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Canada's national laboratory for particle and nuclear physics

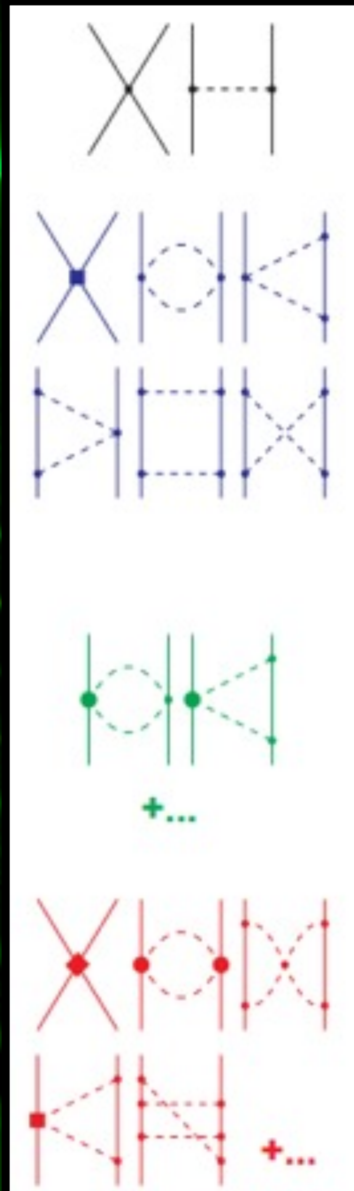
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Understanding the proton radius puzzle:

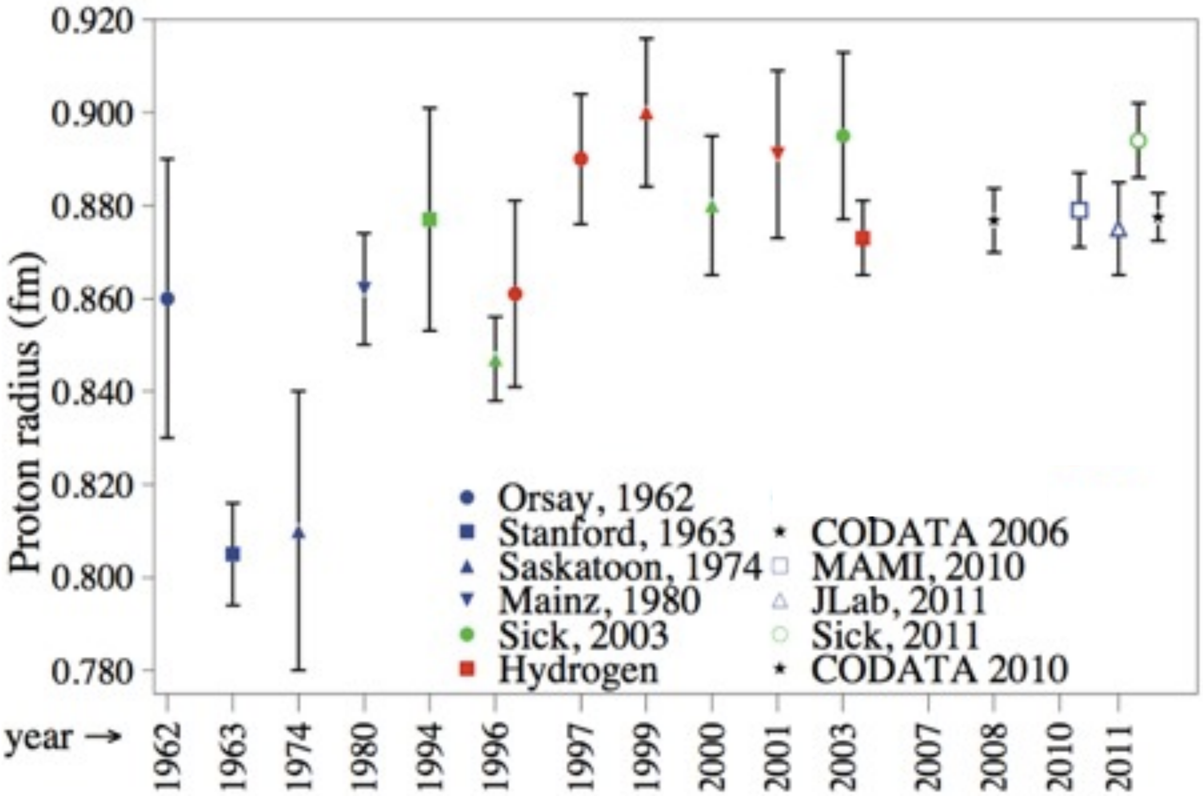
Nuclear structure corrections in light muonic atoms

Sonia Bacca

In collaboration with:
Nir Barnea, Javier Hernandez, Chen Ji and Nir Nevo Dinur

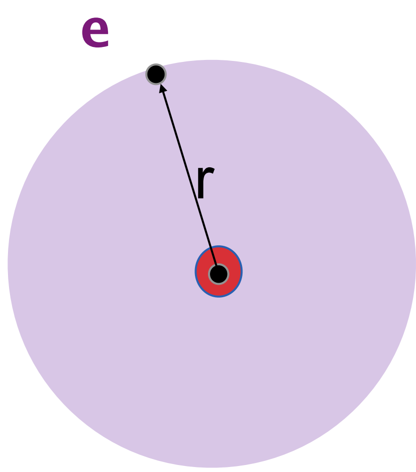


The Proton Charge Radius



Pic from Pohl *et al.*, Ann.Rev.Nucl.Part.Sci. 63 (2013)

$$r_p^2 = -6 \left. \frac{dG}{dQ^2} \right|_{Q^2=0}$$



CODATA2010:
r_p= 0.8775(51)fm

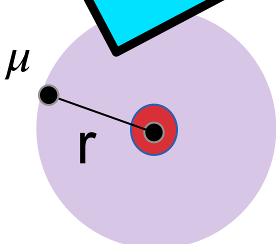
Mohr, *et al.* Rev. Mod Phys. (2012)

