Weakly Bound few-particle systems in 3 and 2 dimensions

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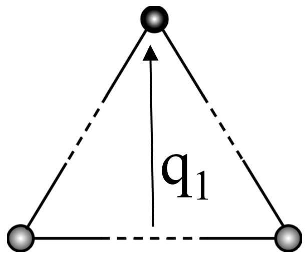
- Aksel Jensen - Aarhus
- Nikolaj Zinner – Aarhus
- Dimtri Fedorov - Aarhus

INT, Seattle, May 8, 2014

Weakly bound system wave function & contact interaction (3d)

Three-boson wave function:

$$(E - H_0)\psi = 0$$

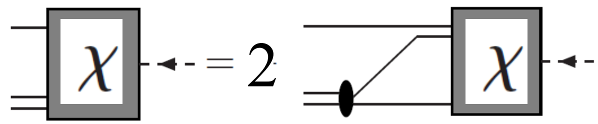


$$\psi = \int d^3q_1 \frac{\exp\{i[E-(3/4)q_1^2]^{1/2}R_1\}}{R_1} e^{i\mathbf{q}_1\cdot\mathbf{r}_1}\chi(\mathbf{q}_1)$$

$$+ \quad (1 \rightarrow 2) \; + \; (1 \rightarrow 3)$$

Zero-range 3-boson equation: Thomas-Efimov effect (3d)

Skorniakov and Ter-Martirosian equations (1956)



$(\hbar = m = 1)$

$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm\sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}\vec{y}^2}} \int d^3x \left(\frac{1}{\epsilon_3 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} - \frac{1}{1 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} \right) \chi(\vec{x})$$

$\epsilon_3 = E_3 / \mu^2_{(3)}$ $\quad\epsilon_2 = E_2 / \mu^2_{(3)}$ $\quad\mu^2_{(3)} = 1$

Thomas collapse: $\mu^2_{(3)} \rightarrow \infty$

Efimov effect: $E_2 \rightarrow 0$

$\epsilon_2 = E_2 / \mu^2_{(3)}$

Thomas-Efimov effect!

S.K. Adhikari, A. Delfino, T. Frederico, I.D. Goldman, and L. Tomio, Phys. Rev. A **37**, 3666 (1988).

Method: Hamiltonian for Subtracted 3B equations (3d)

TF, Delfino, Tomio, Yamashita PPNP 67, 939 (2012)

n-subtracted T-matrix equation (for Dirac-delta n=1)

$$T(E) = V^{(n)}(E,-\mu^2) + (-1)^n (E+\mu^2)^n V^{(n)}(E,-\mu^2) G_0^{(+)}(E) G_0^n(-\mu^2) T(E)$$

Invariance of T-matrix by dislocations of the subtraction point: $\frac{\partial V^{(n)}}{\partial \mu^2} = -V^{(n)} \frac{\partial G_n^{(+)}(E;-\mu^2)}{\partial \mu^2} V^{(n)}$

Renormalized Hamiltonian: $H_{\mathcal{R}} = H_0 + V_{\mathcal{R}}$

$$V_{\mathcal{R}} = [1 + V^{(n)} G_0^{(+)}(E)\ (1 - (-1)^n (\mu^2 + E)^n G_0^n(-\mu^2))]^{-1} V^{(n)}$$

$$\frac{\partial V_{\mathcal{R}}}{\partial \mu^2} = 0 \quad \text{and} \quad \frac{\partial H_{\mathcal{R}}}{\partial \mu^2} = 0$$

Subtracted-Faddeev equations 3B: $T_k(E) = t_{(ij)}\left(E - \frac{q_k^2}{2m_{ij,k}}\right) [1 + (G_0^{(+)}(E) - G_0(-\mu_3^2))\ (T_i(E) + T_j(E))]$

Adhikari,TF,Goldman, PRL74 (1995) 487

$$H_{\mathcal{R}I}^{(3B)} = \sum_{(ij)} V_{\mathcal{R}(ij)}^{(2B)} + V_{\mathcal{R}}^{(3B)}.$$

Scale invariance at the unitary limit (3d)

$$\epsilon_2 = \epsilon_3 = 0 \text{ and } \mu_3 \to \infty$$

s-wave: $$\chi(y) = \frac{4}{\pi\sqrt{3}y}\int_0^\infty dx\, x^2\chi(x)\int_{-1}^{1} dz \frac{1}{x^2+y^2+x\,y\,z}$$

Solution: $$\chi(y) = y^{s-2}$$

Efimov equation: $$1 = \frac{8}{\sqrt{3}s}\frac{\sin(\pi s/6)}{\cos(\pi s/2)} \qquad s = \pm i s_0 \qquad s_0 \approx 1.00624$$

$$\chi(y) = a_+ y^{\imath s_0 - 2} + a_- y^{-\imath s_0 - 2}$$

G.S. Danilov, Sov. Phys. JETP 13 (1961) 349

$$\chi(y) = y^{-2}\sin(s_0 \ln y + c)$$

One *parameter* to fix the solution → *3-body scale*
in 2d no 3-body scale and $\chi(y)$?

Zero-range 3-boson equation in 2d

$$f(\mathbf{q}) = -\frac{\pi^{-1}}{\ln\sqrt{E_3+\frac{3}{4}q^2}-\ln\sqrt{E_2}}\int d^2k\,\frac{f(\mathbf{k})}{-E_3-\mathbf{q}^2-\mathbf{k}^2-\mathbf{k}\cdot\mathbf{q}}$$

Efimov effect disappears: $E_3^{(0)} = 16.52 E_2 \quad E_3^{(1)} = 1.270 E_2$

$$f(q) \underset{q\to\infty}{\longrightarrow} \frac{\ln q}{q^2}$$

Bellotti et al PRA87, 013610 (2013)

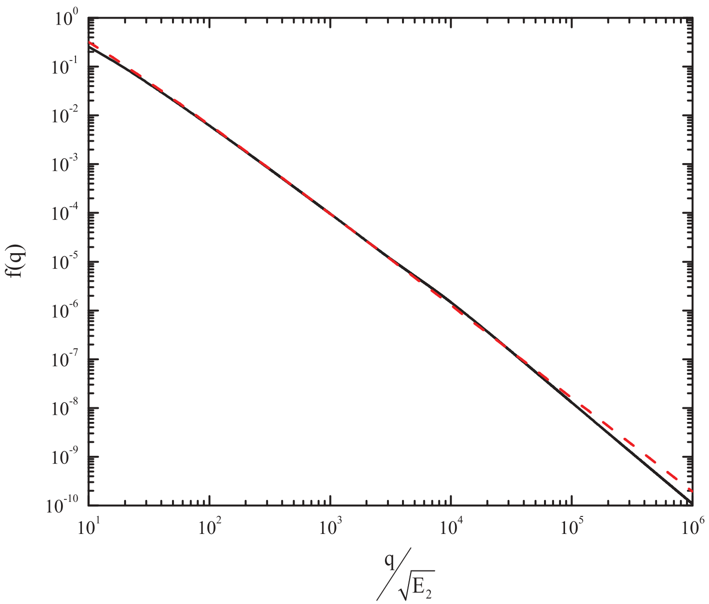
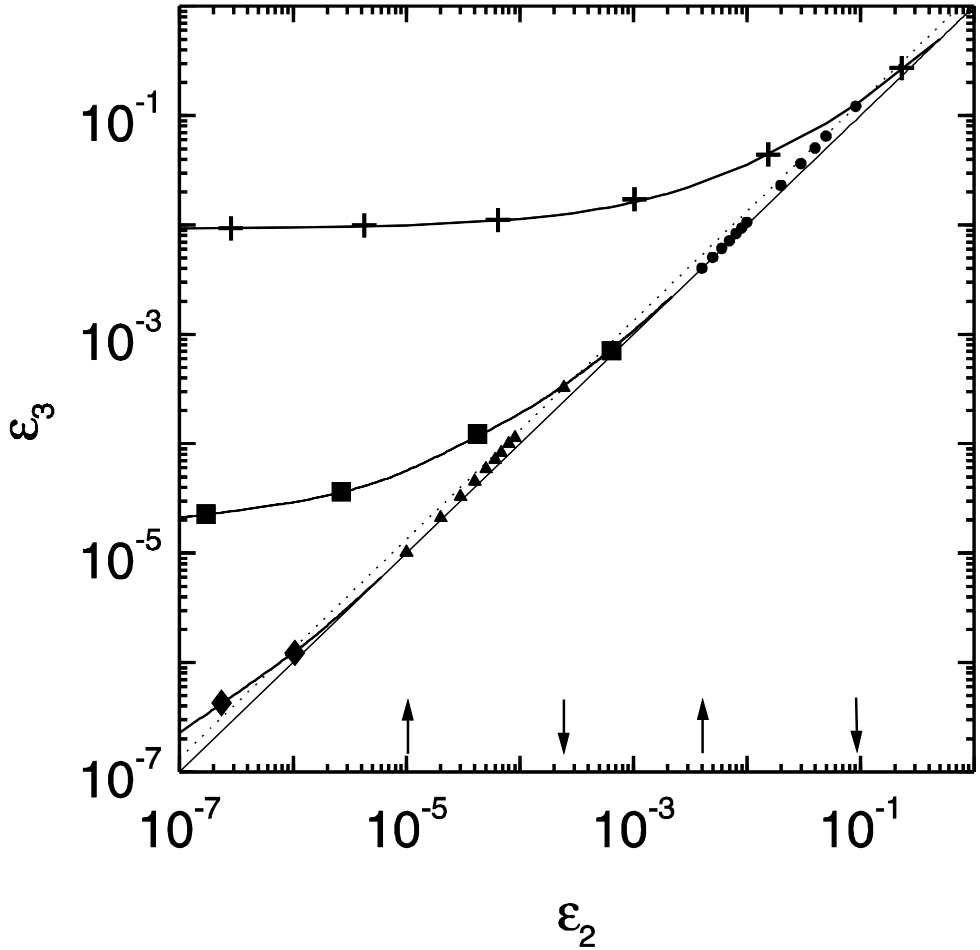


FIG. 3. (Color online) Spectator function $f(q)$ for the ground state calculated numerically (black solid line) and using the ansatz $f(q) = A_0\frac{\ln q}{q^2}$ (red dashed line). The solid (black) line tends to oscillate around the dashed (red) one as $q \to \infty$ due to finite numerical precision.

