

“Three-body Physics in Quenched Unitary Bose gases”

Jose P. D’Incao

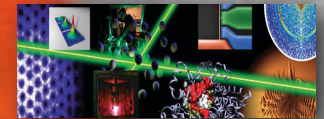
JILA, Dept. of Physics, U. of Colorado at Boulder
Dept. Physics, Kansas State University

Collaborators:

Jia Wang (UConn), Yujun Wang (KSU),
Guido Pupillo (Strasbourg), Brett Esry (KSU),
John Bohn (JILA), Chris H. Greene (Purdue)



NIST
University of Colorado



KANSAS STATE
UNIVERSITY
Department of Physics

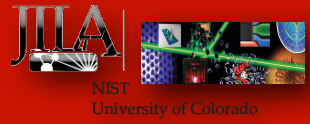


National Science
Foundation

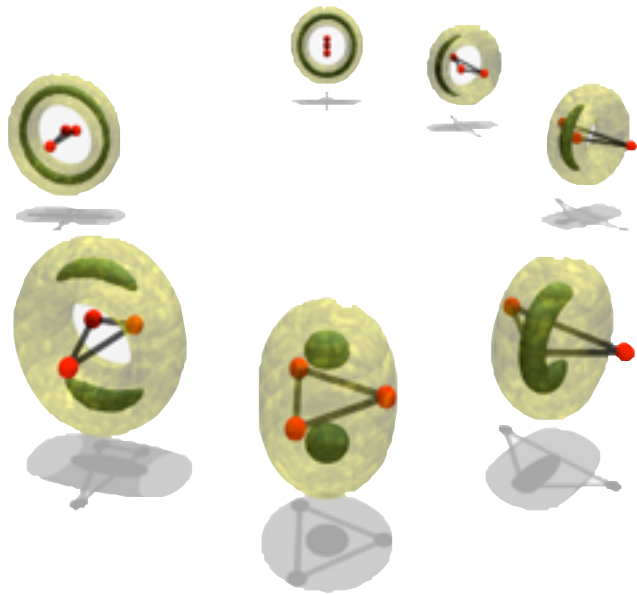


Air Force Office of
Scientific Research
MURI

A Few-body Perspective in Ultracold Quantum Gases



Universal Few-Body Physics

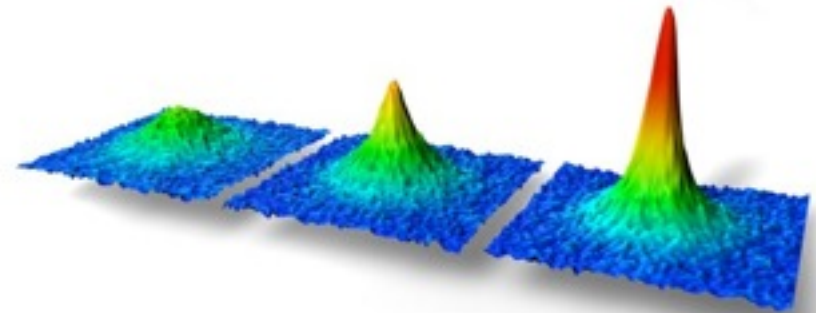


From the theoretical side:

- ✓ Signatures of Efimov Physics,
- ✓ Four- and More-bodies universal states,
- ✓ New families of universal states,
- ✓ Approaching a quantitative level (exps.)

Ultracold Quantum Gases

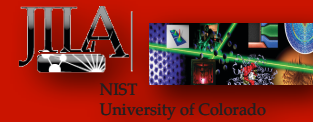
($<peV$)



From the experimental side:

- ✓ clean and accurate experiments
- ✓ **CONTROL** of interactions (B -field or E -field)
- ✓ can explore the universal regime (low T and strong interactions)

A Few-body Perspective in Ultracold Quantum Gases



A new research venue?

PHYSICAL REVIEW LETTERS
 PUBLISHED ONLINE 10 FEBRUARY 2010
Observation of Heteronuclear Atomic Efimov Resonances
 G. Bantini¹, C. Walter², F. Rebelli¹, J. Casan³, G. Thalhammer¹, M. Jepsich¹, and P. Fritsch³
¹ZARA, European Laboratory for Non-linear Spectroscopy, Via San Giuseppe 1, 50125 Florence, Italy
²ITPA, European Laboratory for Non-linear Spectroscopy, Via San Giuseppe 1, 50125 Florence, Italy
³Center for Integrated Quantum Science, University of Colorado, Boulder, Colorado 80509, USA

PHYSICAL REVIEW LETTERS
 PUBLISHED ONLINE 10 FEBRUARY 2010
Observation of an Efimov-like trimer resonance in ultracold atom-dimer scattering
 S. Knopp¹, F. Fortnagel¹, M. Moré², M. Benninger², H. Schöbl¹, H.-C. Nägerl¹, and R. Grimm²

PHYSICAL REVIEW LETTERS
 PUBLISHED ONLINE 10 FEBRUARY 2010
Evidence for Universal Four-Body States Tied to an Efimov Trimer
 F. Fortnagel¹, S. Knopp¹, M. Benninger², W. Hans¹, J. P. D'Incao³, H.-C. Nägerl¹, and R. Grimm²

PHYSICAL REVIEW LETTERS
 PUBLISHED ONLINE 10 FEBRUARY 2010
Observation of an Efimov system
 M. Zaccanti¹, B. DeSalvo¹, C. D'Errico¹, M. Fattori², M. Jona-Lasinio¹, and G. Modugno¹

PHYSICAL REVIEW LETTERS
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Universality in Three- and Four-Body Bound States of Ultracold Atoms
 Scott E. Pollack¹, Daniel Dries, Randall G. Hulet

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Observation of Universality in Ultracold ⁷Li Three-Body Recombination
 Nils-Göran Ericsson¹, Zeno Steiner¹, Servaas Kokkilahti¹, and Lev Efimov²

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Signatures of universal four-body phenomena and their relation to the Efimov effect
 J. Von Stecher¹, J. P. D'Incao², and Chris H. Greene²

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Radio-Frequency Association of Efimov Trimers
 Sander Lesaffre¹, Yiyang B. Cheng^{1,2,3}, Prashant Srivastava^{2,3}, J. J. Gerber⁴, and M. H. Anderson^{1,2}

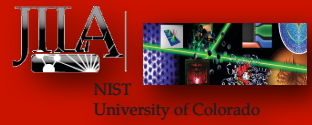
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Toolkit for Exploring Few-body Physics ...



For N particles ...

$$\hat{H} = -\frac{1}{2\mu} \nabla_T^2 + \sum_{i < j} V(r_{ij})$$

... angles + set of non-compact
coordinates $r_{ij} \rightarrow [0, \infty]$

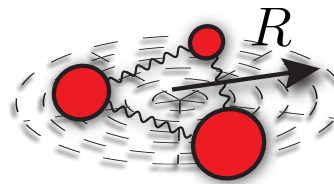
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... the hyperspherical way !!!

$$\hat{H} = -\frac{1}{2\mu} \frac{d^2}{d^2 R} + \frac{\Lambda^2(\Omega)}{2\mu R^2} + V(R, \Omega)$$



hyperradius R : overall size
(collective motion)

$$R \rightarrow [0, \infty]$$

hyperangles $\{\Omega\}$: internal motion

$$\{\Omega\} \rightarrow [0, \infty \pi]$$

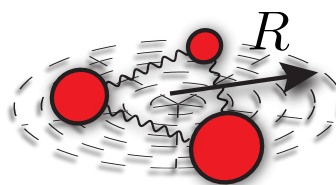
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(Democratic hyperangles:

Smith-Whitten, Johnson, Kuppermann, Aquilanti)

Fragmentation thresholds

Symmetrization is simpler

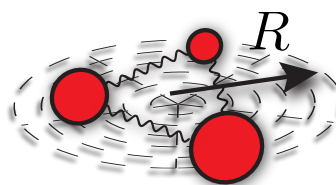
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$$R \rightarrow [0, \infty]$$

Adiabatic representation:

$$\Psi(R; \Omega) = \sum_{\nu} F_{\nu}(R) \Phi_{\nu}(R; \Omega)$$

$$\left[\frac{\Lambda(\Omega)}{2\mu R^2} + V(R, \Omega) \right] \Phi_{\nu}(R; \Omega) = U_{\nu}(R) \Phi_{\nu}(R; \Omega)$$

↑
(effective potential)

hyperangles $\{\Omega\}$: internal motion

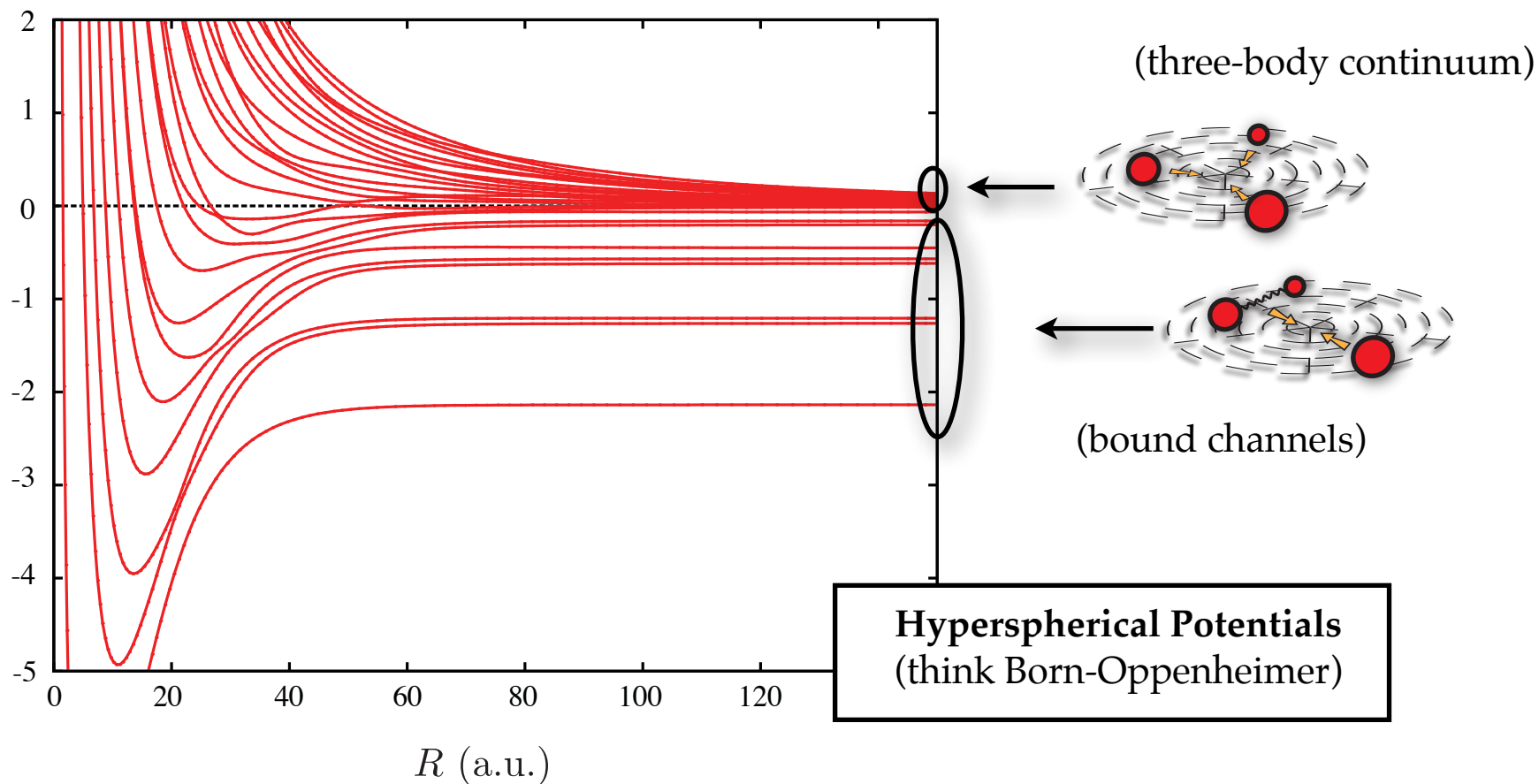
$$\{\Omega\} \rightarrow [0, \infty \pi]$$

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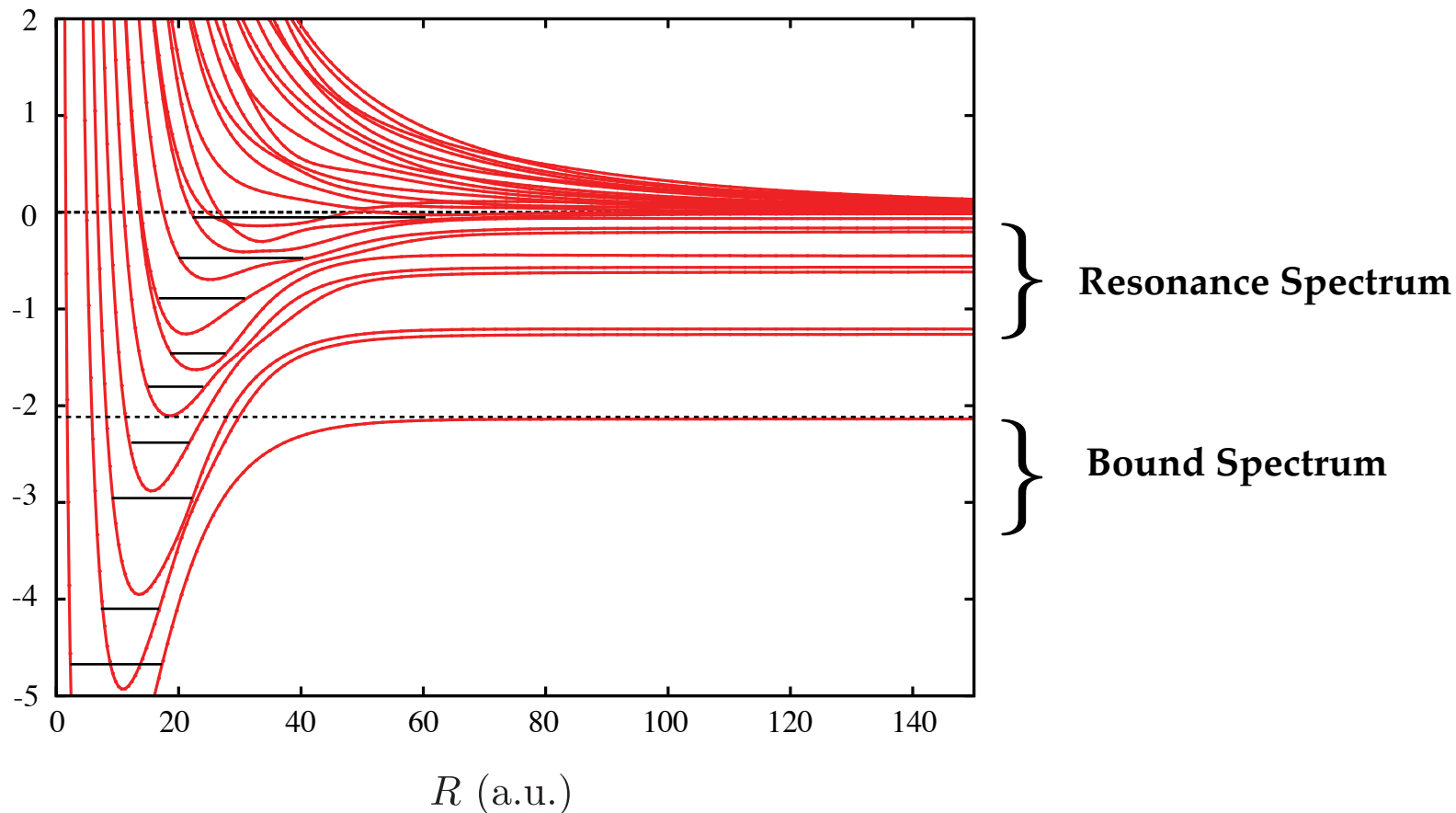
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Bound and Scattering Properties

$$\left[-\frac{1}{2\mu} \frac{d^2}{dR^2} + U_\nu(R) - E \right] F_\nu(R) + \sum_{\nu'} W_{\nu\nu'}(R) F_{\nu'}(R) = 0$$

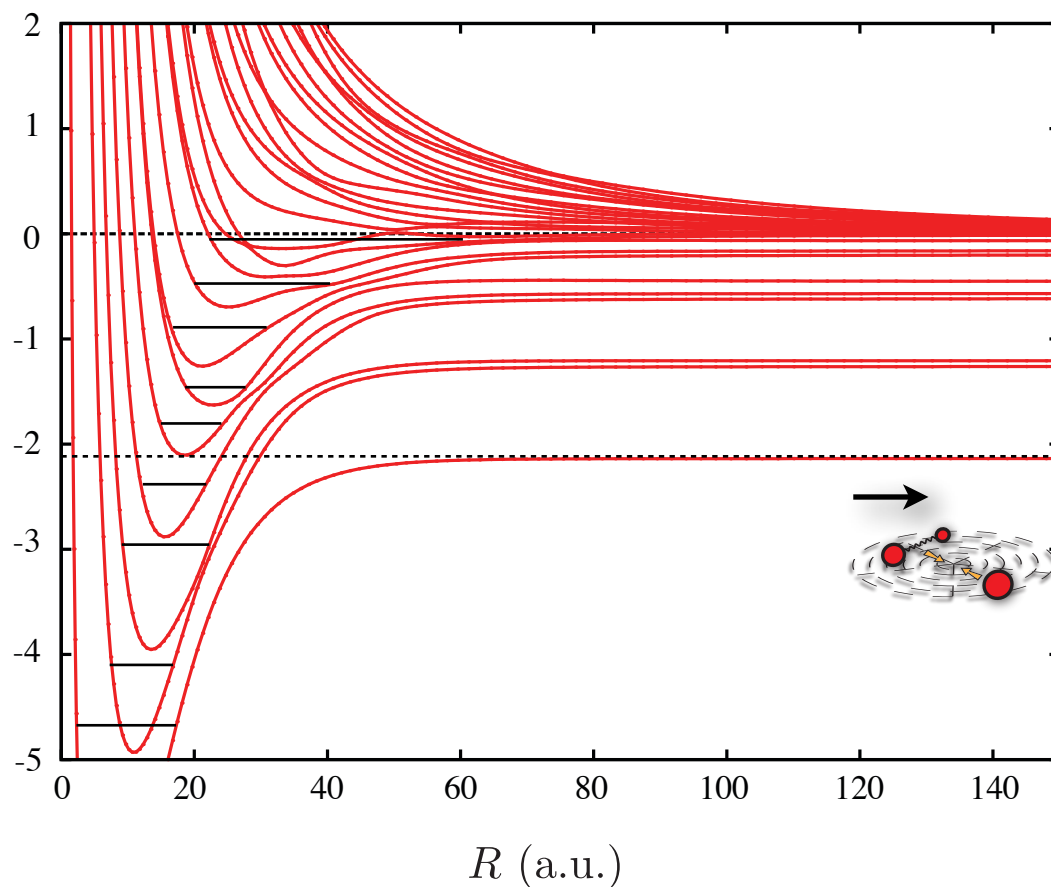
(Hyperradial Schrodinger Equation)



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Ultracold Few-body Collisions

$W_{\nu\nu'}(R)$: non-adiabatic couplings
(drive inelastic transitions)

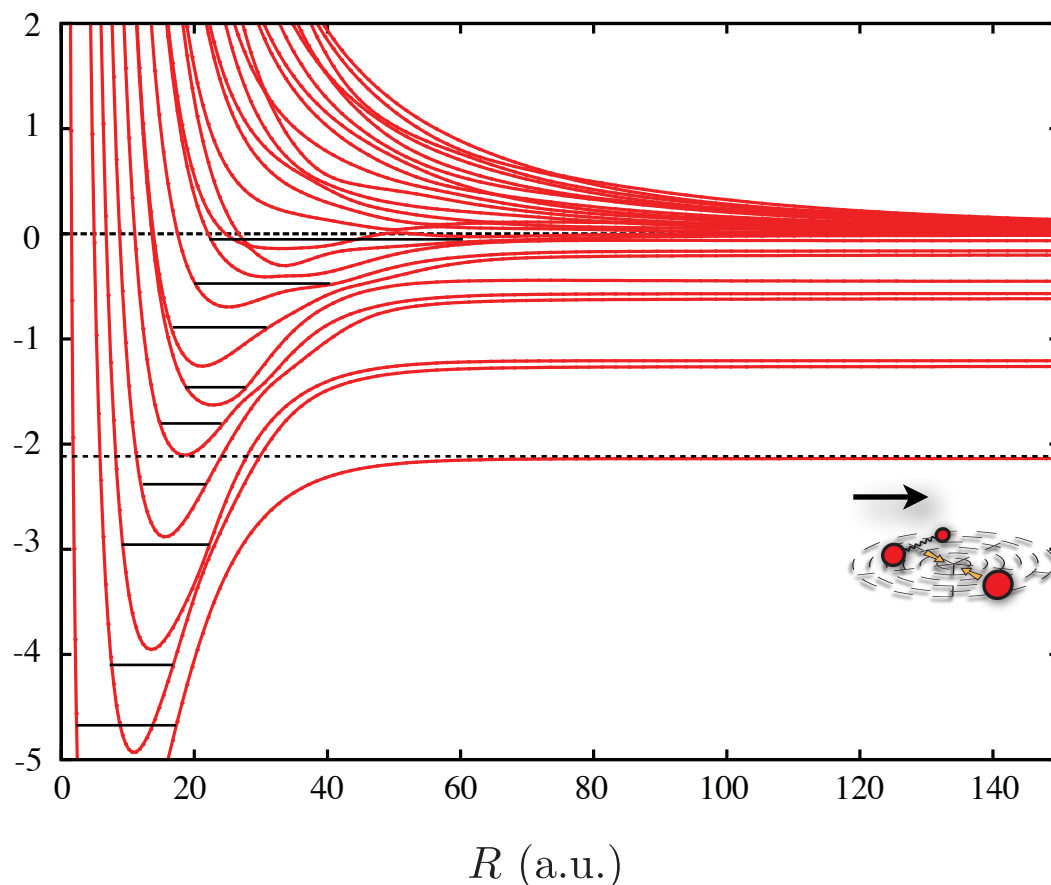
Typical length & energy scales:

- Van der Waals length: $r_0 \approx 100a_0$
- Temperature: $T \approx 100\text{nK}$

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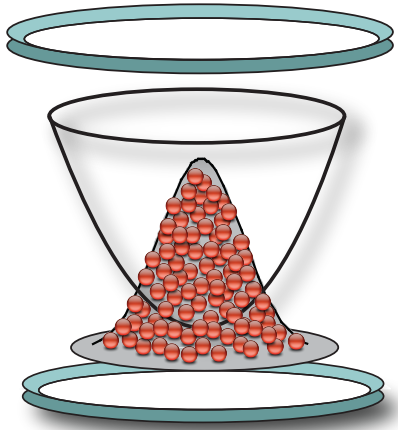
→ Solve Schrödinger equation for $R \approx 10^6 a_0$
(0.05mm!!!)

