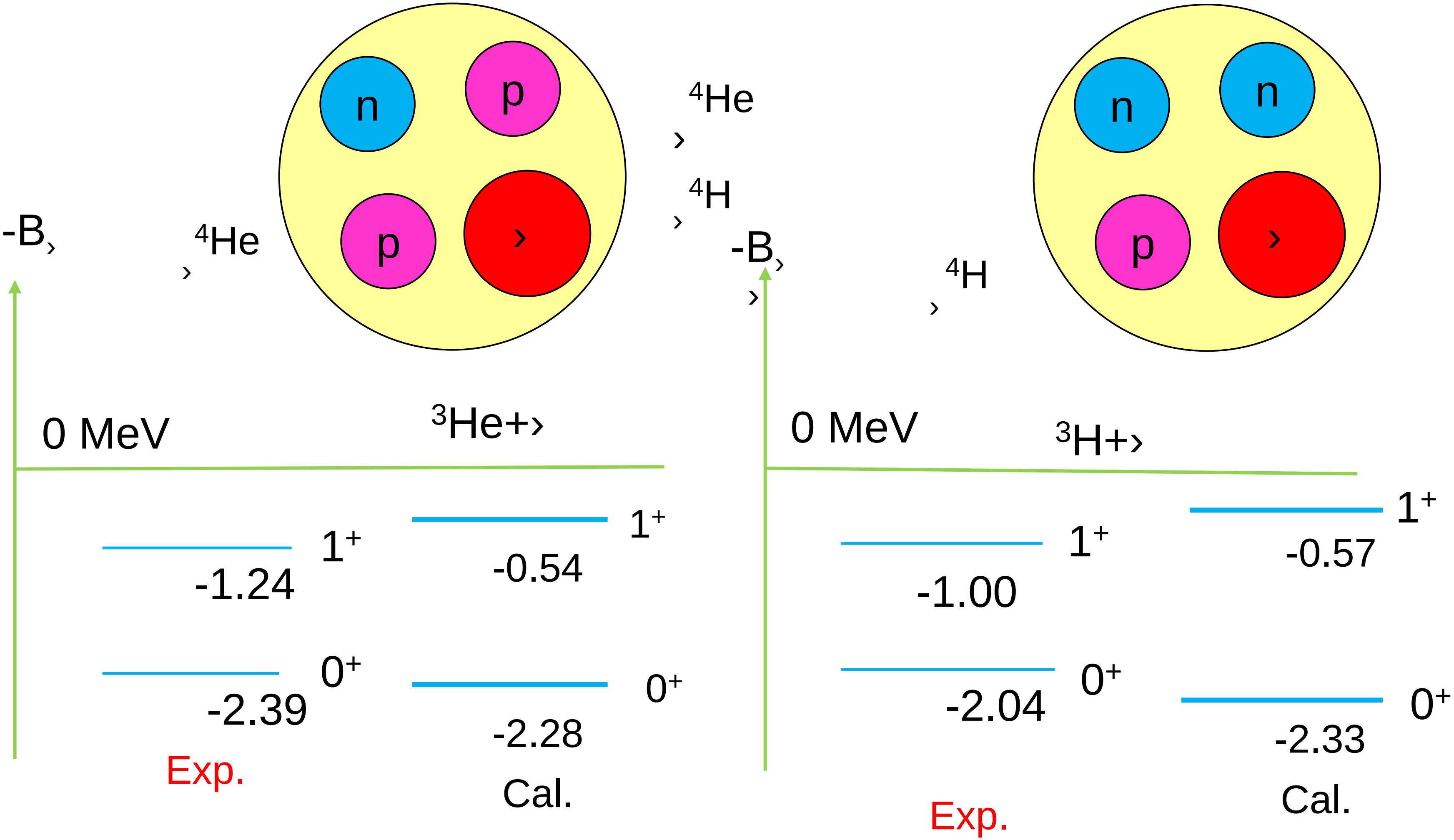
To take into account of › particle to be converted into £ particle, we should perform below calculation using realistic hyperon(Y)-nucleon(N) interaction.



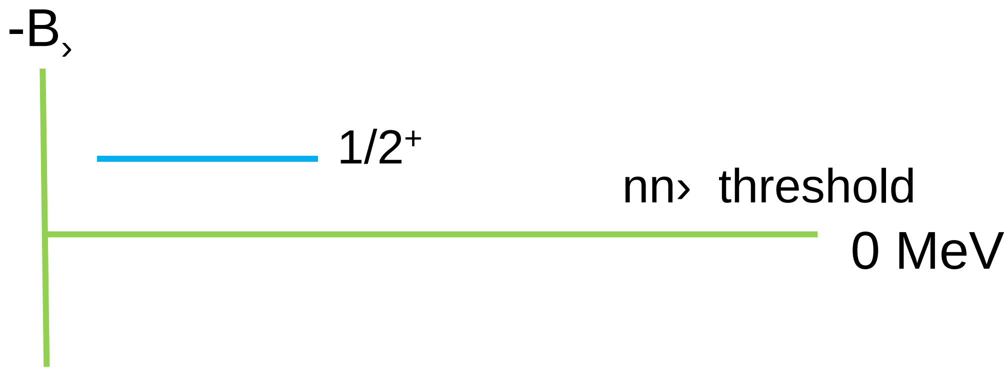
YN interaction: Nijmegen soft core '97f potential (NSC97f) proposed by Nijmegen group

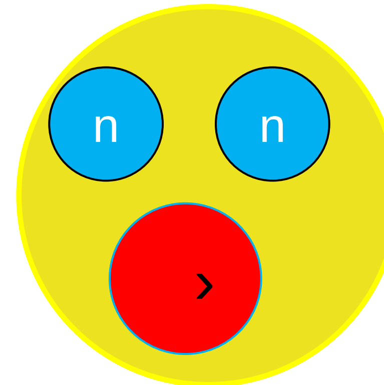
reproduce the observed binding energies of ${}^{3}_{›}$H, ${}^{4}_{›}$H and ${}^{4}_{›}$He


n
p
›
³H
›
-B›
0 MeV
d+›
1/2⁺
-0.13 ±0.05 MeV
Exp.
1/2⁺
-0.19 MeV
Cal.



What is binding energy of nn› ?





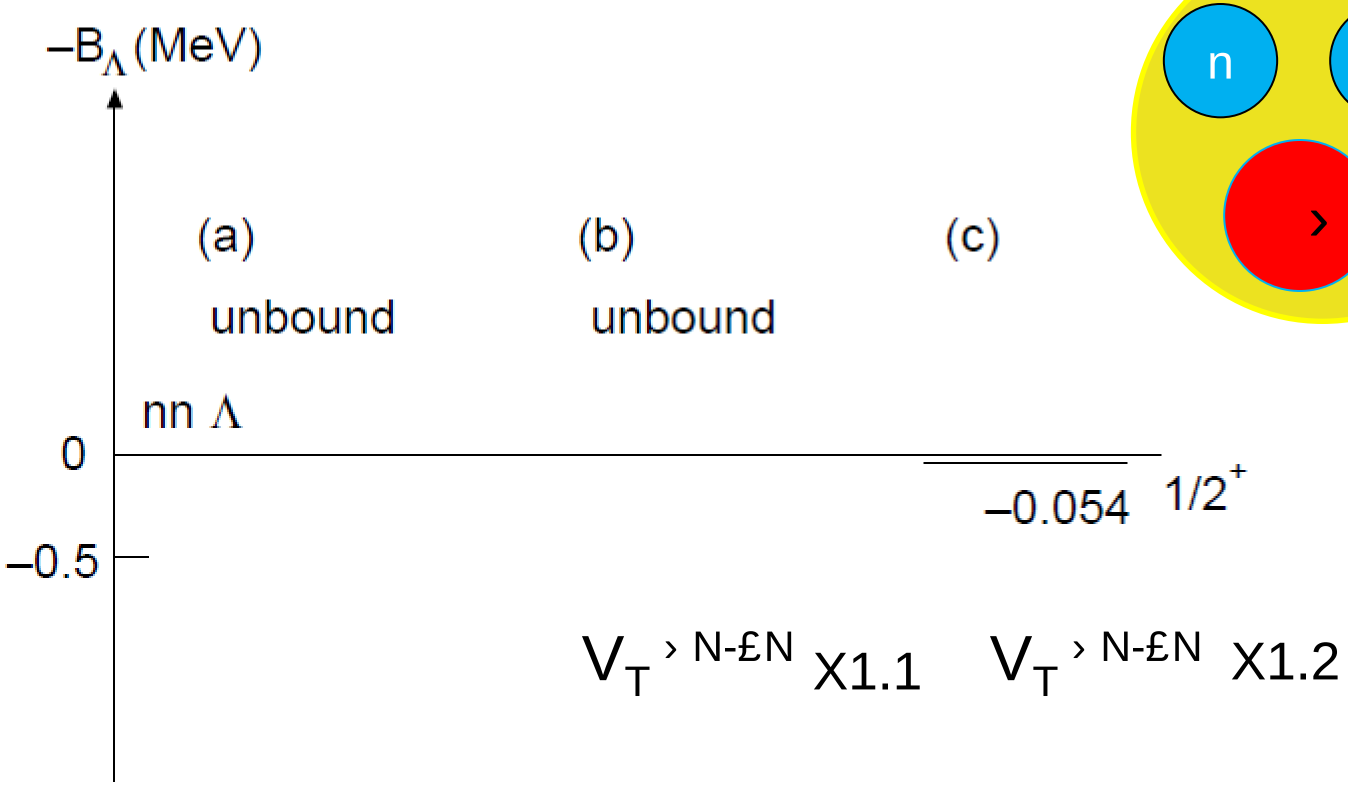
We have no bound state in nn› system.
This is inconsistent with the data.

