nucleon-α scattering in EFT

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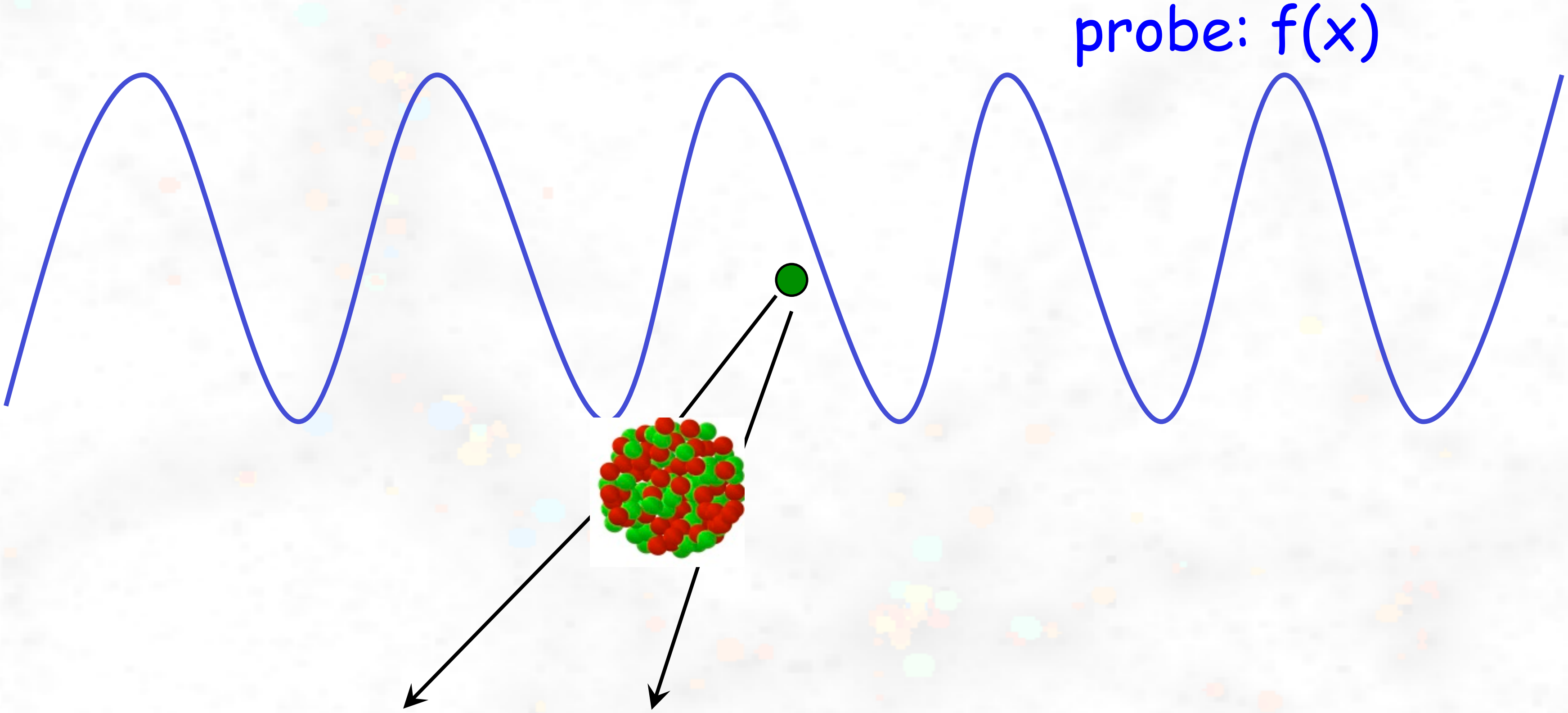
Effective Theories



$$f(x) = f(0) + \mathbf{p} \cdot \left(\nabla f\right)\Big|_0 + \mathbf{Q} \cdot \left(\nabla^2 f\right)\Big|_0 + \cdots$$

monopole:

dipole:

quadrupole:

controlled precision

Low energy scattering

Scattering amplitude

$$f(\theta) = \sum_l (2l+1) P_l(\cos\theta)\, T_l(k)$$

Bethe, Peierls, 1935
Bethe, 1949

T-matrix

$$T_l(k) = \frac{1}{k} e^{i\delta_l(k)} \sin\delta_l(k)$$
$$= \frac{1}{k\cot\delta_l(k) - ik}$$

$$k^{2l+1}\cot\delta_l = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 - \frac{\wp_l}{4} k^4 + \cdots$$

Effective Range Expansion (ERE)

"monopole": (scattering length)

"dipole": (effective range)

"quadrupole": (shape parameter)

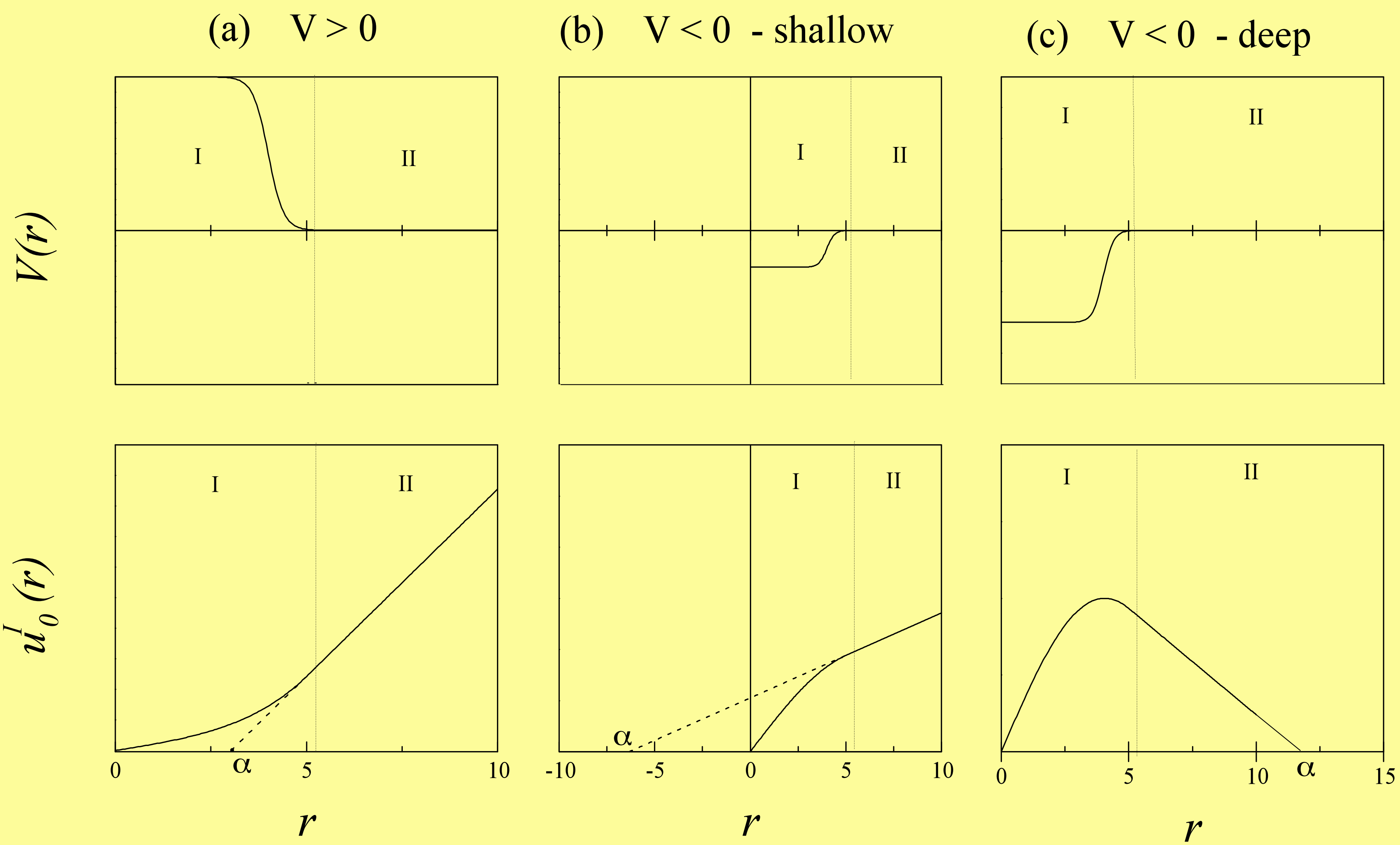
$$u(r) \sim k(r-a)$$

$$a \sim R\left(1 + \frac{1}{L(R)}\right) \qquad \text{sharp surface } (L = \text{logarith. der.})$$

$$\frac{1}{2} r_l \sim \int \left[u^2_{asymp}(r) - u^2(r)\right] dr$$

$$\vdots$$

Low energy scattering



Nucleon-Nucleon Scattering

$^1S_0$ nn – channel
$$a_0^{nn} = -18.8\,\mathrm{fm}$$
$$r_0^{nn} = 1.7\,\mathrm{fm}$$

$^1S_0$ np – channel
$$a_0^{np}(I=1) = -23.7\,\mathrm{fm}$$
$$r_0^{np}(I=1) = 2.73\,\mathrm{fm}$$

$^3S_1$ np – channel
$$a_0^{nn}(I=0) = 5.74\,\mathrm{fm}$$
$$r_0^{nn}(I=0) = 1.73\,\mathrm{fm}$$

particularly large scattering lengths

$a \gg r_{NN} \sim 1\,\mathrm{fm}$

(unnatural)

$a > 0$ existence of a bound state (deuteron)

$$u_0(r) \sim e^{-ikr} - S_0(k)e^{ikr}$$
$$S_0 \sim \frac{i}{-1/a + r_0k^2/2 - ik}$$

pole on imaginary axis ($k = i\kappa$)

$$-1/a - r_0\kappa^2/2 + \kappa = 0$$

(deuteron)

$$\kappa = -0.2137\,\mathrm{fm}^{-1}; \qquad E_B = \frac{\hbar^2\kappa^2}{m_N} = 2.23\,\mathrm{MeV}$$

Coulomb Interaction

Bethe, 1949
Jackson, Blatt, 1950

$$e^{-ikr},\ \ e^{ikr} \ \rightarrow F(kr), G(kr)$$

$$F \sim C_\eta\left[1 - r/a_B + \cdots\right]$$

$$G \sim \left(1/C_\eta\right)\left[1/kr + 2\eta\left(h_\eta + 2\gamma - 1 + \ln 2r/a_B\right) + \cdots\right]$$

match logarithimic derivative:

$$k\cot\delta\, C_\eta^2 + \frac{2}{a_B}\left(h_\eta - \ln\frac{a_B}{2R} + 2\gamma - 1\right)$$

$$\sim -\frac{1}{R}\left(1 + \frac{1}{L(R)}\right) = -\frac{1}{a_S}$$

$$\gamma = 0.577215\ldots,\quad a_B = 1/m\alpha,\quad \eta = 1/ka_B$$

$$C_\eta^2 = 2\pi\eta/\left(e^{2\pi\eta} - 1\right),\qquad h_\eta = \operatorname{Re} H(i\eta)$$

$$H(x) = \psi(x) + 1/2x - \ln x$$

definition of pp-scatttering length, a_C:

$$k\cot\delta\, C_\eta^2 + \frac{2}{a_B}h_\eta = -\frac{1}{a_C} + \cdots \longrightarrow -\frac{1}{a_S} = -\frac{1}{a_C} - \frac{2}{a_B}\left(\ln\frac{a_B}{2R} + 1 - 2\gamma\right)$$

$$a_C = a_0^{pp} = -7.82\,\mathrm{fm}$$

$$r_0^{pp} = 2.83\,\mathrm{fm}$$

$$a_S = -17\,\mathrm{fm} \sim a_0^{nn}$$

small difference from a_0^{nn} = -18.8 fm due to $m_n \neq m_p$

S-wave scattering – pionless EFT (spin-isospin indices not shown)

Invariance: (a) parity, (b) Galilean, (c) time reversal, (d) particle number

$$\mathbf{L}_{EFT} \sim N^{+}\left(i\partial_t + \frac{\nabla^2}{2m_N}\right)N + \left(\frac{\mu}{2}\right)^{4-D}\left\{-C_0\left(N^{+}N\right)^2 + \frac{C_2}{8}\left[N^{+}N\left(N^{+}\overleftrightarrow{\nabla}^2 N\right) + h.c. + \cdots\right]\right\}$$

π-less EFT

$\delta(r) + \text{higher derivatives of } \delta(r)$

$N^T = (p\ \ n) = \text{isopin doublet}; \quad \overleftrightarrow{\nabla}^2 = \overleftarrow{\nabla}^2 - 2\overleftarrow{\nabla}\cdot\vec{\nabla} + \vec{\nabla}^2$