Unitary Quantum gases

($n|a|^3 \gg 1$)

Unitary Quantum Gases ($n|a|^3 \gg 1$)

Unitary Bose Gases (fundamental interest)

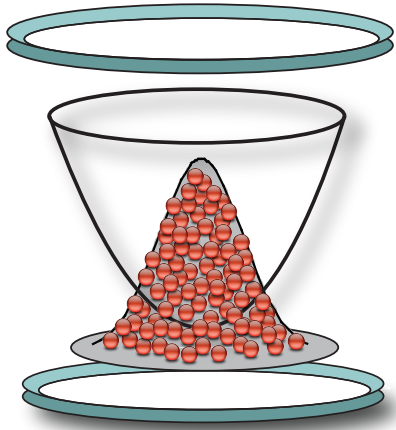


Unitary Fermi Gases: BEC-BCS crossover, Superfluidity ...
(Duke, Innsbruck, JILA, MIT, Rice, ...)

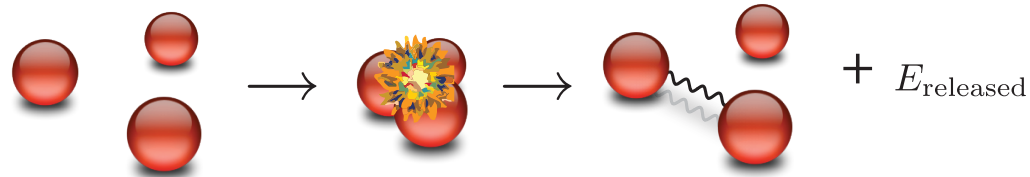
Stable and Universal state !!!

[see Giorgini, Pitaevskii &
Stringari, RMP (2008)]

Unitary Bose Gases (fundamental interest)



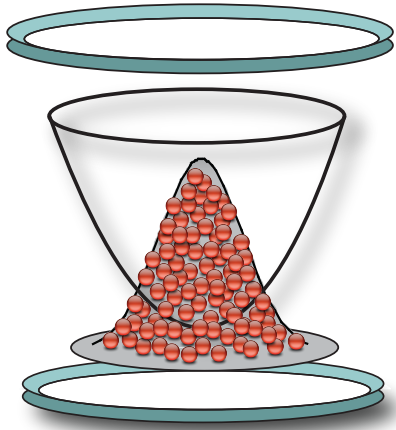
Three-Body Losses: (three-body recombination)



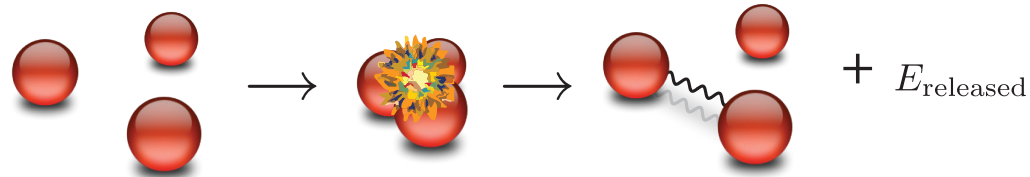
Fermions: $L_3 \propto T a^6$ (0 for $T \rightarrow 0$)

Bosons: $L_3 \propto a^4$ (const. for $T \rightarrow 0$)

Unitary Bose Gases (fundamental interest)



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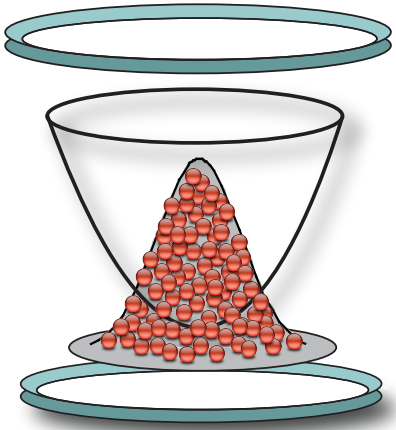
$$\frac{\tau_{\text{loss}}}{\tau_{\text{mb}}} \gg 1$$

Bosons: $L_3 \propto a^4$ (const. for $T \rightarrow 0$)

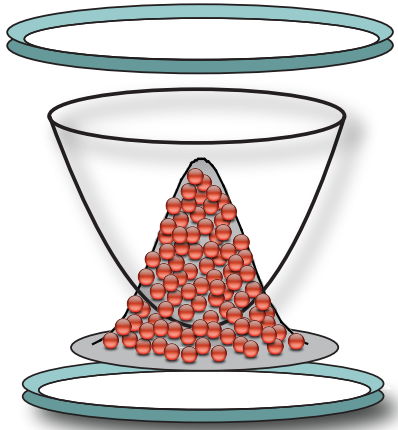
$$\frac{\tau_{\text{loss}}}{\tau_{\text{mb}}} \ll 1$$

[τ_{mb} : timescale for many-body physics]

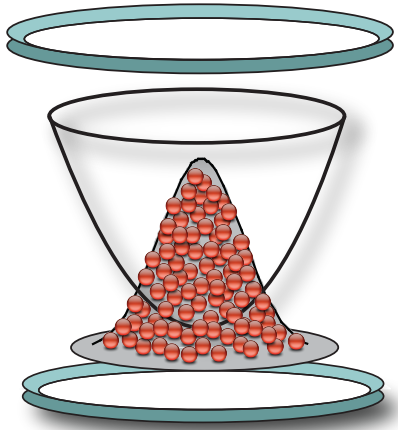
Unitary Bose Gases (fundamental interest)



Unitary Bose Gases (fundamental interest)



Unitary Bose Gases (**fundamental interest**)

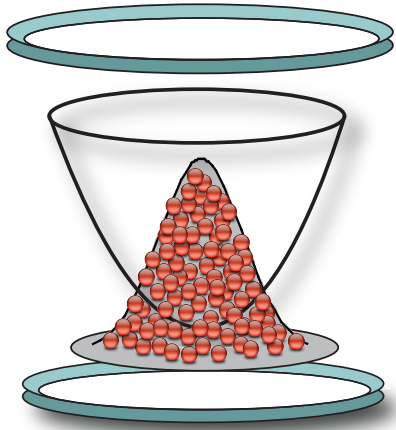


Efimov Physics (~1970, nuclear physics)

appearance of an *attractive* or *repulsive* three-body effective interaction ... in the strongly interacting regime ($|a| \gg r_0$)

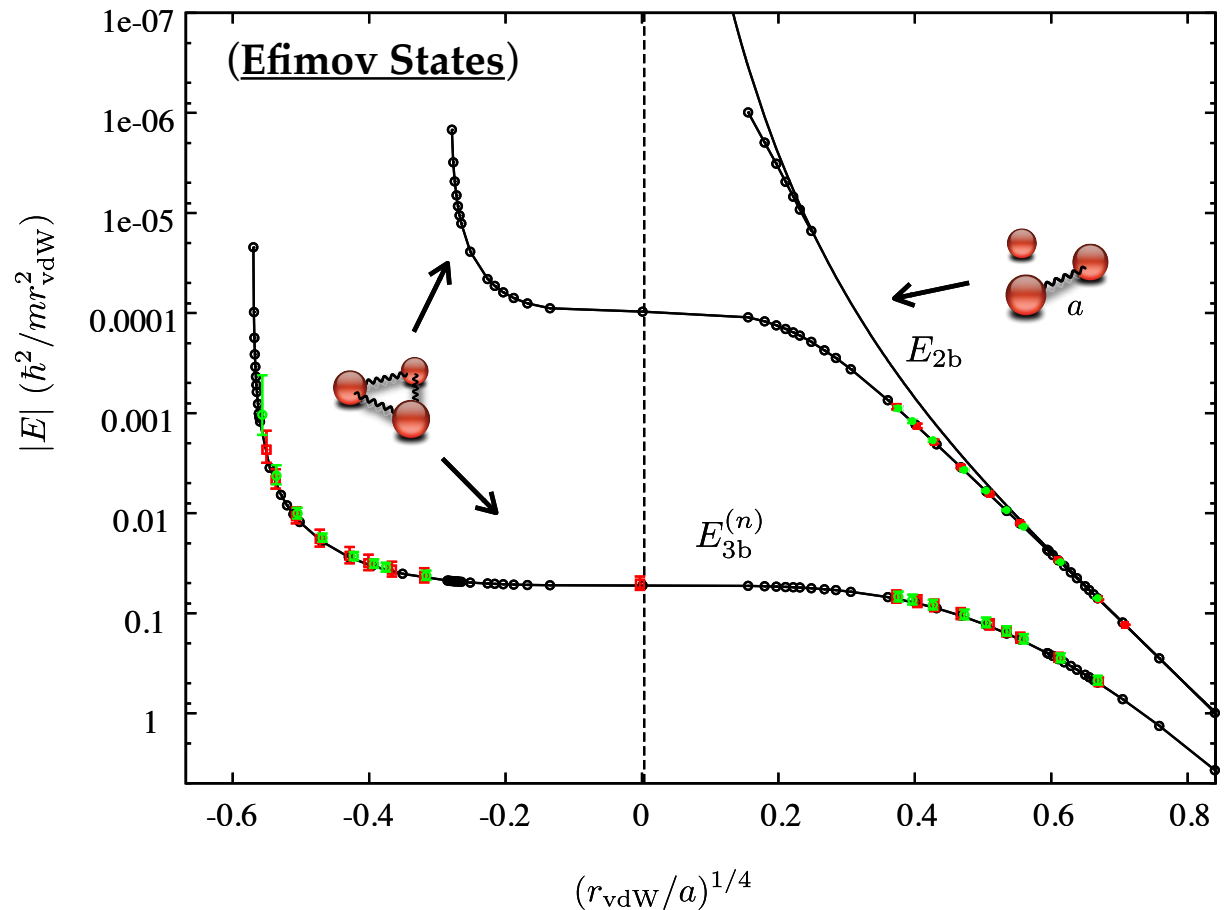
- ✓ **Repulsive Interactions (Fermions):** Long lifetimes
- ✓ **Attractive interactions (Bosons):** infinite number of weakly bound three-body states (**Efimov effect**)

Unitary Bose Gases (fundamental interest)

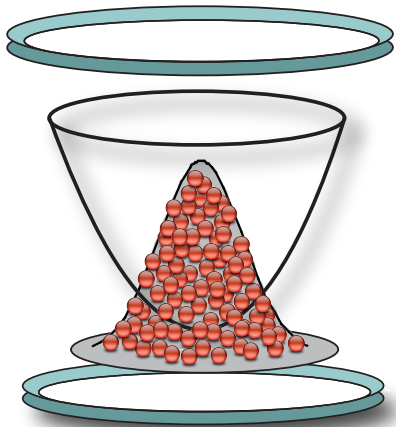


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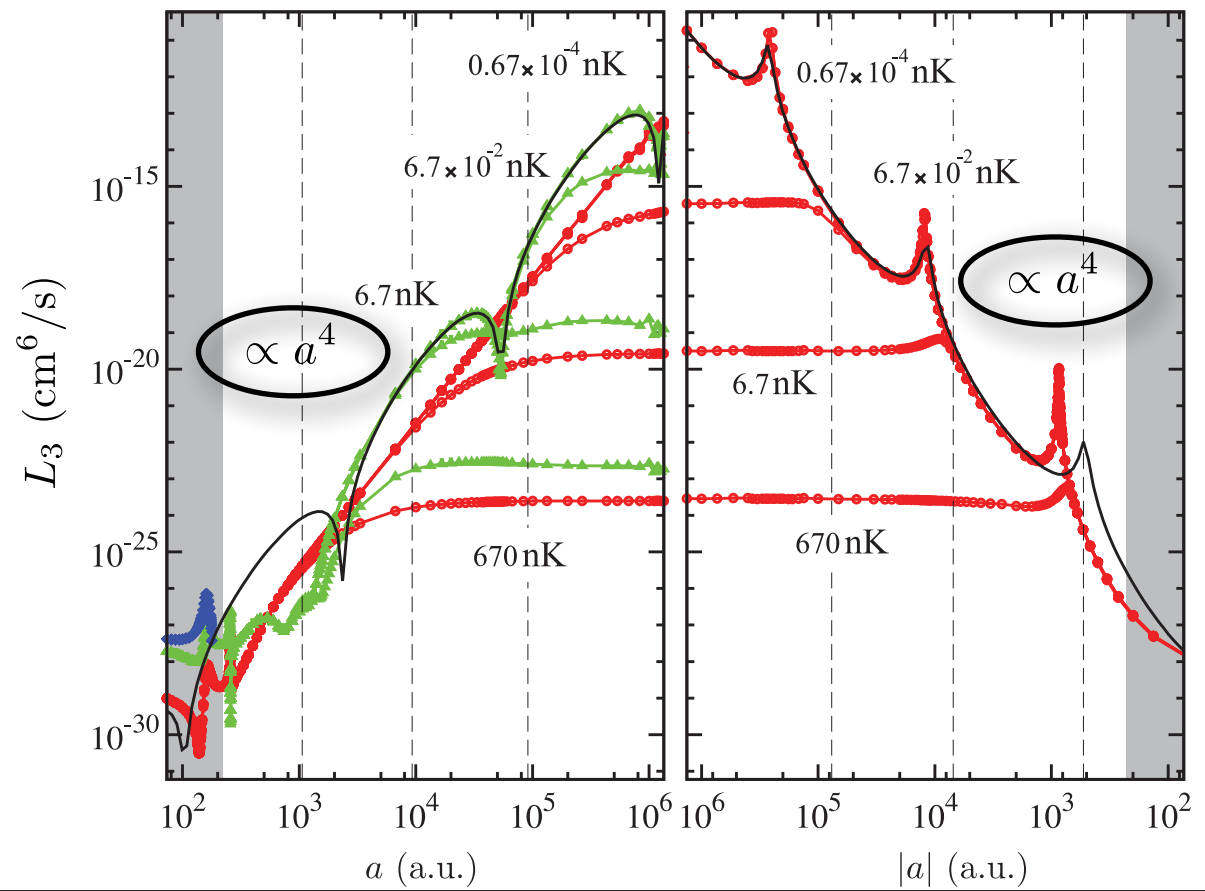
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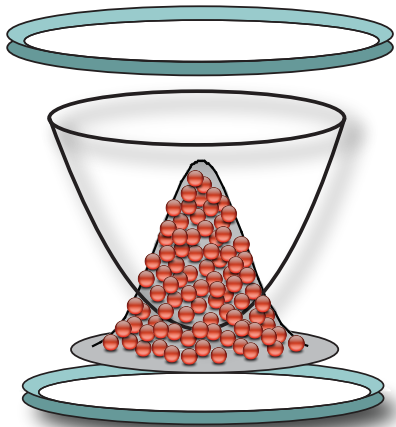
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D'Incao, Greene & Esry, JPB (2009)



Unitary Bose Gases (fundamental interest)



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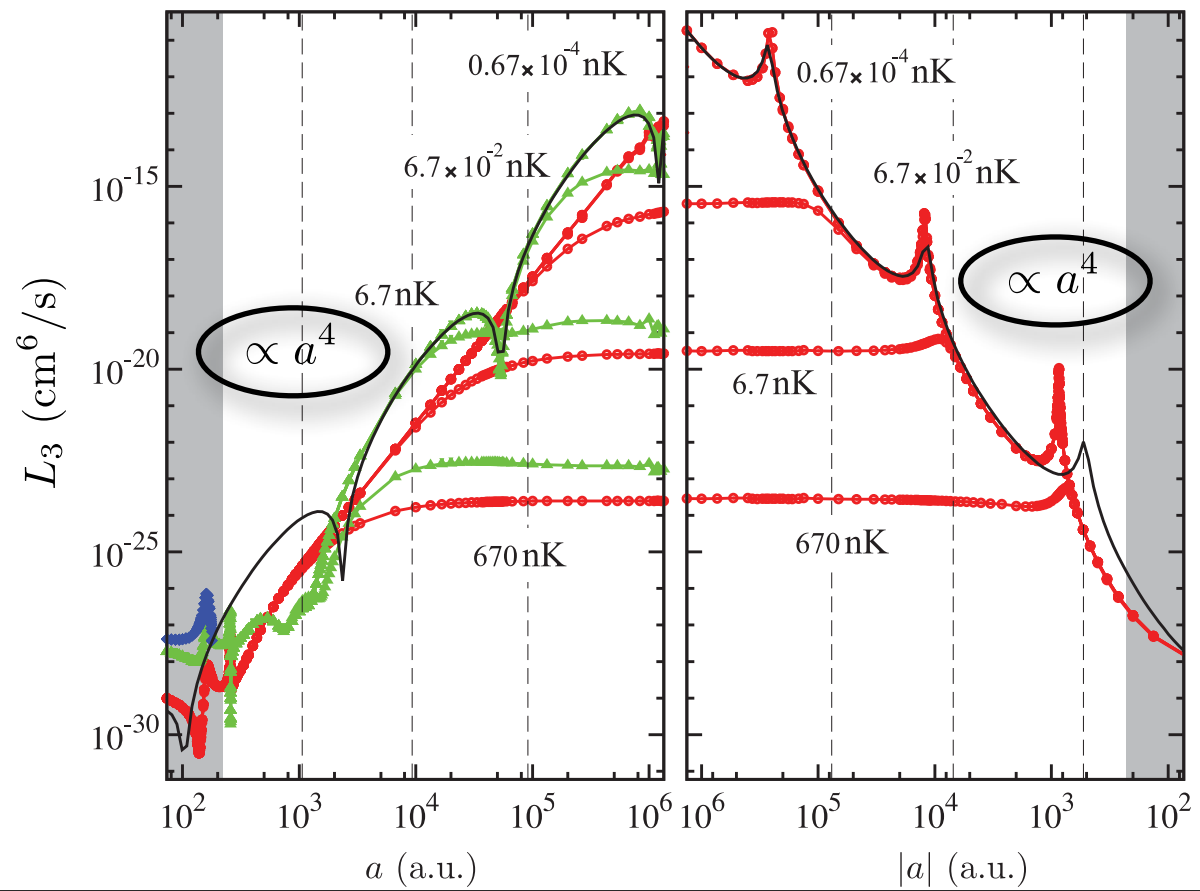
... at finite temperatures ($k|a| \gg 1$)

$$L_3 = \frac{36\sqrt{3}\pi^2}{m^3} \frac{(1 - e^{-4\eta})}{(k_B T)^2} \hbar^5$$

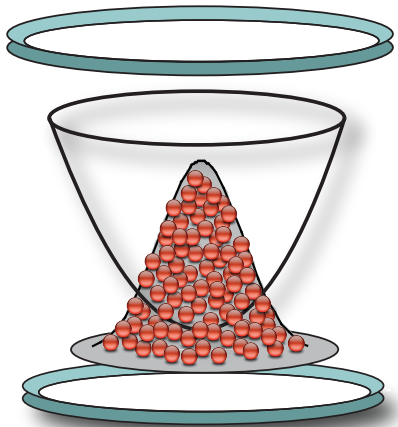
η : 3-body inelasticity parameter

[See Rem et. al, PRL (2013),
D'Incao & Esry, PRL (2004)]

D'Incao, Greene & Esry, JPB (2009)



Unitary Bose Gases (fundamental interest)