Efimov States – Bound and virtual states (3 identical bosons) (3d)



Bound-states (lines with simbols)

With Plus - Fundamental state

With squares – 1st. excited

With diamonds – 2nd. excited

Virtual-states (just simbols)

circles – 1st. Excited state

triangles – 2nd. Excited state

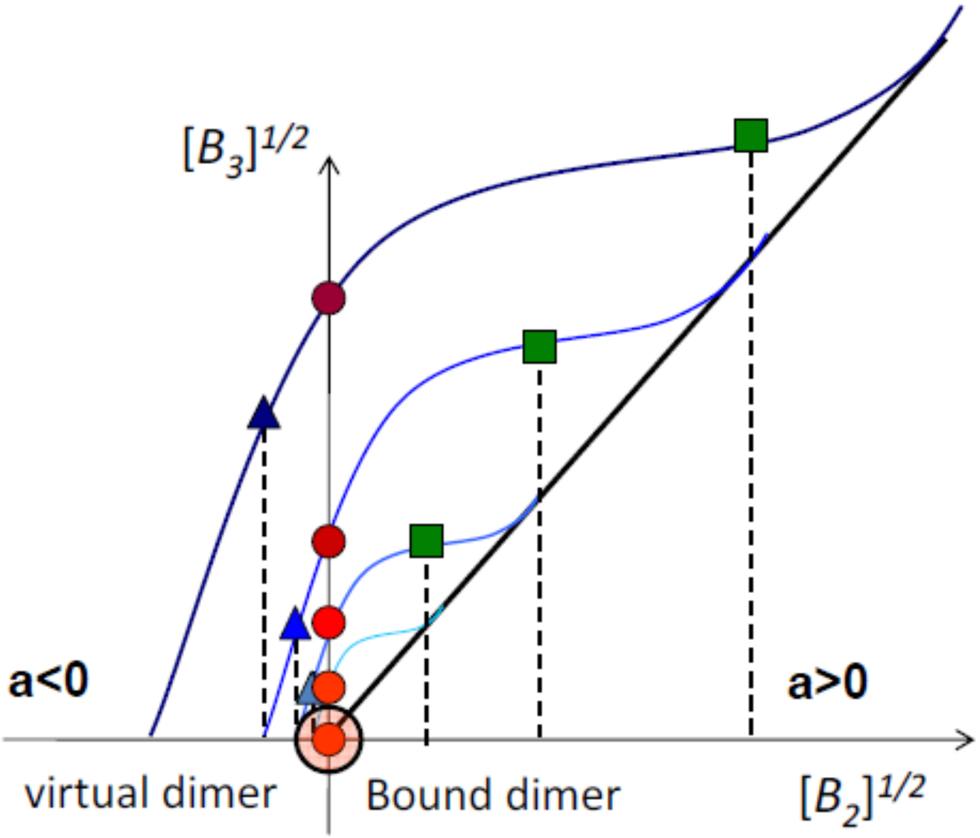
ε_2 bound

↓ the virtual state starts (dotted line) $\varepsilon_3 = \frac{4}{3}\varepsilon_2$

↑ Virtual-state turns to an excited state (continuous line) $\varepsilon_3 = \varepsilon_2$

S. K. Adhikari, A. C. Fonseca and LT *Phys. Rev.* **C27**, 1826 (1983).

Efimov Plot



Scaling limit & limit cycle

$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)}(\pm\sqrt{\epsilon_2})$$

$$\xi \equiv \pm\sqrt{\epsilon_2} = \pm(E_2\epsilon_3^{(N)}/E_3^{(N)})^{1/2}$$

$$\frac{E_3^{(N+1)}}{E_3^{(N)}} = \lim_{N\to\infty}\frac{\epsilon_3^{(N+1)}(\xi)}{\epsilon_3^{(N)}} = \mathcal{F}\left(\pm\sqrt{\frac{E_2}{E_3^{(N)}}}\right)$$

Scaling function

$$\mathcal{F}(0) = e^{2\pi/s_0} = 1/515$$

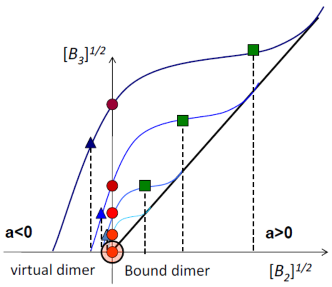
Efimov 1970

Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

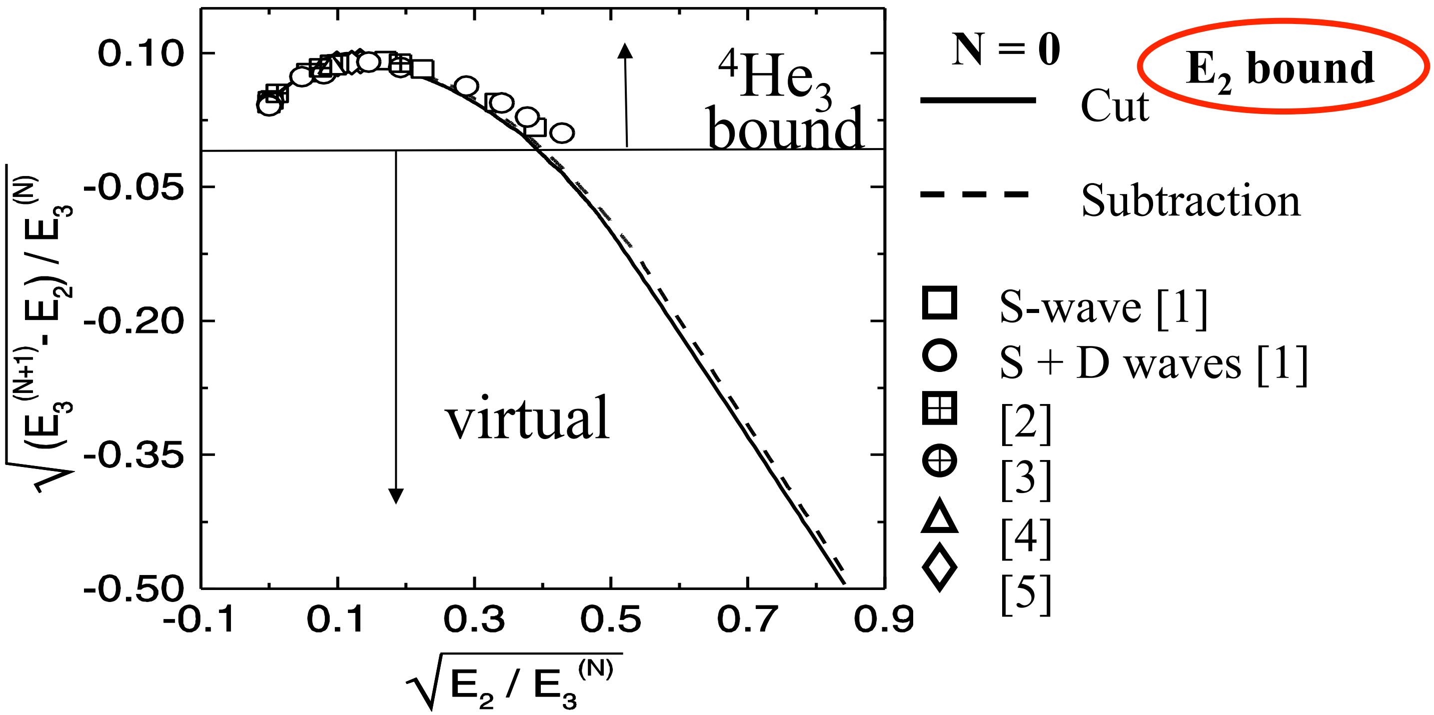
Limit cycle:
Mohr et al Ann.Phys. 321 (2006)225

Efimov States – Bound and virtual states (3 identical bosons)

T. Frederico, LT, A. Delfino and E. A. Amorim. *Phys. Rev.* **A60**, R9 (1999).



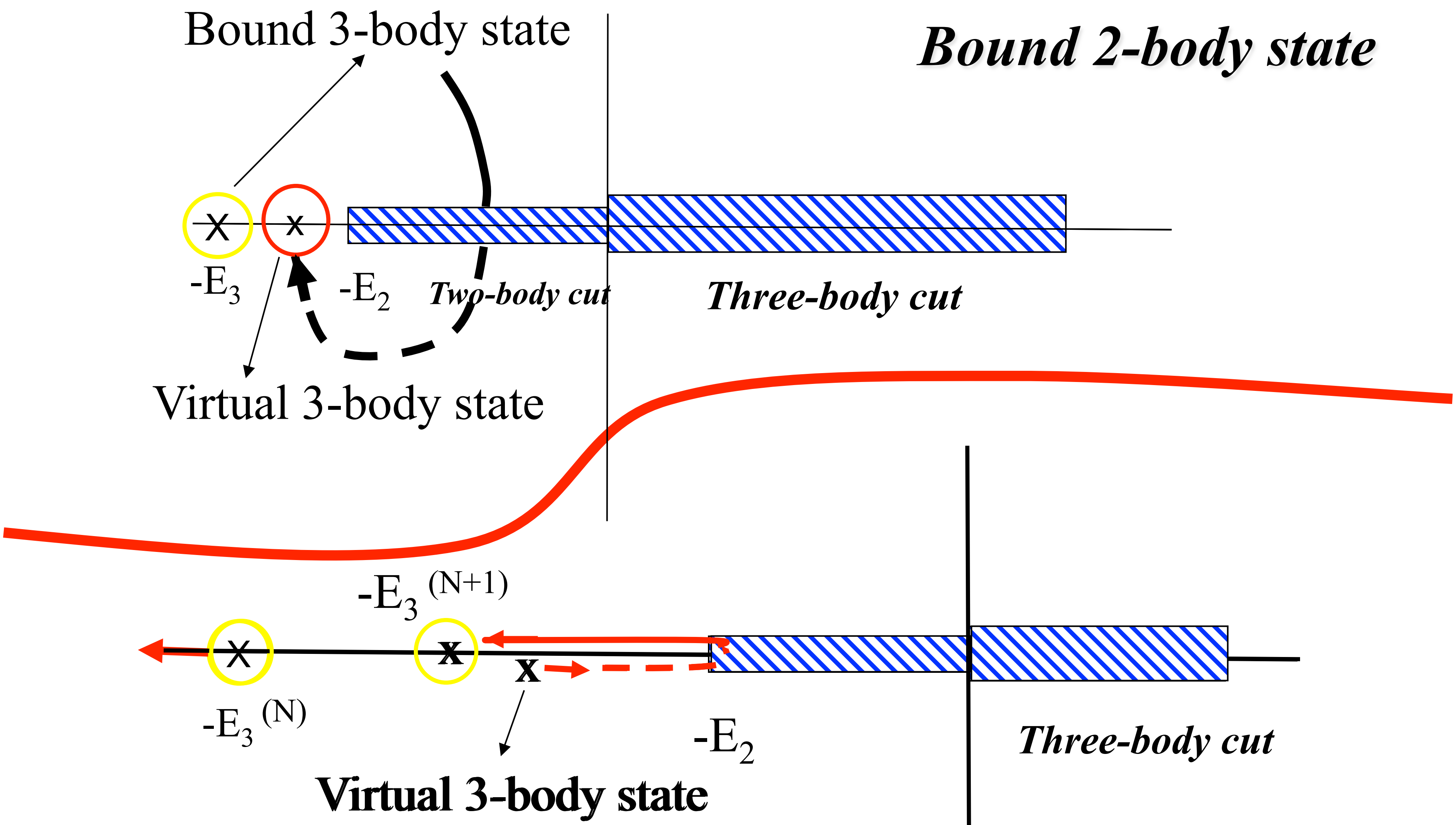
Scaling plot



[1] Th. Cornelius e W. Glöckle. *J. Chem. Phys.* **85**, 1 (1996).
[2] S. Huber. *Phys. Rev.* **A31**, 3981 (1985).
[3] P. Barletta e A. Kievsky. *Phys. Rev.* **A64**, 042514 (2001).
[4] D. V. Fedorov e A. S. Jensen. *J. Phys.* **A34**, 6003 (2001).
[5] E. A. Kolganova, A. K. Motovilov e S. A. Sofianos. *Phys. Rev.* **A56**, R1686 (1997).

