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Perfect screening of the Efimov effect by the dense Fermi sea

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ENS

Laboratoire Kastler Brossel Physique quantique et applications

Outline

- **Perfect screening of the Efimov effect by the dense Fermi sea.**

S. Endo, M. Ueda, arXiv:1309.7797 (2013)

- **Universality the 3-body parameter**
  - **Different universality classes of universal 3-body parameter**

P. Naidon, S. Endo, M. Ueda, arXiv:1208.3912 (2012)

P. Naidon, S. Endo, M. Ueda, PRL. **112**, 105301 (2014)

Pascal Naidon (RIKEN)

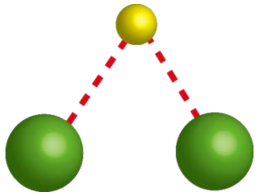
Masahito Ueda (Univ. Tokyo)

Efimov physics as a universal 3-body phenomenon

- **Efimov states appear universally in resonantly interacting 3-body systems** V. Efimov, Phys. Lett. B **33**, 563 ((1970)
  Nucl. Phys. A **210**, 157 (1973).
  - **3 identical bosons**

    T. Kraemer *et al.*, Nature **440** 315 (2006).
  - **Hetero-nuclear systems**

    2 identical bosons + 1 particle

    ^{87}Rb ^{41}K : G. Barontini, *et al.*, PRL **103**, 043201 (2009).

    ^{133}Cs ^{6}Li : S.-K. Tung *et al.*, arXiv:1402.5943 (2014)
    R. Pires *et al.*, arXiv:1403.7246 (2014)

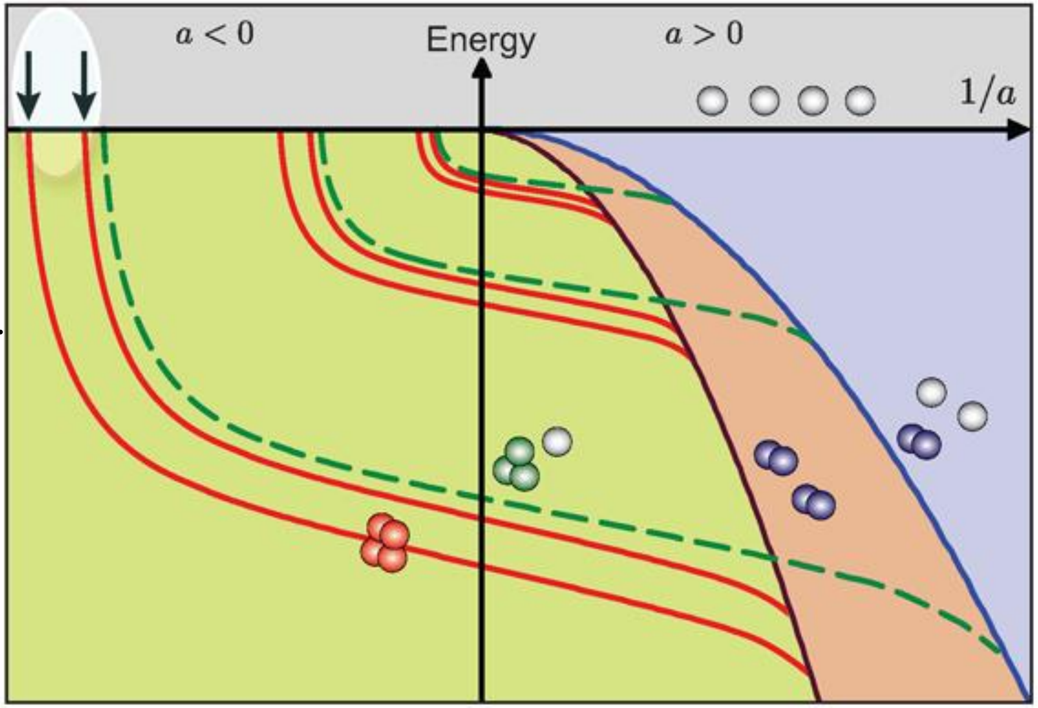
    2 identical fermions + 1 particle when M/m>13.6
  - **3 distinguishable particles when more than one interaction is resonant**

    ^{6}Li T. B. Ottenstein *et al.*, PRL. **101**,203202 (2008).
    S. Nakajima *et al.*, PRL. **106**,143201 (2011).
    J. R.Williams *et al.*, PRL. **103**, 130404 (2009).

Extending the Efimov scenario to more-body

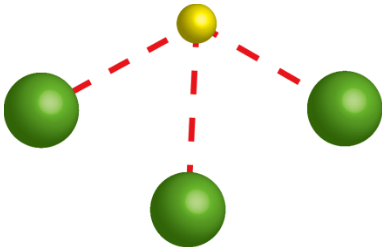
- **4-body, 5-body,….**
  - **Efimov associated N-body bound states for bosons**

F. Ferlaino, R. Grimm, Physics **3**, 9 (2010)
G. J. Hanna, D. Blume, PRA **74,** 063604 (2006).
H. W. Hammer, L. Platter, EPJ. A **32**, 113 (2007)
J. Stecher *et al*., Nature Physics **5**, 417 (2009)
A. Deltuva, EPL**. 95**, 43002 (2011)
M. R. Hadizadeh *et al*., PRL. **107,** 135304 (2011).
M. Gattobigio *et al*., PRA **84**, 052503 (2011)
and many others…



- **Four-body universal tetramer and Efimov tetramers in a mass imbalanced Fermi system**

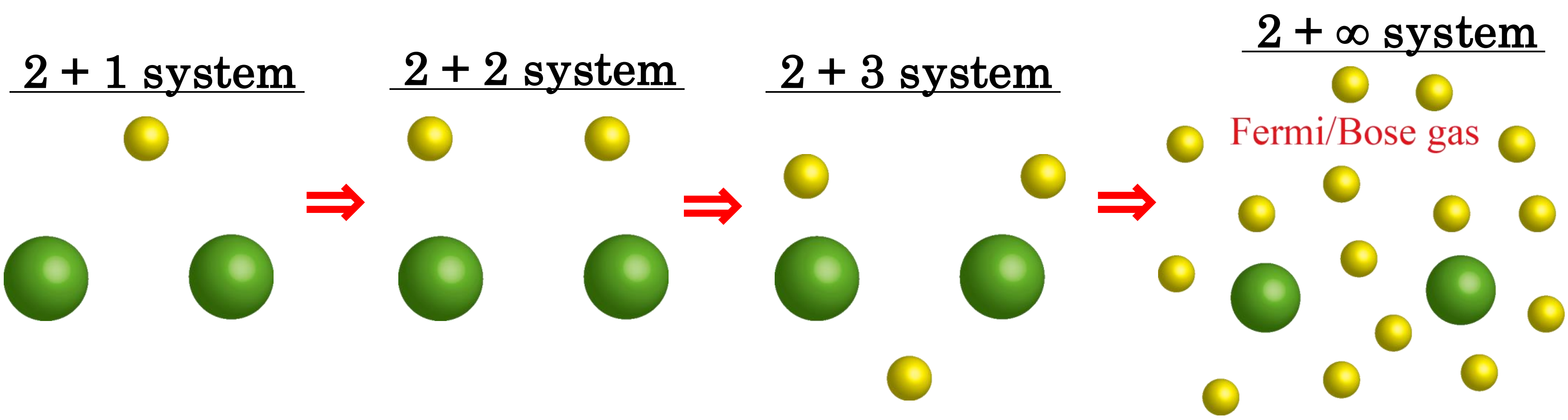
Y. Castin, *et al*., PRL. **105,** 223201 (2010)
D. Blume, PRL. **109**, 230404 (2012)



Many-body effect on the Efimov physics

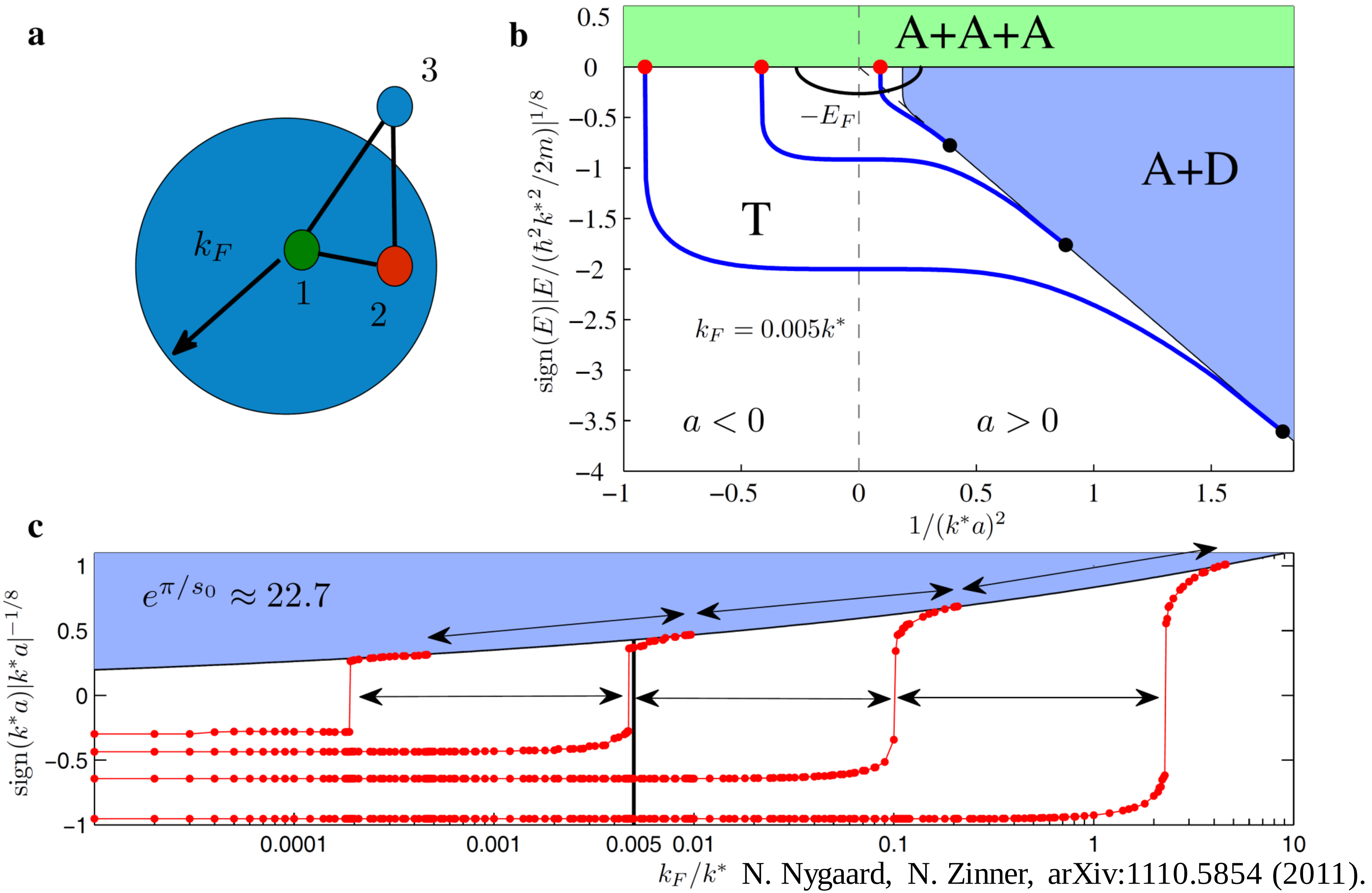
- **Resonantly interacting few-body system immersed in a many-body background**
  - **Natural extension of the universal few-body physics to many-body**
  - **How the Efimov physics affected by the many-body background?**

(c.f.) Unitary Bose gas P. Makotyn *et al*., Nature Physics **10,** 116 (2014).