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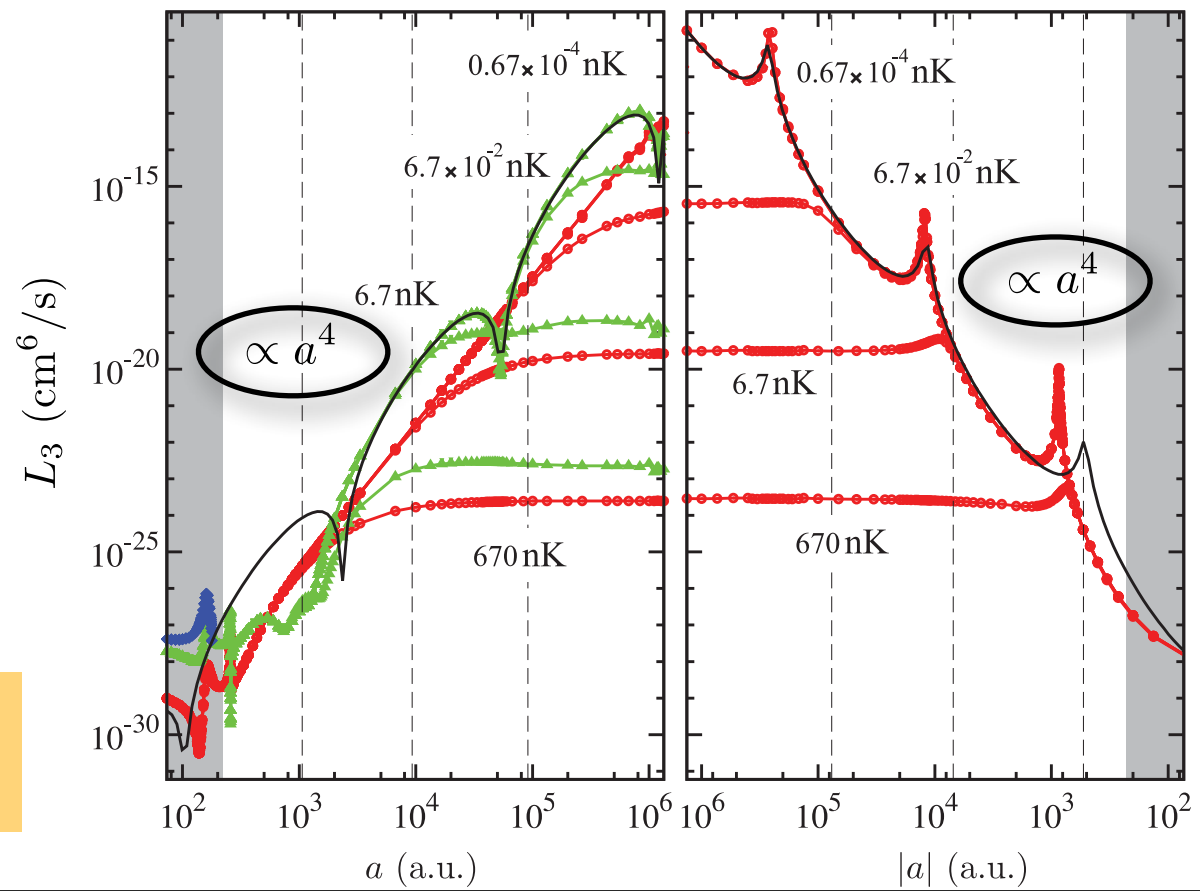
η : 3-body inelasticity parameter

(for ^{85}Rb : $\eta \approx 0.06$)

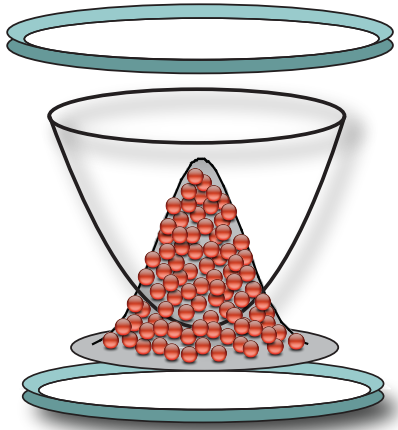
$$L_3 \approx 3 \times 10^{-20} \text{ cm}^6/\text{s}$$

$$\tau = 1/n_{pk}^2 L_3 \approx \underline{0.4\mu\text{s} !!}$$

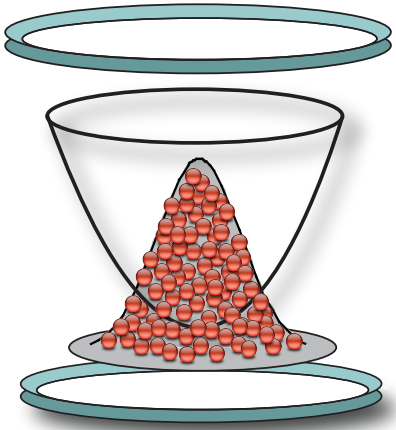
D'Incao, Greene & Esry, JPB (2009)



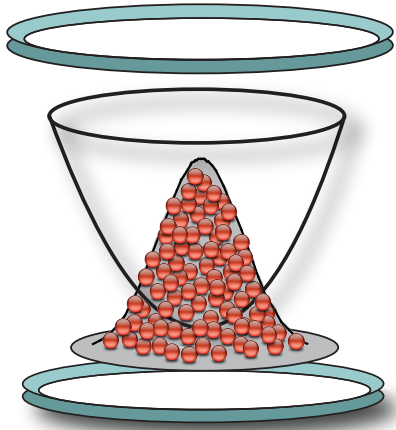
Unitary Bose Gases (fundamental interest)



Unitary Bose Gases (**fundamental interest**)



Unitary Bose Gases (**fundamental interest**)



Experimental Scenario

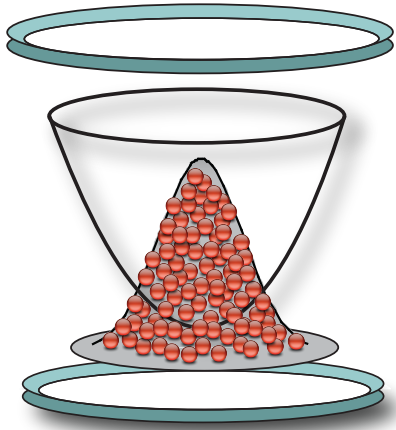
Lifetime of the Bose Gas with Resonant Interactions (7Li)

Rem, Grier, Ferrier-Barbut, Eismann, Langen, Navon, Khaykovich, Werner, Petrov, Chevy, and Salomon
[Phys. Rev. Lett. **110**, 163202 \(2013\)](#)

Stability of a Unitary Bose Gas (39K)

Fletcher, Gaunt, Navon, Smith, and Hadzibabic
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Unitary Bose Gases (**fundamental interest**)



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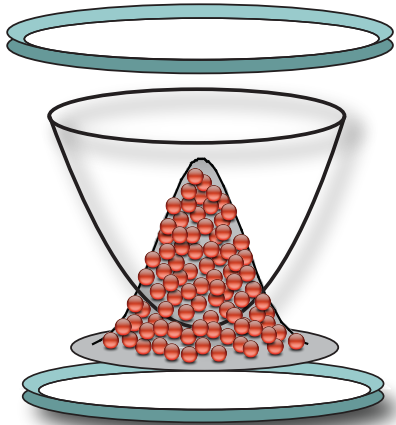
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... but still $n|a|^3 < 1$

Unitary Bose Gases (fundamental interest)

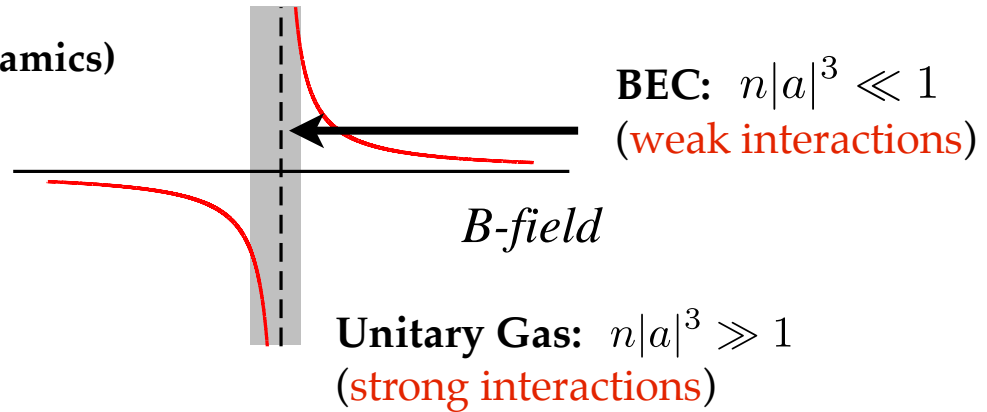


Experimental Scenario

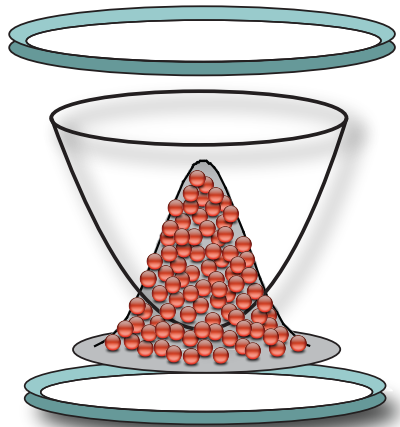
Universal dynamics of a degenerate unitary Bose gas (85Rb)

Makotyn, Klauss, Goldberger, Cornell, and Jin
Nature Phys. **10**, 116 (2014)

(Quench Dynamics)



Unitary Bose Gases (fundamental interest)

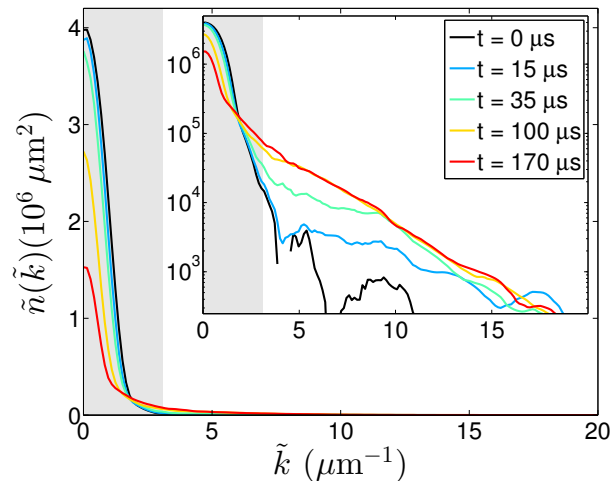
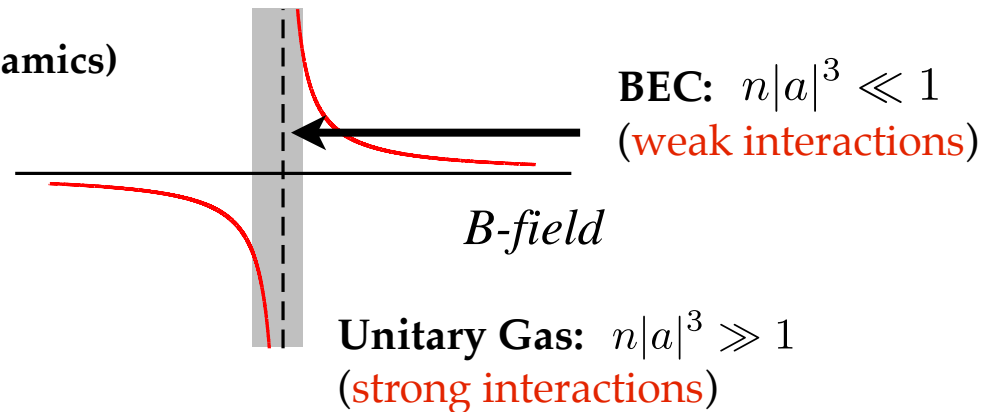


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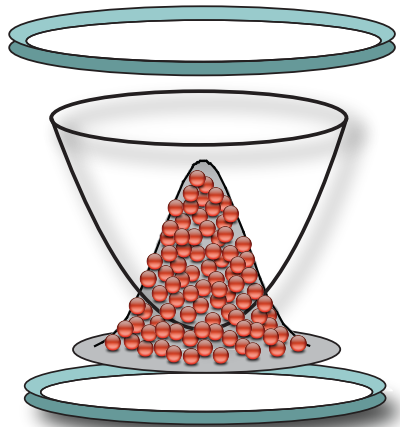
(Quench Dynamics)



Found:

- **Non-equilibrium dynamics**
- **metastable Bose gas (strongly interacting Bose liquid)**
- **dynamics at unitary (Tan's contact interaction)**
- **open new ways to explore unitary regime**

Unitary Bose Gases (fundamental interest)

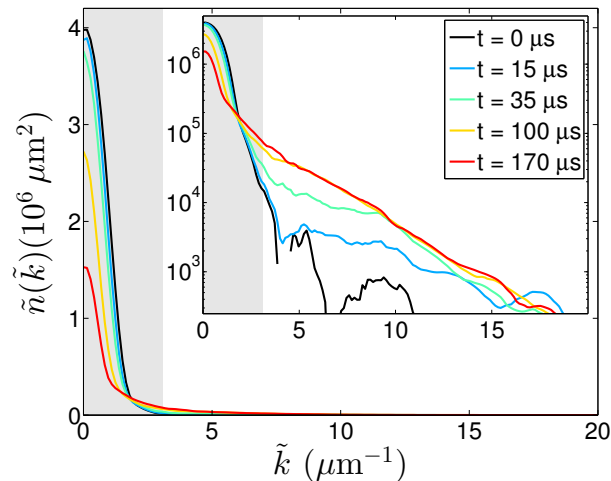
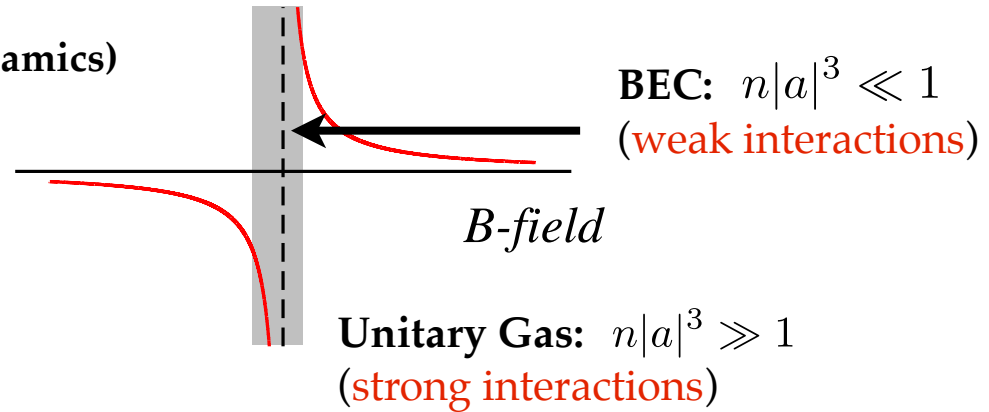


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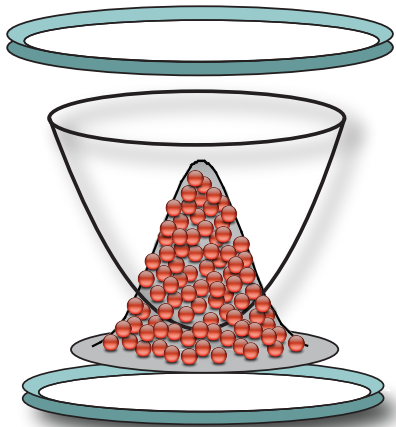
(Quench Dynamics)



Key ingredient: lifetimes $\sim 0.6\text{ms}!!$

$$\frac{\tau_{\text{loss}}}{\tau_{\text{mb}}} \gg 1 \longrightarrow \text{What !?}$$

Unitary Bose Gases (fundamental interest)



Theoretical Scenario

Quench dynamics of a strongly interacting resonant Bose gas

Yin, Radzihovsky, [PRA 88, 063611 \(2013\)](#)

Quenching to unitarity: Quantum dynamics in a 3D Bose gas

Sykes, Corson, D’Incao, Koller, Greene, Rey, Hazzard, Bohn, [PRA 89, 021601 \(2014\)](#)

Two-body and three-body Contacts for identical bosons near unitarity

Smith, Braaten, Kang, Platter, [PRL 112, 110402 \(2014\)](#)

Nonequilibrium states of a quenched Bose Gas

Ben, Ling, [arXiv:1401.2390 \(2014\)](#)

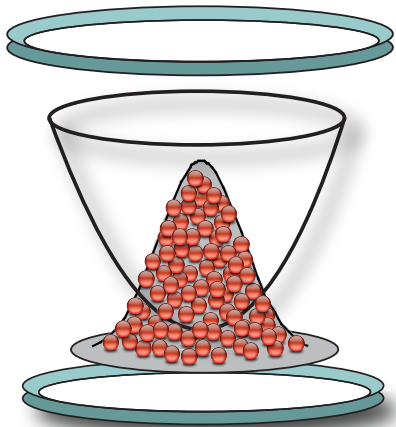
Momentum distribution of a dilute unitary Bose gas with three-body losses

Laurent, Leyronas, Chevy, [arXiv:1312.0079 \(2013\)](#)

Equilibrating dynamics in quenched Bose gases: characterizing multiple time regimes

Racon, Levin, [arXiv:1403.0141 \(2014\)](#)

Unitary Bose Gases (fundamental interest)



Theoretical Scenario

Quench dynamics of a strongly interacting resonant Bose gas

Yin, Radzihovsky, [PRA 88, 063611 \(2013\)](#)

Quenching to unitarity: Quantum dynamics in a 3D Bose gas

Sykes, Corson, D’Incao, Koller, Greene, Rey, Hazzard, Bohn, [PRA 89, 021601 \(2014\)](#)

Two-body and three-body Contacts for identical bosons near unitarity

Smith, Braaten, Kang, Platter, [PRL 112, 110402 \(2014\)](#)

Nonequilibrium states of a quenched Bose Gas

Ben, Ling, [arXiv:1401.2390 \(2014\)](#)

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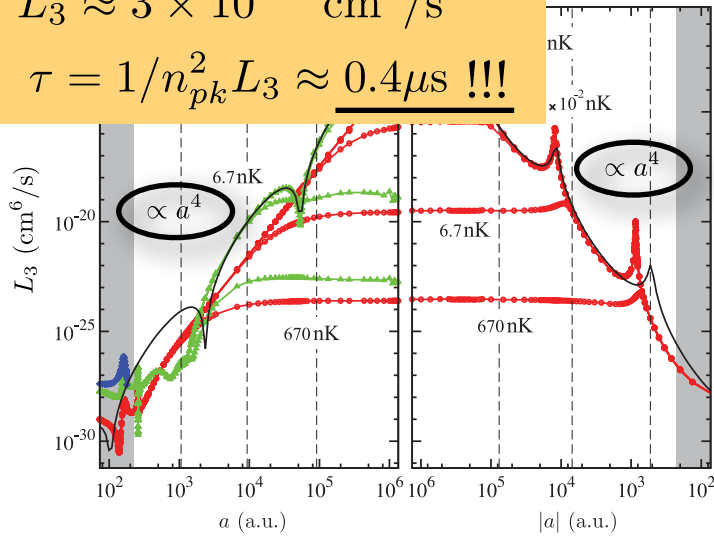
Unitary Quantum Gases ($n|a|^3 \gg 1$)

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(for ^{85}Rb : $\eta \approx 0.06$)

$$L_3 \approx 3 \times 10^{-20} \text{ cm}^6/\text{s}$$

$$\tau = 1/n_{pk}^2 L_3 \approx 0.4 \mu\text{s} !!!$$



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 PRA **89**, 021601 (2014)

Losses for a unitary gas: ($n|a|^3 \gg 1$)

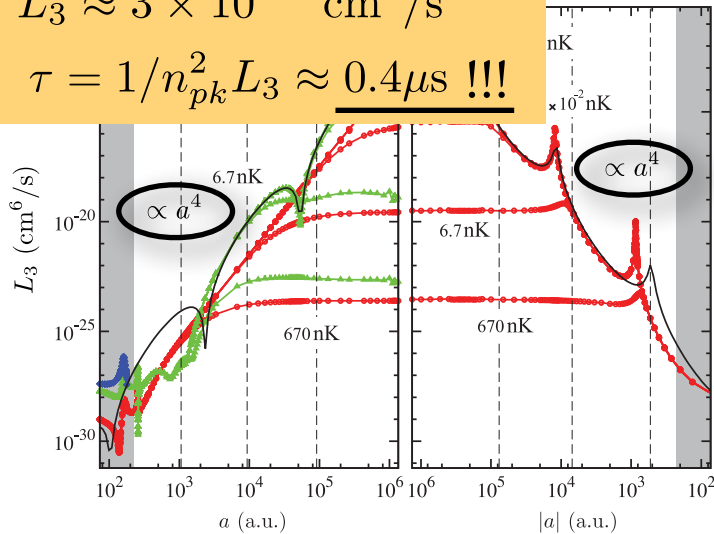
$$k_B T \rightarrow E_F = \frac{(6\pi^2 n)^{2/3} \hbar^2}{2m} \quad \text{and} \quad \langle L_3 \rangle = \frac{\int L_3 n^3 d^3 r}{\int n^3 d^3 r}$$