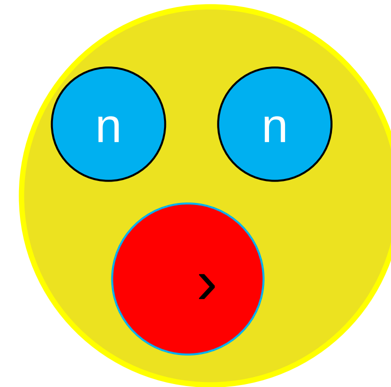
Now, we have a question.

Do we have a possibility to have a bound state in nn› system tuning strength of YN potential ?

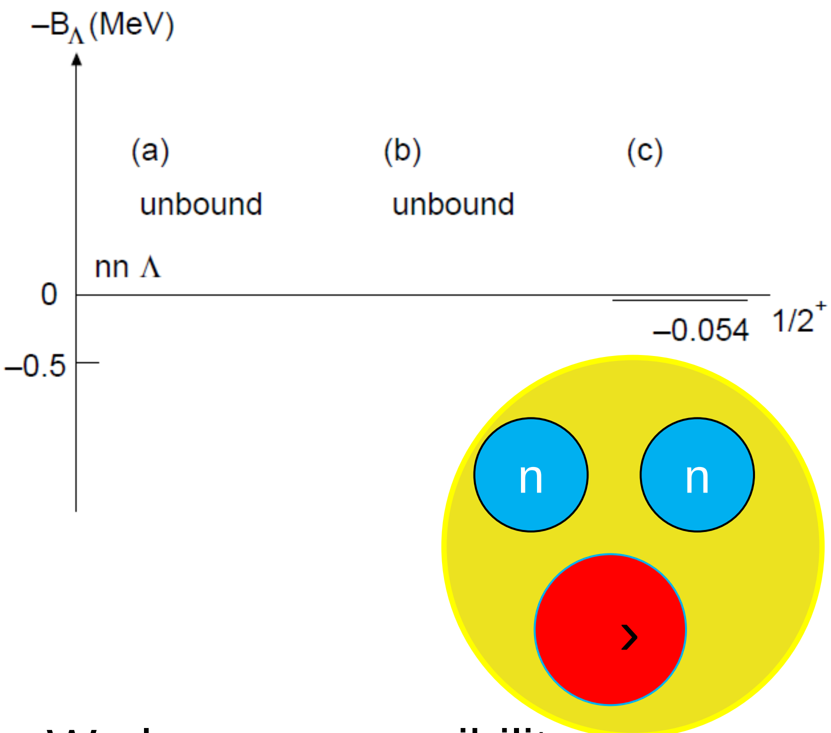
It should be noted to maintain consistency with the binding energies of $_{›}^{3}$H and $_{›}^{4}$H and $_{›}^{4}$He.

$V_T^{\text{› N-£N}}$ X1.1, 1.2





When we have a bound state in nn› system, what are binding energies of $^{3}_{›}$H and A=4 hypernuclei?



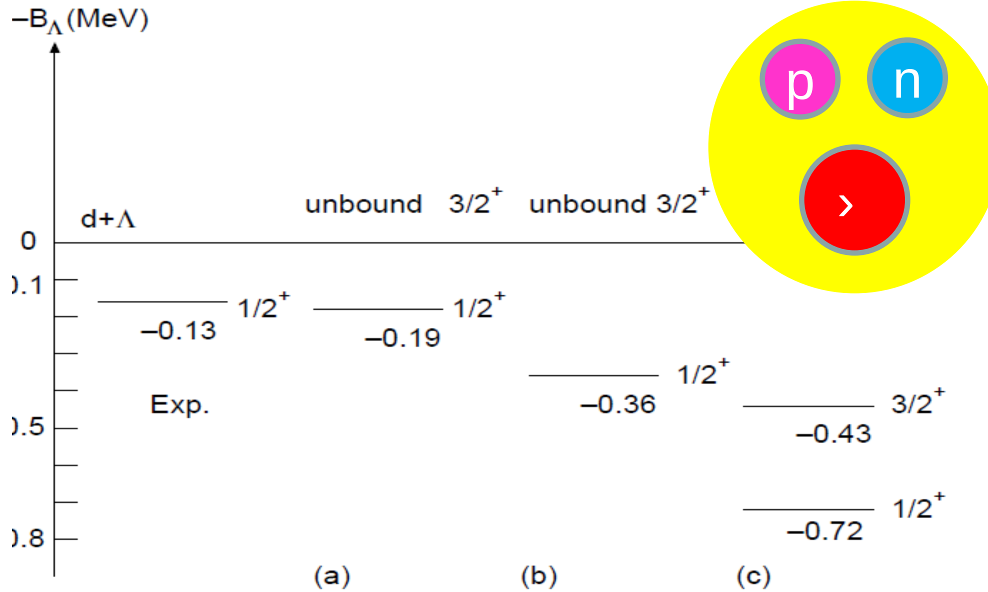
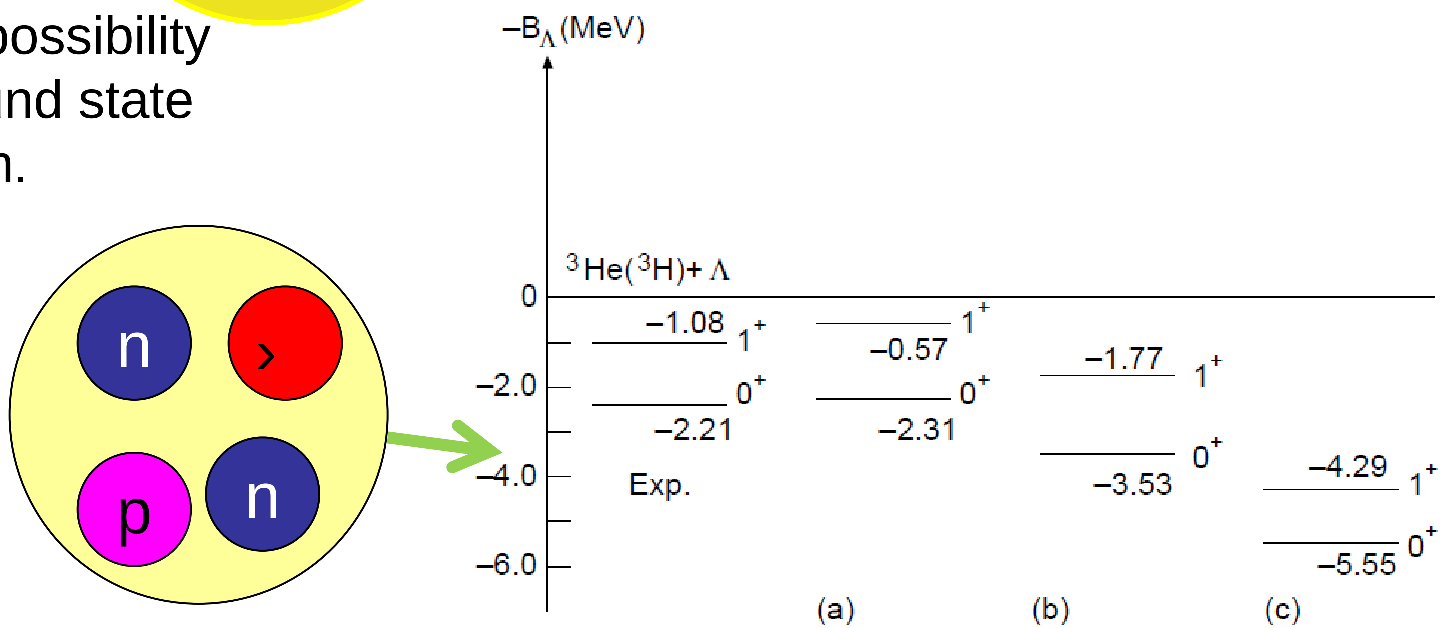


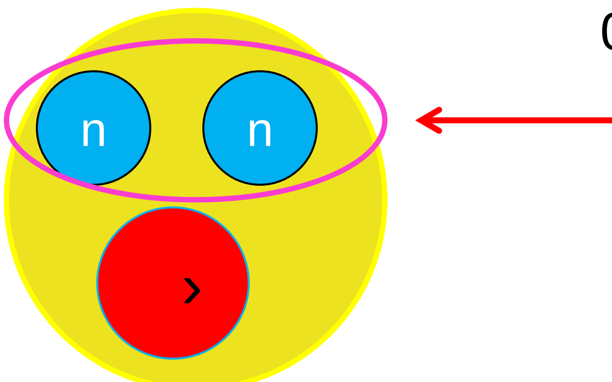
FIG. 3: Calculated Λ-separation energy for $^3_\Lambda$H with (a) $^3V^T_{N\Lambda-N\Sigma}$ × 1.00, (b) $^3V^T_{N\Lambda-N\Sigma}$ × 1.10, and (c) $^3V^T_{N\Lambda-N\Sigma}$ ×

We have no possibility to have a bound state in nn› system.