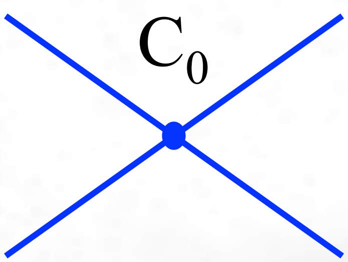
$\frac{\mu}{2} = \text{arbitrary mass to make } C_{2n}\nabla^{2n} \text{ same dimension for any D}$

Weinberg, 1991

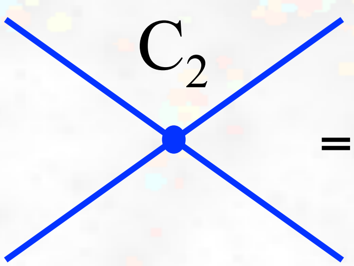
Short-range physics (quarks, gluons) encoded in C_0, C_2, …

Feynman rules:

$$iS_N = \frac{i}{q_0 - \mathbf{q}^2/2m + i\varepsilon}$$



$= -i(\mu/2)^{4-D}C_0$

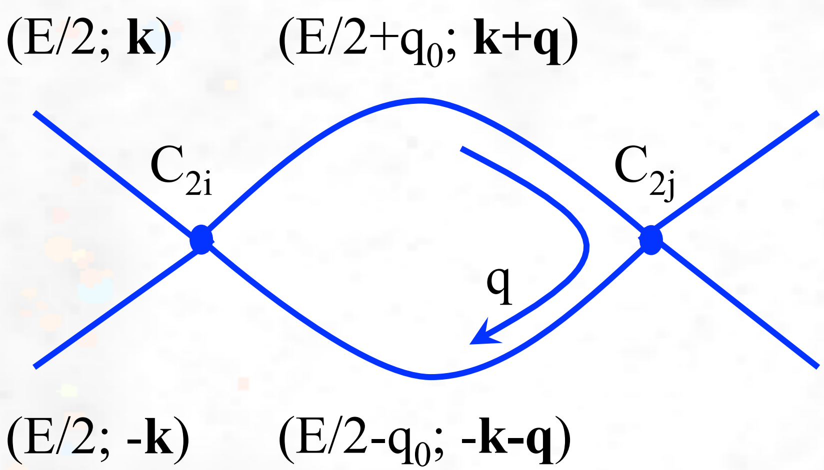


$= -i(\mu/2)^{4-D}C_2k^2$

$$iT_{tree} = -i(\mu/2)^{4-D}C_0 - i(\mu/2)^{4-D}C_2k^2 + \cdots$$

$$= -i(\mu/2)^{4-D}\sum_{n=0}^{\infty} C_{2n}k^{2n}$$

Loops

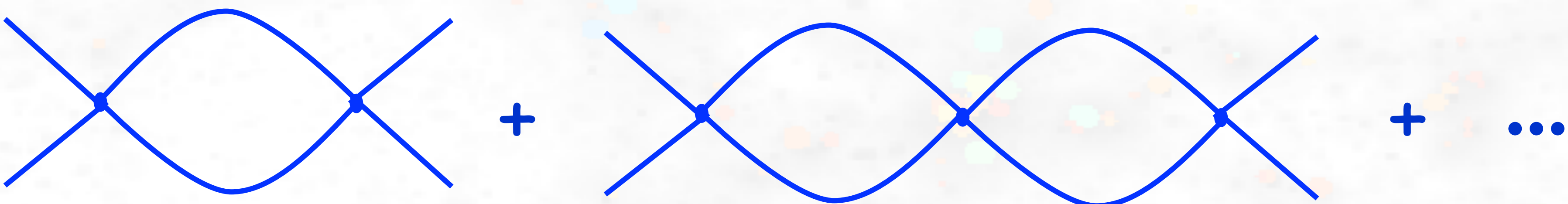


$$= -i(\mu/2)^{4-D} C_{2i} C_{2j} I_{i+j}$$

residues + minimal subtraction scheme*

$$I_n = (\mu/2)^{4-D} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{\mathbf{q}^{2n}}{E - \mathbf{q}^2/m_N + i\varepsilon}; \qquad m_N E = k^2$$

$$I_n^{MS} = i\left(\frac{m_N}{4\pi}\right) k^{2n+1}$$



$$T = T_{tree} + T_{loops}$$

$$= -\left(\sum_l C_{2l} k^{2l}\right)\left\{1 + \sum_m \left(-i\frac{m_N}{4\pi}\right) C_{2m} k^{2m} + \sum_{m,n} \left(-i\frac{m_N}{4\pi}\right)^2 C_{2m} k^{2m} C_{2n} k^{2n} + \cdots\right\} = \frac{-\sum_n C_{2n} k^{2n}}{1 + i\frac{m_N}{4\pi} k \sum_n C_{2n} k^{2n}}$$

---

* MS = subtract any 1/(D - 4) pole before taking the D → 4 limit.

Key points

$$\exp(i\theta) = \cos\theta + i\sin\theta$$

"mathematical jewel for physicists"
(Feynman Lectures of Physics)

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

"mathematical jewel for quantum field theorists"

Power counting

$M_{hi} \sim m_\pi$

$$C_{2n} \sim \frac{4\pi}{m_N} \frac{1}{M_{hi}^{2n+1}}$$

higher derivative contact terms suppressed

naturalness: physical parameters with dimension $(\text{mass})^d$ scale as $(M_{hi})^d$.

$$C_0 I_0 \sim C_0 \frac{m_N}{4\pi} k \sim \frac{k}{M_{hi}}$$

loops also suppressed

$\Rightarrow$ **EFT series perturbative: can be organized in powers of k/M_{hi}**

Matching to ERE

$$T_{EFT} = -C_0\left\{1 - i\frac{m_N}{4\pi}C_0 k - \left[\left(\frac{m_N}{4\pi}\right)^2 C_0 - \frac{C_2}{C_0}\right]k^2 + \cdots\right\}$$

$$T_{ERE} = -\frac{4\pi}{m_N}a\left[1 - iak - \left(a^2 - \frac{ar_0}{2}\right)k^2 + \cdots\right]$$

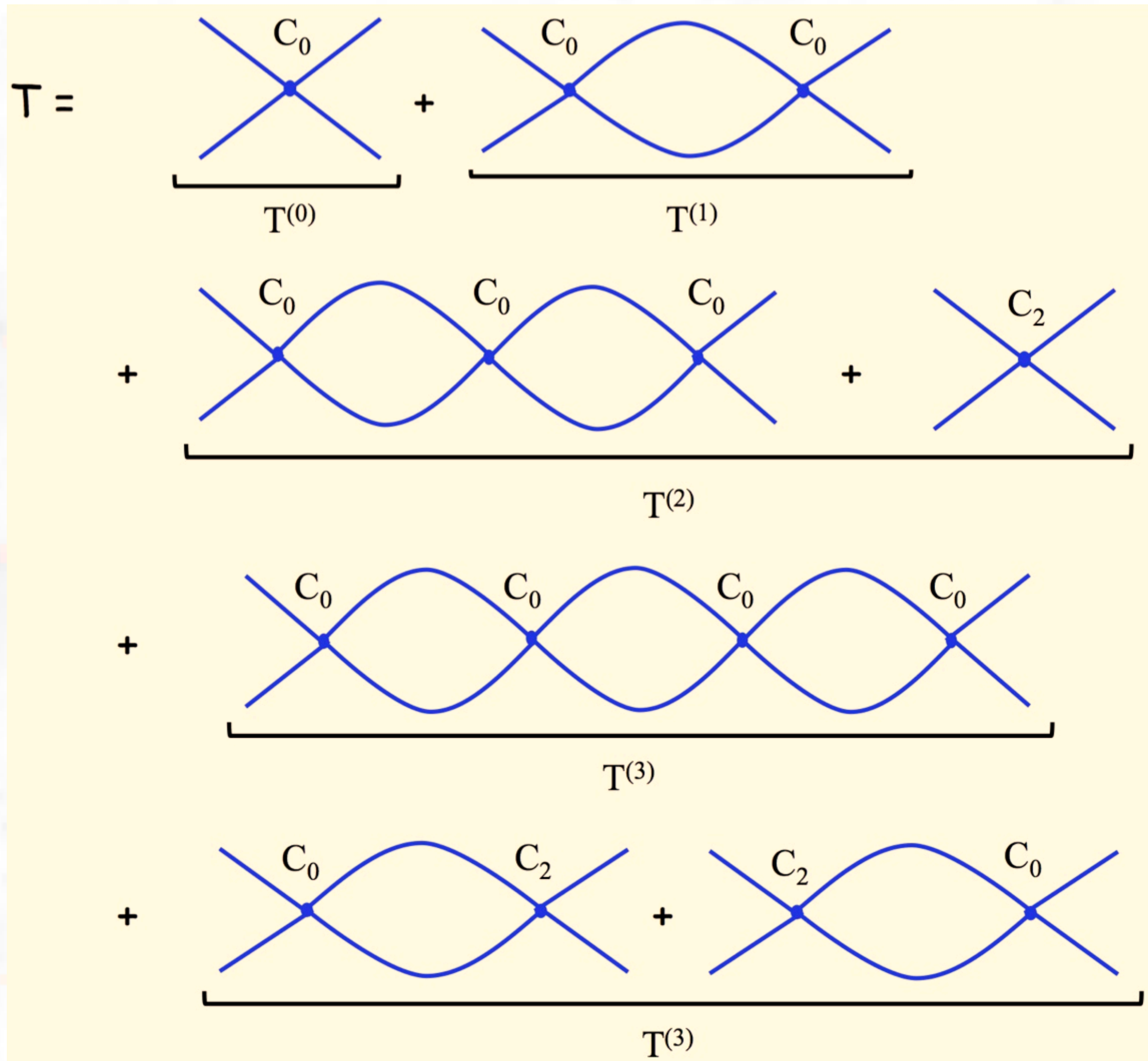
$$\Rightarrow \quad C_0 \sim \frac{4\pi}{m_N}a$$

$$C_2 \sim C_0\frac{ar_0}{2}$$

valid for $a, r_n \sim \dfrac{1}{M_{hi}}$

(natural case)

in general $C_{2n} \sim \dfrac{4\pi}{m_N M_{hi}}\dfrac{1}{M_{hi}^{2n}}$



Unnatural case (large a, shallow bound states)

van Kolck, 1997
Gegelia, 1998
Kaplan, Savage, Wise, 1998

deuteron, halo nuclei

T expansion in terms of **ka** fails for **k ~ 1/a**

→ use ERE keeping all orders in ka :

$$T_{\text{eff.range}} = -\frac{4\pi}{m_N}\frac{1}{1/a+ik}\left[1+\frac{r_0/2}{1/a+ik}k^2+\frac{(r_0/2)^2}{(1/a+ik)^2}k^4+\cdots\right]$$

To match with above EFT expansion scale as (**k**$^{-1}$, **k**^{0}, **k**^{1}, …)

1 – Expansion should be:

$$T=\sum_{n=-1}^{\infty}T_n\,;\qquad T_n\sim O(k^n)$$

2 – Use PDS regularization* scheme (Kaplan, Savage, Wise):

- subtract not only 1/(D-4) poles corresponding to log divergences but also poles of lower dimension **D** (e.g., I_n has a pole at **D = 3**) by adding a counterterm:

$$\delta I_n=-\frac{m_N(m_N E)^n\mu}{4\pi(D-3)}\quad\Rightarrow\quad I_n^{PDS}=I_n+\delta I_n=-k^2\left(\frac{m_N}{4\pi}\right)(\mu+ik)$$