Fermi sea effect on the Efimov physics

- **Fermi sea suppresses the Efimov effect when** $E \sim E_F$



N. Nygaard, N. Zinner, arXiv:1110.5854 (2011).

Many-body background effect on the Efimov physics

- **Many-body background tends to suppress the Efimov effects for various 3-body systems.**
  - **3 component Fermi system in which one of them is degenerate fermions** N. Nygaard, N. Zinner, arXiv:1110.5854 (2011).
  - **2 heavy particles in a light Fermi sea**

    D. J. MacNeill F. Zhou, PRL. **106**, 145301 (2011).

    Y. Nishida, PRA. 79, 013629 (2009).
  - **3 component Fermi system in which all components are degenerate Fermions**

    P. Niemann, H. W. Hammer, Phys. Rev. A **86**, 013628 (2012).
  - **2 heavy bosons in a BEC of light particles**

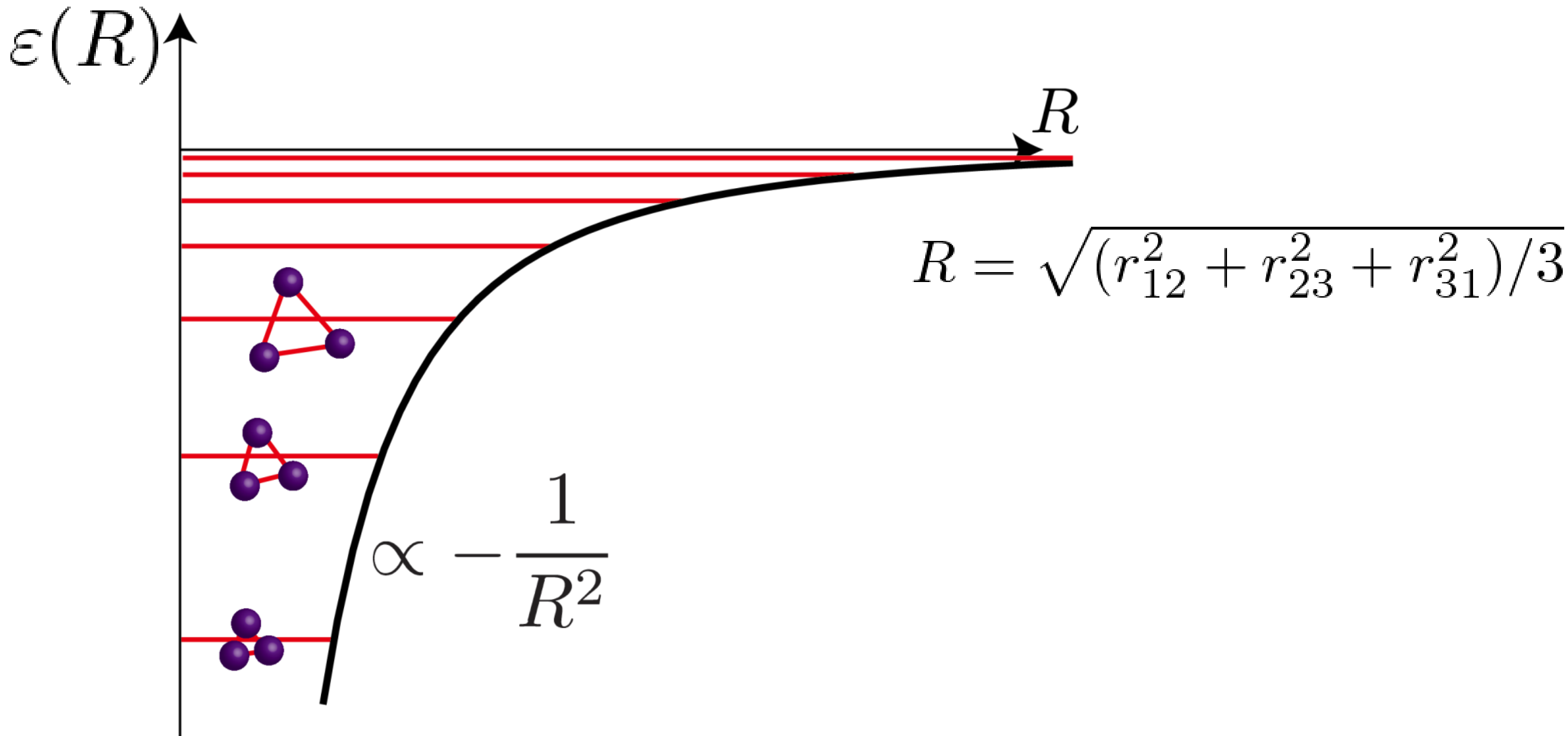
    N. T. Zinner, EPL. **101**, 60009 (2013).

Why a many-body background suppresses the Efimov effect?

- **3-body problem** $1/a = 0$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial R^2} + \varepsilon(R)\right] f(R) = E f(R)$$

Infinite number of 3-body bound states

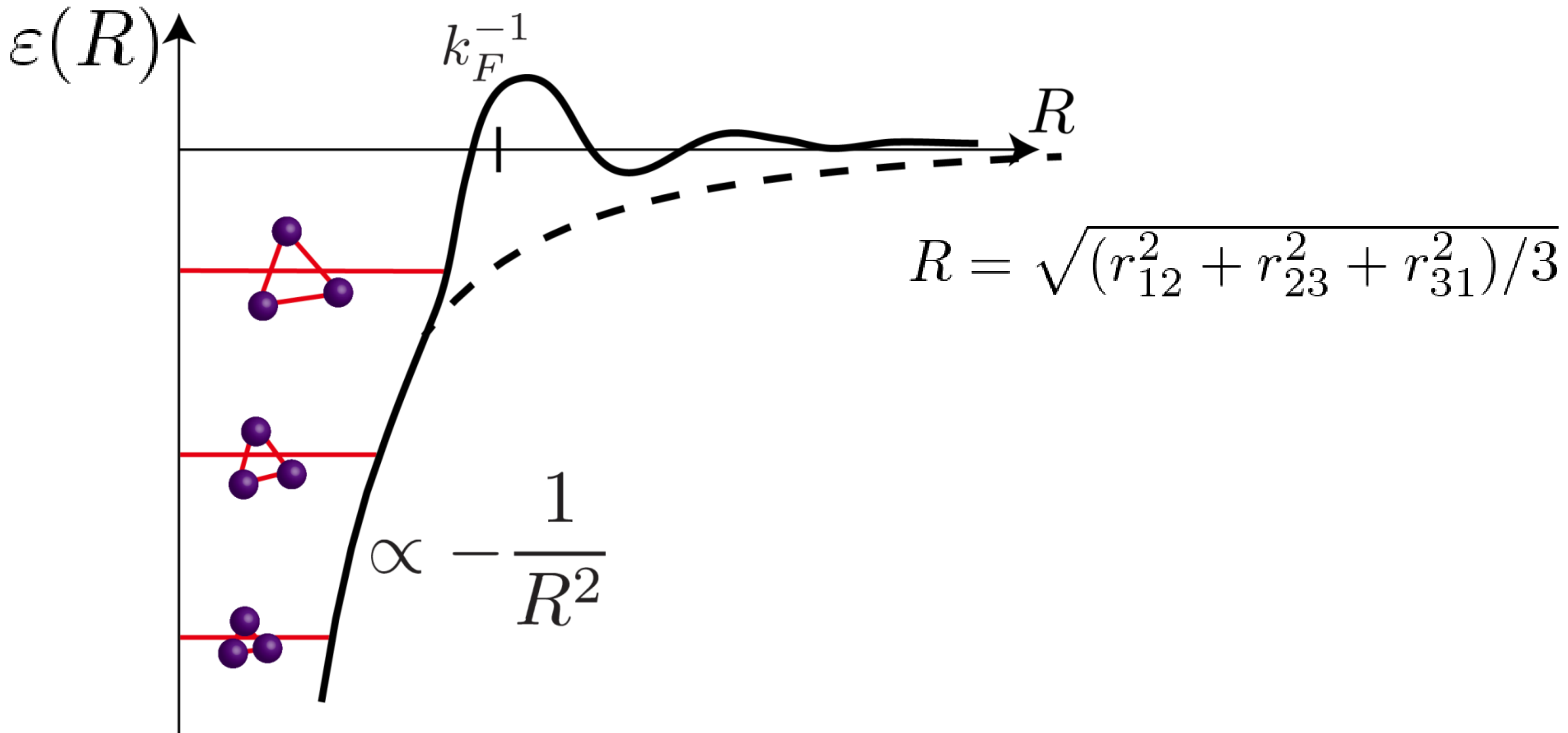


Why a many-body background suppresses the Efimov effect?