Question: If we tune $^1S_0$ state of nn interaction,
Do we have a possibility to have a bound state in nn› ?
In this case, the binding energies of ^{3}H and ^{3}He reproduce the observed data?

Some authors pointed out to have dineutron bound state in nn system. Ex. H. Witala and W. Gloeckle, Phys. Rev. C85, 064003 (2012).



T=1, $^1S_0$ state
I multiply component of $^1S_0$ state by 1.13 and 1.35. What is the binding energies of nn› ?

PHYSICAL REVIEW C **85**, 064003 (2012)

Di-neutron and the three-nucleon continuum observables

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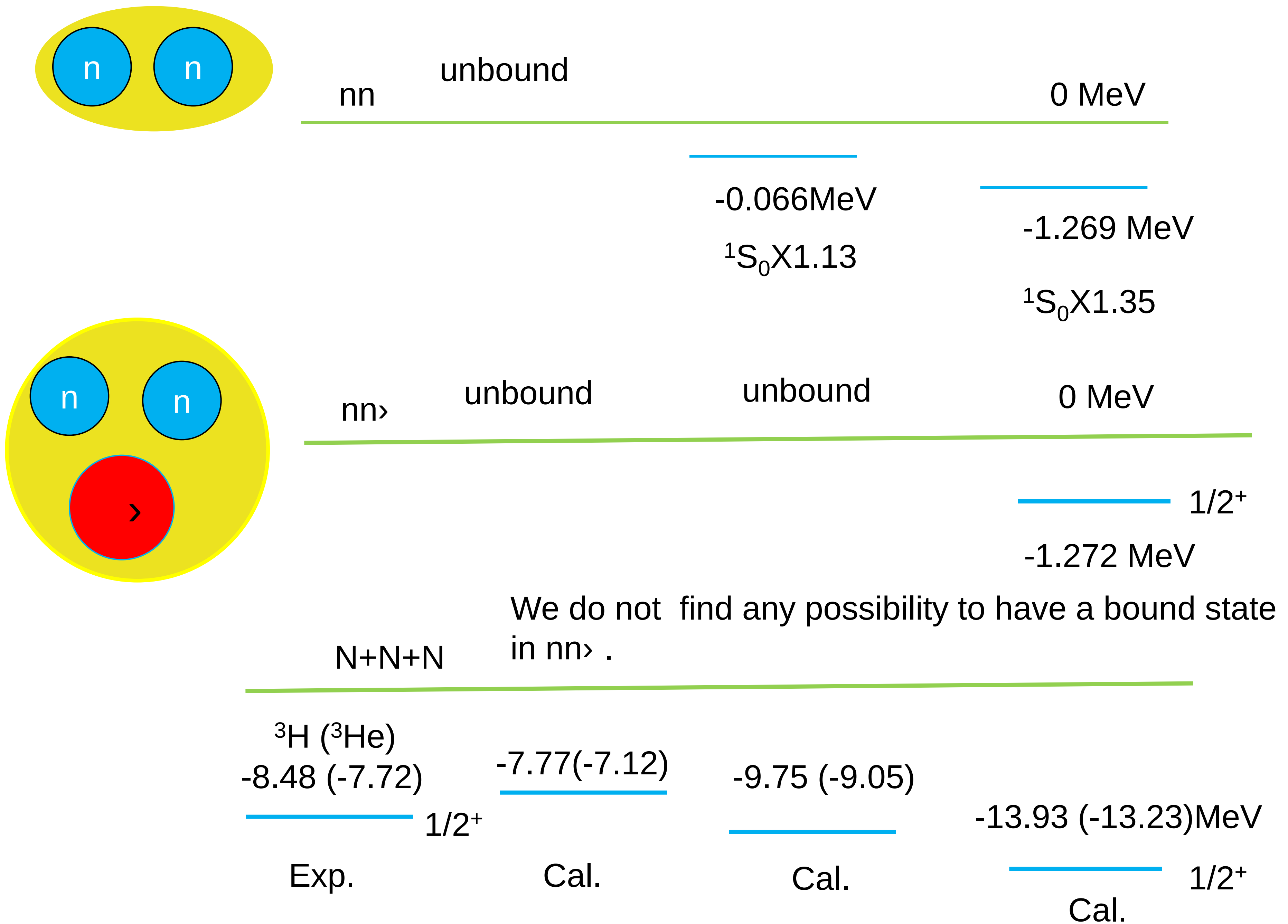
(Received 24 April 2012; published 25 June 2012)

We investigate how strongly a hypothetical $^1S_0$ bound state of two neutrons would affect observables in neutron-deuteron reactions. To that aim we extend our momentum-space scheme of solving the three-nucleon Faddeev equations and incorporate in addition to the deuteron also a $^1S_0$ di-neutron bound state. We discuss effects induced by a di-neutron on the angular distributions of the neutron-deuteron elastic scattering and deuteron breakup cross sections. A comparison to the available data for the neutron-deuteron total cross section and elastic scattering angular distributions cannot decisively exclude the possibility that two neutrons can form a $^1S_0$ bound state. However, strong modifications of the final-state-interaction peaks in the neutron-deuteron breakup reaction seem to disallow the existence of a di-neutron.

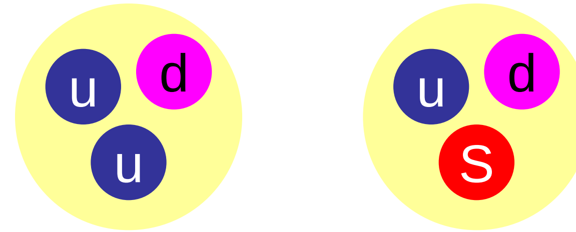

n
n
nn
unbound
0 MeV
-0.066MeV
1S0X1.13
-1.269 MeV
1S0X1.35
n
n
›
nn›
unbound
unbound
0 MeV
1/2+
-1.272 MeV
We do not find any possibility to have a bound state in nn› .
N+N+N
3H (3He)
-8.48 (-7.72)
1/2+
Exp.
-7.77(-7.12)
Cal.
-9.75 (-9.05)
Cal.
-13.93 (-13.23)MeV
1/2+
Cal.

Summary of hypernuclear part:

Motivated by the reported observation of data suggesting
a bound state nn› , we have calculated the binding energy
of this hyperucleus taking into account › N-£N explicitly.
We did not find any possibility to have a bound state in this
system. However, the experimentally they reported
evidence for a bound state.
As long as we believe the data, we should consider additional
missing elements in the present calculation. But, I have no idea.
Unfortunately, they did not report binding energy.
I hope that further experimental data is needed.

Outline of my talk

(1) Introduction



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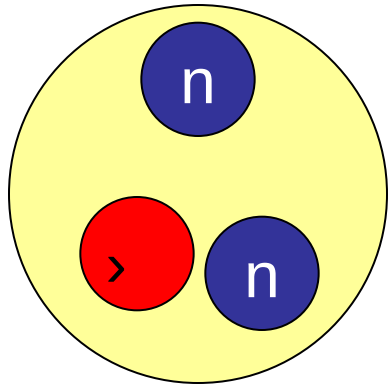
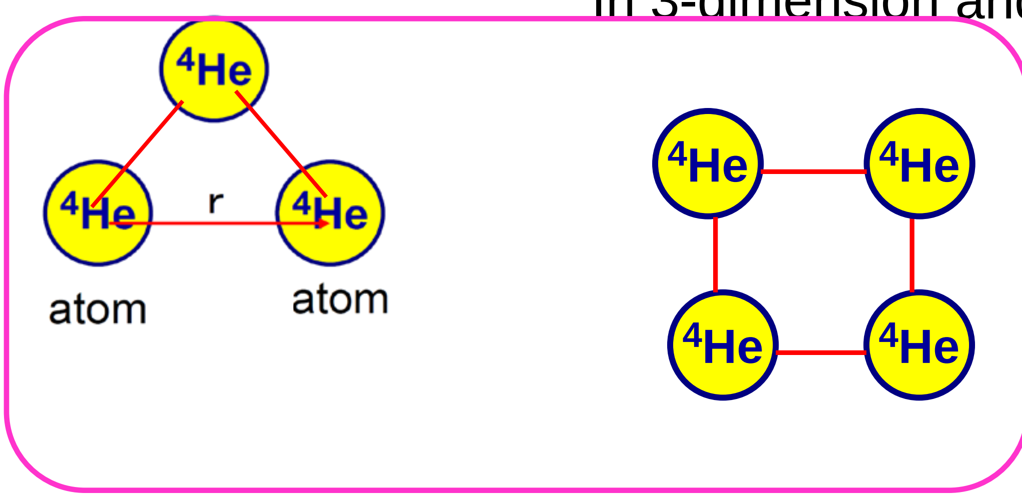


^{3}n
›

In 3-dimension and 2-dimension world

(2)



LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential

Efimov effects in 4He atoms

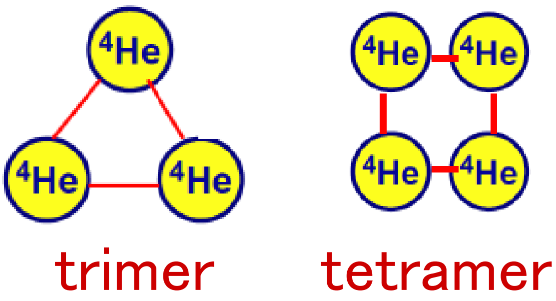
One of the most fundamental theoretical issues

from the view point of few-body problems:

to perform accurate calculations of the

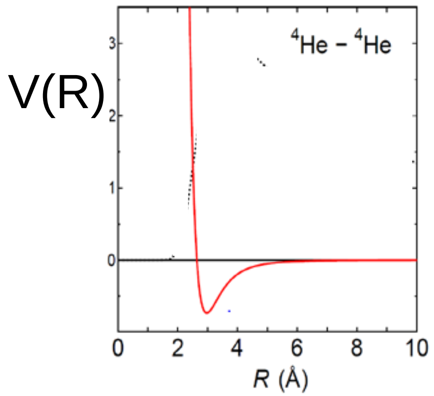
3- and 4-body 4He-atom systems

using realistic 4He-4He potentials.