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$$L_3 = \frac{36\sqrt{3}\pi^2}{m^3} \frac{(1 - e^{-4\eta})}{(k_B T)^2} \hbar^5$$



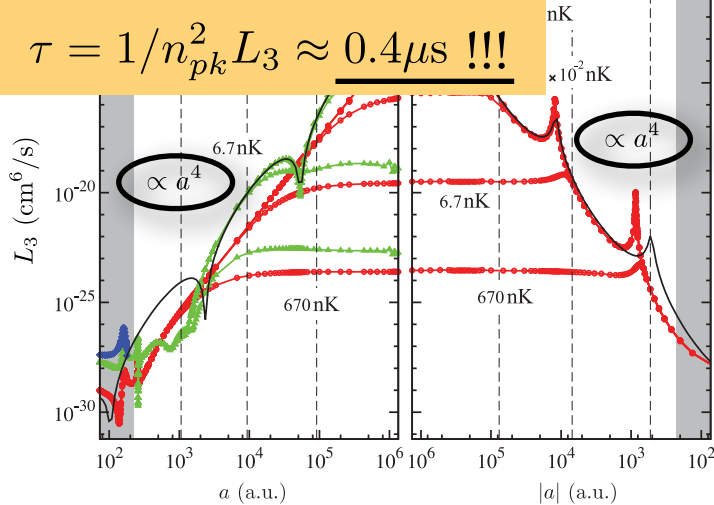
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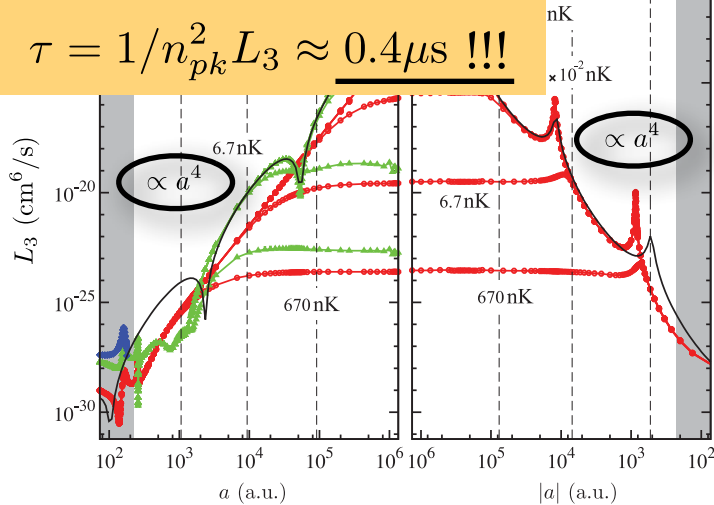
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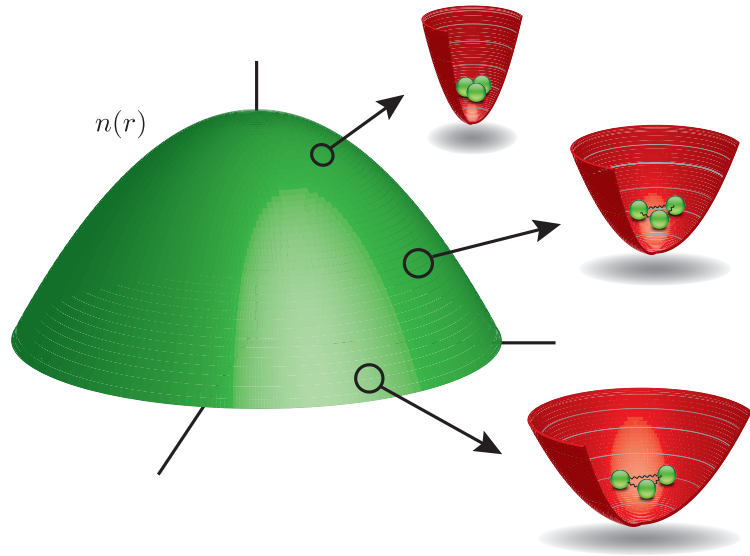
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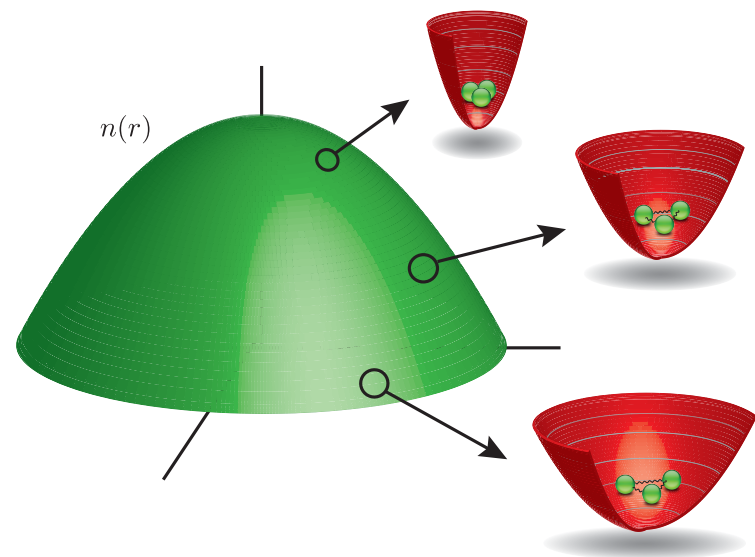
Local properties are important !
Scattering for $n|a|^3 \gg 1$?
What about Efimov states ?

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Local model for three-body losses in an unitary Bose gas





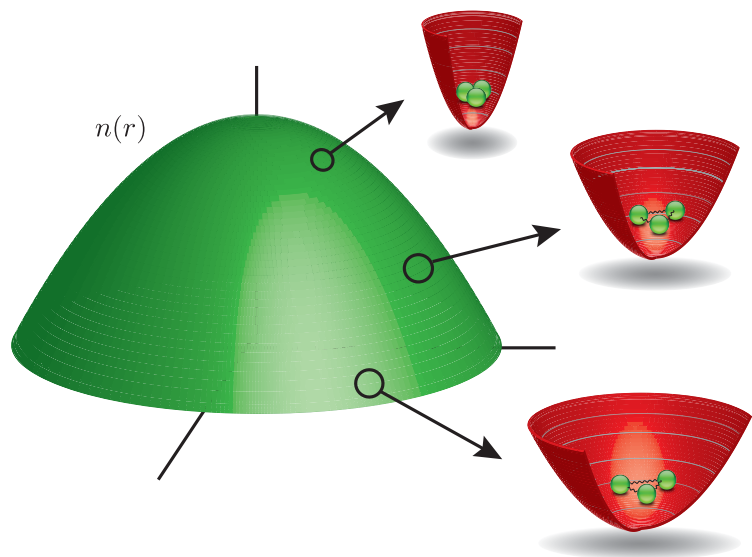
Solve the three-body problem (hyperspherical) in a harmonic trap whose frequency is determined by the “local” density:

$$n = \frac{9 \sqrt{\pi/2}}{128 a_{\text{ho}}^3} \rightarrow \hbar\omega_{\text{ho}} = \hbar^2 \frac{16(2/\pi)^{1/3}}{3^{2/3}m} n^{2/3}$$

Borca, Blume & Greene, NJP (2003)

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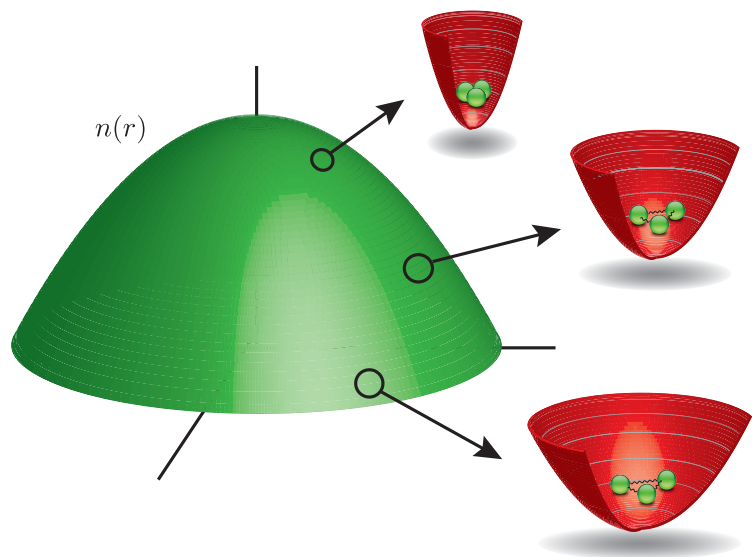
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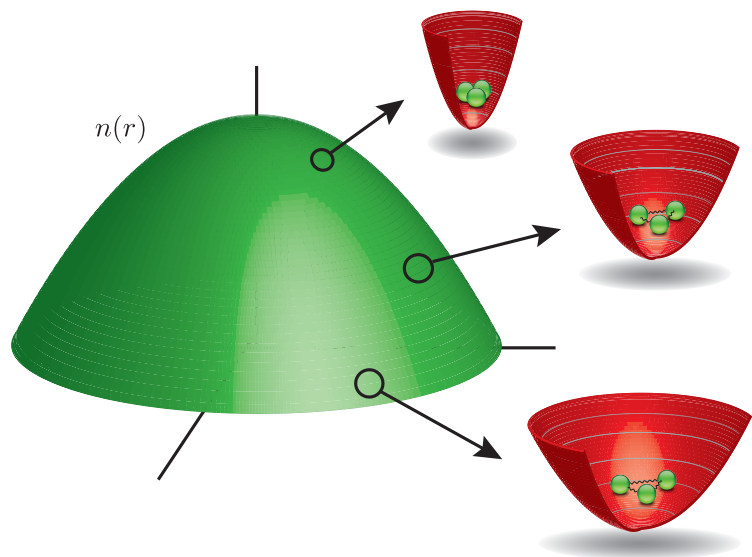
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c_β^2 : Population

$\tau_\beta = \hbar/\Gamma_\beta$: Lifetime

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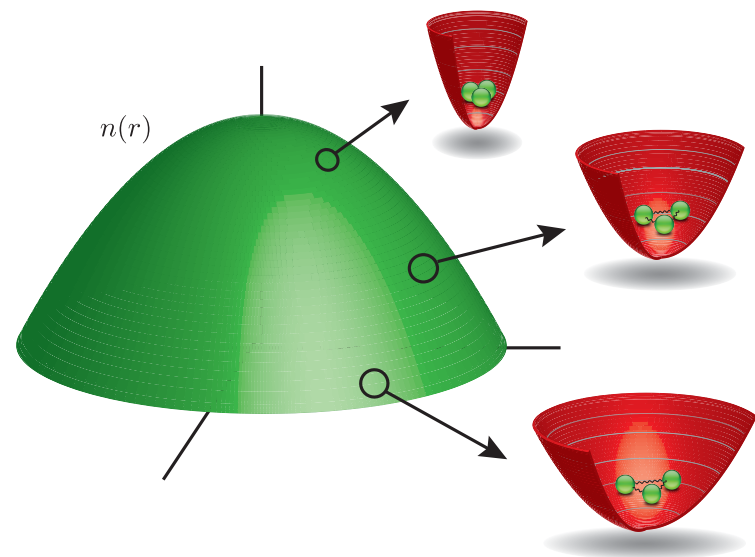
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$$\langle \tau^* \rangle \approx \underline{1.1 \text{ ms !!!}}$$



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Separation of timescales (universal relation)

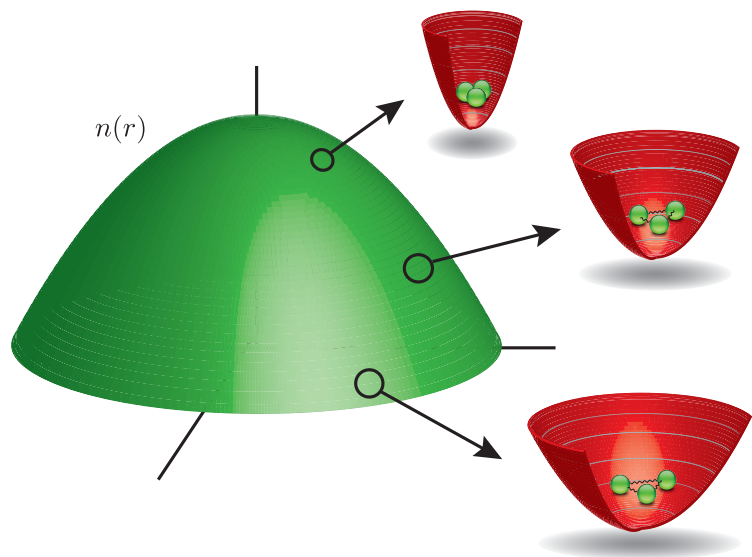
$$\frac{\tau_{\text{loss}}}{\tau_C} \approx \frac{1.89}{\eta}$$

^{85}Rb ($\eta \approx 0.06$): 31.5

^7Li ($\eta \approx 0.21$): 9.0

^{133}Cs ($\eta \approx 0.08 - 0.19$): 23.6–9.9

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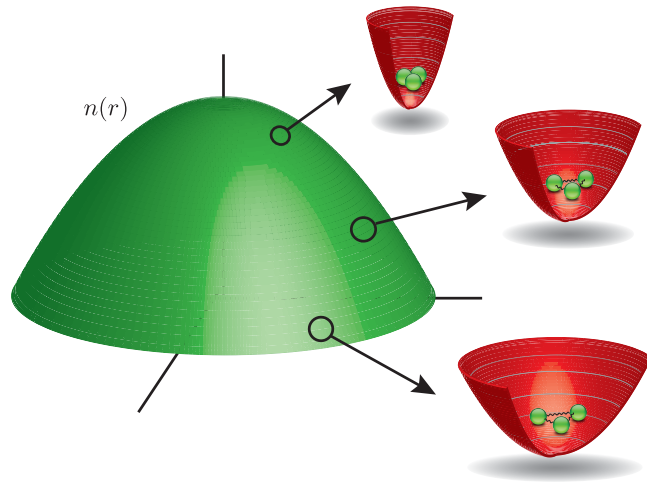
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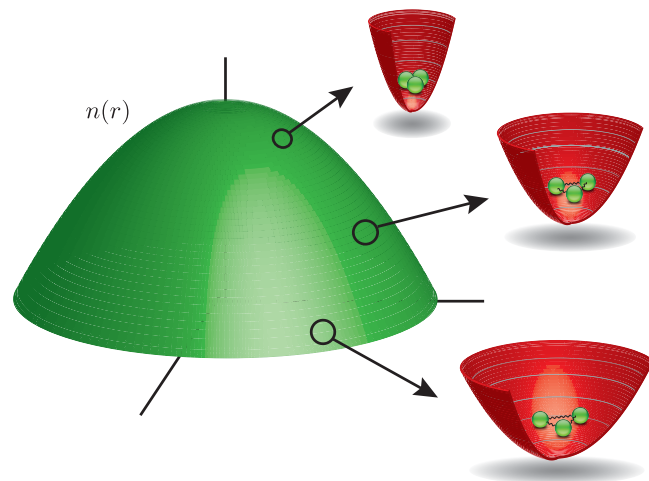
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Three-body inelasticity parameter !!!

Efimov states in Unitary Bose gases !?

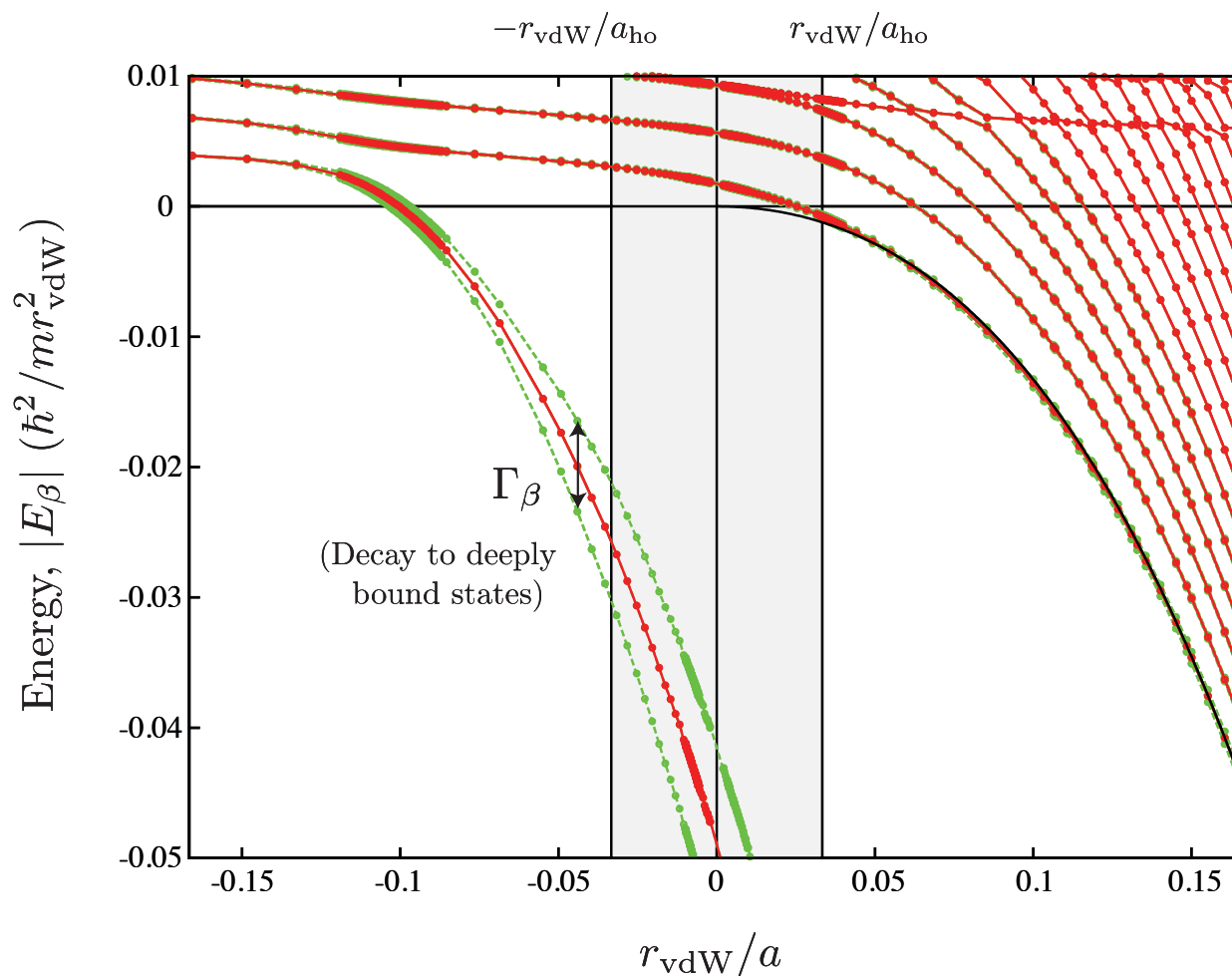


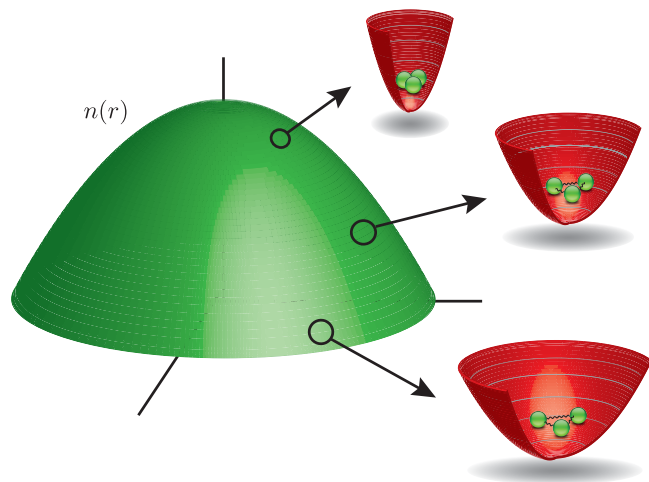
Efimov states and dynamics in ultracold unitary Bose gases
J. P. D’Incao and J. L. Bohn (in preparation)