- **3-body problem in the Fermi sea** $T = 0$ $1/a = 0$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial R^2} + \varepsilon(R)\right] f(R) = Ef(R)$$

Number of 3-body bound states ~ $\log\left(\frac{\kappa^*}{k_F}\right)$



Few-body physics embedded in a background —theoretical challenge

- **Efimov effect: genuine 3-body effect**

→ One must accurately incorporates the 3-body correlation while treating the many-body background

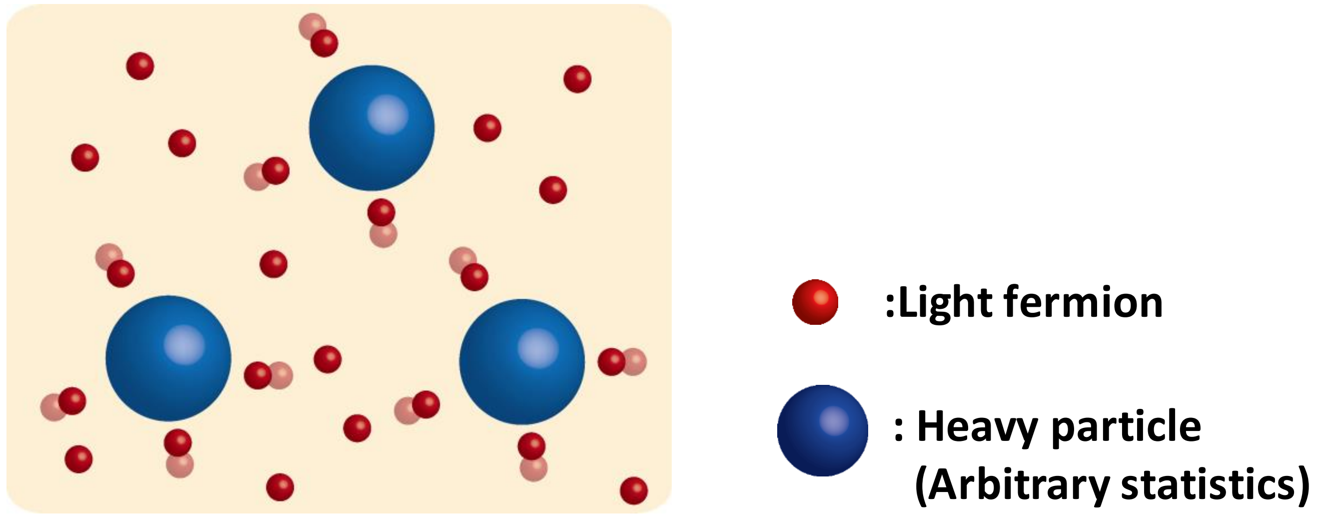
- **Suppression of the Efimov effect by the many-body background shown for**
  - **3-body system immersed in the Fermi/Bose gas**
  - **Numerical**

- **What about *N*-body system in a many-body background?**
- **Is there any analytical approach?**

Our setup

- N_H heavy particles immersed in a single-component Fermi gas

  → N_H is arbitrary as long as $O(N_H) \sim 1$
- Heavy particles: arbitrary statistics.
- Heavy-light interaction: contact s-wave interaction.
- Light-light interaction: non-interacting.



Born-Oppenheimer description of heavy particles in the Fermi sea

- **Consider highly mass imbalanced system** $M/m \gg 1$

  ⇒ Born-Oppenheimer approximation

- **Light fermions: non-interacting ⇒ Slater product of impurity problem**

$$\Psi_{\text{tot}} = \Psi_H(\boldsymbol{R_1}, \boldsymbol{R_2}, ..., \boldsymbol{R_{N_H}})\phi_{\boldsymbol{R}}(\boldsymbol{r_1}, \boldsymbol{r_2}, ...) \qquad \boldsymbol{R} = (\boldsymbol{R_1}, \boldsymbol{R_2}, ..., \boldsymbol{R_{N_H}})$$

Heavy particles' WF　　Light fermions' WF

$$\phi_{\mathbf{R}}(r_1, r_2 ...) = \mathcal{A} \prod_i \phi_{\mathbf{R}}^{(i)}(r_i)$$

← Solution of single-particle Schrodinger equation under N_H fixed impurities

$$V_{\text{eff}}(\boldsymbol{R_1}, \boldsymbol{R_2}, ..., \boldsymbol{R_{N_H}}) = E(\boldsymbol{R}) - \lim_{|\boldsymbol{R_{ij}}| \to \infty} E(\boldsymbol{R})$$

$$E(\boldsymbol{R}) = \sum_i \varepsilon_i(\boldsymbol{R})$$

- **Many-body problem reduces to one-body problem**

  ⇒ Scattering theory <u>(non-central potential)</u>

Main theorem

- **For any N_H and α , we show**

$$\lim_{k_F \to +\infty} V_{\text{eff}}(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_{N_H}) = 0,$$

Main theorem

- For any N_H and a , we show

$$\lim_{k_F \to +\infty} V_{\text{eff}}(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_{N_H}) = 0,$$

<u>**What does this relation means?**</u>

1. **The effective interaction between the heavy particles gets weak as finally vanishes as the number of the light fermions is increased.** $R_{ij}, |a| \gg k_F^{-1} \to 0$

2. **Heavy particles cannot form bound states**

⇒ No N-body Efimov states

No Efimov associated N-body bound states

No universal (Kartavtsev Malykh) N-body bound states

in the presence of a dense Fermi sea.

Step 1 of the proof: Friedel's sum rule

- **Friedel's Sum Rule (non-central potential)**

J. Friedel, *Phil. Mag*, **43**, 153 (1952).
R. G. Newton, *J. Math. Phys.* , **18**, 1348 (1977).

$$N_I - N_0 = \frac{1}{\pi} \sum_n \delta_n(k_F)$$

Change in the number of fermions induced by N_H impurity potentials

Phase shift induced by N_H impurity potentials

$$\mathcal{S}(k)\boldsymbol{v}_n(k) = e^{2i\delta_n(k)}\boldsymbol{v}_n(k)$$

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- **Using the thermodynamic relation** $\left(\frac{\partial \Omega}{\partial \mu}\right) = -N$ **, and integrating the both sides by** $\mu_I \approx \mu_0 = \frac{\hbar^2 k_F^2}{2m}$ **(Valid with** $O(V^{-\frac{1}{3}})$ **accuracy)**

$$E(\boldsymbol{R}) = \Omega_I - \Omega_0 = -\frac{\hbar^2}{\pi m} \sum_n \int_0^{k_F} k dk \delta_n(k) + \sum_{\varepsilon_i < 0} \varepsilon_i(\boldsymbol{R})$$

Sum of continuum and bound-state contributions

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Sum of continuum and bound-state contributions

- **N_H=1 : Fumi's theorem.** J. Fumi, *Phil. Mag*, **46**, 1007 (1955).

  Central potential $n \to (\ell, m_\ell)$

- **N_H=2:** Y. Nishida, *PRA*. **79**, 013629 (2009).

  Parity is the good quantum number $n \to +/-$

Step 2 of the proof: Fredholm Determinant

- **Fredholm Determinannt (Jost Function)** $D(k)$ R. G. Newton, *J. Math. Phys.* , **18**, 1348 (1977).

$$\sum_n \delta_n(k) = \frac{i}{2} \log\left[\frac{D(k)}{D^*(k)}\right] + \mathrm{Const}$$

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- **<u>Property 1:</u>** $D^*(k) = D(-k)$ **if** k **is real**

$$\nabla_{\boldsymbol{R}_i} E(\boldsymbol{R}) = -\frac{i\hbar^2}{2\pi m} \int_{-k_F}^{k_F} k dk \frac{\nabla_{\boldsymbol{R}_i} D(k)}{D(k)} + \nabla_{\boldsymbol{R}_i} E^{\mathrm{BS}}(\boldsymbol{R}).$$

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- **Property 2:** $D(k)$ **is analytical and** $\lim_{|k|\to\infty} D(k) = 1$ **when** $\text{Im}k \geq 0$
- **Property 3: Zero of** $D(k)$ **has one to one correspondence with bound state**

$$D(k = i\kappa) = 0 \iff E^{\text{BS}} = -\frac{\hbar^2\kappa^2}{2m}$$

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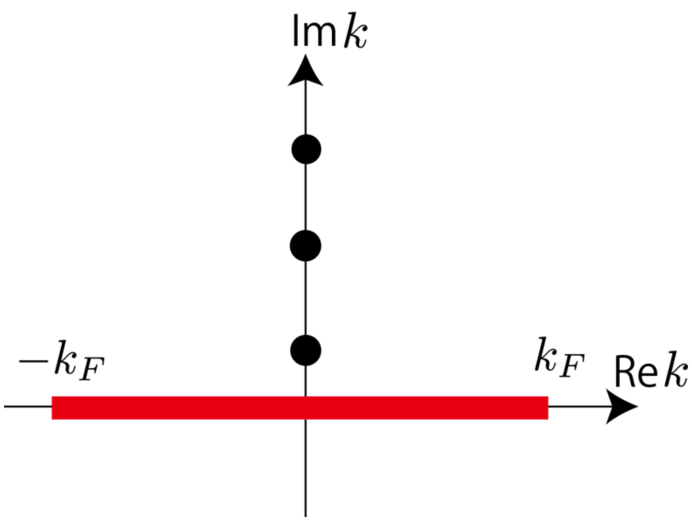

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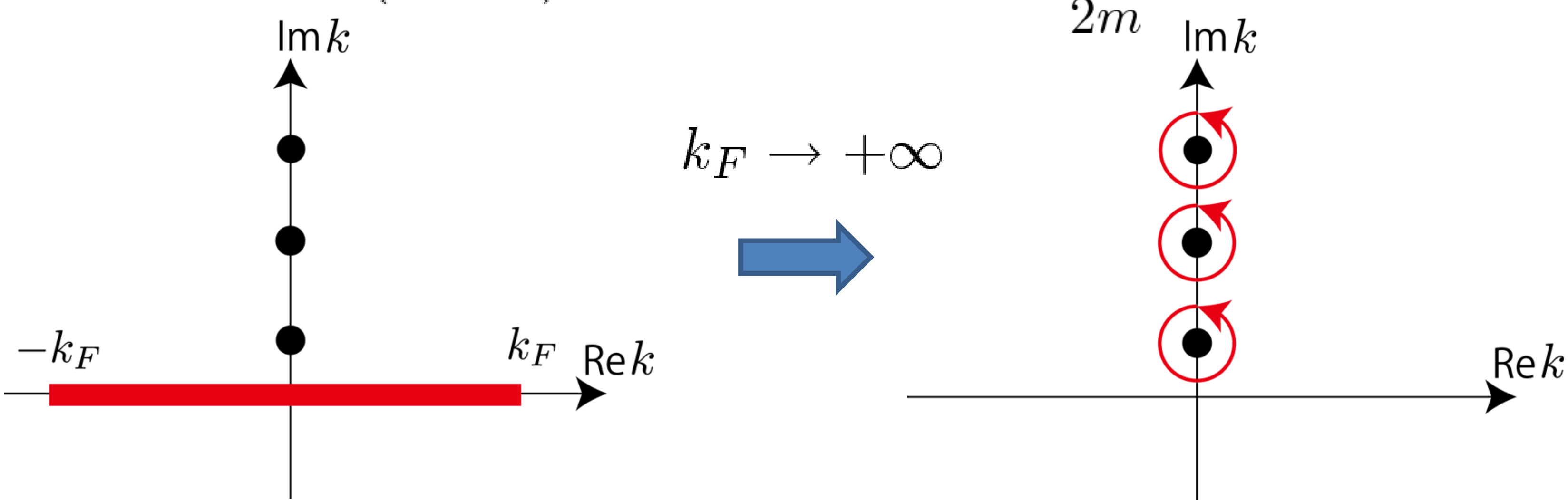

