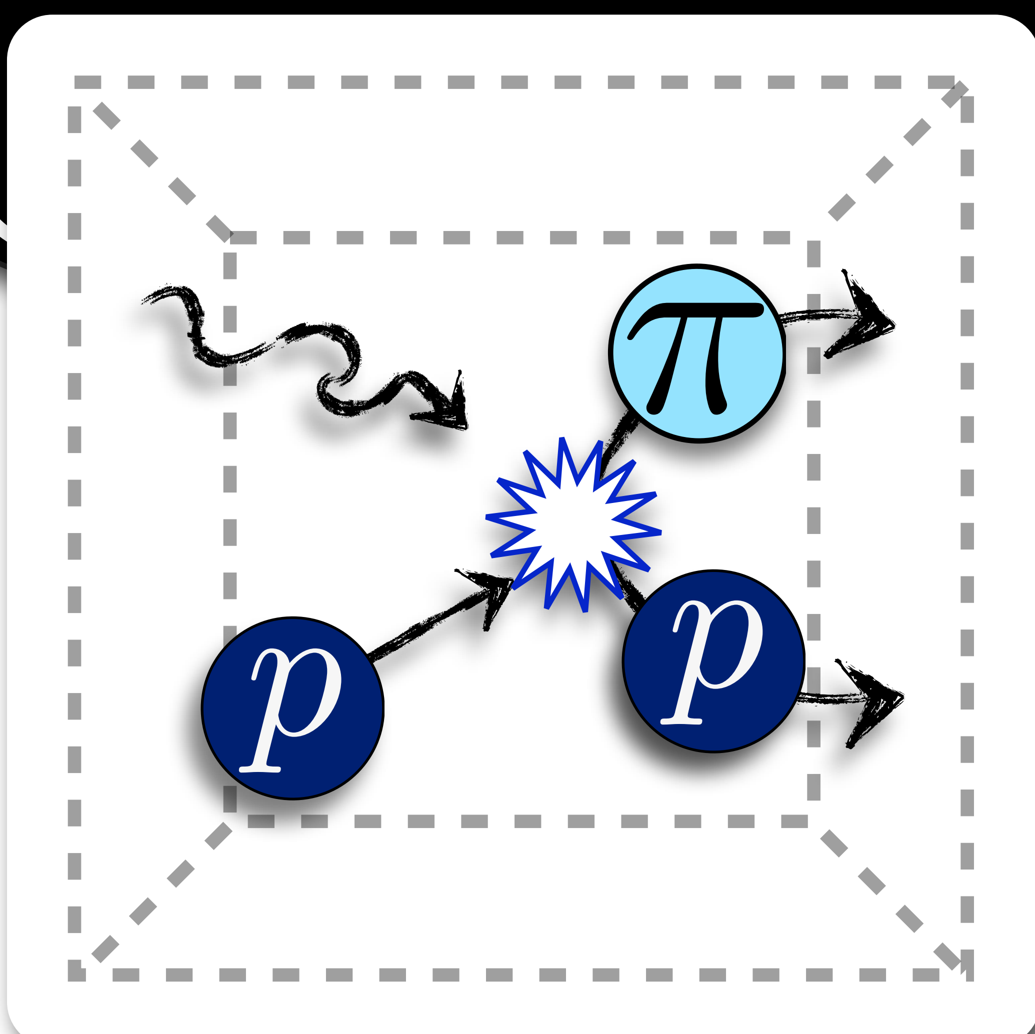
INT 14-1

Lattice QCD & strongly interacting few-body systems

A universal result and not-so-universal implications

Raúl Briceño



Jefferson Lab

Goals

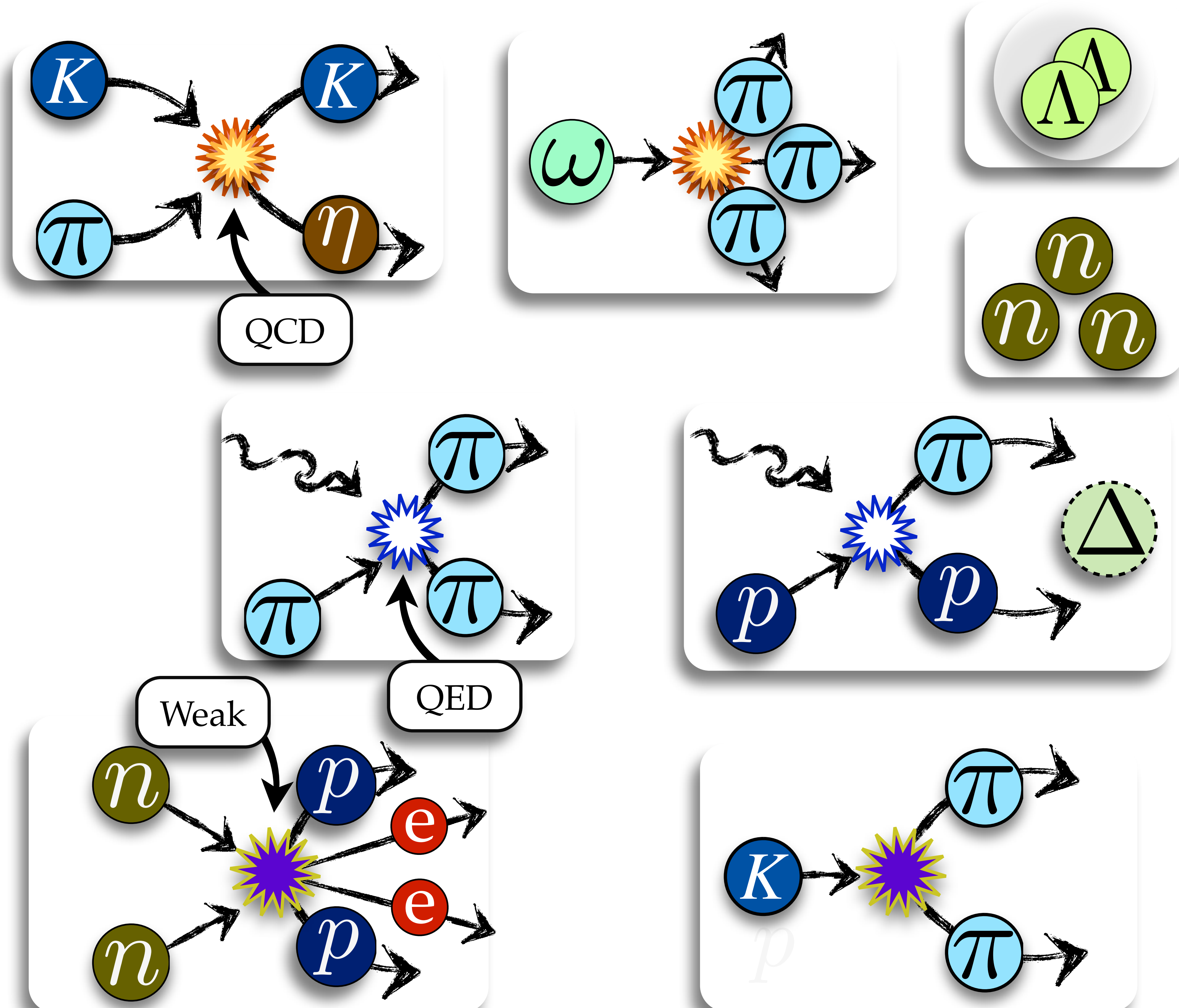
- Spectroscopy / scattering:
- Form factors:
- Fundamental symmetries:

Institute for Nuclear Theory
Workshop INT-13-53W

Nuclear Reactions from Lattice QCD
March 11-12, 2013
Organizers: RB, Zohreh Davoudi, Thomas Luu

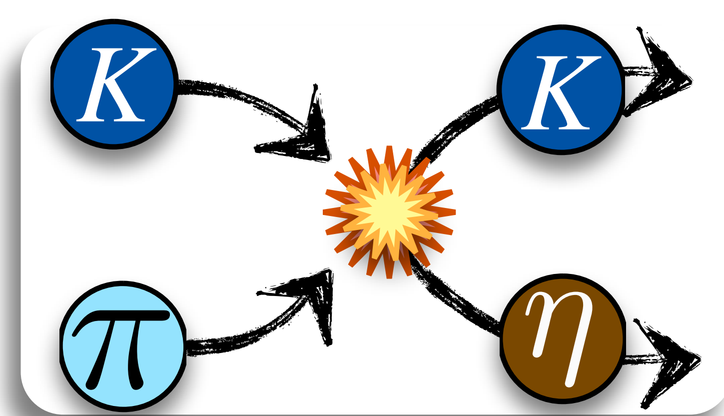
Goals

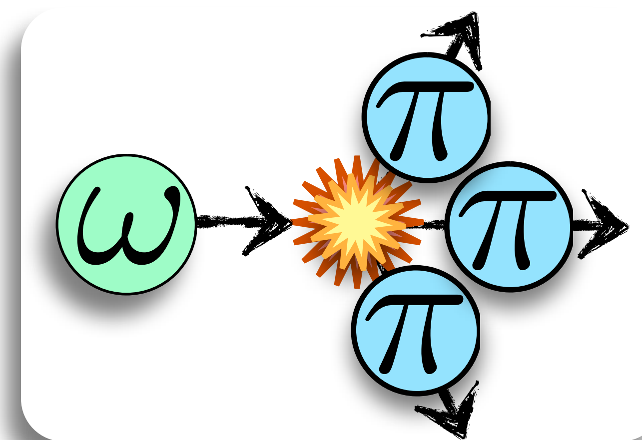
- Spectroscopy / scattering:
- Form factors:
- Fundamental symmetries:

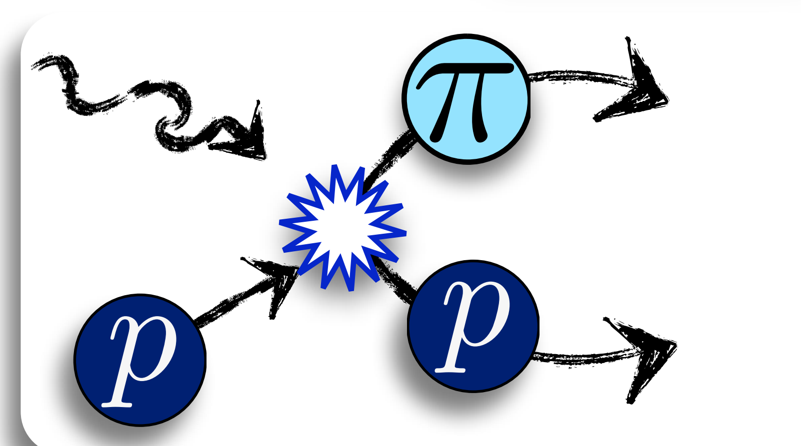


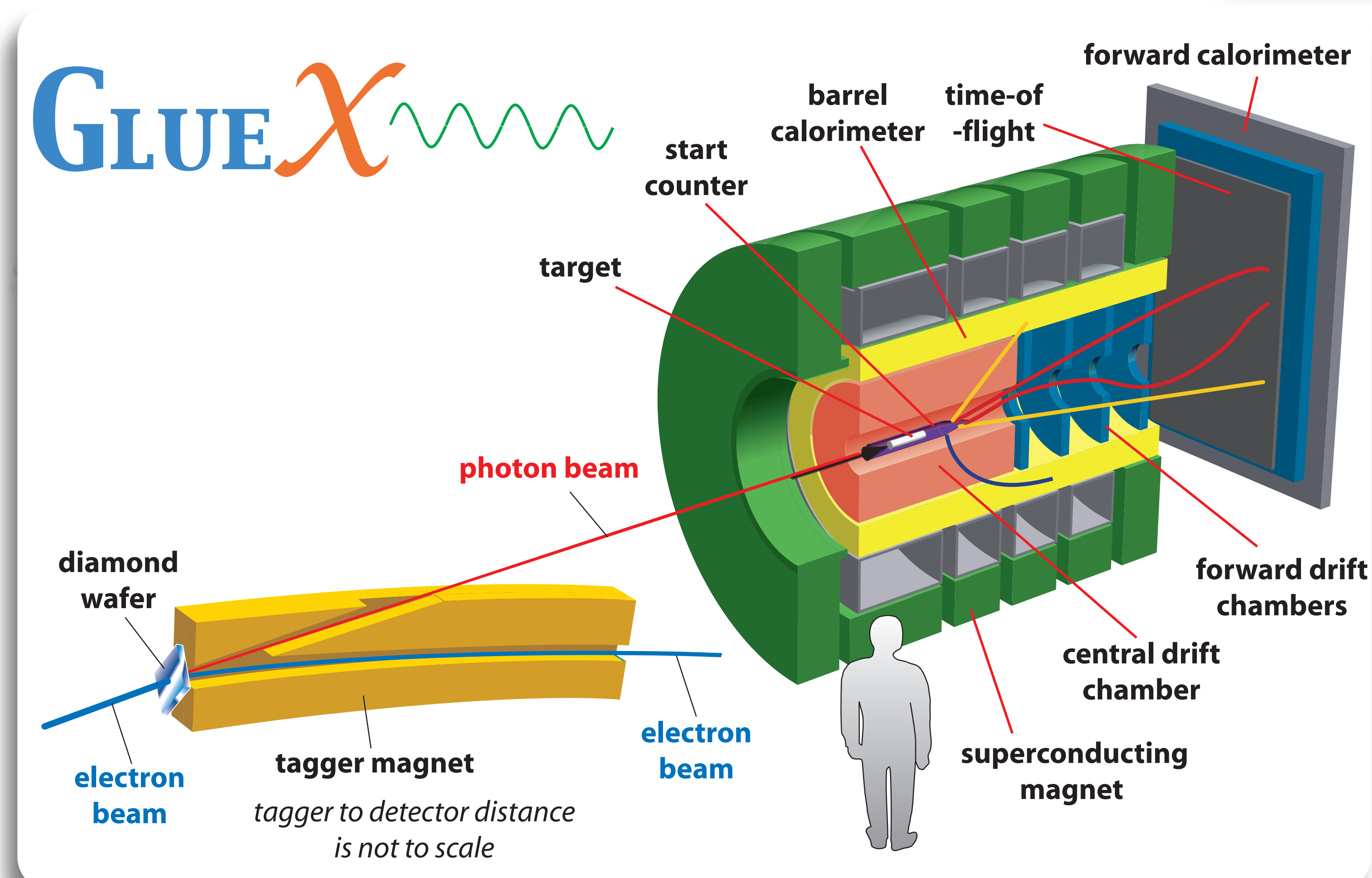
Goals

Spectroscopy:



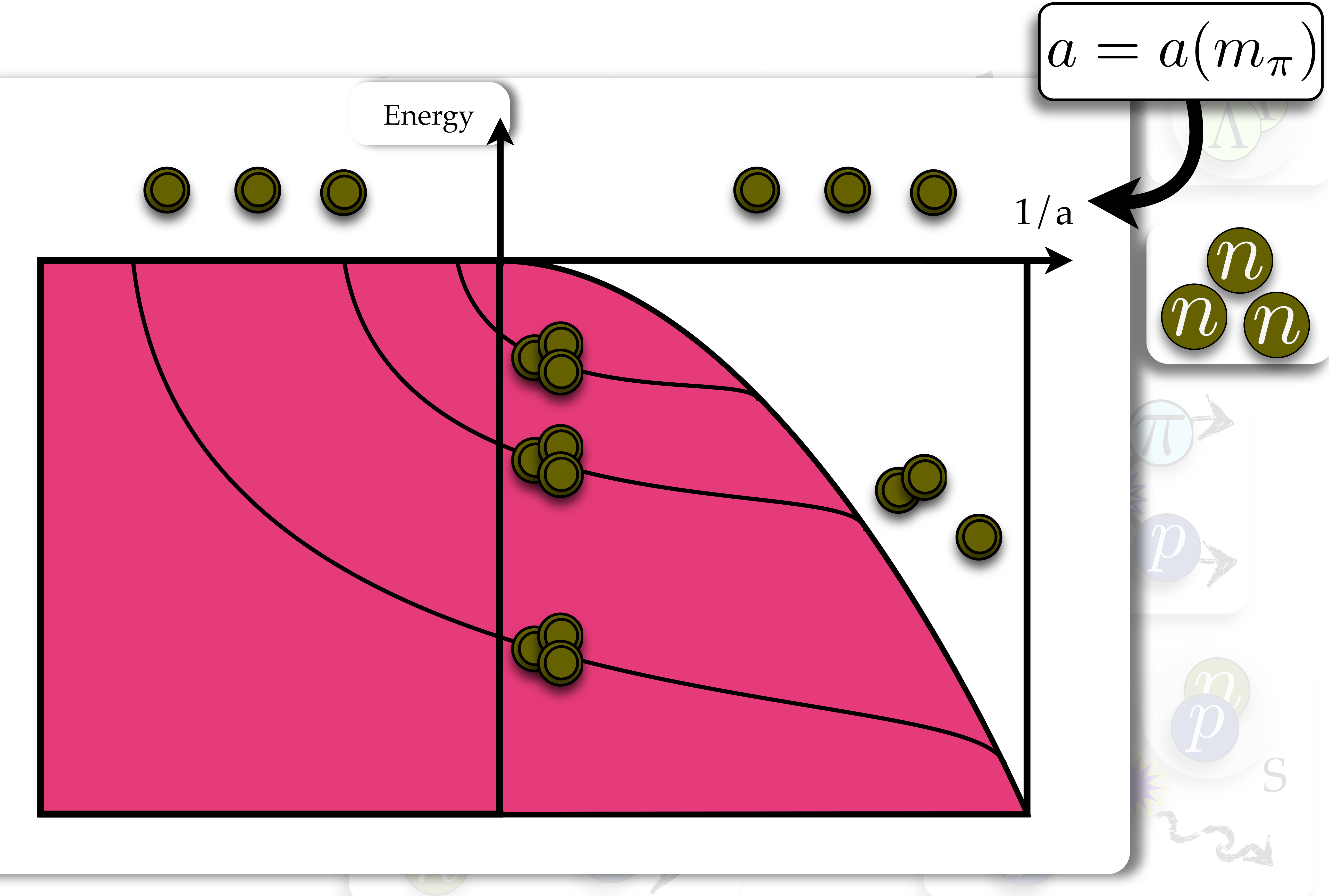






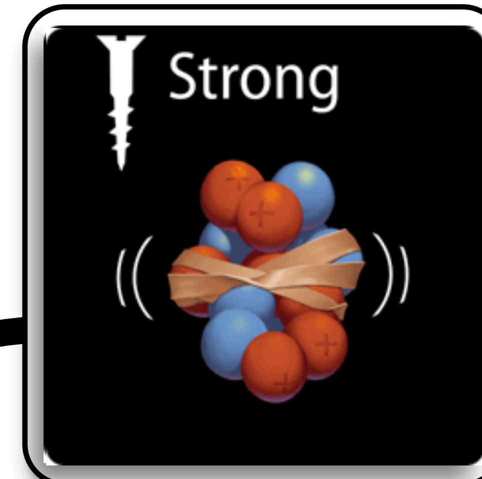
Jefferson Lab

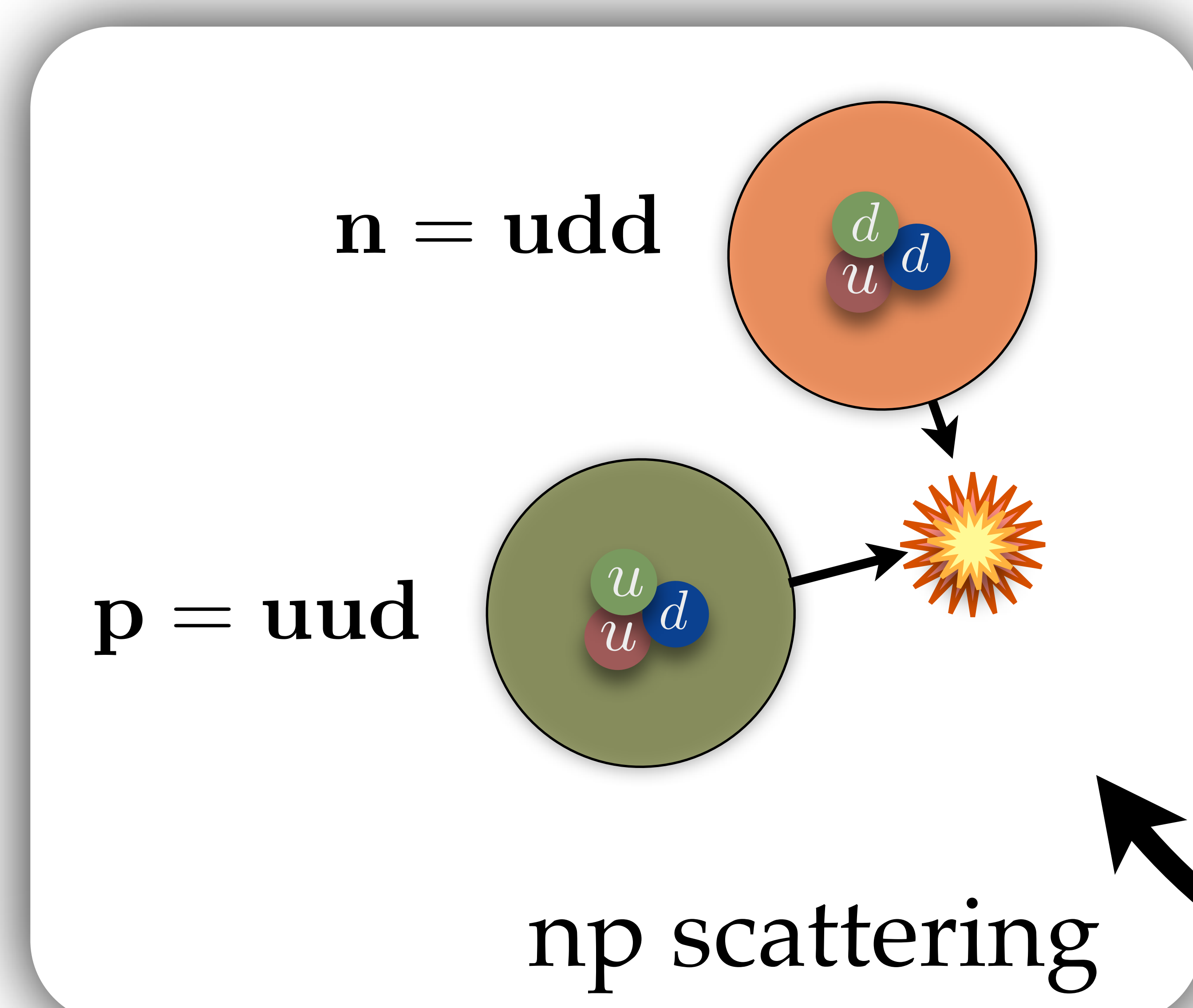
Goals



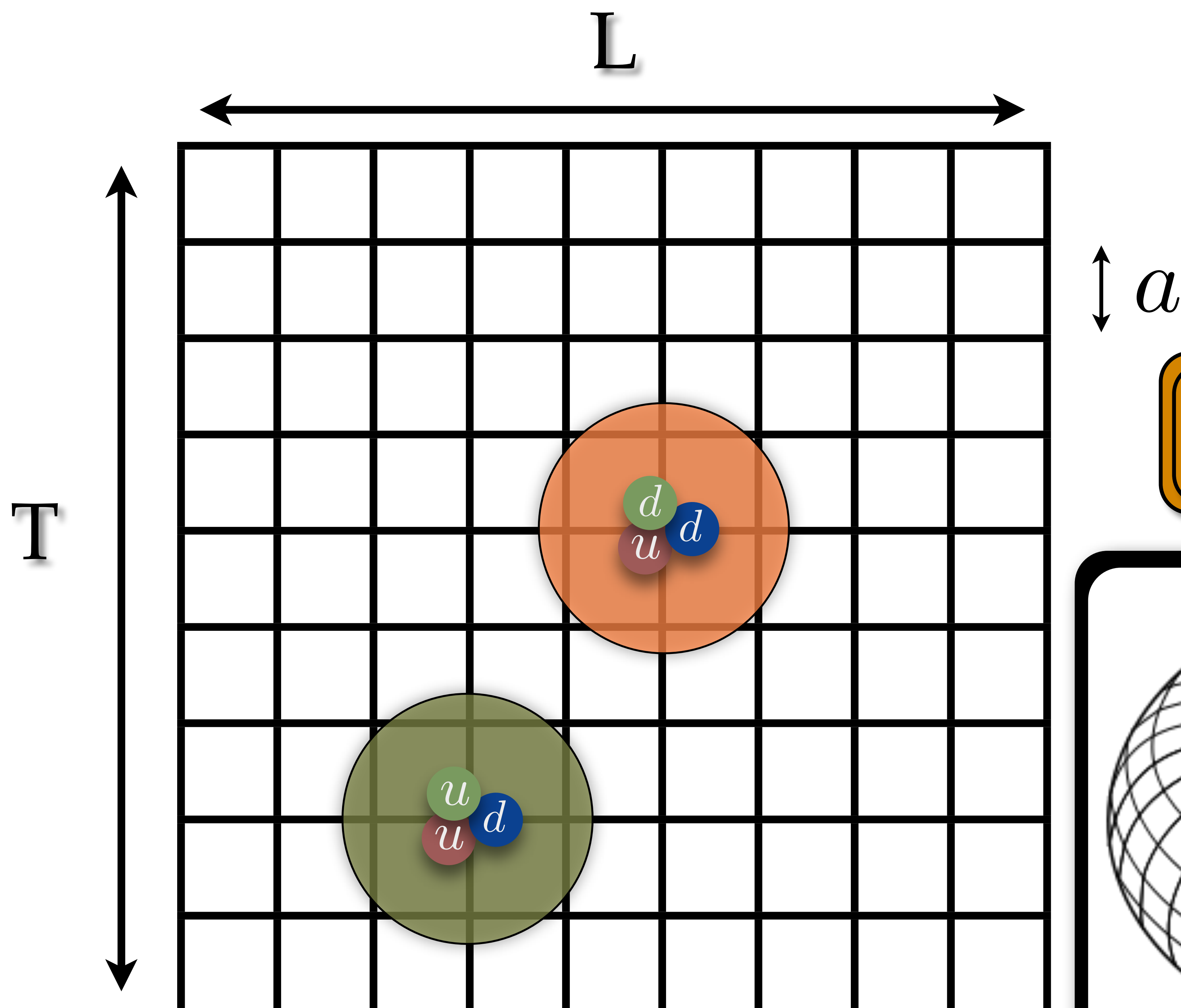
Lattice QCD

$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m}_{\mathrm{f}}\right) \psi_{\mathbf{f}} - \frac{1}{4} \mathrm{tr}\left(\mathrm{GG}\right)$$



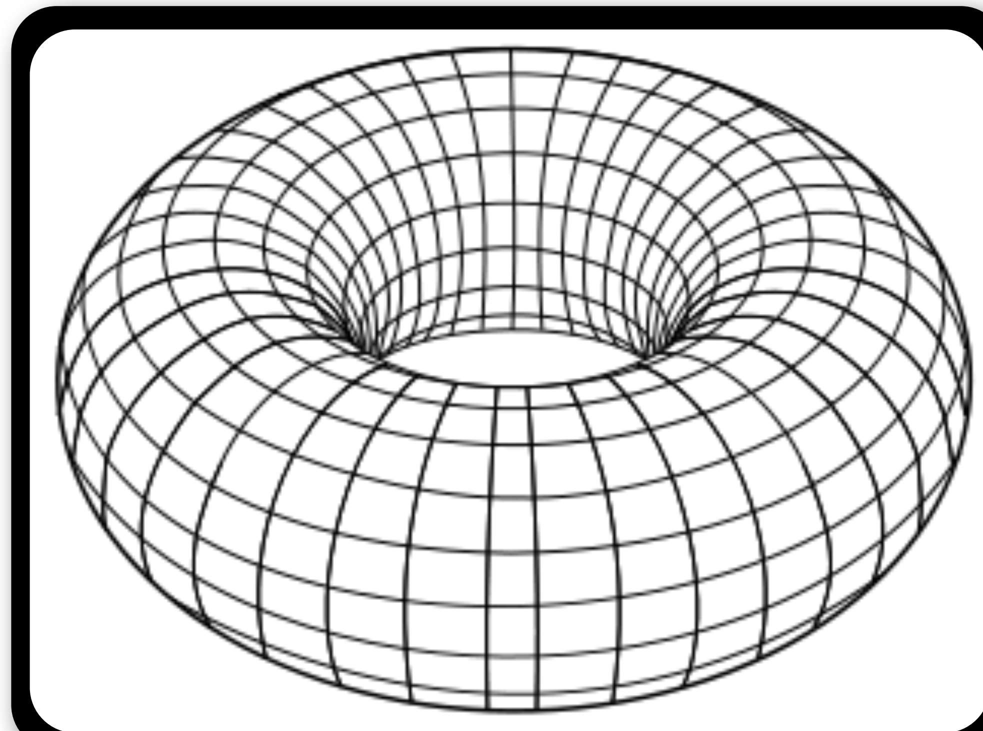


LQCD: Finite Euclidean Spacetime



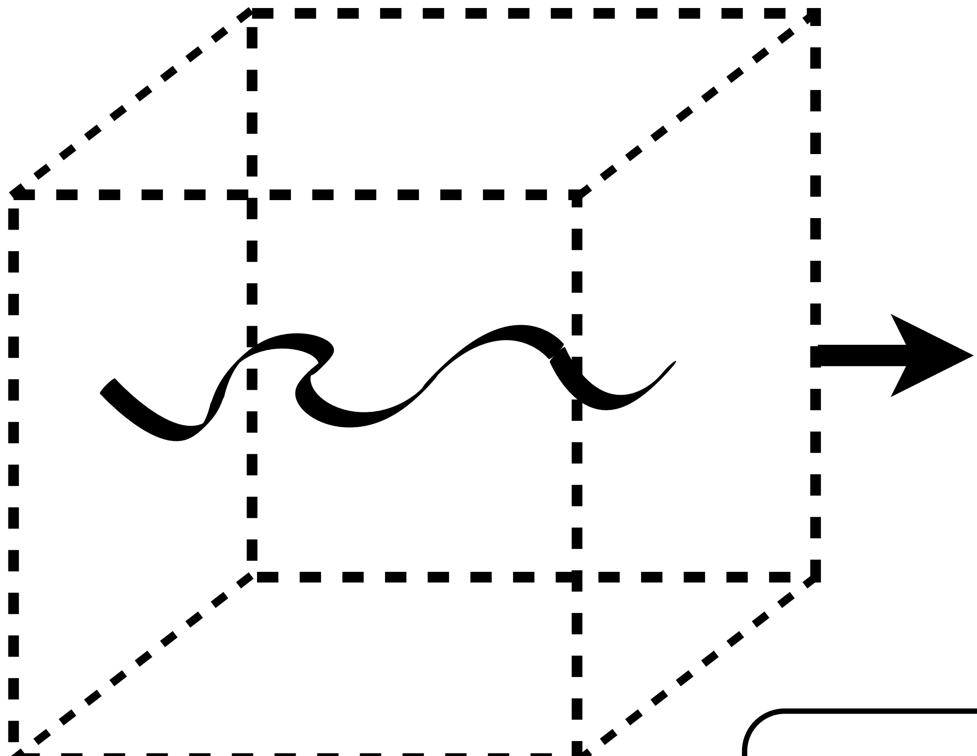
$t \to i\tau$

2D torus



neutron-proton in a 4D torus

LQCD: Finite Euclidean Spacetime



Euclidean Spacetime

- Hadronic spectra: $E(a, L, T, m_q)$
- Matrix elements: $\mathcal{A}(a, L, T, m_q)$

- **Formalism**
- Limits: $(a \to 0, L \to \infty, T \to \infty, m_q \to m_q^{phys})$

- Physics: hadron masses, decay constants, scattering parameters, form factors,...

Minkowski Spacetime

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0 | \mathcal{O}'_{\lambda'}(x_0, \mathbf{P}) \mathcal{O}^\dagger_\lambda(y_0, \mathbf{P}) | 0 \rangle$$

- Operators could be different
- Must have same quantum numbers

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0 - y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}_\lambda(0, \mathbf{P})|0\rangle$$

The spectrum!

Evaluate using Monte Carlo techniques:

$$C(x_0 - y_0, \mathbf{P}) = \frac{1}{Z_{Eucl.}} \int \mathcal{D}[U, q, \bar{q}]\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})\ e^{-S_{\mathrm{Eucl.}}}$$

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^{\dagger}_{\lambda}(y_0, \mathbf{P})|0\rangle$$
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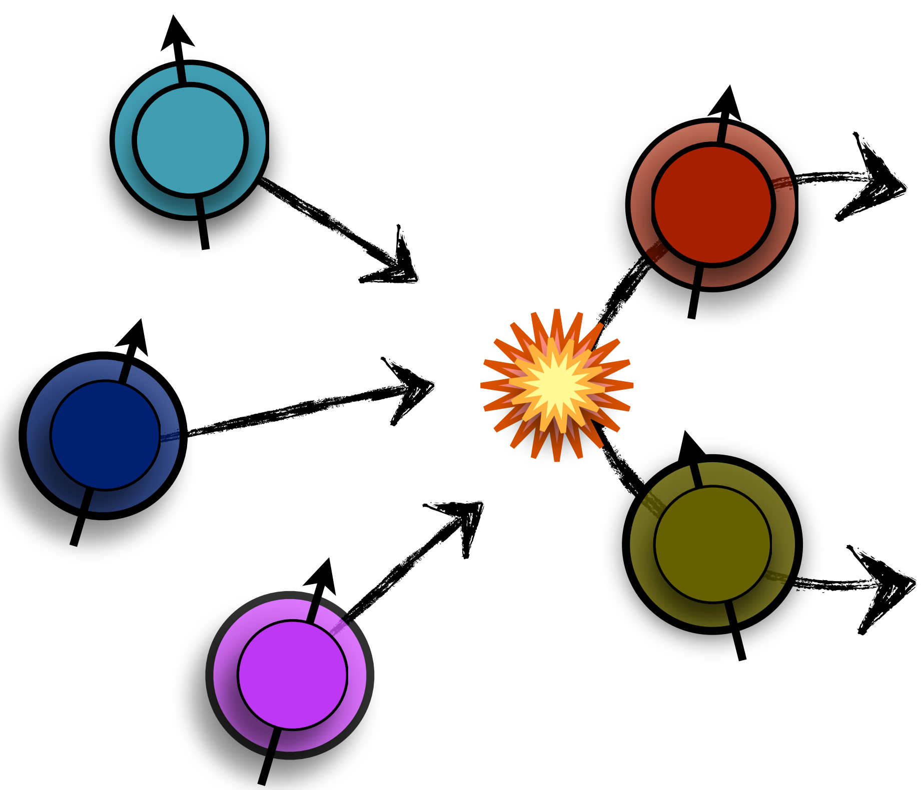
e.g. N-N′ particle in a infinite volume

$|Jm_J, P, LS, a\rangle$



$|Jm_J, P, L'S', b\rangle$

J=angular momentum
P=Parity
L=orbital angular momentum
S=spin
a=flavor content,...

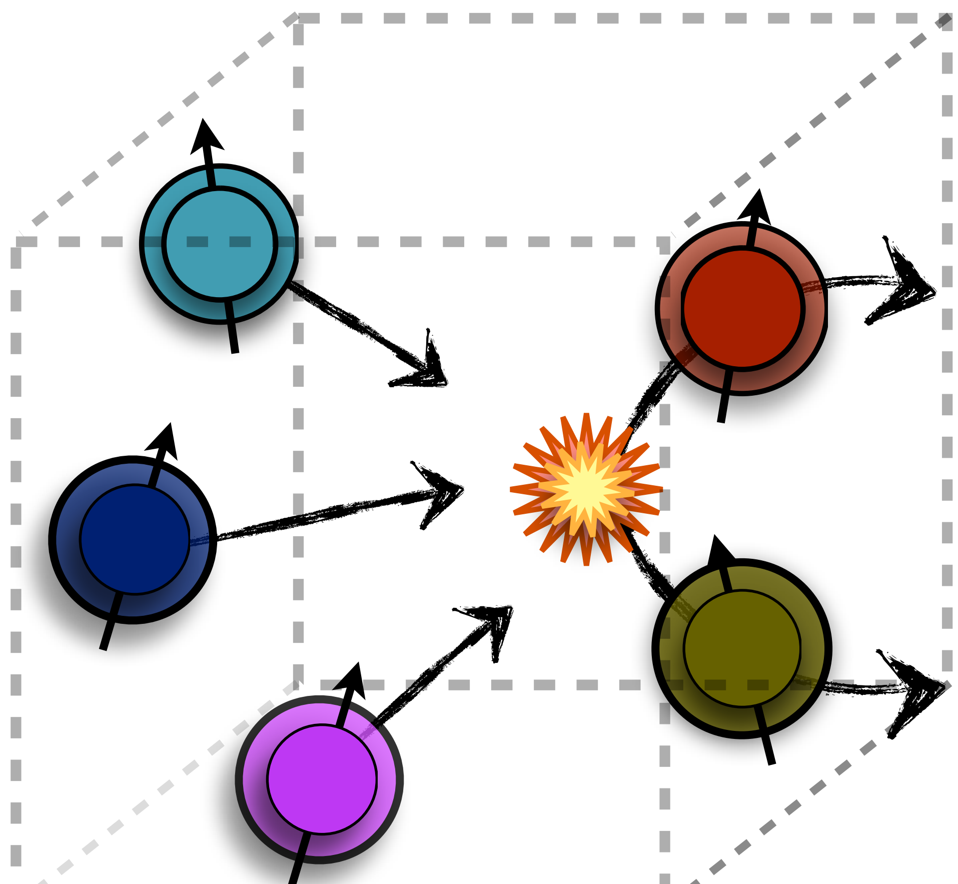
Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})|0\rangle$$

$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0-y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}_\lambda(0, \mathbf{P})|0\rangle$$

e.g. N-N′ particle in a finite volume

$|Jm_J, P, LS, a\rangle$



$|J'm_{J'}, P', L'S', b\rangle$

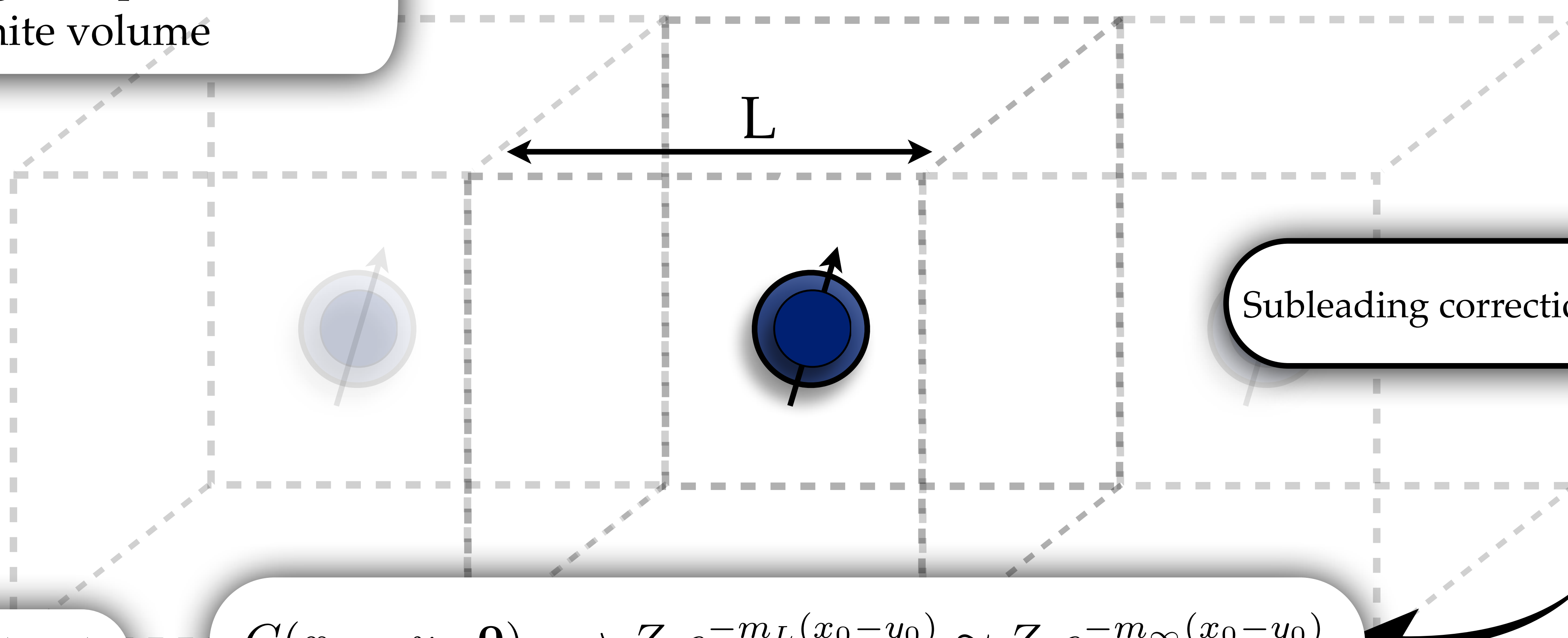
J=angular momentum
P=Parity
L=orbital angular momentum
S=spin
a=flavor content,...

Everything can vary!

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0 - y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}_\lambda(0, \mathbf{P})|0\rangle$$

e.g. one-particle in a finite volume



Subleading corrections

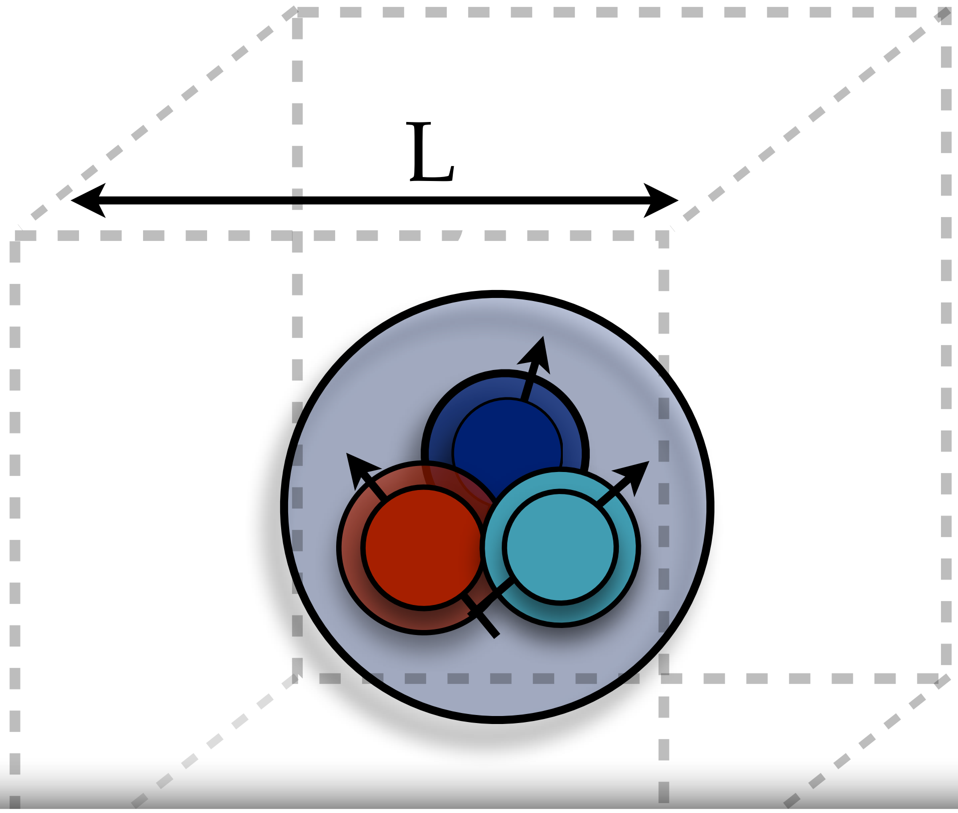
Lüscher (1985)

$$C(x_0 - y_0, \mathbf{0}) \longrightarrow Z_0 e^{-m_L(x_0 - y_0)} \approx Z_0 e^{-m_\infty(x_0 - y_0)}$$