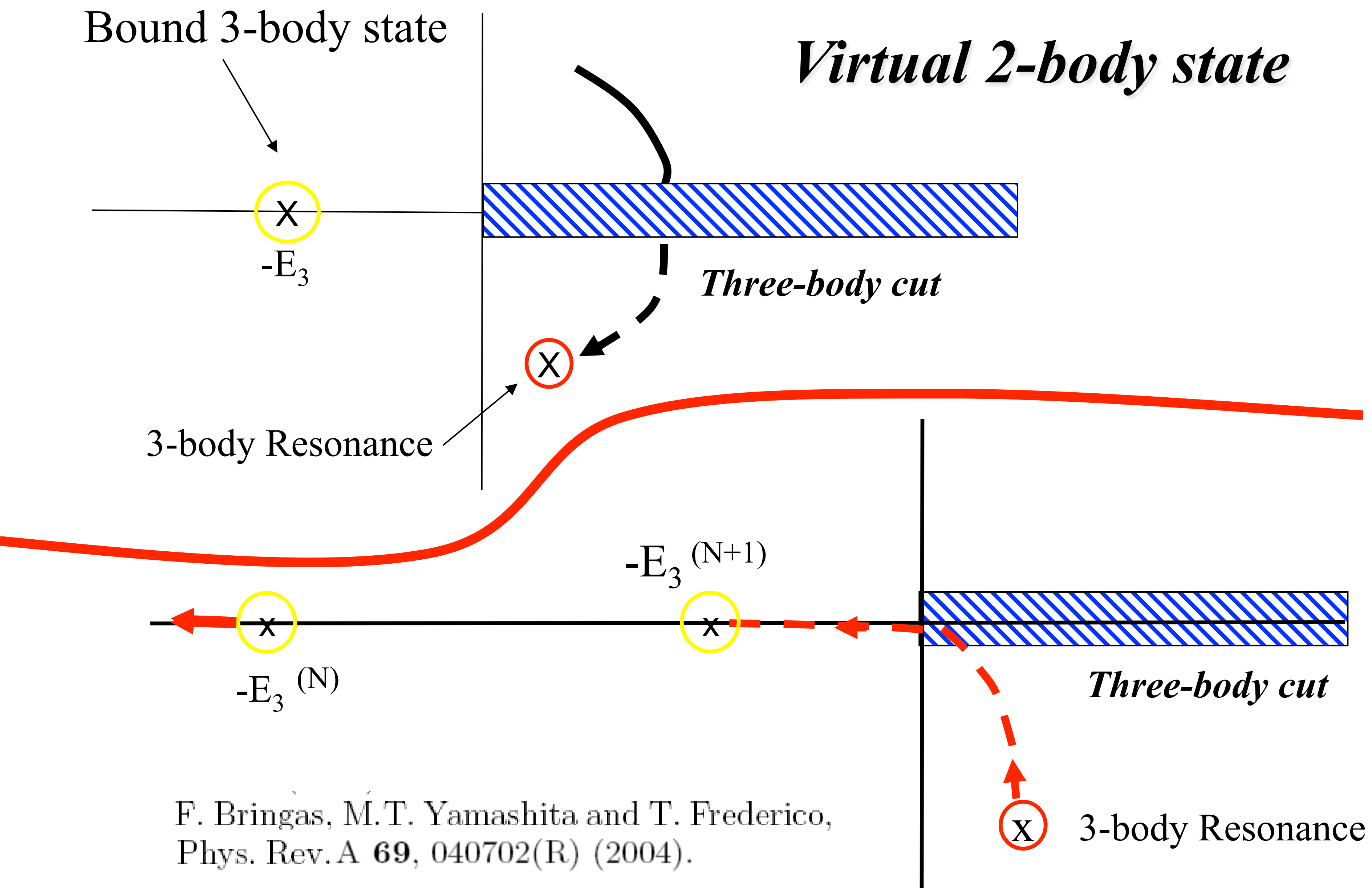
Range correction: Thogersen, Fedorov, Jensen PRA78(2008)020501(R)

Three-bosons: analytic structure & Efimov state trajectory



S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).

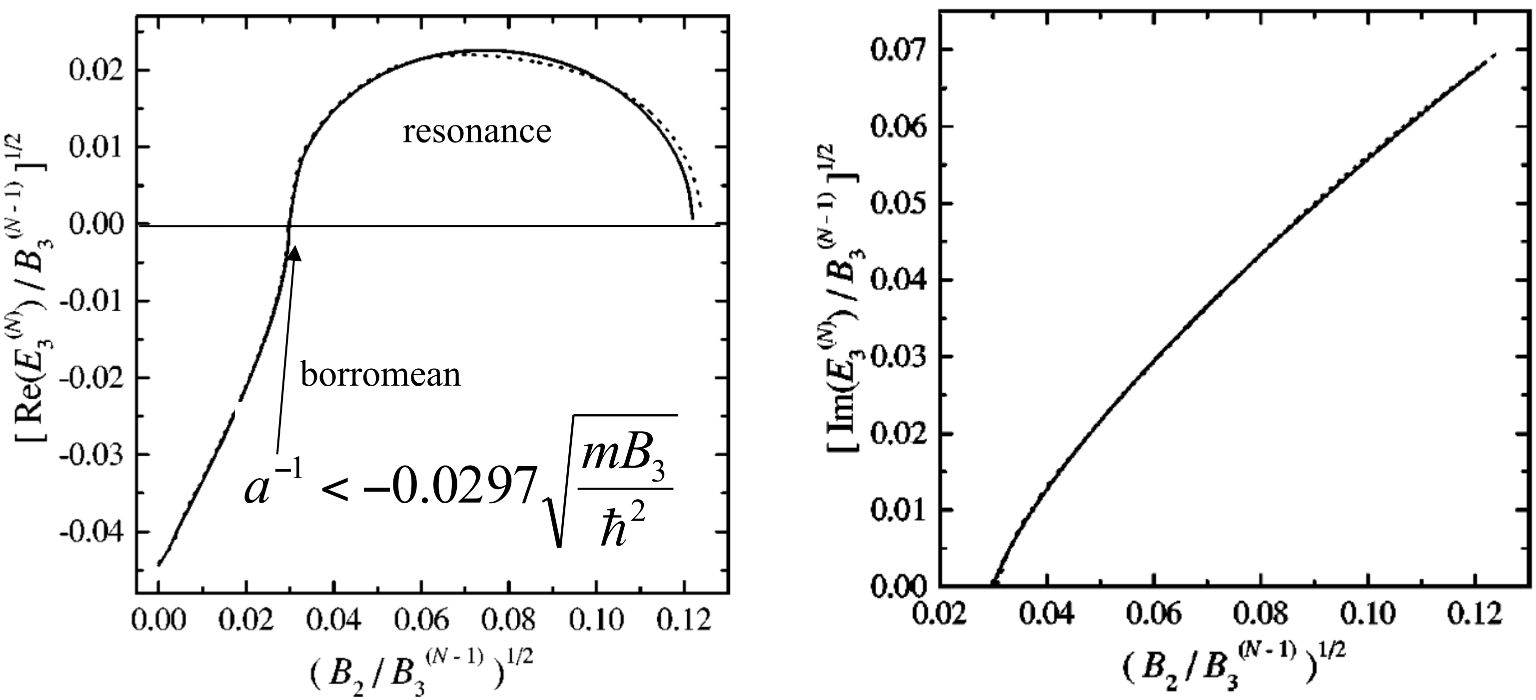
Borromean configuration: analytic structure & Efimov state trajectory



F. Bringas, M.T. Yamashita and T. Frederico,
Phys. Rev. A **69**, 040702(R) (2004).

Efimov state trajectory: borromean case

S-wave three-boson resonance



F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

Evidence of continuum resonances in recombination of ultracold Cs atoms

T. Kraemer et al, Nature **440**, 315 (2006)

Evidence of continuum resonances in ultracold cesium gas

T. Kraemer et al, Nature **440**, 315 (2006)

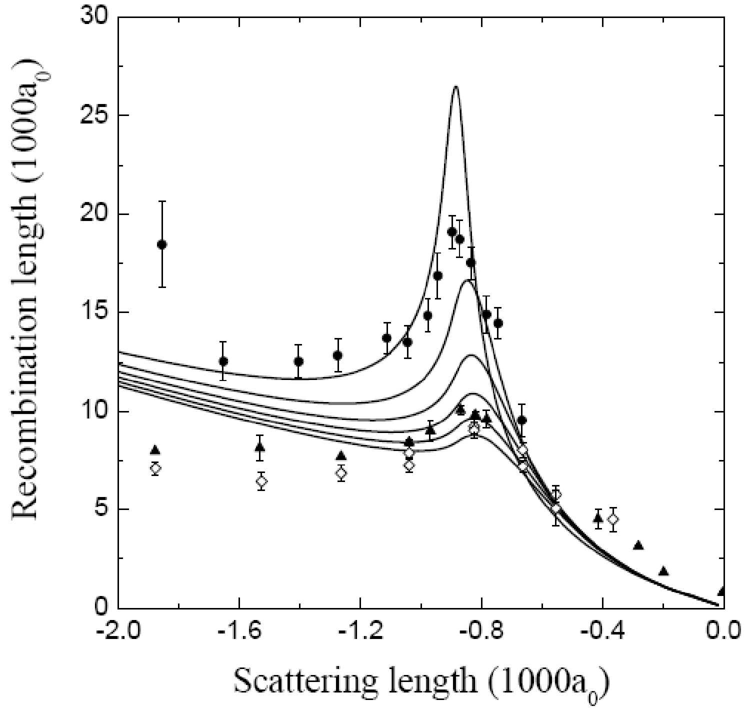
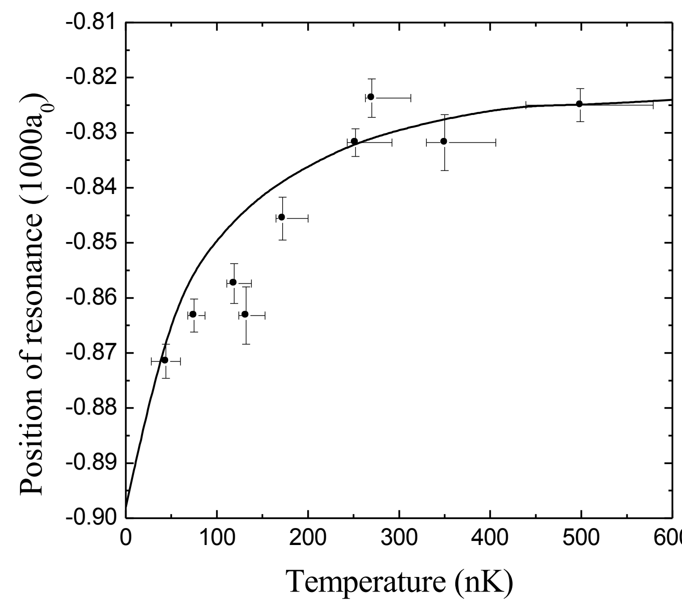


FIG. 2: Recombination length ($\rho_3 = [2m/(\sqrt{3}\hbar)\langle L_3\rangle_T]$) in the cesium trapped gas as a function of the scattering length and temperature. The solid curves from up to bottom are the theoretical results for $T = 10$ nK, 100 nK, 200 nK, 300 nK, 400 nK and 500 nK. The symbols are the experimental results for $T = 10$ nK (full circles), 200 nK (full triangles) and 250 nK (open diamonds) from Ref. [4].