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$$\lim_{k_F\to\infty} \nabla_{\boldsymbol{R}_i} V_{\mathrm{eff}}(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_{N_H}) = 0$$

$$\Longrightarrow \lim_{k_F\to\infty} V_{\mathrm{eff}}(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_{N_H}) = 0$$

Physical Origin of vanishing interaction

- **Density modulation around a single heavy impurity:**

  1. **Screening of bound state by the continuum.**

$$N_I - N_0 = \frac{1}{\pi}\sum_n \delta_n(k_F)$$

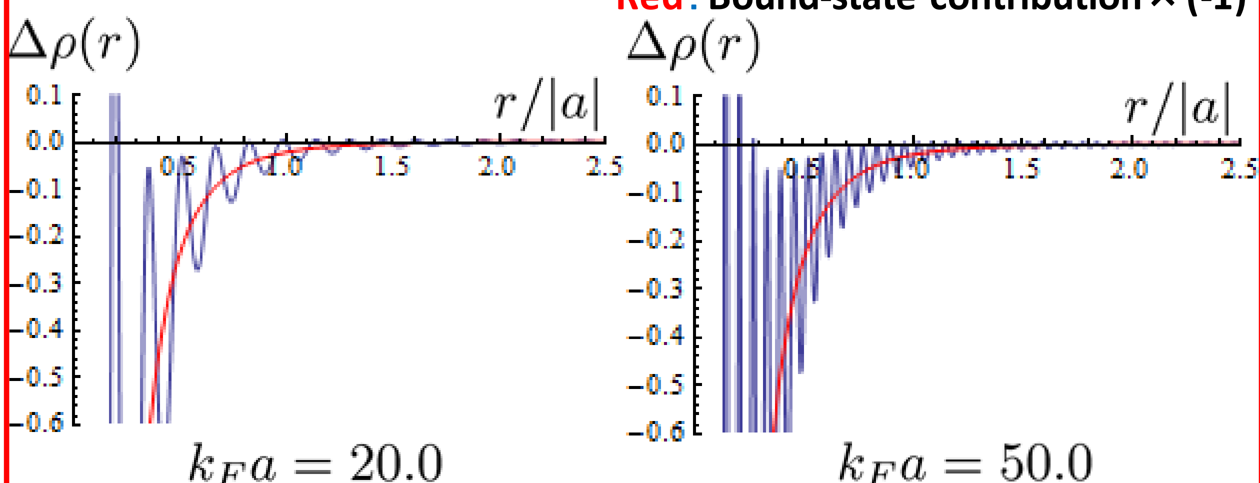
$$\Rightarrow \int d^3r\,\Delta\rho_B(r) + \int d^3r\,\Delta\rho_c(r) = \frac{1}{\pi}\delta_0(k_F) \rightarrow 0$$

  2. **Fast Friedel's oscillation when** $k_F \rightarrow \infty$

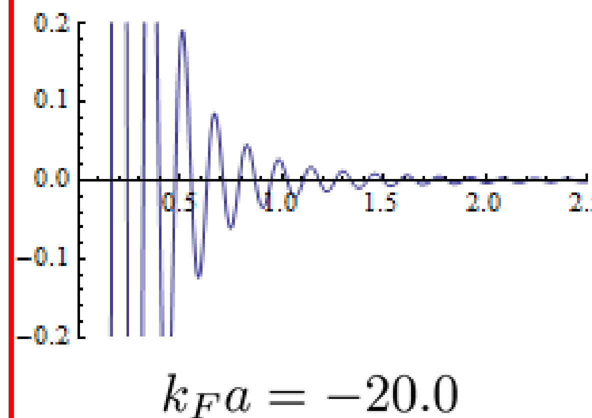
a>0 (with a bound state)

Blue : Continuum's contribution

Red : Bound-state contribution × (-1)



a<0 (No bound state)



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  2. **Fast Friedel's oscillation when** $k_F \to \infty$

- **For N_H=2, one can directly show when** $k_F^{-1} \ll |a|, R_{12}$

$$V_{\rm eff}(R_{12}) = -\frac{\hbar^2}{2\pi m k_F R_{12}^3}\cos 2k_F R_{12} - \frac{\hbar^2}{4\pi m k_F^2 R_{12}^4}\sin 2k_F R_{12} + \frac{\hbar^2}{\pi m R_{12}^3 k_F^2 a}\sin 2k_F R_{12} + \frac{\hbar^2}{m}O\left(\frac{1}{k_F^3 a R_{12}^4}, \frac{1}{k_F^3 a^3 R_{12}^2}, \frac{1}{k_F^3 a^5}, \ldots\right).$$

which vanishes at $k_F \to \infty$

Summary of this section

SE, M. Ueda, arXiv:1309.7797 (2013)

- **We consider N_H heavy particles immersed in a Fermi sea of the light fermions.**

- **With the Born-Oppheneimer approximation, we have shown that the effective interaction between the heavy particles exactly vanishes when** $R_{ij}, |a| \gg k_F^{-1} \to 0$

- **Formation of *N*-body bound states, including the Efimov states, are suppressed by the Fermi sea**

Outline

- Perfect screening of the Efimov effect by the dense Fermi sea.

  S. Endo, M. Ueda, arXiv:1309.7797 (2013)

- Universality the 3-body parameter
  - Different universality classes for 2-body potentials with different long-range tail.

  P. Naidon, S. Endo, M. Ueda, arXiv:1208.3912 (2012)

  P. Naidon, S. Endo, M. Ueda, PRL. **112**, 105301 (2014)

Pascal Naidon (RIKEN)

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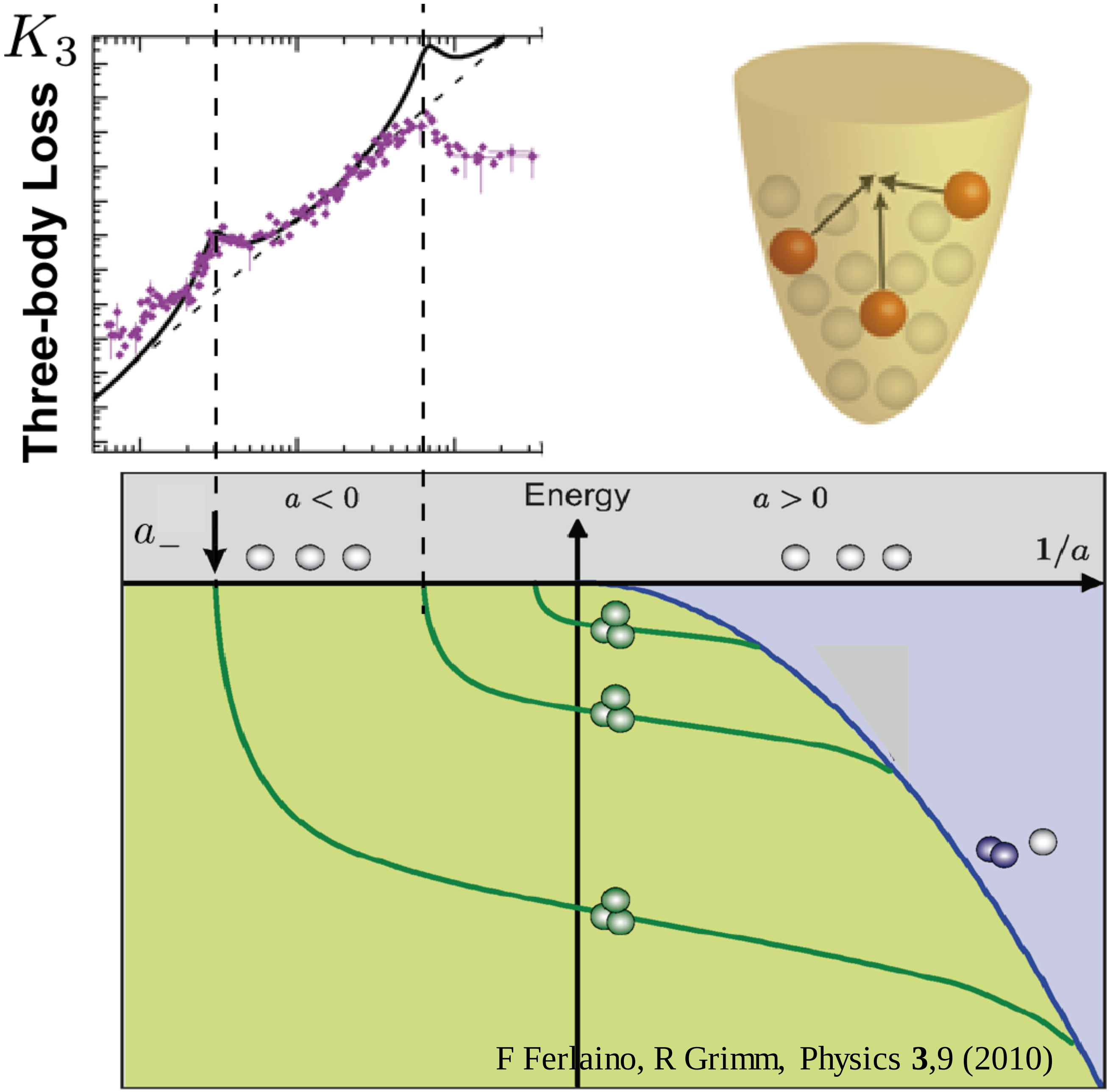
Universal 3-body parameter



F Ferlaino, R Grimm, Physics **3**,9 (2010)