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0-y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}_\lambda(0, \mathbf{P})|0\rangle$$

e.g. three-particle bound state in a finite volume



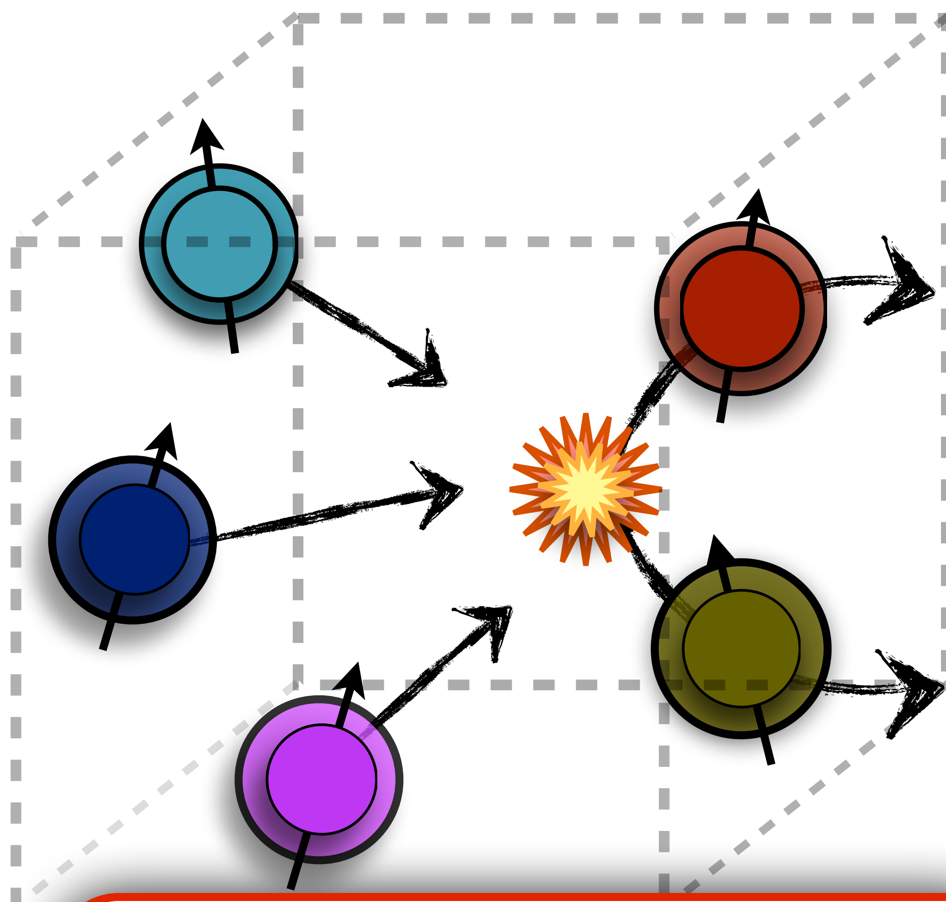
- Typically larger corrections
- Dictated by size of bound state

$$C(x_0 - y_0, \mathbf{0}) \longrightarrow Z_0 e^{-m_{B,L}(x_0-y_0)} \approx Z_0 e^{-m_{B,\infty}(x_0-y_0)}$$

Correlation functions

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, \mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0-y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}_\lambda(0, \mathbf{P})|0\rangle$$

$|\Lambda\mu, a\rangle$

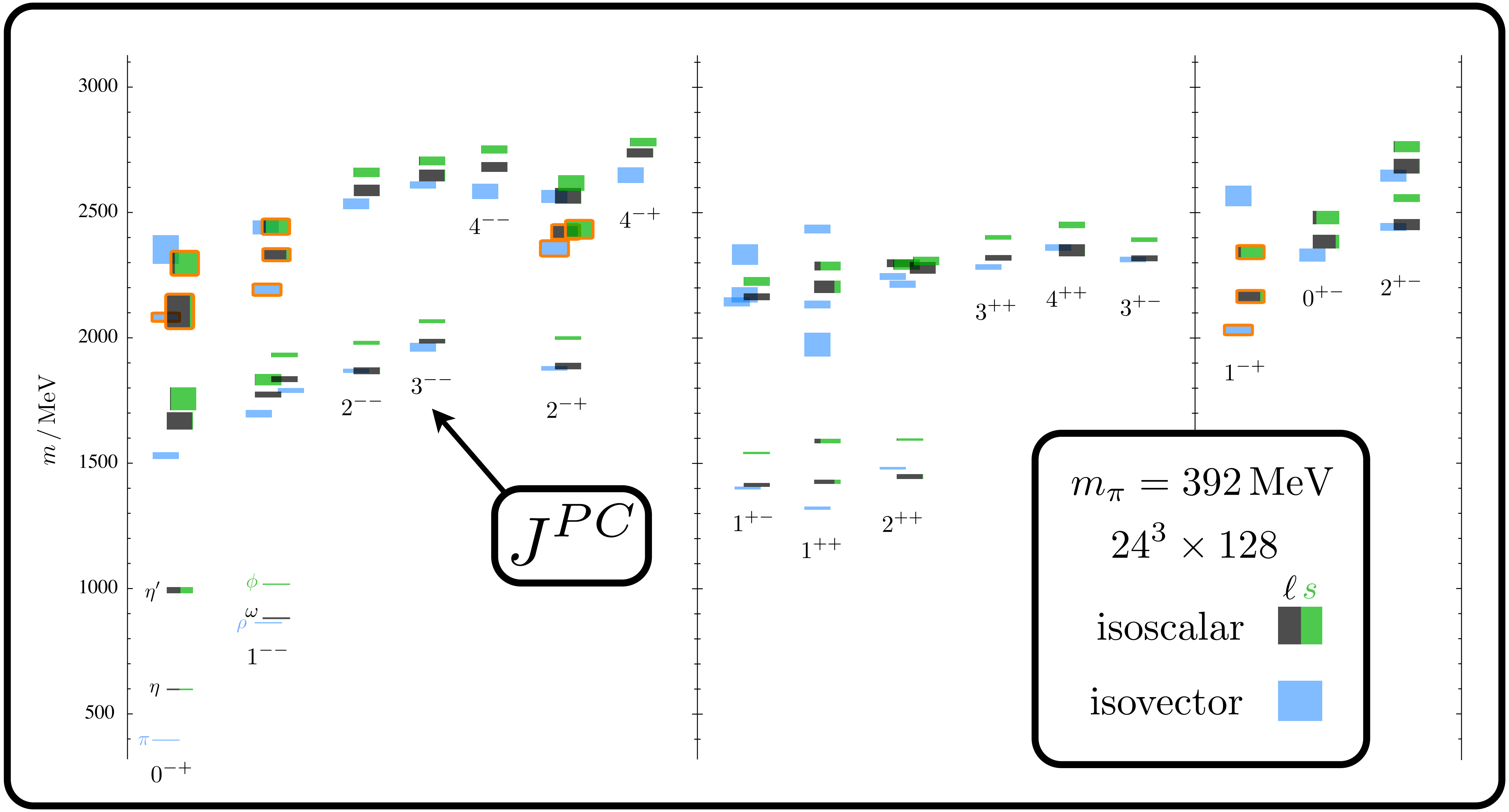


$|\Lambda\mu, b\rangle$

In general: not so straightforward

The state of the art

in the meson sector



Hadron Spectrum Collaboration: [PRD] arXiv:1309.2608 [hep-lat]
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

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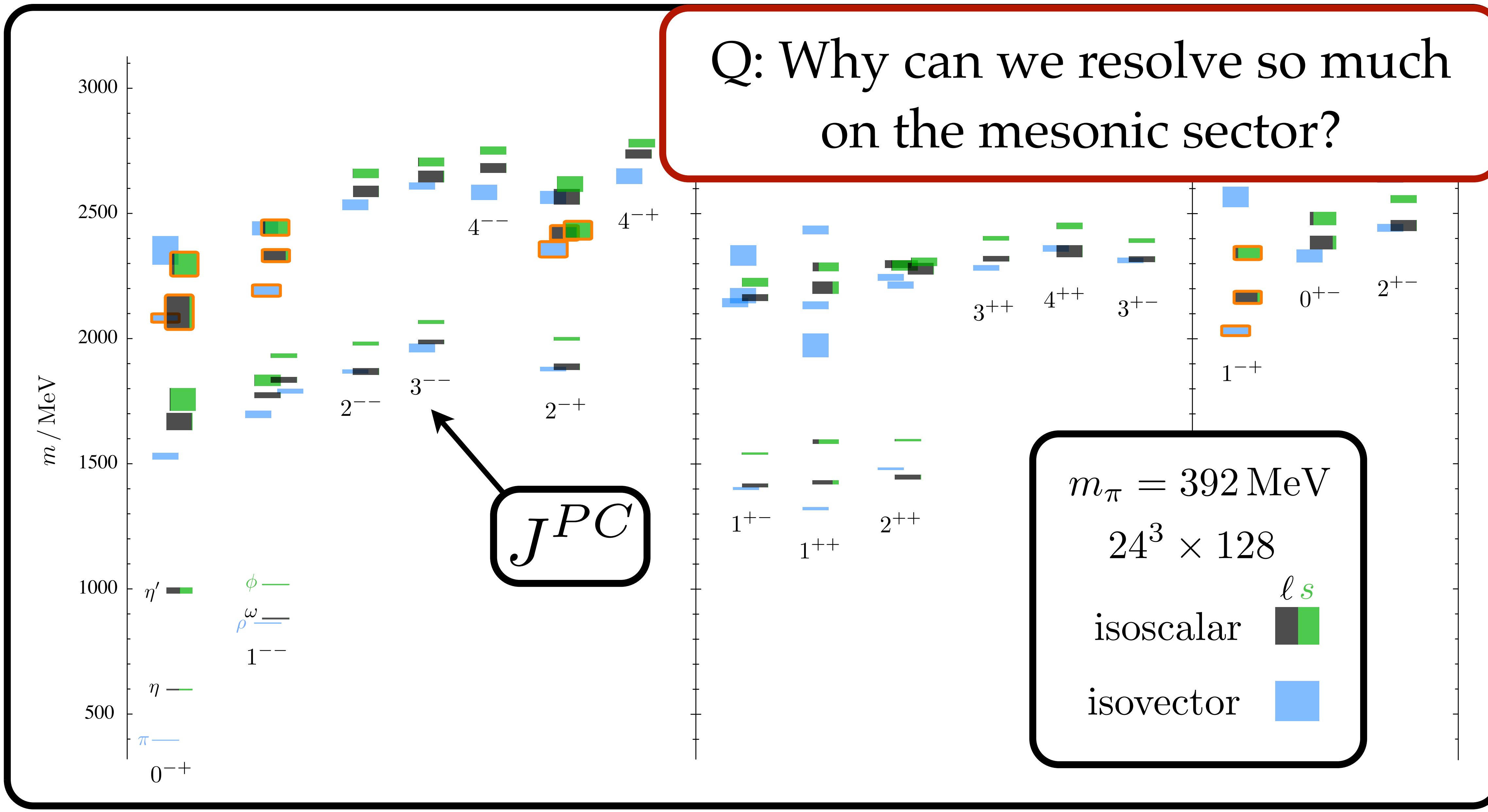
in the meson sector



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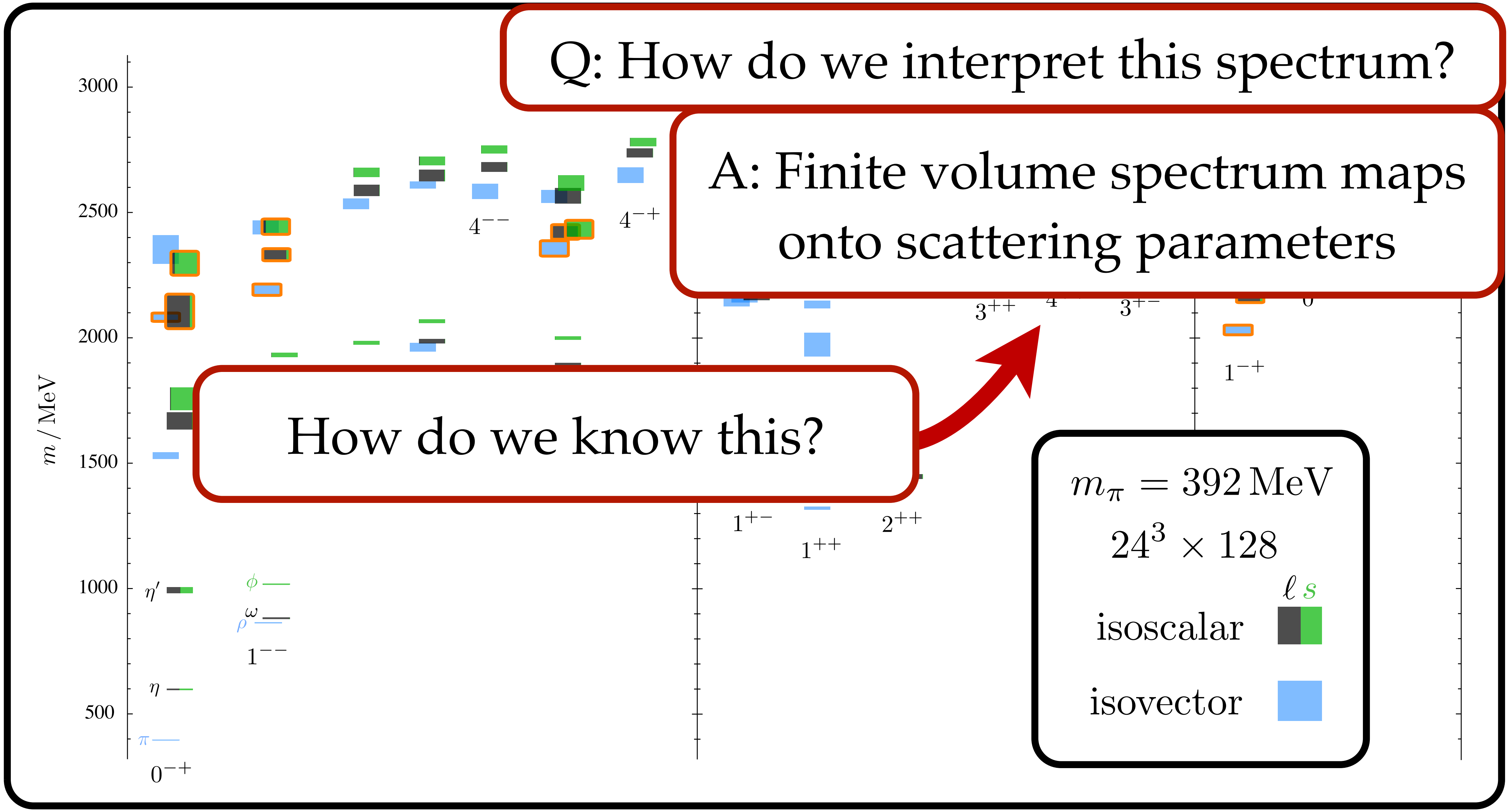


Hadron Spectrum Collaboration: [PRD] arXiv:1309.2608 [hep-lat]
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

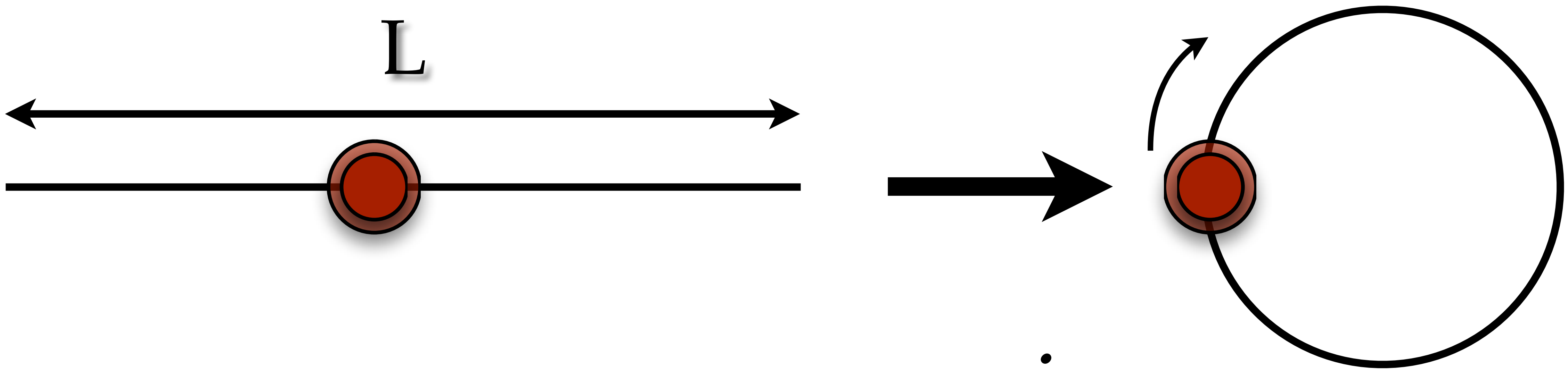
A long & *incomplete* list

- **M. Lüscher (1086), (1991)** *(“Lüscher Formalism”)*
- L. Maiani and M. Testa (1990)
- K. Rummukainen and S. A. Gottlieb (1995)
- S. Beane, P. Bedaque, A. Parreno, and M. Savage (2004), (2005)
- P. Bedaque (2004)
- X. Li and C. Liu (2004)
- W. Detmold and M. J. Savage (2004)
- X. Feng, X. Li, and C. Liu (2004)
- N. H. Christ, C. Kim, and T. Yamazaki (2005)
- C. Kim, C. Sachrajda, and S. R. Sharpe (2005)
- V. Bernard, M. Lage, U.-G. Meissner, and A. Rusetsky (2008)
- N. Ishizuka (2009)
- S. Bour, S. Koenig, D. Lee, H.-W. Hammer, and U.-G. Meissne (2011)
- Z. Davoudi and M. J. Savage (2011) (2014)
- L. Leskovec and S. Prelovsek (2012)
- M. Gockeler, R. Horsley, M. Lage, U.-G. Meissner, P. Rakow (2012)
- K. Polejaeva and A. Rusetsky (2012)
- M. T. Hansen and S. R. Sharpe (2012), (2013)
- **RB** and Z. Davoudi (2012), (2013)
- N. Li and C. Liu (2013)
- P. Guo, J. Dudek, R. Edwards, and A. P. Szczepaniak (2013)
- **RB**, Z. Davoudi, and T. C. Luu (2013)
- **RB**, Z. Davoudi, T. C. Luu and M. J. Savage (2013) (2013)
- V. Bernard, M. Lage, U.-G. Meissner, and A. Rusetsky (2011)
- N. Li, S. Y. Li, C. Liu (2014)
- **RB (2014)**
- Ning Li, Song-Yuan Li, Chuan Liu (2014)
- …

Plenty of purple and gold!

Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



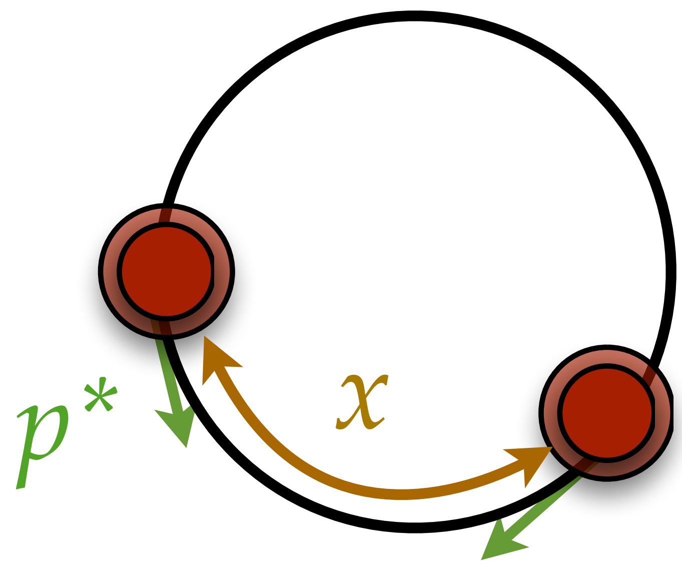
$$\phi(x) \sim e^{ipx}$$

Periodicity: $\phi(L) = \phi(0)$

Quantization condition: $L\, p_n = 2\pi n$

Reinventing the *quantum-mechanical* wheel

Two particles:



infinite volume scattering phase shift

$$\psi(x) \sim e^{ip^* x + i2\delta(p^*)}$$

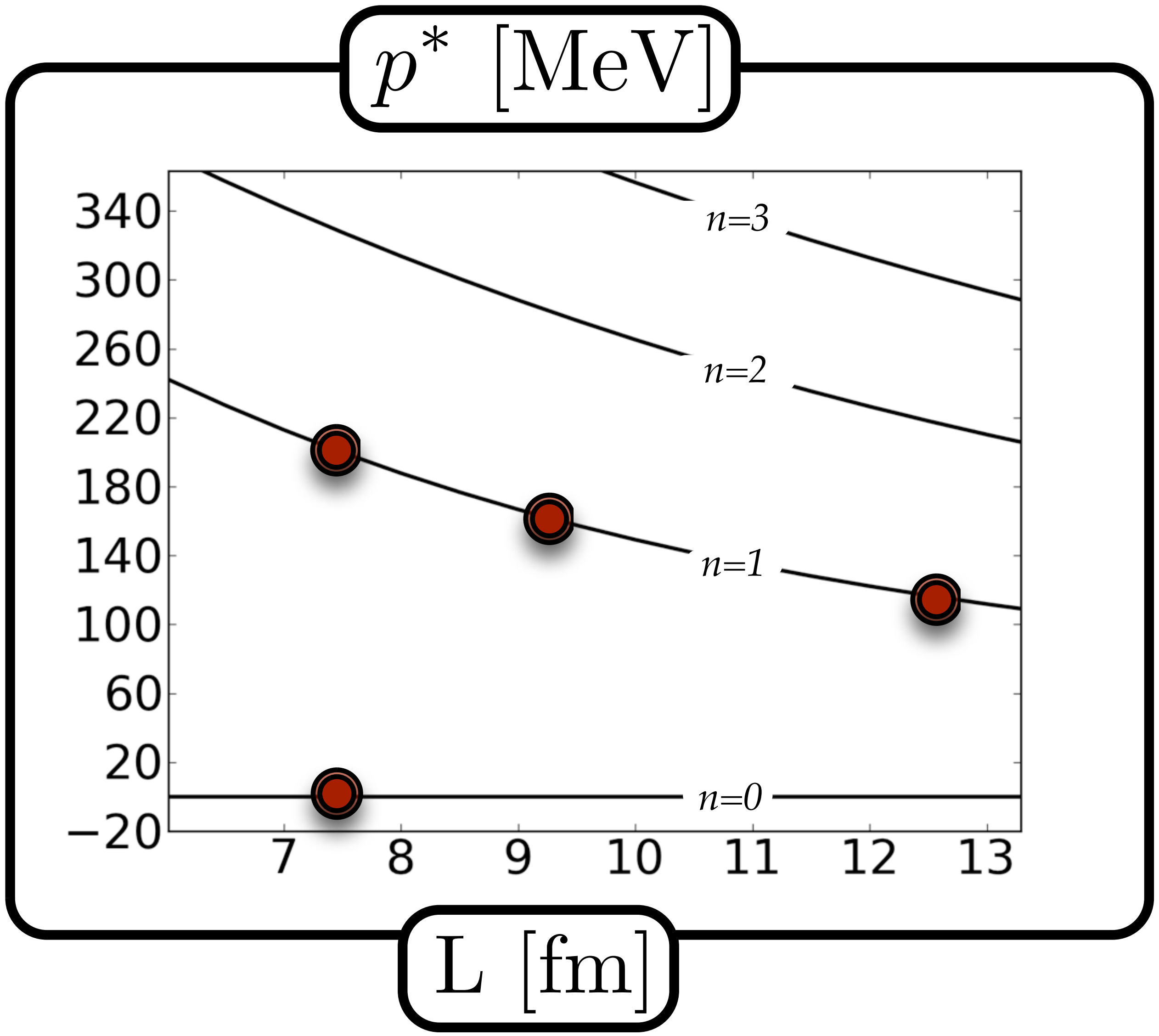
Asymptotic wavefunction

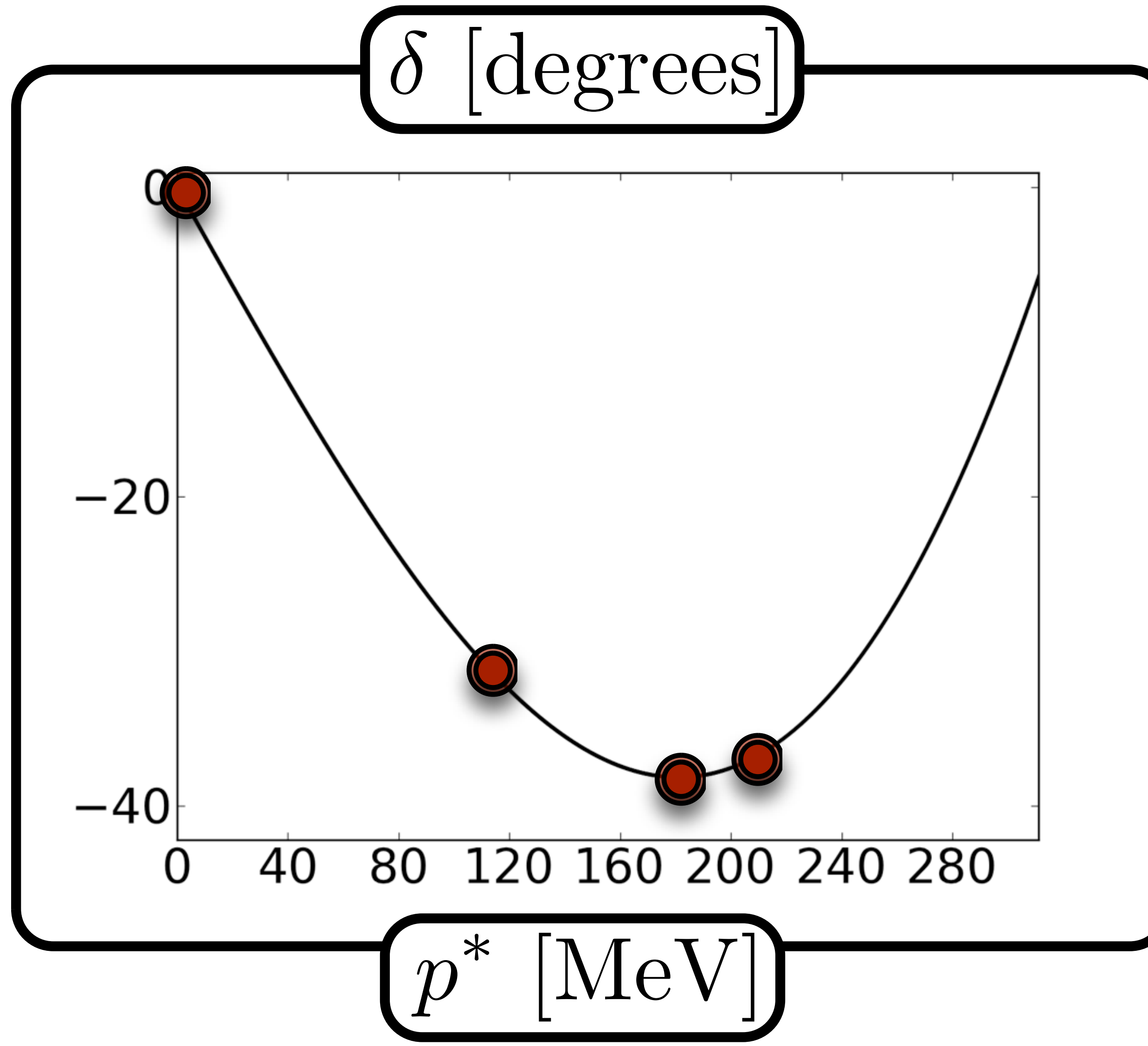
Periodicity: $\psi(L) = \psi(0)$

Quantization condition: $L\ p_n^* + 2\delta(p_n^*) = 2\pi n$

Reinventing the *quantum-mechanical* wheel

$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

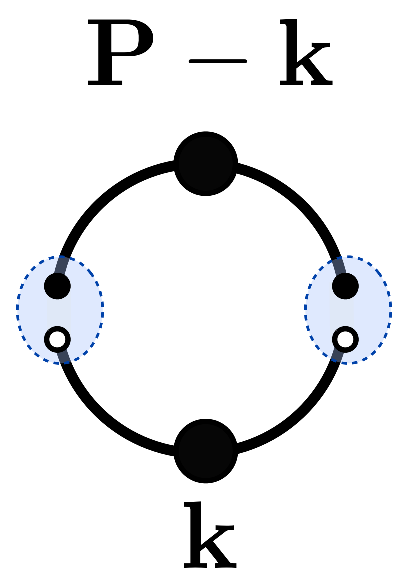




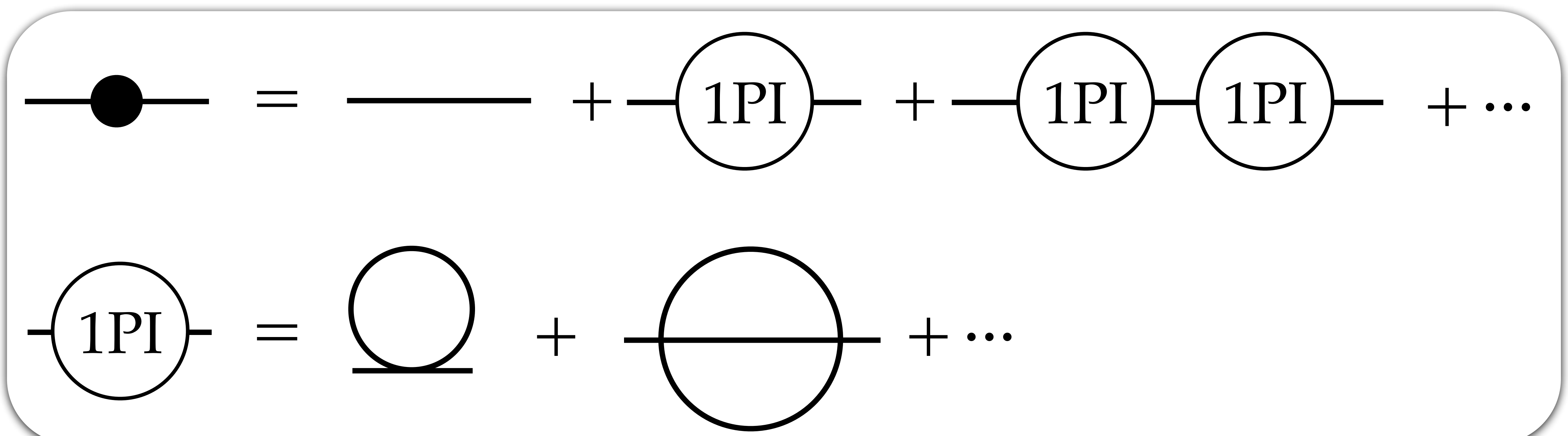
Sketch of 3+1D result

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\Lambda\mu}(x_0, \mathbf{P})\mathcal{O}^\dagger_{\Lambda\mu}(y_0, \mathbf{P})|0\rangle$$

Fourier transform...

$C(P, k) =$ 

$+ \cdots$



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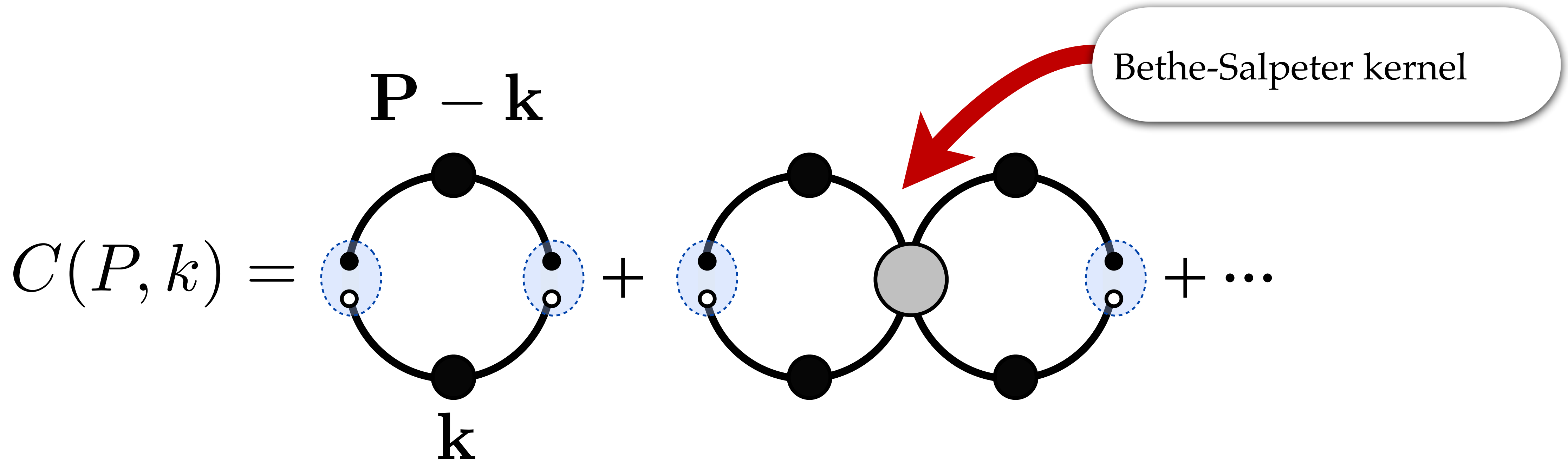
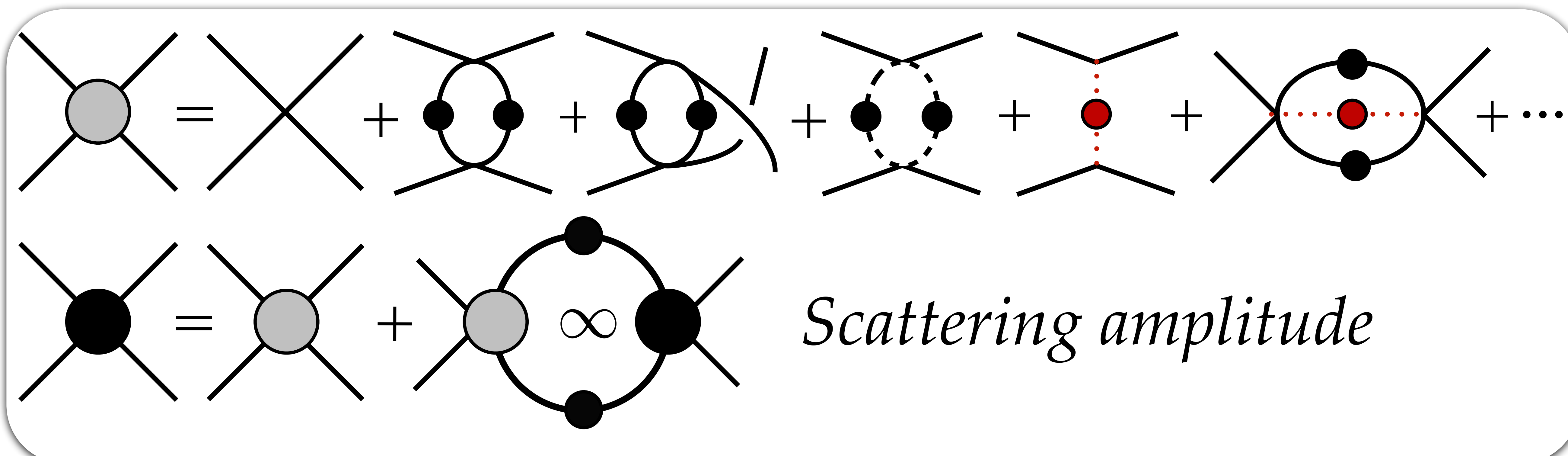
$$C(x_0 - y_0, \mathbf{P}) = \langle 0 | \mathcal{O}'_{\Lambda\mu}(x_0, \mathbf{P}) \mathcal{O}^{\dagger}_{\Lambda\mu}(y_0, \mathbf{P}) | 0 \rangle$$





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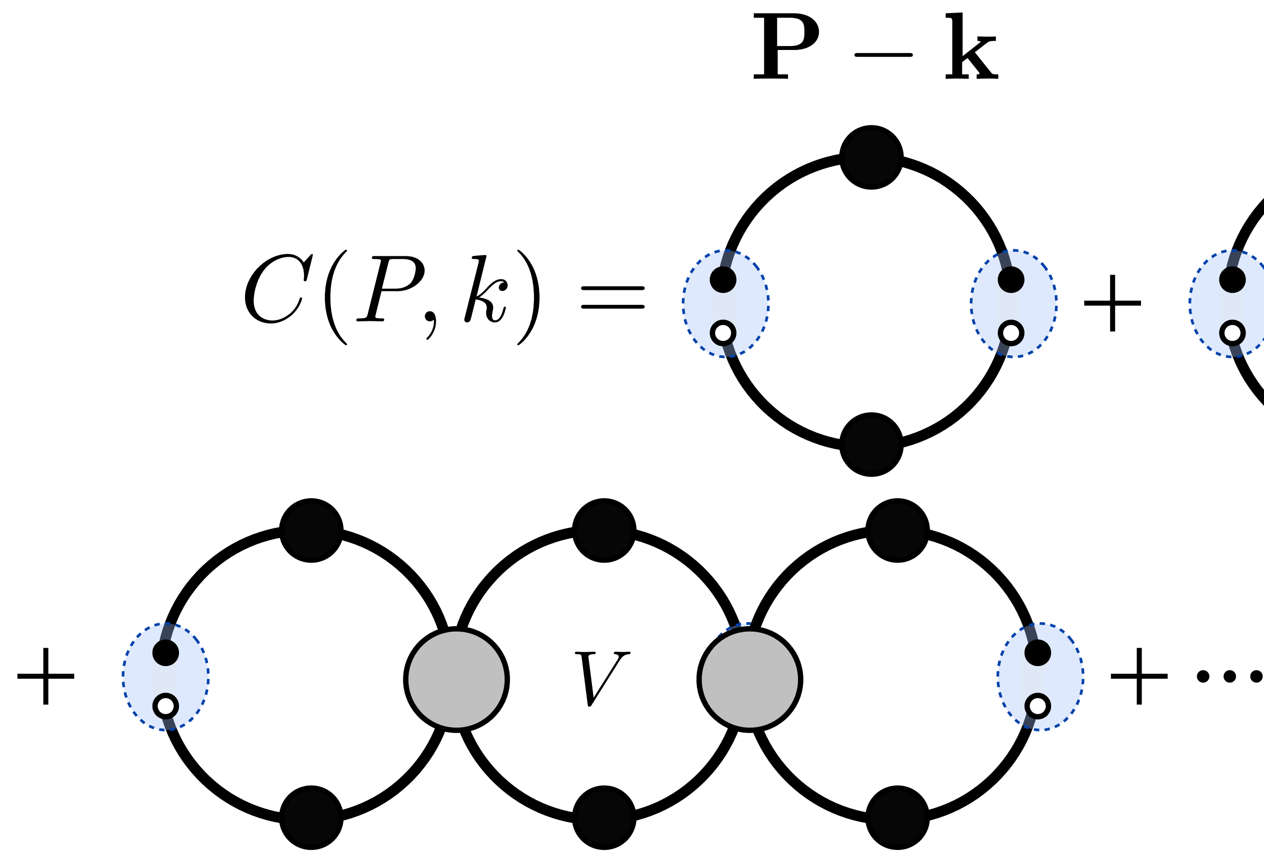
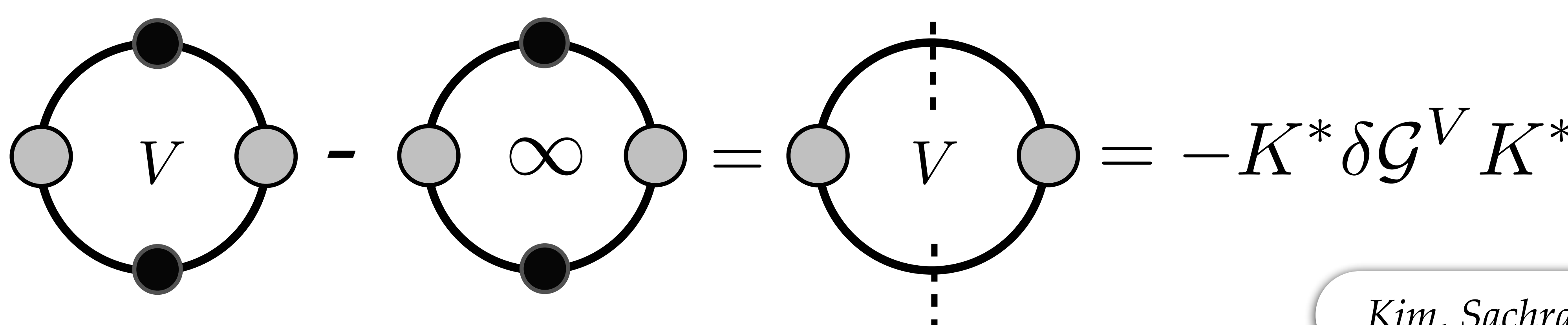


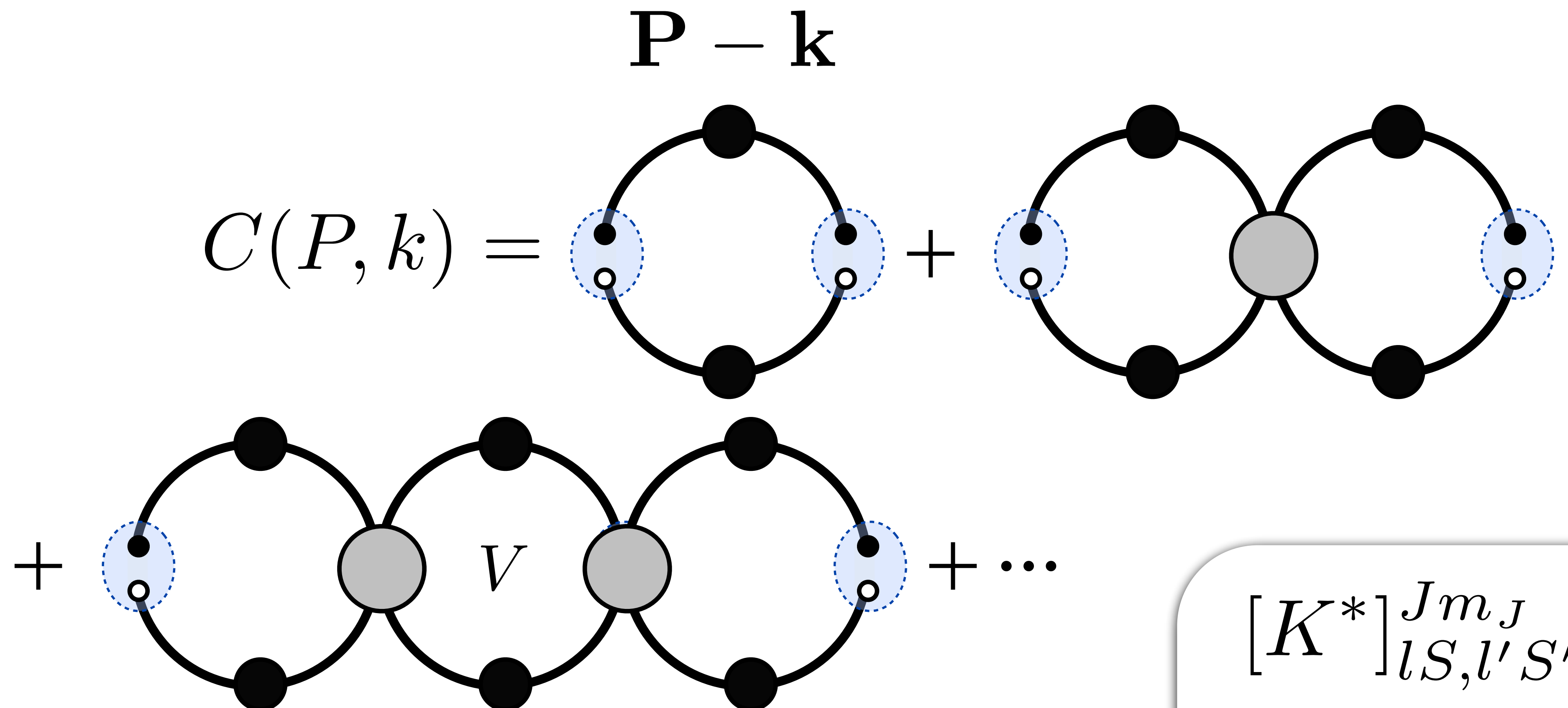
On-shell state can sample boundaries of your volume



Kim, Sachrajda, and Sharpe (2005)

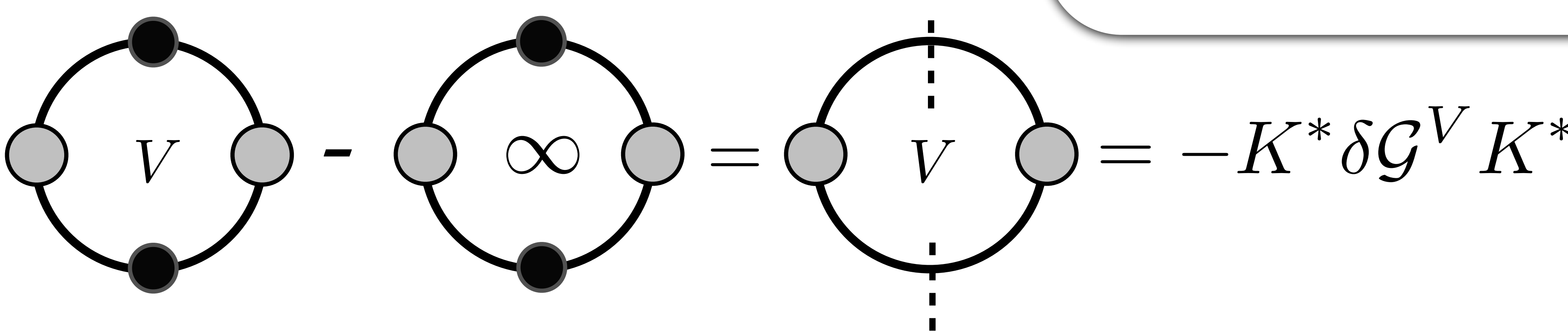
Sketch of 3+1D result

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\Lambda\mu}(x_0, \mathbf{P})\mathcal{O}^\dagger_{\Lambda\mu}(y_0, \mathbf{P})|0\rangle$$