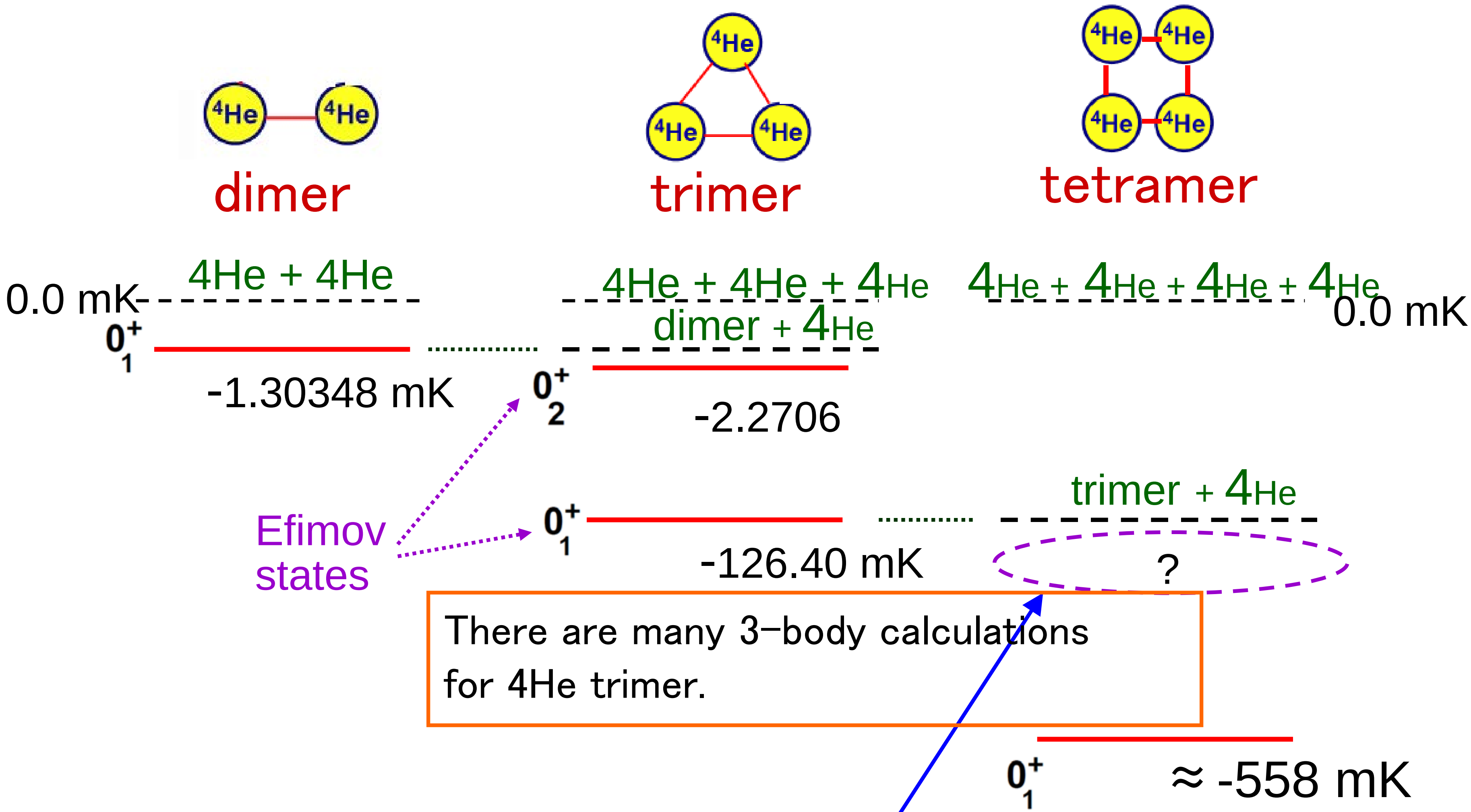


This subject has been intensively studied
by nuclear physicists, atomic physicists and
quantum chemists.

Realistic potential



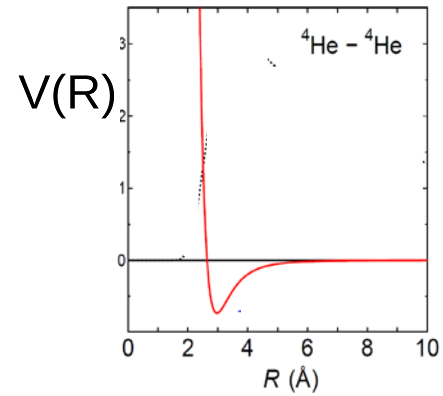
^{4}He-atom clusters --- Level structure known before our calculation :



There was a few calculation or estimation for very weakly bound state near the trimer +4He threshold.

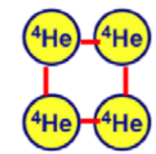
← Numerical difficulty:

The difficulty for the 4-body calculations of the 4He tetramer is



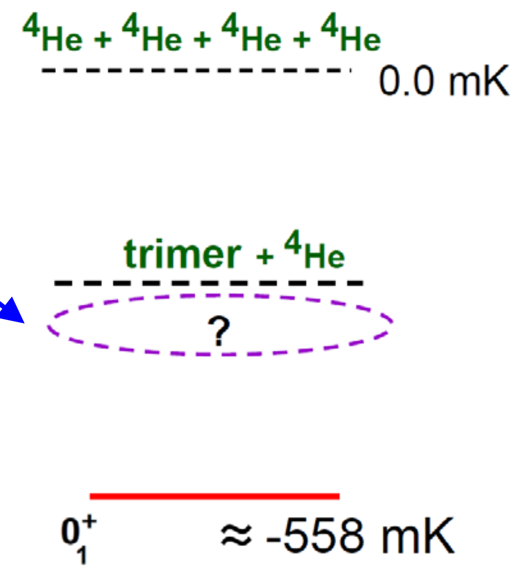
1) the realistic 4He-4He potential has an extremely strong repulsive core.

The core part of the atomic potential is ～1000 times higher than that of the nucleon-nucleon potential.



tetramer

2) the very weakly-bound excited state should have a very long-range tail, which is also difficult to treat.



There are 5 calculations of tetramer using realistic pair potentials (LM2M2, TTY).

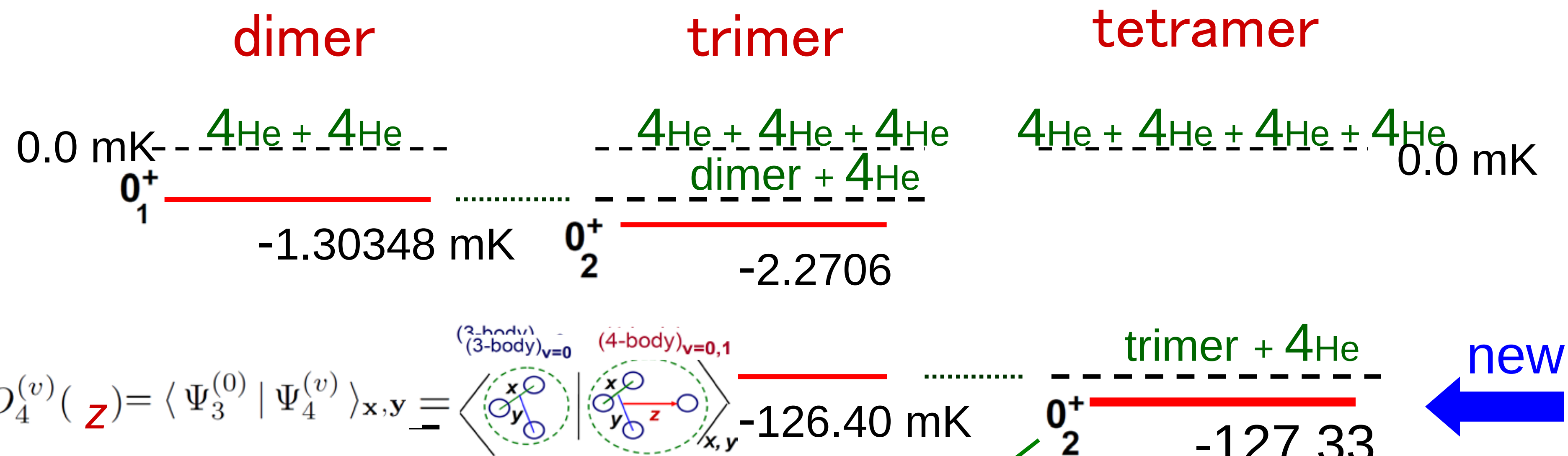
Method	^{4}He tetramer binding energies Reference	potential	ground state (mK)	excited state (mK)
Monte Carlo	Lewerenz (1977)	TTY	558	
Monte Carlo	Bressanini *et al.* (2000)	TTY	559.1	
Monte Carlo	Blume and Greene (2000)	LM2M2	557	133
Faddeev	Lazauskas and Carbonell (2006)	LM2M2	557.5	127.5
Correlated potential harmonic expansion	Das *et al.* (2011)	TTY	558	178

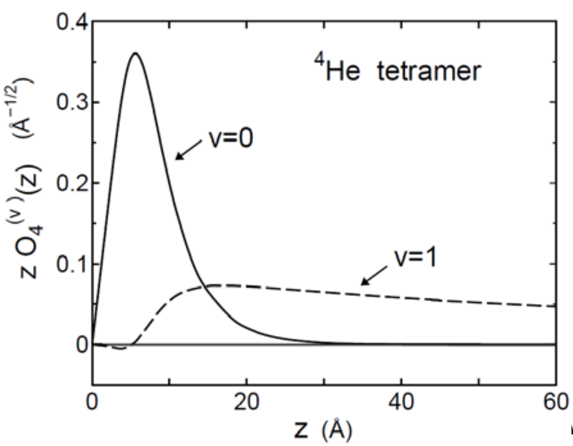
cf. Trimer g.s. = 126.40

This value was not obtained by bound-state calculation, but was extrapolated from atom-trimer scattering calculation.