Position of the maximum of the recombination length as a function of the temperature. Experimental data from B. Engeser et al., *in preparation*.



M.T. Yamashita et al. / Physics Letters A 363 (2007) 468

Single particle densities: mass imbalanced AAB systems (3d)

Yamashita et al PRA **87**, 062702 (2013)

$a \gg r_0$ Physical observables do not depend on the details of the short-range interaction

The two-body contact parameter

$$C \equiv \lim_{k \to \infty} k^4 n(k)$$

short-range two-body correlations

↕

many-body thermodynamics

Bosons 1D
Olshanii/Dunjko (2003)

Two-component Fermi gases
Tan(2008)

Experimentally verified
Stewart (2010), Kuhnle (2010)

One more contact parameter...

Let's consider a bosonic system in the universal regime ($a \gg r_0$)

Three-body contact parameter $n(k) \rightarrow \frac{C}{k^4} + f(k) D$

Functional form depends on dimensionality

Experimentally verified Wild et al. (2012)

3-boson system: Subleading terms averages to zero- Werner and Castin PRA 83, 063614 (2011)

$$n_{3D}(q) \rightarrow \frac{C}{q^4} + \frac{\sin^2[s\ln(q/q_3^*)]}{q^5} D_3 \;+ \ldots..$$

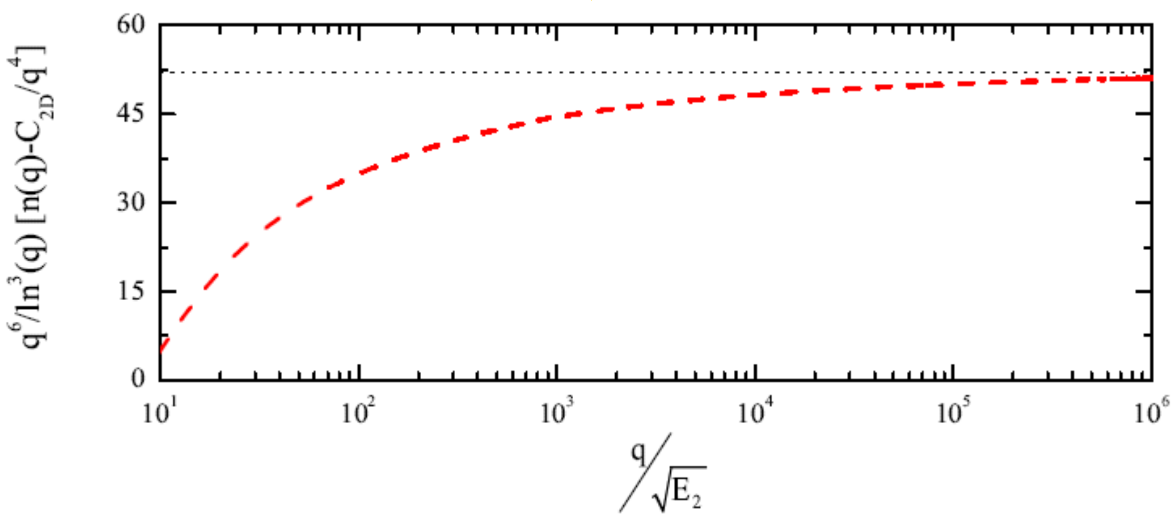
accidental cancellation of the average ?

What happens in the 3-boson case if we change to 2d?

Belotti et al PRA87, 013610 (2013)

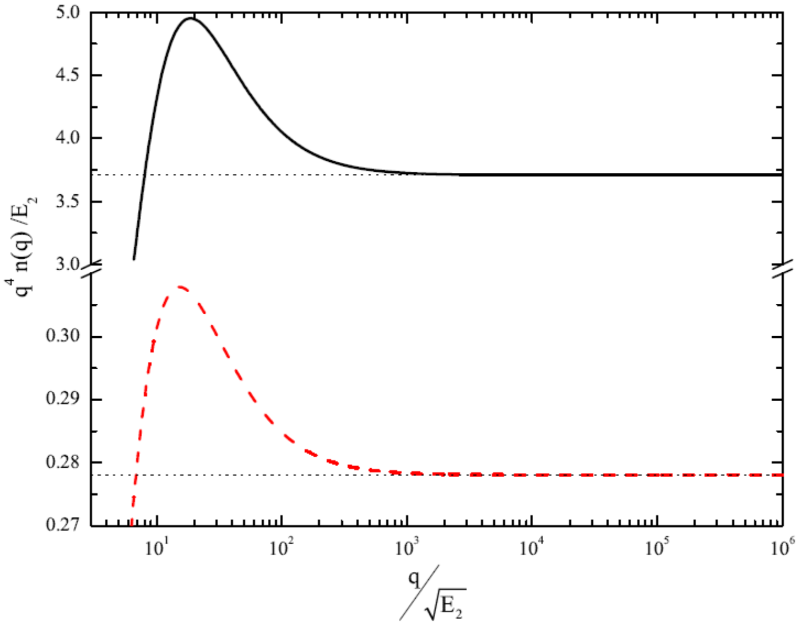
$$E_3^{(0)} = 16.52 E_2 \quad E_3^{(1)} = 1.270 E_2$$

$$n_{3\mathrm{D}}(q) \to \frac{C}{q^4} + \frac{\sin^2[s \ln(q/q_3^*)]}{q^5} D_3 + \ldots \quad n_{2\mathrm{D}}(q) \to \frac{C^{(n)}}{q^4} + \frac{\ln^3(q/q_2^*)}{q^6} D_2^{(n)}$$



$D_2^{(n)}$ depends weakly on (n)

Leading-order term: two-body contact parameter 2d



$$n_3^0(q) \to \frac{3.71E_2}{q^4}$$

$$n_3^1(q) \to \frac{0.28E_2}{q^4}$$

Dividing $C^{(n)}$ by the "natural scale" E_3

$$\frac{C^{(1)}}{E_3^{(1)}} = \frac{0.28E_2}{1.270E_2} = 0.219 \qquad \frac{C^{(0)}}{E_3^{(0)}} = \frac{3.71E_2}{16.52E_2} = 0.224$$

$$\frac{C}{E_3} = 0.222 \pm 0.003$$

Mass imbalanced AAB systems in 3d

AAB wave function:

$$\langle \vec{q}_B \vec{p}_B | \Psi \rangle = \frac{\chi_{AA}(q_i) + \chi_{AB}(q_j) + \chi_{AB}(q_k)}{E_3 + H_0} = \frac{\chi_{AA}(q_B) + \chi_{AB}(|\vec{p}_B - \frac{\vec{q}_B}{2}|) + \chi_{AB}(|\vec{p}_B + \frac{\vec{q}_B}{2}|)}{E_3 + H_0}$$

Spectator functions SKM subtracted equations:

$$\chi_{AA}(y) = 2\tau_{AA}(y;E_3)\int_0^\infty dx \frac{x}{y} G_1(y,x;E_3)\chi_{AB}(x)$$

$$\chi_{AB}(y) = \tau_{AB}(y;E_3)\int_0^\infty dx \frac{x}{y}\left[G_1(x,y;E_3)\chi_{AA}(x) + \mathcal{A}G_2(y,x;E_3)\chi_{AB}(x)\right]$$

Mass imbalanced AAB systems in 3d

Spectator functions 3d

$$\tau_{AA}(y;E_3) \equiv \frac{1}{\pi}\left[\sqrt{E_3+\frac{\mathcal{A}+2}{4\mathcal{A}}y^2}\mp\sqrt{E_{AA}}\right]^{-1}$$

$$\tau_{AB}(y;E_3) \equiv \frac{1}{\pi}\left(\frac{\mathcal{A}+1}{2\mathcal{A}}\right)^{3/2}\left[\sqrt{E_3+\frac{\mathcal{A}+2}{2(\mathcal{A}+1)}y^2}\mp\sqrt{E_{AB}}\right]^{-1}$$

$$G_1(y,x;\epsilon_3) \equiv \ln\frac{2\mathcal{A}(\epsilon_3+x^2+xy)+y^2(\mathcal{A}+1)}{2\mathcal{A}(\epsilon_3+x^2-xy)+y^2(\mathcal{A}+1)} - \ln\frac{2\mathcal{A}(\mu^2+x^2+xy)+y^2(\mathcal{A}+1)}{2\mathcal{A}(\mu^2+x^2-xy)+y^2(\mathcal{A}+1)}$$

$$G_2(y,x;\epsilon_3) \equiv \ln\frac{2(\mathcal{A}\epsilon_3+xy)+(y^2+x^2)(\mathcal{A}+1)}{2(\mathcal{A}\epsilon_3-xy)+(y^2+x^2)(\mathcal{A}+1)} - \ln\frac{2(\mathcal{A}\mu^2+xy)+(y^2+x^2)(\mathcal{A}+1)}{2(\mathcal{A}\mu^2-xy)+(y^2+x^2)(\mathcal{A}+1)}$$

For $\epsilon_2=\epsilon_3=0$ and $\mu_3\to\infty$

$$\chi_{AA}(y)=c_{AA}\, y^{-2+\imath s} \qquad \chi_{AB}(y)=c_{AB}\, y^{-2+\imath s}$$

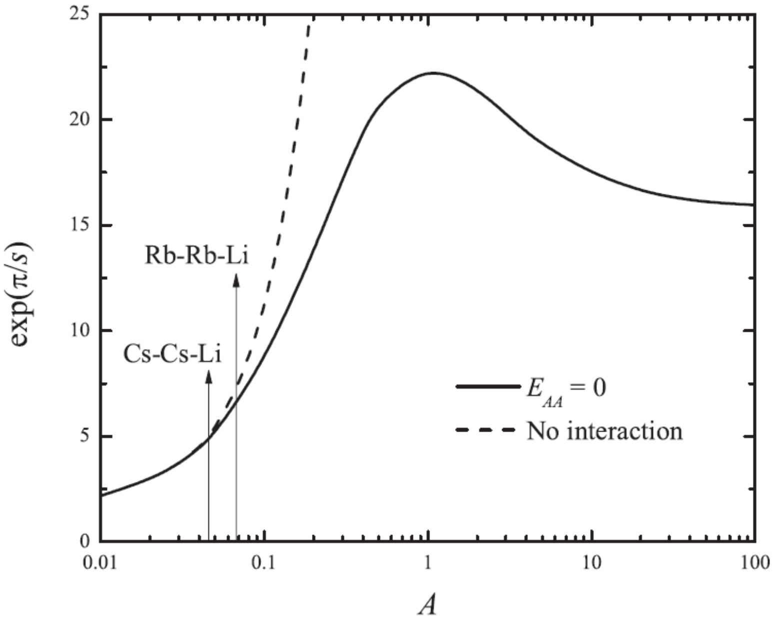


FIG. 1. Scaling parameter s as a function of $\mathcal{A} = m_B/m_A$ for $E_{AA} = 0$ and $E_{AB} = 0$ (resonant interactions) (solid line) and for the situation where $E_{AB} = 0$ but with no interaction between AA (dashed line). The arrows show the corresponding mass ratios for ^{133}Cs-^{133}Cs-^{6}Li and ^{87}Rb-^{87}Rb-^{6}Li.