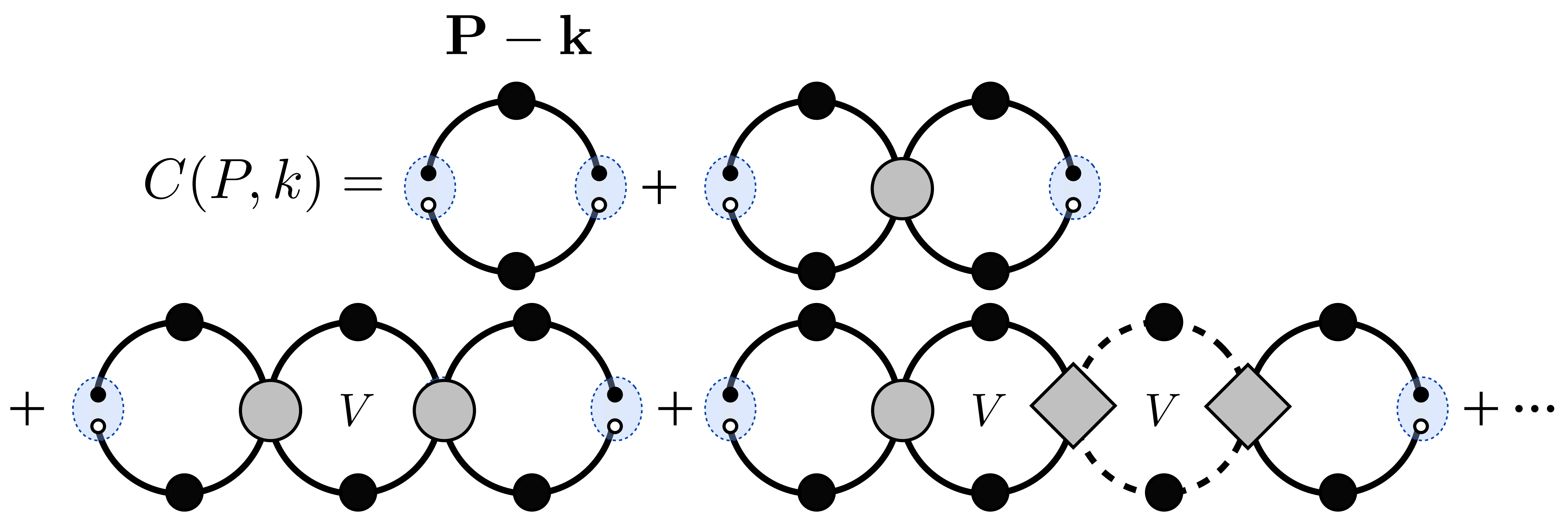
$[K^*]^{Jm_J}_{lS,l'S'}$

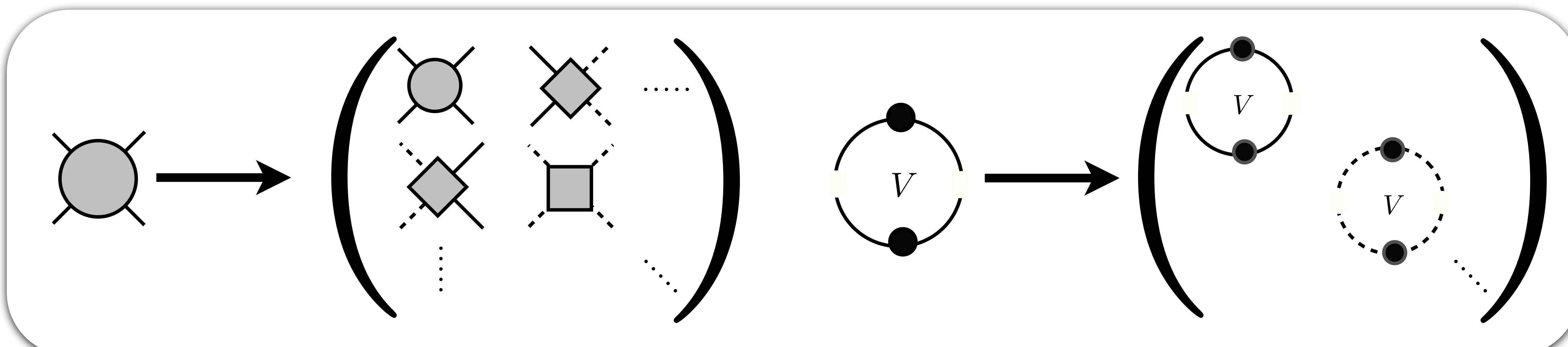
$\left[\delta\mathcal{G}^V\right]_{Jm_J,lS;J'm_{J'},l'S}$



Sketch of 3+1D result

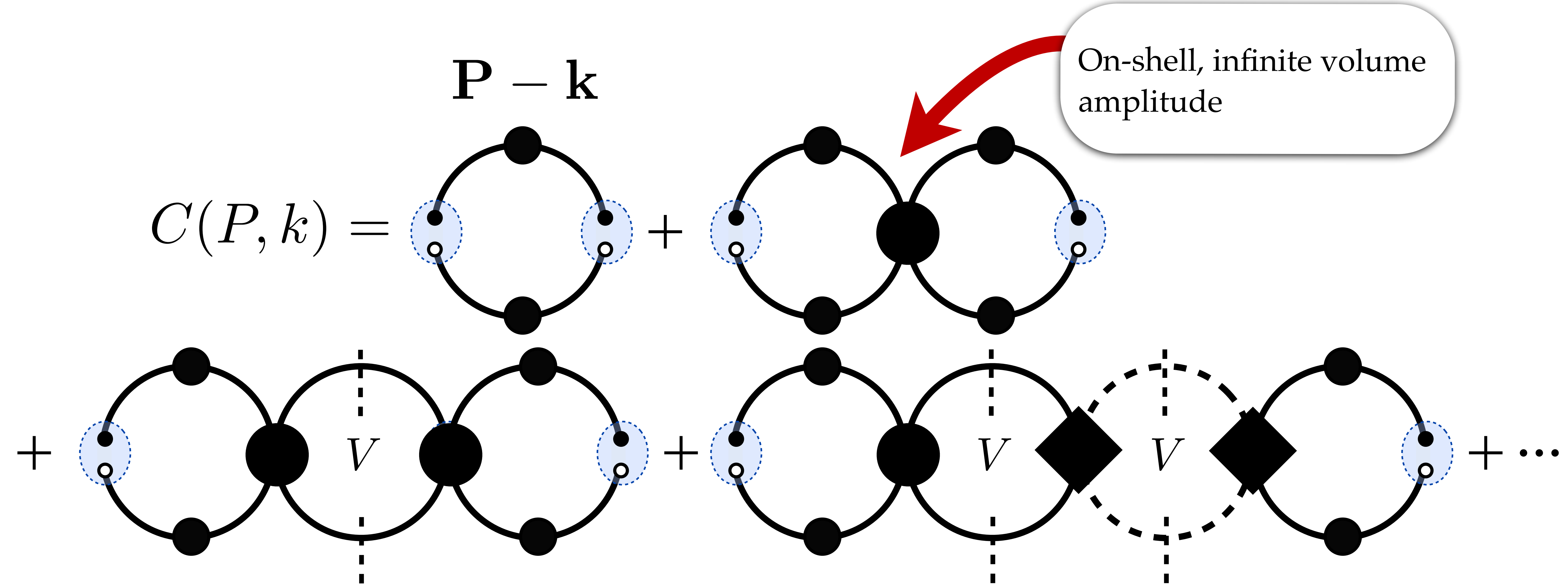
$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\Lambda\mu}(x_0, \mathbf{P})\mathcal{O}^{\dagger}_{\Lambda\mu}(y_0, \mathbf{P})|0\rangle$$





Sketch of 3+1D result

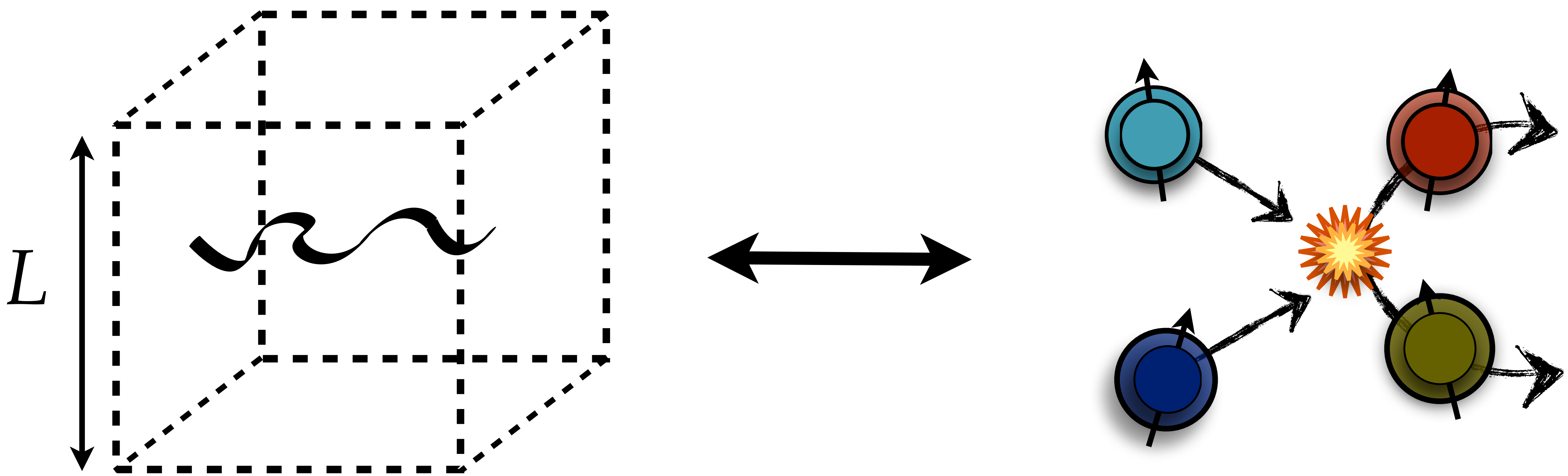
$$C(x_0 - y_0, \mathbf{P}) = \langle 0 | \mathcal{O}'_{\Lambda\mu}(x_0, \mathbf{P}) \mathcal{O}^{\dagger}_{\Lambda\mu}(y_0, \mathbf{P}) | 0 \rangle$$



$$= \left(\frac{\#}{\mathcal{M}^{-1} + \delta\mathcal{G}^{V}} + \cdots \right)$$

3+1D result

$$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^{V}\right]=0$$



RB (PhD Thesis) : arXiv:1311.6032 [hep-lat]

RB, Zohreh Davoudi* [PRD] : arXiv:1204.1110 [hep-lat]

RB, Zohreh Davoudi* [PRD] : arXiv:1212.3398 [hep-lat]

RB, Zohreh Davoudi*, Tom Luu** [PRD] : arXiv:1305.4903 [hep-lat]

RB, Zohreh Davoudi*, Tom Luu**, Martin Savage* [PRD] : arXiv:1309.3556 [hep-lat]

RB, Zohreh Davoudi*, Tom Luu**, Martin Savage* : arXiv:1311.7686 [hep-lat]

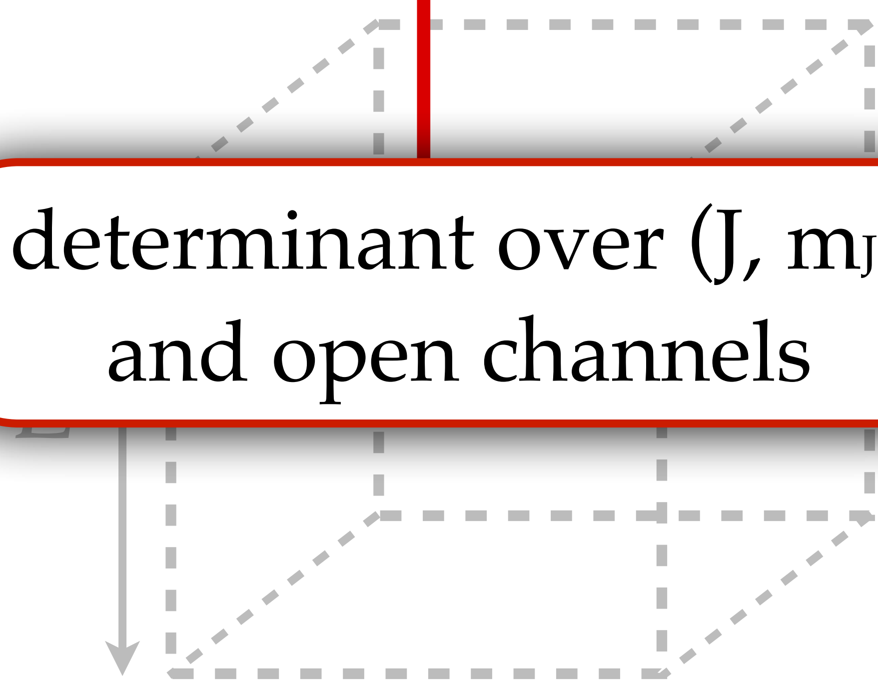
RB [Accepted at PRD] : arXiv:1401.3312 [hep-lat]

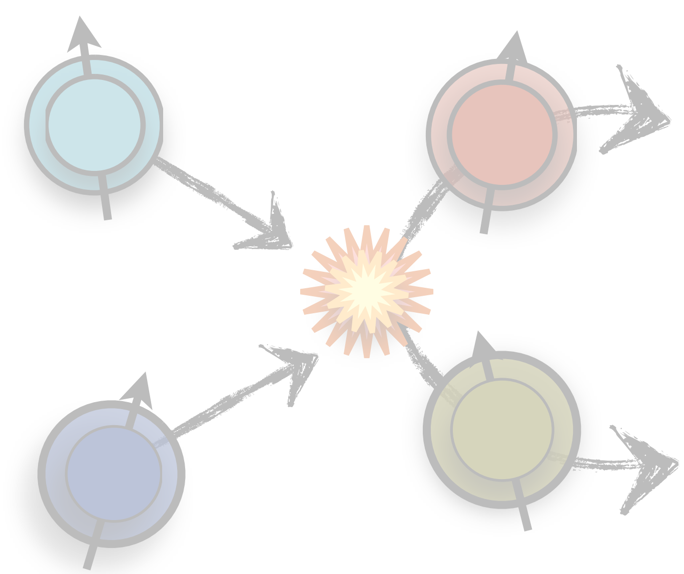
*UW, ** Jülich

3+1D result

$$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^{V}\right]=0$$

determinant over (J, m_J) and open channels





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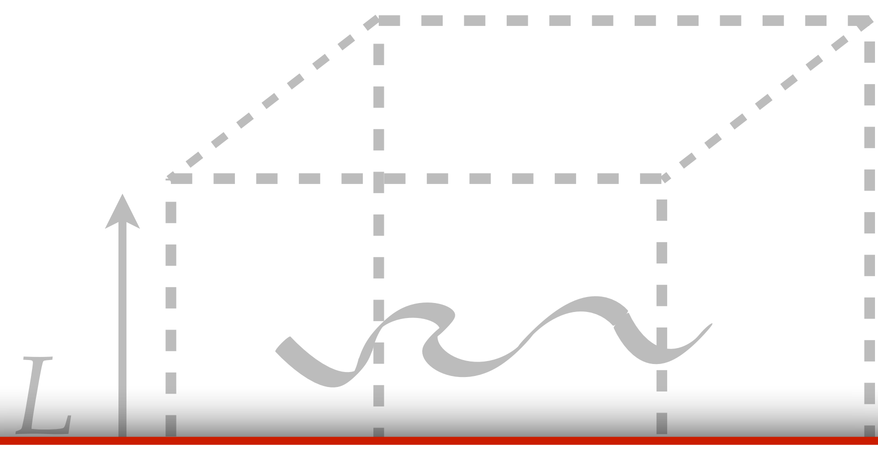
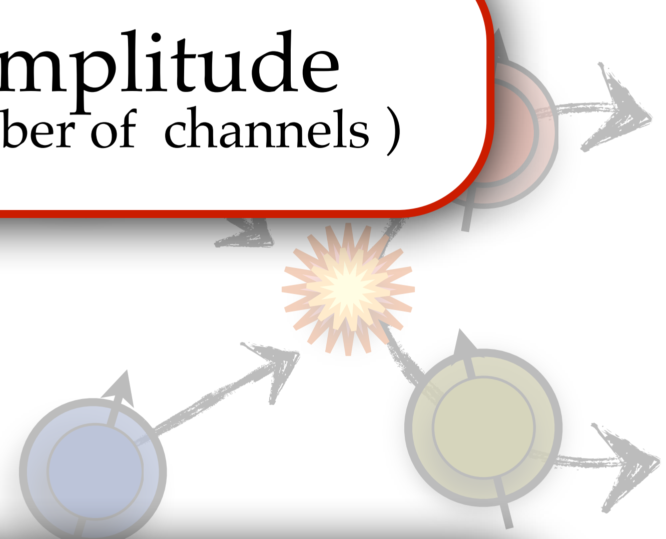
*UW, ** Jülich

3+1D result

$$\det\ [\mathcal{M}^{-1} + \delta\mathcal{G}^V] = 0$$

Scattering amplitude
(Can couple any number of channels)





e.g. positive parity, isosinglet, two-nucleon channel (deuteron...)

$$\begin{pmatrix} \mathcal{M}_1^{S} & \mathcal{M}_1^{SD} & 0 & \\ \mathcal{M}_1^{DS} & \mathcal{M}_1^{D} & 0 & \\ 0 & 0 & \mathcal{M}_3^{D} & \\ & & & \ddots \end{pmatrix}$$

11.6032 [hep-lat]
1204.1110 [hep-lat]
1212.3398 [hep-lat]
1305.4903 [hep-lat]
1309.3556 [hep-lat]
1311.7686 [hep-lat]
:1401.3312 [hep-lat]

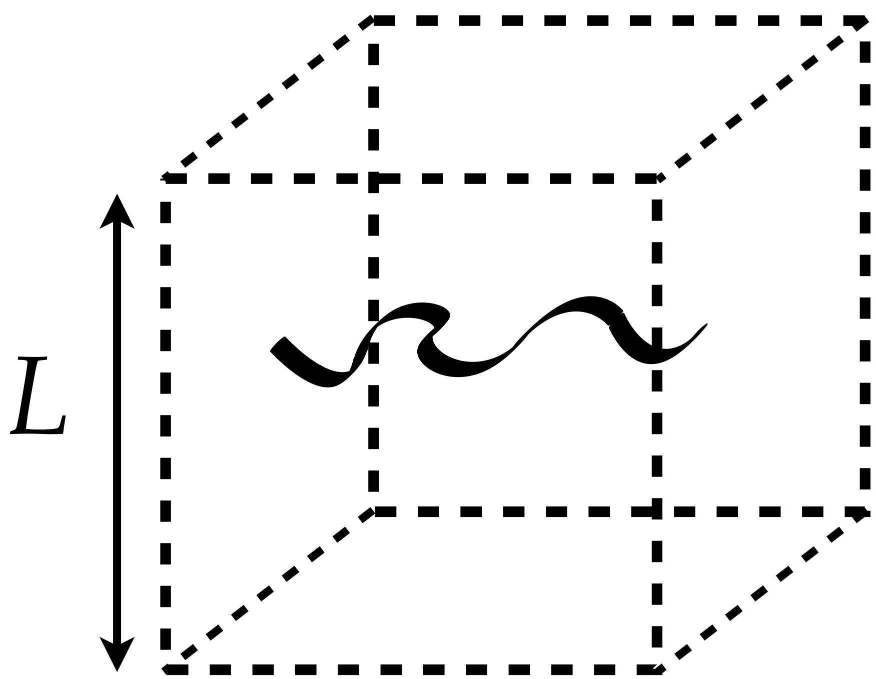
*UW, ** Jülich

3+1D result

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

$$\begin{pmatrix} \delta G^V_{00} & \delta G^V_{01} & \delta G^V_{02} & \\ \delta G^V_{10} & \delta G^V_{11} & \delta G^V_{12} & \\ \delta G^V_{20} & \delta G^V_{21} & \delta G^V_{22} & \\ & & & \ddots \end{pmatrix}$$

Typically a sparse matrix



e.g. S-wave at rest

$$k^* \cot\delta_S = \frac{1}{\pi L}\sum_{\mathbf{n}} \frac{1}{\mathbf{n}^2 - (k^* L/2\pi)^2}$$

RB (PhD Thesis) : arXiv:1311.6032 [hep-lat]

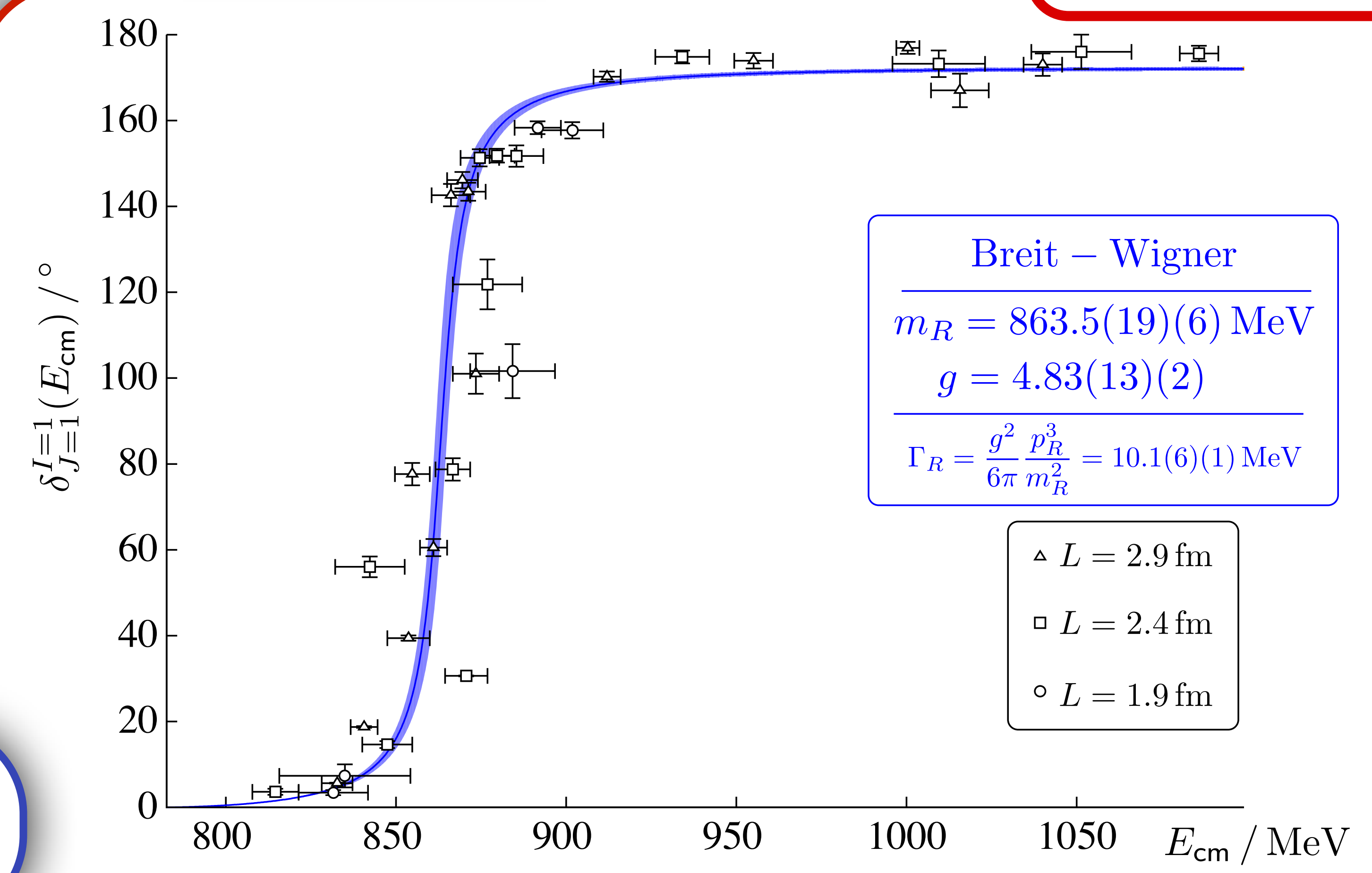
3+1D result

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

In goes finite volume spectra

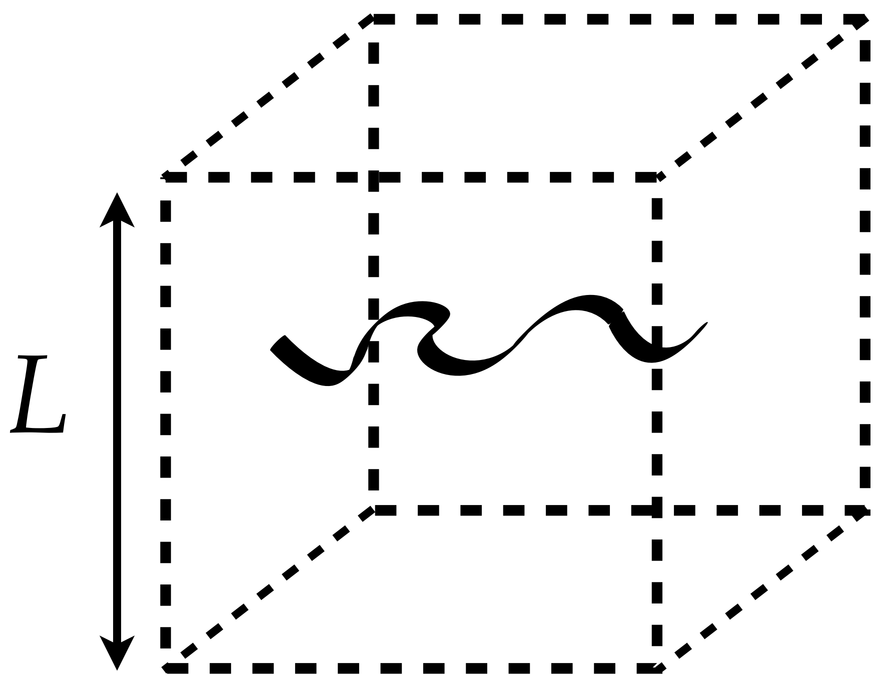
out goes...