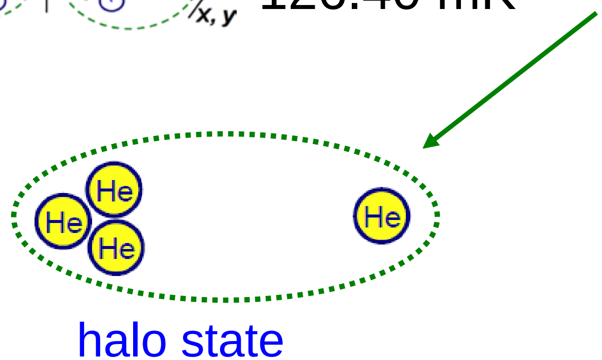
So, we intended to confirm this value by our bound-state calculation.

We confirmed this level structure of 4He-atom clusters (2012).

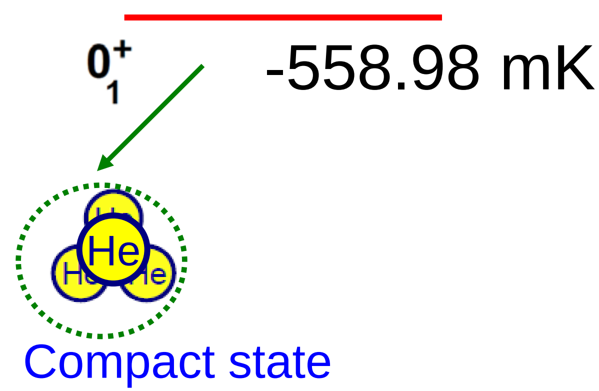




E. Hiyama *et al*., PRC 70 (2004)



We found that this state is a typical halo state having this configuration



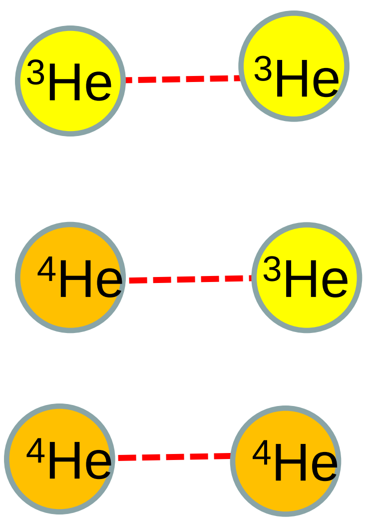
How about 2-dimensions?

In the terms of using realistic potential,

For example, S. Kilic and L. Vranjes, Journal of Low Temperature Physics, Vol. 134, 713(2004).
L. Vranjes and S. Kilic, Phys. Rev. A 65, 042506 (2001).

Variation and diffusion Monte Calro method => N=2 to 6-body problems
Potential: SAPT96 potential

2-body problem



3-body problem

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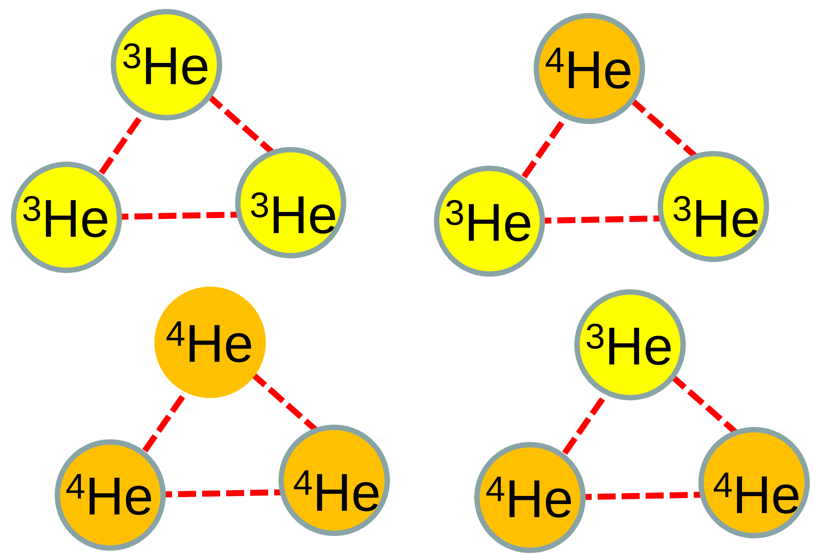
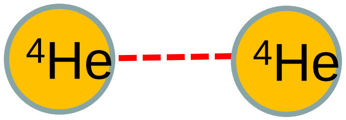


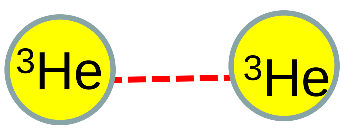
I am interested in calculating trimers and tetramer systems using various types of realistic potentials.
I will show you our preliminary results here.

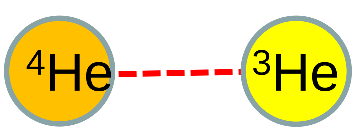
LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential

Dimer (2-body problem)



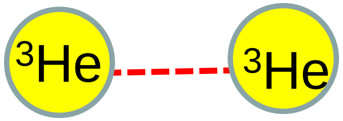
	2 dimensions		3D
Potential	Our result	Kilic et al.(2004)	
SAPT96	-40.84 mK	-41 mK	-1.744 mK
LM2M2	-38.39 mK		-1.309 mK
TTY	-38.35 mK		-1.316 mK
HFD-B3-FCI1	-38.99 mK		-1.448 mK
CCSAPT07	-39.49 mK		-1.564 mK
PCKJS	-39.69 mK		-1.615 mK
HFD_B	-40.00 mK		-1.692 mK





You see that the dimer binding energy in 2D is less bound than that in 3D. Because Kinetic energy in 2D is smaller than that in 3D.
Also, scattering length in 2D is much smaller than that in 3D.
Scattering length in 2D: 41.90 ú effective range:21.96 ú
Scattering length in 3D: ～190 ú

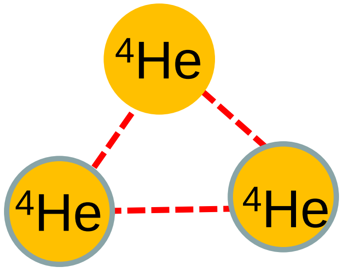
Dimer (2-body problem)

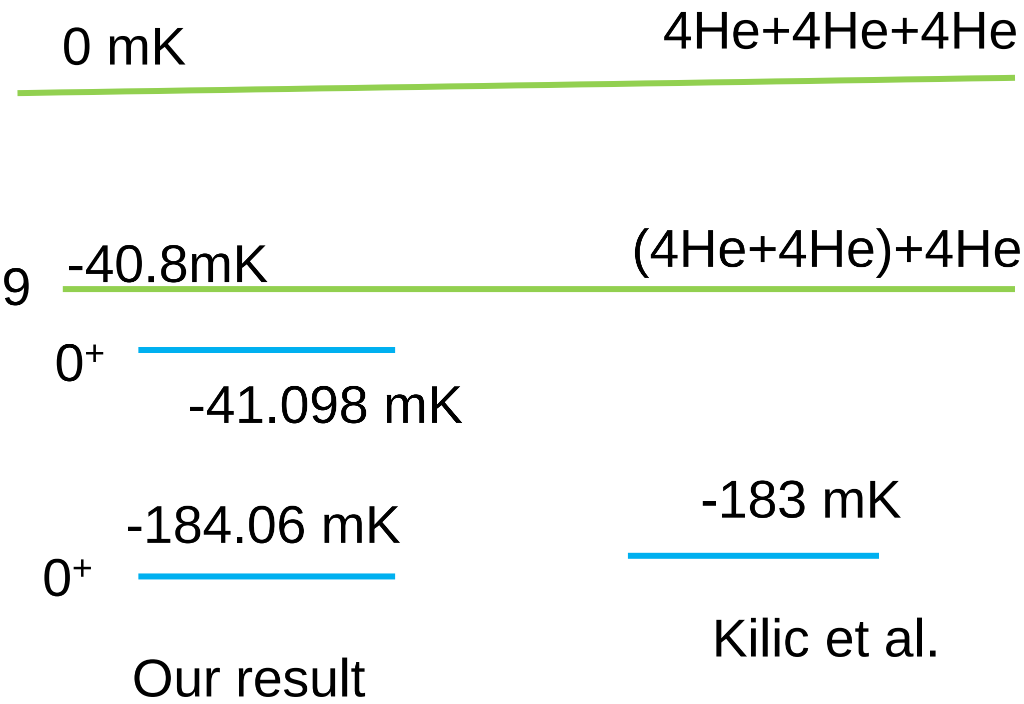


Potential	2 dimensions: Our result	2 dimensions: Kilic et al.(2004)	3D
SAPT96	-0.02 mK	-0.02 mK	No bound
LM2M2	-0.013 mK		
TTY	-0.013 mK		
HFD-B3-FCI1	-0.014 mK		
CCSAPT07	-0.016 mK		
PCKJS	-0.016 mK		
HFD_B	-0.018 mK		