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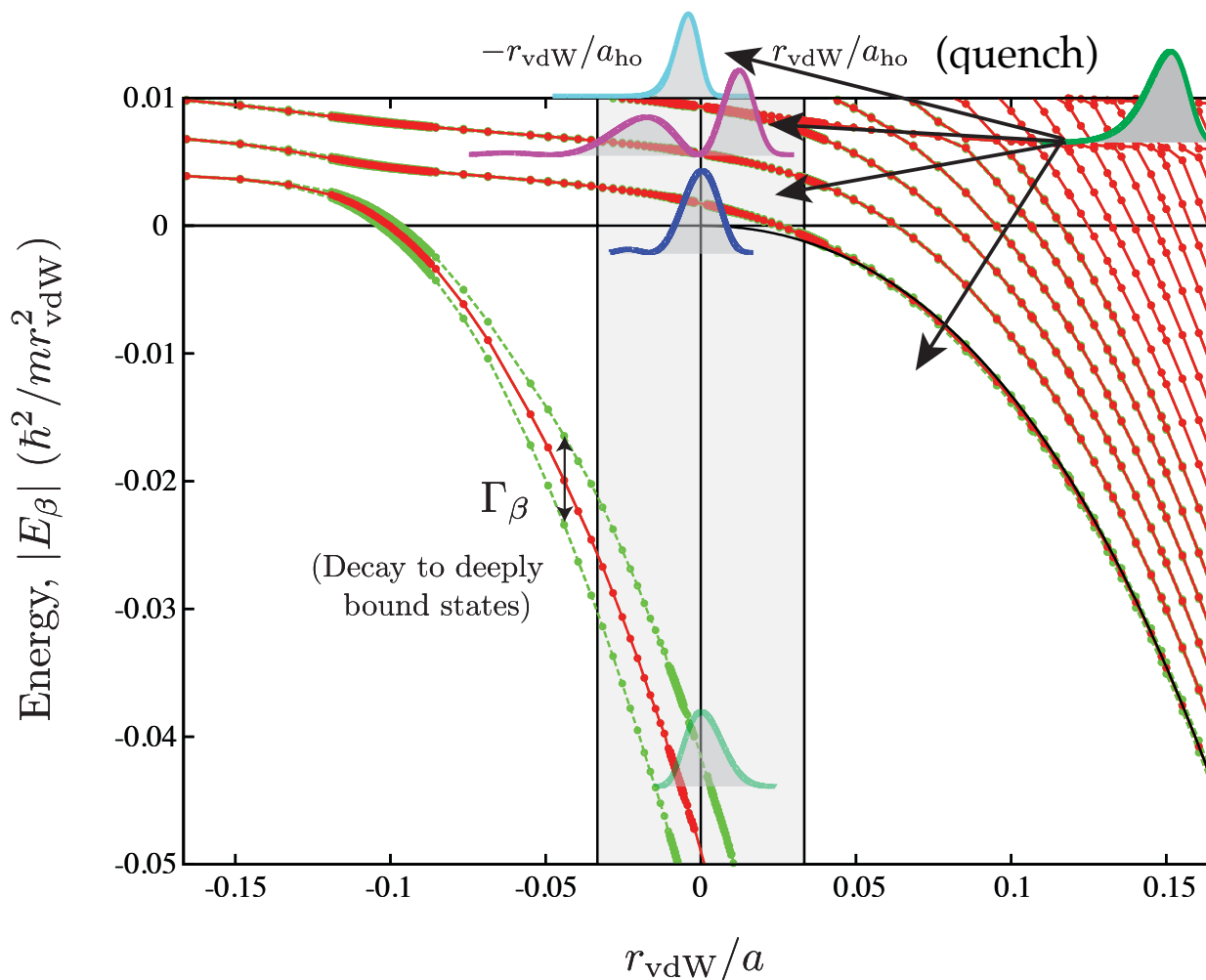
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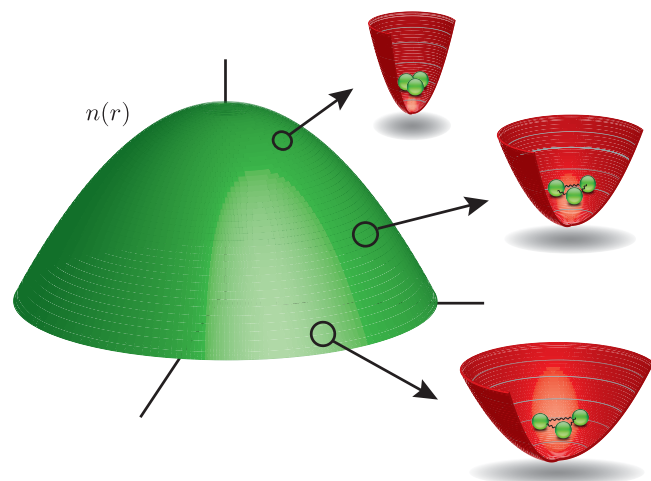


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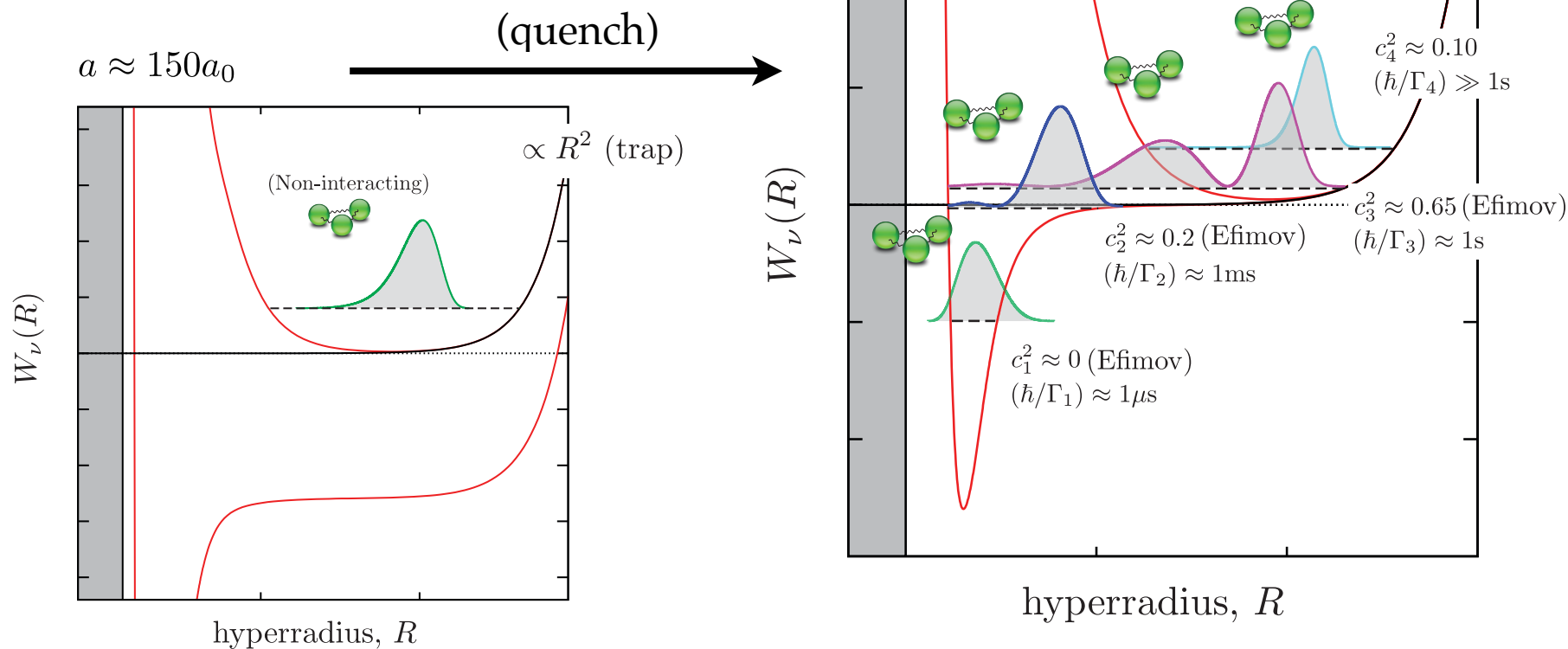
Local model for three-body losses in an unitary Bose gas



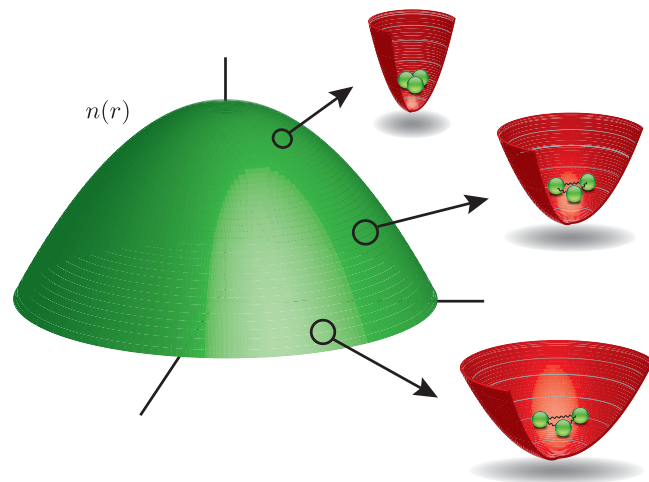
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(In the hyperspherical picture)

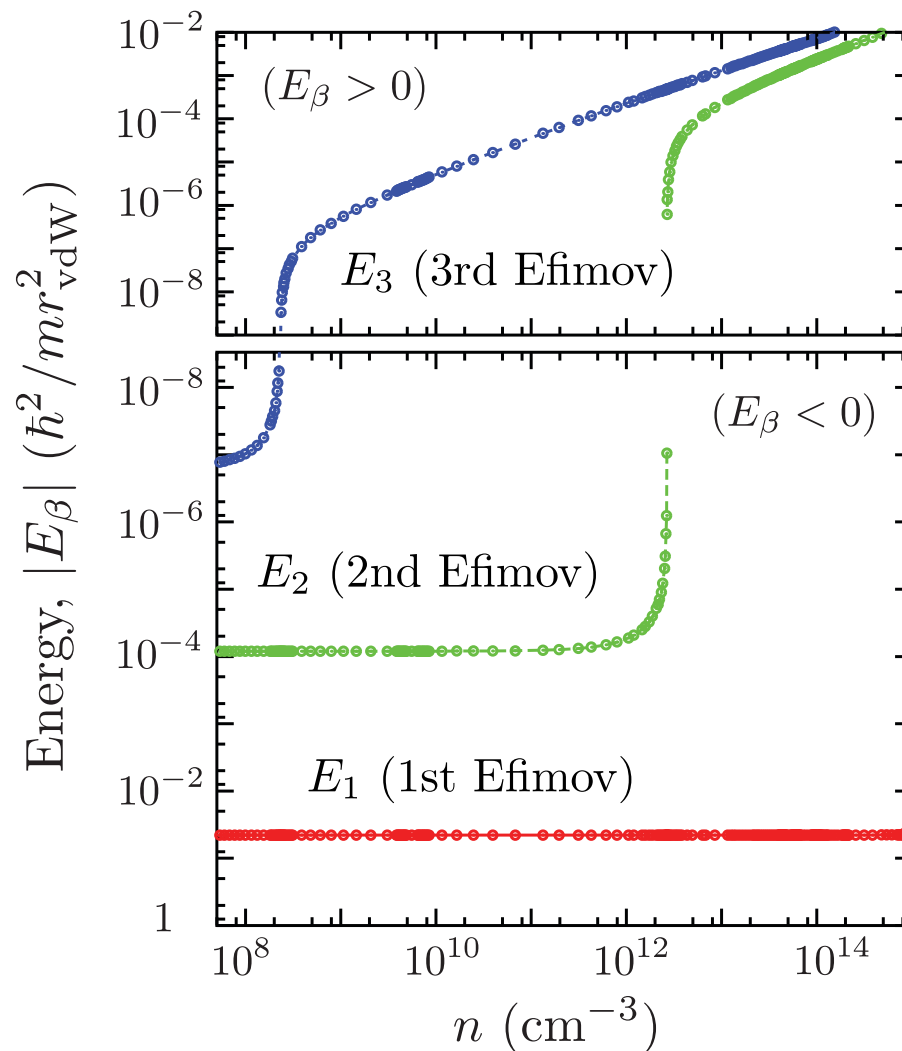


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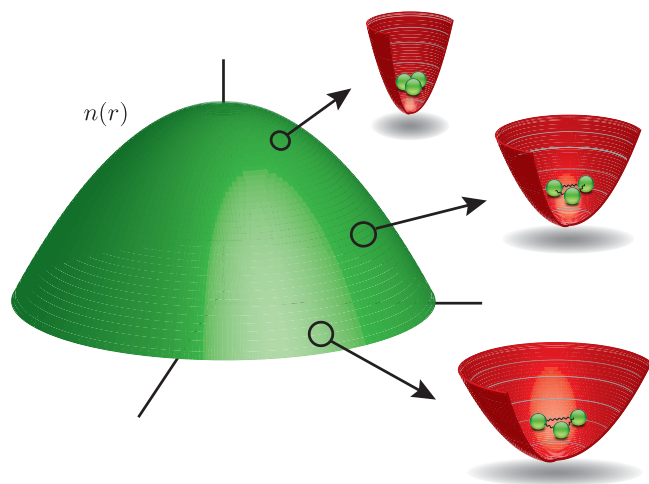


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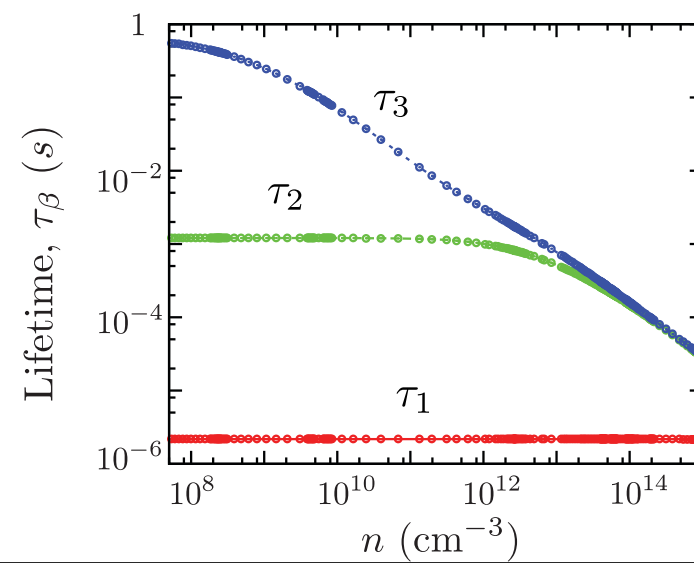
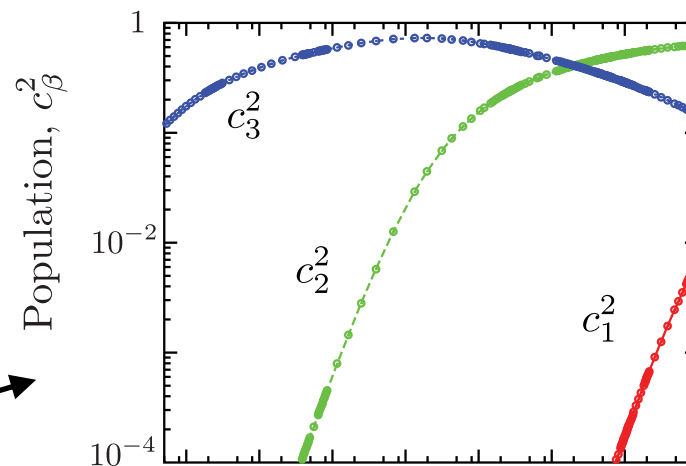
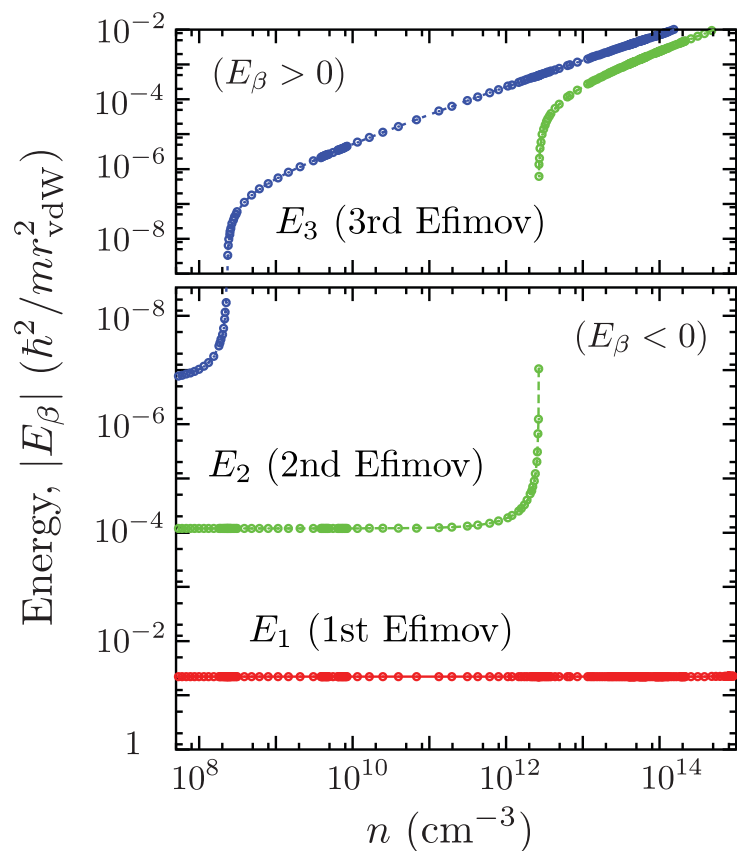


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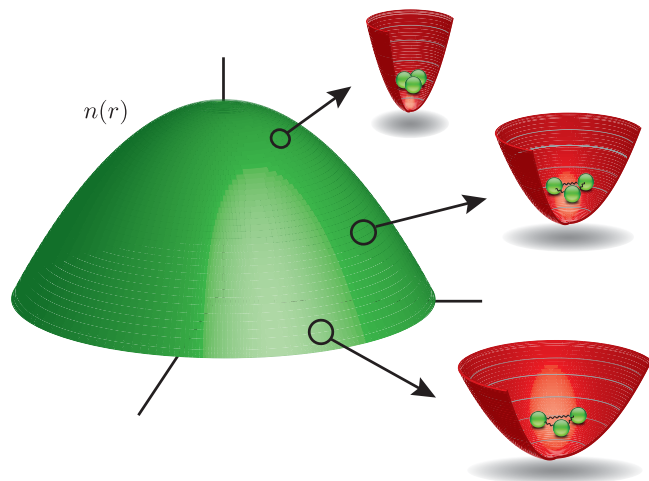


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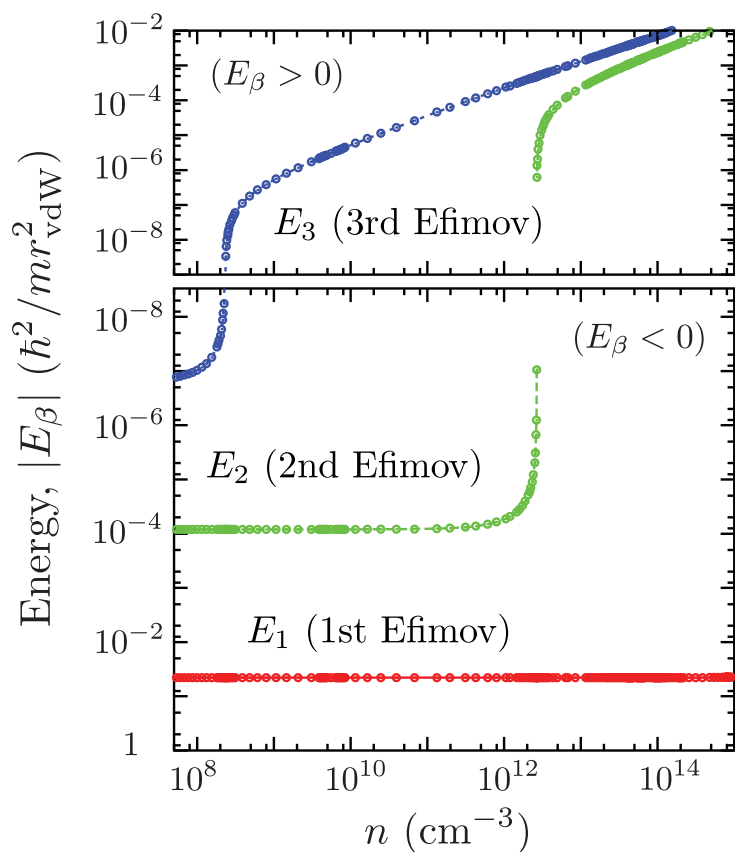