Asymptotic forms

$$\chi_{AA}(q) = c_{AA} \frac{\sin[s \ln(q/q^*)]}{q^2}$$

$$\chi_{AB}(q) = c_{AB} \frac{\sin[s \ln(q/q^*)]}{q^2}$$

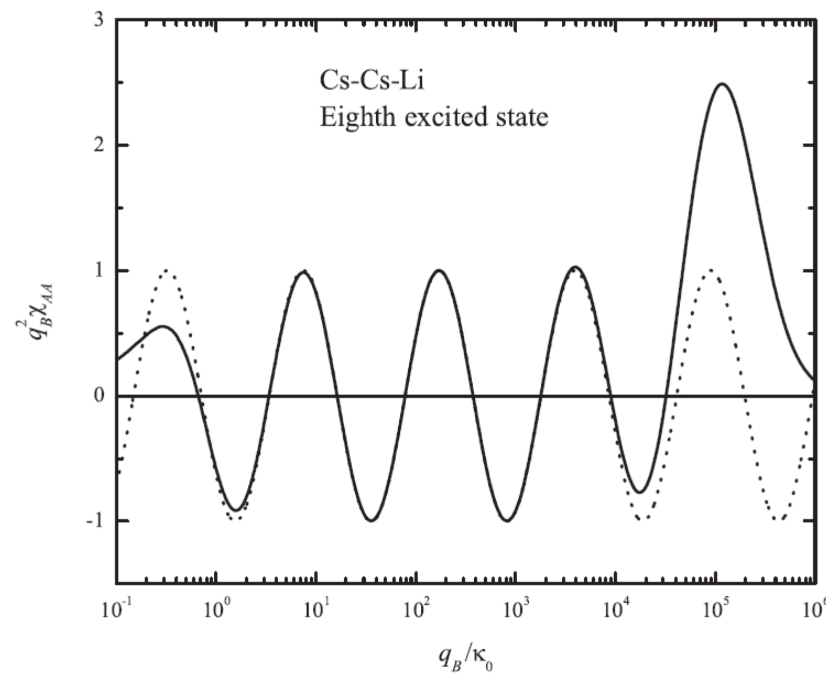
Asymptotic window ($\hbar = m_A = \mu = 1$)

$$\sqrt{E_3} \ll q \ll \mu$$

$$\kappa_0 \equiv \sqrt{E_3}$$

$$1 \ll \frac{q}{\kappa_0} \ll \frac{1}{\kappa_0}$$

Asymptotic region?



3D Momentum distribution AAB systems 3d

$$\langle \vec{q}_B \vec{p}_B | \Psi \rangle = \frac{\chi_{AA}(q_i) + \chi_{AB}(q_j) + \chi_{AB}(q_k)}{E_3 + H_0} = \frac{\chi_{AA}(q_B) + \chi_{AB}(|\vec{p}_B - \frac{\vec{q}_B}{2}|) + \chi_{AB}(|\vec{p}_B + \frac{\vec{q}_B}{2}|)}{E_3 + H_0}$$

$$n(q_B) = \int d^3p_B |\langle \vec{q}_B \vec{p}_B | \Psi \rangle|^2 \qquad n(q_B) = \sum_{i=1}^{4} n_i(q_B)$$

$$n_1(q_B) = |\chi_{AA}(q_B)|^2 \int d^3p_B \frac{1}{(E_3 + p_B^2 + q_B^2 \frac{\mathcal{A}+2}{4\mathcal{A}})^2} = \pi^2 \frac{|\chi_{AA}(q_B)|^2}{\sqrt{E_3 + q_B^2 \frac{\mathcal{A}+2}{4\mathcal{A}}}},$$

$$n_2(q_B) = 2 \int d^3p_B \frac{\left|\chi_{AB}(|\vec{p}_B - \frac{\vec{q}_B}{2}|)\right|^2}{(E_3 + p_B^2 + q_B^2 \frac{\mathcal{A}+2}{4\mathcal{A}})^2} = 2 \int d^3q_A \frac{|\chi_{AB}(q_A)|^2}{(E_3 + q_A^2 + \vec{q}_A \cdot \vec{q}_B + q_B^2 \frac{\mathcal{A}+1}{2\mathcal{A}})^2}$$

$$n_3(q_B) = 2\, \chi_{AA}^*(q_B) \int d^3p_B \frac{\chi_{AB}(|\vec{p}_B - \frac{\vec{q}_B}{2}|)}{(E_3 + p_B^2 + q_B^2 \frac{\mathcal{A}+2}{4\mathcal{A}})^2} + c.c.$$

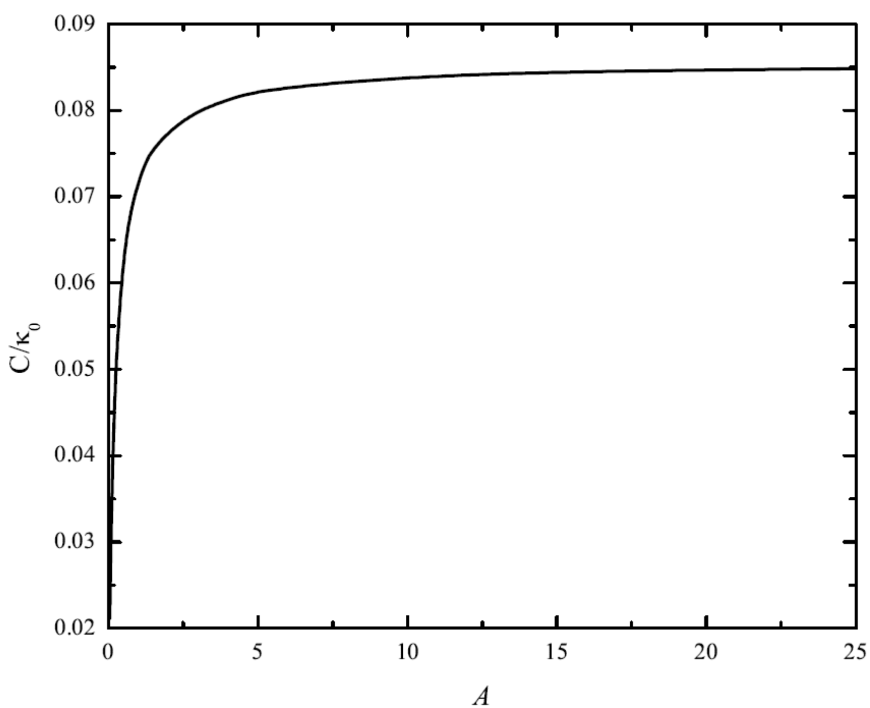
$$n_4(q_B) = \int d^3p_B \frac{\chi_{AB}^*(|\vec{p}_B - \frac{\vec{q}_B}{2}|)\chi_{AB}(|\vec{p}_B + \frac{\vec{q}_B}{2}|)}{(E_3 + p_B^2 + q_B^2 \frac{\mathcal{A}+2}{4\mathcal{A}})^2} + c.c.$$

The asymptotic limit in n_2 gives the leading-order term $\frac{C}{q_B^4}$

$$n_2(q_B) = 2\int d^3q_A \frac{|\chi_{AB}(q_A)|^2}{\left(q_A^2 + \vec{q}_A\cdot\vec{q}_B + q_B^2\,\frac{\mathcal{A}+1}{2\mathcal{A}}\right)^2} = \frac{8\mathcal{A}^2}{q_B^4\,(\mathcal{A}+1)^2}\int d^3q_A\ |\chi_{AB}(q_A)|^2$$

$$+\int d^3q_A\,|\chi_{AB}(q_A)|^2\left[\frac{2}{\left(q_A^2 + \vec{q}_A\cdot\vec{q}_B + q_B^2\,\frac{\mathcal{A}+1}{2\mathcal{A}}\right)^2} - \frac{8\mathcal{A}^2}{(\mathcal{A}+1)^2}\frac{1}{q_B^4}\right]$$

and C is simply given by $C = \frac{8\mathcal{A}^2}{(\mathcal{A}+1)^2}\int d^3q_A\,|\chi_{AB}(q_A)|^2$



	C / κ_0
^{133}Cs-^{133}Cs-^{6}Li	0.0274
^{87}Rb-^{87}Rb-^{6}Li	0.0211
$A = 1$	0.0715

x $3(2\pi)^3 = 53.197$

"Exact" value: 53.097

Werner and Castin PRA 83, 063614 (2011)

Analysis of subleading terms 3d

Werner and Castin PRA 83, 063614 (2011)

The non-oscillatory term of order q_B^{-5} coming from n_1 to n_4 cancels for $\mathcal{A}=1$

After averaging out the oscillating part:

$$\langle n_1(q_B)\rangle = \frac{\pi^2}{q_B^5}\ |c_{AA}|^2\ \sqrt{\frac{\mathcal{A}}{\mathcal{A}+2}}$$