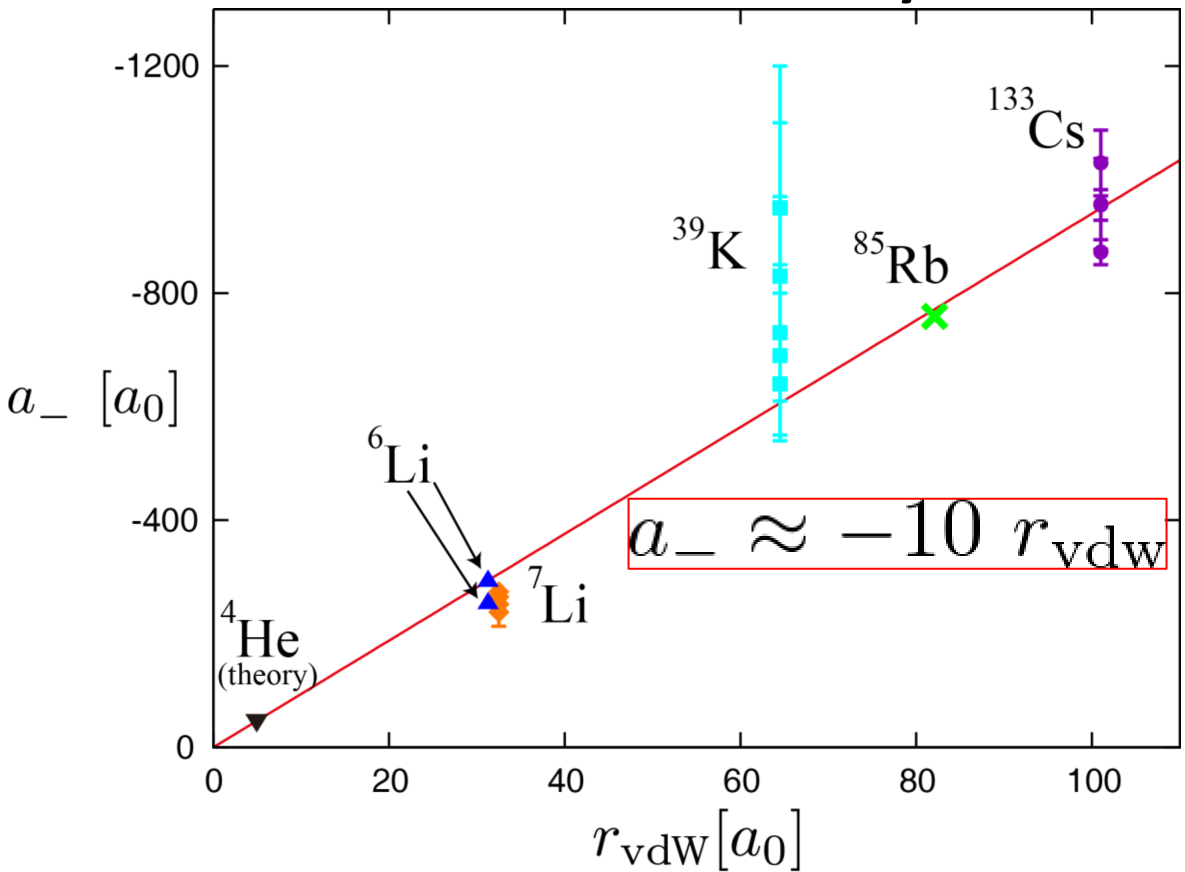
Universal 3-body parameter

- **3-body parameter measured for various atomic species**

⇒Universally characterized by the van der Waals length!

M. Berninger, *et al*. PRL. **107**, 120401 (2012)

- **Theoretical calculation for ⁴He (Gaussian expansion) is also consistent with the universality.** P. Naidon, E. Hiyama, M. Ueda, PRA **86**, 012502 (2012)



$$r_{vdw} = \frac{1}{2}\left(\frac{mC_6}{\hbar^2}\right)^{\frac{1}{4}}$$

van der Waals length

a_0 Bohr radius

Theoretical work on the universal 3-body parameter

- Broad Feshbach Resonance

C. Chin, arXiv:1111.1484 (2011)

P. Naidon, E. Hiyama, M. Ueda, PRA **86**, 012502 (2012)

J. Wang, J. P. D'Incao, B. D. Esry, C. H. Greene, PRL. **108**, 263001 (2012)

P. K. Sørensen, D. V. Fedorov, A. S. Jensen, N. T. Zinner, PRA **86**, 052516 (2012).

- Narrow Feshbach Resonance

R. Schmidt, S. P. Rath, W. Zwerger, EPJ. B **85**, 1 (2012)

D. S. Petrov, PRL. **93**, 143201 (2004).

- Heteronuclear system

Y. Wang, J. Wang, J. P. D'Incao, and C. H. Greene, PRL. **109**, 243201 (2012).

- Van der Waals Universality

Y. Wang, P. S. Julienne arxiv:1404.0483 (2014)

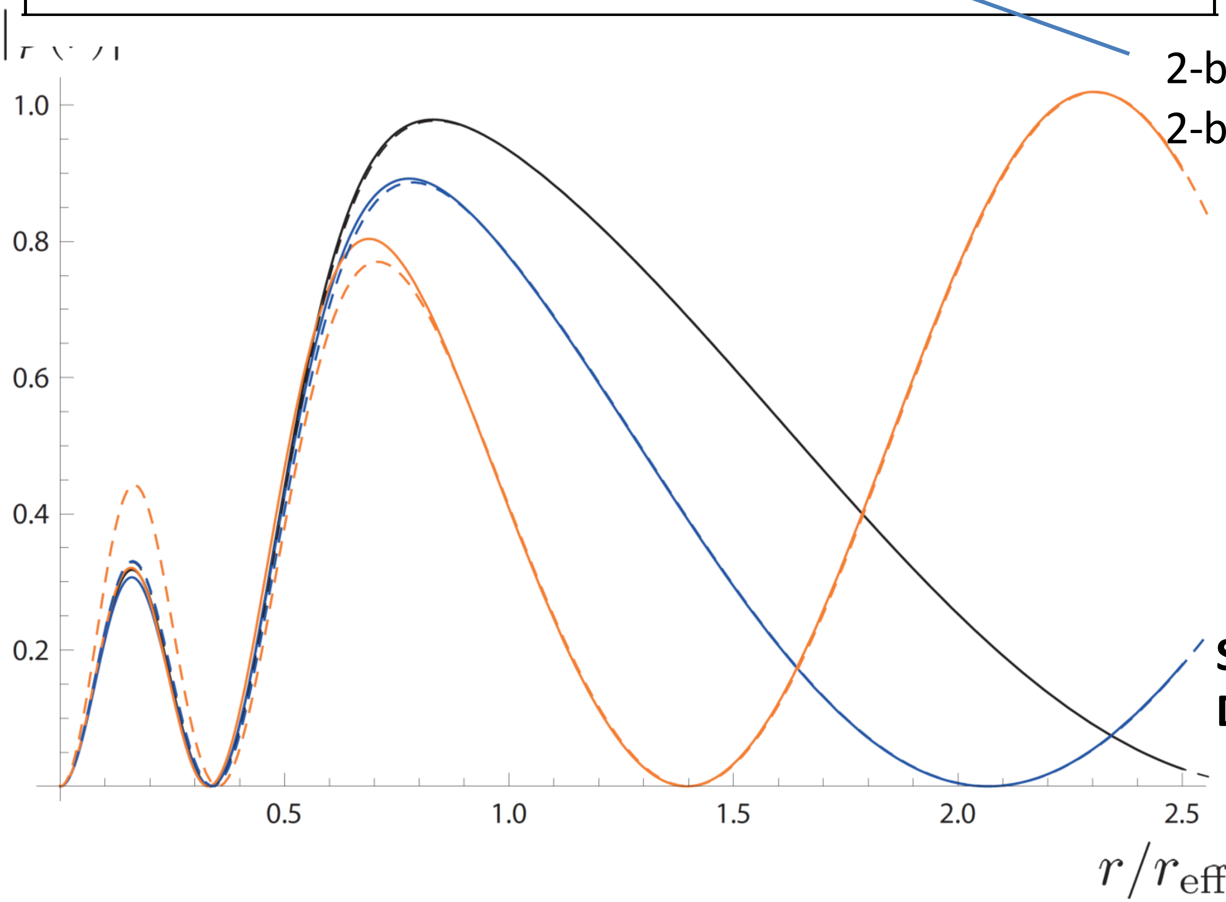
Separable potential

- Separable potential is constructed to exactly reproduce the 2-body wave function at E=0.

$$\frac{\phi(q)}{\phi(0)} = 1 + q\int_0^\infty dr \left(\frac{r}{a}(1-\frac{a}{r}-\psi_0(r))\right)\sin(qr)$$

$$V = \frac{\hbar^2}{2\mu}\xi|\phi\rangle\langle\phi|$$

2-body wave function for the 2-body potential in question (E=0)



Black: $E=\frac{\hbar^2}{2mr_{\mathrm{eff}}^2}$

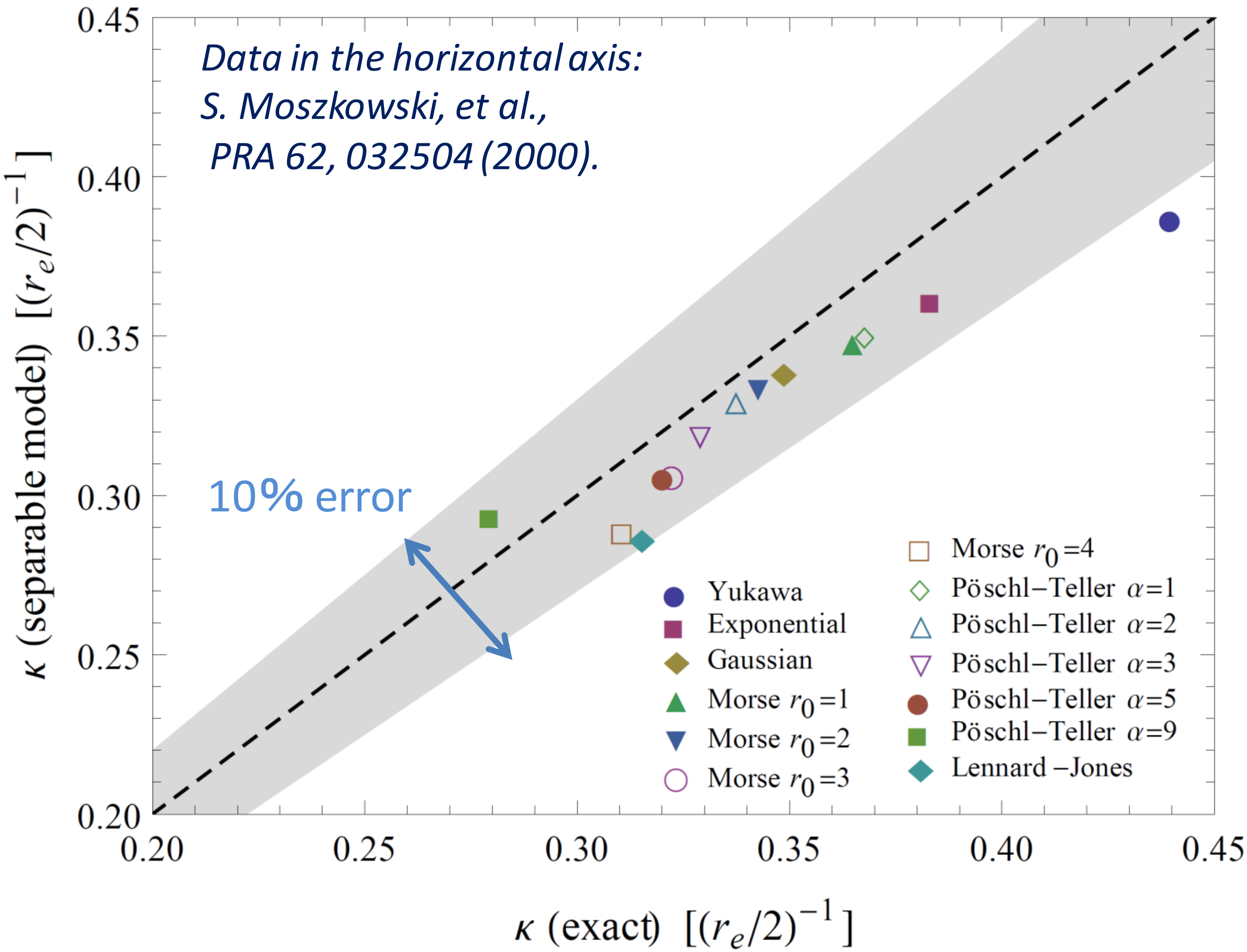
Blue: $E=\frac{\hbar^2}{mr_{\mathrm{eff}}^2}$