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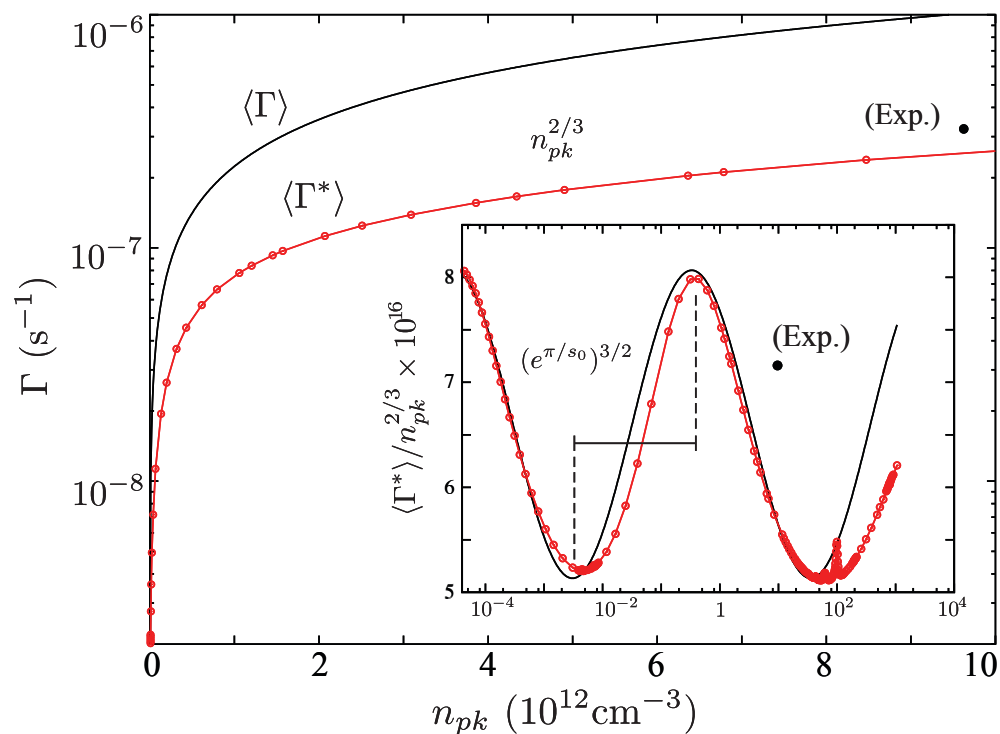
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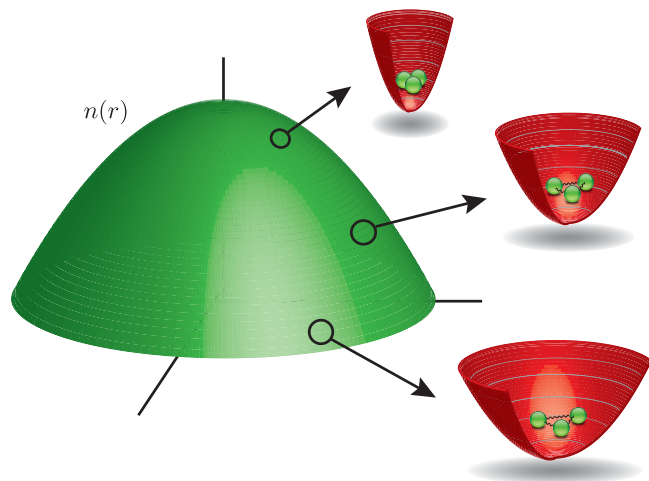
J. P. D’Incao and J. L. Bohn (in preparation)



Efimov oscillations:

$$\langle \Gamma \rangle \approx [A + B \sin^2(s_0 \ln n^{1/3} + \phi)] n^{2/3}$$





Efimov states and dynamics in ultracold unitary Bose gases

J. P. D’Incao and J. L. Bohn (in preparation)

Bottom Line:

>>> Separation of timescales

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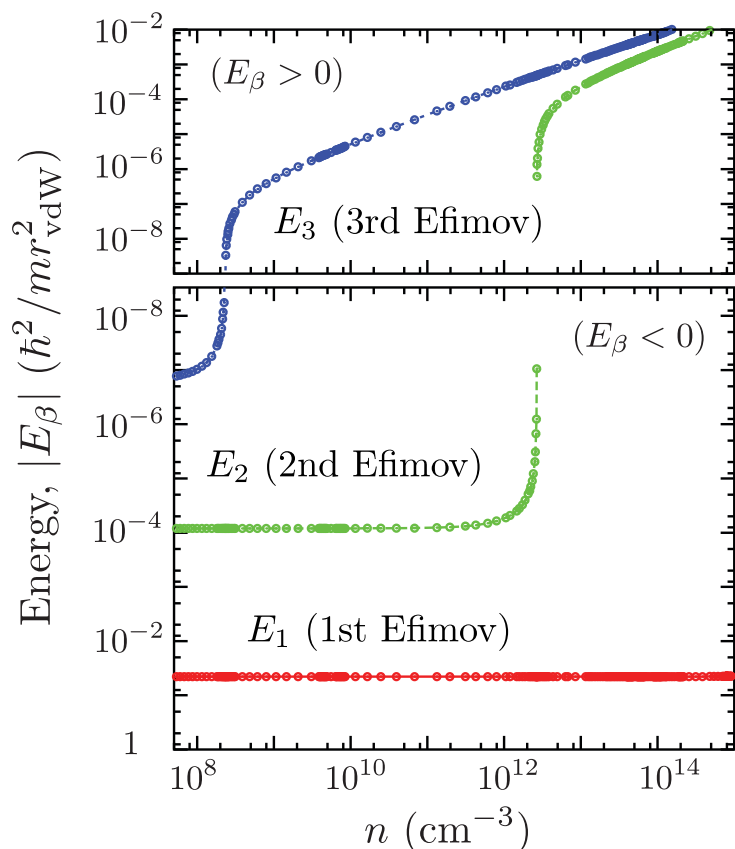
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>>> Lifetime controlled by Efimov states

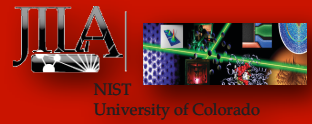


Can we control Efimov state lifetime ?

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(with G. Pupillo, J. Wang, C. H. Greene)

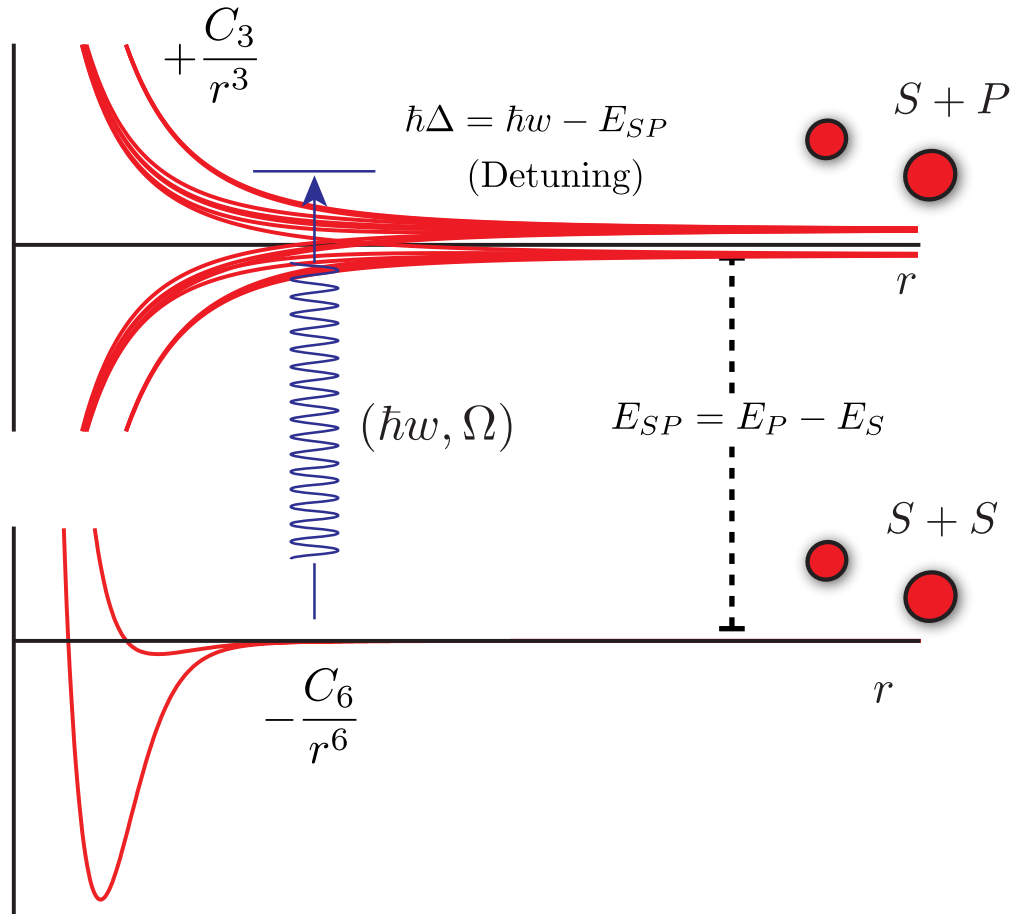
Laser assisted collisions



(with G. Pupillo, J. Wang, C. H. Greene)

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Schematic Two-body potentials:



Atom-Laser interaction

$$\hat{W}(t) = \hbar\Omega \left(|S\rangle\langle P| + |P\rangle\langle S| \right) \cos(\omega t)$$

$\Omega \equiv$ Rabi frequency

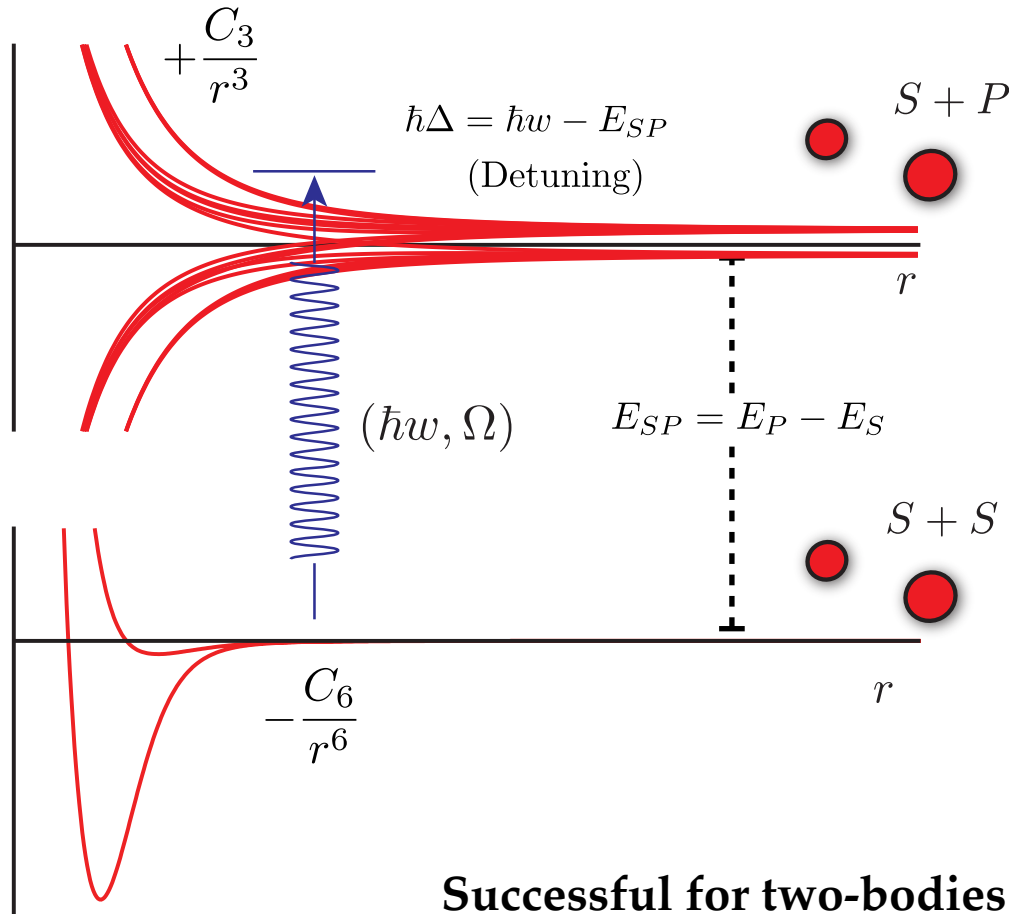
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Blue-Shielding:

blue-detuned Laser = coupling to a repulsive $1/r^3$ interaction = prevents atoms to approach short distances

(with G. Pupillo, J. Wang, C. H. Greene)

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Successful for two-bodies

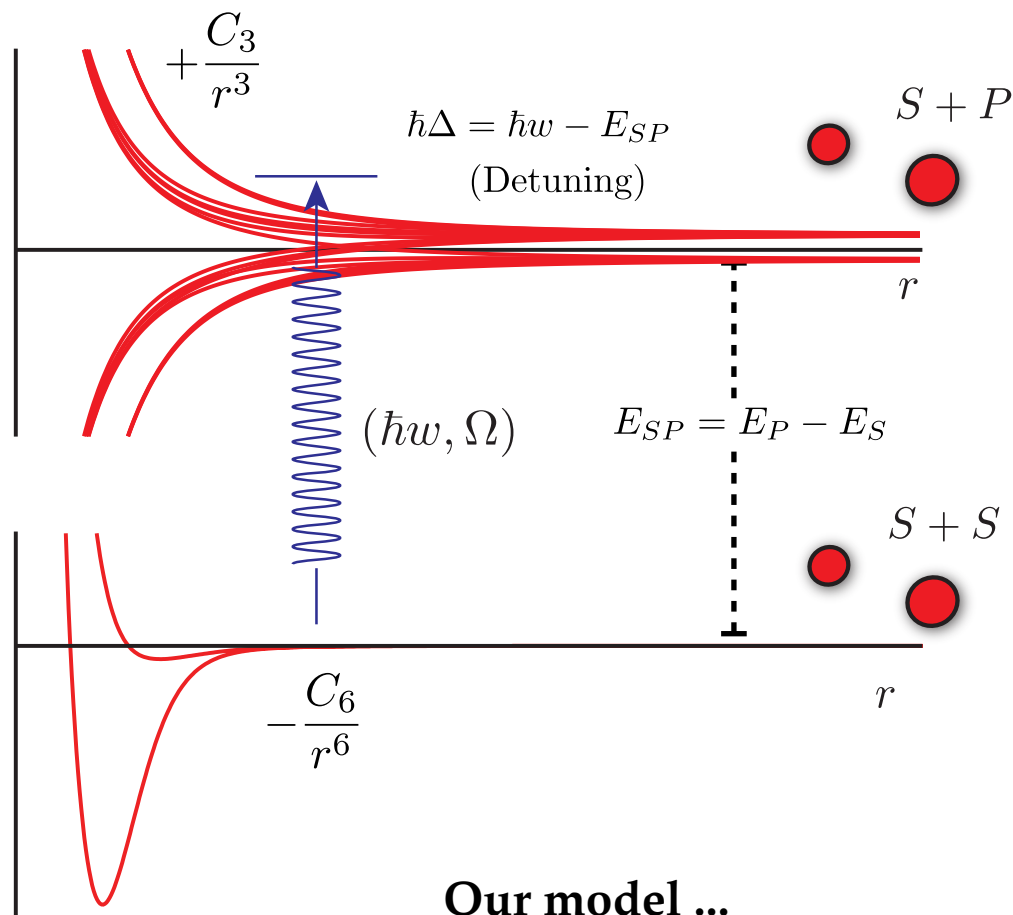
Suominen, Holland, Burnett, Julienne, PRA **51**, 1446 (1995);

Weiner, Bagnato, Zilio, Julienne, RMP **71**, 1 (1999);

Gorshkov, Rabl, Pupillo, Micheli, Zoller, Lukin, Buchler, PRL **101**, 073201 (2008)

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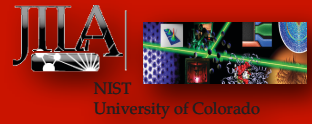
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Our model ...

- $S + S$ Interaction : $v_{SS}(r) = -\frac{C_6}{r^6} \left(1 - \frac{\lambda_*^6}{r^6} \right)$, $(a \rightarrow \pm\infty)$
- $S + P$ Interaction : $v_{SP}(r) = +\frac{C_3}{r^3}$

Extension to three-bodies ?

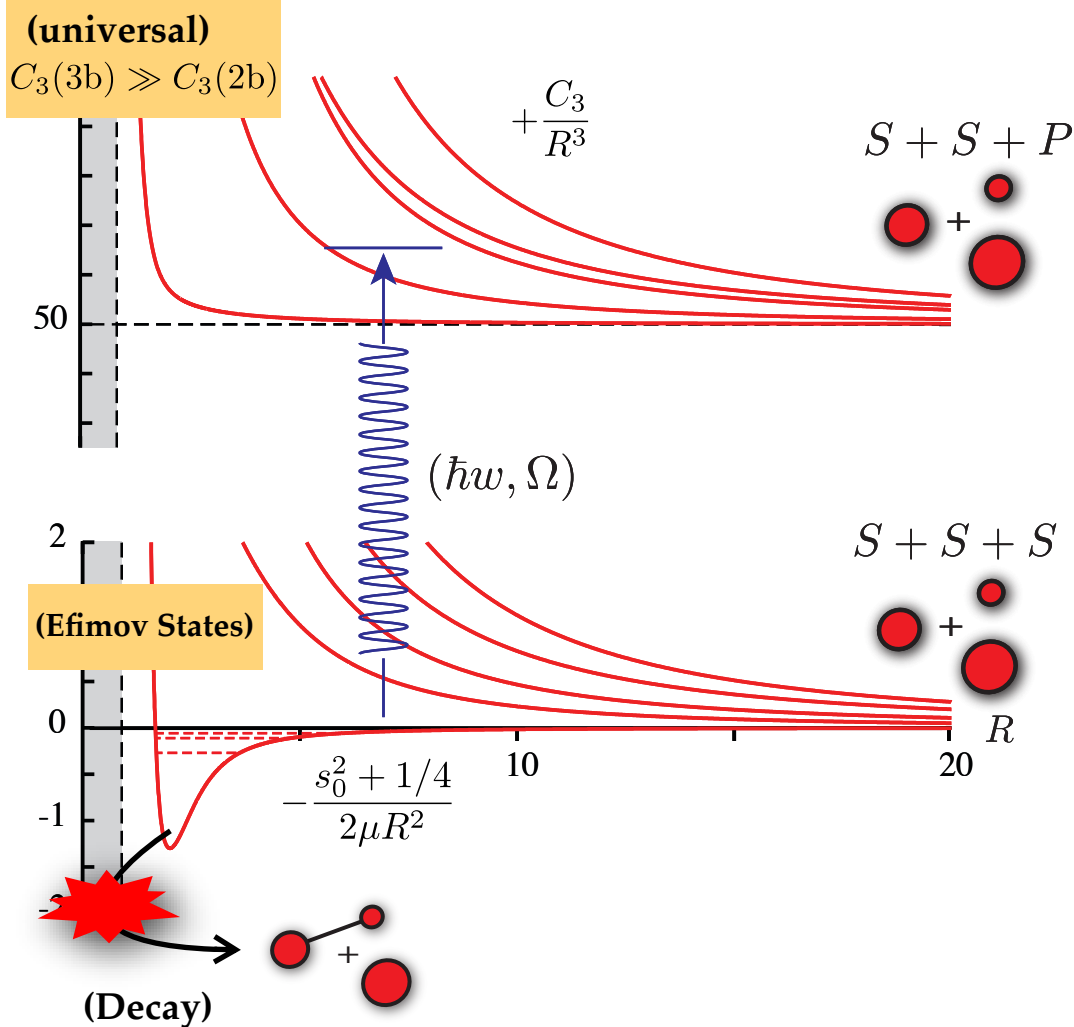
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Hyperspherical Three-body potentials: ($a \rightarrow \pm\infty$)



Hyperspherical Adiabatic Hamiltonian:

$$\hat{\mathcal{H}}_{\text{ad}}(\vec{R}, t) = \hat{H}_{\text{ad}}(\vec{R}) + \hat{W}(t) - i\hbar \frac{d}{dt}$$

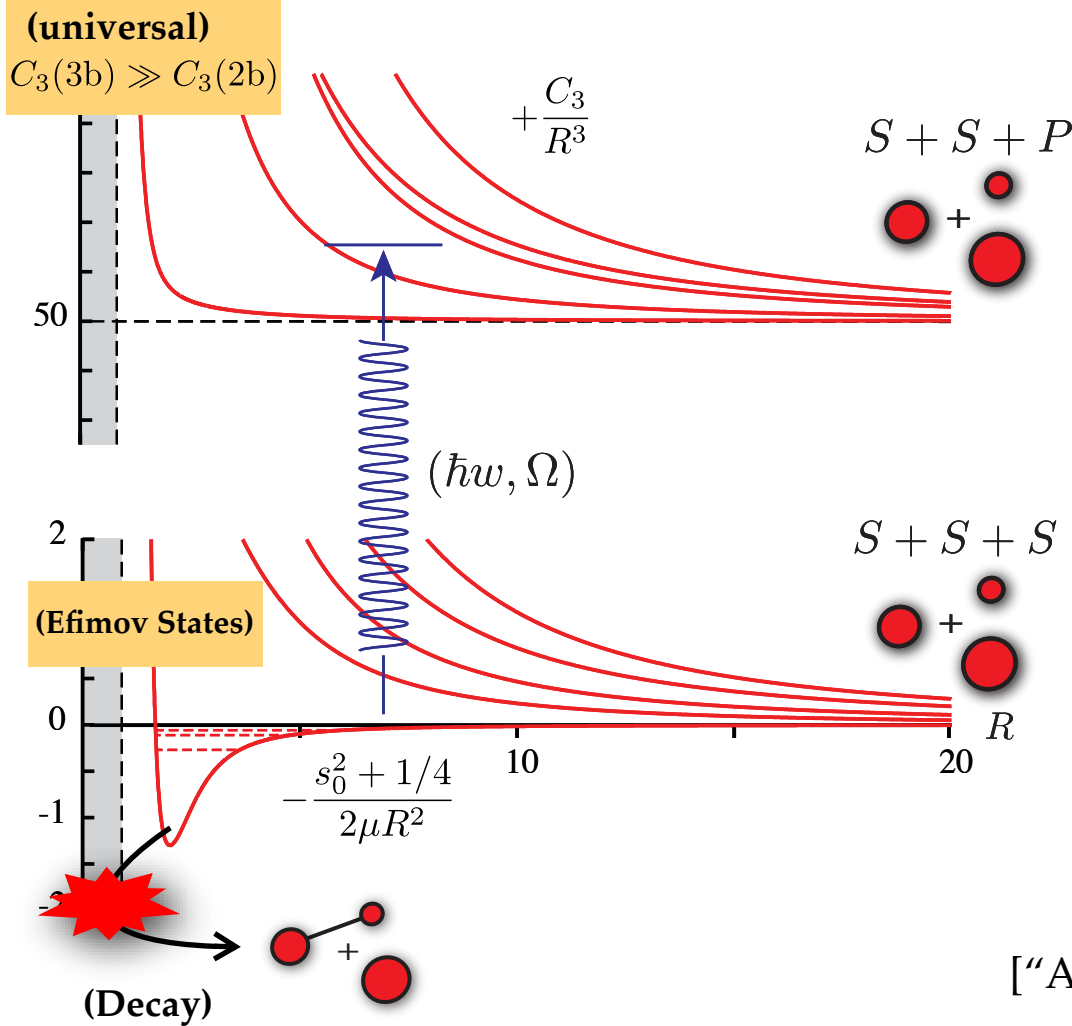
(Bare Interactions) ←

(External Field) ↓

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(External Field)

$$\hat{W}(t) = \hat{W}_1(t) + \hat{W}_2(t) + \hat{W}_3(t)$$

$$\hat{\mathcal{H}}_{\text{ad}}(\vec{R}, t)\Phi_\gamma(\vec{R}, t) = U_\gamma(R)\Phi_\gamma(\vec{R}, t)$$