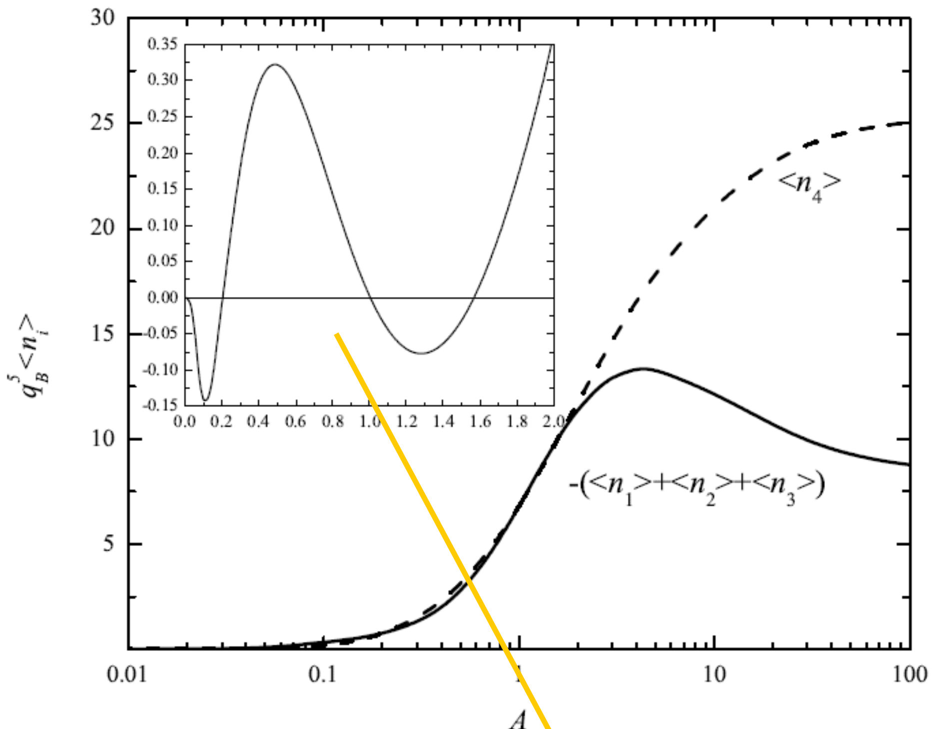
$$\langle n_2(q_B)\rangle = -\frac{8\pi^2\,|c_{AB}|^2}{q_B^5}\frac{\mathcal{A}^3(\mathcal{A}+3)}{(\mathcal{A}+1)^3\sqrt{\mathcal{A}(\mathcal{A}+2)}}$$

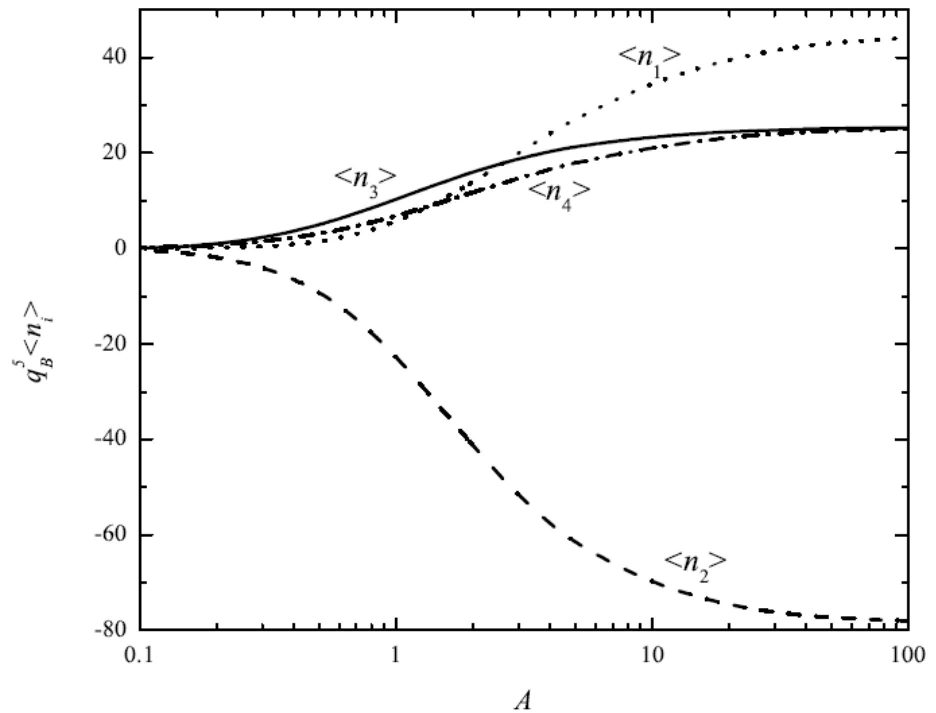
$$\langle n_3(q_B)\rangle = \frac{4\pi^2 c_{AA}\ c_{AB}}{q_B^5\cosh\left(\frac{s\pi}{2}\right)}\left\{\sqrt{\frac{\mathcal{A}}{\mathcal{A}+2}}\cos\left(s\ln\sqrt{\frac{\mathcal{A}+1}{2\mathcal{A}}}\right)\cosh\left[s\left(\frac{\pi}{2}-\theta_3\right)\right]+\sin\left(s\ln\sqrt{\frac{\mathcal{A}+1}{2\mathcal{A}}}\right)\sinh\left[s\left(\frac{\pi}{2}-\theta_3\right)\right]\right\}$$

$$\tan\theta_3 = \sqrt{\frac{\mathcal{A}+2}{\mathcal{A}}}\quad \text{for}\quad 0 \le \theta_3 \le \pi/2$$

$$\langle n_4(q_B)\rangle = \frac{8\pi^2|c_{AB}|^2\mathcal{A}^2}{s\, q_B^5\ \cosh\left(\frac{s\pi}{2}\right)}\left\{\sinh\left[s\left(\frac{\pi}{2}-\theta_4\right)\right] - \frac{s\,\mathcal{A}}{\sqrt{\mathcal{A}(\mathcal{A}+2)}(\mathcal{A}+1)}\cosh\left[s\left(\frac{\pi}{2}-\theta_4\right)\right]\right\}$$

$$\tan\theta_4 = \sqrt{\mathcal{A}(\mathcal{A}+2)} \text{ for } 0 \le \theta_4 \le \pi/2$$





The nonoscillatory term of order q_B^{-5} coming from n_1 to n_4 cancels for A =1. Cancellation of the subleading nonoscillatory term for other mass ratios:

A = 0.2, 1 and 1.57

Single particle densities: mass imbalanced ABC systems (2d)

Bellotti et al New J. Phys. 16, 013048 (2014) & JPB46, 055301(2013)

$$\lim_{q\to\infty} f_\alpha(q) \to \frac{\Gamma}{m_{\beta\gamma}} \frac{\ln q}{q^2}$$

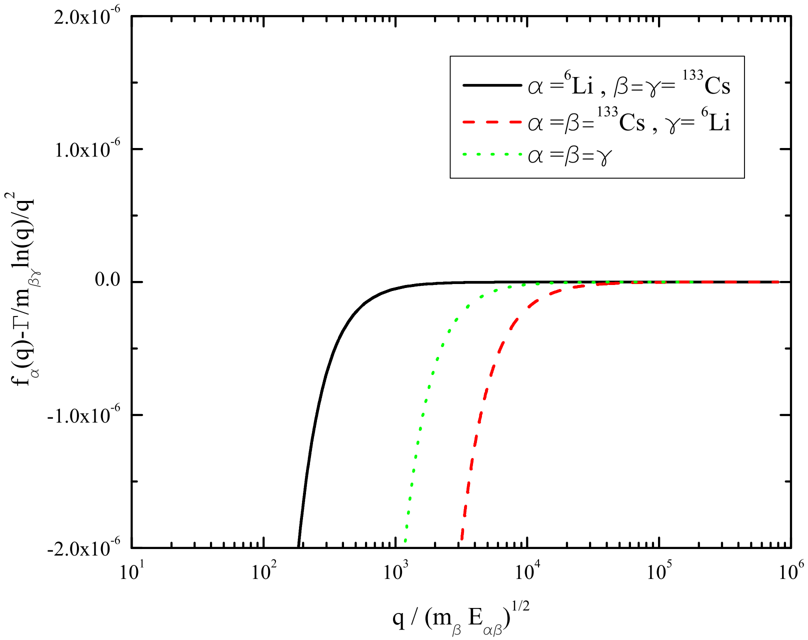


Figure 1. The difference $f_\alpha(q) - \frac{\Gamma}{m_{\beta\gamma}} \frac{\ln q}{q^2}$ as a function of the momentum q. We see that (19) exactly describes the asymptotic spectator function within our accuracy.

$$n(q_\alpha) \to \frac{C}{q^4} + \frac{D \ln^2(q_\alpha)}{q_\alpha^6}$$

Four-bosons in 3d with zero-range interaction

Yamashita, Tomio, Delfino & Frederico
Four-boson scale near a Feshbach resonance. Europhys. Lett.75 (2006) 555

•***Tetramer ground state moves as a short-range scale collapses to zero with the trimer is fixed***!
•coupling between a closed and open channels→ many-body forces in the open channel

•***Tetramer is fixed by the trimer information:***

Platter, Hammer, & Meissner,
Four-boson system with short-range interactions. Phys. Rev. A 70, 52101 (2004).

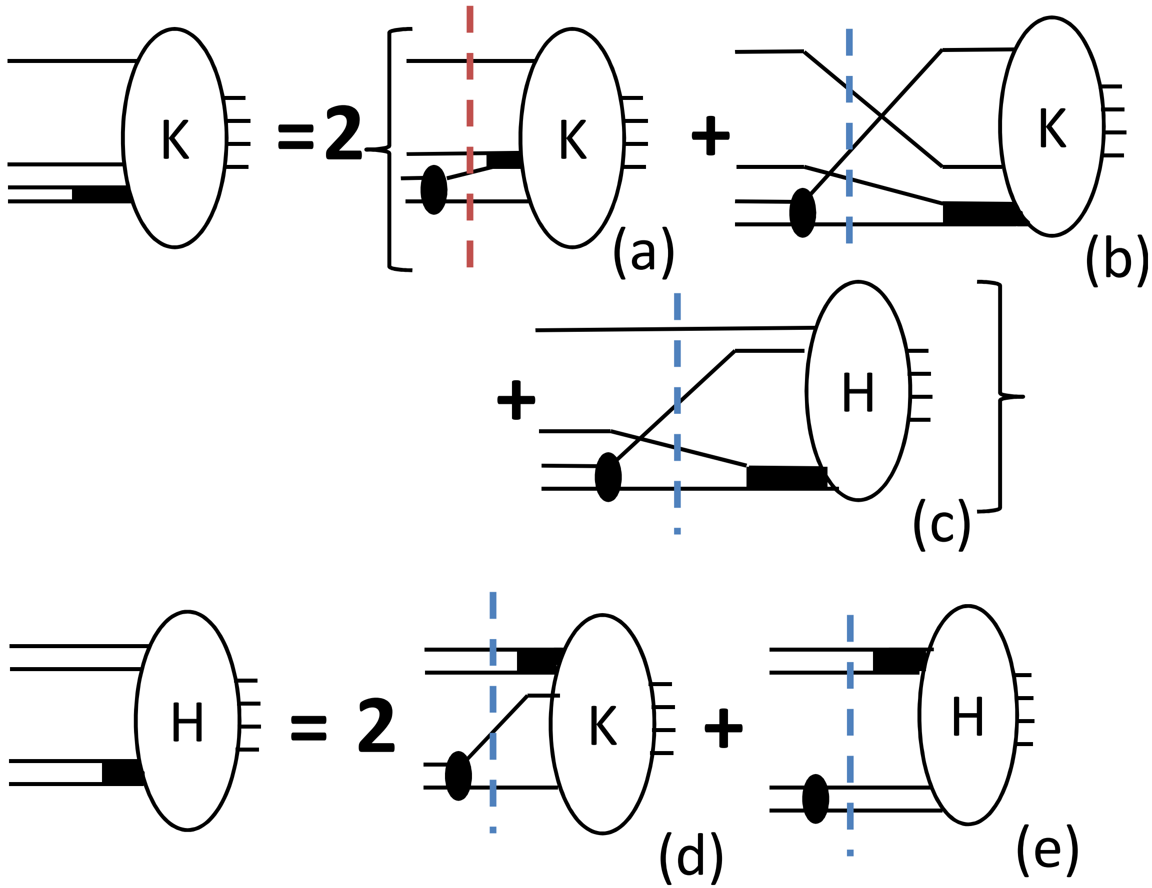
Stecher, D'Incao &Greene,
Signatures of universal four-body phenomena and their relation to the Efimov physics Nat.Phys. 5(09)417

Deltuva
Efimov physics in bosonic atom-trimer scattering, Phys. Rev. A 82, 040701(R) (2010)

Gattobigio, Kievsky,Viviani, Birse, Hiyama…

4-body force near the Feshbach resonance?