Orange: $E=\frac{3\hbar^2}{mr_{\mathrm{eff}}^2}$

Solid: Original 2-body potential
Dotted: Separable potential

3-body parameter with the separable potential

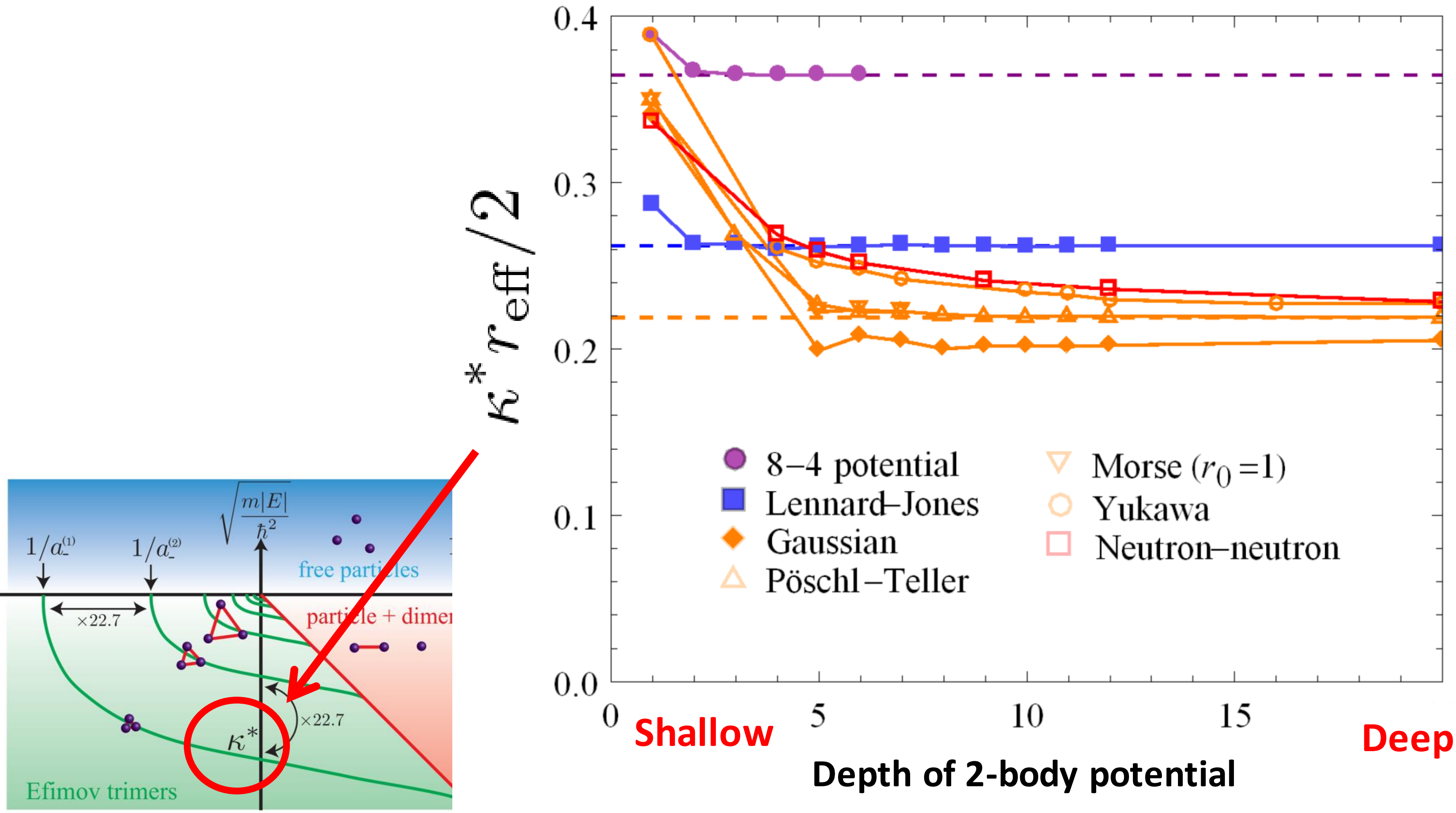
- Our separable potential accurately reproduces 3-body parameter for various types of shallow 2-body potentials.



3-body parameter for various classes of potentials

- 3-body parameter mostly characterized by effective range

$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\rm eff}}{2}\right)^{-1}$$



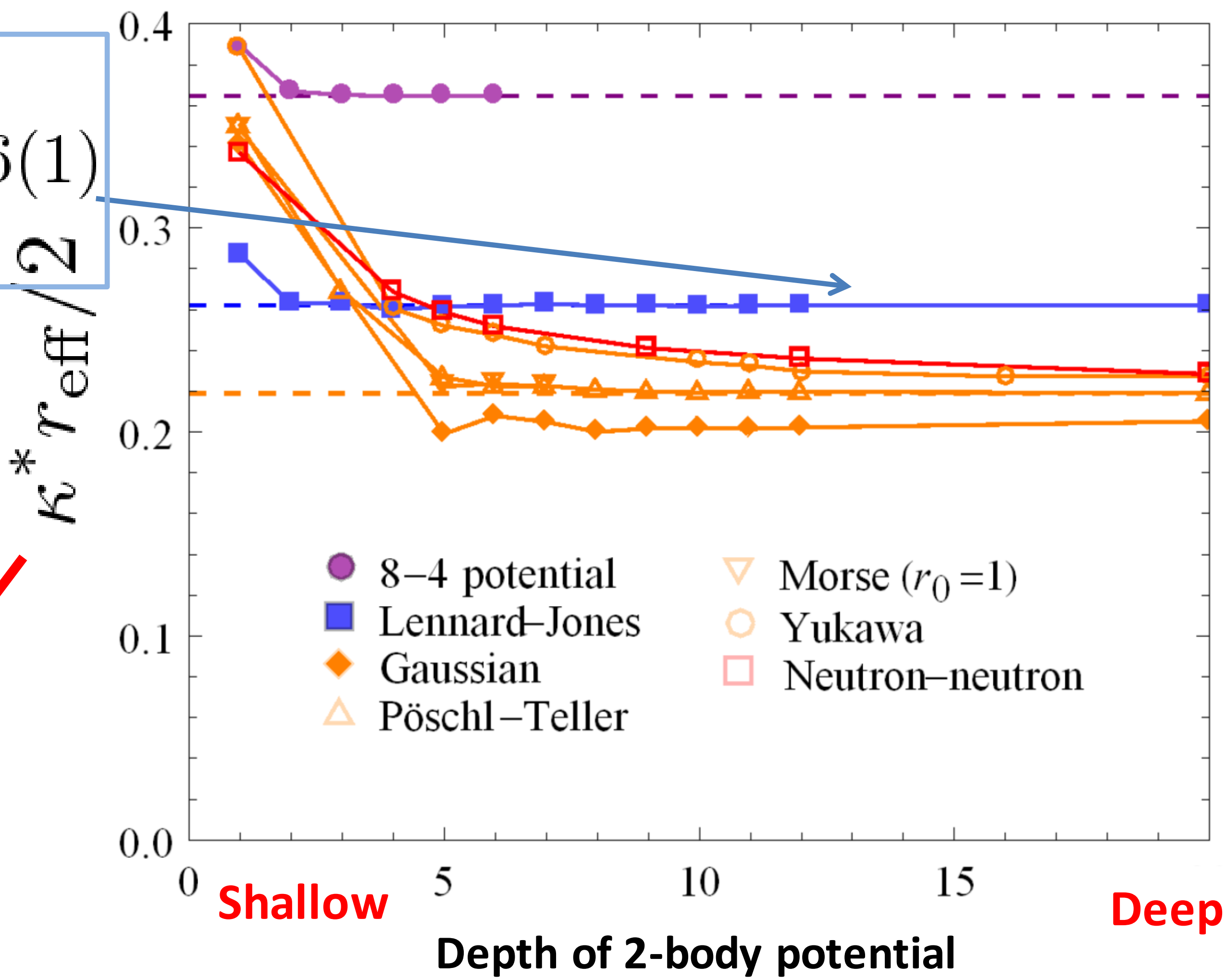
3-body parameter for various classes of potentials

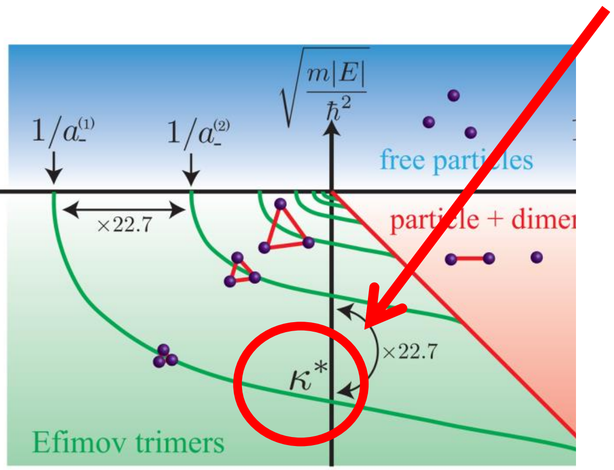
- 3-body parameter mostly characterized by effective range