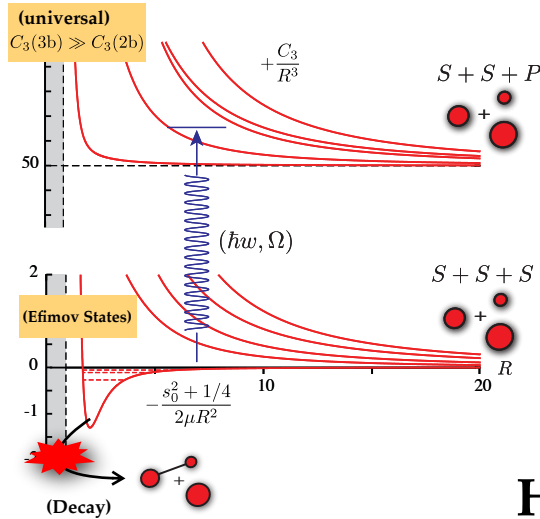
$U_\gamma(R) \equiv$ Three-body dressed Potentials

["Adiabatic Floquet Formalism for few-body systems",
 J. P. D'Incao, in preparation]

[See Shih-I Chu and Dmitry A. Telnov, Phys. Rep. 390, 1 (2004)]

(with G. Pupillo, J. Wang, C. H. Greene)

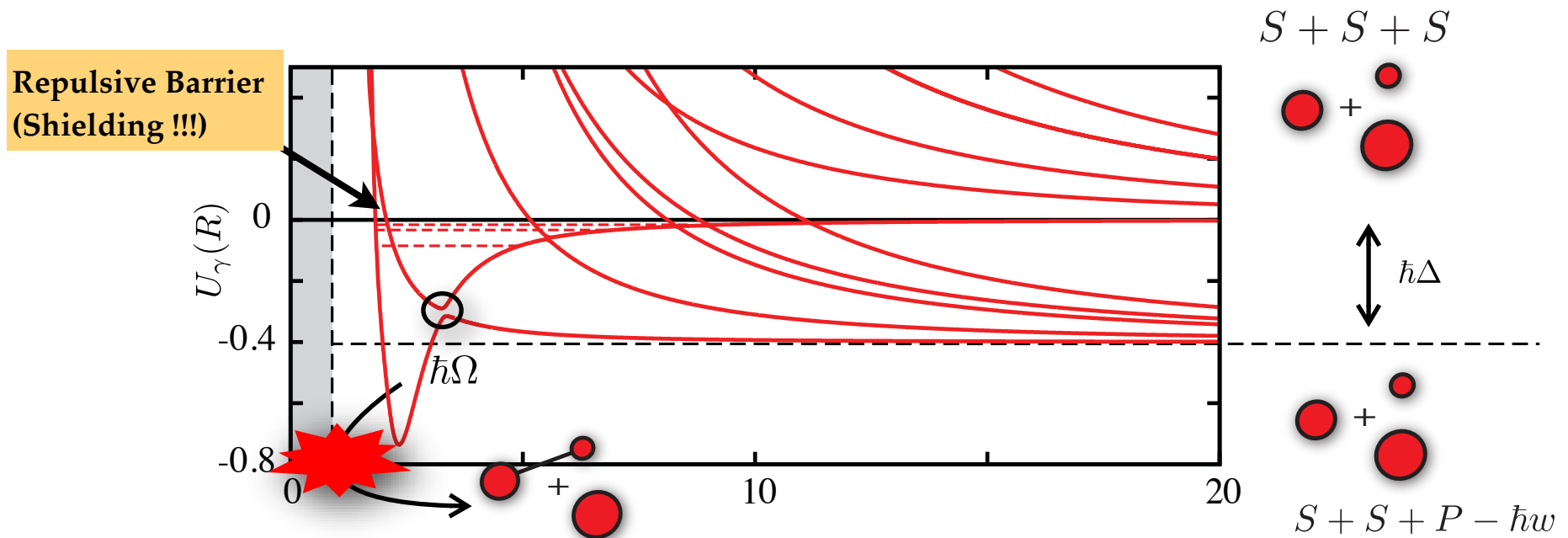
Hyperspherical Three-body potentials: ($a \rightarrow \pm\infty$)



Hyperspherical Adiabatic Hamiltonian:

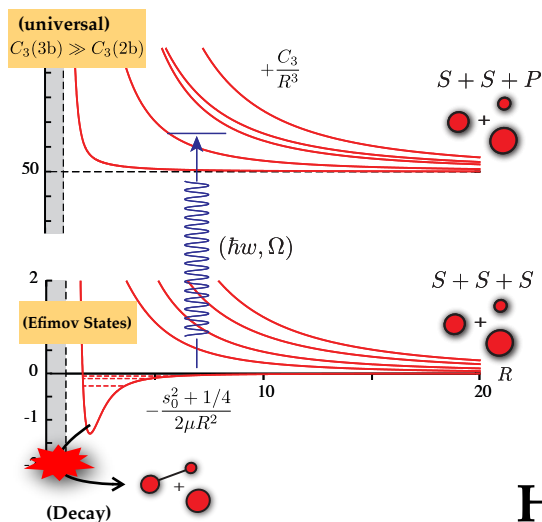
$$\hat{\mathcal{H}}_{\text{ad}}(\vec{R}, t) = \hat{H}_{\text{ad}}(\vec{R}) + \hat{W}(t) - i\hbar \frac{d}{dt}$$

Hyperspherical Dressed Three-body potentials:



(with G. Pupillo, J. Wang, C. H. Greene)

Hyperspherical Three-body potentials: ($a \rightarrow \pm\infty$)

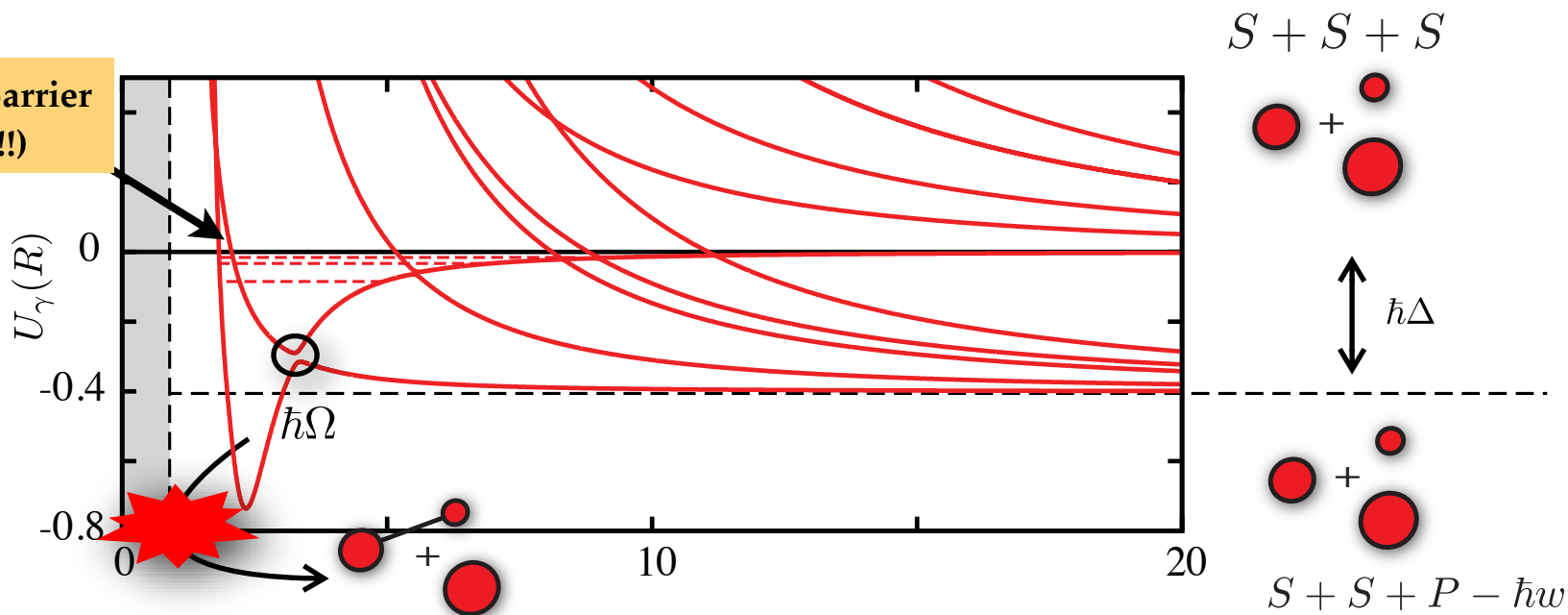


Ω and $\Delta =$ Controllable parameters

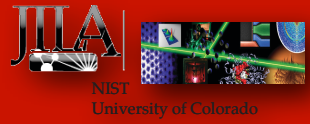
- needs to find an optimal detuning
- keep Rabi frequency as low as possible

Hyperspherical Dressed Three-body potentials:

Repulsive Barrier
(Shielding !!!)



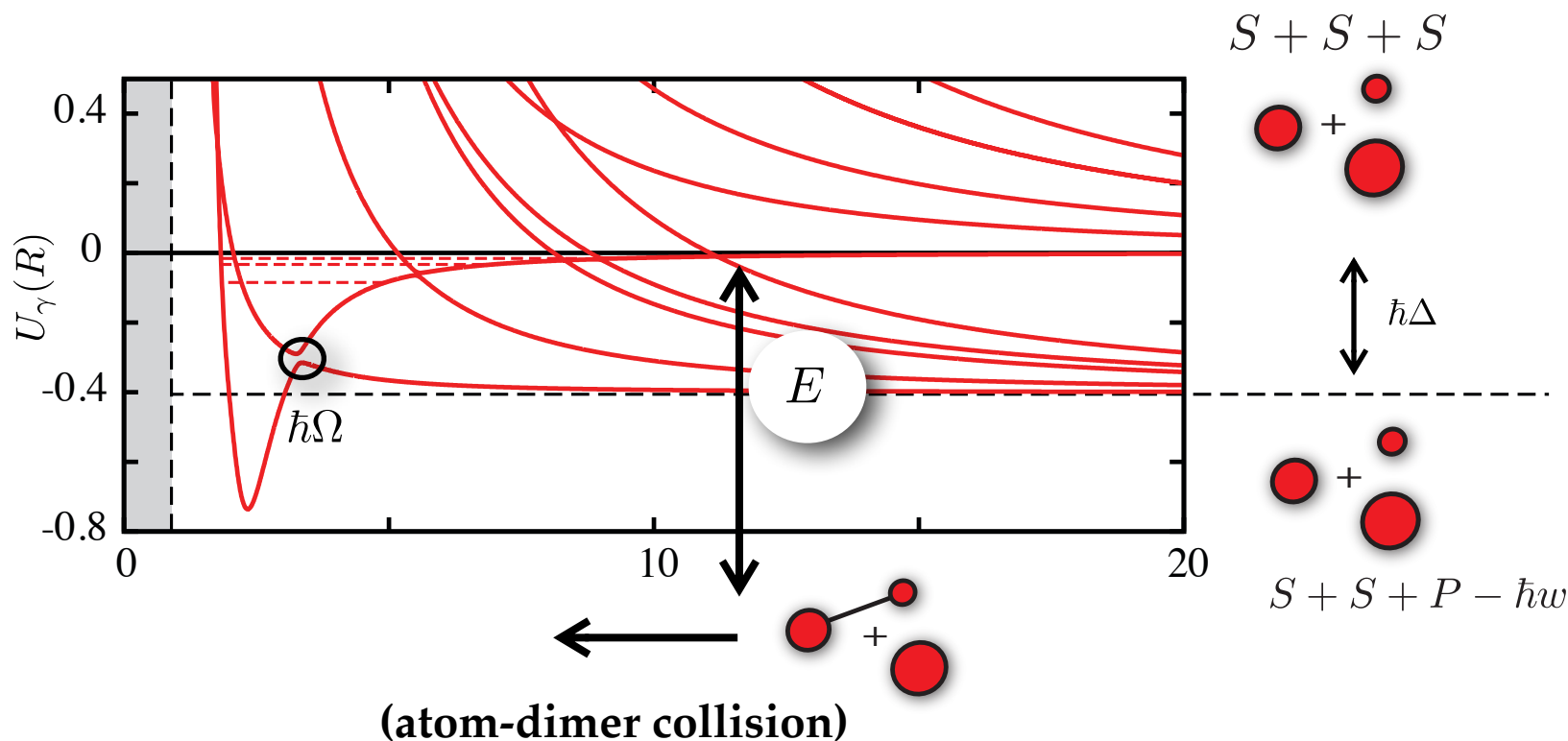
Preliminary Results (Blue-Shielding)



(with G. Pupillo, J. Wang, C. H. Greene)

(with G. Pupillo, J. Wang, C. H. Greene)

Determining Energy and width of Efimov States

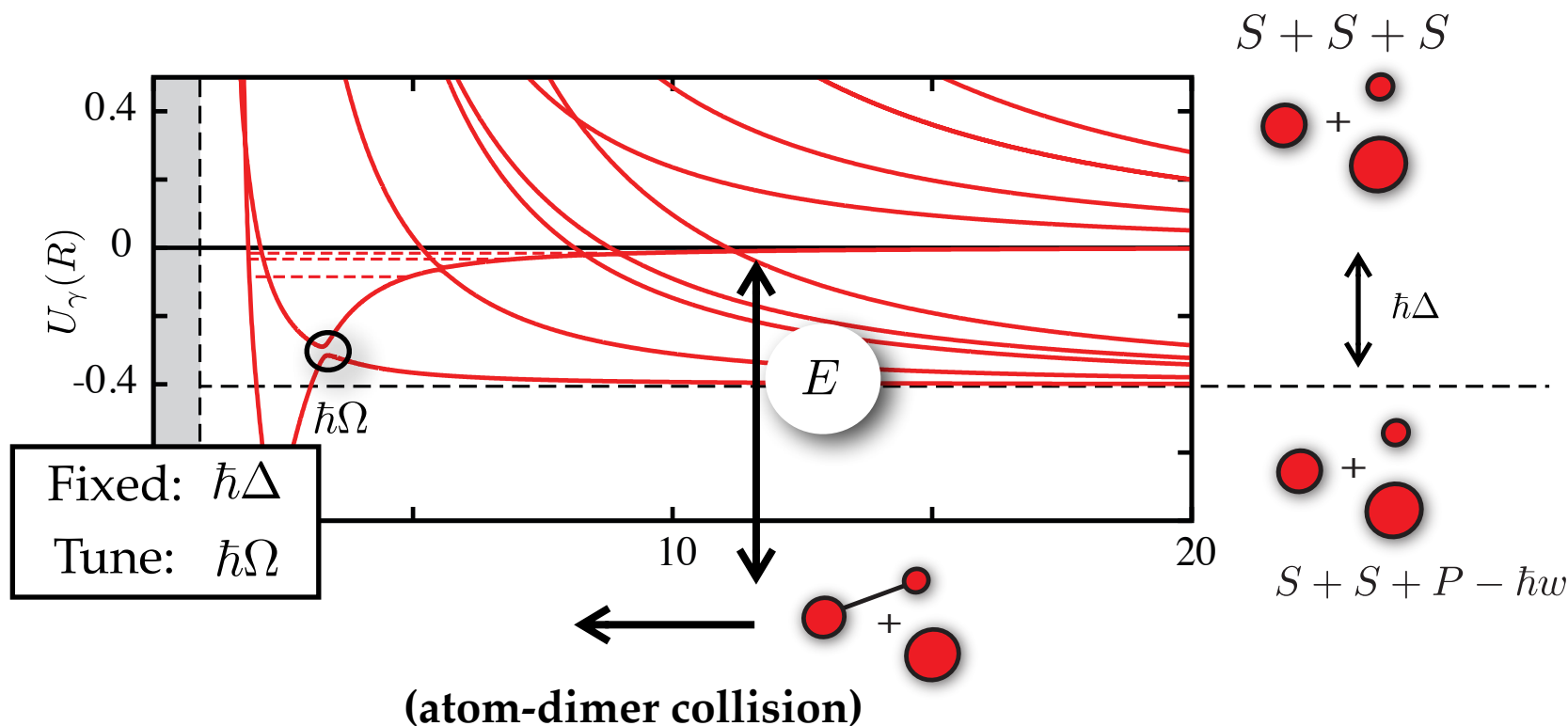


Calculate:

$$\frac{d\delta_{ei}}{dE} = \frac{\Gamma_R/2}{(E - E_R)^2 + (\Gamma_R/2)^2} \begin{cases} E_{\max} = E_R \\ \frac{d\delta_{ei}}{dE} \Big|_{E_{\max}} = \frac{2}{\Gamma_R} (\propto \tau) \end{cases}$$

(with G. Pupillo, J. Wang, C. H. Greene)

Determining Energy and width of Efimov States



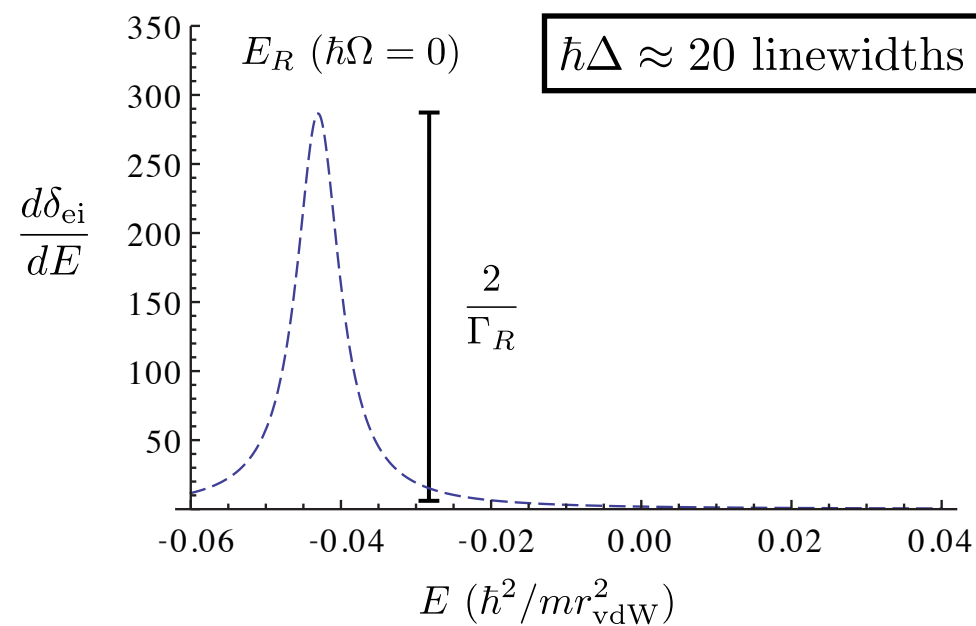
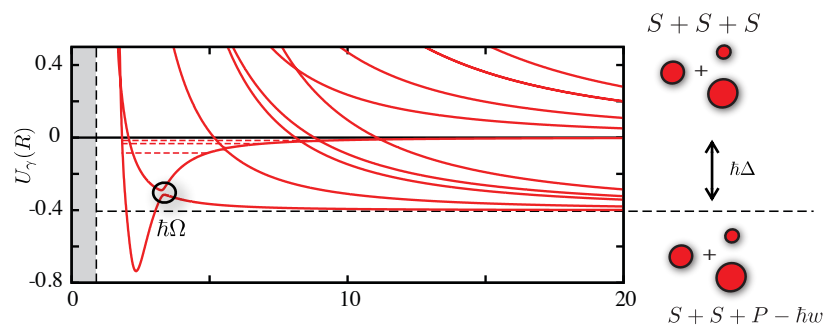
Calculate:

$$\frac{d\delta_{ei}}{dE} = \frac{\Gamma_R/2}{(E - E_R)^2 + (\Gamma_R/2)^2} \quad \left\{ \begin{array}{l} E_{\max} = E_R \\ \frac{d\delta_{ei}}{dE} \Big|_{E_{\max}} = \frac{2}{\Gamma_R} (\propto \tau) \end{array} \right.$$