Four-bosons

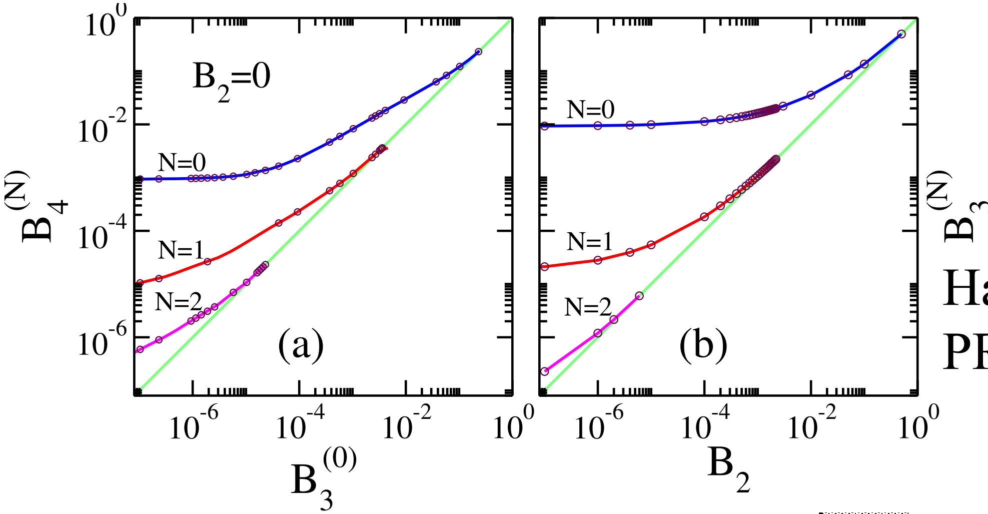


Subtracted Green's Functions: $G_0^{(N)} = \frac{1}{E-H_0} - \frac{1}{-\mu_N^2-H_0}$

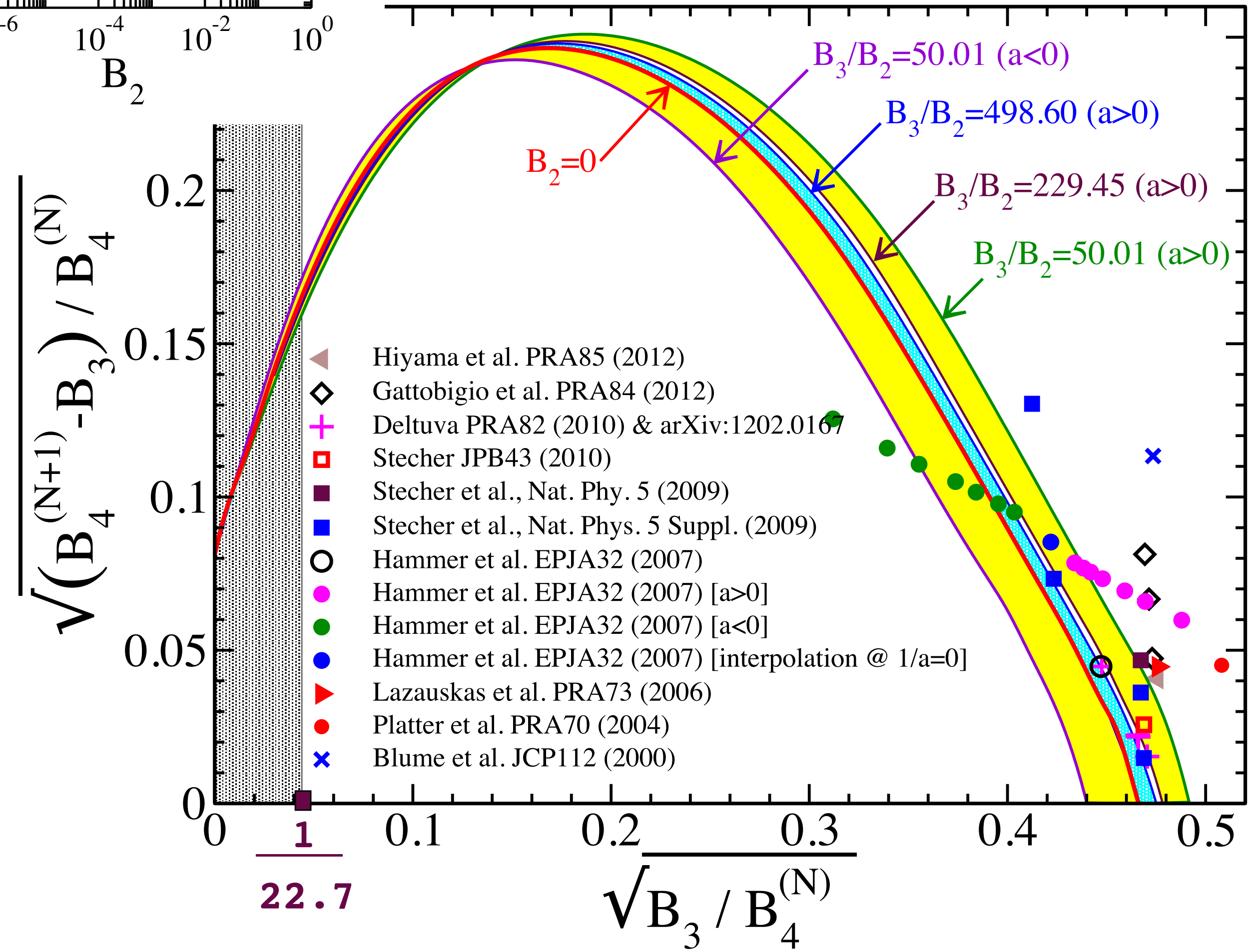
with μ_3 (RED): 3B scale & μ_4 (BLUE): 4B scale

Yamashita, Tomio, Delfino, TF, EPL 75 (2006) 555.

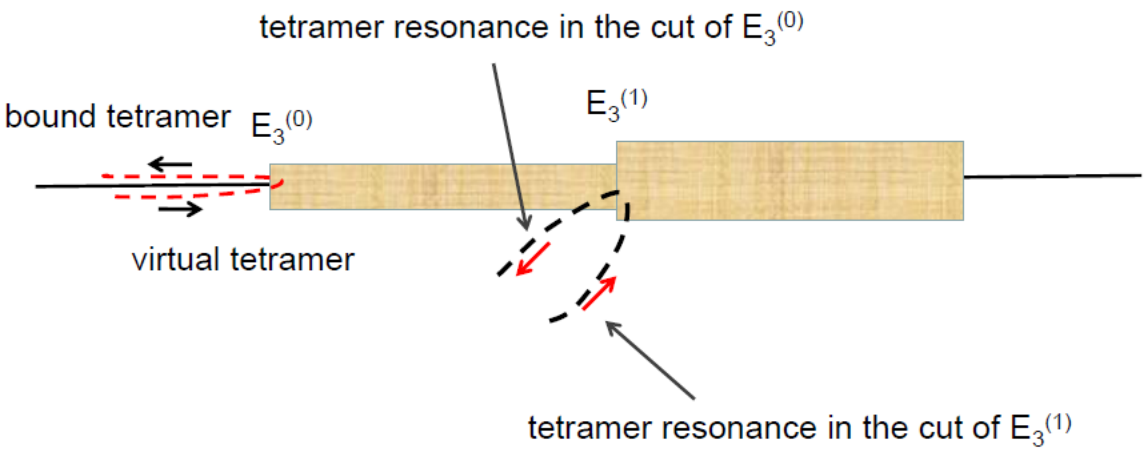
$a = \infty$



Hadizadeh, Yamashita,Tomio, Delfino, TF, PRL107, 135304 (2011)



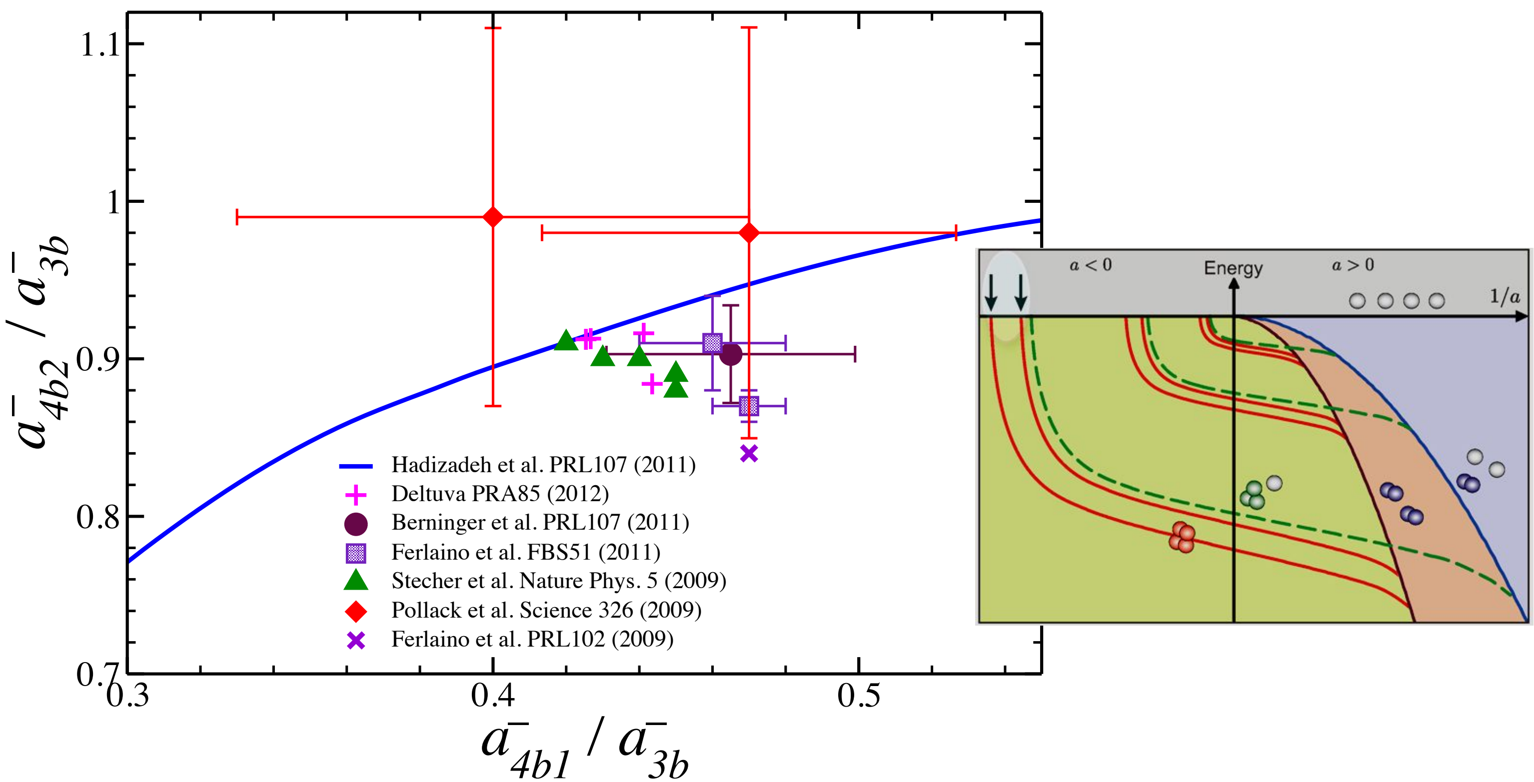
Trajectory of four-boson bound states: one scenario…