Potential	2 dimensions: Our result	2 dimensions: Kilic et al.(2004)	3D
SAPT96	-4.32 mK	-4.3 mK	No bound

SAPT96





L.W. Brush and L.A. Tjon, PRA19, 425 (1979

$E_3^{(1)} = 1.27 E_2$

$E_3^{(0)} = 16.522 E_2$

↕

My calculation

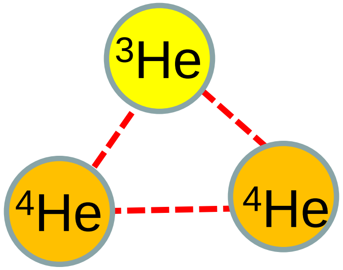
$E_3^{(1)} = 1.007 E_2$

$E_3^{(0)} = 4.511 E_2$

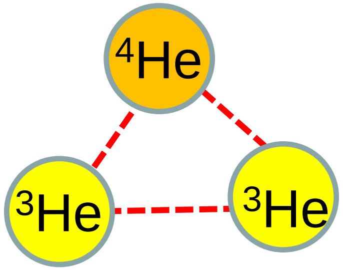
K.Helrich and H-W. Hammer, PRA83,052703(2011)

They pointed out to need effective range correction. $E_3^{(1)} = 1.145 E_2$, $E_3^{(0)} = 10.578 E_2$

We calculated trimer states of 4He using LM2M2 potential et al.
We have two bound states.



SAPT96



0 mK 4He+4He+3He

-40.8 mK (4He+4He)+3He

0^+ -73.88 mK

Our result

-74.3 mK
Kilic et al.

We have no second bound state.

0 mK 3He+3He+4He

-4.3 mK (3He+4He)+3He

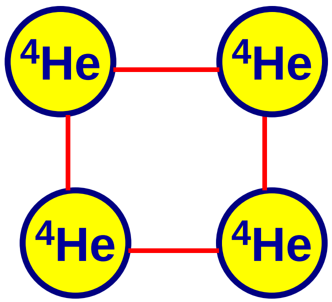
0^+ -14.46 mK

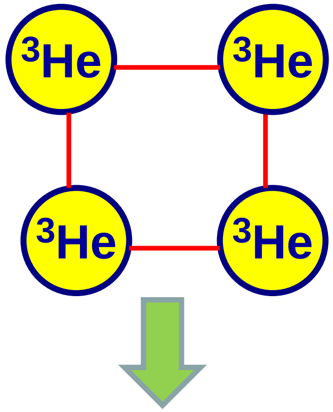
Our result

-14.4 mK
Kilic et al.

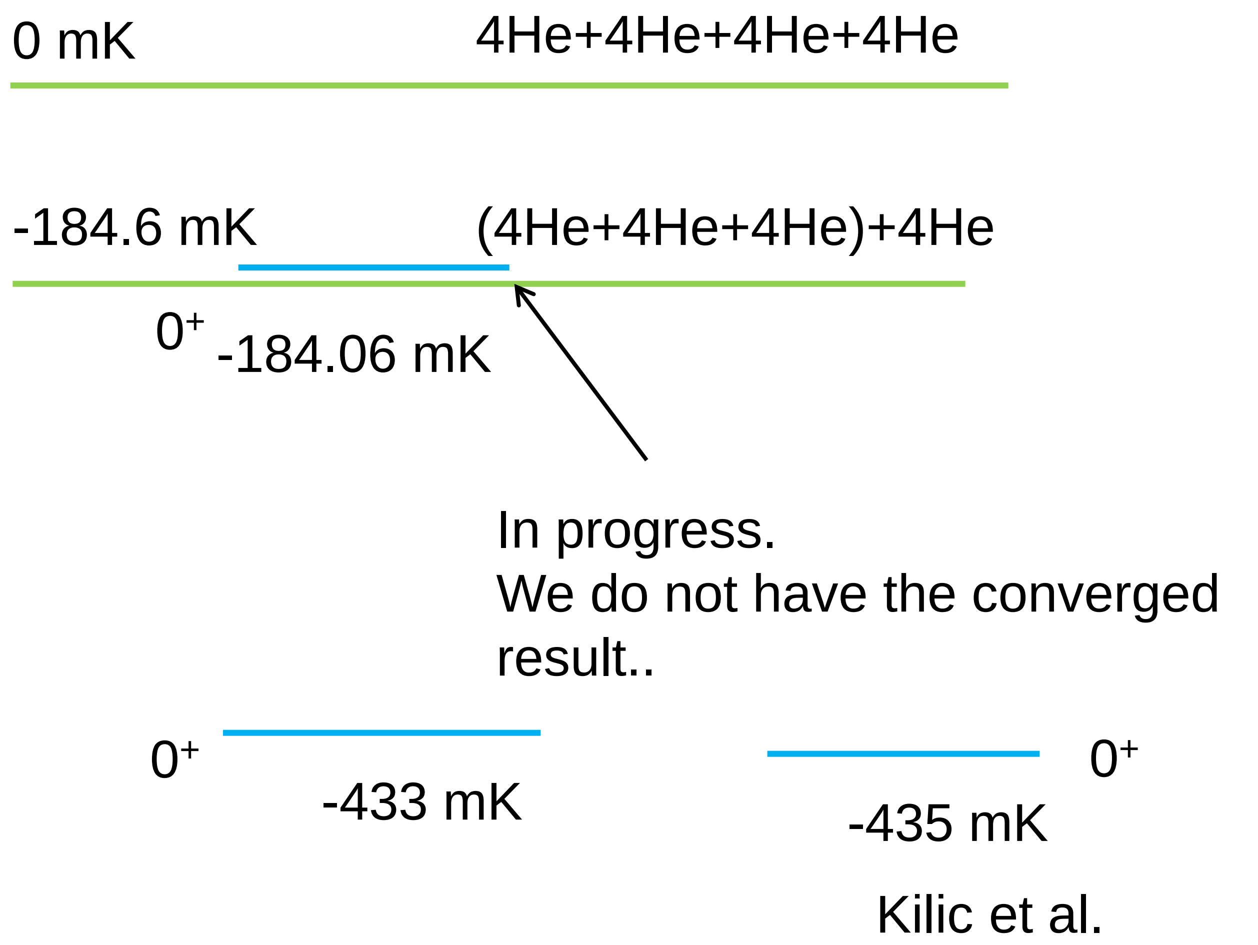
We have no second bound state.

Tetramer system





Not yet
Future plan



The calculation of trimer and tetramer systems
is still in progress.

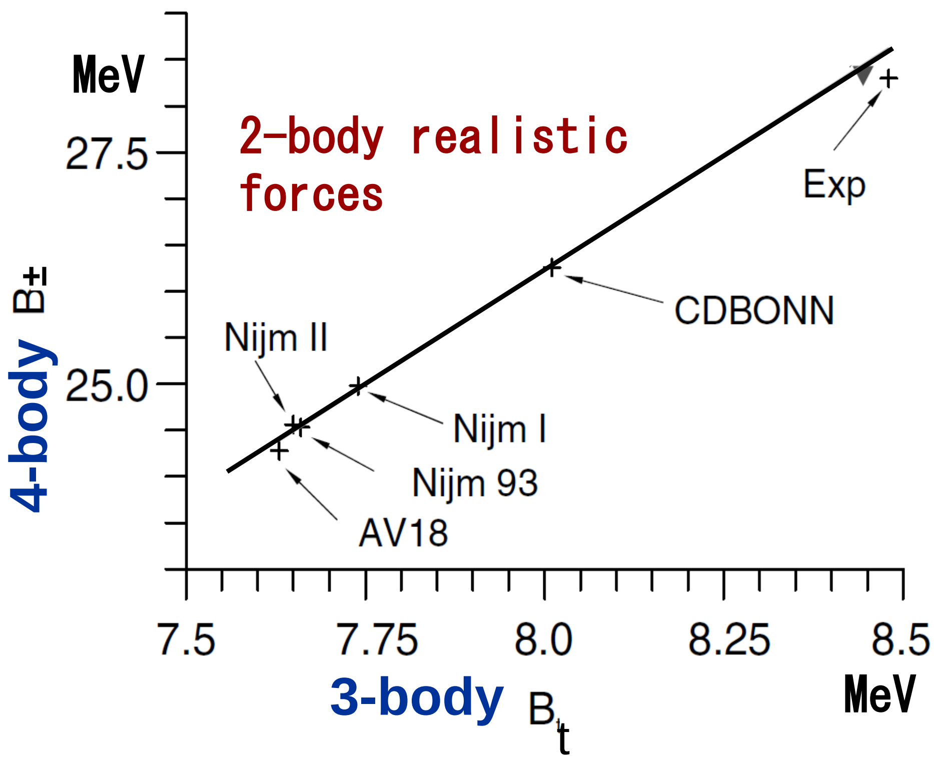
When the calculation od 2D is finished,
I want to compare results in 2D and those in 3D.