Preliminary Results (Blue-Shielding)

(with G. Pupillo, J. Wang, C. H. Greene)

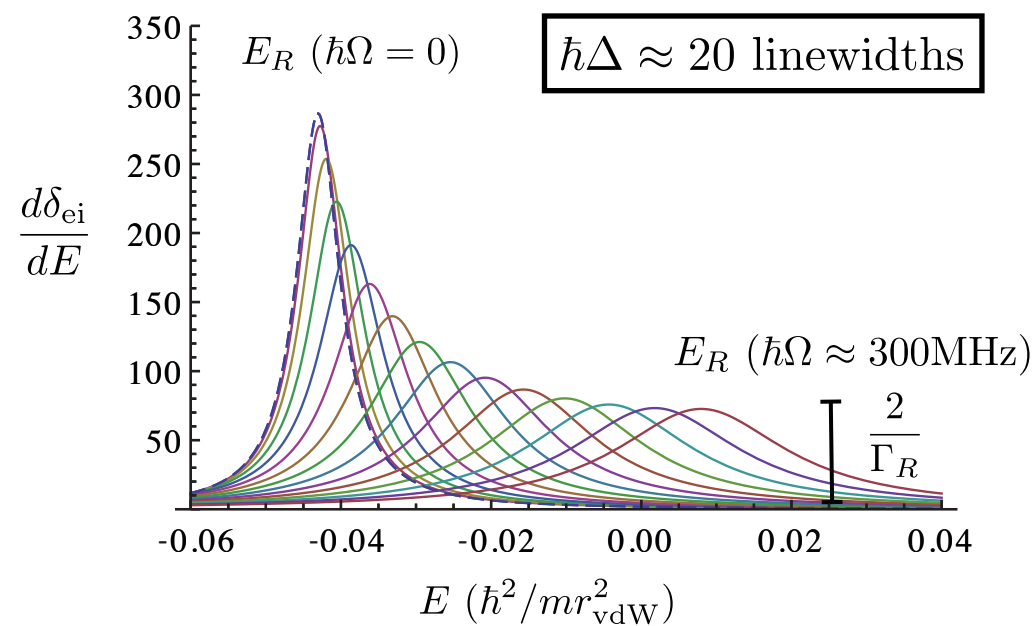
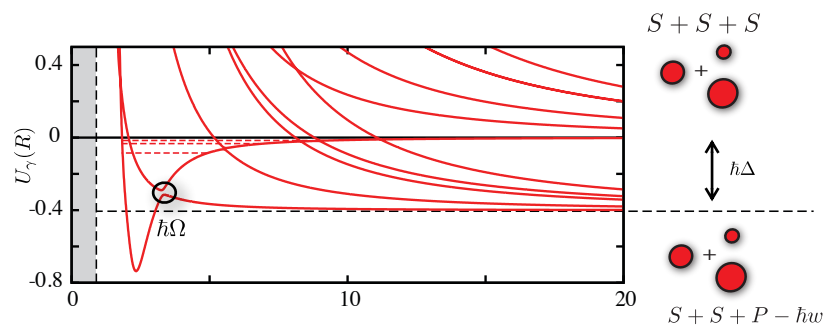
Determining Energy and width of Efimov States



Preliminary Results (Blue-Shielding)

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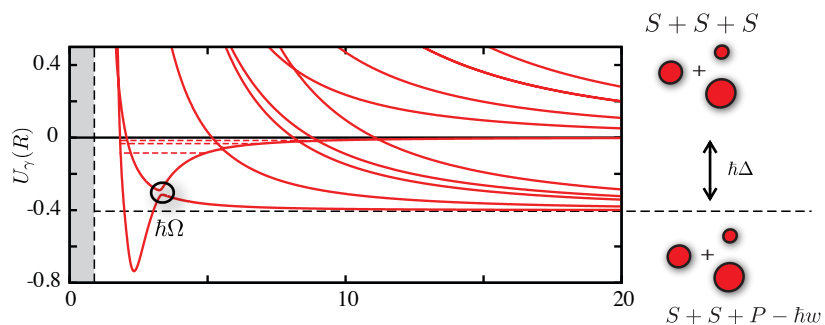
Determining Energy and width of Efimov States



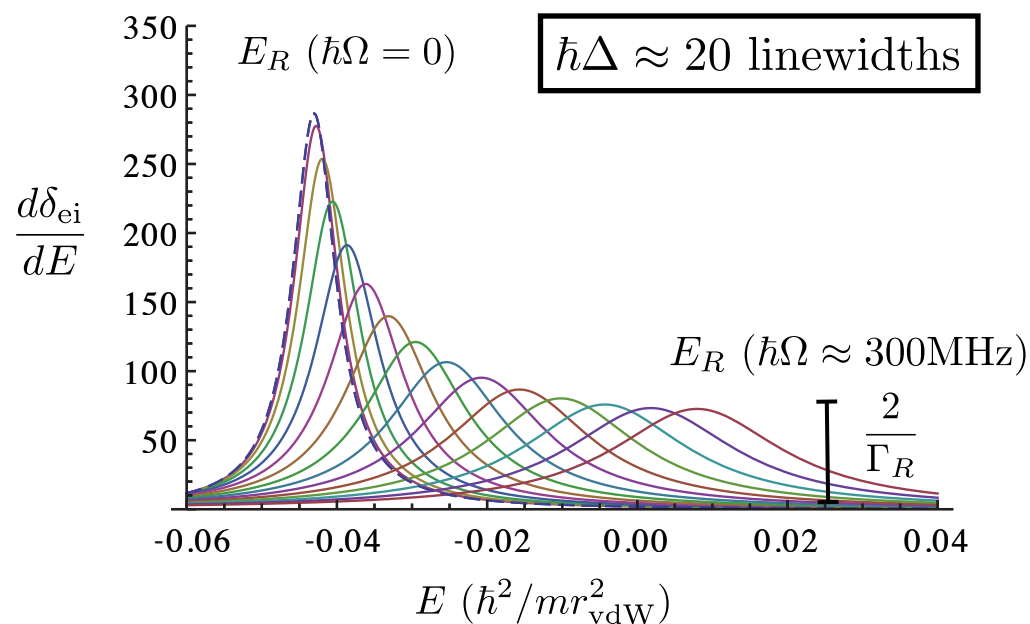
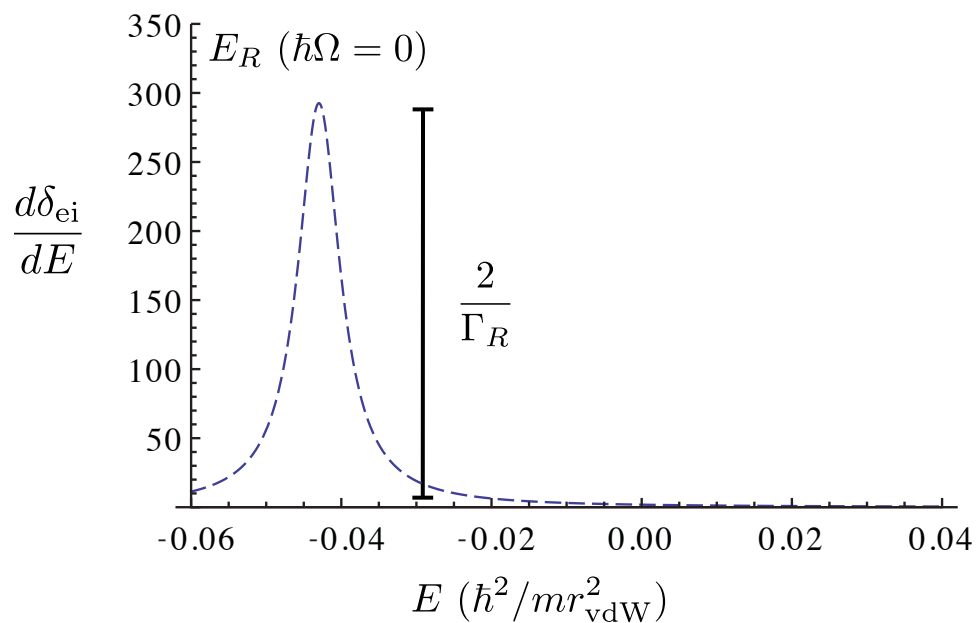
Preliminary Results (Blue-Shielding)

(with G. Pupillo, J. Wang, C. H. Greene)

Determining Energy and width of Efimov States



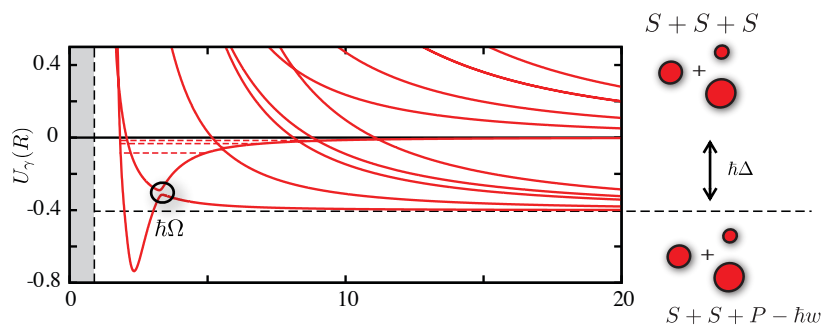
$\hbar\Delta \approx 80$ linewidths



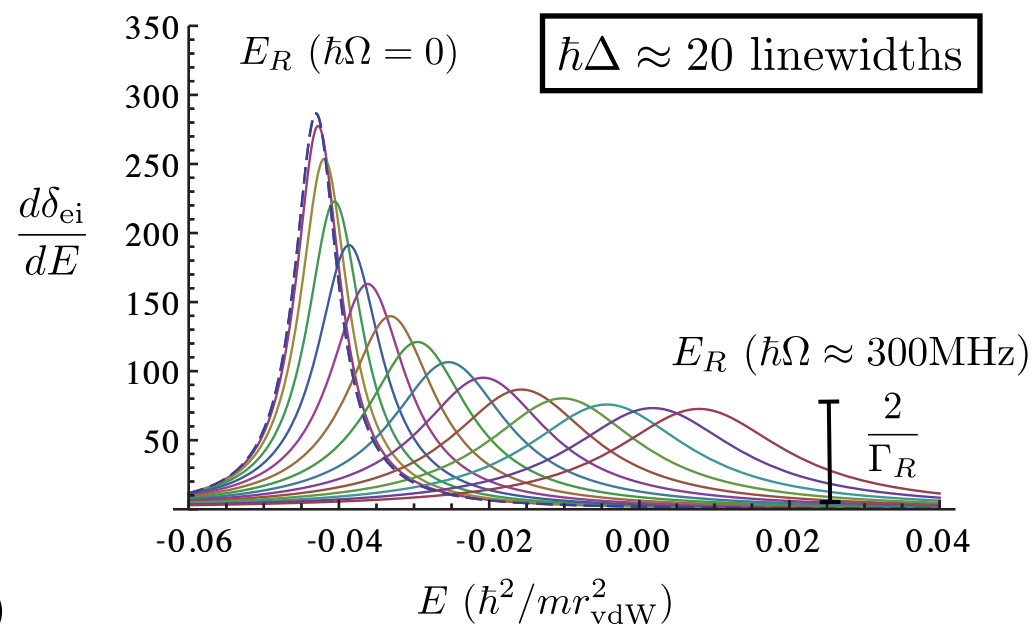
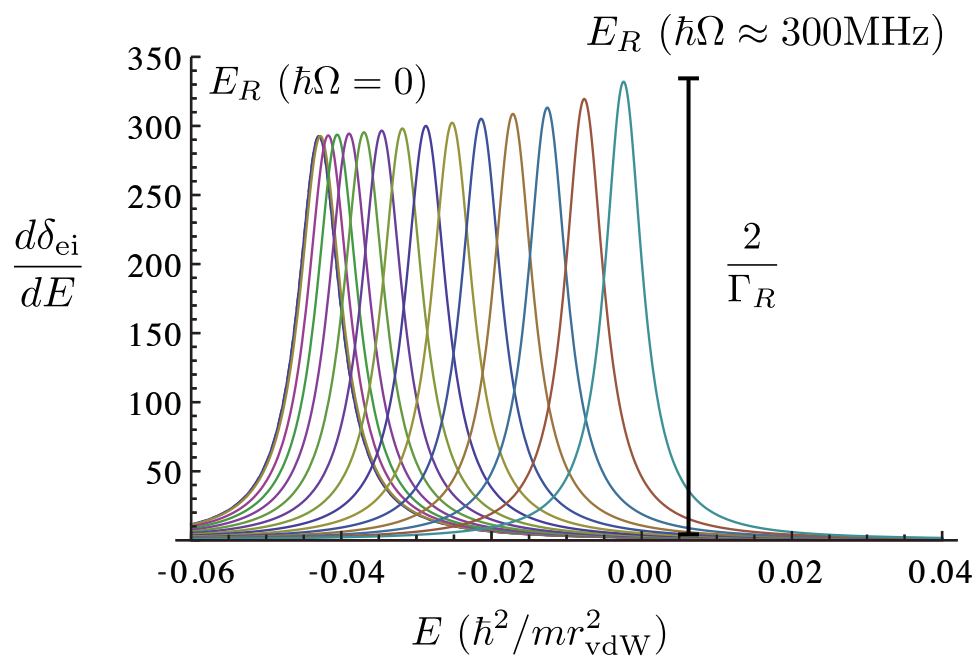
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Determining Energy and width of Efimov States



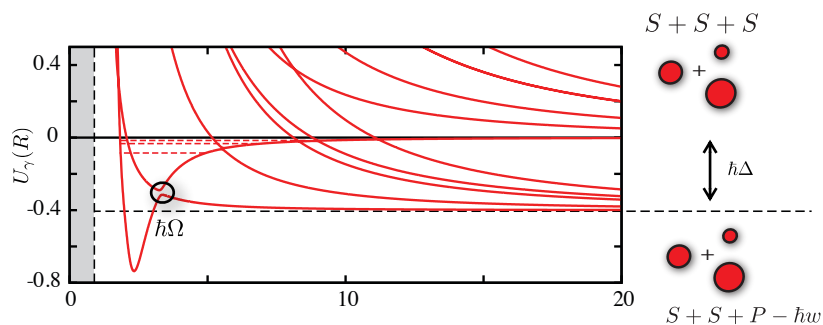
$\hbar\Delta \approx 80$ linewidths



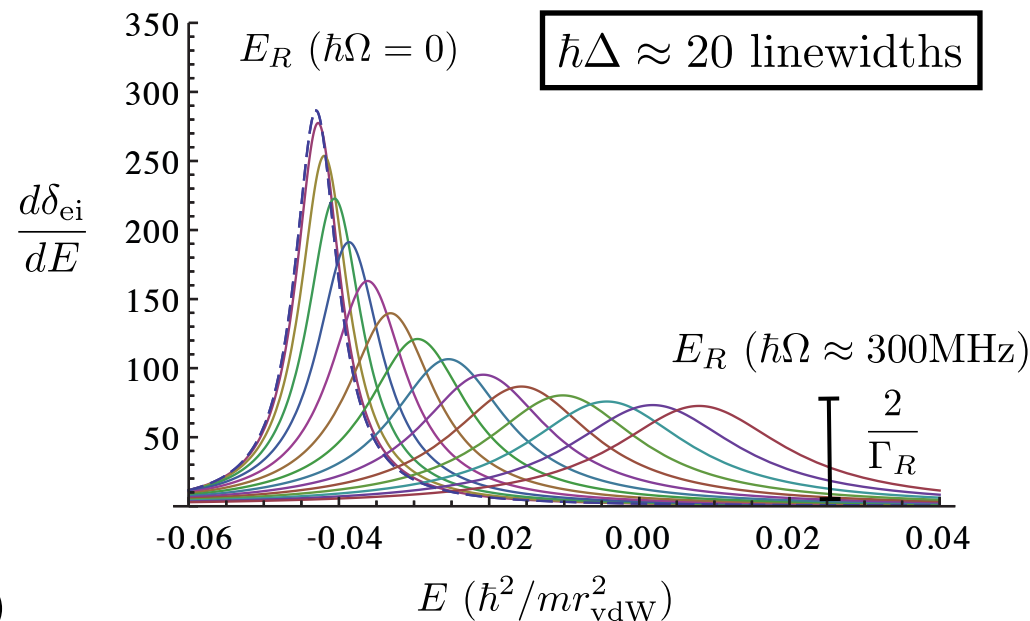
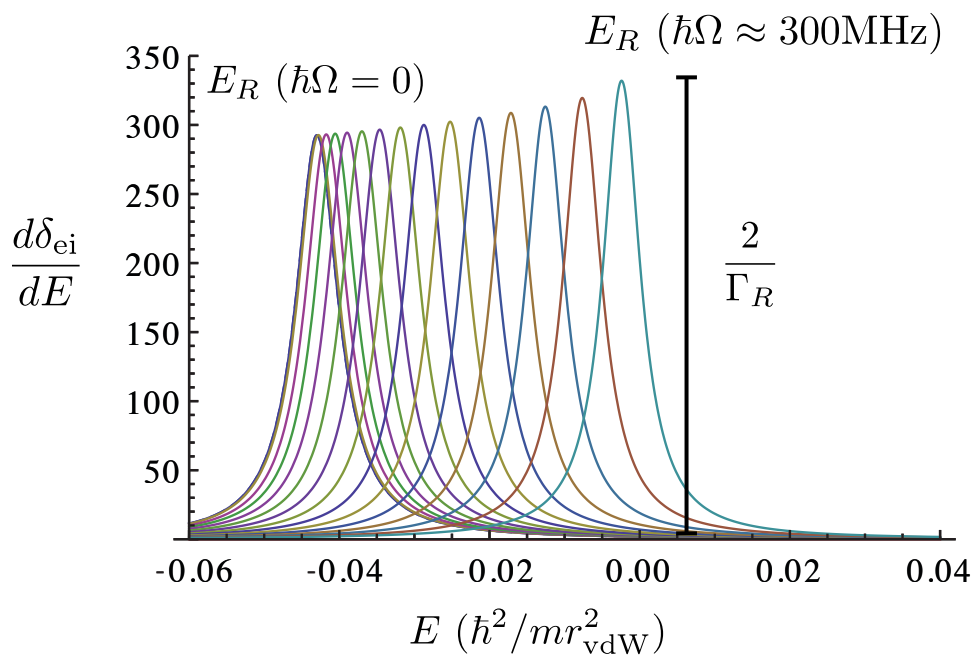
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Determining Energy and width of Efimov States

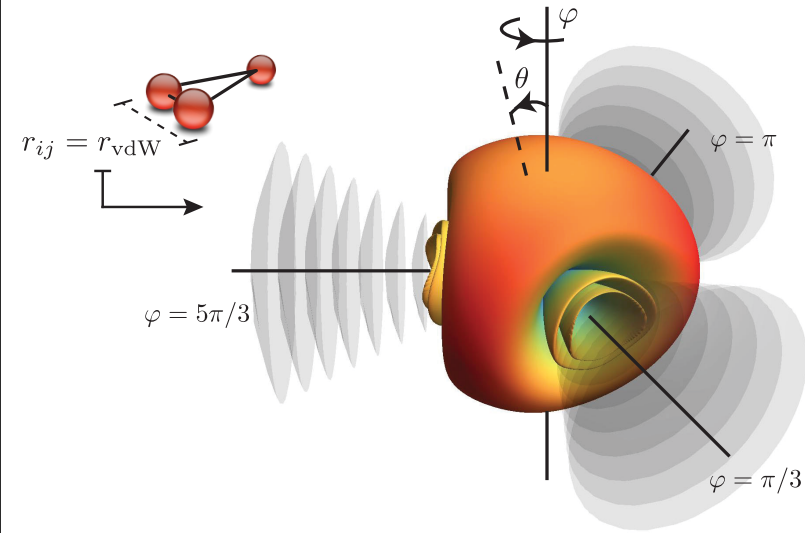


$\hbar\Delta \approx 80$ linewidths



- Tunability of the 3BP (good)
- 10-20% improvement on lifetime (more?)
- (Needs to explore more the parameter space)

Summary



- Both the theoretical and experimental advances in ultracold quantum gases have demonstrated these systems to be ideal candidates to explore universal few-body physics

- Some of the physical aspects of unitary Bose gases can be understood from a few-body perspective

- Quench to unitary: The separation of time scales for losses and many-body physics is favorable and allows interesting regimes to explore universal few-body physics

- The use of a blue-detuned laser seems promising in order to allow for the control of both energy and lifetime of Efimov states. In particular, we expect a better control for Efimov states when the scattering is finite.