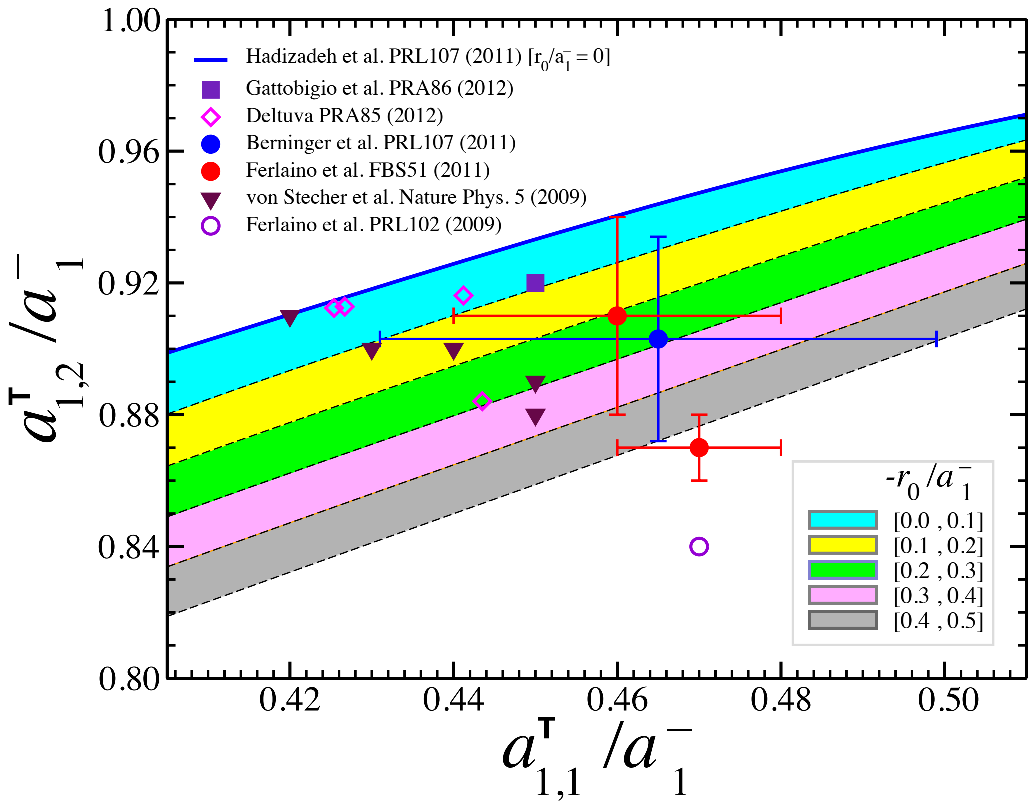
?!

Problem: Position of four-atom resonant recombination

- Positions of four-atom recombination peaks ($a < 0$) where two successive tetramers become unbound (blue-solid line). Cesium atoms wide Feshbach resonances.
- (First point from the left corresponds to $B_4 \simeq 64\, B_3$ at the unitary limit.)



Range correction to the position of 4-atom resonance



$$r_0/a_{3b}^{-} = 0, -0.1, -0.2, -0.3, -0.4, -0.5$$

- von Stecher (priv. comm.) 0.38 vs. $\sim$0.37
- Deltuva (priv. comm.) 0.33 vs. $\sim$0.29

r_0 from the shift of the peaks of the four-atom losses

Ref.	$a^T_{1,1}/a^-_1$	$a^T_{1,2}/a^-_1$	a^-_1 [$R_{\rm vdW}$]	r_0 [$R_{\rm vdW}$]
Ferlaino et al PRL'09	0.47	0.84	-8.7(1)	> 5
Berniger et al PRL'11	0.465(34)	0.903(31)	-9.54(28)	2.5 ± 1.7
Ferlaino et al FBS'11	0.47(1)	0.87(1)	-8.71	4.8 ± 1.0
Ferlaino et al FBS'11	0.46(2)	0.91(3)	-9.64	2 ± 2

- $R^{Cs_2}_{\rm vdW} = 101.0\, a_0$ [Chin et al RMP82(2010)]
- $\bar{a}^{Cs_2} \simeq 0.955978\, R^{Cs_2}_{\rm vdW} = 96.5\, a_0$.
- $3.5 < r_0 < 4.3\, R_{\rm vdW}$
- Weighted average for the fitted r_0 values: $3.9 \pm 0.8\, R_{\rm vdW}$

$$r_0 \simeq 2.9179\, \bar{a} \left[\left(\frac{\bar{a}}{a} \right)^2 + \left(\frac{\bar{a}}{a} - 1 \right)^2 \right]$$

Gribakin and Flambaum PRA48 (1993)

Dimensional crossover transitions 3d→2d

Yamashita et al arXiv:1404.7002 [cond-mat.quant-gas]

Periodic boundary conditions: $$p_z = \frac{2\pi n}{L} = \frac{n}{R}\ ,$$

with $n = 0, \pm 1, \pm 2\ldots$ and $L = 2\pi R$.

$$f(\vec{q}_\perp, n) = -2\,\tau_R\left(E_3 - \frac{3}{4}(q_\perp^2 + \frac{n^2}{R^2})\right) \times \sum\!\!\!\!\!\!\int \frac{d^2p_\perp}{R}\left[g_{0R}(E) - g_{0R}(-\mu^2)\right] f(\vec{p}_\perp, m)\quad ,$$

$$\tau_R(E) = -R\left[\pi \ln\left(\frac{\sinh \pi\sqrt{-E}R}{\sinh \pi\sqrt{-E_2}R}\right)\right]^{-1}$$

where

$$g_{0R}^{-1}(E) = E - q_\perp^2 - p_\perp^2 - \vec{q}_\perp\cdot\vec{p}_\perp - \frac{n^2}{R^2} - \frac{m^2}{R^2} + \frac{n\,m}{R^2}.$$

Change in the two-body bound state-energy with R:

$$\lim_{\Lambda\to\infty}\left\{\int_{-\infty}^{\infty} dy \ln\left[\frac{-E_2^{3D}R^2 + y^2}{-E_2^{3D}R^2 + y^2 + (\Lambda\,R)^2}\right] - \sum_{n=-\infty}^{\infty}\ln\left[\frac{E\,R^2 - n^2}{E\,R^2 - n^2 - (\Lambda\,R)^2}\right]\right\} = 0$$

$$\sqrt{-E_2} = \frac{1}{\pi R}\sinh^{-1}\frac{e^{\pi\,R/a}}{2}$$

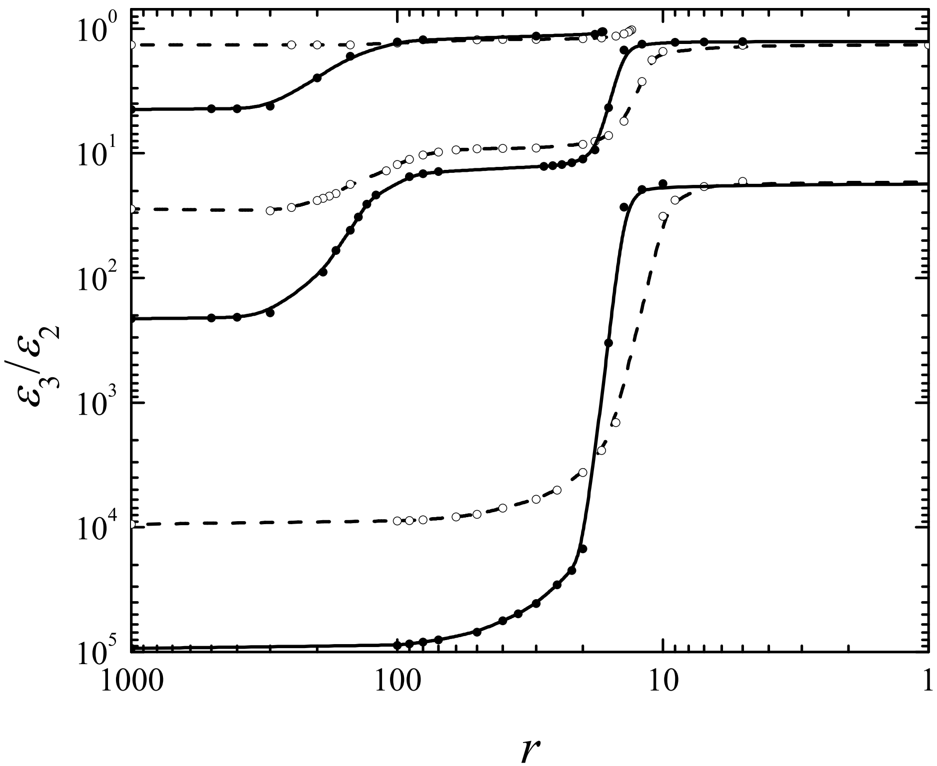


FIG. 2: ϵ_3/ϵ_2 as a function of r, for $\epsilon_2^{3D} = 10^{-7}$ (full circles) and 10^{-6} (empty circles). The solid and dashed lines are guides to the eye. As we approach the 2D limit ($r \to 0$), higher excited states disappear and only the ground and first excited states remain. Note that the values of r and ϵ_3/ϵ_2 increase from right to left and top to bottom respectively.

Summary

- Zero-range model 3B and 4B systems in 3d:
  Scaling functions & limit cycles & correlation between observables
  3B threshold conditions for excited states and resonances
  3B borromean configuration: **Efimov state→ resonance**
  3B at least one subsystem is bound: **Efimov state→ virtual state**

- 3d: q_B^{-5} nonoscillatory term cancels for A = 0.2, 1 and 1.57

- 2d/3d: Analytical forms for the asymptotic spectator functions and momentum distribution for general A

- Functional form changes drastically when passing from 3d to 2d

- 4B scaling function and position of the resonance (range corrections) in 3d

- Dimensional crossover transitions 3d→2d:
  sharp transition of Efimov state energies