$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\rm eff}}{2}\right)^{-1}$$

Van der Waals potential

$$a_-/r_{\rm vdw} = -10.86(1)$$
$$\kappa^* r_{\rm vdw} = 0.187(1)$$





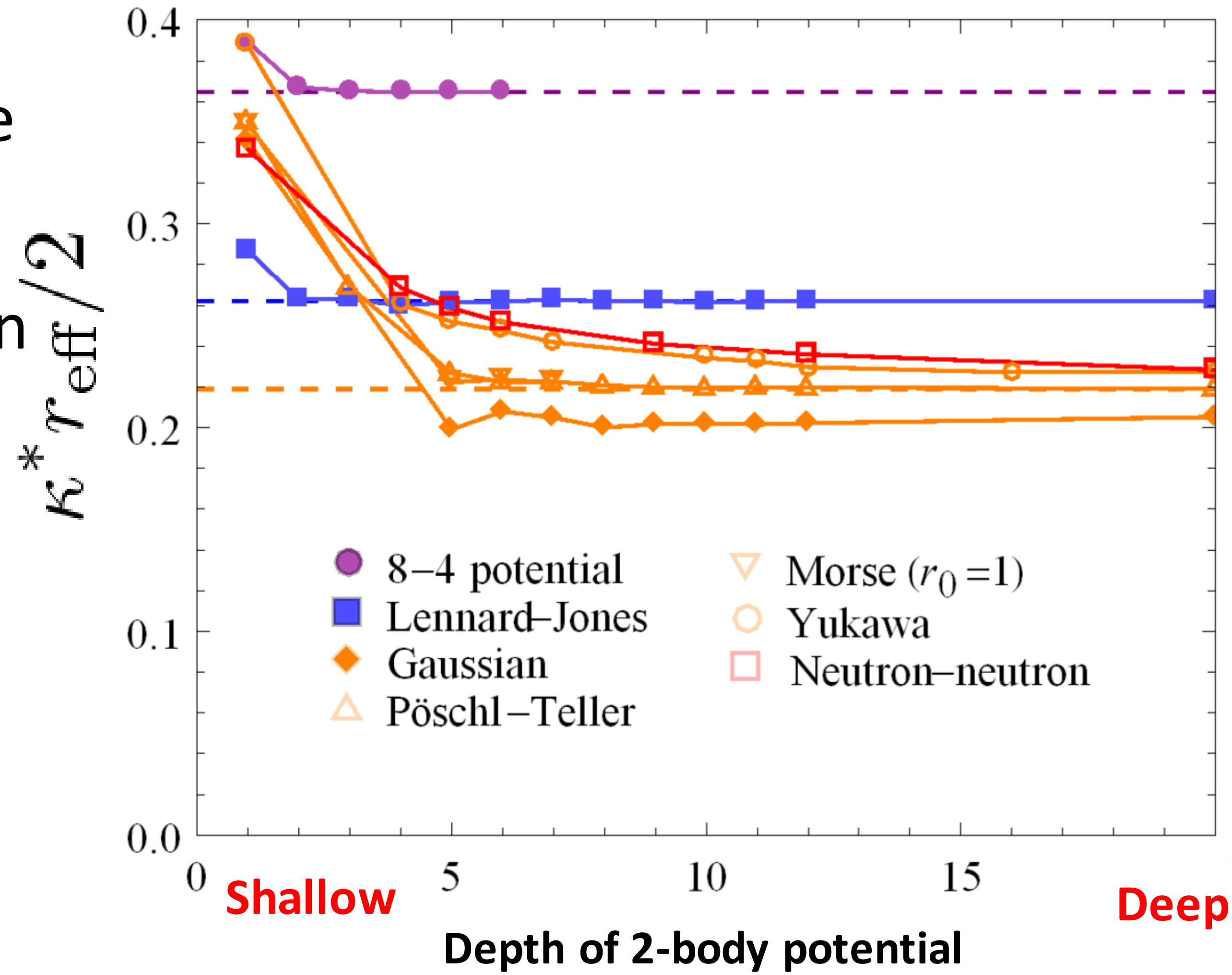
3-body parameter for various classes of potentials

- 3-body parameter mostly characterized by effective range

$$\kappa^* = (0.2 - 0.4) \left(\frac{r_{\text{eff}}}{2} \right)^{-1}$$

- Small difference

➡ Difference in pair correlation



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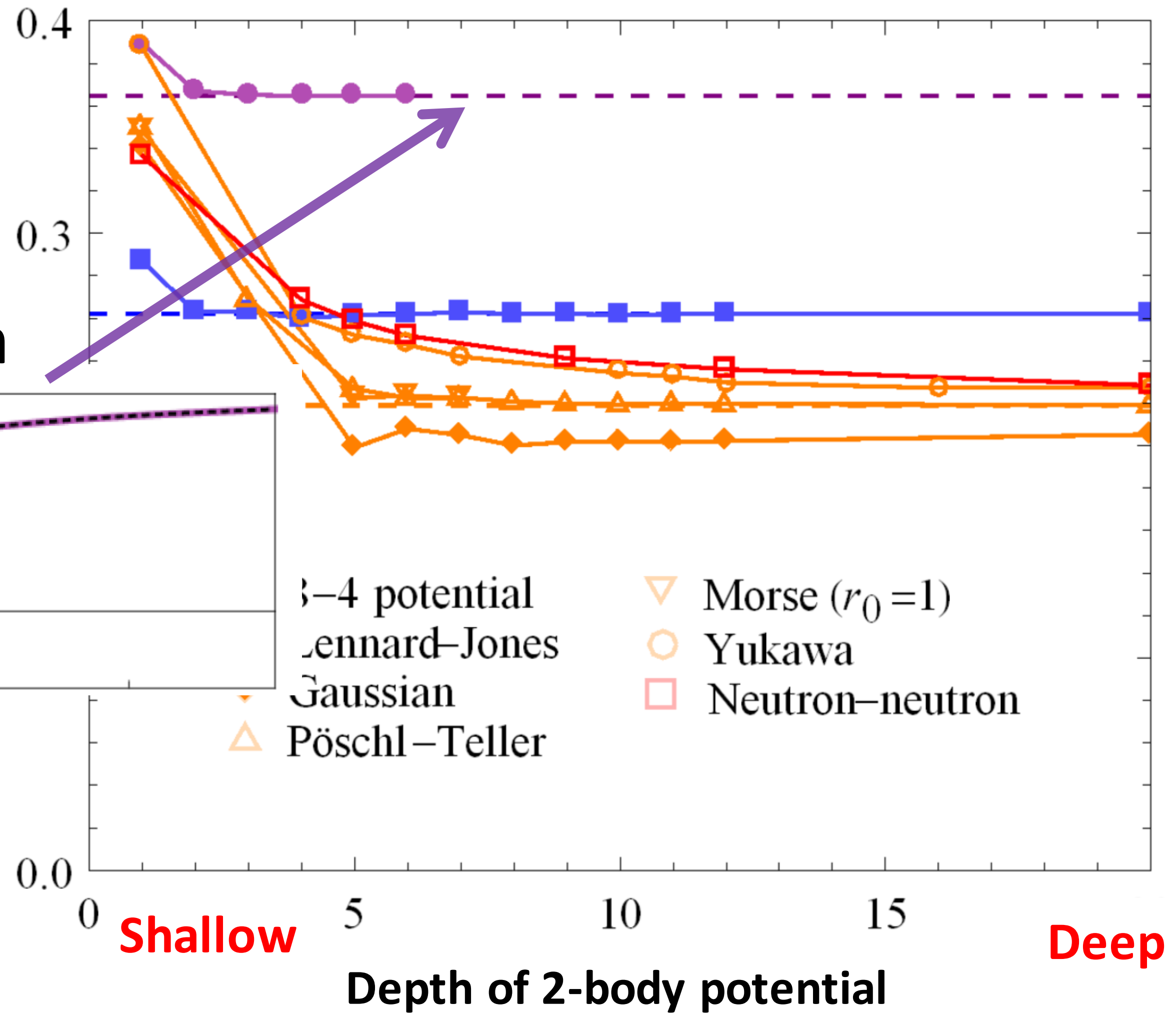
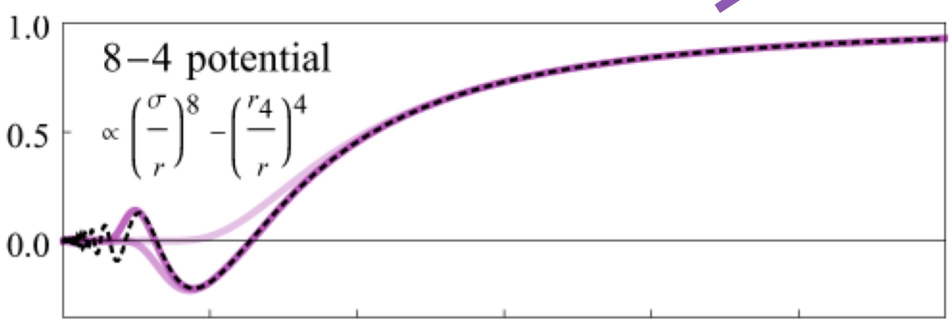
$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\mathrm{eff}}}{2}\right)^{-1}$$