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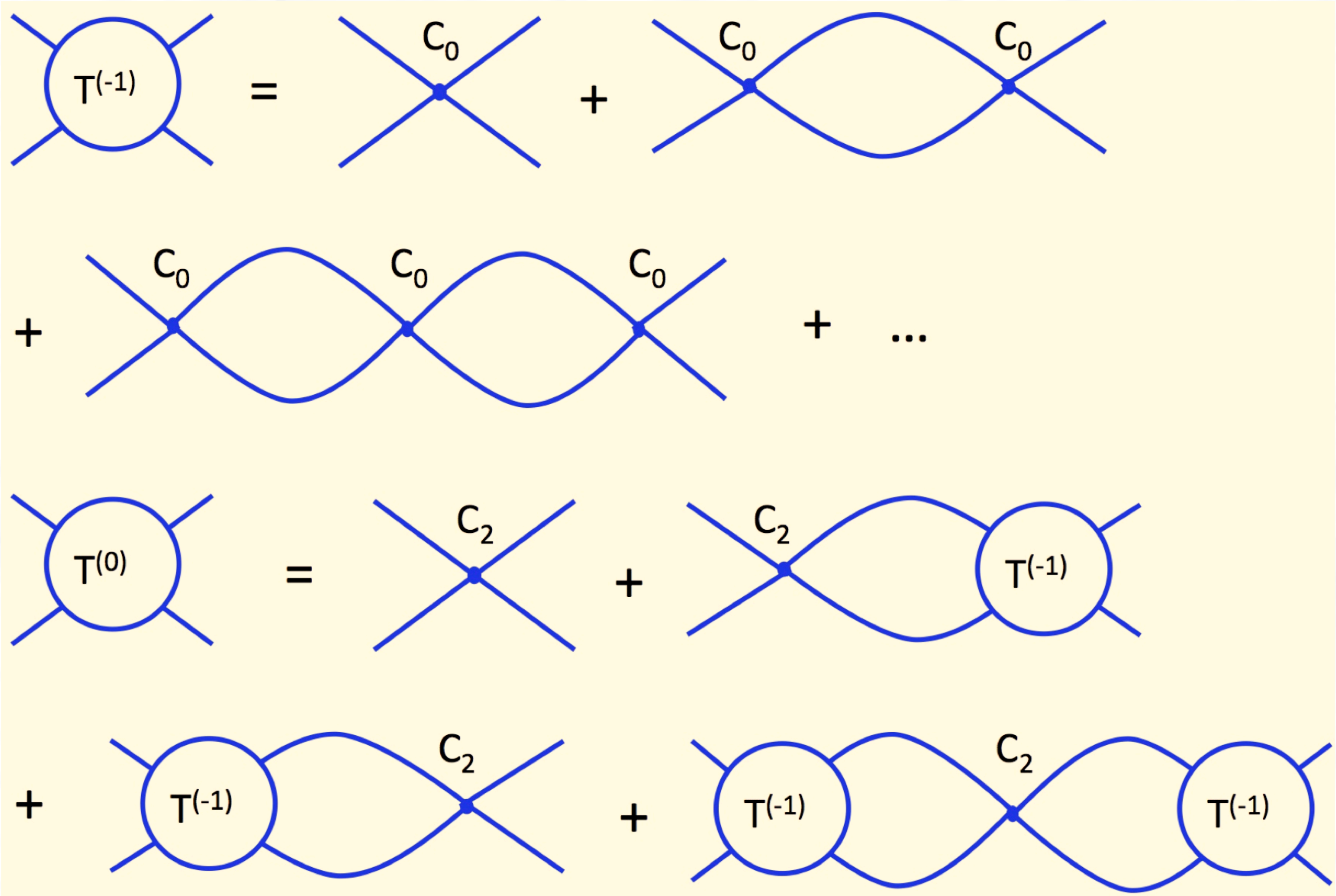
* PDS = Power Divergence Subtraction.

Unnatural case (leading and subleading terms)



Using subtraction scheme →

$$T_{-1} = -C_0\left[1+\frac{C_0 m_N}{4\pi}(\mu+ik)\right]^{-1}$$

$$T_0 = -C_2 k^2\left[1+\frac{C_0 m_N}{4\pi}(\mu+ik)\right]^{2}$$

Unnatural case (matching to ERE)

$$C_0(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right); \qquad C_2(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right)^2 \frac{r_0}{2}; \qquad \cdots$$

$$T_1 = -\frac{\left(C_2 k^2\right)^2 m_N(\mu + ik)/4\pi}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^3} - \frac{C_4 k^4}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^4}; \qquad \cdots\cdots$$

power counting: $$C_{2n} \sim \frac{4\pi}{m_N}\frac{1}{M_{hi}^n \mu^{n+1}}$$

T-matrix for physics at **k ~ 1/a** scale: has a pole in **k = i**κ corresponding to real or virtual bound states κ **~ i/a + higher order corrections**

T_{EFT} should not depend on **μ** → **renormalization group equations**

e.g. $$\mu \frac{d}{d\mu}\left(\frac{1}{T}\right) = 0$$

$$\mu \frac{d}{d\mu} C_{2n} = \frac{m_N}{4\pi} \sum_{m=0}^{n} C_{2m} C_{2(n-m)}$$

with the boundary condition that $C_0(0) = 4\pi a/m_N$

Unnatural case (matching to ERE)

$$C_0(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right); \qquad C_2(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right)^2 \frac{r_0}{2}; \qquad \cdots$$

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$$T_1 = -\frac{\left(C_2 k^2\right)^2 m_N(\mu + ik)/4\pi}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^3} - \frac{C_4 k^4}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^4}; \qquad \ldots\ldots$$

power counting:

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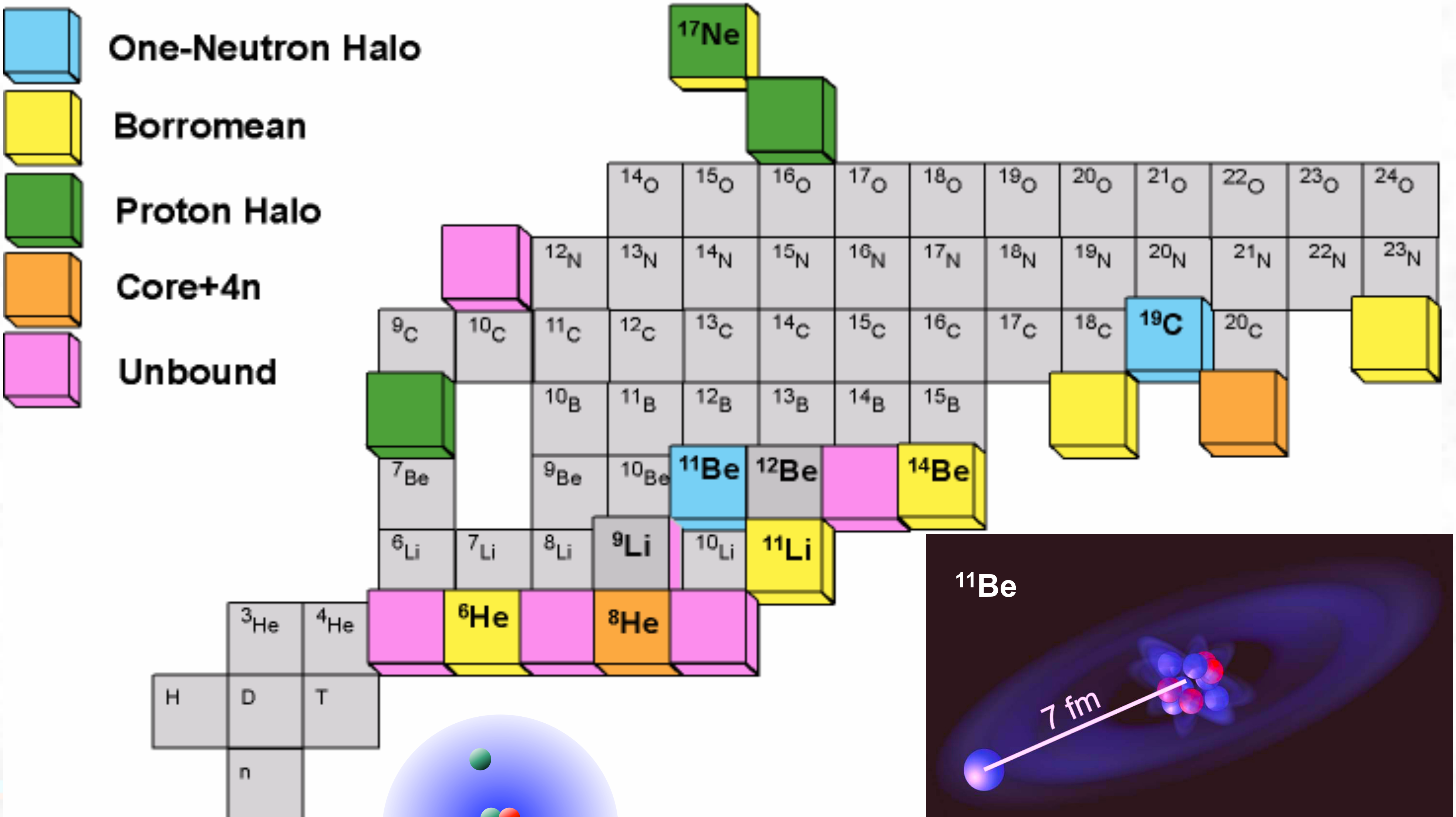
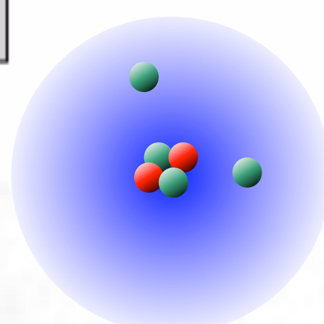
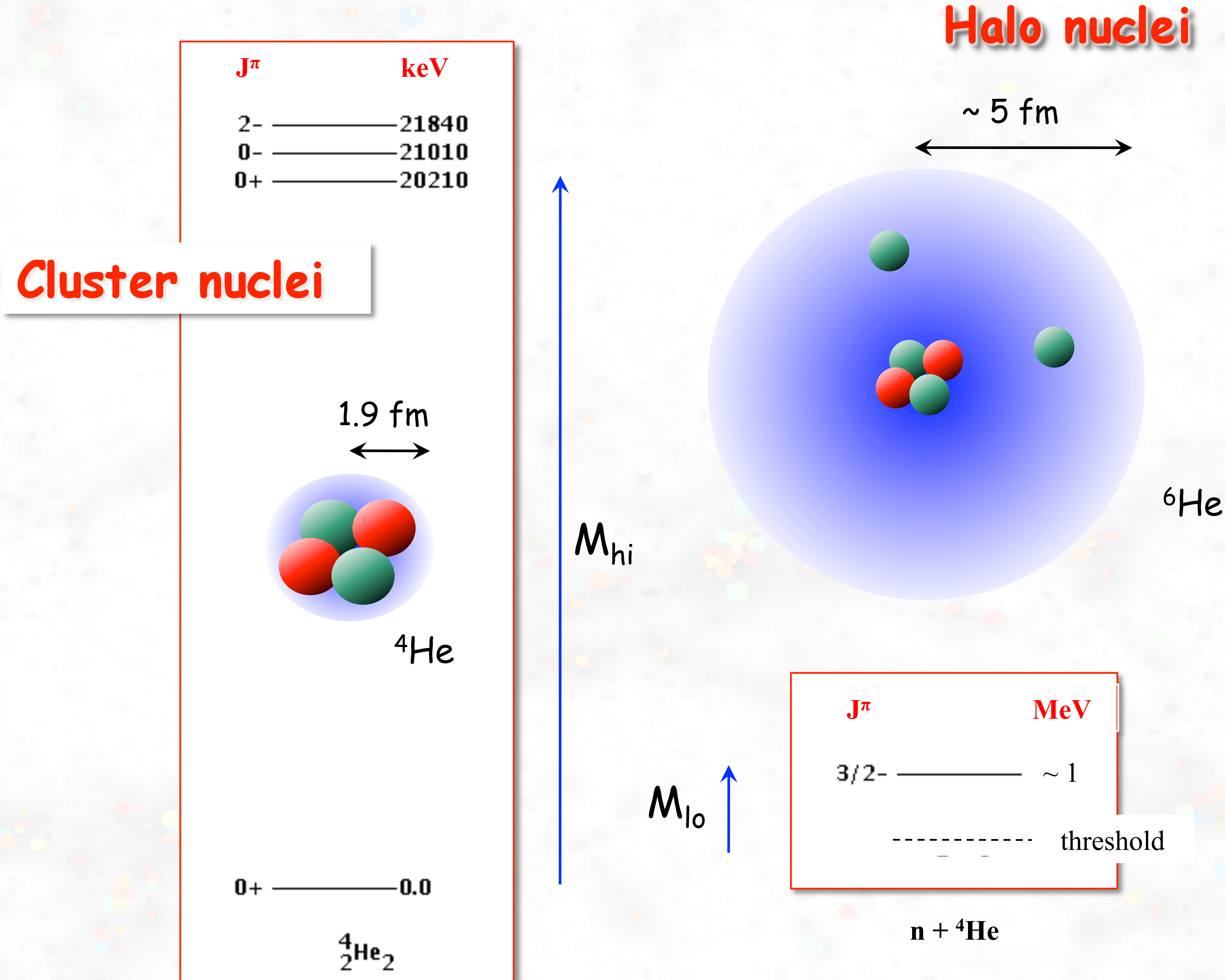
Halo nuclei


One-Neutron Halo
Borromean
Proton Halo
Core+4n
Unbound
17Ne
14O 15O 16O 17O 18O 19O 20O 21O 22O 23O 24O
12N 13N 14N 15N 16N 17N 18N 19N 20N 21N 22N 23N
9C 10C 11C 12C 13C 14C 15C 16C 17C 18C 19C 20C
10B 11B 12B 13B 14B 15B
7Be 9Be 10Be 11Be 12Be 14Be
6Li 7Li 8Li 9Li 10Li 11Li
3He 4He 6He 8He
H D T
n
11Be
7 fm





Halo nuclei
Cluster nuclei
Jπ
keV
2- 21840
0- 21010
0+ 20210
1.9 fm
4He
0+ 0.0
4He2
Mhi
~ 5 fm
6He
Mlo
Jπ
MeV
3/2- ~ 1
threshold
n + 4He