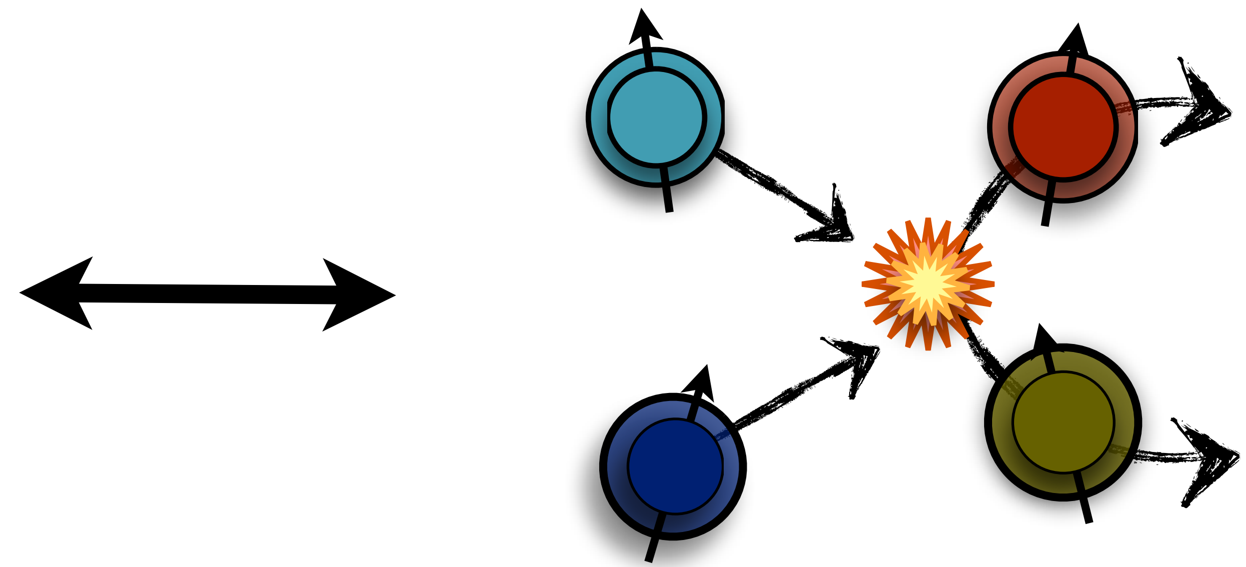
$m_\pi = 392\,\mathrm{MeV}$



[PRD] arXiv:1212.0830 [hep-ph] [PRD]
J. Dudek, R. Edwards & C. Thomas (2012)

3+1D result



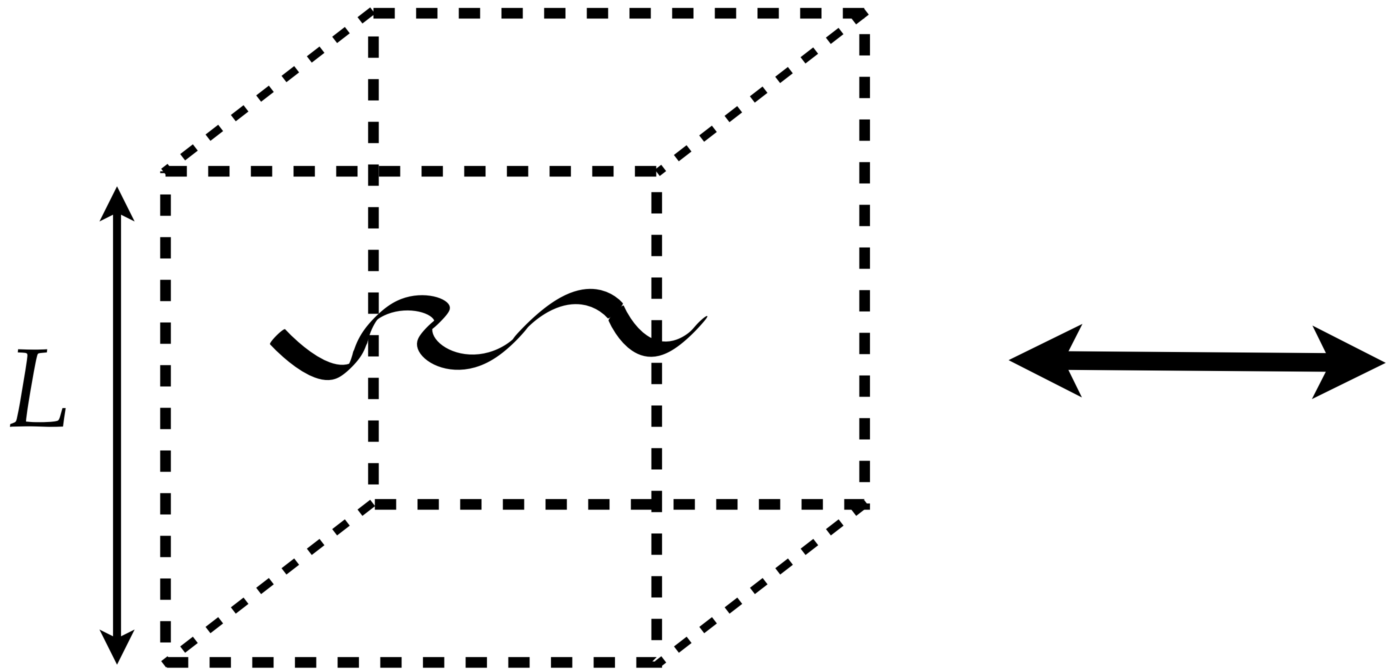


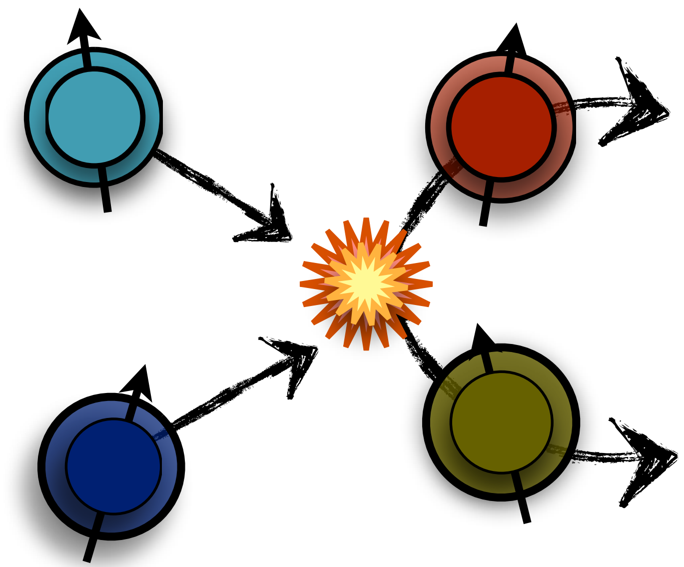
"just a consequence of quantum mechanics"

- Model independent & non-perturbative
- Universal: nuclear physics, atomic physics, etc
- Arbitrary quantum numbers: relativity, spin, masses, momenta, angular momentum, inelasticities, etc
- General volumes with any boundary conditions: periodic, anti-periodic , or any linear combination on any rectangular prism

RB [Accepted at PRD] : arXiv:1401.3312 [hep-lat]

3+1D result





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☑ Nuclear theory homework assignment by M. J. S. (2008)

RB [Accepted at PRD] : arXiv:1401.3312 [hep-lat]

A long & *incomple*... ist

- **M. Lüscher (1086), (1991)**
- L. Maiani and M. Testa (1990)
- K. Rummukainen and S. A. Gottlieb (1995)
- S. Beane, P. Bedaque, A. Parreno, and M. Savage (2004), (2005)
- P. Bedaque (2004)
- X. Li and C. Liu (2004)
- W. Detmold and M. J. Savage (2004)
- X. Feng, X. Li, and C. Liu (2004)
- N. H. Christ, C. Kim, and T. Yama...
- C. Kim, C. Sachrajda, and S. R...
- V. Bernard, M. Lage, U.-G... ...A. Rusetsky (2008)
- N. Ishizuka (2009)
- S. Bour, S. Koe... ...Hammer, and U.-G. M...
- Z. Dav... ...age (2011) (2014)
- L. Les... ...ovsek (2012)
- M... ...sley, M. Lage, U.-G. Meissner, P.
- ...and A. Rusetsky (2012)
- ...nsen and S. R. Sharpe (2012), (2013)
- ...and Z. Davoudi (2012), (2013)
- N. Li and C. Liu (2013)
- P. Guo, J. Dudek, R. Edwards, and A. P. Szczepaniak (2013)
- **RB**, Z. Davoudi, and T. C. Luu (2013)
- **RB**, Z. Davoudi, T. C. Luu and M. J. Savage (2013) (2013)
- V. Bernard, M. Lage, U.-G. Meissner, and A. Rusetsky (2011)
- N. Li, S. Y. Li, C. Liu (2014)
- Ning Li, Song-Yuan Li, Chuan Liu (2014)
- ...

Checks out!

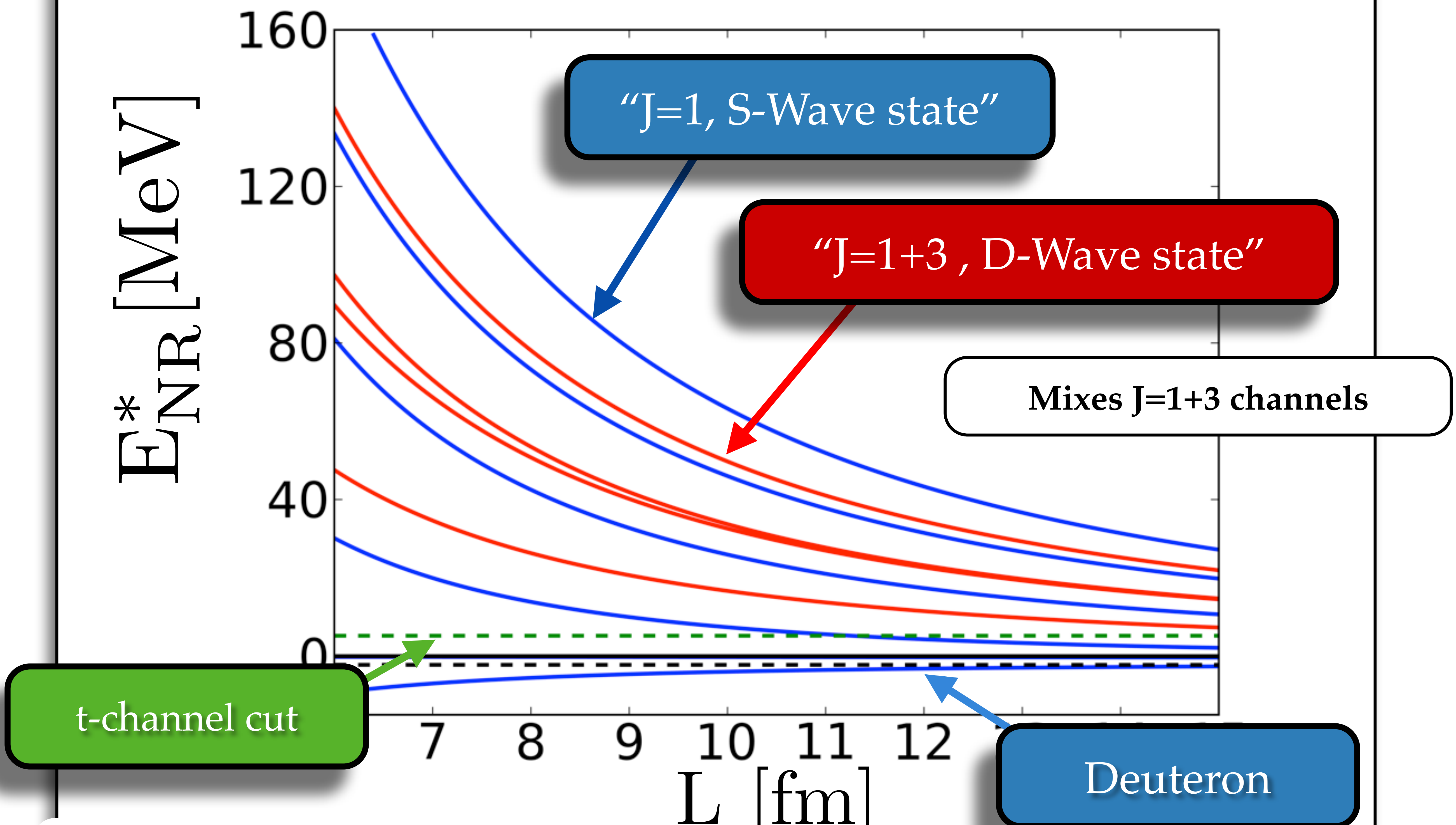
Independent derivation for multichannel baryon-baryon using Dirac fermions (two weeks later)

Predicted T1 Spectrum

$\mathbf{d} = (0,0,0)$

$E^*_{NR} = E^* - 2\, m_N$

$m_\pi \sim 140$ MeV



RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

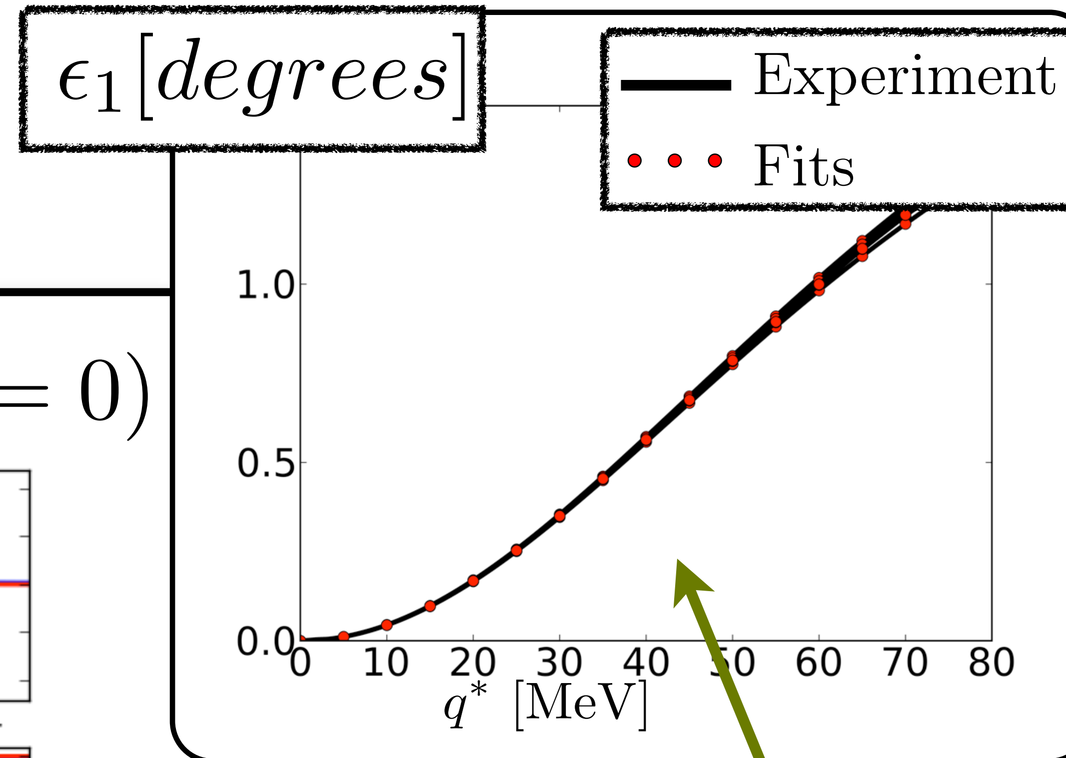
Input physical phase shifts [http://nn-online.org]

T_1 Excited states

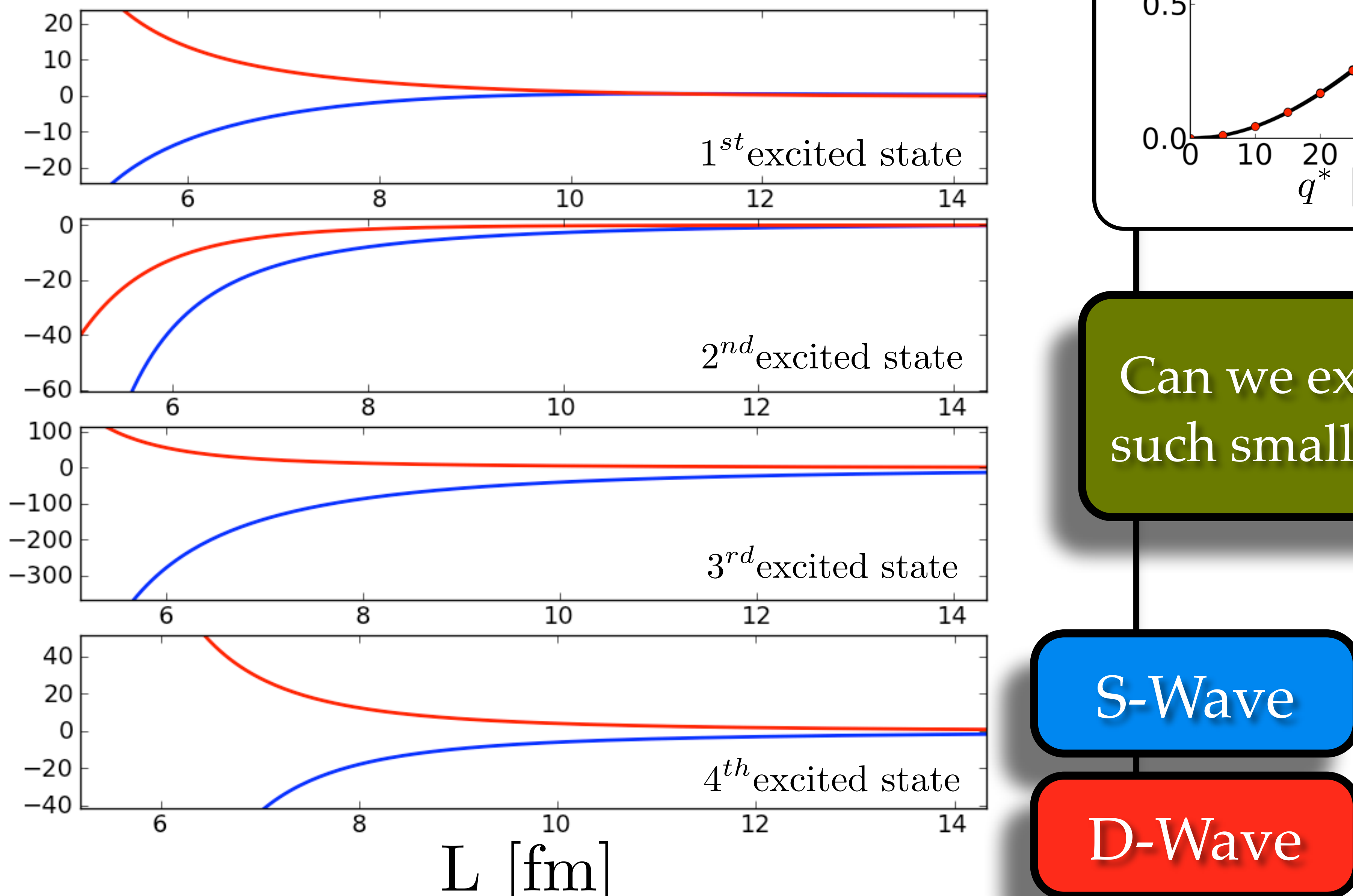
$$\mathbf{d} = (0,0,0)$$

$$\mathcal{M}_{J=1} = \begin{pmatrix} \mathcal{M}^{S} & \mathcal{M}^{SD} \\ \mathcal{M}^{DS} & \mathcal{M}^{D} \end{pmatrix}$$