- Small difference

➡ Difference in 2-body correlation





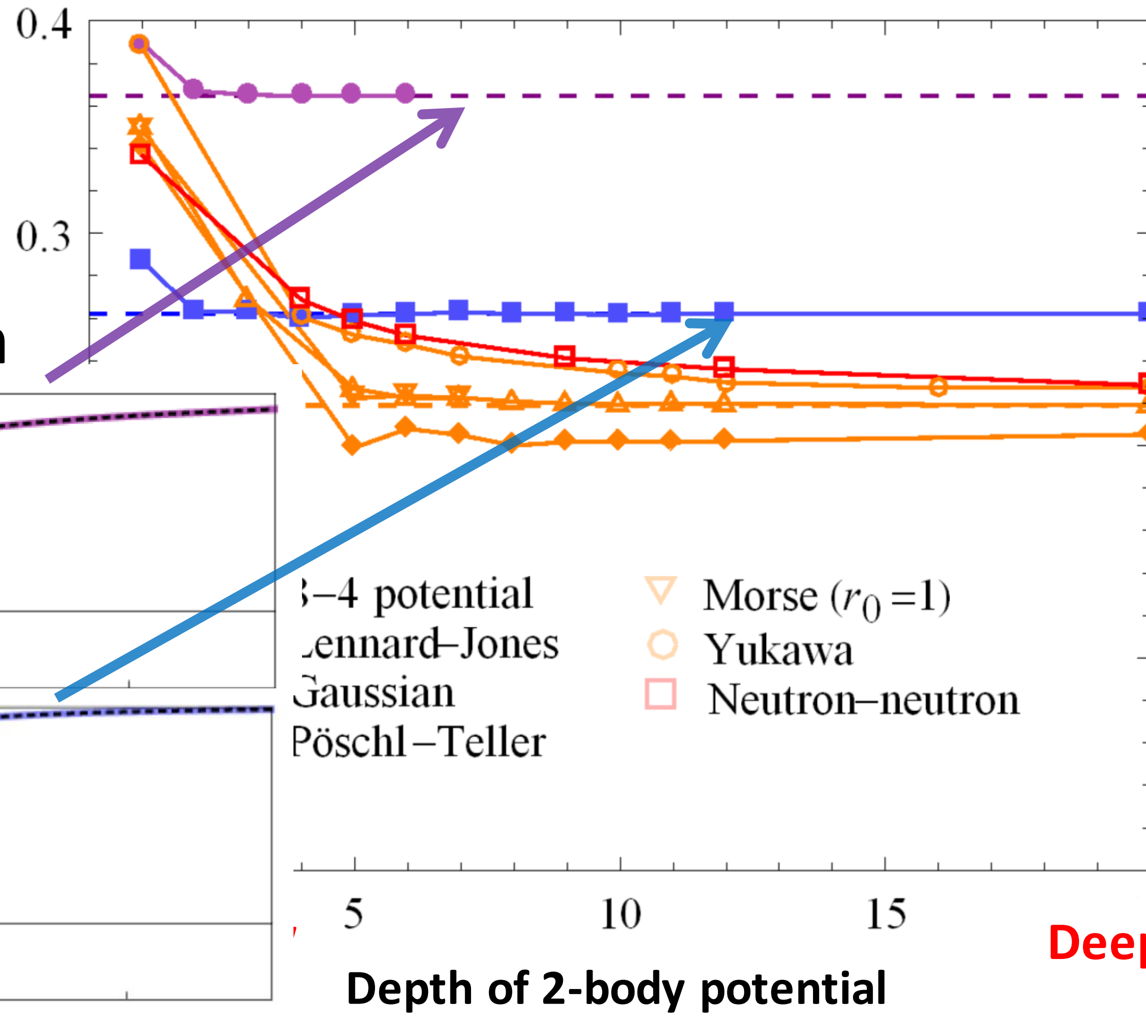
3-body parameter for various classes of potentials

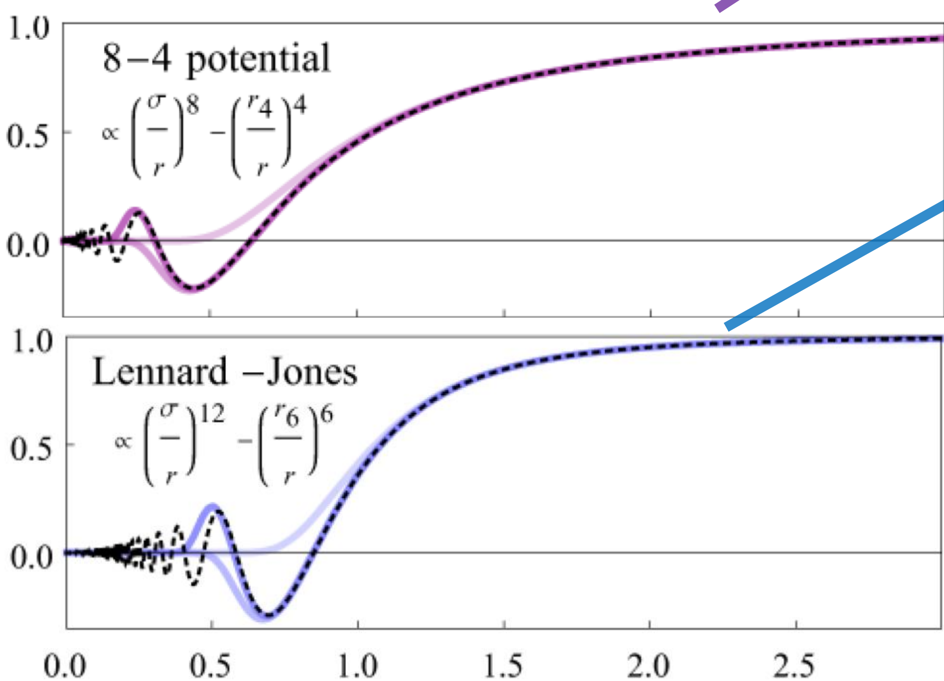
- 3-body parameter mostly characterized by effective range

$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\mathrm{eff}}}{2}\right)^{-1}$$

- Small difference

➡ Difference in 2-body correlation





3-body parameter for various classes of potentials

- 3-body parameter mostly characterized by effective range

$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\rm eff}}{2}\right)^{-1}$$

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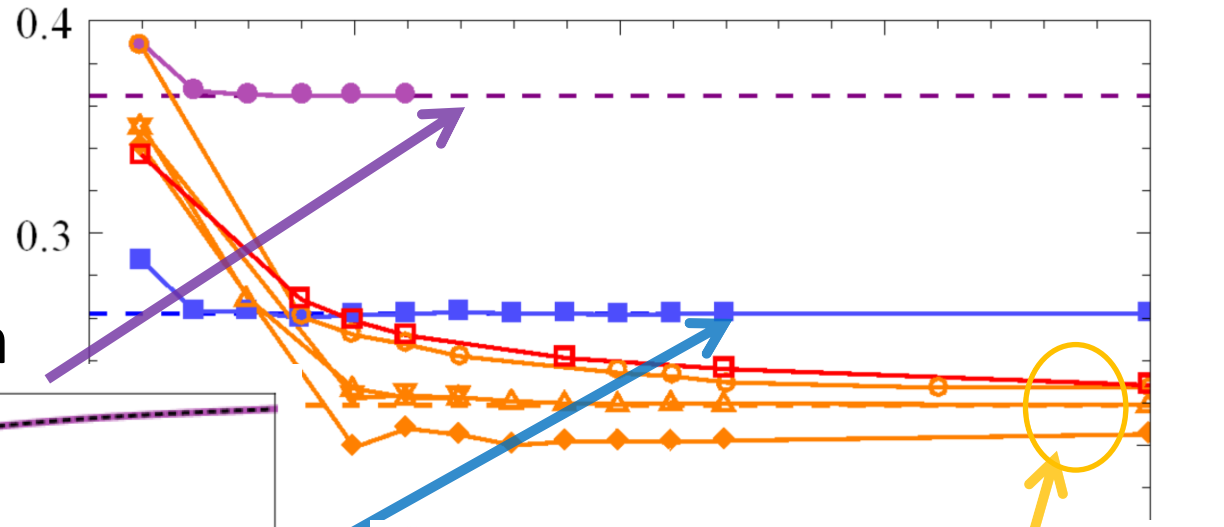

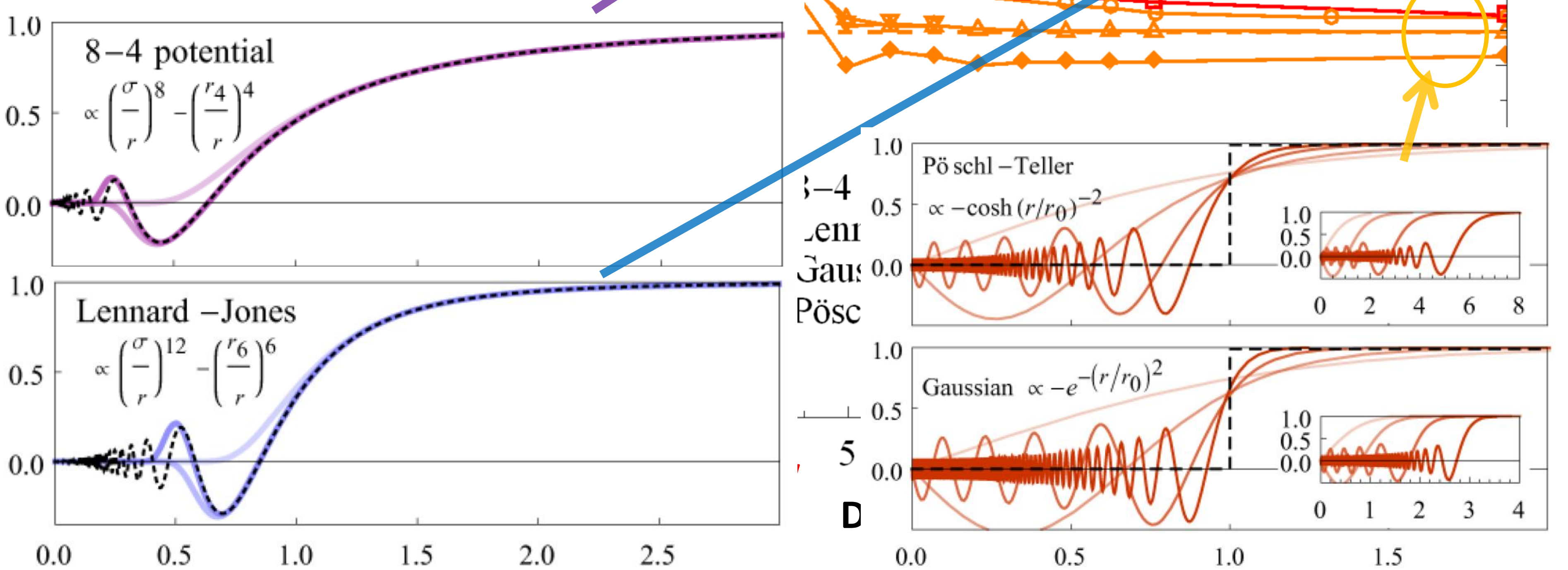

Summary

P. Naidon, S. Endo, M. Ueda, arXiv:1208.3912 (2012)
P. Naidon, S. Endo, M. Ueda, PRL. **112**, 105301 (2014)

- Separable potential, constructed to reproduce the pair correlation at E=0, accurately describes the 3-body parameter

- Several universality classes of the 3-body parameter exist, depending on the tails of the 2-body potentials

Pascal Naidon (RIKEN)

Masahito Ueda (Univ. Tokyo)