Spinless n + ^{4}He

CB, Hammer, van Kolck, 2002

Goal: use EFT to reproduce ERE for p-wave

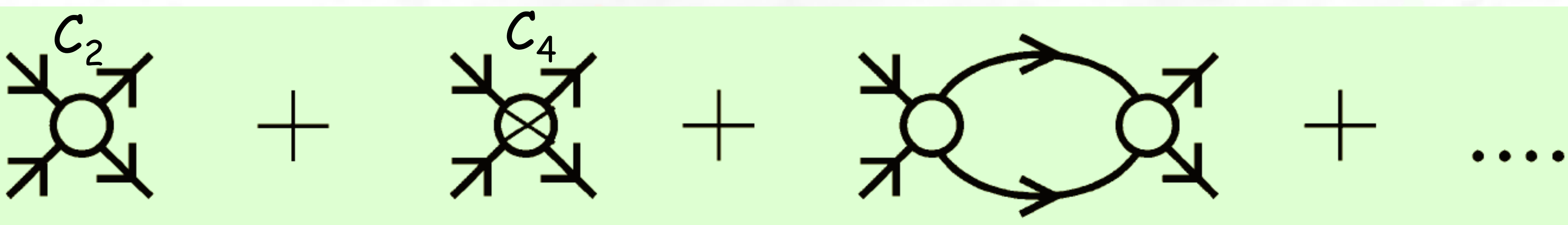
$$T_1(k,\cos\theta) = -\frac{12\pi a_1}{m} k^2 \cos\theta \left\{1 + \frac{a_1 r_1}{2} k^2 - i a_1 k^3 + \frac{a_1}{4}\left(a_1 r_1^2 - \wp_1\right) k^4 + \cdots\right\}$$

Natural case, assuming spinless particles – most general p-wave interaction:

$$L_{EFT} \sim N^+\left(i\partial_0 + \frac{\nabla^2}{2m}\right)N + \frac{C_2^p}{8}\left(N\overleftrightarrow{\nabla}N\right)^+\left(N\overleftrightarrow{\nabla}N\right) - \frac{C_4^p}{64}\left[\left(N\overleftrightarrow{\nabla}^2\overleftrightarrow{\nabla}_i N\right)^+\left(N\overleftrightarrow{\nabla}_i N\right) + h.c.\right] + \cdots$$

$\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ Galilean derivative

$\Rightarrow$ $$iS(p_0,\mathbf{p}) = \frac{i}{p_0 - \mathbf{p}^2/2m + i\varepsilon}$$ propagator

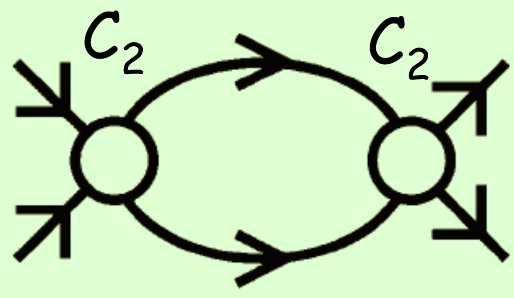


(a) (b) (c)

$$iT_{1(a)} = -iC_2^p\,\mathbf{k}\cdot\mathbf{k}' \qquad iT_{1(b)} = -iC_4^p\,k^2\,\mathbf{k}\cdot\mathbf{k}'$$

Spinless n + ^{4}He

Natural case:



$$iT_{1(c)} = \left(-iC_2^p\right)^2 \int \frac{d^4q}{(2\pi)^4} \frac{i\mathbf{q}\cdot\mathbf{k}}{E/2 - q_0 - \mathbf{q}^2/2m + i\varepsilon} \frac{i\mathbf{q}\cdot\mathbf{k}'}{E/2 - q_0 - \mathbf{q}^2/2m + i\varepsilon}$$

$$= \left(C_2^p\right)^2 imk'_i k_j \int \frac{d^3q}{(2\pi)^3} \frac{q_i q_j}{q^2 - k^2 + i\varepsilon}$$

$$\Rightarrow \quad iT_{1(c)} = \left(C_2^p\right)^2 \frac{im}{6\pi^2} \mathbf{k}\cdot\mathbf{k}' \left[L_3 + k^2 L_1 + \frac{\pi}{2} ik^3\right]$$

L_1 and L_3 are (ultraviolet) infinities that can be absorbed in C_2 and C_4

Matching to ERE $\rightarrow$

$$T_{1(a)} \quad \rightarrow \quad C_2^p = 12\pi \frac{a_1}{m}$$

$$T_{1(b)} \quad \rightarrow \quad C_4^p = C_2^p r_1 \frac{a_1}{2}$$

\+ $\quad T_{1(c)}$ reproduces 3rd term of ERE expansion

Spinless n + ^{4}He

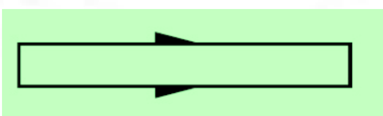
Unnatural case (shallow state) – p waves: with C^p_2 and C^p_4 to all orders

Kaplan, 1997 → introduce auxiliary field **d** (**dimeron**) which reproduces same physics as original EFT Lagrangian

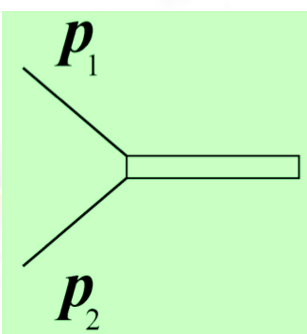
$$L_{EFT} \sim N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m}\right)N + \eta_1 d_i^{+}\left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta_1\right)d_i + \frac{g_1}{4}\left[d_i^{+}\left(N\overleftrightarrow{\nabla}N\right) + h.c.\right] + \cdots$$

1- parameters $\eta_1 = \pm 1$, g_1 and Δ_1 fixed from matching

2- advantage: get quicker to the answer, appropriate for large a's



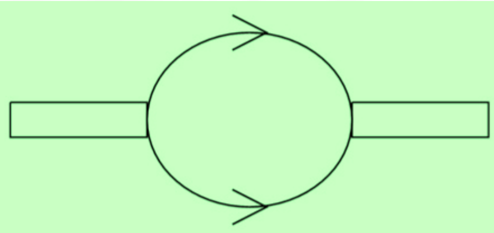
dimeron propagator: $$iD_1^0(p_0,\mathbf{p})_{ij} = \frac{i\eta_1\delta_{ij}}{p_0 - \mathbf{p}^2/4m - \Delta_1 + i\varepsilon}$$



Feynman rules:

nucleon-dimeron vertex: $$V_{N-d} = \frac{g_1}{4}\left(\mathbf{p}_1 - \mathbf{p}_2\right)$$

Spinless n + ^{4}He

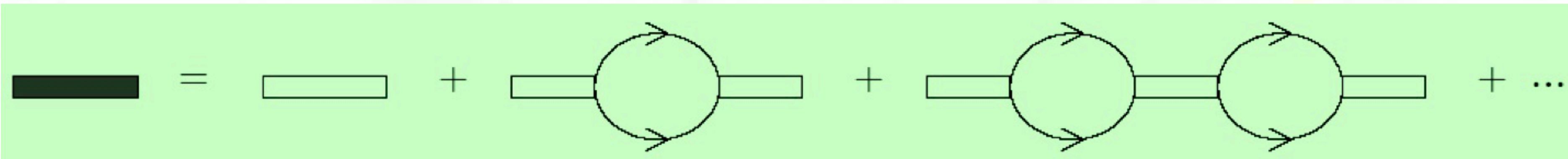


$$-i\Sigma_1 = i\delta_{ij}\frac{mg_1^2}{12\pi}\left[\frac{2}{\pi}L_3 + \frac{2}{\pi}L_1\left(mp_0 - \mathbf{k}^2/4\right) + i\left(mp_0 - \mathbf{k}^2/4\right)^{3/2}\right]$$

self-energy Σ_1

infinite constants absorbed into g_1 and Δ_1

full dimeron propagator:



$$iD_1^0 = iD_1^0 + iD_1^0\left(-\Sigma_1\right)iD_1^0 + \cdots = \frac{iD_1^0}{1-\Sigma_1 D_1^0}$$

attach external legs to full dimeron propagator →

$$T_{\mathrm{EFT}}^{(\mathrm{p-wave})} = \frac{12\pi}{m}k^2\left(\eta_1\frac{12\pi\Delta_1^R}{m\left(g_1^R\right)^2} - \eta_1\frac{12\pi\Delta_1^R}{m\left(g_1^R\right)^2}k^2 - ik\right)$$

With renormalized parameters g^R, Δ^R

Now match to

$$T_{\mathrm{EFE}}^{(\mathrm{p-wave})} = \frac{12\pi}{m}k^2\left(-\frac{1}{a_1} + \frac{r_1}{2}k^2 - ik^3\right)^{-1}$$

to get η_1^R, g_1^R and Δ_1^R

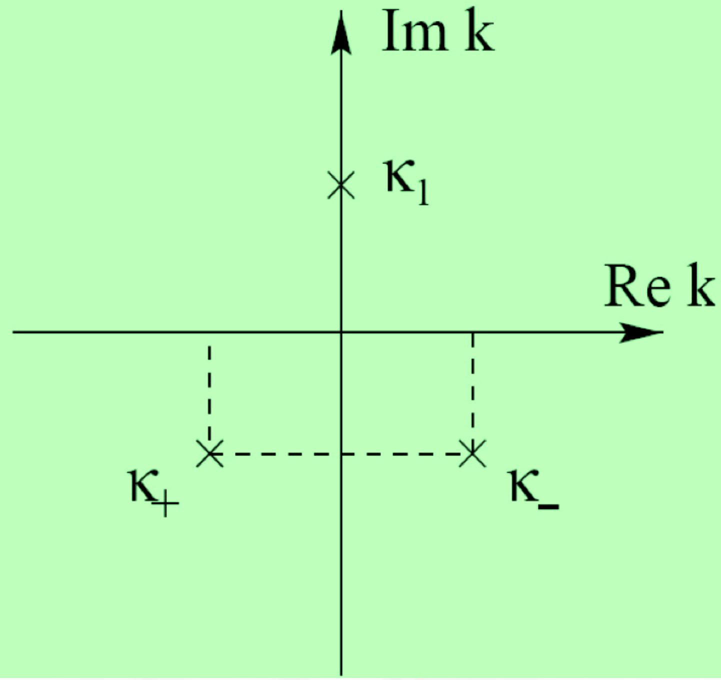
Spinless n + ^{4}He

Pole structure

for $a_1, r_1 < 0$ (e.g., n + ^{4}He)

$$-\frac{1}{a_1} - \frac{r_1\kappa^2}{2} - i\kappa^3 = 0; \quad \Rightarrow \quad \kappa_1 = i\gamma_1; \quad \kappa_\pm = i(\gamma \pm i\tilde{\gamma})$$

bound-state resonance



$$\delta_1 = \frac{1}{2}\arctan\left(\frac{2\sqrt{EB}}{E-B}\right) - \arctan\left(\frac{\Gamma(E)}{2(E-E_0)}\right);$$

$$E = \frac{k^2}{2m}; \quad E_0 = \frac{\gamma^2 + \tilde{\gamma}^2}{2m}; \quad \Gamma(E) = -4\gamma\sqrt{\frac{E}{2m}}; \quad B = \frac{\gamma_1^2}{2m}$$

n (with spin) + ^{4}He

CB, Hammer, van Kolck, 2002

Partial wave $l_\pm$	$a_{l\pm}$ [fm^{1+2l}]	$r_{l\pm}$ [fm^{1-2l}]	$\mathcal{P}_{l\pm}$ [fm^{3-2l}]
0^+	2.4641(37)	1.385(41)	--
1^-	-13.821(68)	-0.419(16)	--
1^+	-62.951(3)	-0.8819(11)	-3.002(62)