$\mathcal{M}^{SD} \rightarrow \sim \sin\epsilon_1$



$$\delta E^{*(\mathbb{T}_1)}[keV] = E^{*(\mathbb{T}_1)} - E^{*(\mathbb{T}_1)}(\epsilon_1 = 0)$$

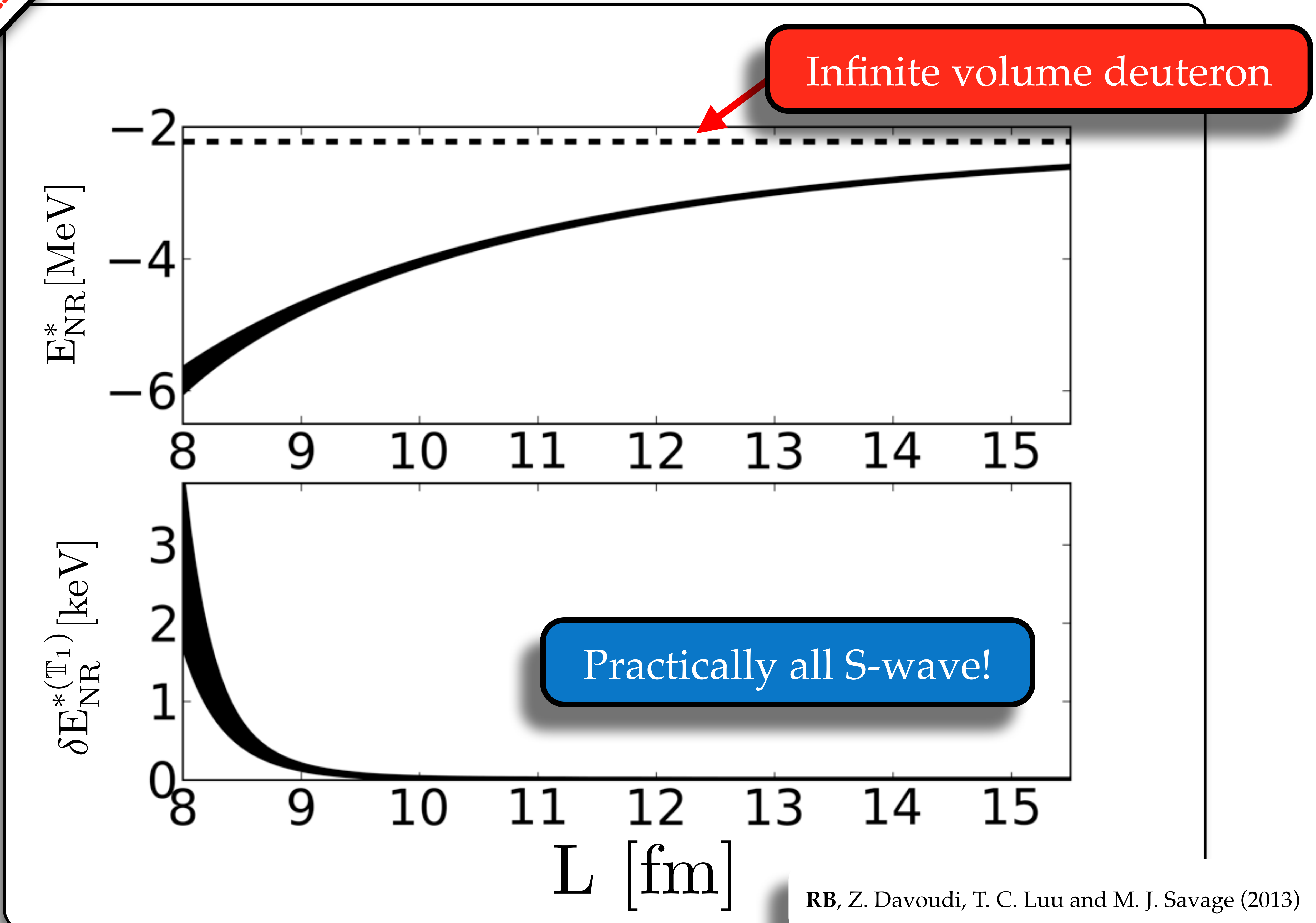


Can we expect to extract such small mixing angle?

RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Deuteron at rest $\mathbf{d} = (0,0,0)$

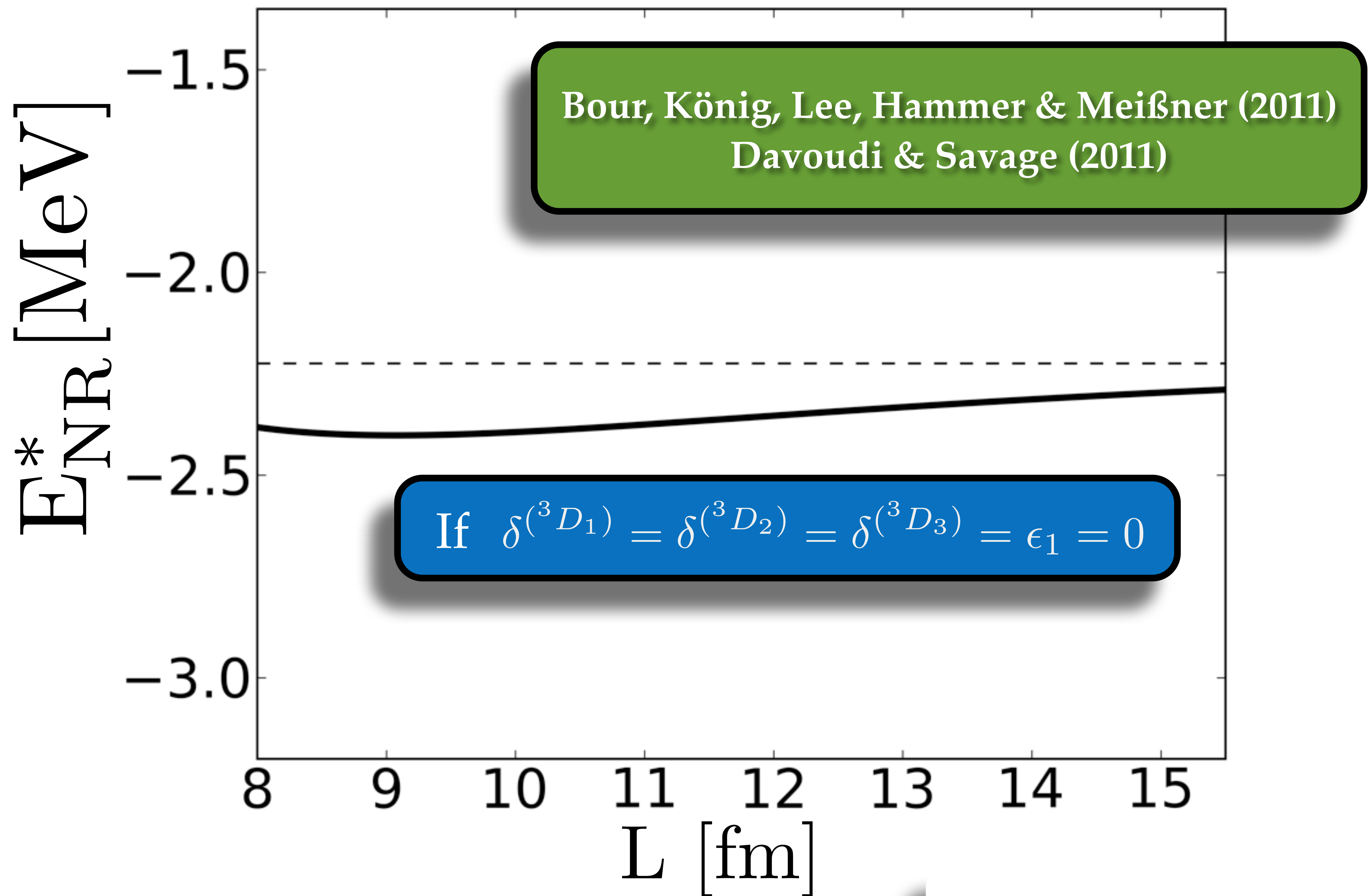
Prediction



RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Boosted deuteron $\mathbf{d} = (0, 0, 1)$

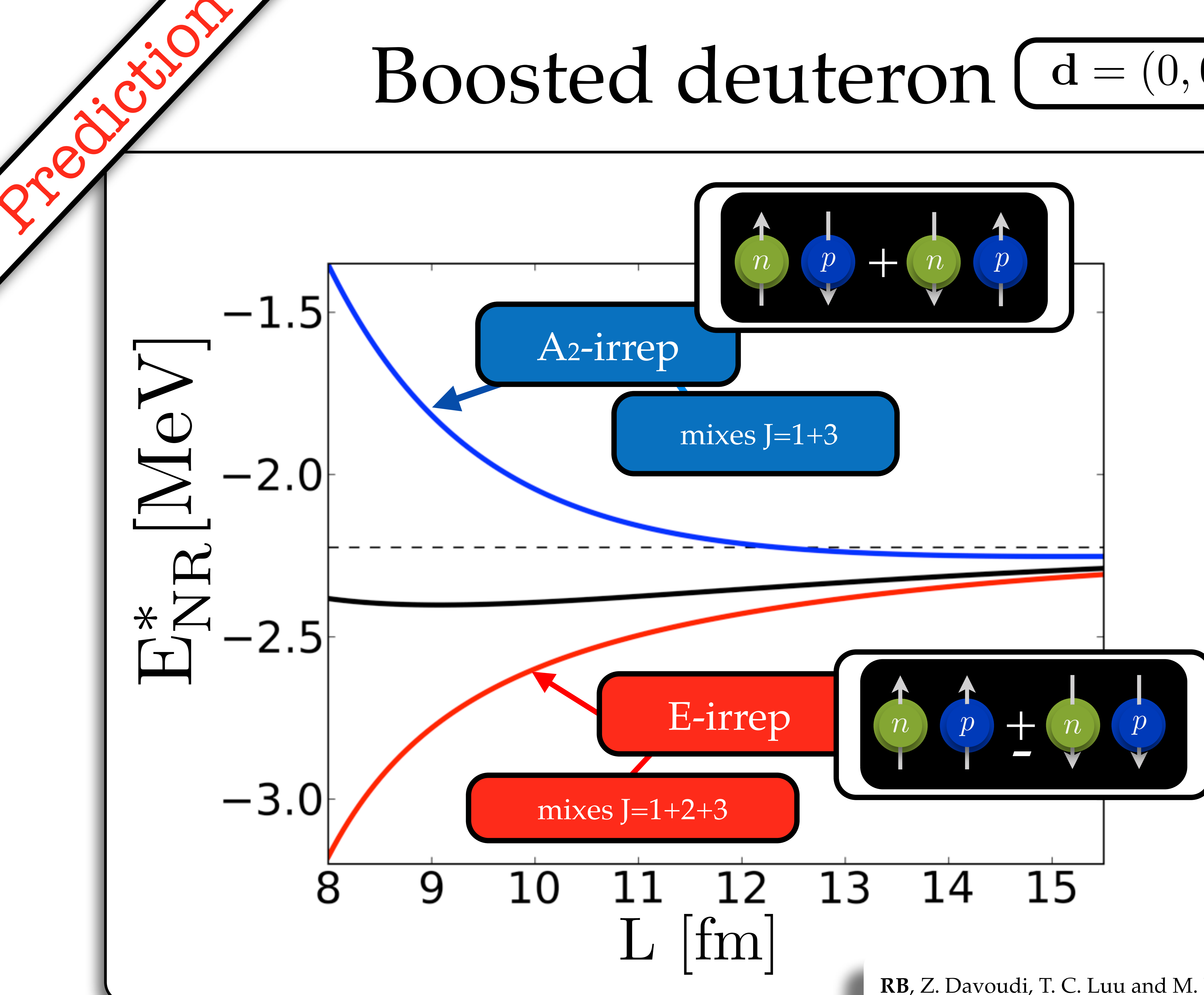
Prediction



RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Boosted deuteron

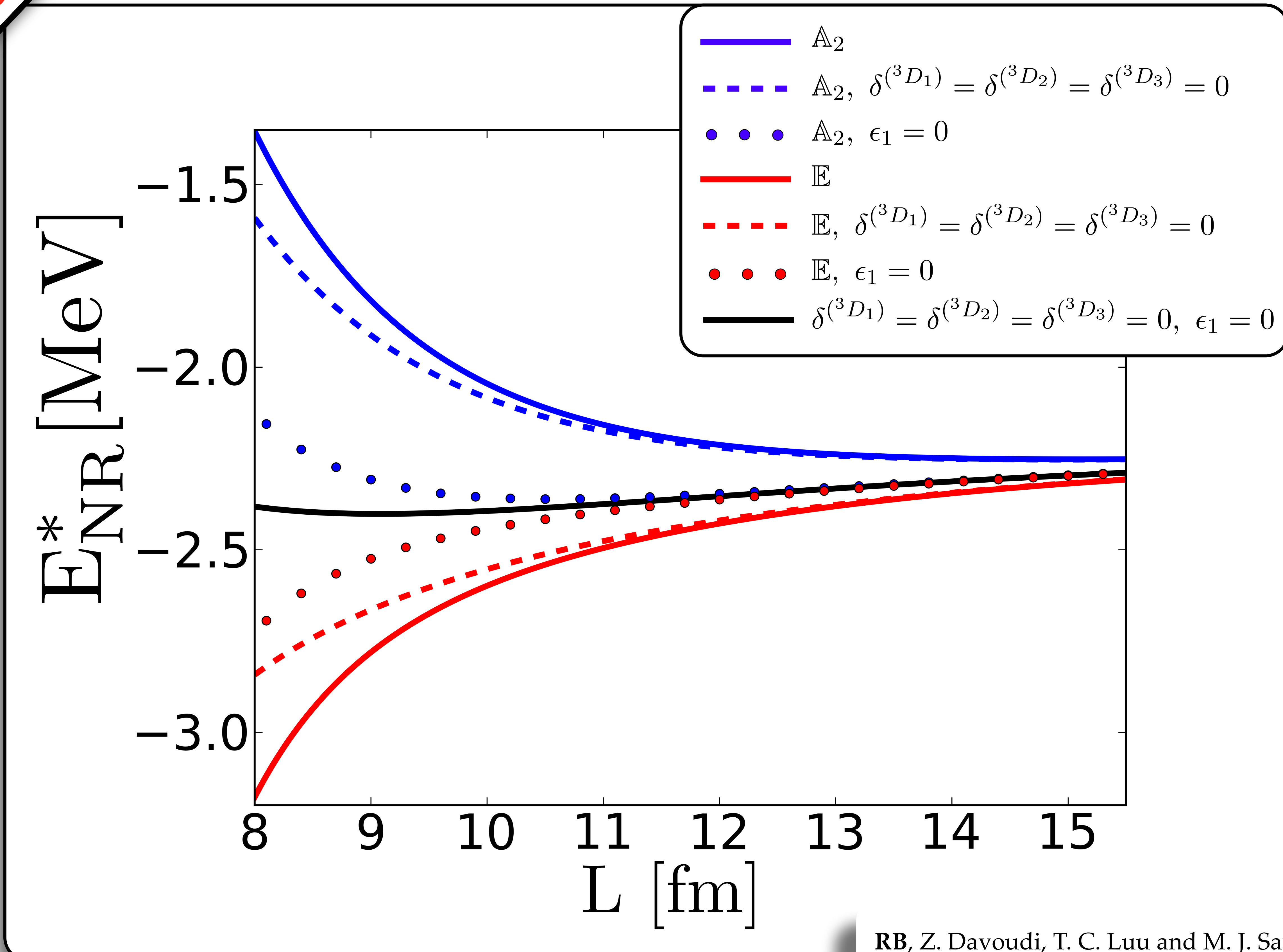
$\mathbf{d} = (0, 0, 1)$



RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Boosted deuteron $\mathbf{d} = (0, 0, 1)$

Prediction

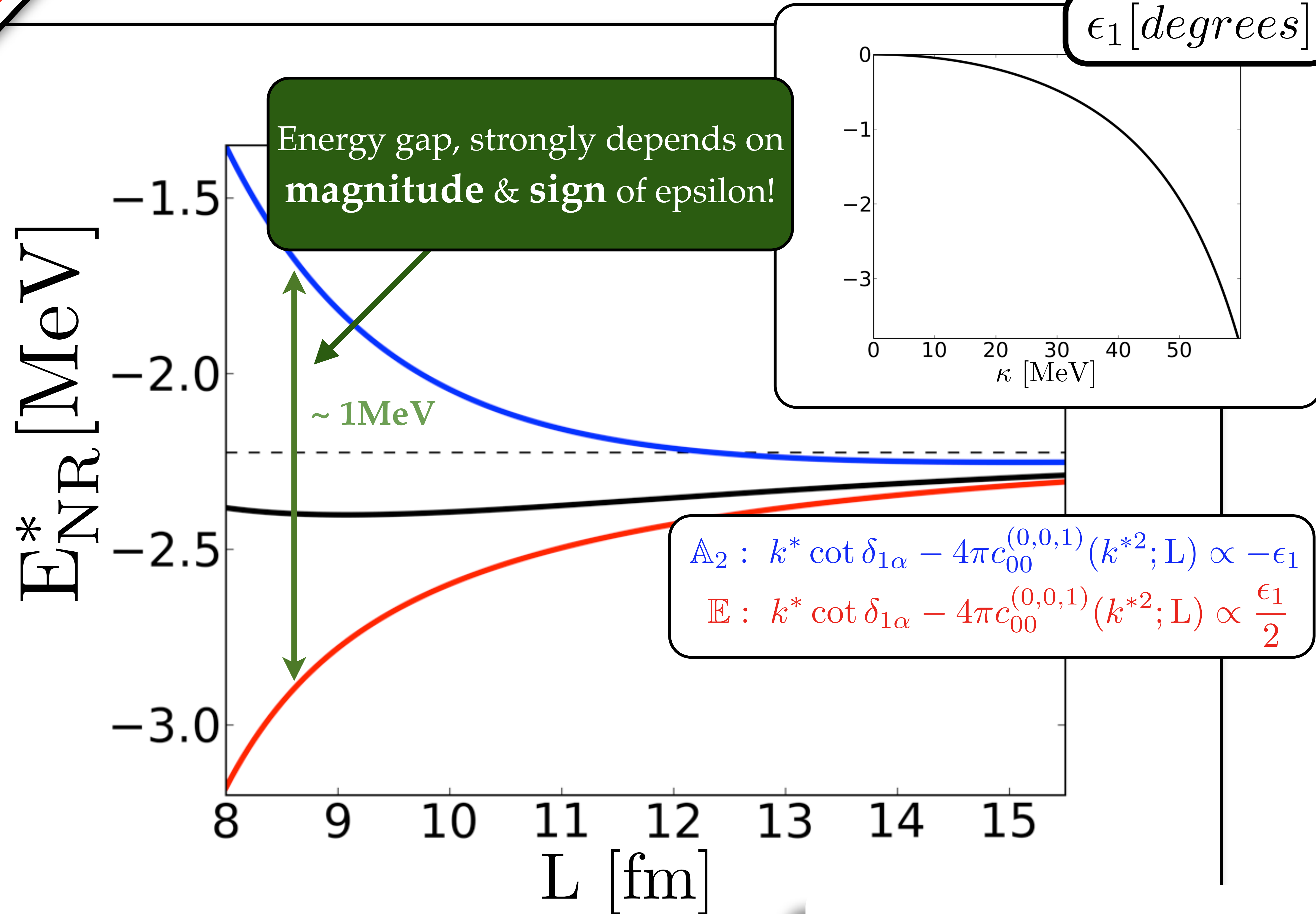


RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Prediction

Boosted deuteron

$\mathbf{d} = (0, 0, 1)$



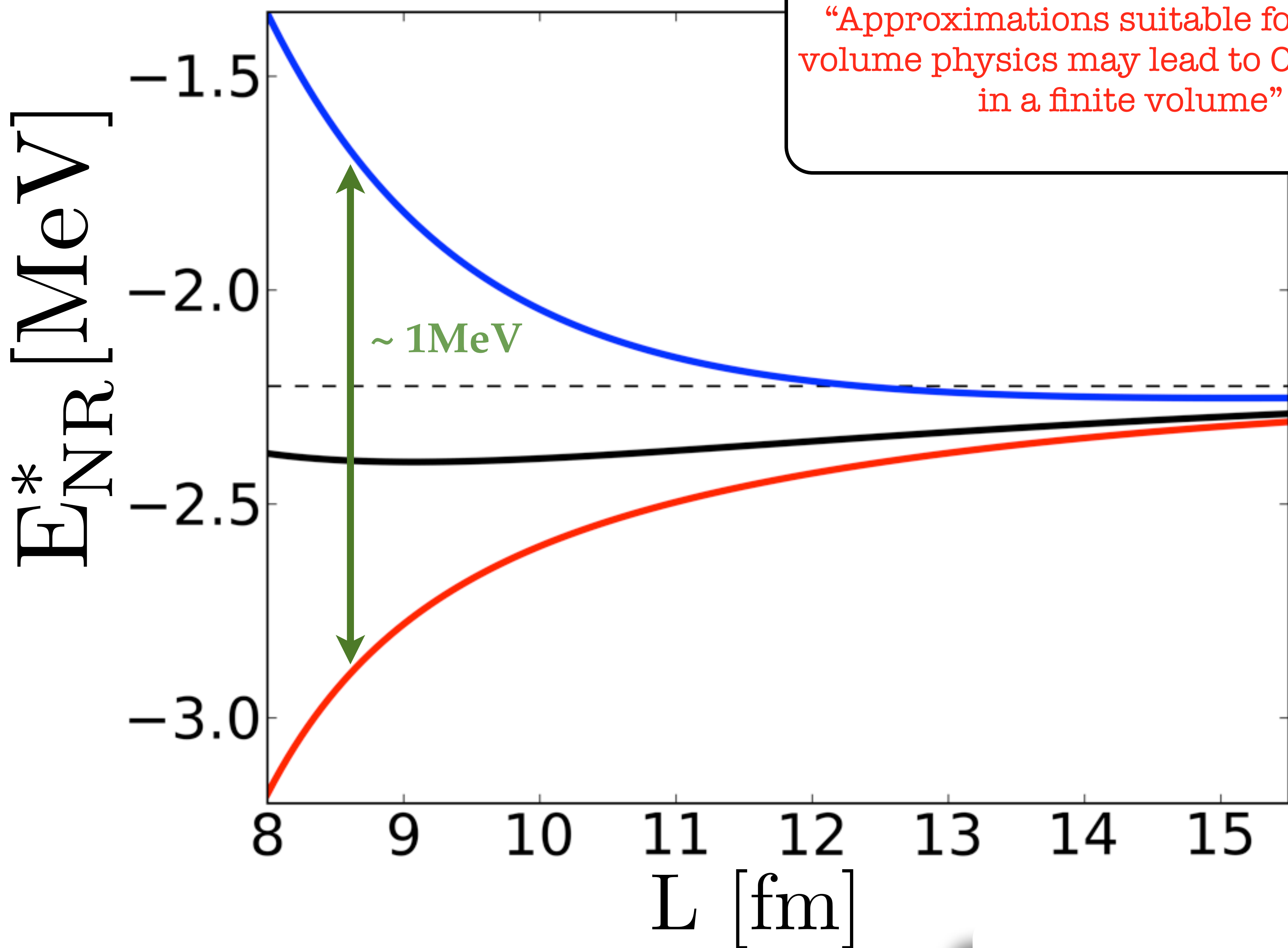
$\mathbb{A}_2: \; k^* \cot \delta_{1\alpha} - 4\pi c_{00}^{(0,0,1)}(k^{*2}; \mathrm{L}) \propto -\epsilon_1$