For example,

3D world, Do we have

Similarlity in **Tjon line** of **atomic** and **nuclear** 4-body systems with large scattering length ?

Correlation between the 3N (^3H) and 4N (^4He) binding energies for different NN potentials is **approximately linear**. This line is called **Tjon line** (well known in nuclear few-body physics).



Slope = 4.8

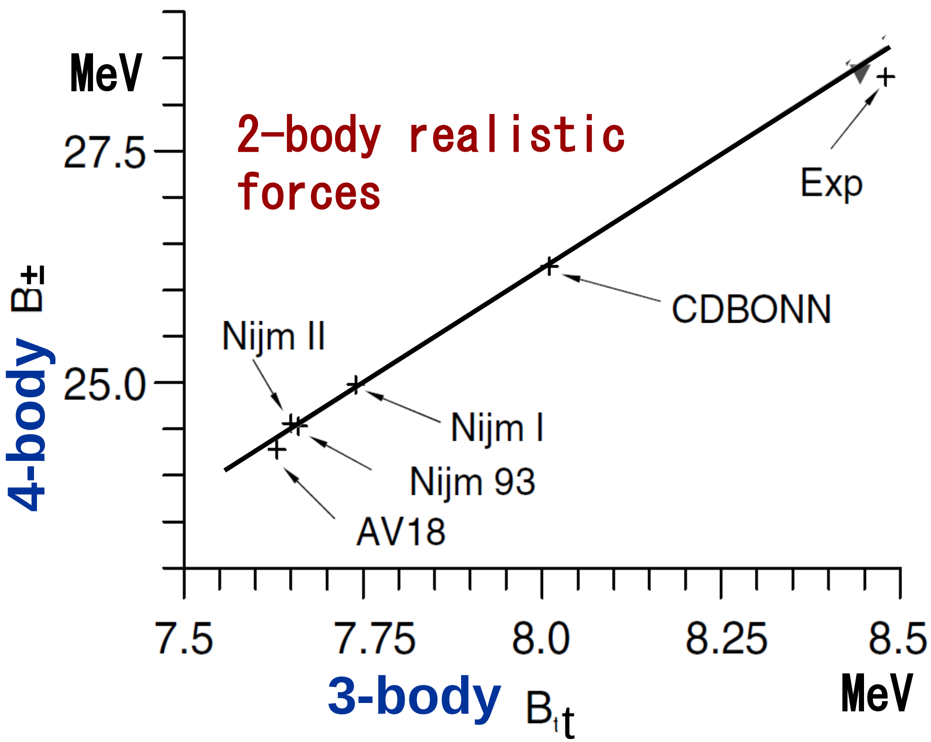
How is this type of correlation in ^{4}He-atom tetramer ?

The slope is universal ?

$B_{\alpha t}$ correlation

Nuclear **Tjon** line

A. Nogga *et al.*, PRL **85** (2000)

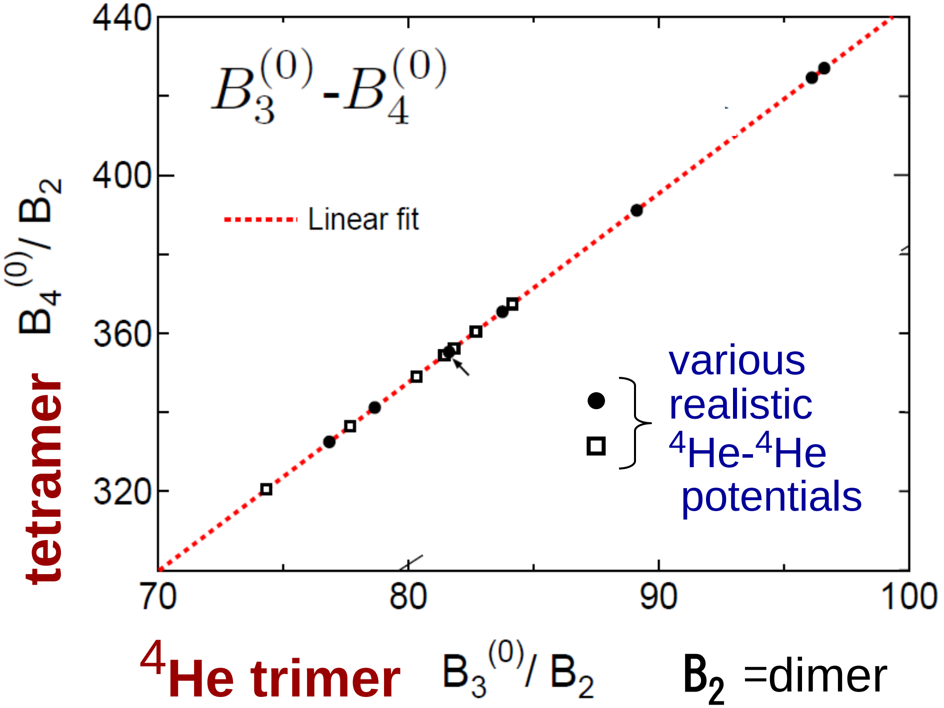


Slope = 4.8

$B_3^{(0)}$-$B_4^{(0)}$ correlation

Atomic **Tjon** line

E. Hiyama & M. K, PRA **85** (2012)



Slope = 4.778

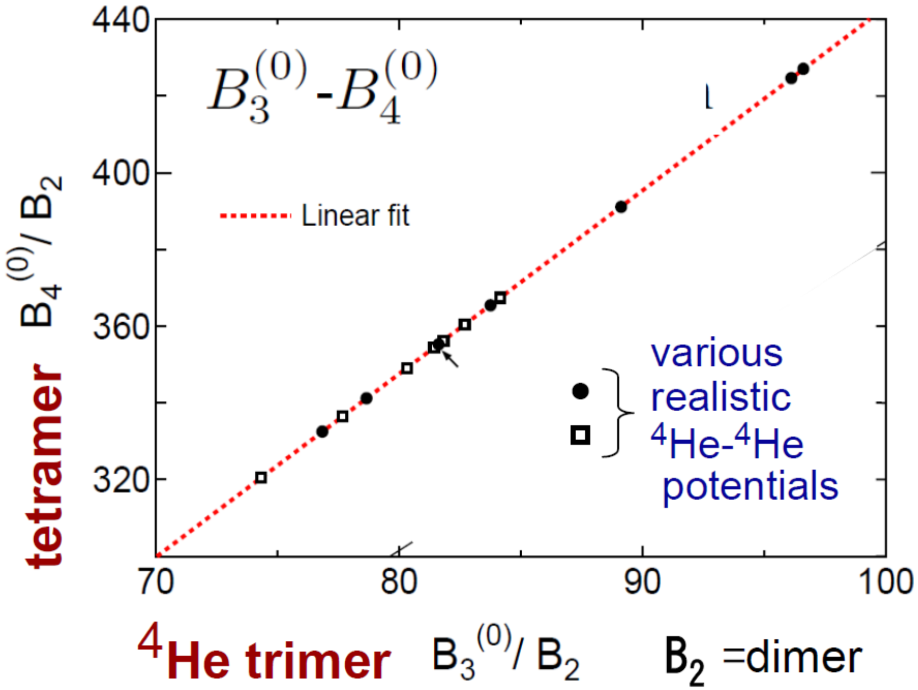
Least square fit

universality

3D world

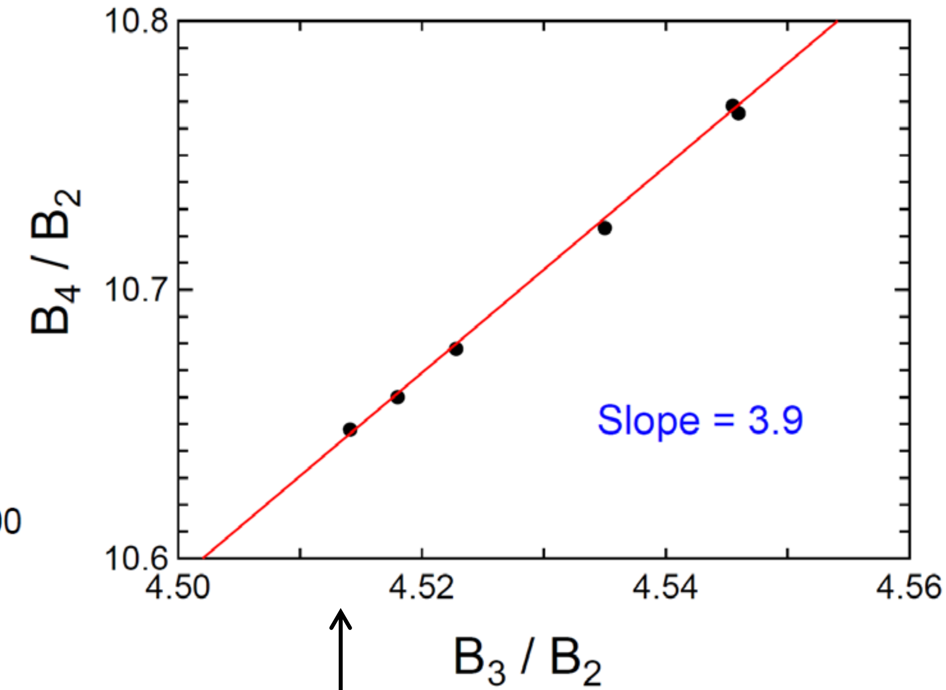
Atomic **Tjon** line

E. Hiyama & M. K, PRA **85** (2012)



Slope = 4.778

2D world

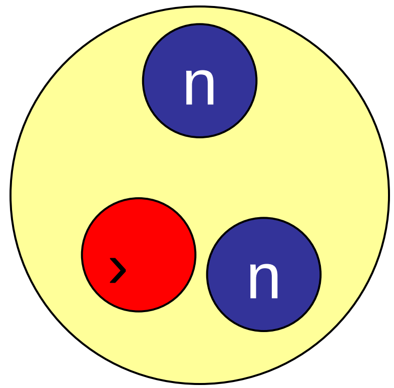


J. A. Tjon, PRA21, 1334 (1980)
Slope=2.9

Further study is in progress.