Arndt, Roper, 1973

n + ^{4}He: $p_{3/2}$ resonance
- shallow: ~ 1 MeV
- has to be treated non-perturbatively
- $p_{1/2}$ weak → perturbatively
- $s_{1/2}$ also perturbatively

neutron spin →

$$T = \frac{2\pi}{m_0}\left(F + i\vec{\sigma}\cdot\hat{n}G\right); \qquad \frac{d\sigma}{d\Omega} = |F(\theta)|^2 + |G(\theta)|^2$$

$$F(k,\theta) = \sum_{l\geq 0}\left[(l+1)f_{l+}(k) + l f_{l-}(k)\right]P_l(\cos\theta)$$

$$G(k,\theta) = \sum_{l\geq 1}\left[f_{l+}(k) - f_{l-}(k)\right]P_l^1(\cos\theta)$$

$$f_{l\pm} = \frac{1}{k\cot\delta_{l\pm} - ik}$$

$n + {}^4He$

parity- and time-reversal-invariant Lagrangians:

$$L_{LO} = \phi^+\left[i\partial_0 + \frac{\nabla^2}{2m_\alpha}\right]\phi + N^+\left[i\partial_0 + \frac{\nabla^2}{2m_N}\right]N + \eta_{1+}t^+\left[i\partial_0 + \frac{\nabla^2}{2(m_\alpha + m_N)} - \Delta_{1+}\right]t$$
$$+\frac{g_{1+}}{2}\left\{t^+\mathbf{S}^+ \bullet \left[N\nabla\phi - (\nabla N)\phi\right] + h.c. - r\left[t^+\mathbf{S}^+ \cdot \nabla(N\phi) + h.c.\right]\right\}$$

$$L_{NLO} = \eta_{0+}s^+\left[-\Delta_{0+}\right]s + +g_{0+}\left[s^+N\phi + \phi^+N^+s\right] + g'_{1+}t^+\left[i\partial_0 + \frac{\nabla^2}{2(m_\alpha + m_N)}\right]t$$

notation: **s, d, t** = $s_{1/2}, p_{1/2}, p_{3/2}$ ϕ = ${}^4He$ scalar field
S_i = 2 x 4 spin-transition matrices

$$S_i S_j^+ = \frac{2}{3}\delta_{ij} - \frac{i}{3}\varepsilon_{ijk}\sigma_k, \qquad S_i^+ S_j = \frac{3}{4}\delta_{ij} - \frac{1}{6}\left\{J_i^{3/2}, J_j^{3/2}\right\} + \frac{i}{3}\varepsilon_{ijk}J_k^{3/2}$$

$$\left[J_i^{3/2}, J_j^{3/2}\right] = i\varepsilon_{ijk}J_k^{3/2}$$

generators of the J = 3/2 representation of the rotation group

n + ^{4}He

α and nucleon propagators (,a,b = spin, α,β = isospin)

$$iS_\phi(p_0,\mathbf{p}) = \frac{1}{p_0 - \mathbf{p}^2/2m_\alpha + i\varepsilon}, \qquad iS_N(p_0,\mathbf{p})^{ab}_{\alpha\beta} = \frac{i\delta_{\alpha\beta}\delta_{ab}}{p_0 - \mathbf{p}^2/2m_N + i\varepsilon}$$

Dimeron propagators

$$iD^0_{1+}(p_0,\mathbf{p})^{ab}_{\alpha\beta} = \frac{i\eta_{1+}\delta_{\alpha\beta}\delta_{ab}}{p_0 - \mathbf{p}^2/2(m_N + m_\alpha) - \Delta_{1+} + i\varepsilon}, \qquad iD^0_{0+}(0,\mathbf{0})^{ab}_{\alpha\beta} = -\frac{i\eta_{0+}\delta_{\alpha\beta}\delta_{ab}}{\Delta_{0+}}$$

$$iD^0_{1-}(p_0,\mathbf{p})^{ab}_{\alpha\beta} = iD^0_{0+}\left(0^+ \to 1^-\right)^{ab}_{\alpha\beta}$$

- Leading contribution for k ~ M_{lo} is ~ $12\pi/mM_{lo}$ solely from the 1+ partial wave with the scattering-length and effective-range terms included to all orders.

- NLO order correction suppressed by M_{lo}/M_{hi} and fully perturbative.

- 1– partial wave still vanishes at NLO.

n + ^{4}He

1+ dimeron propagator, using $$iD = \frac{iD}{1-\Sigma_1 D}$$

+ attaching the external particle lines (m_0 = reduced mass)

$$iD^0_{1+}(p_0,\mathbf{p})^{ab}_{\alpha\beta} = i\eta_{1+}\delta_{\alpha\beta}\delta_{ab}\left(p_0 - \frac{p^2}{2(m_\alpha + m_N)} - \Delta_{1+} + \frac{\eta_{1+}\mu g^2_{1+}}{6\pi}(2m_0)^{3/2}\left[-p_0 + \frac{p^2}{2(m_\alpha + m_N)} - i\varepsilon\right]^{3/2} + i\varepsilon\right)^{-1}$$

$$\Rightarrow \quad T^{LO} = \frac{2\pi}{m_0}k^2(2\cos\theta + i\vec{\sigma}\cdot\hat{n})\sin\theta\left(\eta_{1+}\frac{6\pi\Delta_{1+}}{m_0 g^2_{1+}} - \eta_{1+}\frac{3\pi}{m_0^2 g^2_{1+}}k^2 - ik^3\right)^{-1}$$

Matching to ERE:

$$a_{1+} = -\eta_{1+}\frac{m_0 g^2_{1+}}{6\pi\Delta_{1+}}; \qquad r_{1+} = -\eta_{1+}\frac{6\pi}{m_0^2 g^2_{1+}}$$

$\Rightarrow$ g_{1+}, Δ_{1+}, sign η_{1+} in terms of a_{1+} and r_{1+}

$$F_{LO} = \frac{2k^2\cos\theta}{-1/a_{1+} + r_{1+}k^2/2 - ik^3}, \quad G_{LO} = \frac{k^2\cos\theta}{-1/a_{1+} + r_{1+}k^2/2 - ik^3}$$

n + ^{4}He

NLO contributions come from the shape parameter P_{1+} and the s-wave scattering length a_{0+}

$$T^{NLO} = \frac{\eta_{0+} g_{0+}^2}{2\pi\Delta_{0+}} + \frac{6\pi^2 g_{1+}'}{2m_0^4 g_{1+}^2} \frac{k^6\left(2\cos\theta + i\vec{\sigma}\cdot\hat{n}\right)\sin\theta}{\left(1/a_{1+} - r_{1+}k^2/2 - ik^3\right)^2}$$

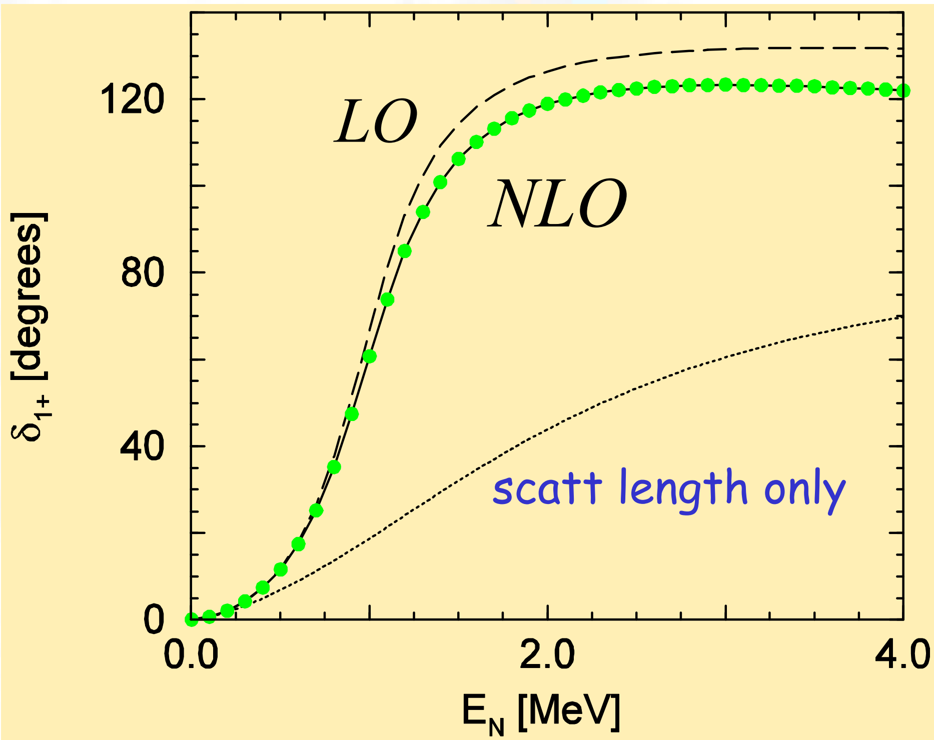
$$a_{0+} = -\eta_{0+}\frac{m_0 g_{0+}^2}{2\pi\Delta_{0+}}\,; \qquad \mathcal{P}_{1+} = \frac{6\pi g_{1+}'}{m_0^2 g_{1+}^2}$$

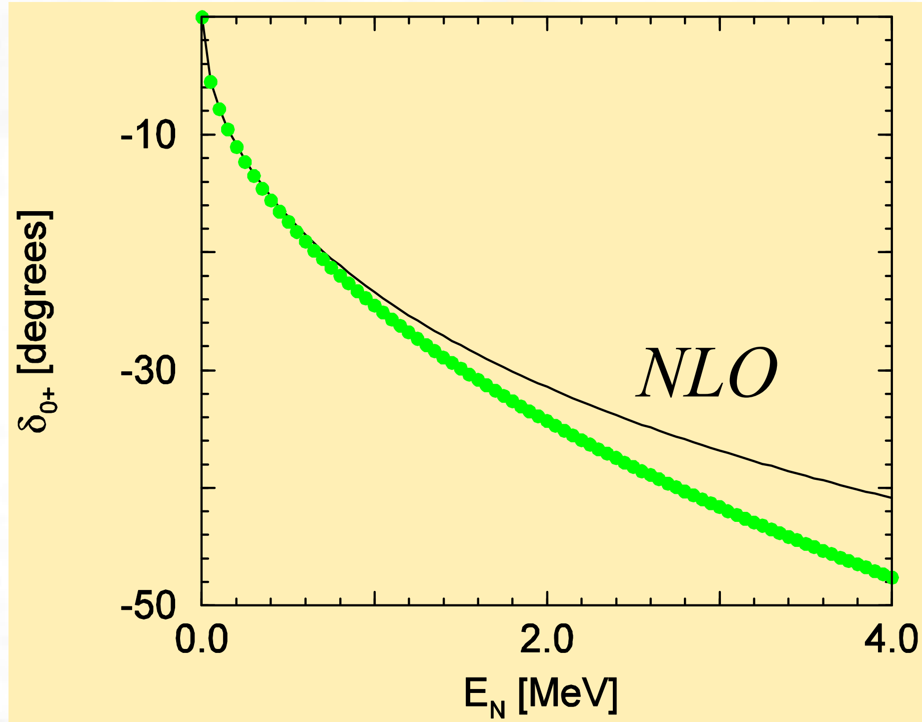
$$F_{NLO} = -a_{0+} + \frac{\mathcal{P}_{1+}}{4}\frac{2k^6\cos\theta}{\left(-1/a_{1+} + r_{1+}k^2/2 - ik^3\right)^2}$$

$$G_{NLO} = \frac{\mathcal{P}_{1+}}{4}\frac{k^6\cos\theta}{\left(-1/a_{1+} + r_{1+}k^2/2 - ik^3\right)^2}$$

n + ^{4}He phase shifts

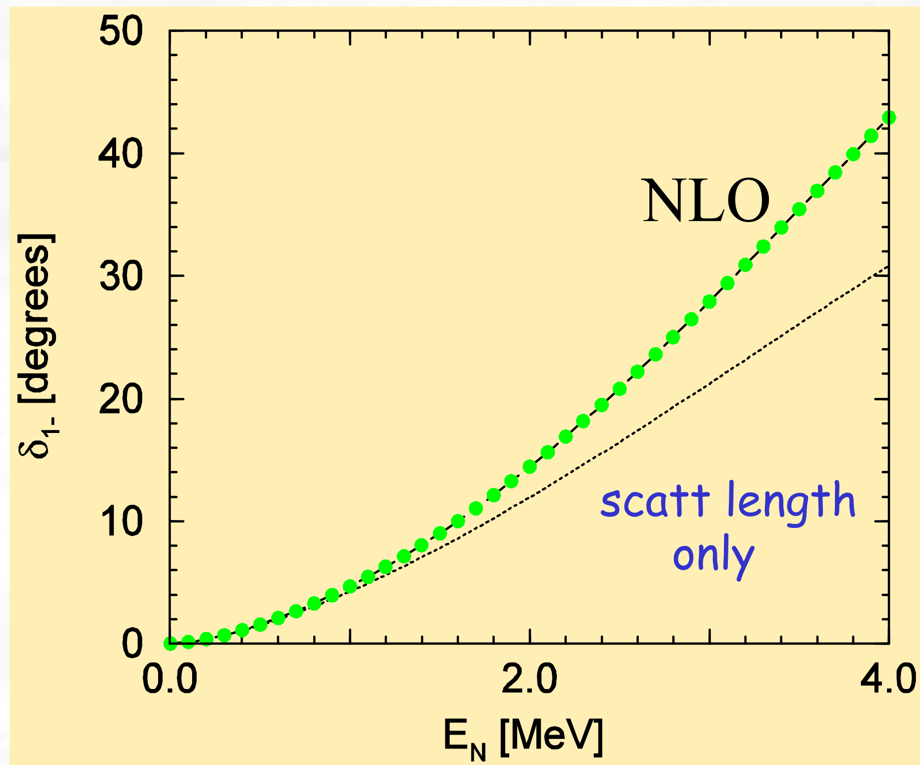
● PSA, Arndt et al. '73



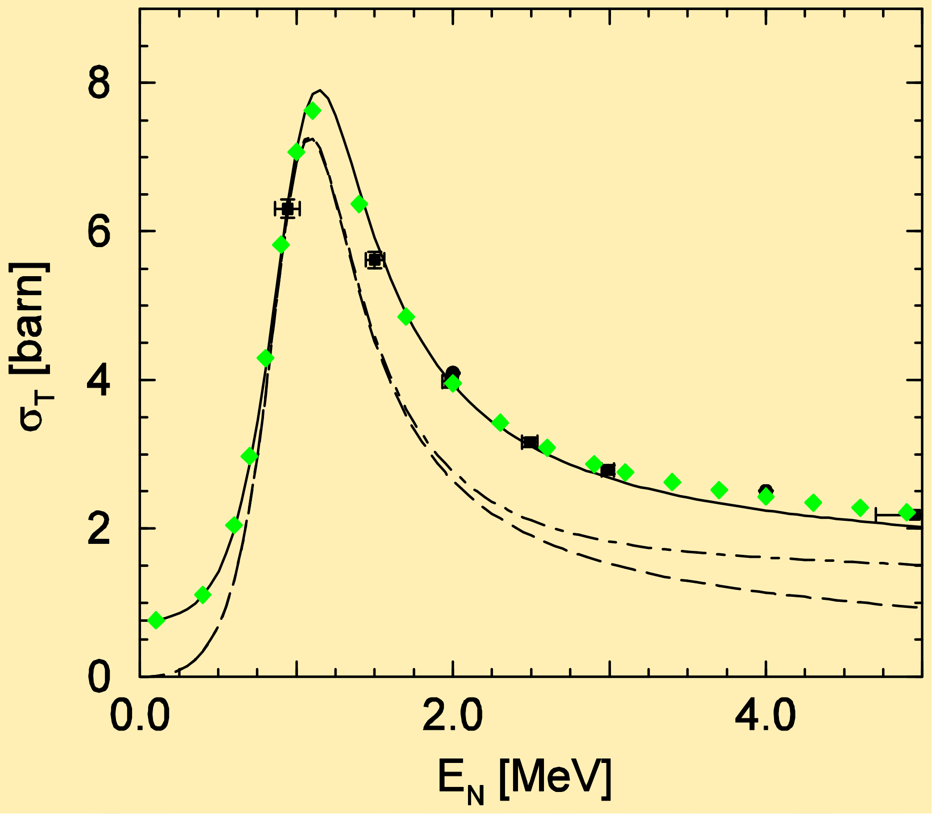


$$E_0 \cong 0.80\,\text{MeV}$$

$$\Gamma(E_0) \cong 0.55\,\text{MeV}$$



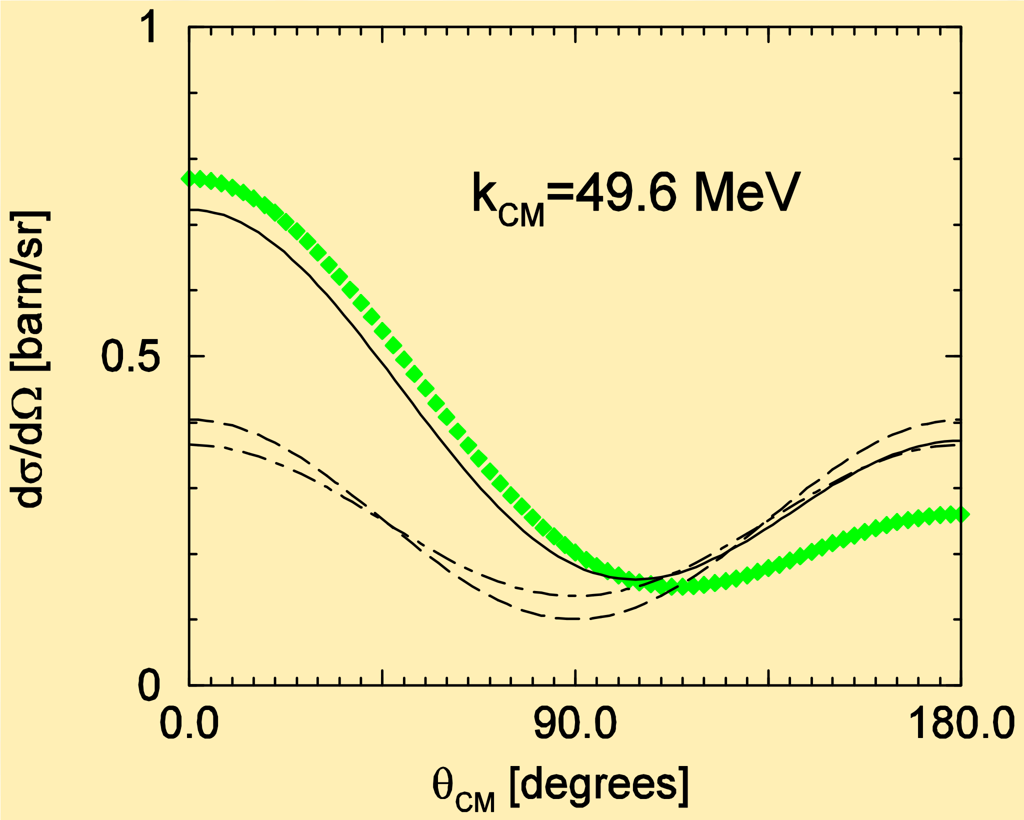
n + ^{4}He cross sections



--- LO

— NLO

-·-·- $LO = p_{1/2}$



◆ NNDC, BNL

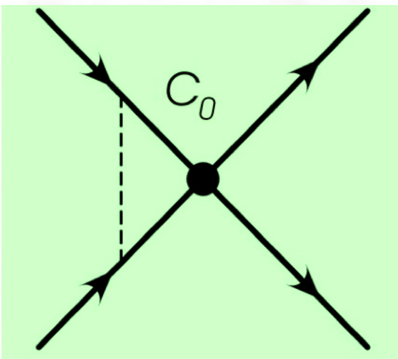
■ Haesner et al. '83

Coulomb Interaction

Kong, Ravndal, 2000

e.g., pp-scattering

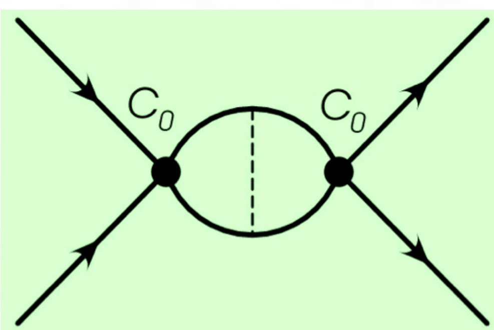
$$L_{EFT} \sim N^{+}\left(i\partial_t + \frac{\nabla^2}{2m_N}\right)N - C_0\left(N^{+}N\right)^2$$



$$\delta T = C_0 \int \frac{d^3q}{(2\pi)^3}\frac{e^2}{\mathbf{k}^2+\lambda^2}\frac{1}{E-(\mathbf{k}-\mathbf{q})^2/m_N + i\varepsilon} \quad \left(\sim C_0\frac{\alpha m_N}{k} = C_0\eta\right)$$

$$= -C_0\eta\left(\frac{\pi}{2} + i\ln\frac{2k}{\lambda}\right) + O(\lambda) \quad \Rightarrow \quad \text{non-perturbative for } k < \alpha m_N$$

➡ external legs strongly influenced by Coulomb repulsion

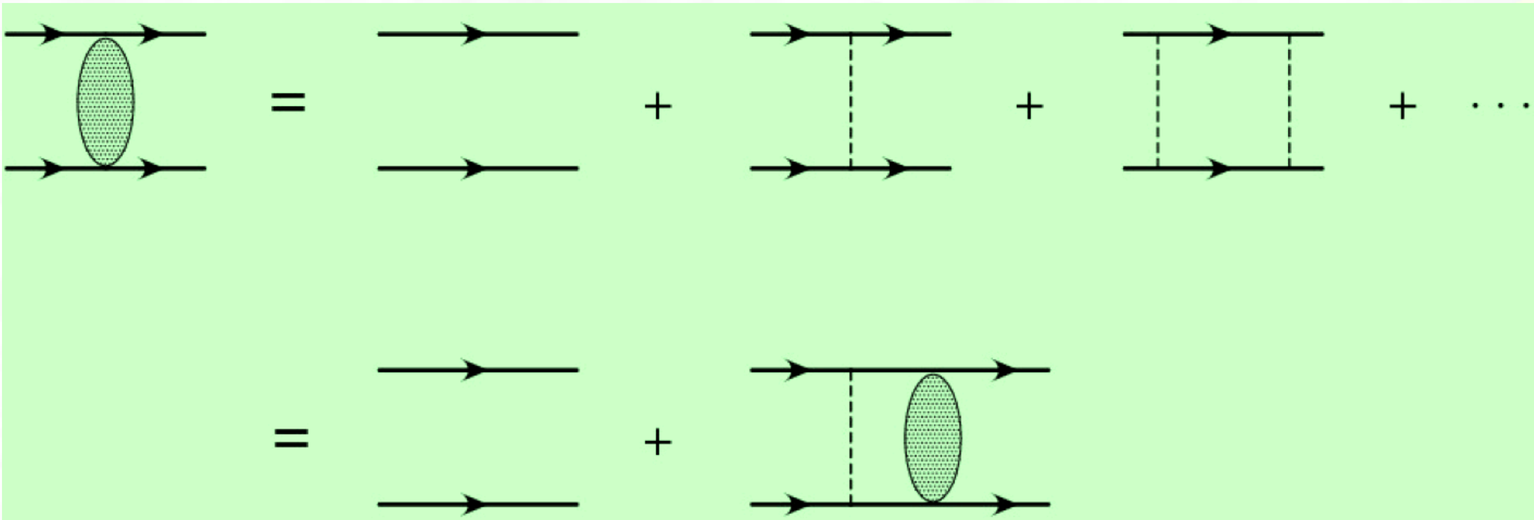


$$\delta I_0 \sim \frac{\eta\, m_N}{8\pi}\left(\frac{1}{\varepsilon} + 2\ln\frac{\mu\sqrt{\pi}}{2k} + \#\right) \quad \Rightarrow \quad \text{non-perturbative for } k < \alpha m_N$$

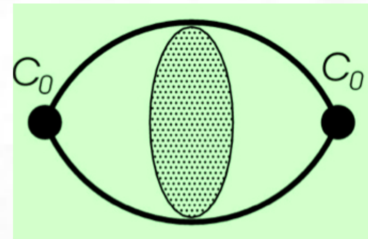
pole at D = 4 → need renorm of C_0

➡ strong interaction also much modified by Coulomb interaction

Coulomb Interaction



+



$$T = C_0 C_\eta^2 e^{2i\sigma_0} + C_0^2 C_\eta^2 e^{2i\sigma_0} J_0(k) + \cdots = C_\eta^2 \frac{C_0 e^{2i\sigma_0}}{1 - C_0 J_0(k)}$$

$$J_0(k) = m_N \int \frac{d^3q}{(2\pi)^3} \frac{2\pi\eta(q)}{e^{2\pi\eta(q)} - 1} \frac{1}{k^2 - q^2 + i\varepsilon}$$

$$= -\frac{\alpha m_N^2}{4\pi}\left[\frac{1}{\varepsilon} + H(\eta) + \ln\frac{\mu\sqrt{\pi}}{\alpha m_N} - \frac{3}{2}(0.5772..)\right] - \frac{\mu m_N}{4\pi}$$

pole at D = 4 → need renorm of C_0

use PDS and get rid of pole at D = 3, too.

Coulomb Interaction

$$-\frac{1}{a_{pp}(\mu)} = -\frac{1}{a_{pp}^{C}} - \frac{2}{a_B}\left(\ln\frac{\mu\sqrt{\pi}}{\alpha m_N} + 1 - \frac{3}{2}\gamma - \frac{1}{2}\mu r_0\right)$$

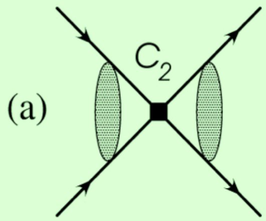
= Jackson, Blatt, 1950

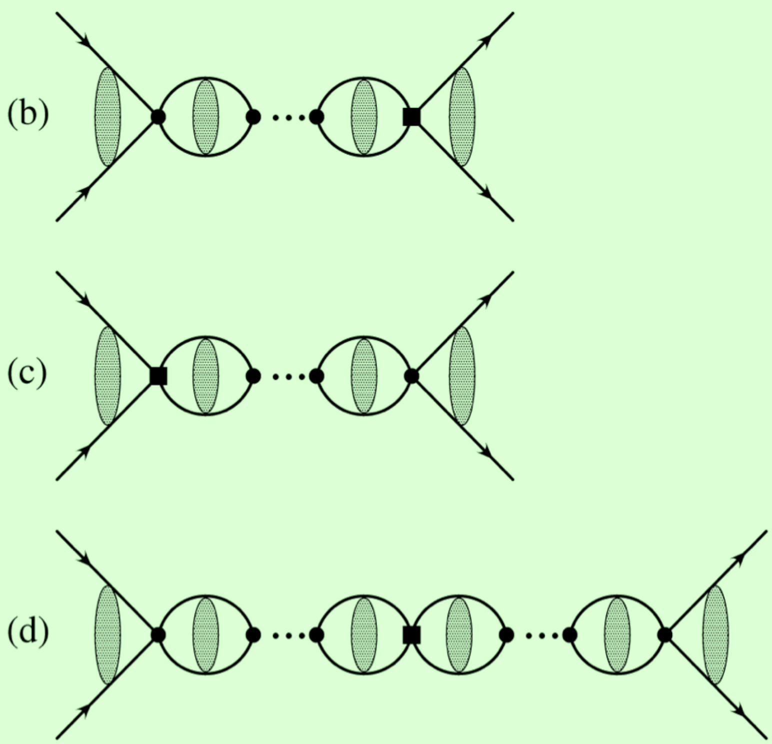
with renormalization of C_0:

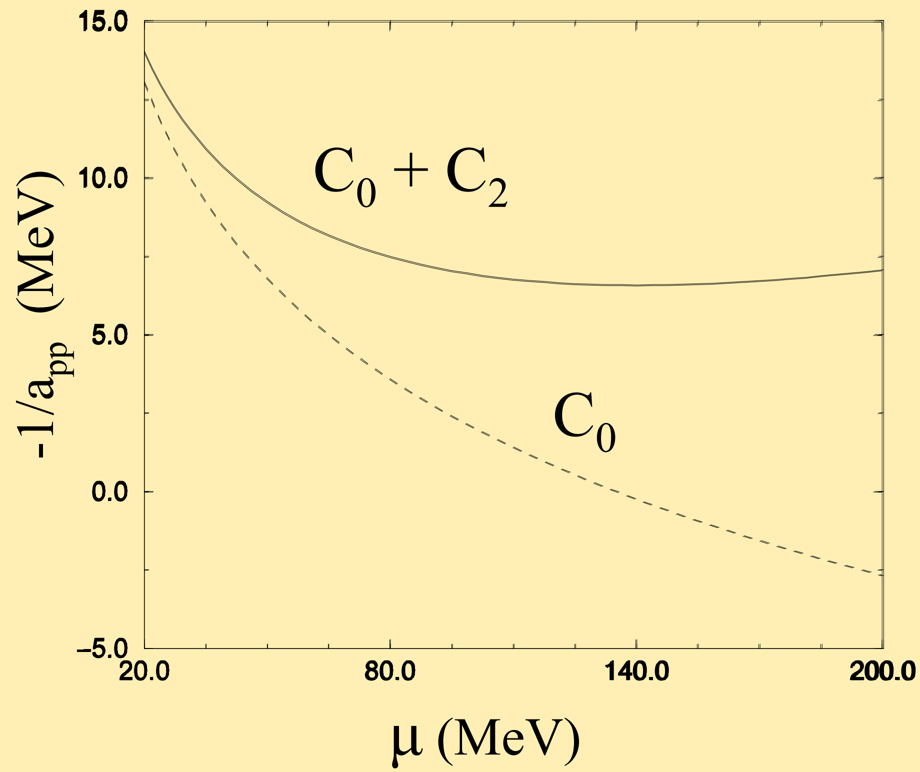
$$\frac{1}{a_{pp}(\mu)} = -\frac{4\pi}{m_N C_0(\mu)} + \mu$$

now add:

$$+\ \mathcal{L}_2 = \frac{C_2}{2}\left[N^{+}NN^{+}\overleftrightarrow{\nabla}^{2}N + h.c. + \cdots\right]$$







$$\Rightarrow\quad a_{pp}(\mu = m_\pi) = -29.9\ \text{fm} \qquad a_{pn}^{exp}(\mu = m_\pi) = -23.7\ \text{fm}$$

Kong, Ravndal, 2000

p-α scattering

Higa, CB, van Kolck, 2014

$$
\begin{aligned}
L_{N\alpha}^{LO} &= \phi^{+}\left[iD_0 + \frac{\mathbf{D}^2}{2m_\alpha}\right]\phi + N^{+}\left[iD_0 + \frac{\mathbf{D}^2}{2m_\alpha}\right]N + \varsigma_{0+}s^{+}\left[-\Delta_{0+}\right]s \\
&+ \varsigma_{1+}t^{+}\left[iD_0 + \frac{\mathbf{D}^2}{2(m_\alpha + m_N)} - \Delta_{1+}\right]t - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ \frac{g_{1+}}{2}\left\{t^{+}\mathbf{S}^{+} \bullet \left[n\mathbf{D}\phi - (\mathbf{D}N)\phi\right] + h.c. - r\left[t^{+}\mathbf{S}^{+}\cdot \mathbf{D}(N\phi) + h.c.\right]\right\} \\
&+ g_{0+}\left[s^{+}N\phi + \phi^{+}N^{+}s\right]
\end{aligned}
$$

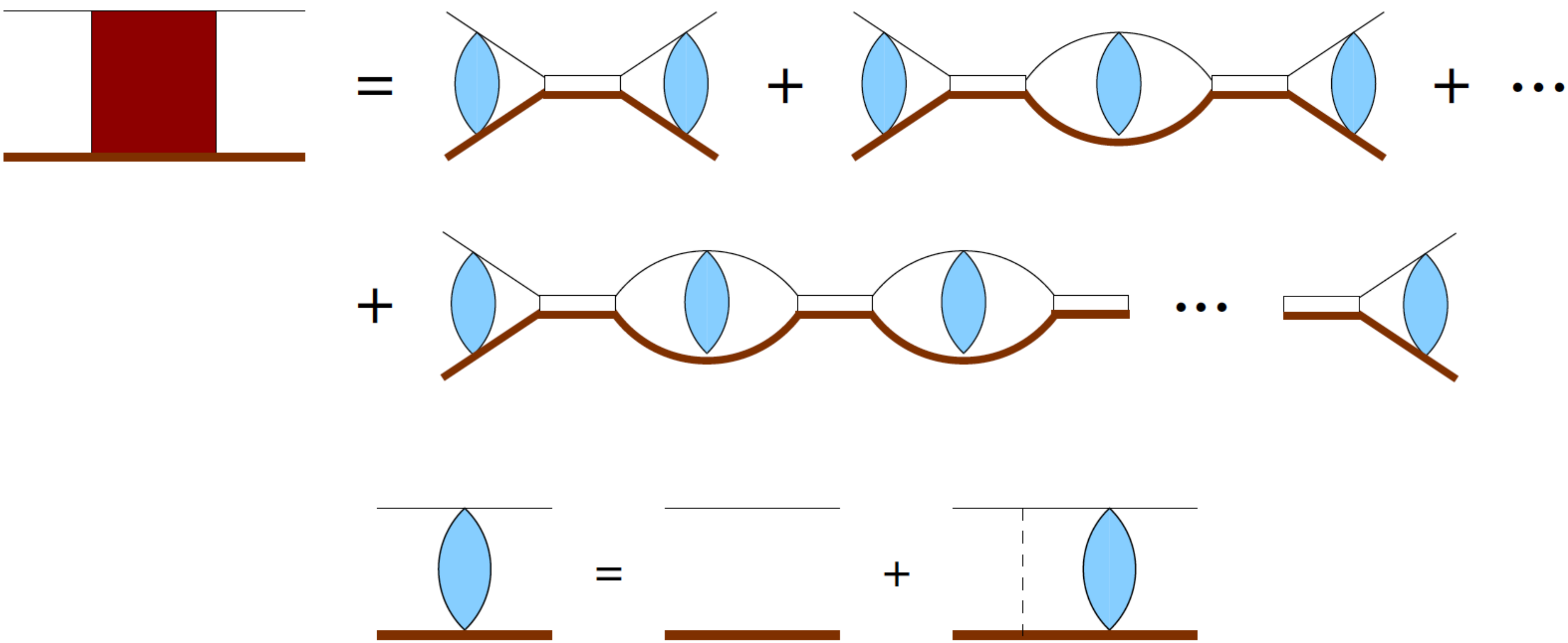
- t and s = dimeron fields coupling $N\alpha$ in $P_{3/2}$ and $S_{1/2}$
- with leading-order coupling constants g_{1+} and g_{0+}
- S_i = 2 x 4 spin-transition matrices between J = 1/2 and J = 3/2
- sign variables ζ_{0+}, ζ_{1+} = ±1 adjusted to reproduce the signs of the respective effective ranges.

$$
r = \frac{m_\alpha - m_N}{m_\alpha + m_N}
$$

$$
D_\mu = \partial_\mu + ieZ\frac{1+\tau_3}{2}A_\mu
$$

p-α scattering

$$L_{N\alpha}^{NLO} = \varsigma_{0+} s^{+}\left[iD_0 + \frac{\mathbf{D}^2}{2(m_\alpha + m_N)} - \Delta_{1+} \right] s + g'_{1+} t^{+}\left[iD_0 + \frac{\mathbf{D}^2}{2(m_\alpha + m_N)} \right]^2 t$$



p-α scattering

k_r (MeV)	k_i (MeV)	E_R (MeV)	$\Gamma_R/2$ (MeV)
51.1	9.0	1.69	0.61

$p_{3/2}$ pα resonance parameters

Csoto, Hale, 1997

$$k_p = k_r - ik_i \qquad k_r \sim M_{lo} \sim 50\ \text{MeV}, \qquad k_i \sim M_{lo}^2/M_{hi} \sim 10\ \text{MeV}$$

a_{0+} (fm)	r_{0+} (fm)
4.97 ± 0.12	1.295 ± 0.082

$S_{1/2}$ pα ERE parameters

Arndt, Long, Roper, 1997

a_{0+} (fm^3)	r_{1+} (fm^{-1})	$\mathcal{P}_{1+}$ (fm)
-44.83 ± 0.51	-0.365 ± 0.013	-2.39 ± 0.15

$P_{1/2}$ pα ERE parameters

$\mathcal{P}_{1+}/4 \sim r_{0+}/2 \sim 1/M_{hi} \rightarrow$ natural

$a_{0+} \sim 1/M_{lo}$, $a_{1+} \sim 1/M_{lo}^3$ and $r_{1+}/2 \sim M_{lo}$ $\rightarrow$ fine tuned

p-α scattering

$$T_{0+} = -\frac{2\pi}{\mu}\frac{C_\eta^2 e^{2i\sigma_0}}{-1/a_{0+} - 2k_C H(\eta)}\left[1 - \frac{r_{0+}k^2/2}{-1/a_{0+} - 2k_C H(\eta)}\right]$$

now pα reduced mass

$$k_C = Z_p Z_\alpha \alpha_{em}\mu$$

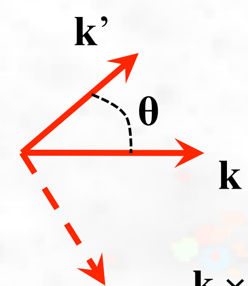
$$T_{1+} = -\frac{2\pi}{\mu}\frac{C_\eta^{(1)2} e^{2i\sigma_1} k^2 P_{1+}(\theta)}{-1/a_{1+} + r_{1+}k^2/2 - 2k_C H(\eta)}\left[1 + \frac{\mathcal{P}_{1+}k^4/4}{-1/a_{1+} + r_{1+}k^2/2 - 2k_C H^{(1)}(\eta)}\right]$$

$$C_\eta^{(1)2} = \left(1+\eta^2\right)C_\eta^2$$

$$H^{(1)}(\eta) = k^2\left(1+\eta^2\right)H(\eta)$$

$$k_C = Z_\alpha Z_p \mu\alpha$$

$$P_{1+}(\theta) = 2\cos\theta + \vec{\sigma}\cdot\hat{n}\sin\theta$$



$$\hat{n} = \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|}$$

$$T^{(LO)} = T_{0+} + T_{01}$$

p-α scattering

$$T_{p\alpha}^{NLO} = -\frac{2\pi}{\mu}\left\{-\frac{r_{0+}k^2}{2}\frac{\left(C_\eta^{(0)}\right)^2 e^{2i\sigma_0}}{\left[-1/a_{0+} - 2k_C H(\eta)\right]^2} + \frac{\mathcal{P}_{1+}k^4}{4}\frac{\left(C_\eta^{(1)}\right)^2 e^{2i\sigma_1}k^2\mathcal{P}_{1+}(\theta)}{\left[-1/a_{1+} + r_{1+}k^2/2 - 2k_C\left(k^2+k_C^2\right)H(\eta)\right]^2}\right\}$$

Matching to ERE: ➡

$$a_{1+} = -\zeta_{1+}\frac{\mu g_{1+}^{(R)2}}{6\pi\Delta_{1+}^{(R)}}, \qquad r_{1+} = -\zeta_{1+}\frac{6\pi}{\mu^2 g_{1+}^{(R)2}}, \qquad \text{and} \qquad \mathcal{P}_{1+} = -\frac{6\pi g_{1+}'}{\mu^3 g_{1+}^{(R)2}}$$

$p_{3/2}$ channel

$$a_{0+} = -\zeta_{0+}\frac{\mu g_{0+}^{(R)2}}{2\pi\Delta_{01+}^{(R)}}, \qquad \text{and} \qquad r_{0+} = -\zeta_{0+}\frac{2\pi}{\mu^2 g_{0+}^{(R)2}}$$

$s_{1/2}$ channel

p-α scattering

$S_{1/2}$ pα ERE fits with EFT

$S_{1/2}$	a_{0+} (fm)	r_{0+} (fm)
LO	7.4 + 8.0 – 2.2	-
NLO	4.81 + 0.05 – 0.21	1.7 + 1.3 – 0.8

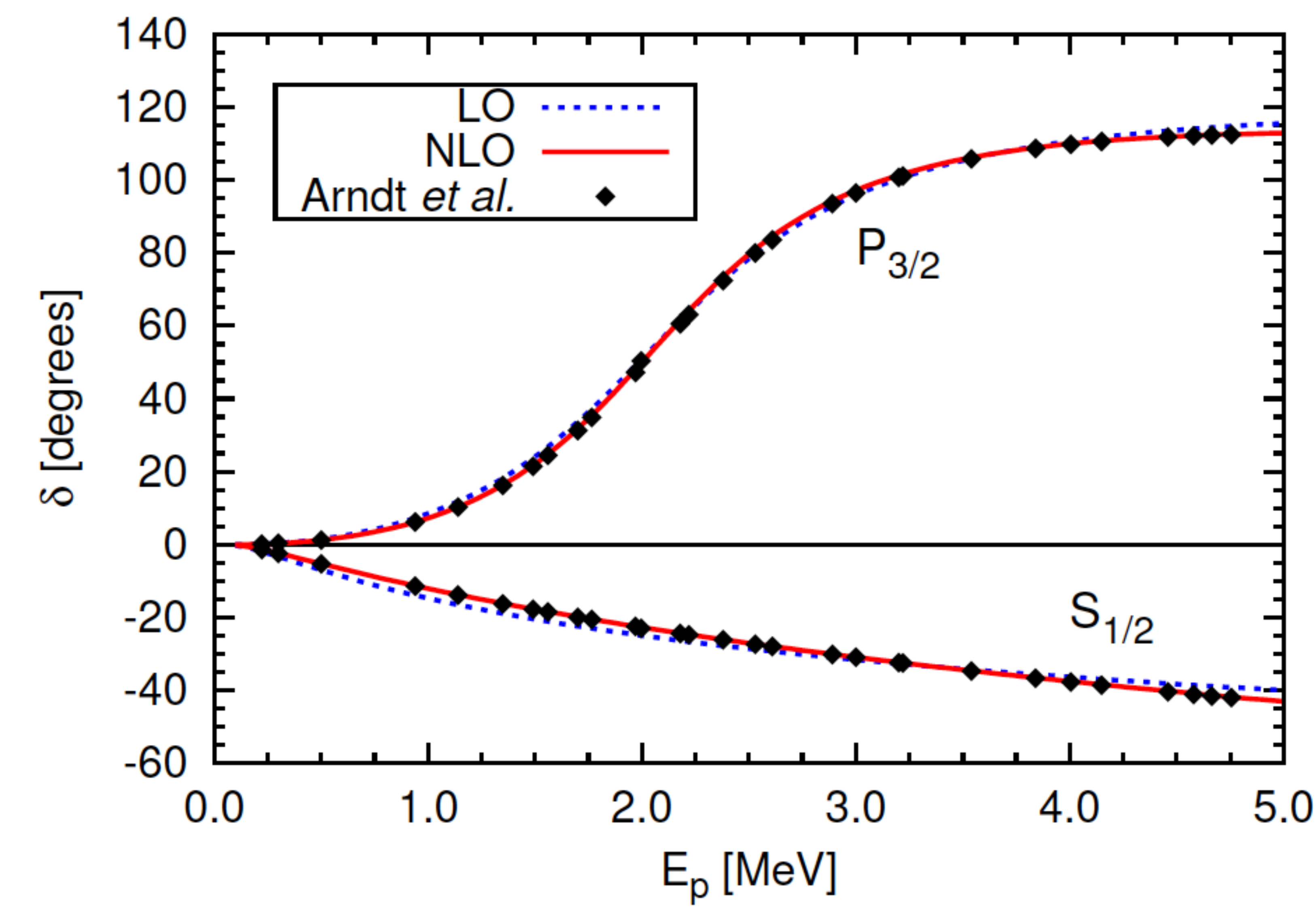
$P_{1/2}$ pα ERE fits with EFT

$P_{3/2}$	a_{1+} (fm^3)	r_{1+} (fm^{-1})	$\mathcal{P}_{1+}$ (fm)
LO	-58.0 + 11.0 – 29.0	-0.15 + 0.14 – 0.09	-
NLO	-44.5 + 1.6 – 0.1	-0.40 + 0.04 – 0.10	-2.8 + 1.0 – 1.8

$P_{3/2}$ pα ERE resonance fits with EFT

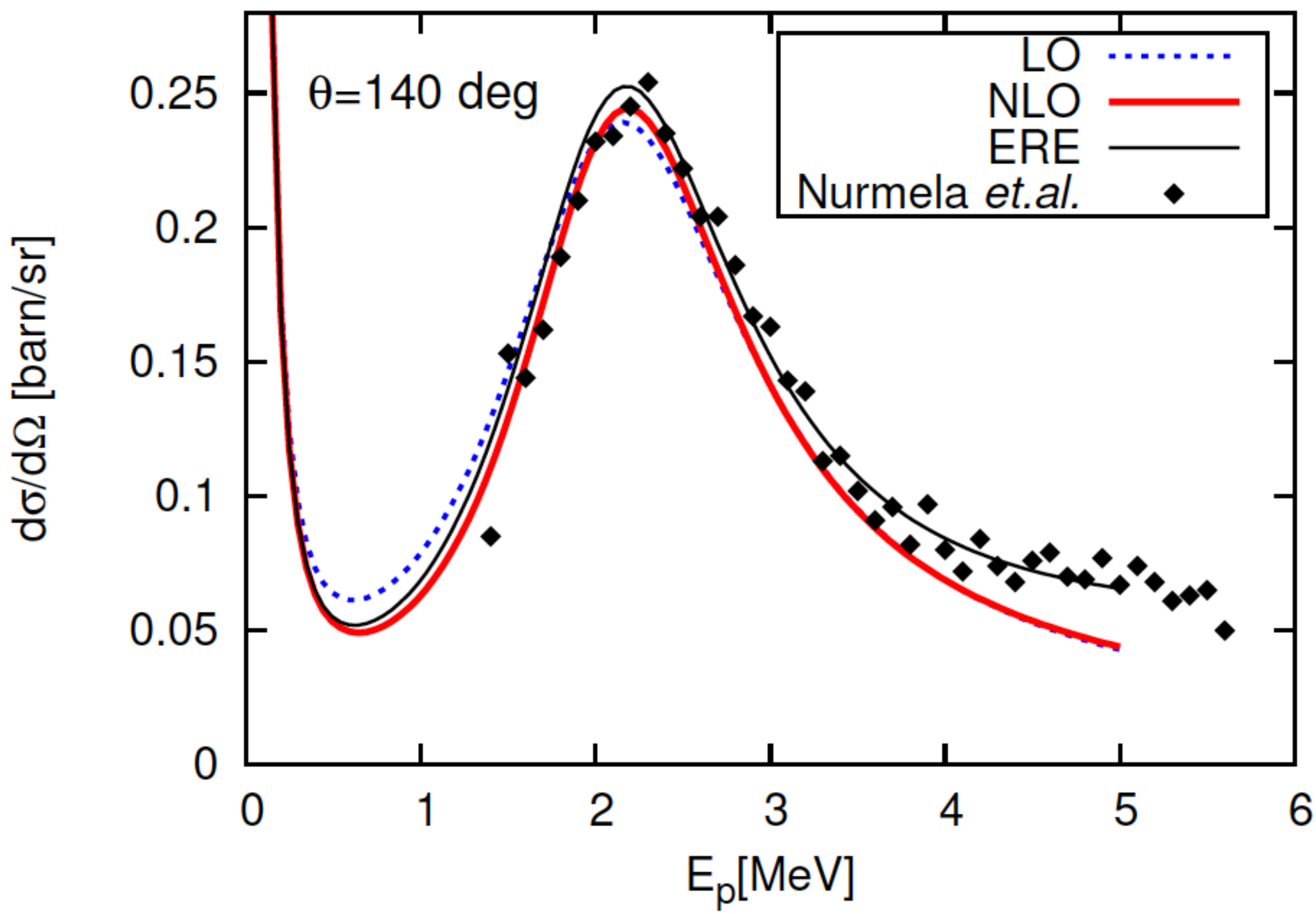
$P_{3/2}$	k_r (MeV)	k_i (MeV)	E_R (MeV)	$\Gamma_R/2$ (MeV)
LO	-50.6 + 1.2 – 2.5	-10.3 + 1.4 – 0.8	1.64 + 0.09 – 0.18	0.70 + 0.12 – 0.09
NLO	-50.7 + 0.5 – 0.6	9.40 + 0.01 – 0.10	1.66 + 0.04 – 0.04	0.63 + 0.01 – 0.02

p-α scattering



EFT results for $S_{1/2}$ and $P_{3/2}$ scattering phase shifts at LO (dotted) and NLO (solid), compared against the partial wave analysis (diamonds).

p-α scattering



EFT at LO (dotted) and NLO (thick solid) for pα elastic cross-section at θ_{lab} = 140^0, compared against the partial wave analysis (thin solid) and measured data points.

Summary

<u>Recent results for pα system:</u>

1 – We include P-waves with resonance and Coulomb interactions.

2 - We perform an expansion of the $P_{3/2}$ amplitude around the resonance pole to extract the resonance properties directly from a fit to the phase shift.

3- Our results at LO and NLO exhibit good convergence and the resonance energy and width are consistent with the ones using the extended R-matrix analysis.

4- Comparison with the differential cross-section at 140° reassures the consistency of the power counting, with $P_{1/2}$ contribution showing up only for proton energies beyond 3.5 MeV.

5 – Final adjustments necessary: a shallow bound state appears in the s-wave
→unitary correction necessary in the absence of Coulomb might need test.

Additional slides

For $P_{3/2}$ resonance amplitude → resonance pole expansion

$$T_{1+} = -\frac{2\pi}{\mu}\frac{C_\eta^{(1)2}e^{2i\sigma_1}k^2P_{1+}(\theta)}{\frac{\bar{r}_{1+}}{2}\left(k^2-k_p^2\right)-2k_C\left[H^{(1)}\left(\frac{k_C}{k}\right)-H^{(1)}\left(\frac{k_C}{k_p}\right)\right]}$$

$$\times\left\{1+\frac{\mathcal{P}_{1+}\left(k^2-k_p^2\right)/4}{\frac{\bar{r}_{1+}}{2}\left(k^2-k_p^2\right)-2k_C\left[H^{(1)}\left(\frac{k_C}{k}\right)-H^{(1)}\left(\frac{k_C}{k_p}\right)\right]}\right\}$$

$$\bar{r}_{1+} = -k_r\left\{\underbrace{2L_i}_{\text{LO}} - \underbrace{2ik_i\mathcal{P}_{1+}}_{\text{NLO}}\right\}$$

L_i, L_r defined at the pole from

$$2k_C H^{(1)}\left(k_C/k_p\right) = k_r^3 L_r + 2ik_r^2k_iL_i$$

$$\frac{r_{1+}}{2} = -k_r\left[L_i - \frac{k_r\mathcal{P}_{1+}}{2}\left(1-\frac{k_i^2}{k_r^2}\right)\right] \cong -k_r\left[\underbrace{L_i}_{\text{LO}} - \underbrace{\frac{k_r\mathcal{P}_{1+}}{2}}_{\text{NLO}}\right]$$

$$\frac{1}{a_{1+}} = -k_r^3\left[L_r + L_i\left(1-\frac{k_i^2}{k_r^2}\right) - \frac{k_r\mathcal{P}_{1+}}{4}\left(1-\frac{k_i^2}{k_r^2}\right)\right] \cong -k_r^3\left[\underbrace{L_r + L_i}_{\text{LO}} - \underbrace{\frac{k_r\mathcal{P}_{1+}}{4}}_{\text{NLO}}\right]$$