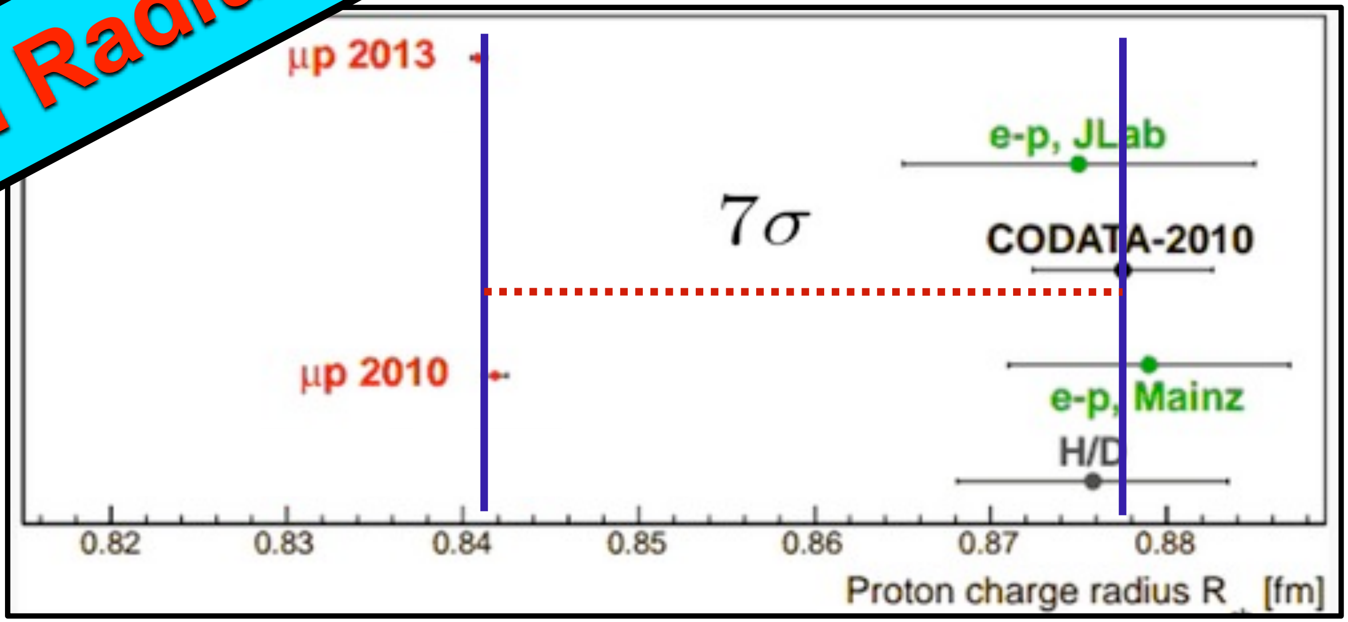
How Small is the Proton?

In 2010, proton charge radius was determined in muonic Hydrogen at PSI from spectroscopy measurements of t Lamb-shift:

Pohl et a **r_p= 0.84184(67) fm**
Ant (2013): r_p= 0.84087(39) fm

Proton Radius Puzzle!





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Origin of the discrepancy?

- Experimental results may be wrong:

"Multiple independent electron-proton experiments agree, and the muonic hydrogen experiment looks more convincing than any of the electron-proton experiments"
Pohl, Gilman, Miller, Pachucki, Ann.Rev.Nucl.Part.Sci. 63 (2013)

Electron scattering experiments are done at finite Q^2 , maybe not small enough

Dispersion analysis:
global fit of n and p give r_p= 0.84(1) with
$\chi^2 \approx 2.2$

Lorenz, Hammer, Meissner, EPJA (2012)

- Exotic hadronic structures?

Birse, McGovern EPJA (2012) vs Miller PLB (2013)

- New physics beyond standard model?

New force carrier, e.g. dark photon, that couples differently with e and μ

Yavin, Pospelov, Carlson etc...

New Experiments to Shed Light on the Puzzle

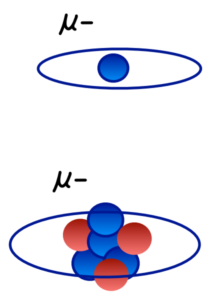
- Higher precision electron scattering experiments

  Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2

- MUSE collaboration (2016)

  measure $e^{\pm}p$ and $\mu^{\pm}p$ to reduce systematic errors

- CREMA collaboration currently measuring Lamb shift in light muonic atoms: Deuterium, Helions



Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E^{2S-2P} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

in a $Z\alpha$ expansion up to 5th order

Charge Radius From the Lamb-Shift

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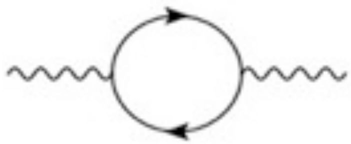
- QED corrections

vacuum polarizations



lepton self energy

relativistic recoil

Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E^{2S-2P} = \delta_{QED} + \delta_{pol} + \boxed{\frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle} - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

- Nuclear structure corrections

Elastic corrections: Finite size

Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

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- Nuclear structure corrections

Elastic corrections: Finite size

Zemach moment

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- Nuclear structure corrections

Elastic corrections: Finite size

Zemach moment

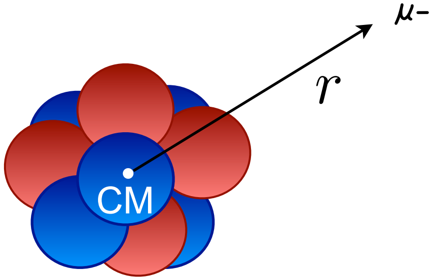
Inelastic corrections: nuclear polarization δ_{pol}

Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E^{2S-2P} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

- Nuclear polarizability corrections



Dipole excitation

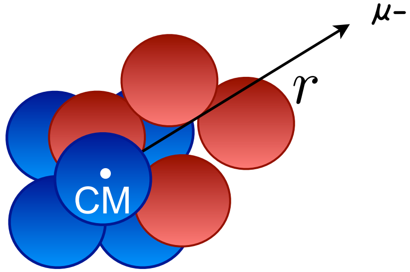
Stronger Coulomb - reduced energy

Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E^{2S-2P} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

- Nuclear polarizability corrections



Dipole excitation

Stronger Coulomb - reduced energy

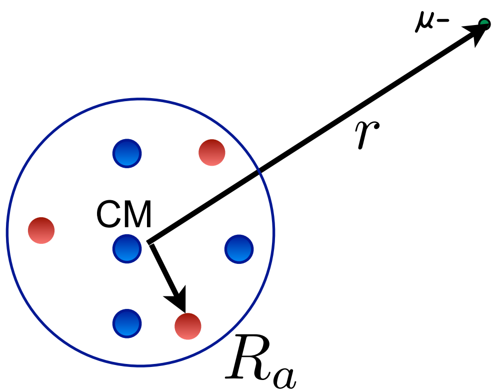
The distorted charge distribution follows the orbiting μ like a “tide”

Nuclear response function $S_{\hat{O}}(\omega) = \frac{1}{2j_0+1}\sum_{N\neq N_0,J} |\left\langle N_0 J_0 || \hat{O} || N J \right\rangle|^2 \delta(\omega - E_N + E_0)$

The Muonic Atom System

$$H = H_N + H_\mu + \Delta V$$

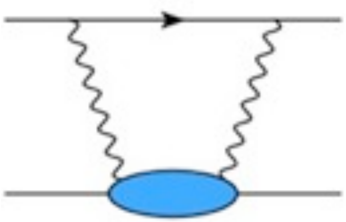
$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

Using perturbation theory at second order one obtains the expression for δ_{pol} up to order $(Z\alpha)^5$



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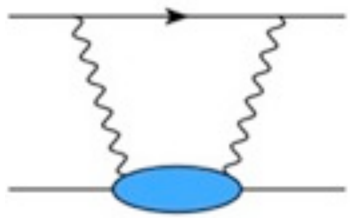
Contributions to δ_{pol}

Non relativistic terms

- Take non-relativistic kinetic energy in muon propagator
- Neglect Coulomb force in the intermediate state
- Expand the muon matrix elements in $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

$\delta^{(0)}$ $\delta^{(1)}$ $\delta^{(2)}$

★ $|\boldsymbol{R}-\boldsymbol{R}'|$ "virtual" distance traveled by the proton between the two-photon exchange

★ Uncertainty principle $|\boldsymbol{R}-\boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$

★ $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} = 0.17$ e.g. for μ-^{4}He

Contributions to δ_{pol}

Non relativistic terms

★ $\delta^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

dominant term, related to the energy-weighted integral of the dipole response function

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

★ $\delta^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$

contains a part that cancels the Zemach moment elastic contribution

cf. Pachucki (2011)
Friar (2013)

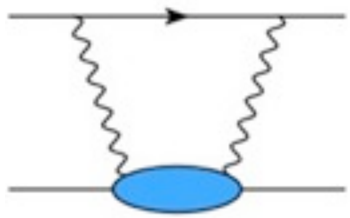
★ $\delta^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

leads to energy-weighted integrals of three different response functions

$$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$$

Contributions to δ_{pol}

Relativistic terms



- Take the relativistic kinetic energy in muon propagator
- Separate in longitudinal and transverse term
- Related to the dipole response function

$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right)\; S_{D_1}(\omega)$$

Coulomb term

- Consider the Coulomb force in the intermediate states
- Naively it is a $\delta^{(0)}_C \sim (Z\alpha)^6$ corrections, but actually logarithmically enhanced

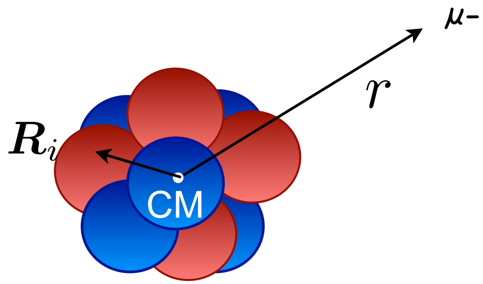
$$\delta^{(0)}_C \sim (Z\alpha)^5 \log(Z\alpha)$$

Friar (1977), Pachucki (2011)

- Related to the dipole response function

Contributions to δ_{pol}

Finite Nucleon Size Corrections



- In point nucleon limit $\Delta V = -\alpha \sum_i^Z \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|}$

- Consider finite nucleon size by including charge distributions

$$\Delta V = -\alpha \sum_i^Z \int d\boldsymbol{R}' \frac{n_p(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|} - \alpha \sum_j^N \int d\boldsymbol{R}' \frac{n_n(\boldsymbol{R}' - \boldsymbol{R}_j)}{|\boldsymbol{r} - \boldsymbol{R}'|}$$

- Low-q approximation of the nucleon form factors

$$G_p^E(q) \simeq 1 - \frac{\langle r_p^2 \rangle}{6} q^2$$

$$G_n^E(q) \simeq -\frac{\langle r_n^2 \rangle}{6} q^2$$

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Nuclear Polarizability Corrections

The accuracy of the extracted radius depends on the accuracy of the nuclear polarizability

$$\Delta E^{2S-2P} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

- Experimental requirement δ_{pol} must be known with 5% accuracy

To estimate the nuclear polarizability one needs information on the excitations of the nucleus → nuclear response function

- Theoretically calculate it
- Extract it from data

Previous Work

- Simple potential models

  μ-^{12}C (Square-well) Rosenfelder '83

  μ-D (Yamaguchi) Lu & Rosenfelder '93

- From experimental photo-absorption cross section

  μ-^{4}He Bernabeu & Karlskog '74; Rinker'76; Friar '77 $\delta_{pol} = -3.1 \text{ meV} \pm 20\%$

  μ-D Carlson, Gorchtein, Vanderhagen 2014 7% Uncertainty

- Zero-range expansion (pion-less EFT)

  μ-D Friar 2013 Accuracy roughly estimated ~ 2%

- State-of-the-art potentials

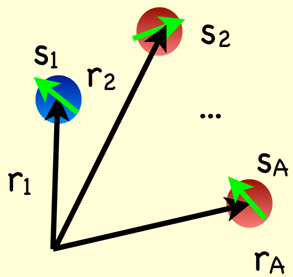
  μ-D (AV14) Leidemann & Rosenfelder '95
  (AV18) Pachucki 2011

  Accuracy < 2% or less

  ⇨ *Ab-initio calculations of the nuclear polarization with state-of-the-art potentials*

Monday, 12 May, 14

Ab-initio Theory Tools



$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN} + V_{3N} + \dots$$

High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

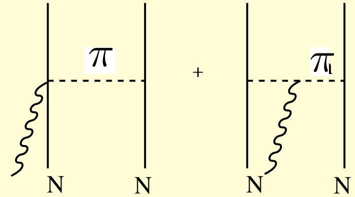
Traditional Nuclear Physics

AV18+UIX, ..., J_2

Effective Field Theory

N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$



two-body currents (or MEC)

subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$S(\omega) \propto |\langle\psi_f|J^\mu|\psi_0\rangle|^2$$

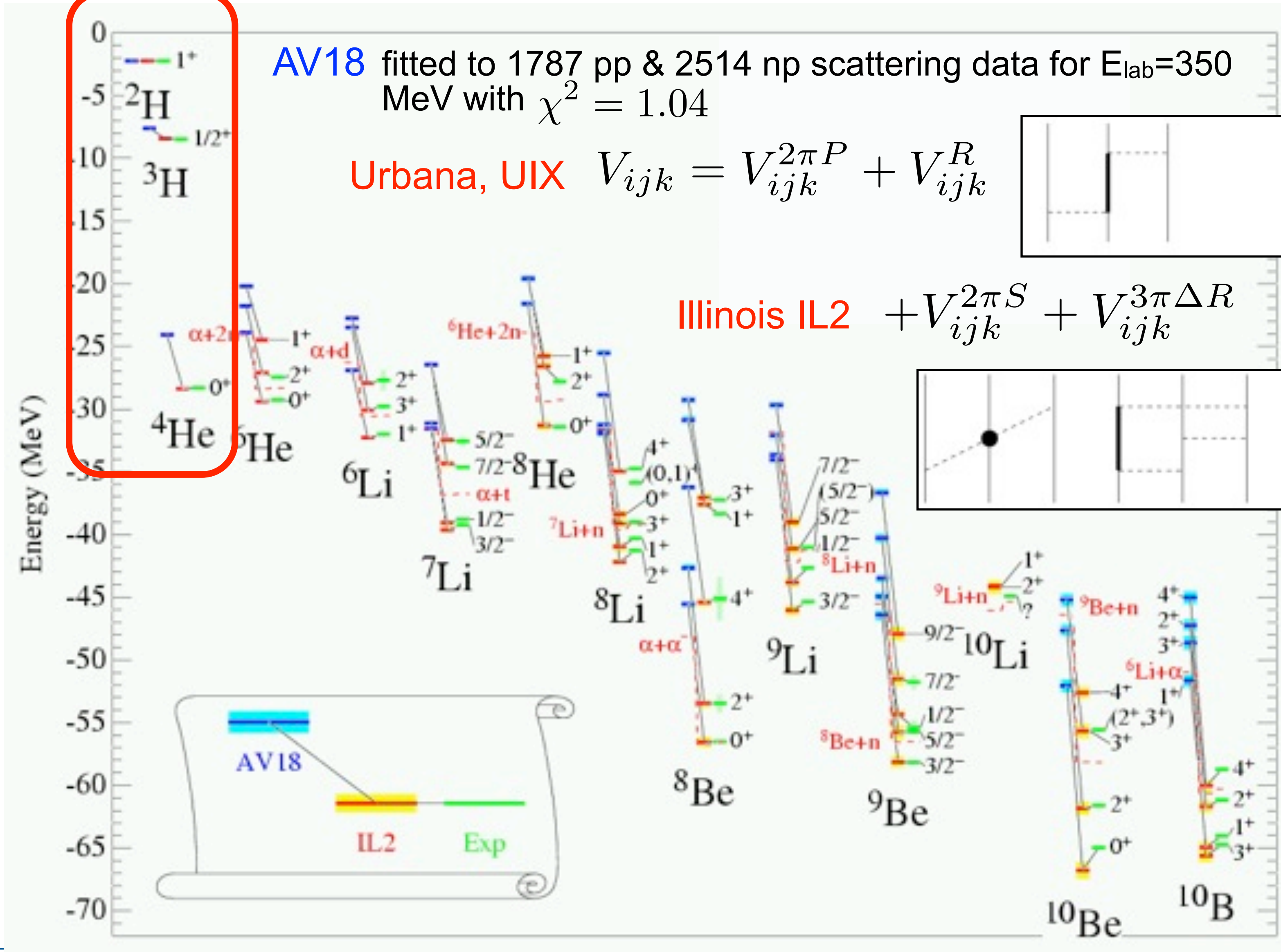
Exact Initial state &
Final state in the continuum at
different energies and for different A

Traditional Potentials

AV18 fitted to 1787 pp & 2514 np scattering data for E_{lab}=350 MeV with $\chi^2 = 1.04$

Urbana, UIX $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$

Illinois IL2 $+V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R}$



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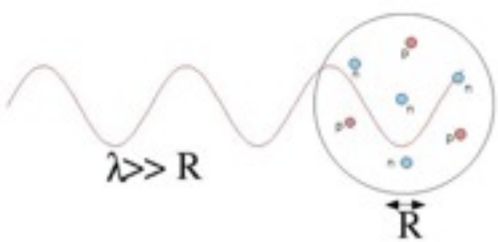
Chiral EFT Potentials

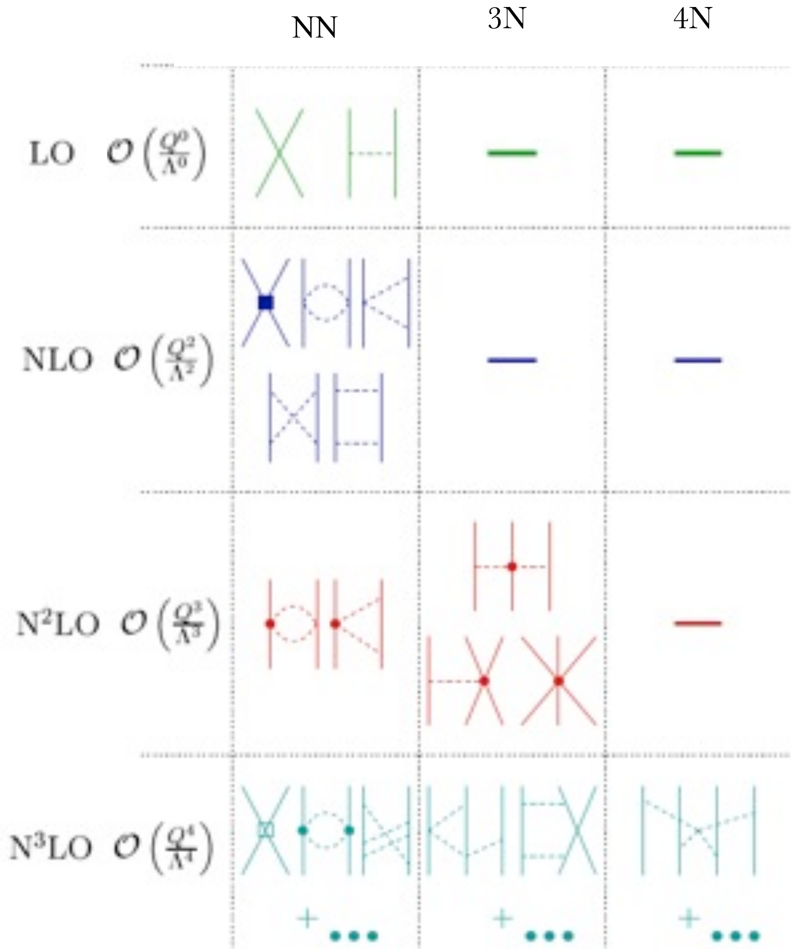


Limited resolution at low energy

Separation of scales

$$\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$$

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \ldots$$



- **Effective Field Theory** of low-energy QCD
- **Nuclear Forces** are built from systematic expansions in Q/Λ
- **Coupling constants** fit to nuclear data

$$V_{NN} > V_{3N} > V_{4N}$$

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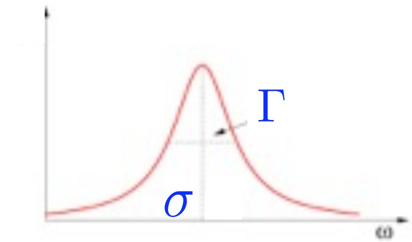
Lorentz Integral Transform Method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Reduce the continuum problem to a bound-state problem

$$S(\omega) = \sum_f \left| \left\langle \psi_f \right| J^\mu \left| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

$$L(\sigma,\Gamma) = \int d\omega \frac{S(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \left\langle \tilde{\psi} | \tilde{\psi} \right\rangle < \infty$$



where $\left|\tilde{\psi}\right\rangle$ is obtained solving

$$(H - E_0 - \sigma + i\Gamma)|\tilde{\Psi}\rangle = J^\mu |\Psi_0\rangle$$

- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since $\langle\tilde{\psi}|\tilde{\psi}\rangle$ is finite, $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

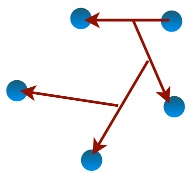
$$L(\sigma,\Gamma) \xrightarrow{\text{inversion}} S(\omega)$$

The exact final state interaction is included in the continuum rigorously!

Monday, 12 May, 14

Hyperspherical Harmonics

$$|\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\ldots,\vec{\eta}_{A-1})\rangle$$



Recursive definition of hyper-spherical coordinates

$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

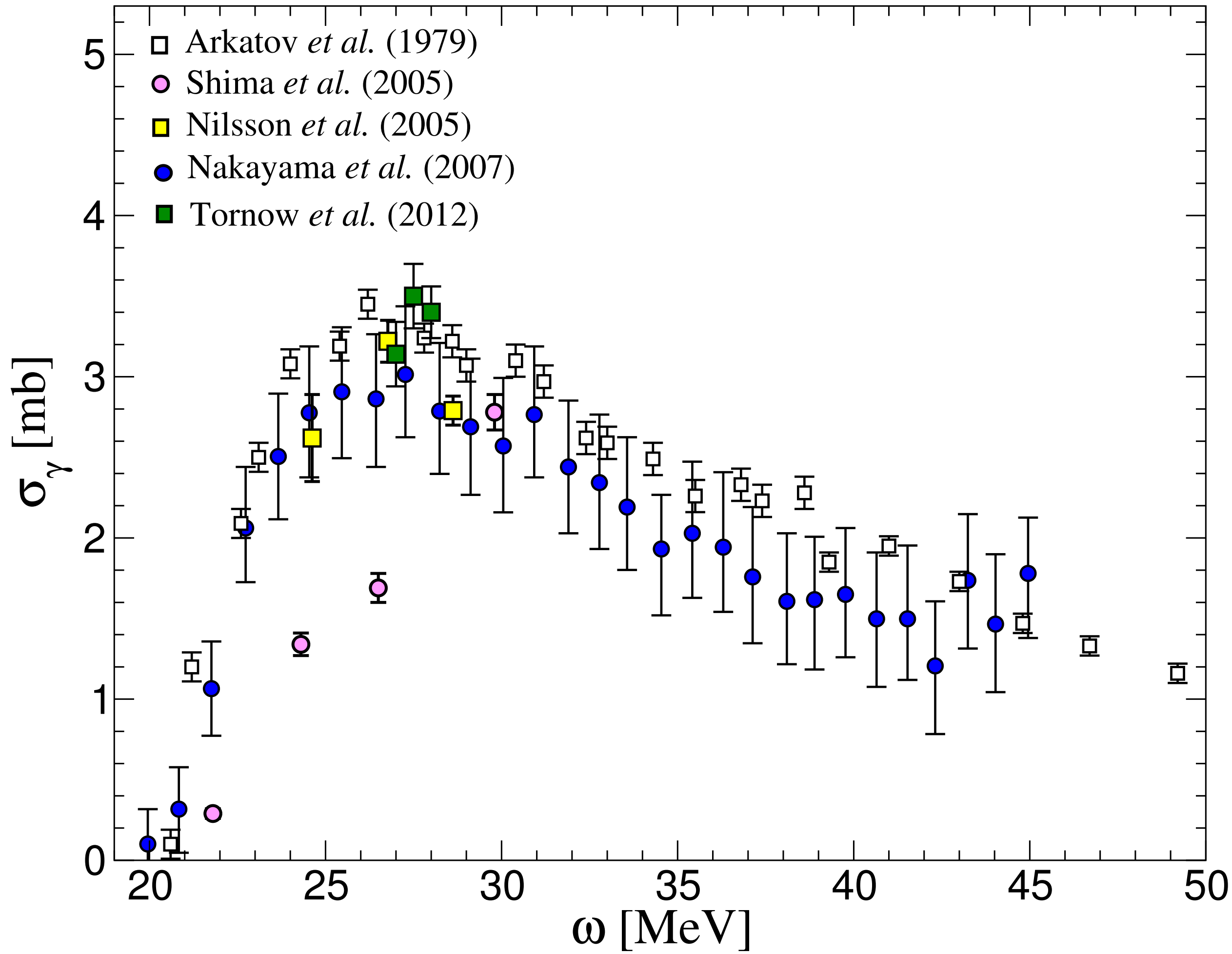
$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right)[\mathcal{Y}^{\mu}_{[K]}(\Omega)\chi^{\bar{\mu}}_{ST}]^a_{JT}$$

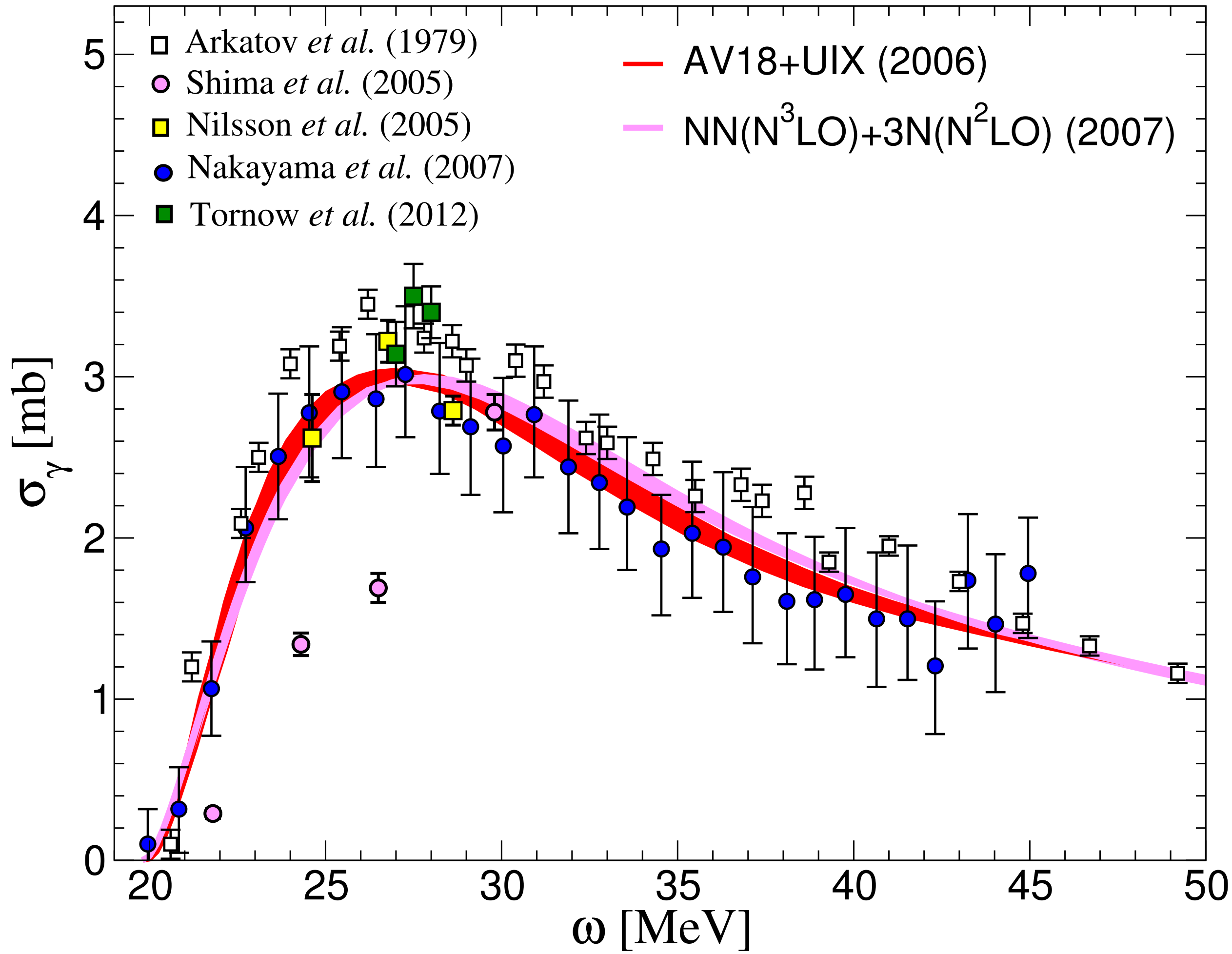
In nuclear physics we are able to use HH for A up to 6 and 7

Antisymmetrization by Barnea and Novoselsky (1998)

^{4}He Photo-absorption cross section



^{4}He Photo-absorption cross section



Theory can be more precise than experiment

Nuclear Polarizability Corrections in $\mu^4\mathrm{He}^+$

We performed the first ab-initio calculations of the nuclear polarizability corrections in muonic Helium

Ji, Nevo Dinur, S.B. and Barnea, PRL **111,** 143402 (2013)

Few-body methods:

Lorenz Integral Transform method

Hyperspherical Harmonics expansion

Hamiltonians:

AV18+UIX and EFT NN(N3LO)+3N(N2LO) (C_D=1 and C_E=-0.029)

We used the difference from the two Hamiltonians to estimate the uncertainty in nuclear physics

Lanczos algorithm:

the nuclear polarization is like an energy-dependent sum rule

This can be calculated directly without first obtaining the response function and the convergence is fast if $g(\omega)$ is smooth

$$\delta_{\hat{O}} \propto \int_{\omega_{th}}^{\infty} g(\omega) S_{\hat{O}}(\omega) d\omega$$

Nevo Dinur, Ji, S.B. and Barnea, arXiv:1403.7651

Nuclear Polarizability Corrections in μ^4He^+

[meV]	AV18/UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Systematic convergence from $\delta^{(0)}$ to $\delta^{(2)}$
- The difference between the two potentials for δ_{pol} is 5.5%
- Uncertainty from nuclear physics

$$\frac{5.5\%}{\sqrt{2}} \rightarrow \pm 4\%$$

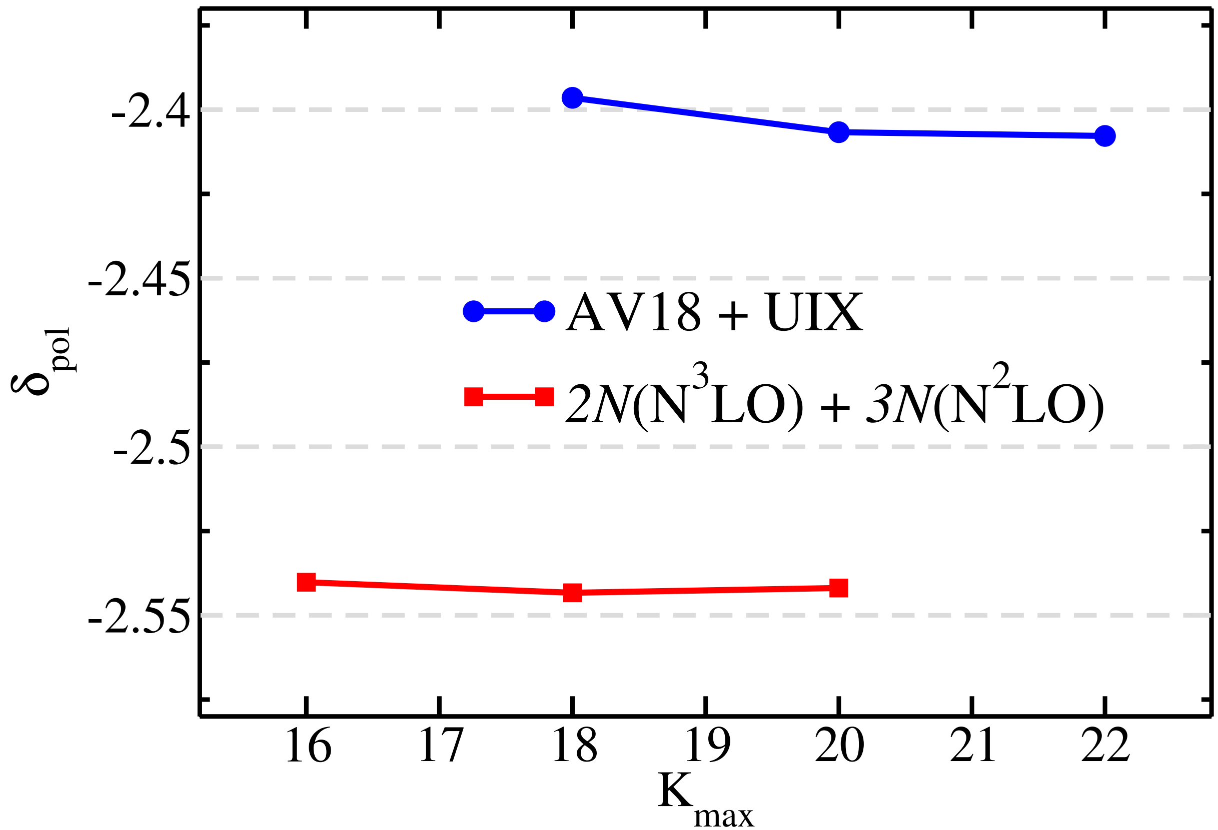
★ C_D=1 and C_E=-0.029

Nuclear Polarizability Corrections in $\mu^4\mathrm{He}^+$

The work is not yet finished ...

Nuclear Polarizability Corrections in $\mu^4\mathrm{He}^+$

Numerical accuracy → HH expansion



0.4%

0.2%

Nuclear Polarizability Corrections in $\mu^4\mathrm{He}^+$

Atomic Physics Uncertainty

- $(Z\alpha)^6$ effects (beyond second order perturbation theory)
- Relativistic and Coulomb effects to multipoles other than dipole
- Higher order nuclear size effects

Combined they give an additional 3-4 %

Nuclear Polarizability Corrections in $\mu^4\text{He}^+$

Error Budget

Nuclear Physics	4%
Numerical Accuracy	0.4%
Atomic Physics	4%
Total	6%

- Dramatic improvement from pervious work based on experimental data

Bernabeu & Karlskog '74; Rinker'76; Friar '77 $\delta_{pol} = -3.1 \text{ meV} \pm 20\%$

- We almost meet the experimental requirement (5% accuracy)

Possible Refinements

To further understand the uncertainty in nuclear physics

- Use chiral EFT at different orders to track the convergence
- At a fixed order vary the cutoff to assess the theoretical error

Epelbaum, Gloeckle, Meissner, NPA (2005)

We will first apply this analysis to μD

Nuclear Polarizability Corrections in μD

Previous Work

PRL 106, 193007 (2011) PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

Nuclear Structure Corrections in Muonic Deuterium

Krzysztof Pachucki
Faculty of Physics, University of Warsaw, Hoża 69, 00-681 Warsaw, Poland
(Received 16 February 2011; published 13 May 2011)

TABLE I: Nuclear structure corrections in muonic deuterium for $2P - 2S$ transition.

correction	value in meV
$\delta_0 E$	1.910
$\delta_{C1} E$	−0.255
$\delta_{C2} E$	−0.006
$\delta_R E$	−0.035
$\delta_{Q0} E$	−0.045
$\delta_{Q1} E$	0.151
$\delta_{Q2} E$	−0.066
$\delta_M E$	−0.016
$\delta_P E$	0.043(3)
ΔE	1.680(16)

with AV18
total error very small and mostly coming from atomic physics

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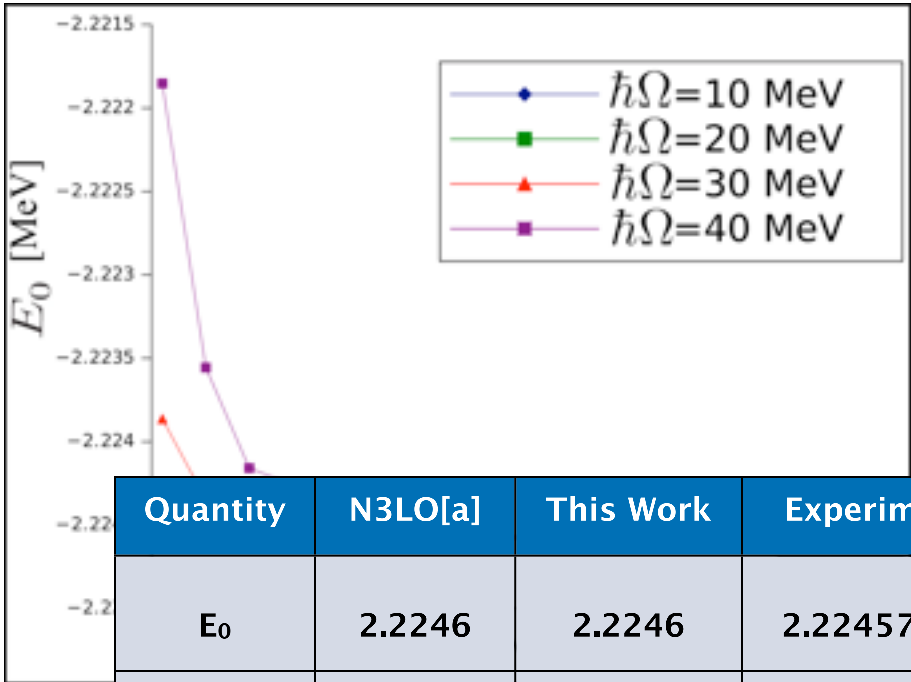
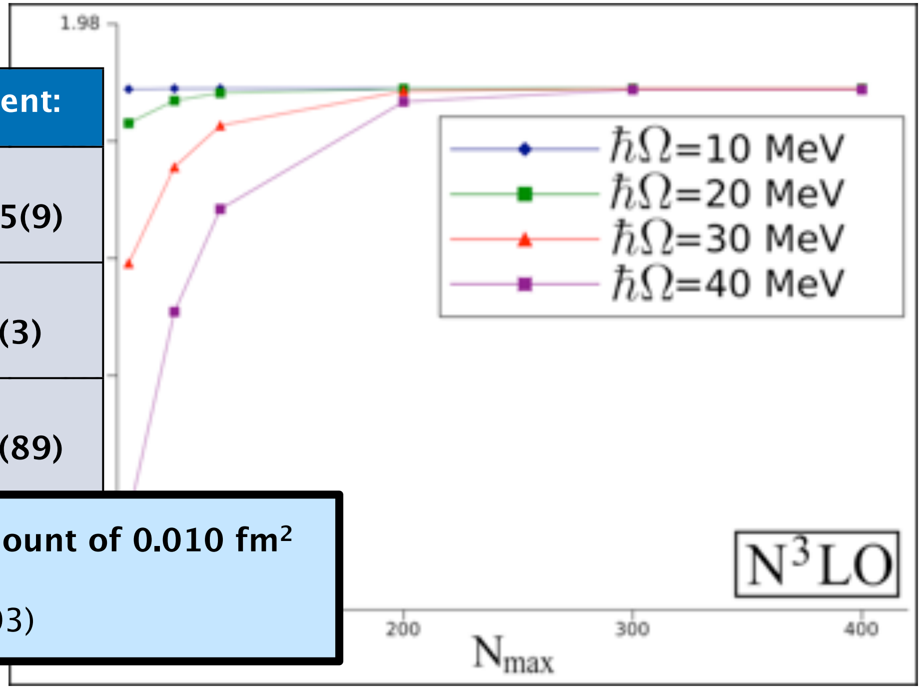
Nuclear Polarizability Corrections in μD

with Javier Hernandez



- Expand on the HO basis $N_{max} = 2n + \ell$
- Can reproduce published observables with AV18 and N3LO



Quantity	N3LO[a]	This Work	Experiment:
E_0	2.2246	2.2246	2.224575(9)
Qd	0.285*	0.285*	0.2859(3)
r	1.978**	1.978**	1.97535(89)

* Including MEC and Relativistic Corrections in the amount of 0.010 fm^2
**Including MEC and relativistic Corrections
[a] D.R Entem, et al Physical Review C 68, 041001(R) (2003)

Nuclear Polarizability Corrections in μD

with Javier Hernandez

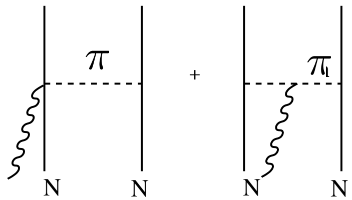
- Magnetic correction

$$\delta_M = \frac{1}{3} m_r^3 \alpha^5 \left(\frac{g_p - g_n}{4 m_p} \right)^2 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2 m_r}} S_{M1}(\omega)$$

- MEC are important in magnetic transitions $S_{M1} \rightarrow \langle \psi_f | \, \mathbf{j} \times \mathbf{q} \, | \psi_0 \rangle$

Appearance of two-body currents in Chiral EFT

Operator	LO	NLO	N2LO	N3LO	N4LO
$\mathbf{j}$	IA-NR	OPE	IA-RC	OPE TPE CT	
ρ	IA-NR	—	IA-RC	OPE	TPE



TRIUMF

Nuclear Polarizability Corrections in μD

Hernandez, Ji, S.B., Nevo-Dinur, Barnea, in preparation

Error Budget

Nuclear Physics	1.1%
Atomic Physics	0.95%
Total	1.5%

Preliminary

- More solid estimate of the total error, that includes the nuclear physics error

- More accurate than evaluation based on experimental data

  μ- D   Carlson, Gorchtein, Vanderhagen (2014)   7% Uncertainty

Conclusions and Outlook

- Lamb shifts in muonic atoms
  - ★ Raise interesting questions about lepton symmetry
  - ★ Connect nuclear, atomic and particle physics
- μ^4He^+
  - ★ We have performed the first ab-initio calculation obtaining $\delta_{pol} = -2.47\text{meV} \pm 6\%$
  - ★ Much more accurate than earlier estimates based on experimental data
- μD
  - ★ Based on EFT expansion we provide more solid estimates of uncertainty
  - ★ Much more accurate than estimates based on experimental data

Our calculations are key to the charge radius extractions of the CREMA experiment

Future

- ★ Investigate μ^3He^+
- ★ Narrow uncertainty in nuclear physics (including MEC, higher order chiral forces, explore correlations of observables)

Thank you!
Merci

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