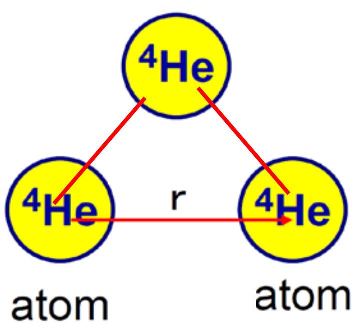
Summary

(2)

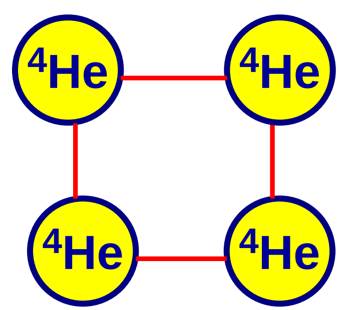


› ^{3}n

(3)



In 3-dimension and 2-dimension world



LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential

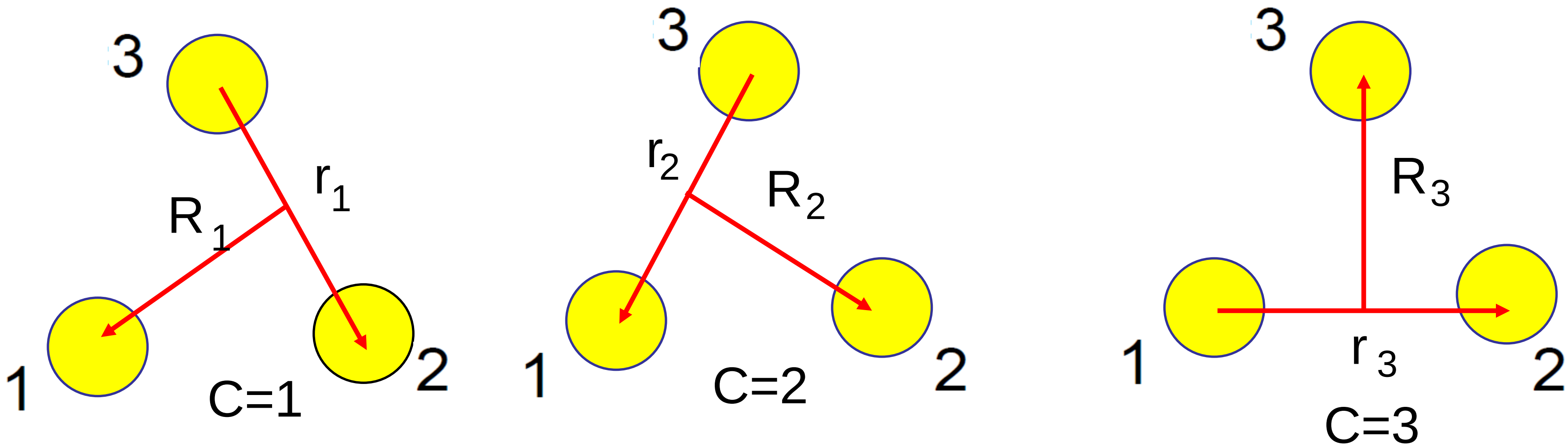
Thank you!

in 2 dimensions. We follow the conventions of Ref. [169] in which the scattering length a and the effective range r_s are defined by

$$\frac{1}{2}\pi \cot \delta_0(k) = \gamma + \ln(\tfrac{1}{2}ka) + \tfrac{1}{4}r_s^2 k^2 + \mathcal{O}(k^4), \tag{414}$$

where $\gamma \simeq 0.577216$ is Euler's constant. The binding energy of the shallow dimer in the scaling limit is given by

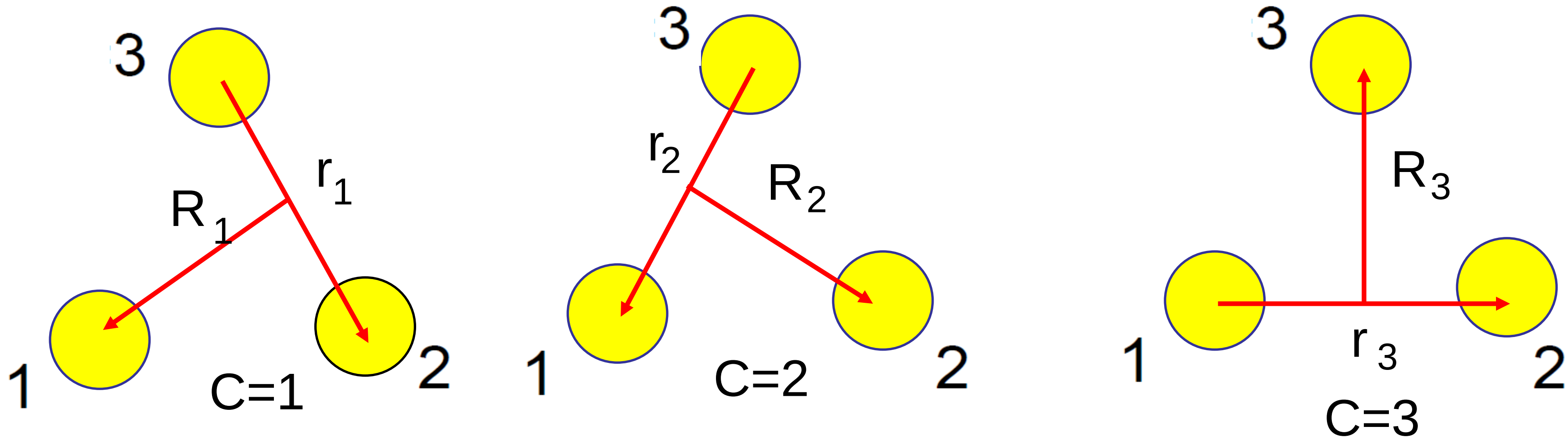
Gaussian Expansion Method (GEM)



$$H = -\frac{\hbar^2}{2\mu_{r_c}}\nabla^2_{\mathbf{r}_c} - \frac{\hbar^2}{2\mu_R}\nabla^2_{\mathbf{R}_C} + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3)$$

$$[\, H - E \,]\, \Psi_{JM} = 0$$

$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathbf{r}_1, \mathbf{R}_1) + \Phi^{(2)}_{JM}(\mathbf{r}_2, \mathbf{R}_2) + \Phi^{(3)}_{JM}(\mathbf{r}_3, \mathbf{R}_3)$$



$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathbf{r}_1, \mathbf{R}_1) + \Phi^{(2)}_{JM}(\mathbf{r}_2, \mathbf{R}_2) + \Phi^{(3)}_{JM}(\mathbf{r}_3, \mathbf{R}_3)$$

Basis functions of each Jacobi coordinate $(c = 1 - 3)$

$$\phi^{(c)}_{\underline{nl}}(r_c)\, Y_{\underline{lm}}(\widehat{\mathbf{r}}_c), \quad \psi^{(c)}_{NL}(R_c)\, Y_{LM}(\widehat{\mathbf{R}}_c)$$

$$\Phi^{(c)}_{JM}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{nl,NL} \underline{C_{NL,lm}}\, \phi^{(c)}_{nl}(r_c)\, \psi^{(c)}_{NL}(R_c)\, [Y_l(\widehat{\mathbf{r}}_c) \otimes Y_L(\widehat{\mathbf{r}}_c)]_{JM}$$

$(c = 1 - 3)$

↑ Determined by diagonalizing H

Radial part : Gaussian function

$$\phi_{nl}(r) = r^l \, e^{-(r/r_n)^2}$$

$$\psi_{NL}(R) = R^L \, e^{-(R/R_N)^2}$$

Gaussian ranges in geometric progression

$$r_n = r_1 \, a^{n-1} \qquad (n = 1 - n_{\max}) \ ,$$

$$R_N = R_1 \, A^{N-1} \qquad (N = 1 - N_{\max})$$

Both the short-range correlations and the exponentially-damped tail are simultaneously reproduced accurately.

Next, by solving eigenstate problem, we get eigenenergy E and unknown coefficients C_n .

$$\left[(H_{in}) - E\,(N_{in}) \right] \left(C_n \right) = 0$$