$\mathbb{E}: \; k^* \cot \delta_{1\alpha} - 4\pi c_{00}^{(0,0,1)}(k^{*2}; \mathrm{L}) \propto \frac{\epsilon_1}{2}$

RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

Boosted deuteron $\mathbf{d} = (0,0,1)$

Prediction

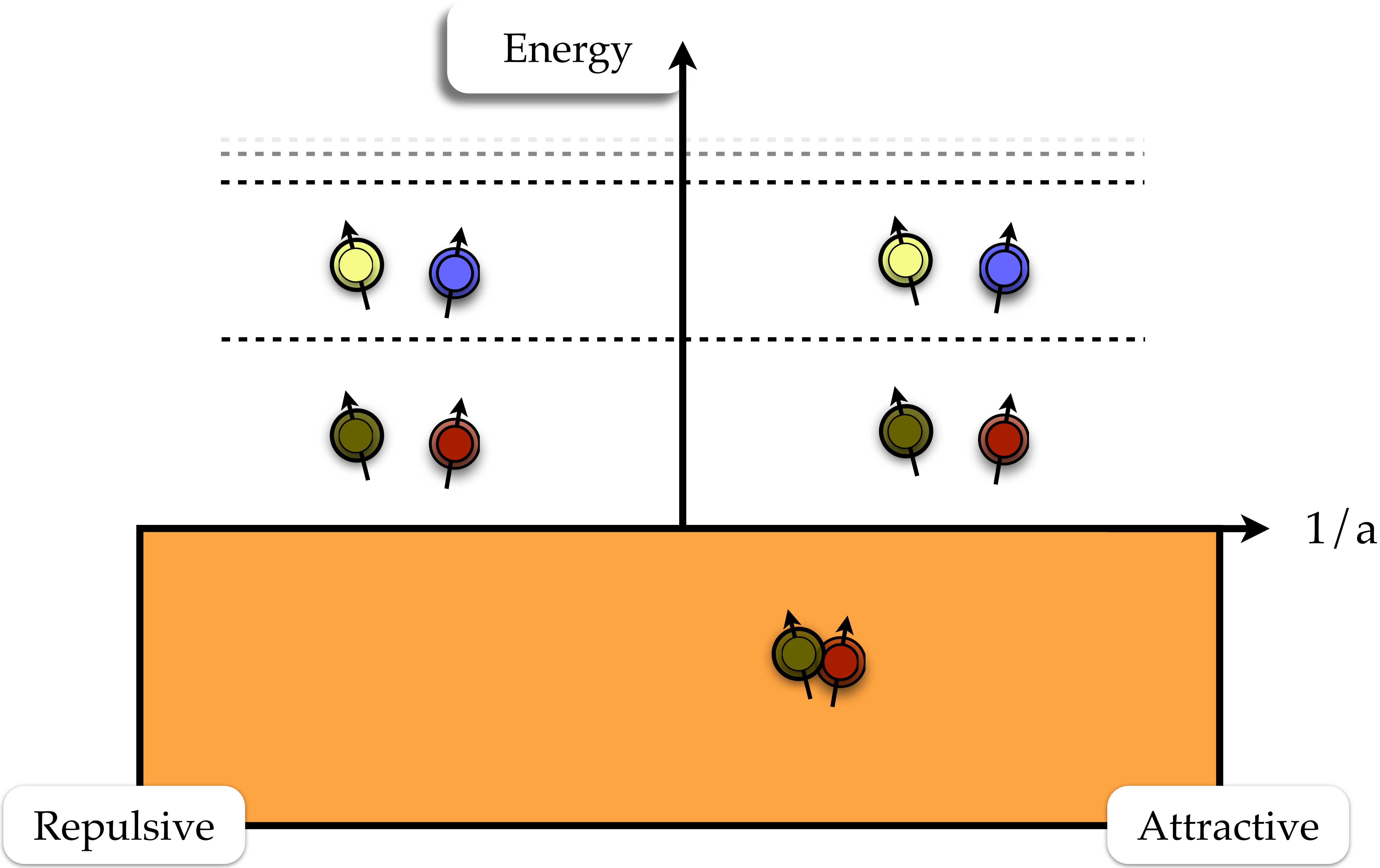


Take home message:

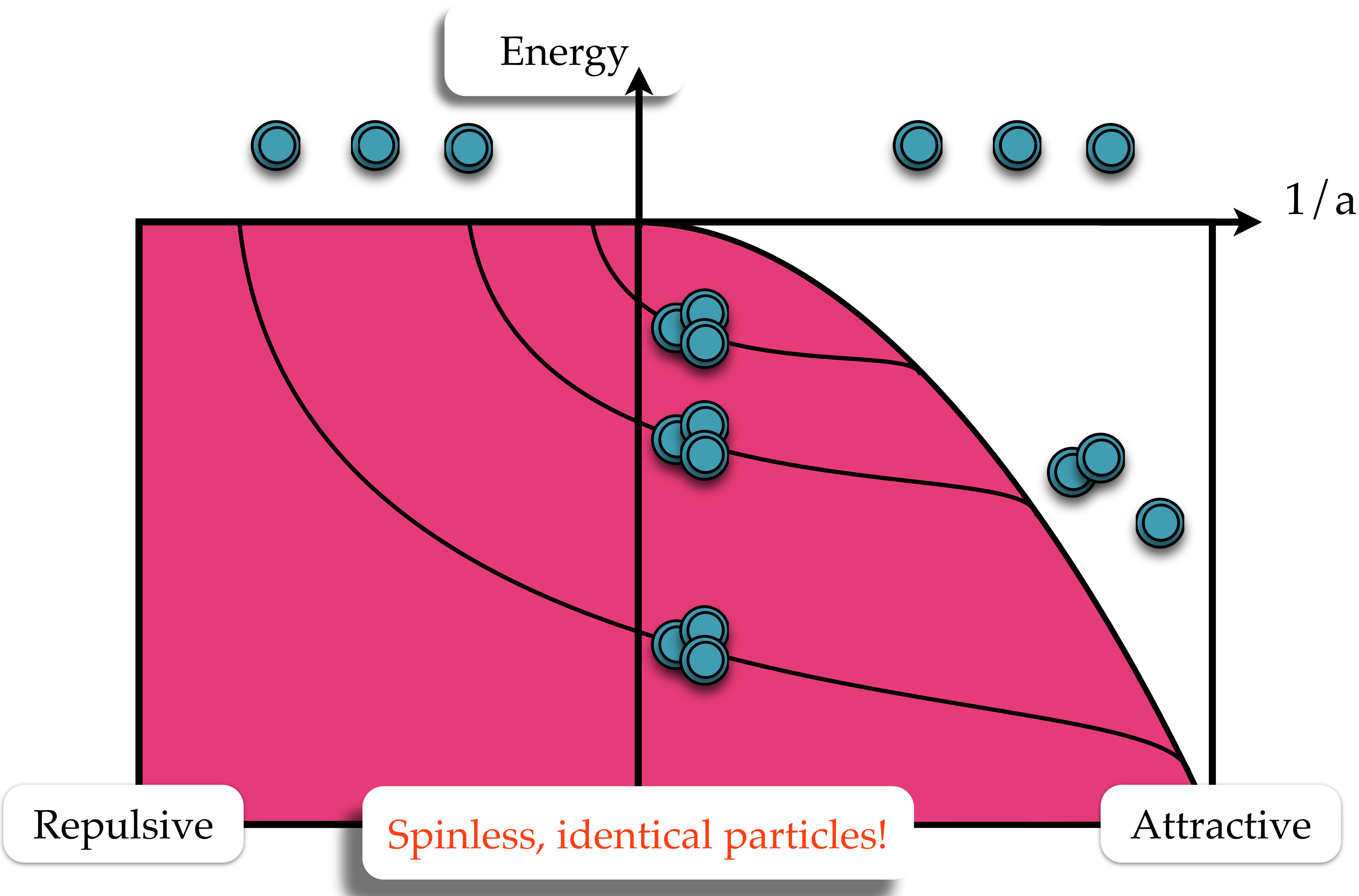
"Approximations suitable for infinite volume physics may lead to O(1) errors in a finite volume"

RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013)

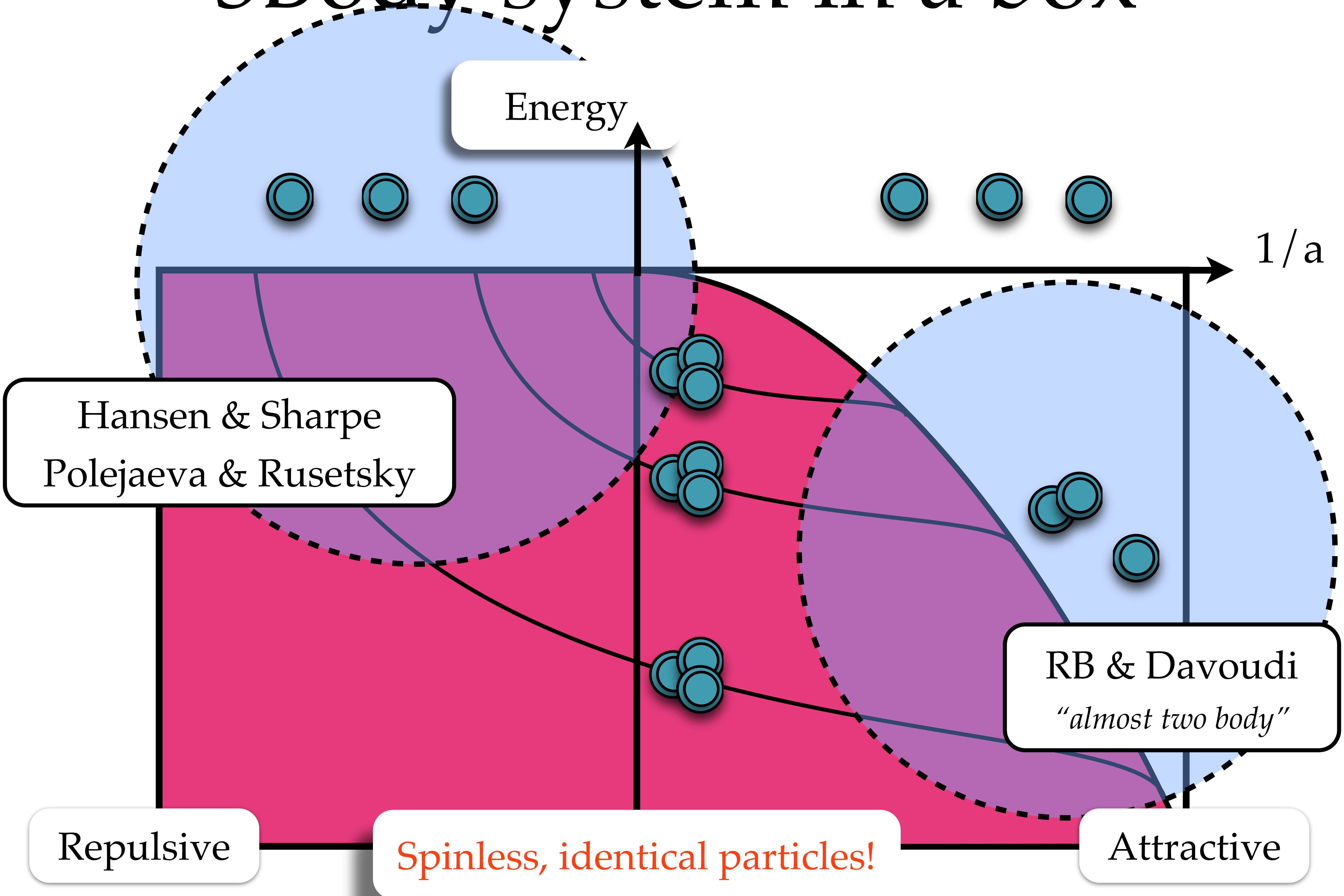
2Body system in a box



3Body system in a box



3Body system in a box



Status
(a very bias estimate)

Formalism, systematics, ...

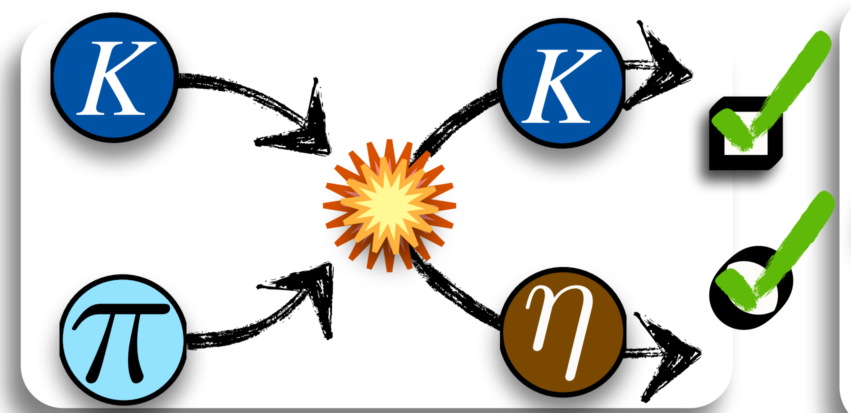
Code development, LQCD calculations

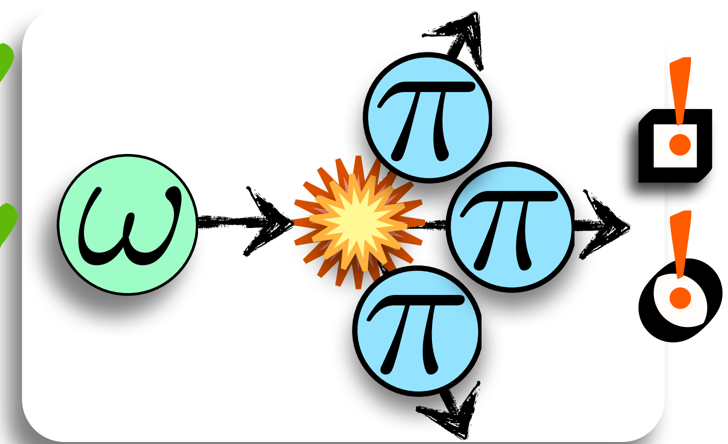
: Under control

: progress made

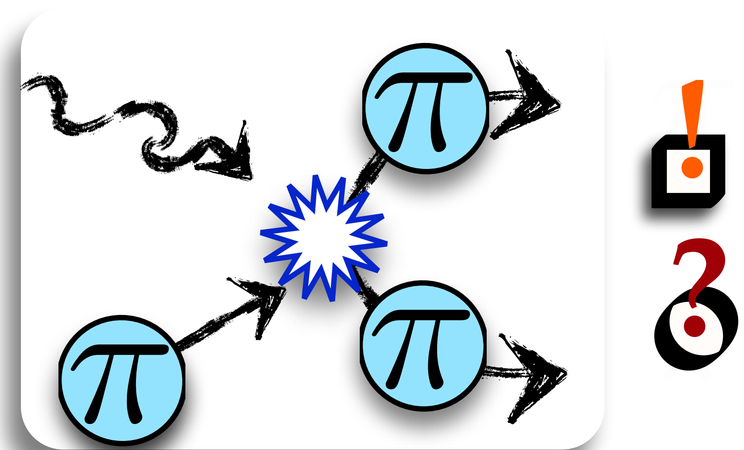
: open problem

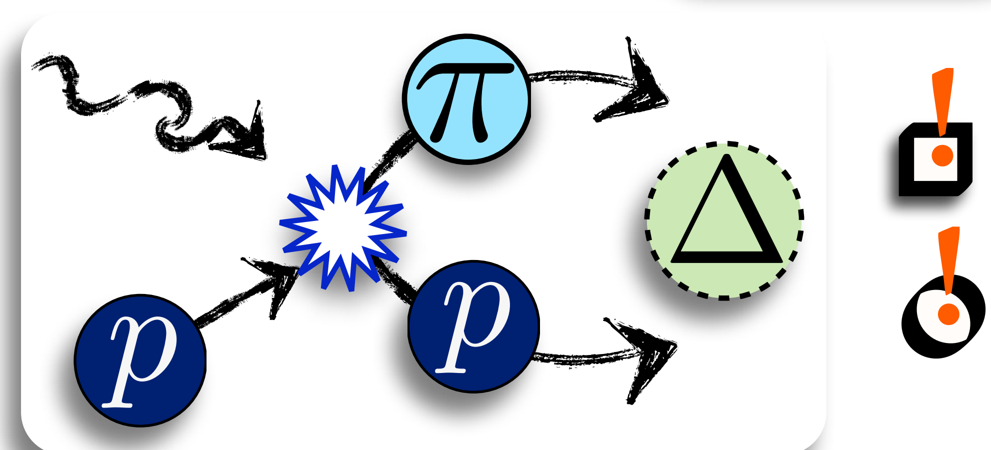
- Spectroscopy / scattering:



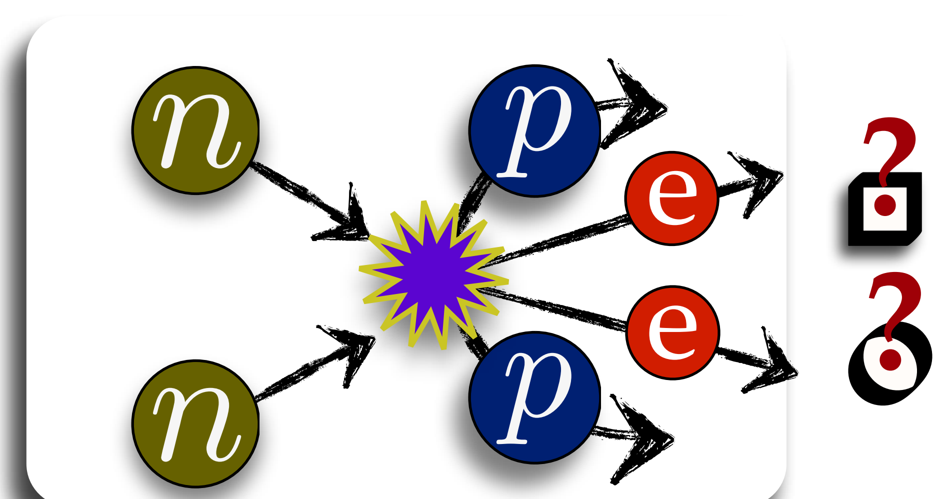


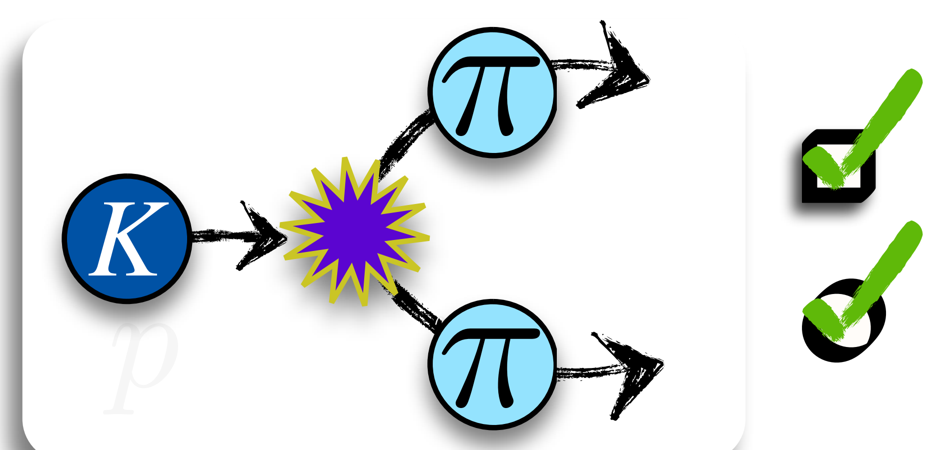
- Form factors:





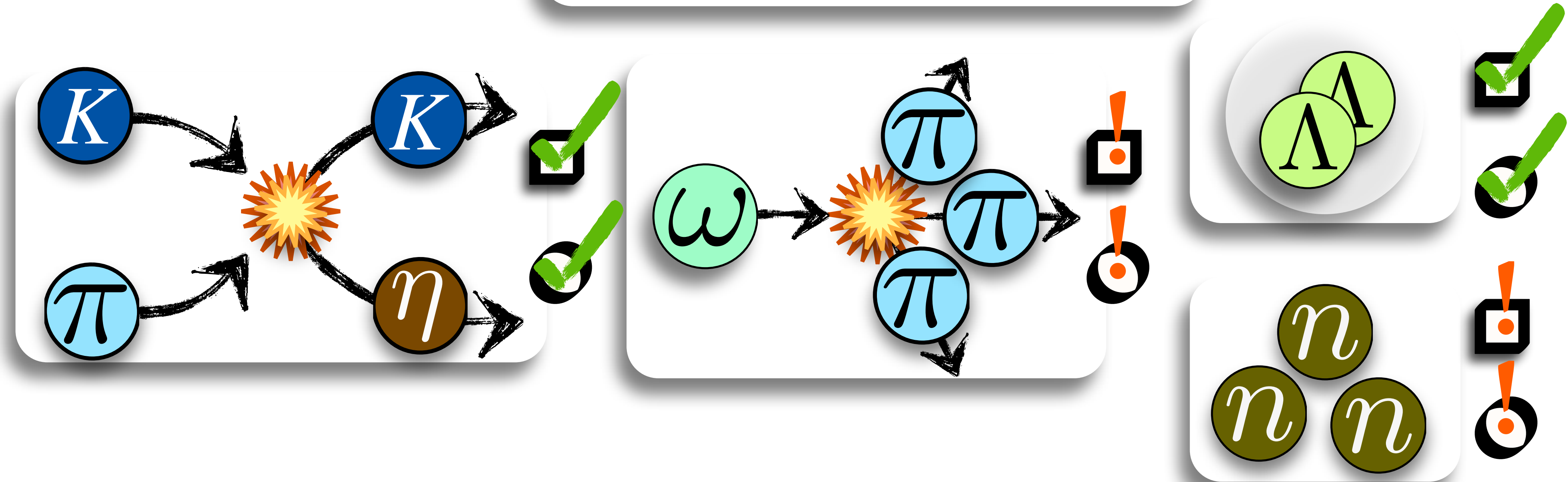
- Fundamental symmetries:





5 year outlook:

- Spectroscopy / scattering:



- Form factors:

- Fundamental symmetries: