Universality of Nucleon-Nucleon correlations

in One- and Two-Body Momentum Distributions

M. Alvioli

National Research Council

Research Institute for Geo-Hydrological Hazards

Perugia, Italy

CONTENTS

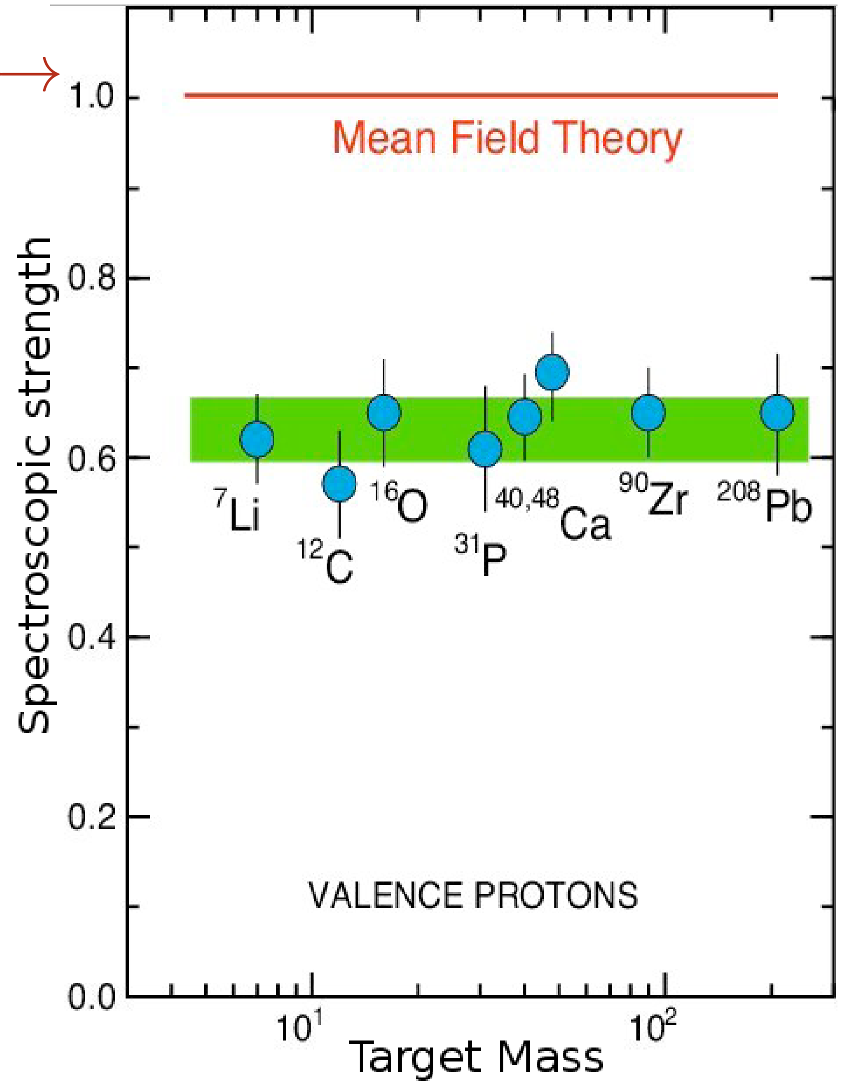
0. Experimental Evidence for Correlations - I

Modern picture of nuclei: *independent particle model breakdown*

Double coincidence $A(e, e'p)X$ measurements:

predictions based on independent particle (mean field) model ⟶

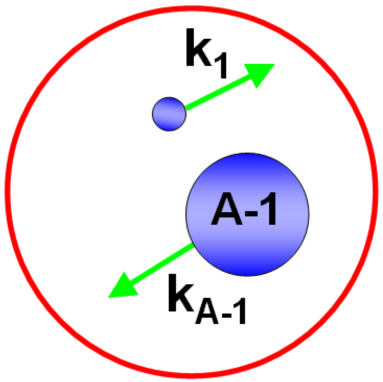
measured strengths ⟶



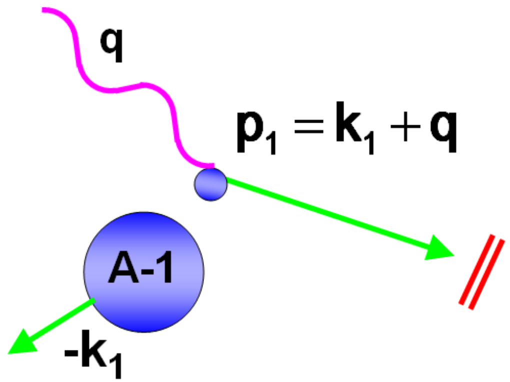
these results suggest that
a significant fraction of nucleon are
not in shell model states

0. Mean Field *vs* Correlated $A(e,e'p)X$

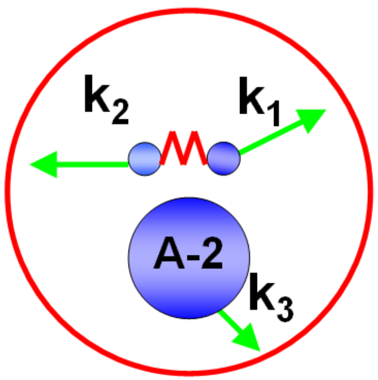
Mean Field picture:



$$\longrightarrow \quad \boldsymbol{k}_1 + \boldsymbol{k}_{A-1} = 0 \quad \longrightarrow$$



Two-Body Correlations picture:

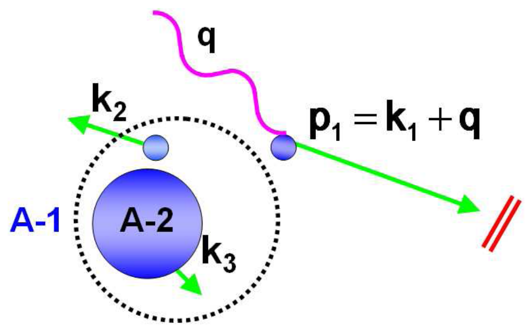


$$\longrightarrow \quad \boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_{A-1} = 0 \quad \longrightarrow$$

$$\boldsymbol{k}_1 \simeq -\boldsymbol{k}_2$$

$$\Downarrow$$

back-to-back nucleons

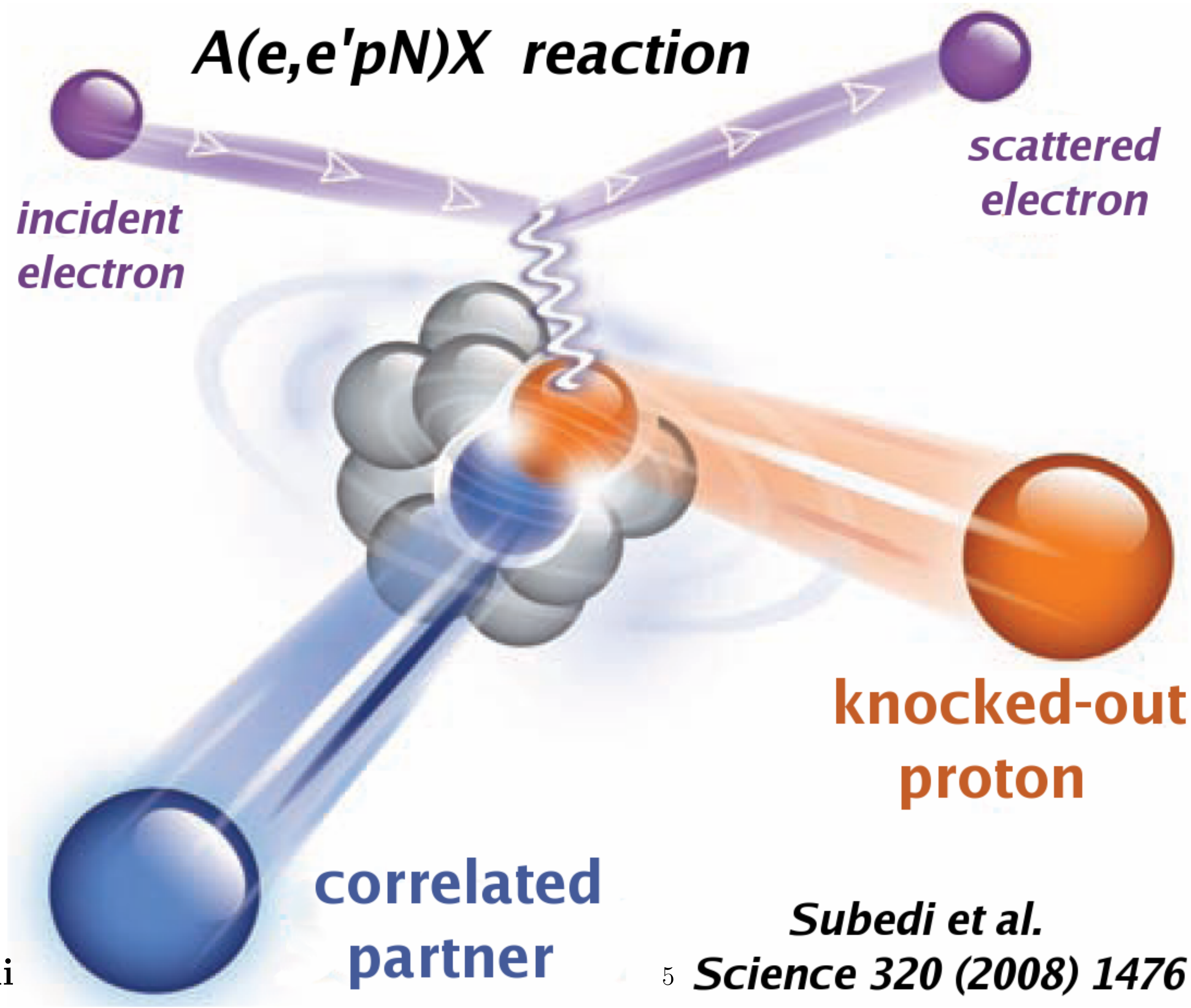


Ciofi, Simula, Frankfurt, Strikman ***PRC44 (1991)***

Ciofi, Simula ***PRC53 (1996)***

0. Experimental Evidence for Correlations - II

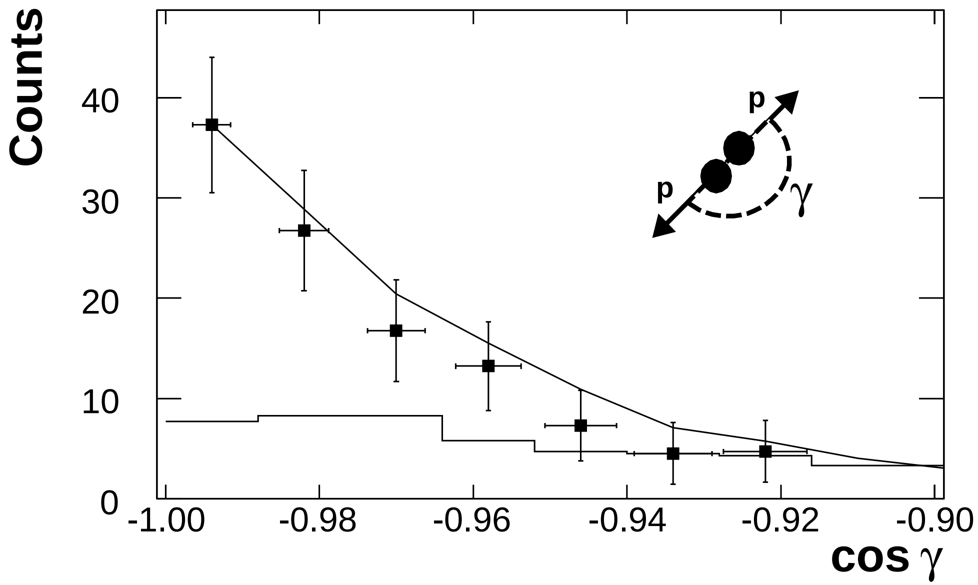
Triple coincidence $A(e,e'pp)X$ and $A(e,e'pn)X$ measurements:



0. Experimental Evidence for Correlations - III

small center of mass momentum,
strong back to back correlation

⇓

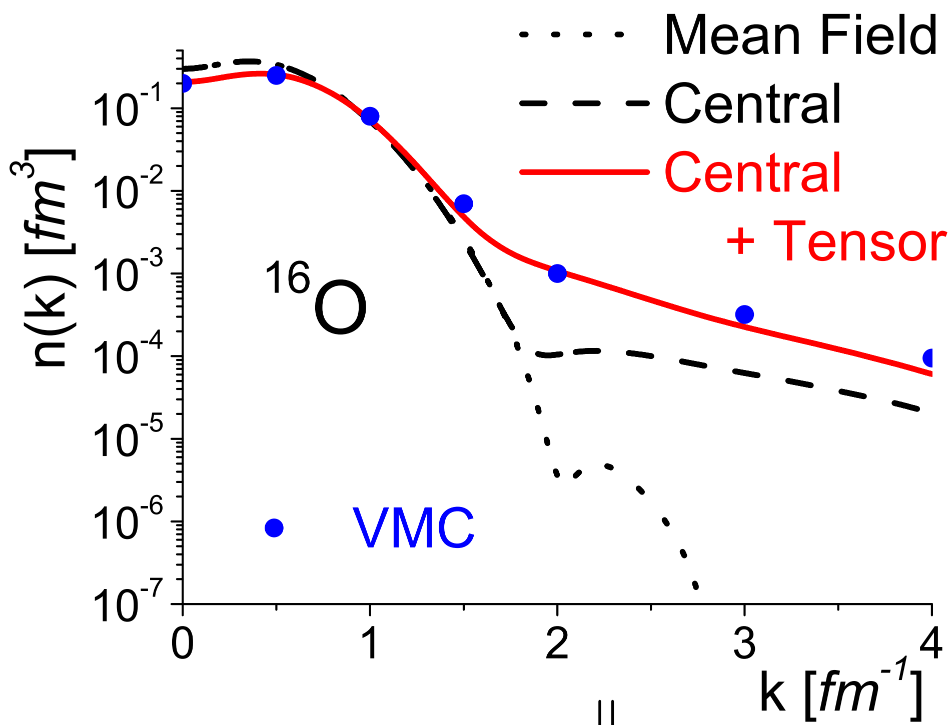


⇓

$A(p, 2p)$ BNL experiment
Tang *et al.*, ***PRL90 (2003)***
and theoretical predictions
Ciofi, Simula ***PRC53 (1996)***

large relative momentum,
strong *pn* dominance

⇓

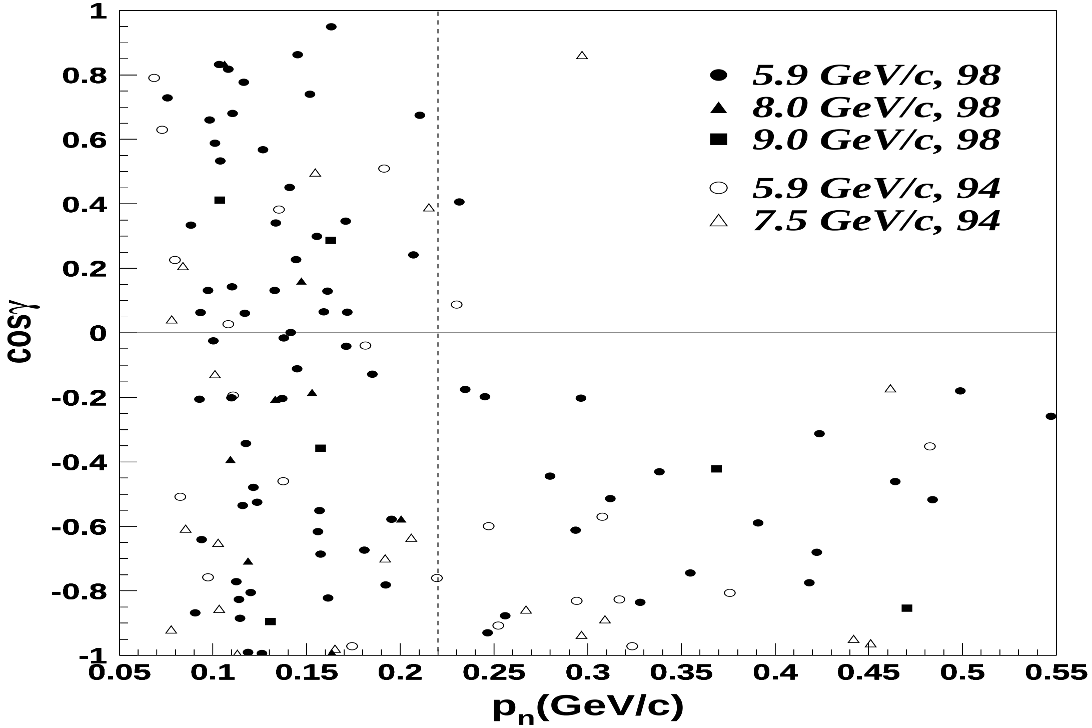


⇓

interpreted as *tensor* correlations
Wiringa *et al.* ***PRC46 (1999)***
Alvioli *et al.*, ***PRC72 (2005)***

0. Experimental Evidence for Correlations - II

$A(p, ppn)X$ measurements for different beam momenta:



- p_n: neutron momentum
- γ: n angle relative to the struck proton direction

- *no correlation* ***below*** Fermi momentum
- *back-to-back correlation* ***above*** Fermi momentum

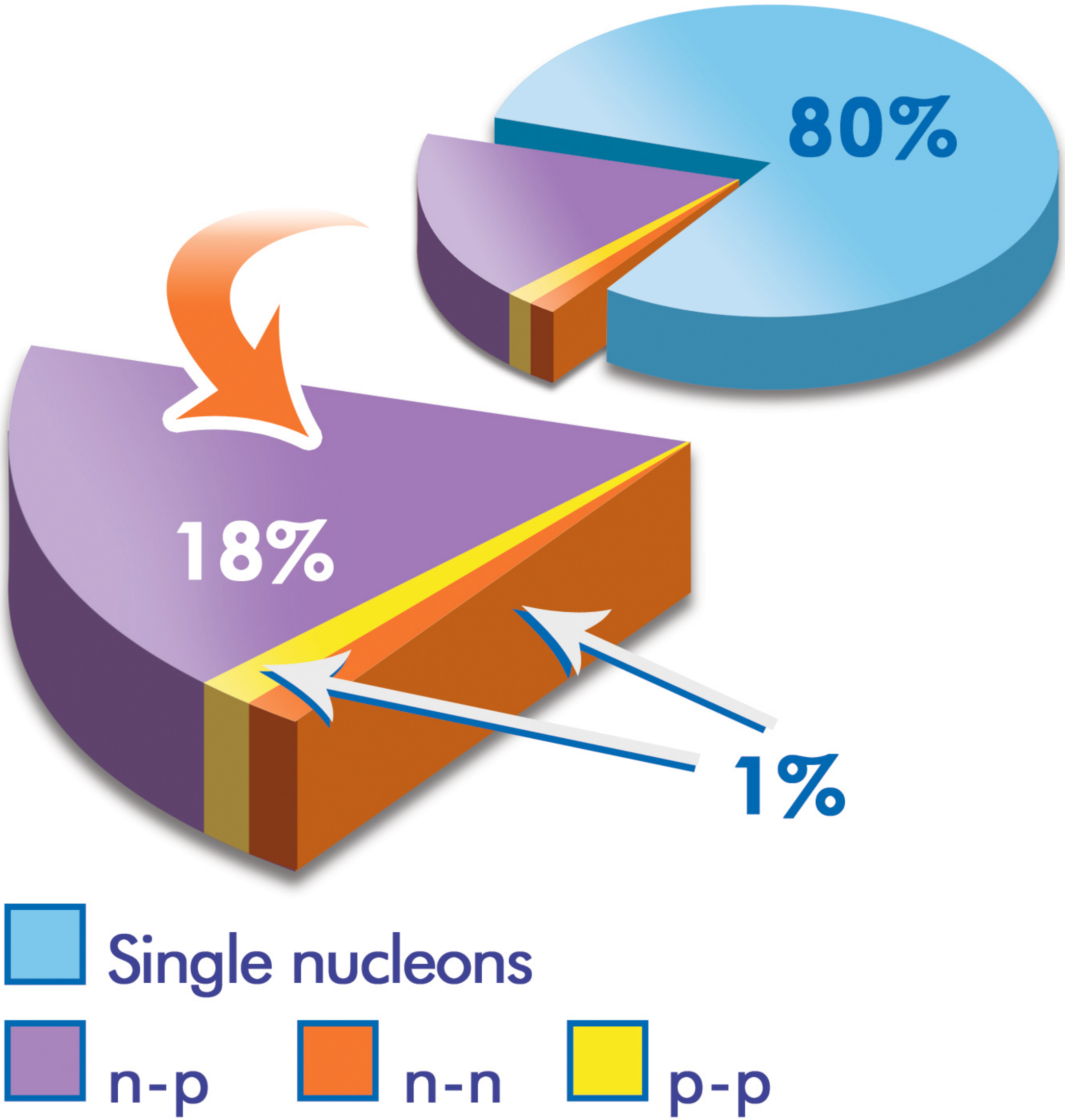
data interpreted within the ***T****wo-****N****ucleon* ***C****orrelation convolution* model

Piasetzky, Sargsian, Frankfurt, Strikman, Watson ***PRL97 (2006)***

data from:

Aclander *et al.*, ***PLB453 (1999)***; Tang *et al.*, ***PRL90 (2003)***

0. Experimental Evidence for Correlations - III



- combined results of experiments on ^{12}C show that independent particle model accounts only for 80% of the nucleons
- 20% of the nucleons are **correlated**; we call these configurations Short Range Correlations (SRC)
- 18% of the nucleons are in a $proton - neutron$ SRC pair!
- theoretical calculations suggest the similar ratios using *tensor* (spin and isospin dependent) correlations!

0. why universality of NN Short Range Correlations?

Observations:

- inclusive $A(e,e')X$ measurements on several targets reveals relative to ^3He show separate plateaux: $1.5 < x < 2$ and $x > 2$
- $^{12}C(e,e'pN)X$ and $^{12}C(p,2p)X$ reveals SRCs with strong dominance of $\boldsymbol{pn}$ *tensor* correlations relative to $\boldsymbol{pp}$;

Forthcoming experiments for the investigation of SRCs:

- investigation of pp $vs.$ pn SRCs with ^4He target
- investigation of pp $vs.$ pn SRCs with ^{40}Ca and ^{48}Ca targets
- planning of three-nucleon emission experiments for 3B correlations search

Hypotheses and models to be tested:

- quantifying 2B SRCs ($i.e.$ region of 2N's balancing momenta)
- quantifying 3B SRCs ($i.e.$ region of 3N's balancing momenta)
- update of TNC convolution model (2N motion factorization in WFs)

0. Nuclear Hamiltonian

- The non-relativistic nuclear many-body problem:

$$\hat{\mathbf{H}}\,\Psi_n \,=\, E_n\,\Psi_n\,, \qquad \hat{\mathbf{H}} \,=\, -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{ij}\hat{v}_{ij} + \ldots$$

- *Exact* ground-state wave functions obtained by various methods are available for ***light nuclei*** $(A \leq 12)$;
  $\Longrightarrow$ calculations will be shown using ^2H, ^3He, ^4He WFs;
- Variational wave functions of nuclei can be obtained with approximated methods; usually difficult to use/generalize
  $\Longrightarrow$ we developed an easy-to-use *cluster expansion* technique for the calculation of basic quantities of ***medium-heavy nuclei***, ^{12}C, ^{16}O, ^{40}Ca;
- An MC generator for ***large nuclei*** such as ^{197}Au and ^{208}Pb to be used for the initialization heavy-ion collisions simulations has been developed
  http://http://users.phys.psu.edu/~malvioli/eventgenerator/

0. Calculation of basic quantities

- one- and two-body densities:

$$\rho_N(\boldsymbol{r}_1, \boldsymbol{r}'_1) \;=\; \sum\!\!\!\!\!\!\int \prod_{j=2}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1, ..., \boldsymbol{x}_A)\, \hat{P}_N\, \Psi_A^o(\boldsymbol{x}'_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_A)$$

$$\rho_{pN}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2) \;=\; \sum\!\!\!\!\!\!\int \prod_{j=3}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1, ..., \boldsymbol{x}_A)\, \hat{P}_p \hat{P}_N\, \Psi_A^o(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_3, ..., \boldsymbol{x}_A)$$

- one- and two-body momentum distributions:

$$n_N(k_1) \;=\; \frac{1}{(2\pi)^3} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1\, e^{-\boldsymbol{k}_1\cdot(\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \rho_N(\boldsymbol{r}_1, \boldsymbol{r}'_1)$$

$$n_{pN}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \;=\; \frac{1}{(2\pi)^6} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1 d\boldsymbol{r}_2 d\boldsymbol{r}'_2\, e^{-\boldsymbol{k}_1\cdot(\boldsymbol{r}_1 - \boldsymbol{r}'_1)}\, e^{-\boldsymbol{k}_2\cdot(\boldsymbol{r}_2 - \boldsymbol{r}'_2)} \;.$$

$$\cdot\, \rho_{pN}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2) \quad\longleftrightarrow\quad n_{pN}^{(2)}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$

0. Two-Body Momentum Distributions

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}' = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$$

$$\boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{R}' = \frac{1}{2}(\boldsymbol{r}'_1 + \boldsymbol{r}'_2)$$

we have

$$n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^6} \int d\boldsymbol{r} d\boldsymbol{r}' d\boldsymbol{R} d\boldsymbol{R}' \, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R}')$$

and

$$n(\boldsymbol{k}) = \int d\boldsymbol{K} \, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r} d\boldsymbol{r}' d\boldsymbol{R} \, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R})$$

$$n(\boldsymbol{K}) = \int d\boldsymbol{k} \, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r} d\boldsymbol{R} d\boldsymbol{R}' \, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}; \boldsymbol{R}, \boldsymbol{R}')$$

$\boldsymbol{K}_{CM} = 0$ corresponds to $\boldsymbol{k}_2 = -\boldsymbol{k}_1$, *i.e. back-to-back* nucleons

0. Using Realistic WFs of large nuclei: *Cluster Expansion*

- Cluster Expansion is a technique to reduce the computational effort in many many-body calculations; we use: $\Psi_o = \hat{F}\Phi_o = \prod_{ij}\sum_n \hat{f}^{(n)}_{ij}\Phi_o$
- Expectation value over Ψ_o of any one- or two-body operator $\hat{Q}$:

$$\frac{\langle\Psi_o|\hat{Q}|\Psi_o\rangle}{\langle\Psi_o|\Psi_o\rangle} = \frac{\langle\hat{F}^\dagger\hat{Q}\hat{F}\rangle}{\langle\hat{F}^2\rangle} = \frac{\langle\prod\hat{f}^\dagger\hat{Q}\hat{f}\rangle}{\langle\prod\hat{f}^2\rangle} = \frac{\langle\hat{Q}\prod(1+\hat{\eta})\rangle}{\langle\prod(1+\hat{\eta})\rangle} =$$

$$= \frac{\langle\hat{Q}\,(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle}{\langle(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle} \simeq \frac{\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle}{1+\langle\sum\hat{\eta}\rangle} =$$

$$\simeq \left[\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle\right]\left(1-\langle\sum\hat{\eta}\rangle+\ldots\right) \simeq \langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle_L$$

- $\langle\hat{\eta}\rangle = \langle\left[\hat{f}^2-1\right]\rangle =$ is the small expansion parameter; $\langle\hat{Q}\rangle \equiv \langle\Phi_o|\hat{Q}|\Phi_o\rangle$
- we end up with ***linked*** clusters; up to 4 particles needed for 2B density, involving up to the square of the correlation operators $\hat{f} = \sum_n f_n(r_{ij})\hat{O}_n(ij)$

• at **first order** of the $\eta-$expansion, the **full correlated** `one-body mixed` density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,,$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int d\boldsymbol{r}_2 \left[H_D(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2) - H_E(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\right]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3 \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\left[H_D(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\rho_o(\boldsymbol{r}_3) - H_E(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}_3)\rho_o^{(1)}(\boldsymbol{r}_3,\boldsymbol{r}'_1)\right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij},r_{kl}) = \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\,f^{(q)}(r_{kl})\,C_{D(E)}^{(p,q)}(r_{ij},r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij},r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij},r_{kl})$ proper functions arising from spin-isospin traces;

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

• at **first order** of the $\eta-$expansion, the **full correlated** `two-body mixed` density matrix expression is as follows:

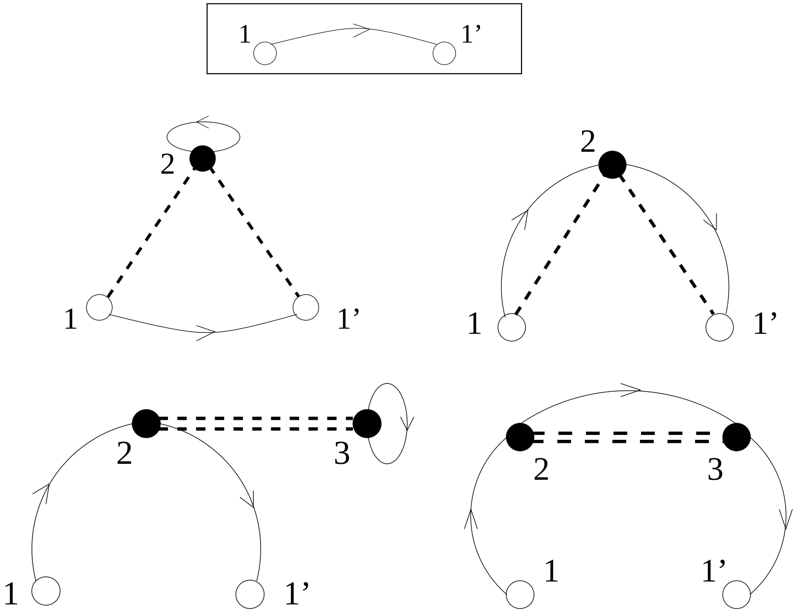
$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

with:

$$\begin{aligned}
\rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= C_D\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - C_E\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\\
\rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\\
\rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \int d\boldsymbol{r}_{\mathbf{3}}\,\hat{\eta}(r_{13},r_{1'3})\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3) + \\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\
&\qquad + \rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\
&\qquad + \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)\,]\\
\rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{4}\int d\boldsymbol{r}_{\mathbf{3}}d\boldsymbol{r}_{\mathbf{4}}\,\hat{\eta}(r_{34})\;\cdot\\
&\quad\cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,]
\end{aligned}$$

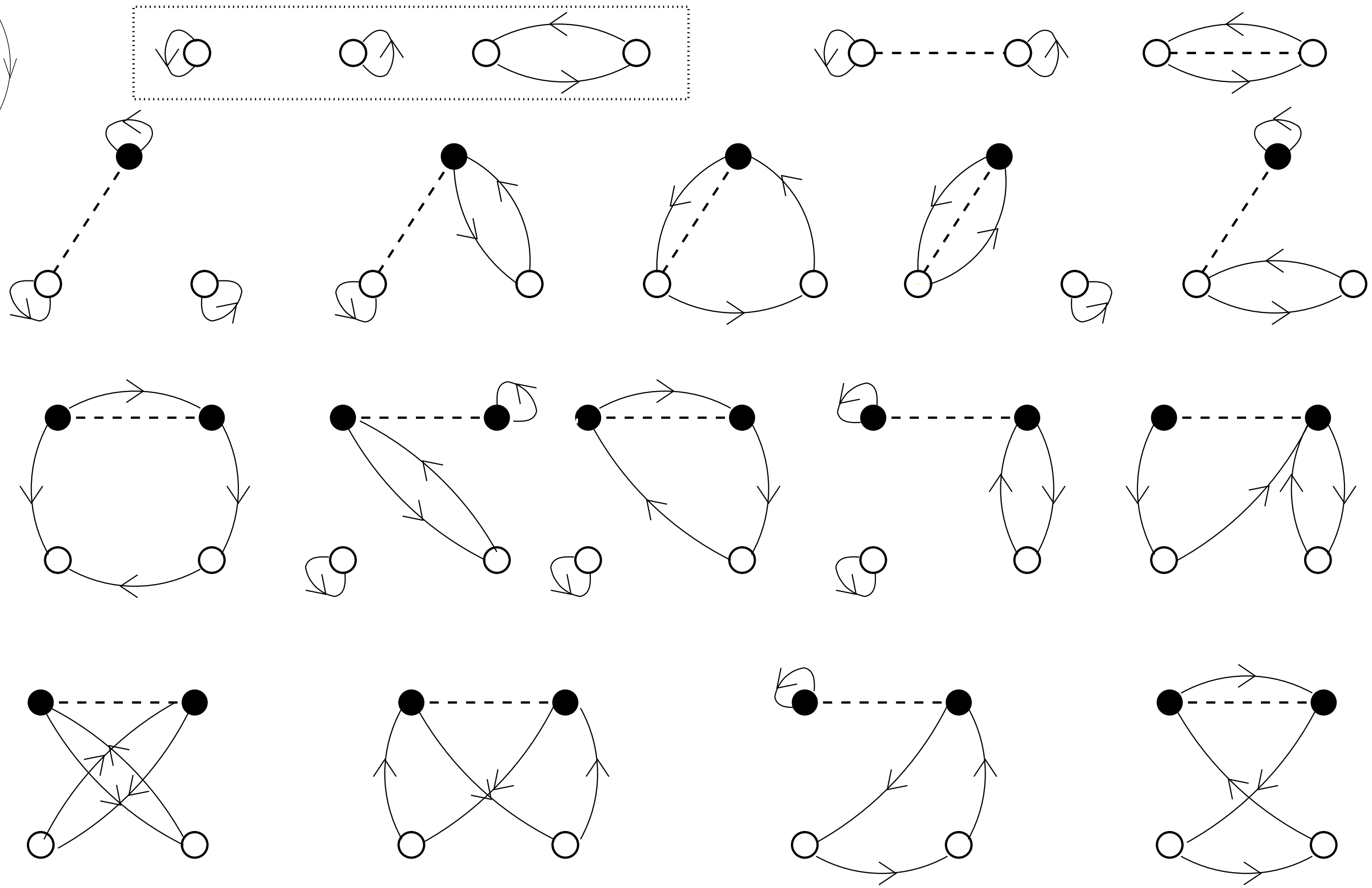
(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

(*Alvioli, Ciofi degli Atti, Morita,* PRL100 (2008))



one-body, *non-diagonal*

$\longleftarrow \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*

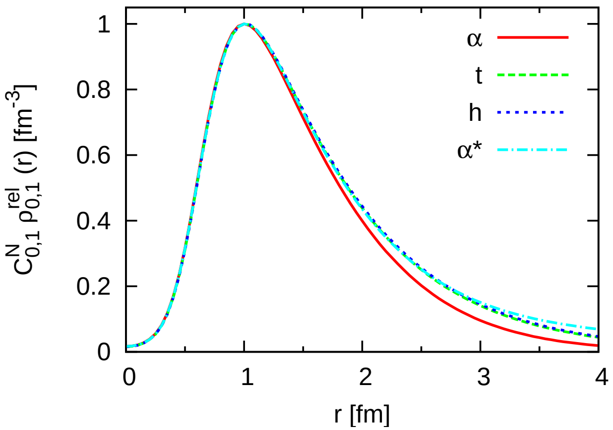


two-body, *diagonal*

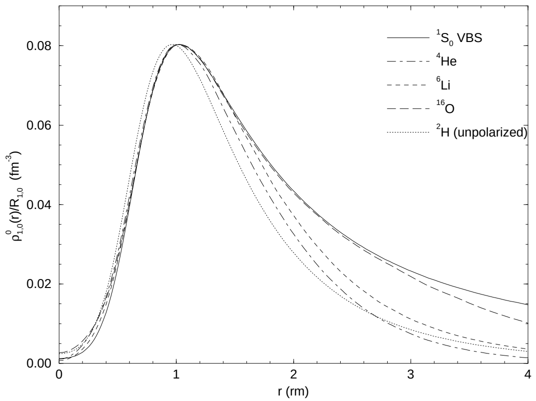
$\rho(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$

1. Correlations signatures in coordinate space densities

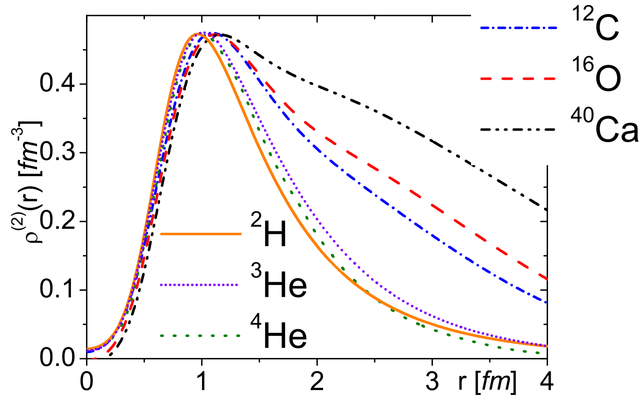
- realistic relative two-body density $\rho(r) = \int d\boldsymbol{R}\,\rho^{(2)}\Big(\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2\Big)$



Feldmeier *et al*, *Phys. Rev. C**84**, (2011) 054003*

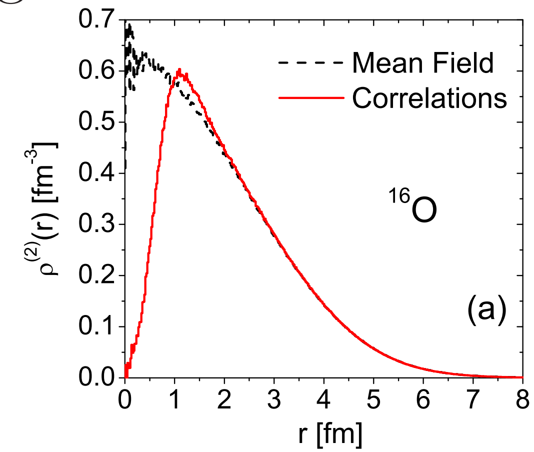


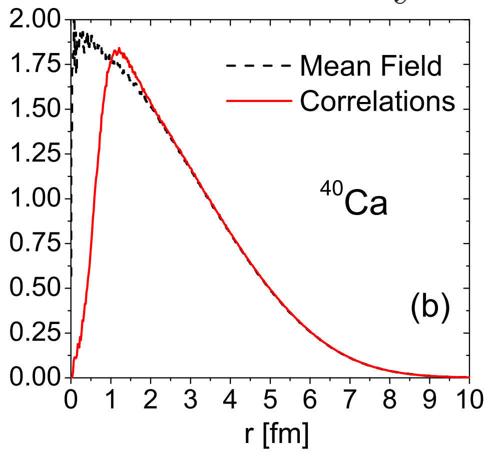
Forest *et al*, *Phys. Rev. C**54** (1996) 646*

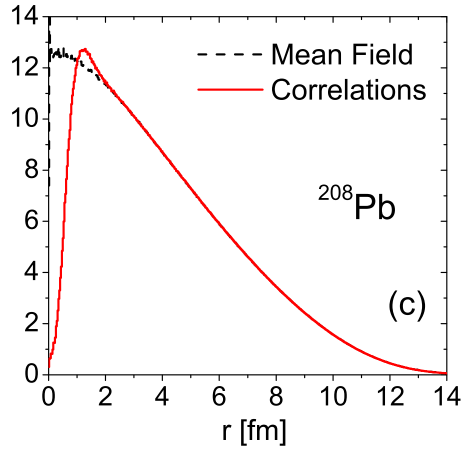


Alvioli *et al*, *Phys. Rev. C**72** (2005)*; *PRL**100** (2008); IJMPE**22** (2013)*

- MC generator to include correlations in heavy nuclei



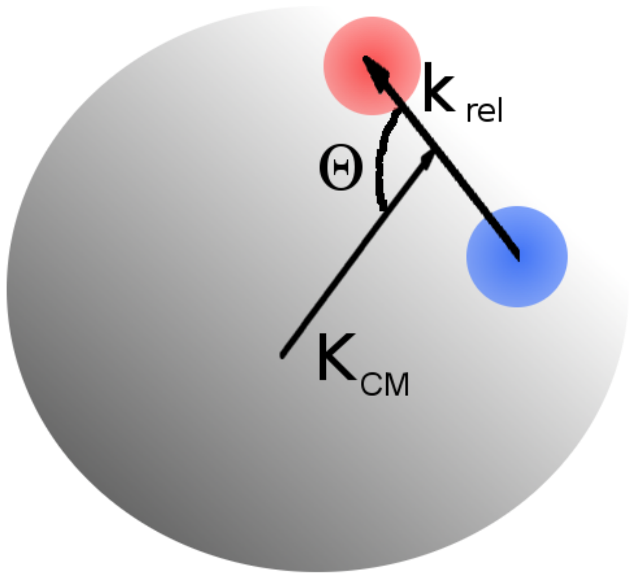




MA, H.-J. Drescher, M. Strikman, *PLB**680** (2009) 225*; MA, M. Strikman, *PRC**83** (2011) 044905*
MA, Eskola et al, *PRC**85** (2012) 034902*; MA, Strikman *PRC**85** (2012) 034902*, *PLB**722** (2013) 347*

1. Motion of a (correlated) pair in the nucleus

- Transform $\rho^{(2)}_{NN}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1; \boldsymbol{r}'_2)$ to momentum space:

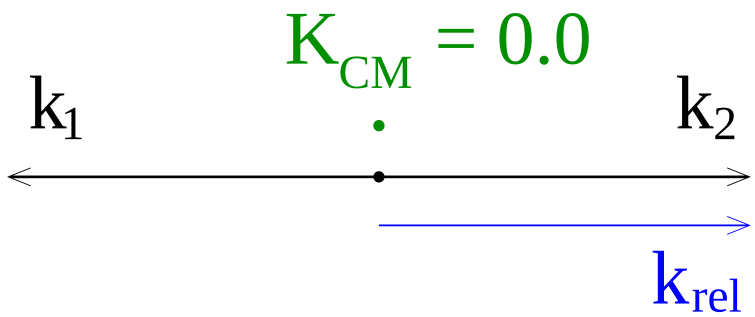


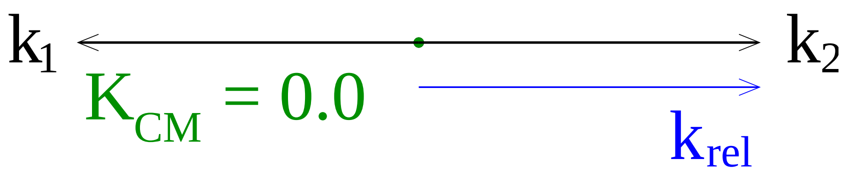
- $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) = n(k_{rel}, K_{CM}, \Theta)$

- Back-to-back nucleons correspond to K_{CM}=0 (Alvioli *et al.*, *Phys. Rev. Lett.* ***100*** *(2008)*)

- We can select any orientation of the two momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2 \longleftrightarrow \boldsymbol{k}_{rel}$, $\boldsymbol{K}_{CM}$

- We discuss: *parallel* and *perpendicular*

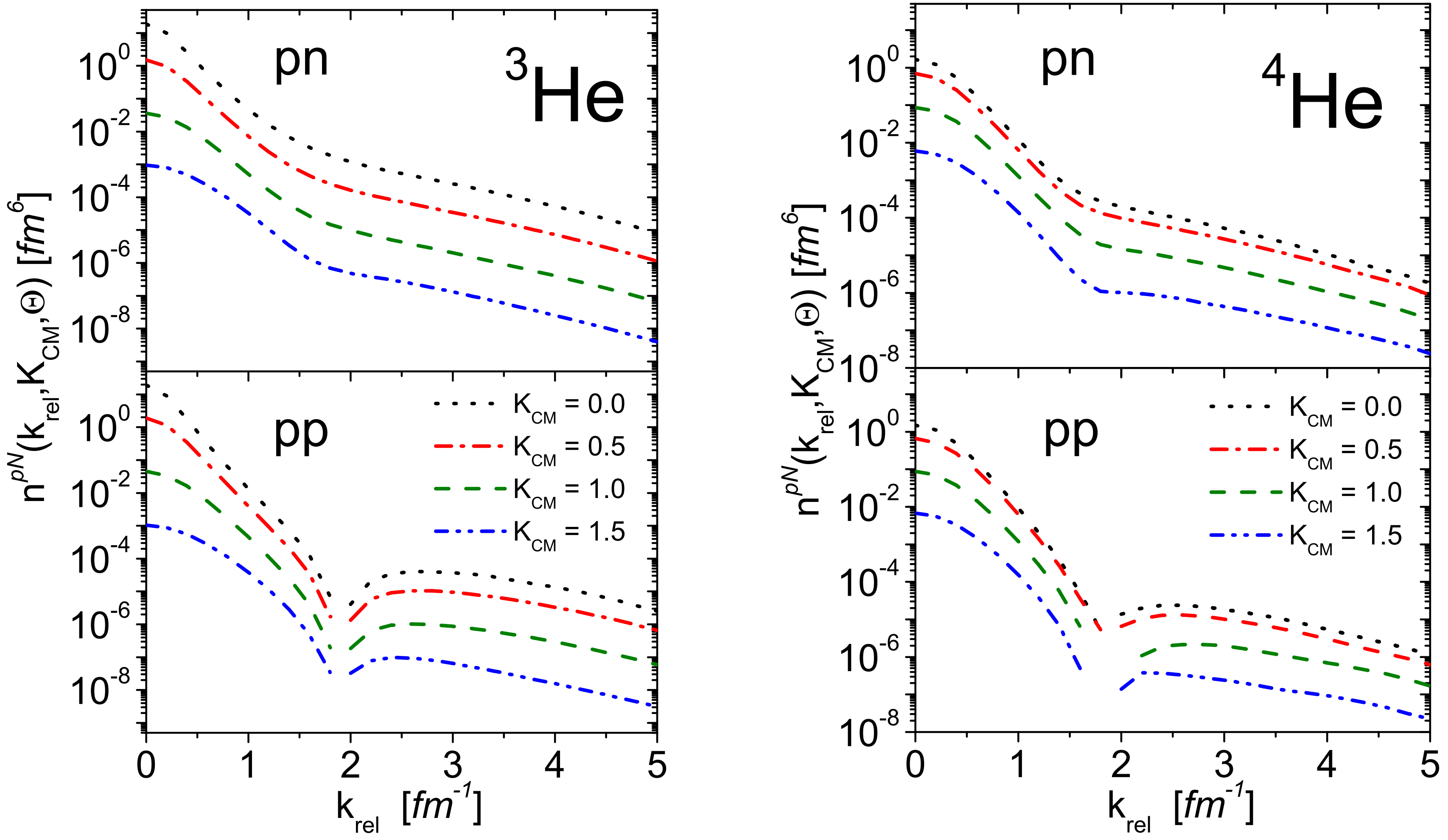




1. Two-Body momentum Distributions of Few-Body Nuclei

Pisa
AV18
+UIX

ATMS
AV8′



here $\boldsymbol{k}_{rel}$ is perpendicular to $\boldsymbol{K}_{CM}$

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

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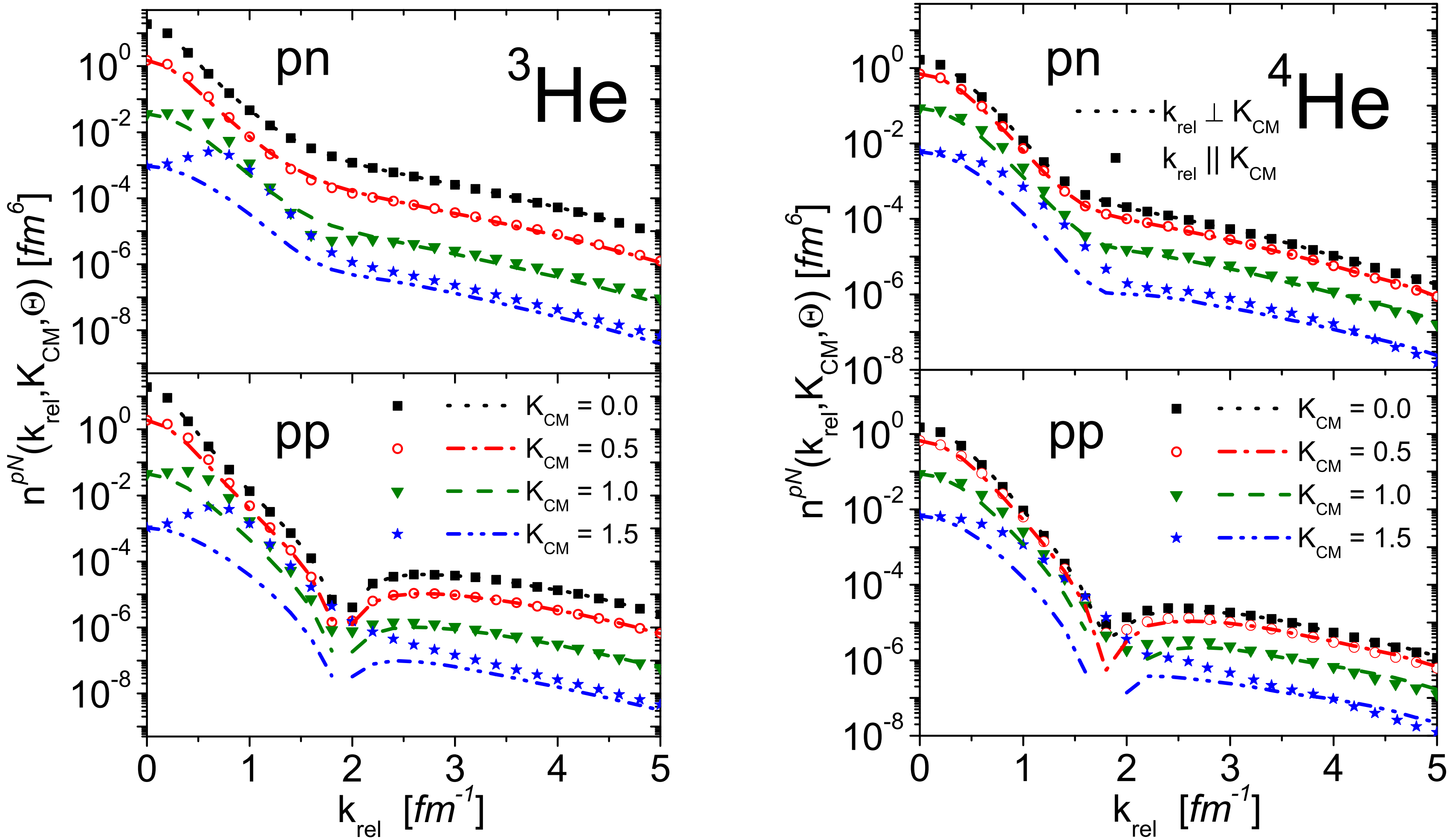

$n^{pN}(k_{rel}, K_{CM}, \Theta)$ is *angle independent* for **large** k_{rel} and **small** K_{CM}

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

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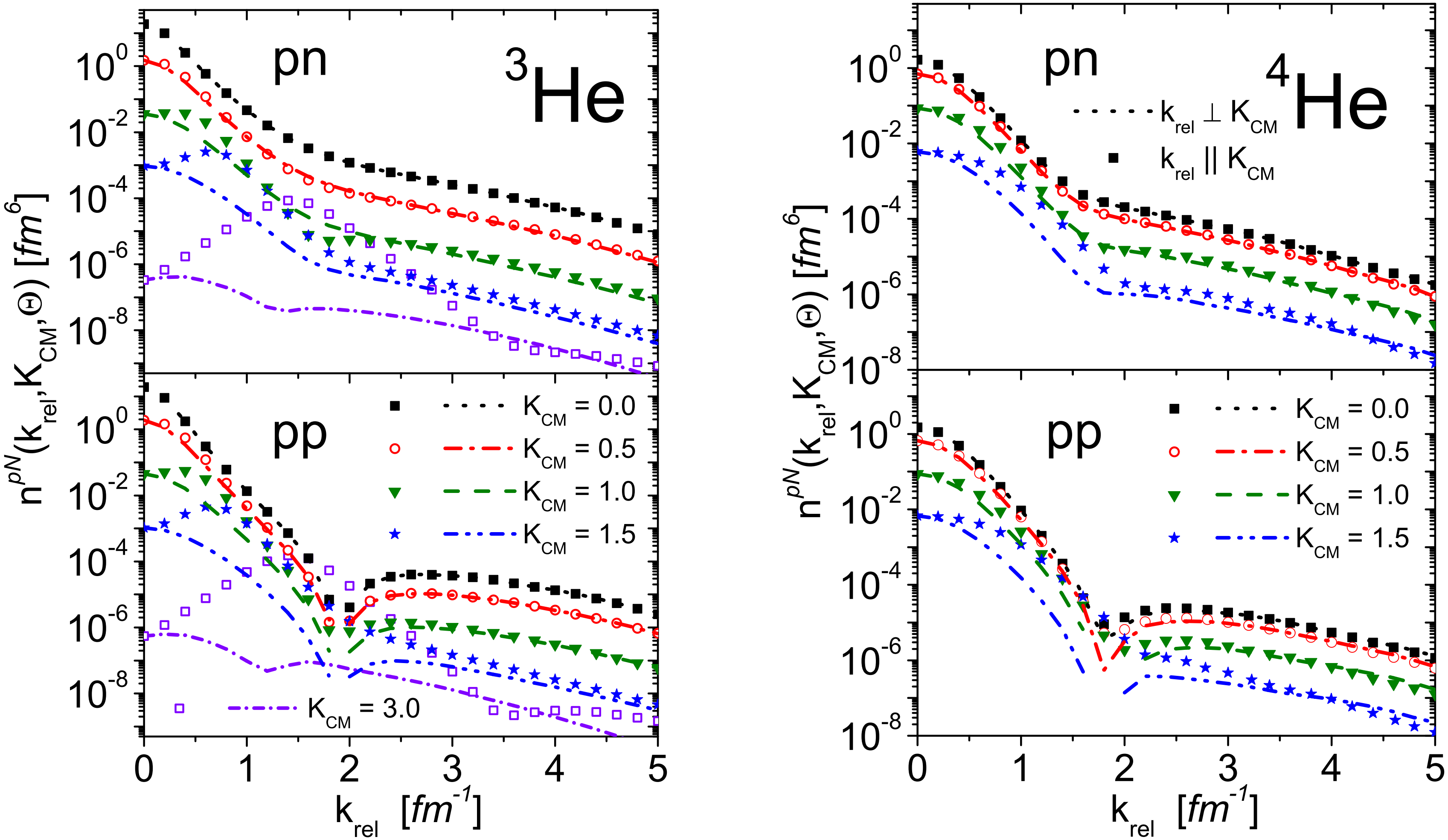
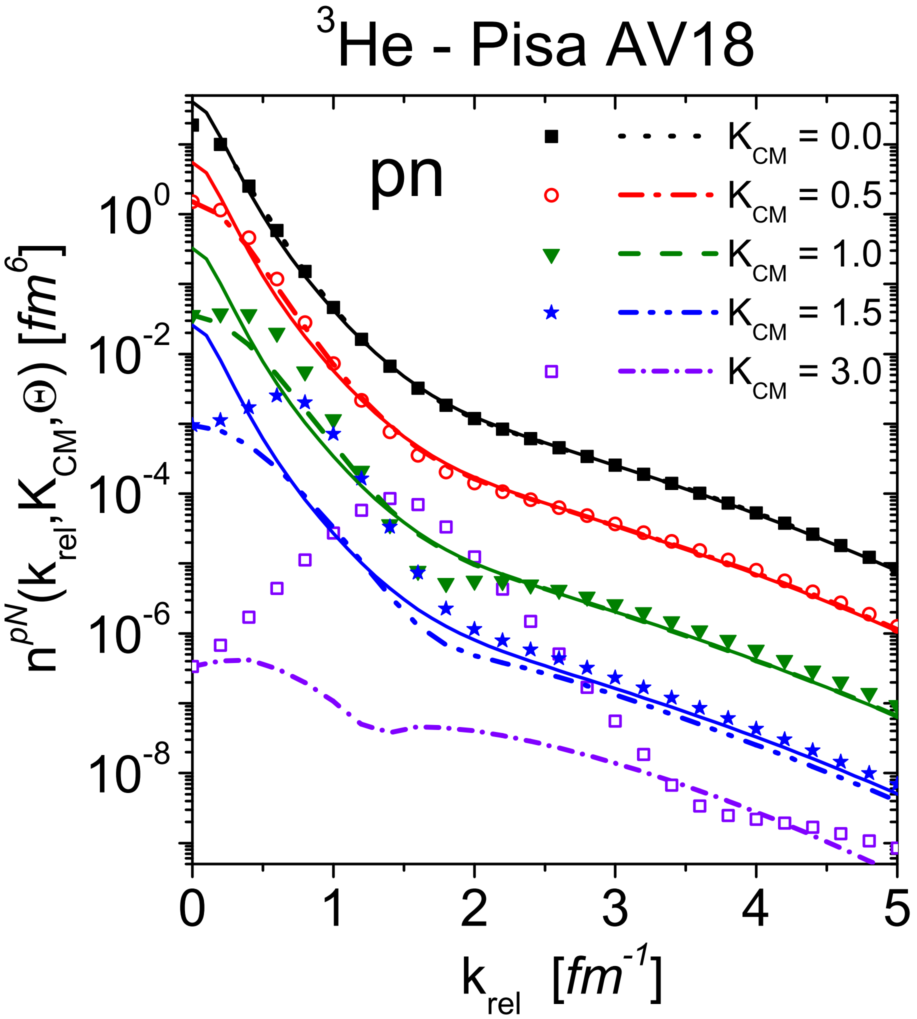
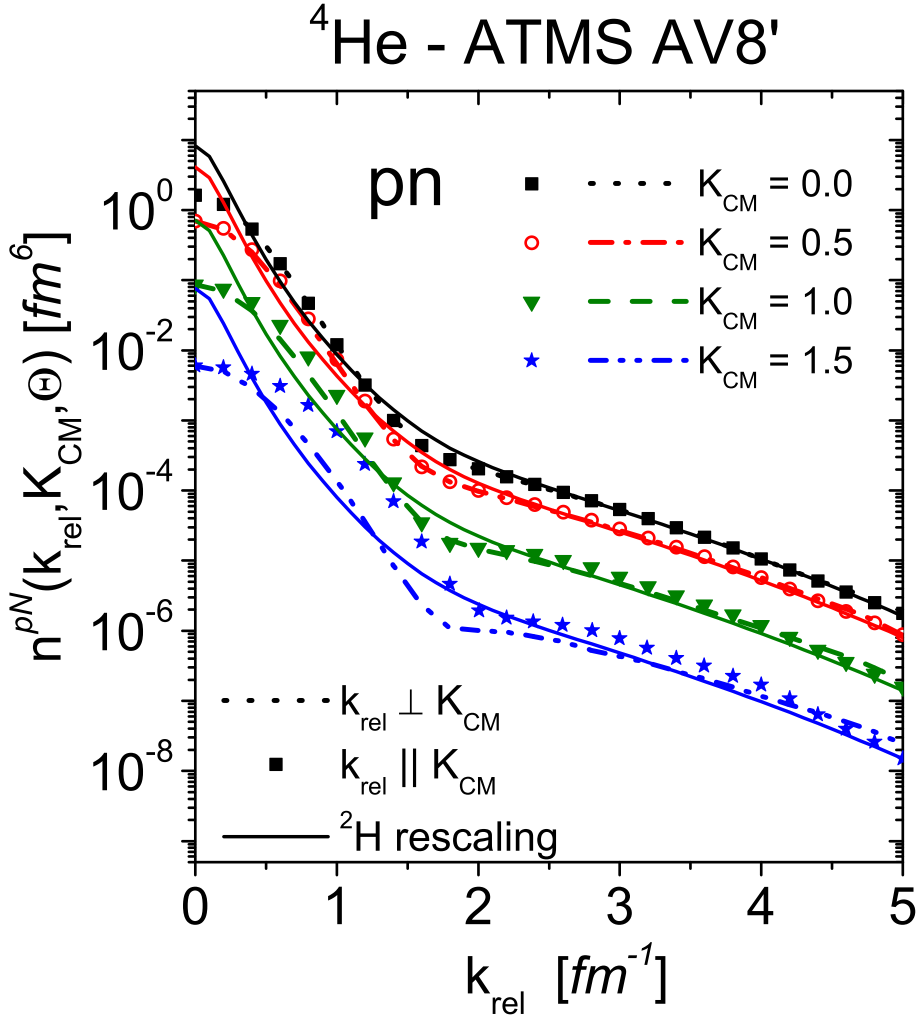


*three-body correlations must be in the **large** K_{CM} region*

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

1. Two-Body momentum Distributions of Few-Body Nuclei



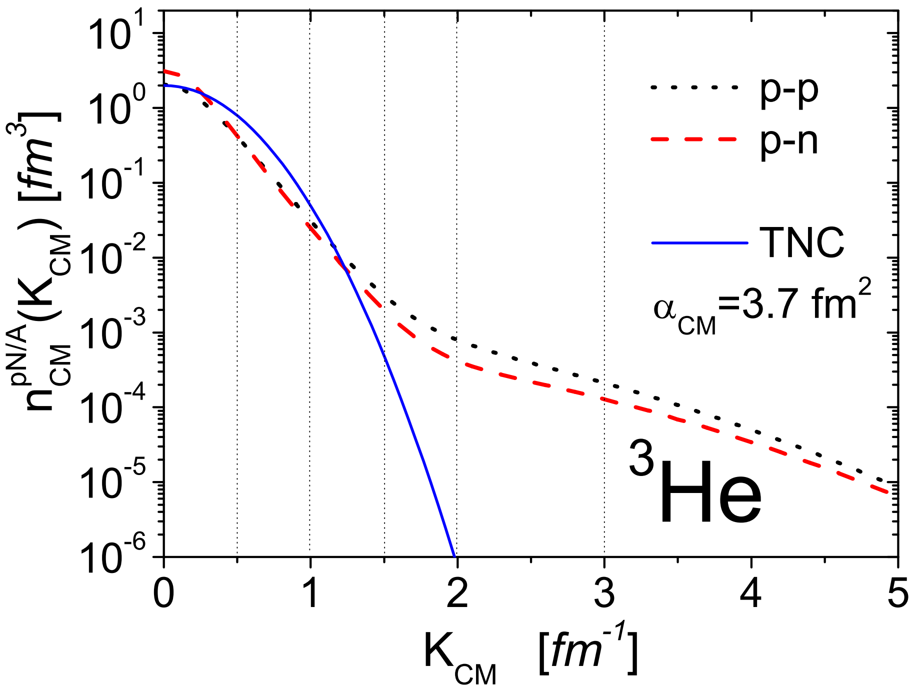


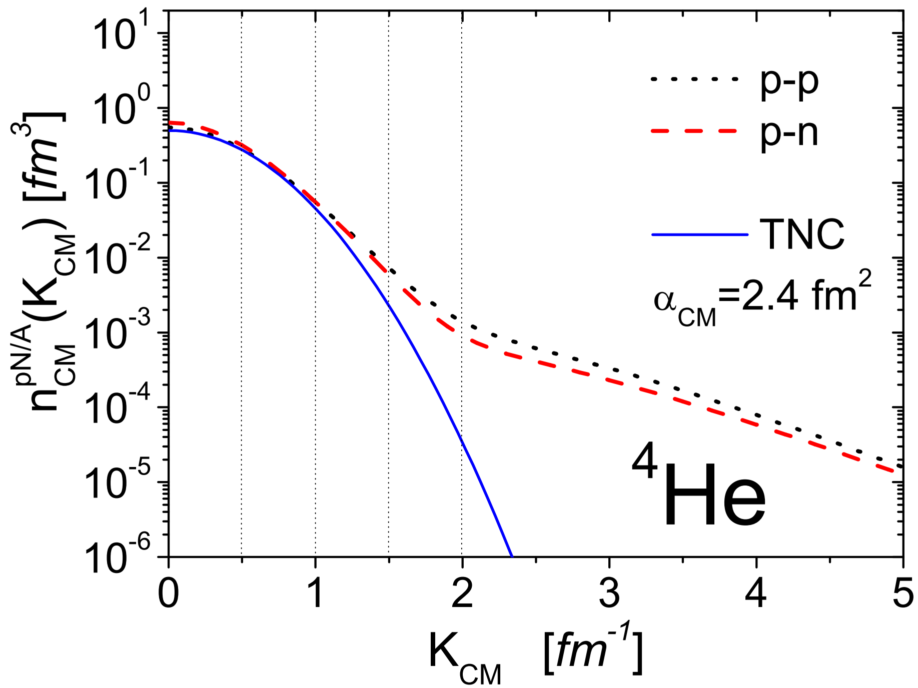
solid curves, the TNC model: rescaling of the deuteron by $n^A_{CM}(K_{CM})$!

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti,
H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

1. Two-Body momentum Distributions of Few-Body Nuclei

the integrated $\boldsymbol{K}_{CM}$ distribution: $n_{CM}(K_{CM}) = \int d\boldsymbol{k}_{rel}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$





- separation between $n(k_{rel}, K_{CM}, \Theta)$ curves corresponds to given K_{CM}
- steepest decrease in ^3He causes the curves to be more distant then in ^4He
- decrease in ^4He can be described by a gaussian at low K_{CM}

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev. C**85** (2012) 021001*

1. Standard Model of Many-Body Nuclei

- The non-relativistic nuclear many-body problem:

$$\hat{\mathbf{H}}\,\Psi_A^n = E_n\,\Psi_A^n\,, \qquad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{ij}\hat{v}_{ij} + \ldots$$

- Ψ_A^o obtained introducing **variational** correlation functions and ϕ_o:

$$\Psi_A^o = \hat{\mathbf{F}}\,\phi_o \longrightarrow \hat{\mathbf{F}} = \hat{S}\prod_{i<j}\hat{f}_{ij} = \hat{S}\prod_{i<j}\sum_n f^{(n)}(r_{ij})\,\hat{O}_{ij}^{(n)}$$

- We use ***cluster expansion*** for densities:

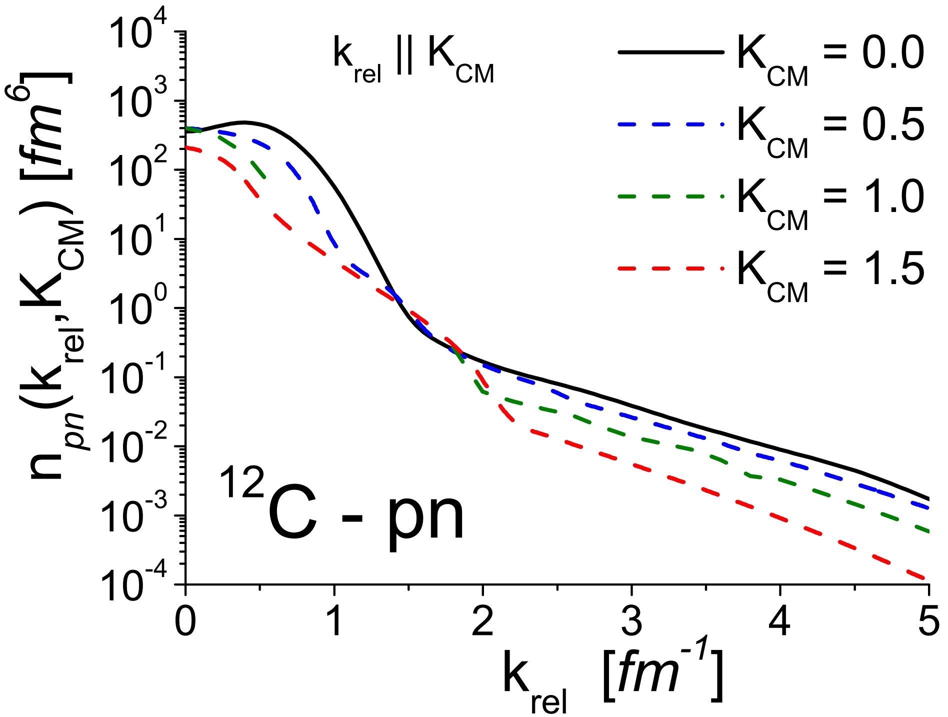
$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int\prod_{j=2}^{A} d\boldsymbol{r}_j\,\Psi_A^{o\dagger}(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_A)\,\Psi_A^o(\boldsymbol{r}'_1,\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)$$

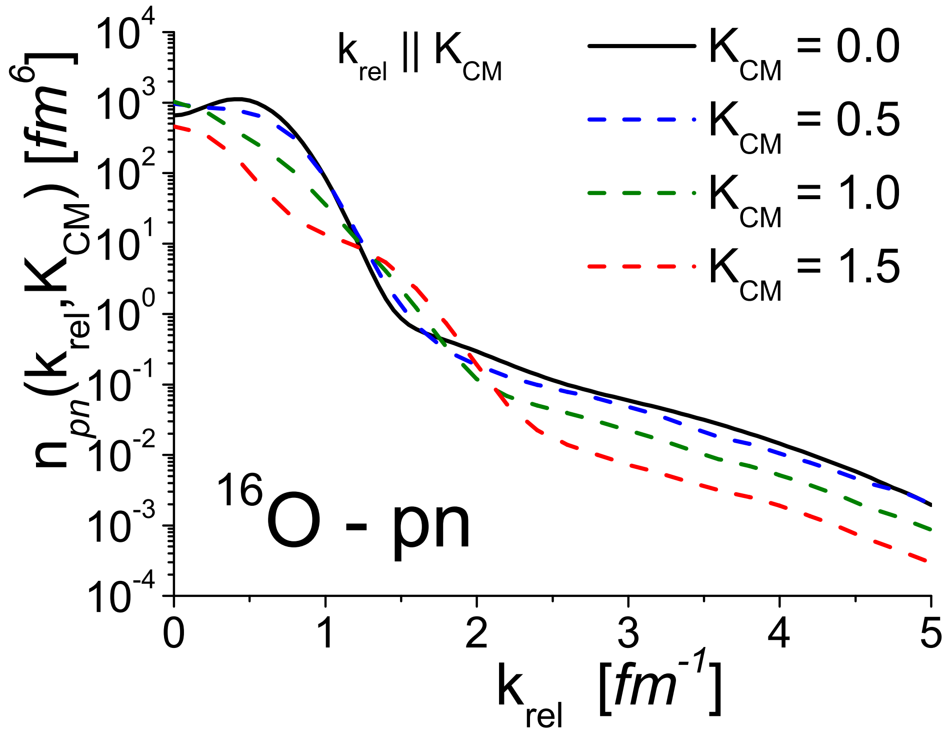
$$\rho_{(n)}^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \int\prod_{j=3}^{A} d\boldsymbol{r}_j\,\Psi_A^{o\dagger}(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)\,\hat{O}_{12}^{(n)}\,\Psi_A^o(\boldsymbol{r}'_1,\boldsymbol{r}'_2,\ldots)$$

(Alvioli, Ciofi degli Atti, Morita: *PRC* ***72*** *(2005)*; *PRL* ***100*** *(2008)*)

1. Two-Body momentum Distributions of Many-Body Nuclei

Obtained using *non-diagonal* two-body densities, $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2)$





M. Alvioli, C.Ciofi degli Atti, H.Morita, *PRL* ***100*** *(2008) 162503*
and M. Alvioli *et al.*; ***work in progress***

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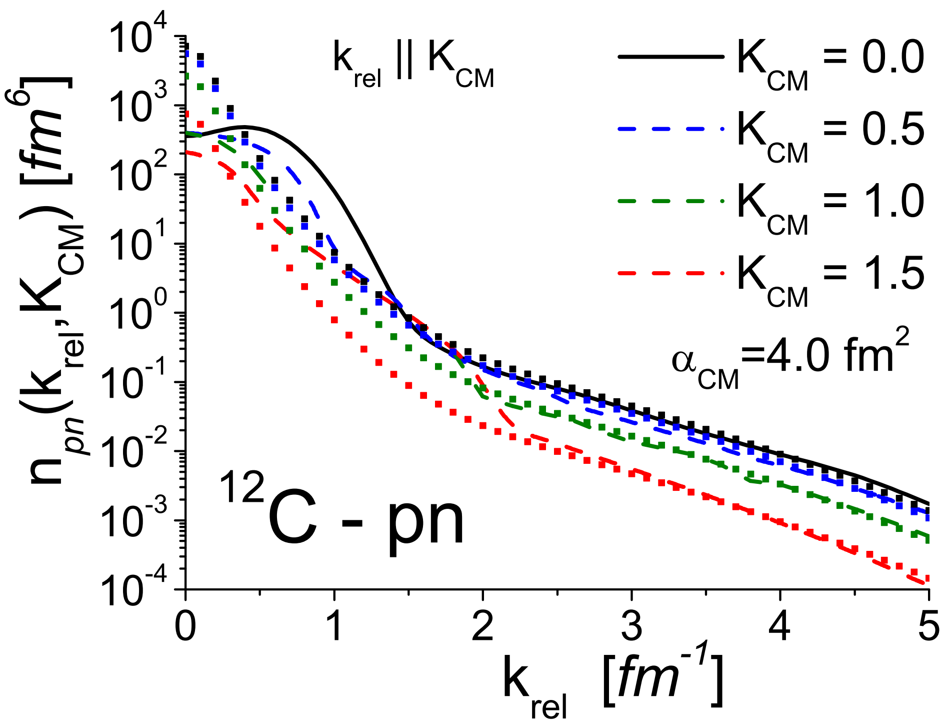
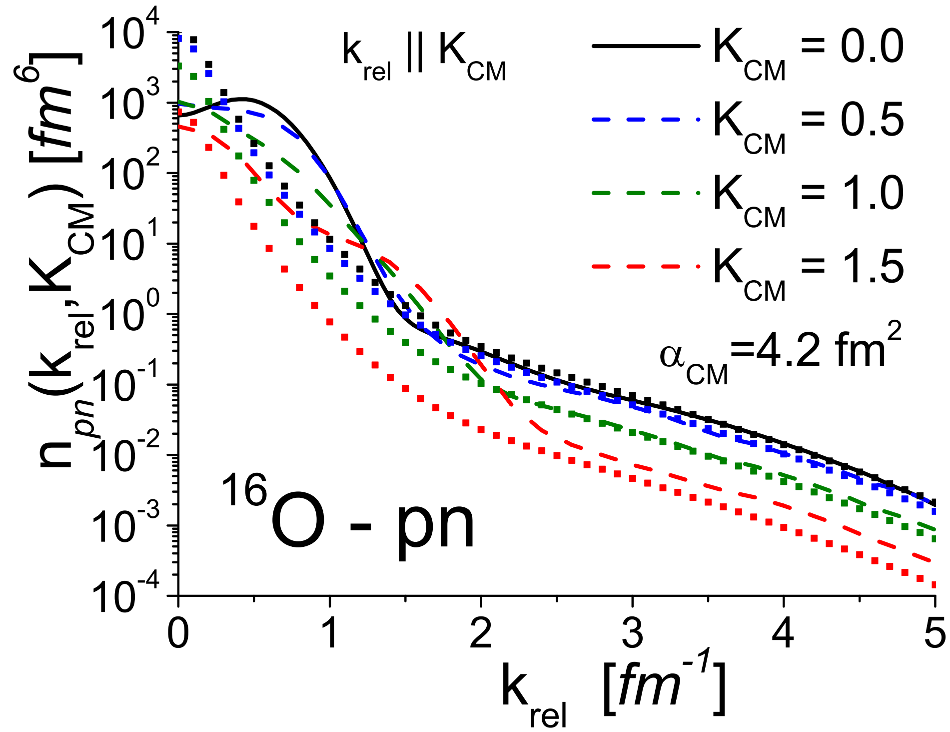
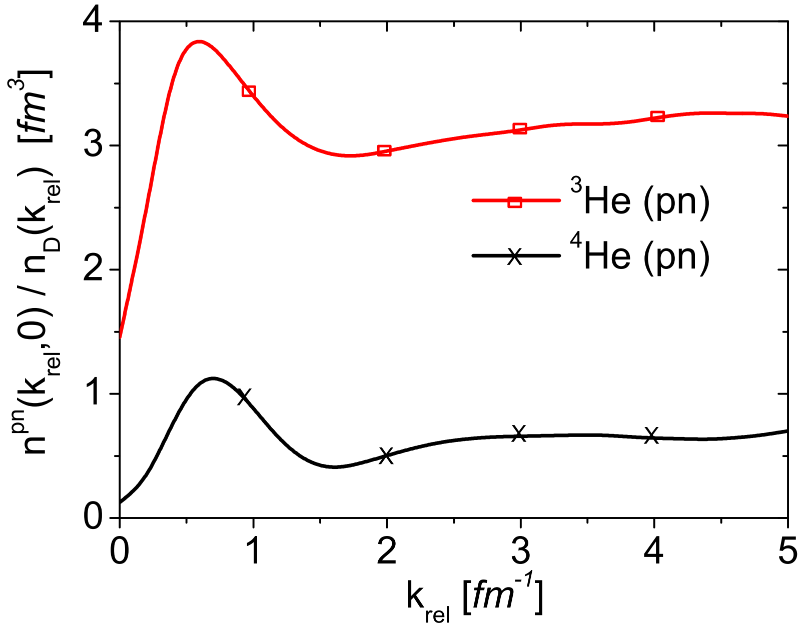




M. Alvioli, C.Ciofi degli Atti, H.Morita, *PRL* ***100*** *(2008) 162503*
and M. Alvioli *et al.*; ***work in progress***

- symbols are the rescaled deuteron with α_{CM} gaussian parameters
- same behaviour & conclusions as in the few-body case (***universality of NN correlations***)

1. Two-Body Distributions: a closer look to deuteron scaling



M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev. C**85** (2012) 021001*

- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?

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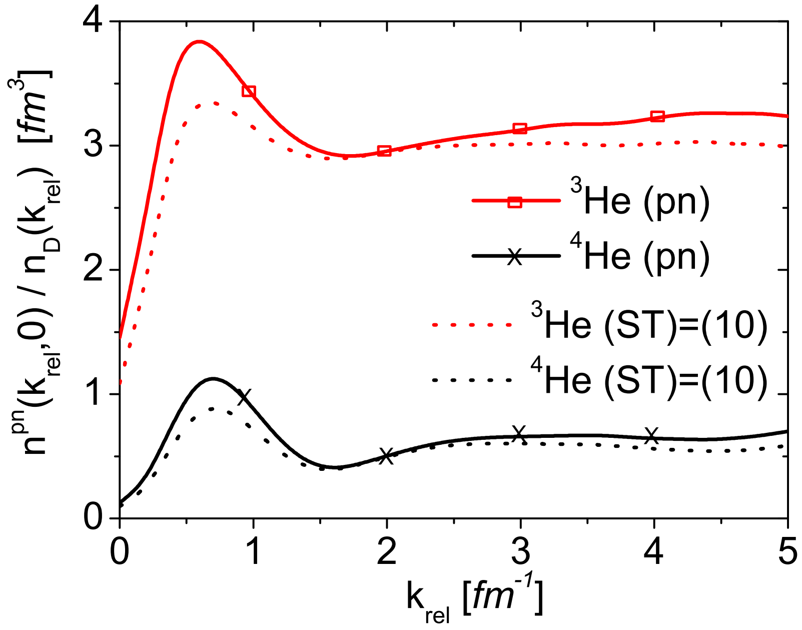

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

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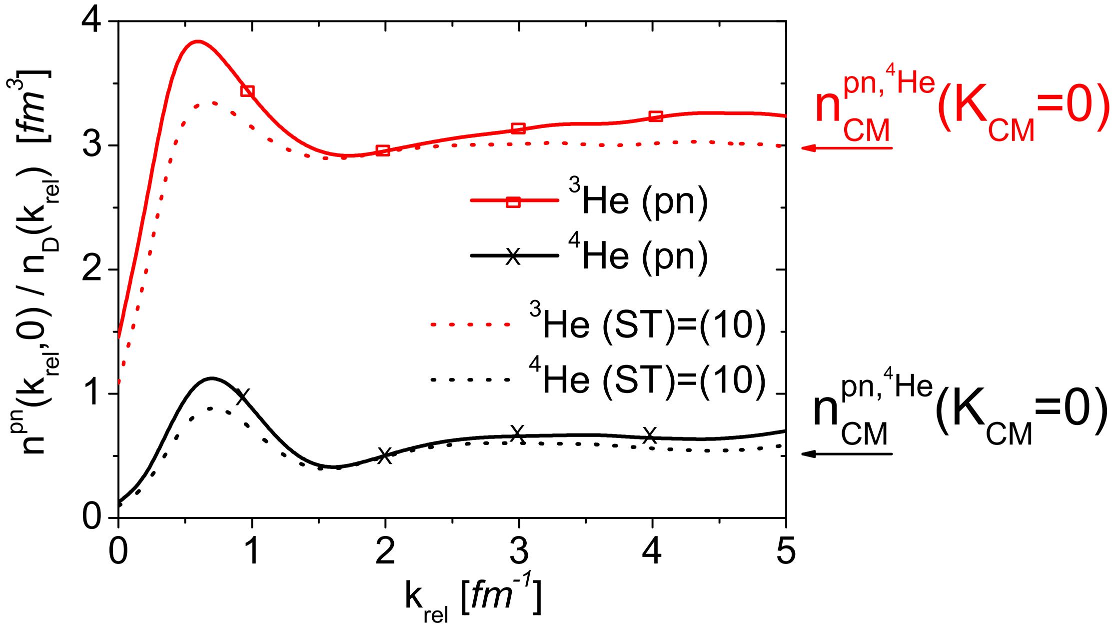
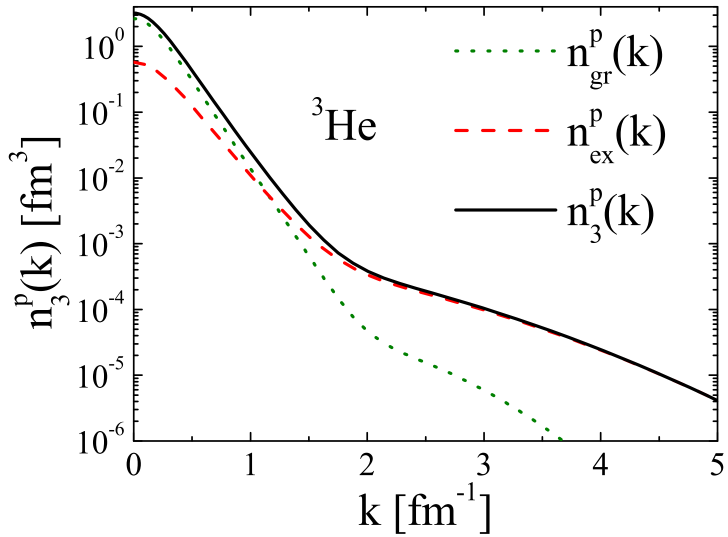


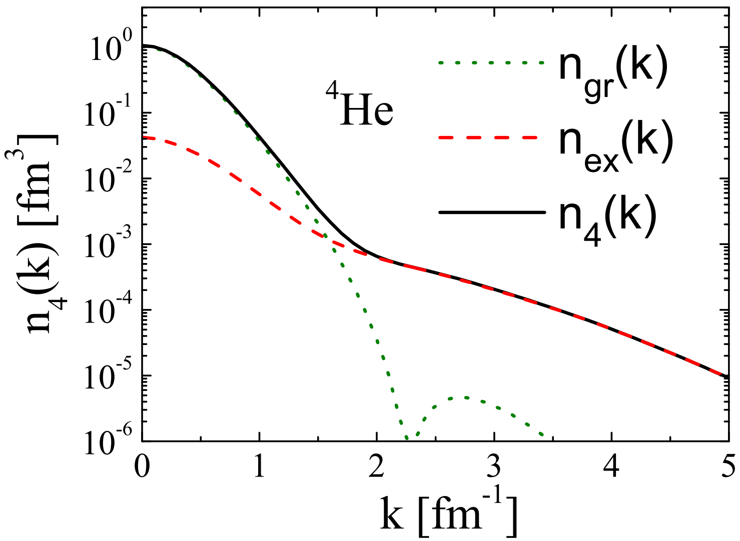
M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?
- Including only pairs with deuteron-like quantum numbers (ST)=(10) we find exact scaling!
- $n(\boldsymbol{k}_{rel}, 0)/n^D(\boldsymbol{k}_{rel}) \simeq n^D(k_{rel})n_{CM}(0)/n^D(k_{rel}) = n_{CM}(K_{CM} = 0)$!

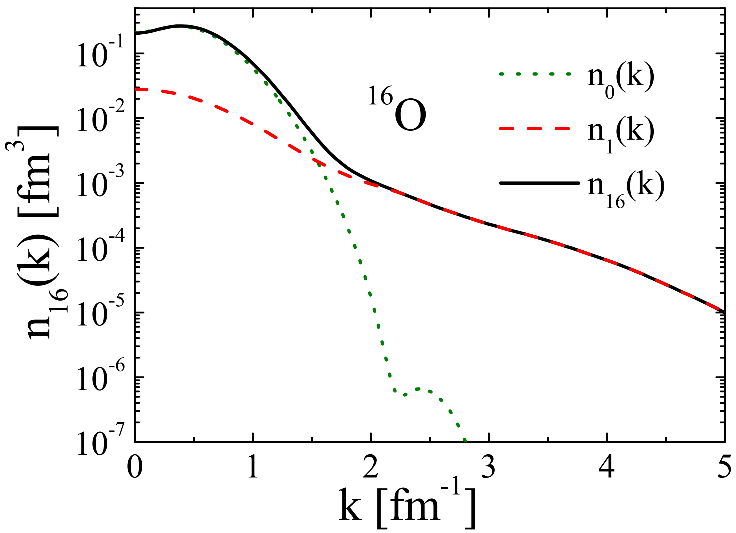
2. Defining the 2B correlation region

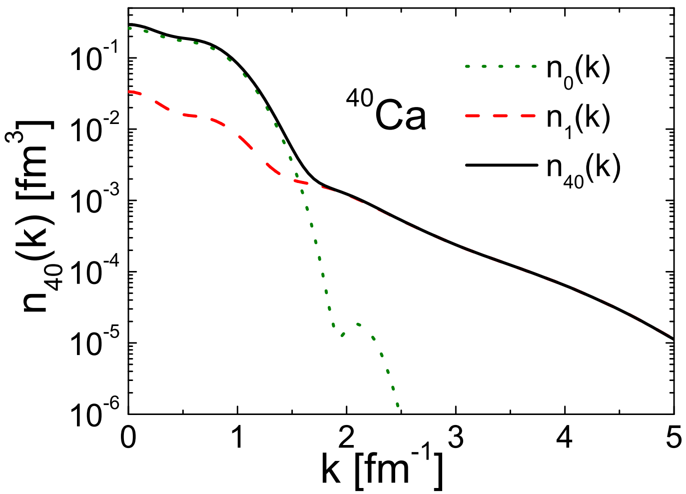
- One-body momentum distribution





$$n(k) = n_{gr}(k) + n_{ex}(k) = n_0^{TNC}(k) + n_1^{TNC}(k)$$

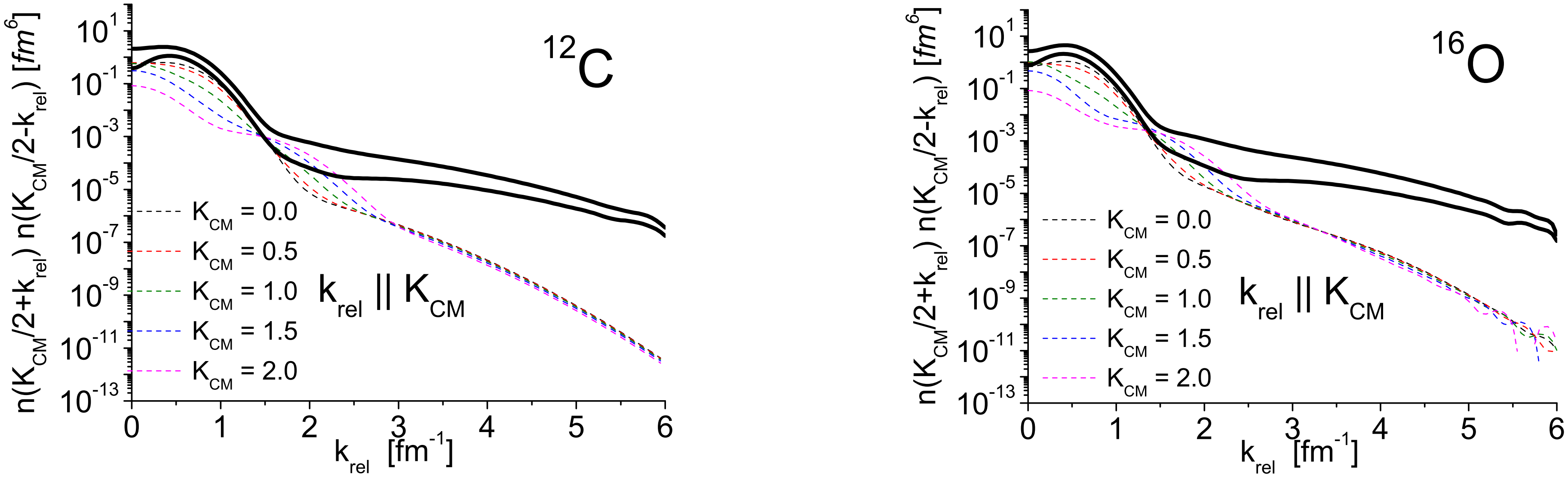




2. Two-Body Distributions: defining the 2B correlation region

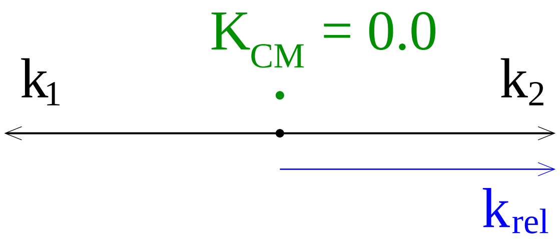
- The simplest ansatz for no correlations 2B mom dis - **M**ean **F**ield:

$$|\psi(\boldsymbol{k}_1 \ldots \boldsymbol{k}_2)|^2 = \prod_{j=1}^{A} n^{(1)}(\boldsymbol{k}_j) \quad \longrightarrow \quad n^{(2)}_{MF}(\boldsymbol{k}_1, \boldsymbol{k}_2) = n^{(1)}(\boldsymbol{k}_1)\, n^{(1)}(\boldsymbol{k}_2)$$

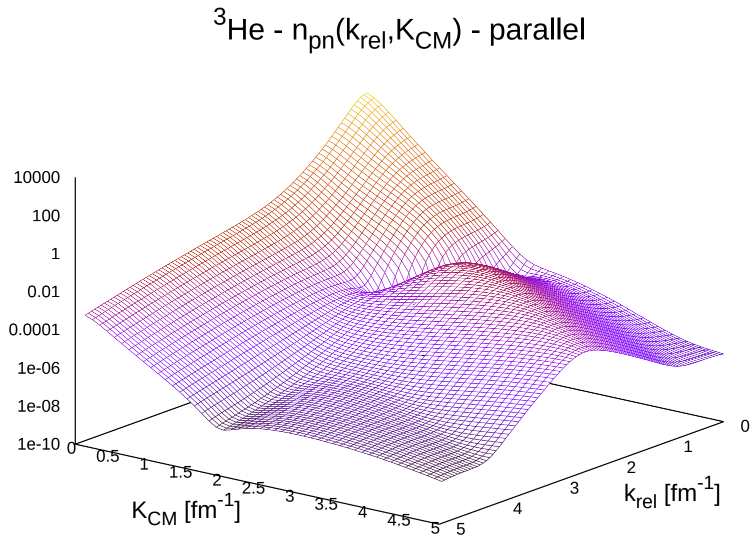


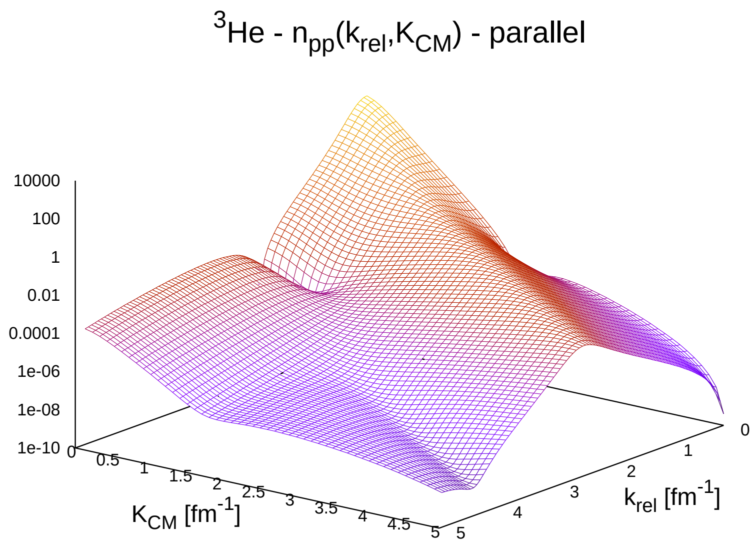
2. Two-Body Distrs: defining the 2B & 3B correlation region

parallel



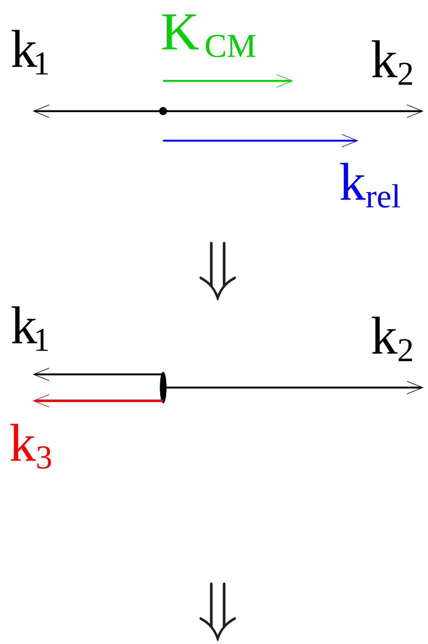
AV18 + UIX
^3He wave function
from
Nogga *et al.*,
PRC ***67*** *(2003)*





add a third nucleon:

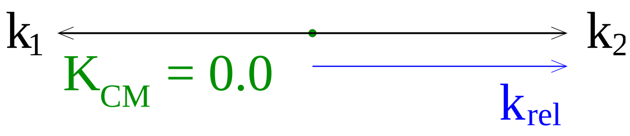
$$k_1 + k_3 = k_2$$
$$|\boldsymbol{k}_1| = |\boldsymbol{k}_3|$$



$$\boxed{K_{CM} = 2\,k_{rel}}$$

2. Two-Body Distrs: defining the 2B & 3B correlation region

perpendicular



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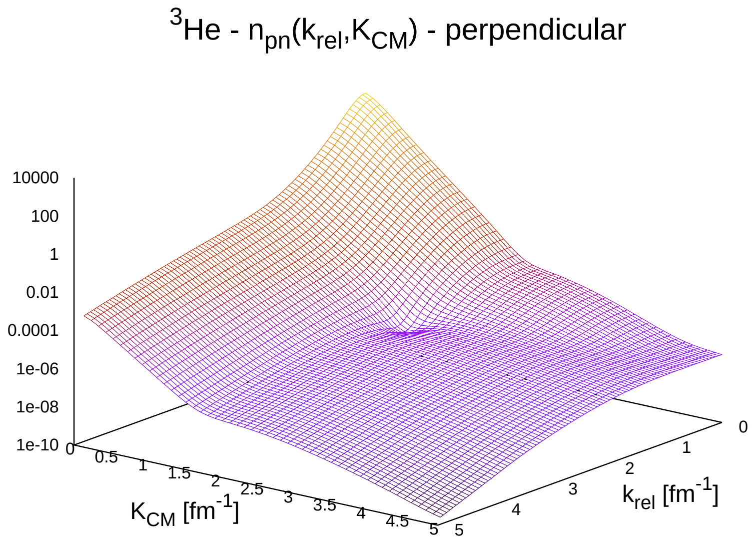
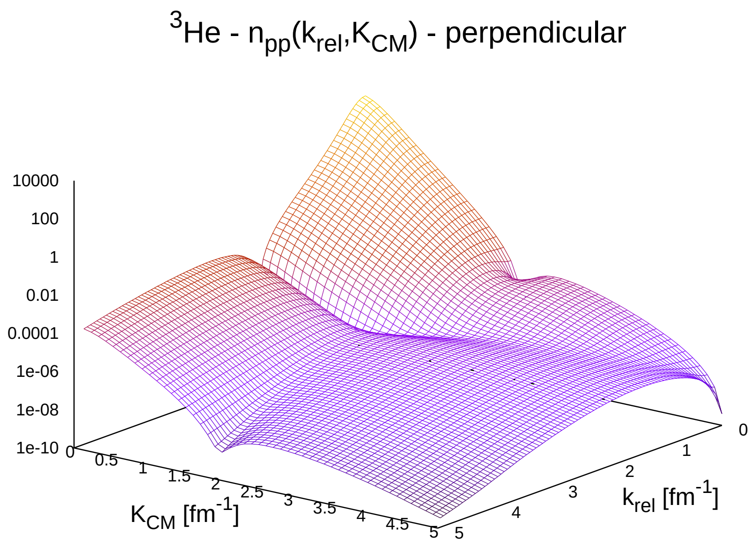
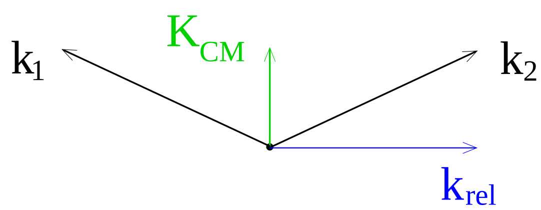




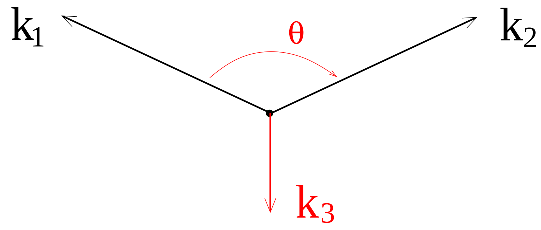
add a third nucleon:

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$$

$$|\boldsymbol{k}_3| = |\boldsymbol{K}_{CM}|$$



$$\Downarrow$$

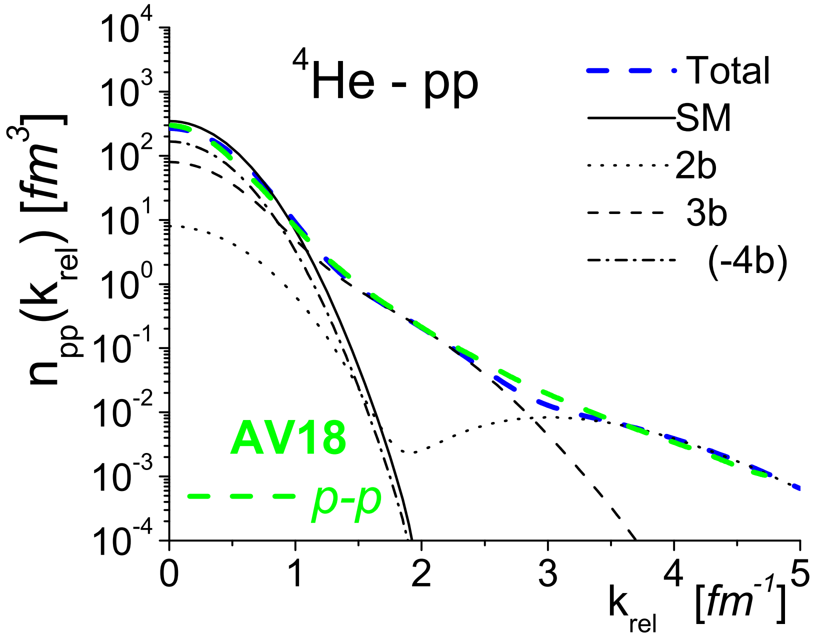


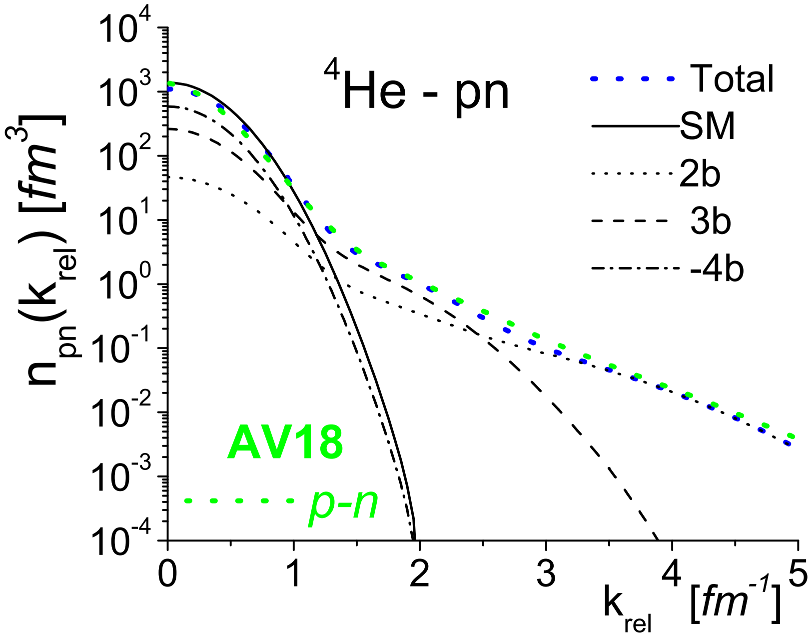
$$\Downarrow$$

$$\boxed{K_{CM} = \frac{2}{\tan\theta/2}\, k_{rel}}$$

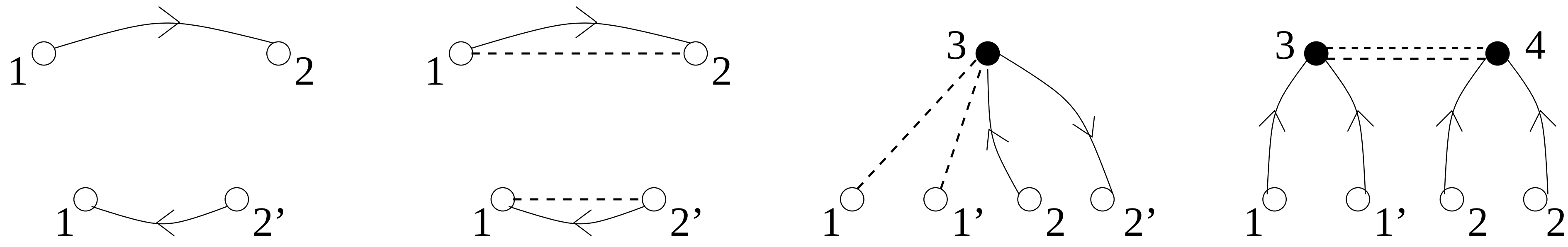
example: many-body contributions in ^4He 2BMD

$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$



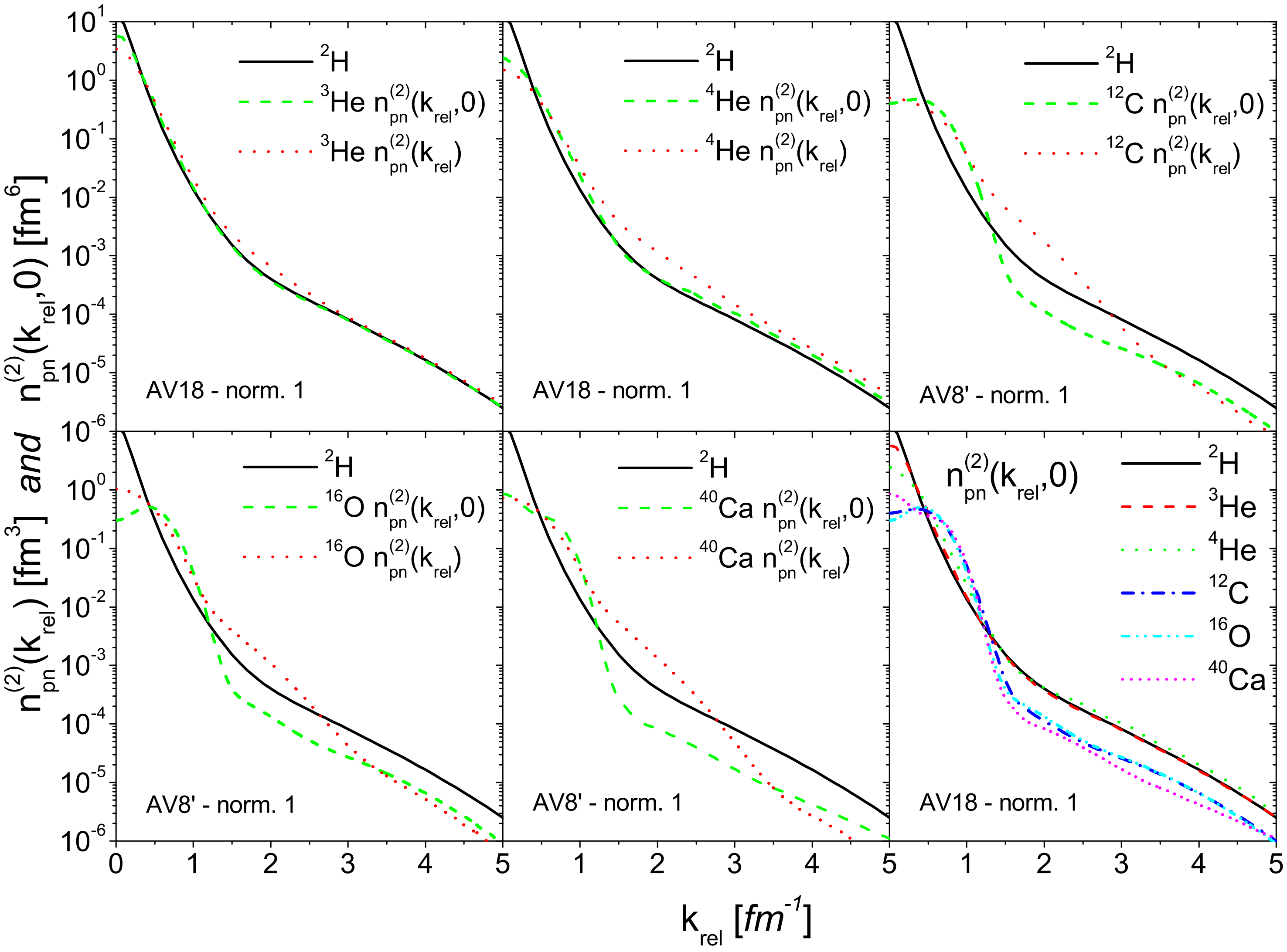


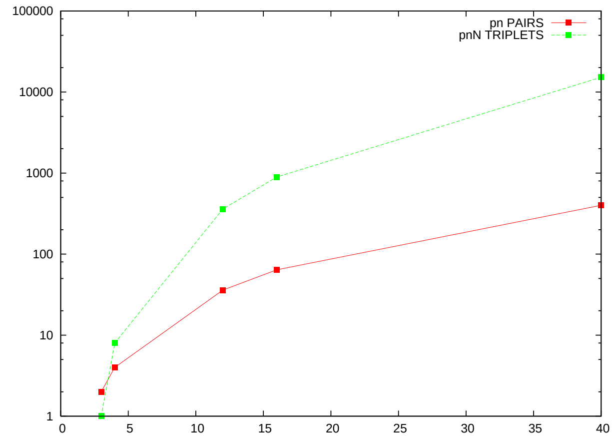
(**AV18**: *Schiavilla at al.* **PRL98** (2007))



Shell Model | two-body | three-body | four-body

2. Defining the 2B & 3B correlation region in k_{rel}, K_{CM}





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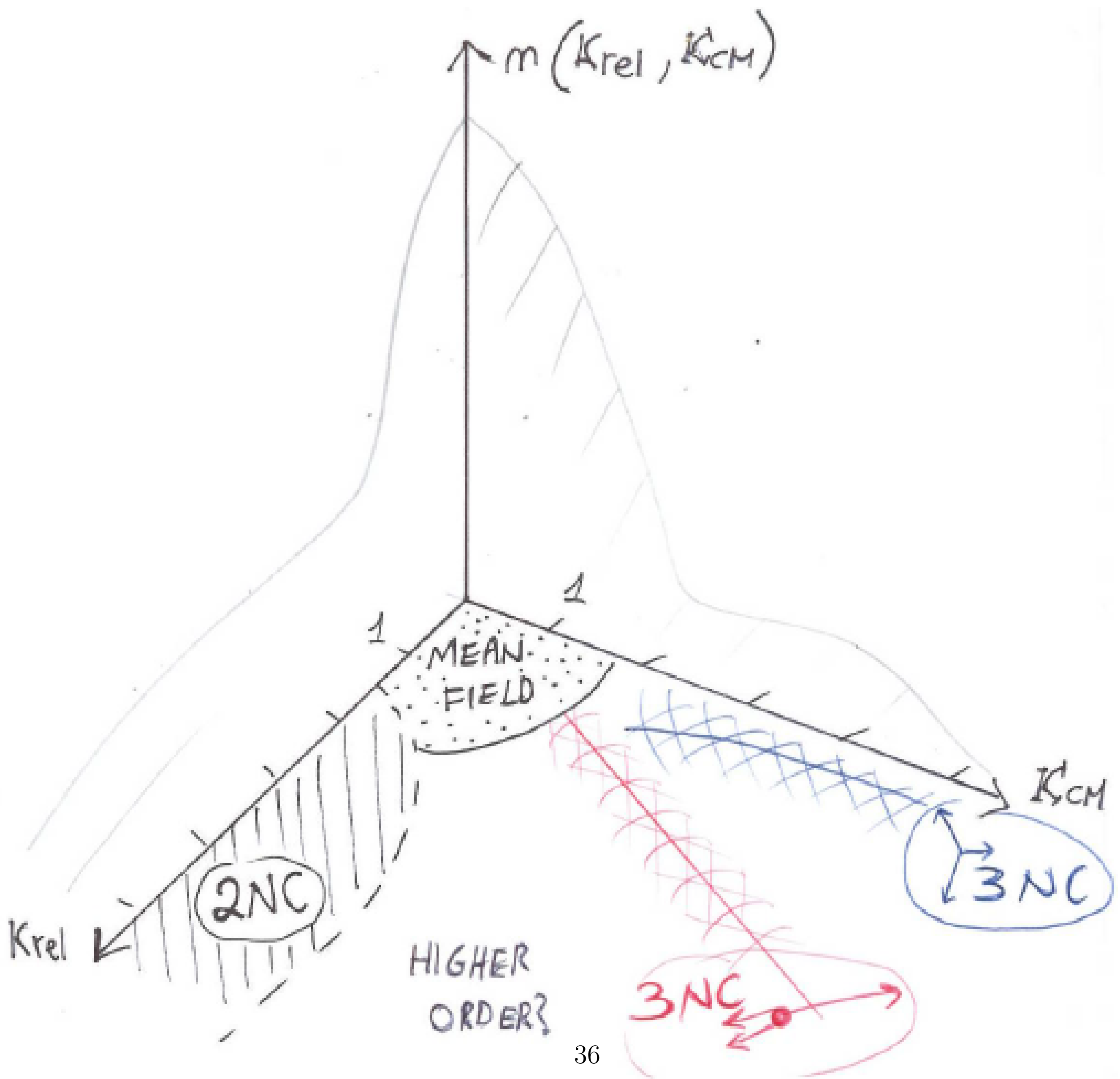


$$P_{pN} = \frac{\int_a^b dk_{rel}\, k_{rel}^2\, n_{pN}(\boldsymbol{k}_{rel}, 0)}{\int_a^b dk_{rel}\, k_{rel}^2\, \left(n_{pp}(\boldsymbol{k}_{rel}, 0) + n_{pn}(\boldsymbol{k}_{rel}, 0)\right)}; \quad 0 < P_{pN} < 1$$

- integration over the whole k_{rel} range: $(a, b) = [0, \infty]$

A	4	12	16	40
P_{pp} (%)	19.7	30.6	29.5	31.0
P_{pn} (%)	81.3	69.4	70.5	69.0

(*Alvioli, Ciofi degli Atti, Morita PRL**100** (2008)*)

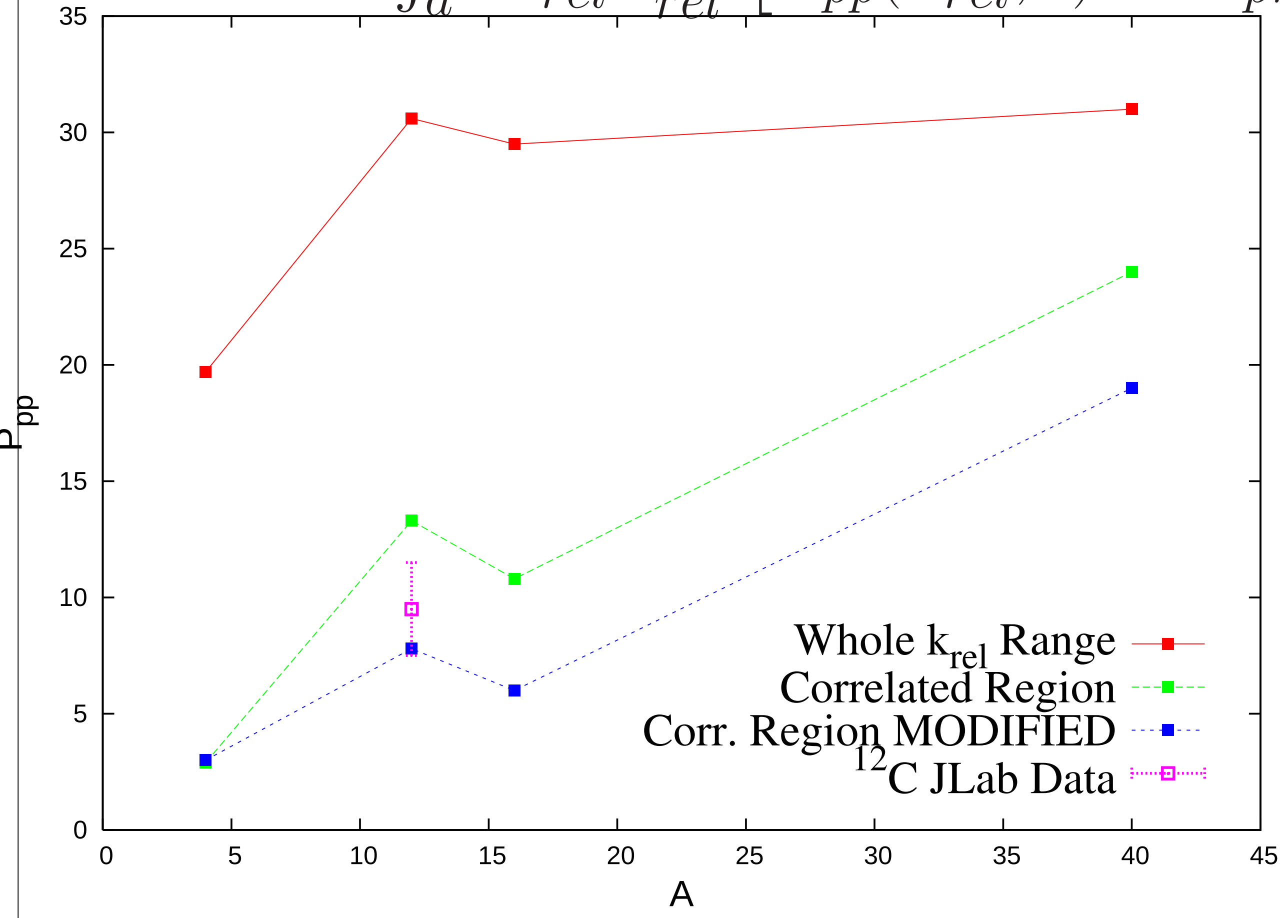
- *correlation region*: $(a, b) = [1.5, 3.0]\ fm^{-1}$

A	4	12	16	40
P_{pp} (%)	2.9	13.3	10.8	24.0
P_{pn} (%)	97.1	86.7	89.2	76.0

$P_{pN}^{A=4}$ in agreement with *Schiavilla et al., PRL98 (2007)*
(extracted from published figures, $AV18$: $P_{pp} \simeq 3\%$, $P_{pn} \simeq 97\%$)
$P_{pp} \simeq 10 - 13\%$ consistent with *Shneor et al., PRL99 (2007)*
(extracted from ${}^{12}C(e, e'pp)X \,/\, {}^{12}C(e, e'p)X$)

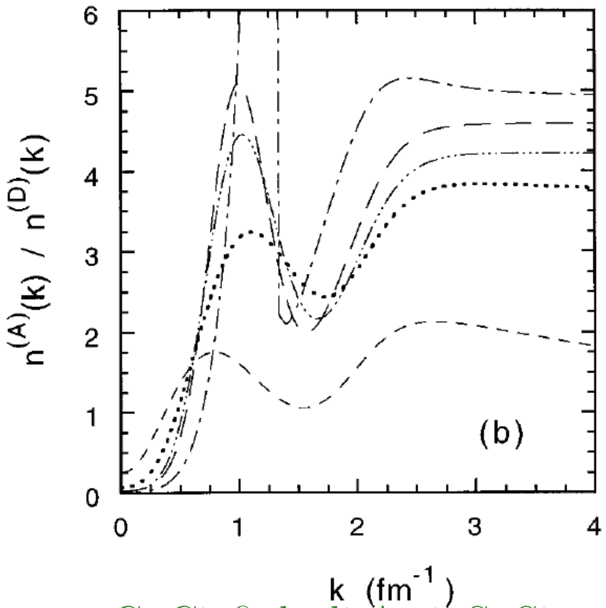
1. ONE data point ☺: pp pair relative probabilities (at $K_{CM} = 0$)

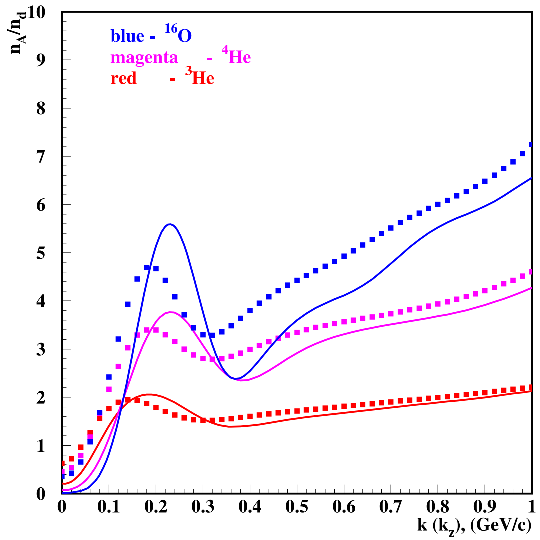
$$P_{pp} = \frac{\int_a^b dk_{rel}\, k_{rel}^2\, n_{pp}(\boldsymbol{k}_{rel}, 0)}{\int_a^b dk_{rel}\, k_{rel}^2\, \left[n_{pp}(\boldsymbol{k}_{rel}, 0) + n_{pn}(\boldsymbol{k}_{rel}, 0 \right]}; \quad 0 < P_{pN} < 100$$

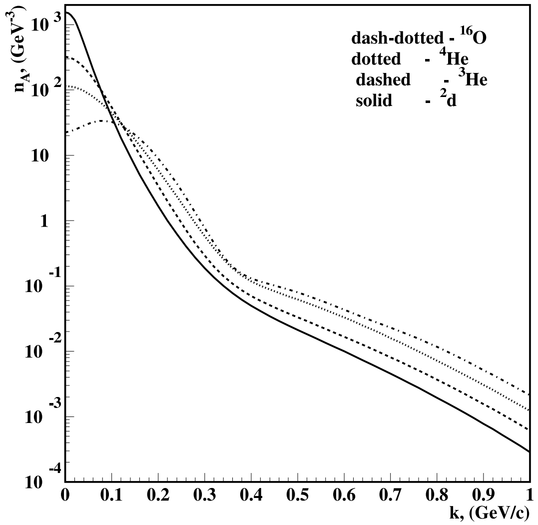


- *mean field, uncorrelated nucleons prediction*
- Alvioli, Ciofi degli Atti, Morita, *PRL* ***100*** *(2008)*
- Alvioli, Strikman, *unpublished*
- Shneor et al., *PRL* ***99*** *(2007)*

3. One-Body Momentum distributions & convolution model







C. Ciofi degli Atti, S. Simula, *Phys. Rev. C**53** (1996) 1689*

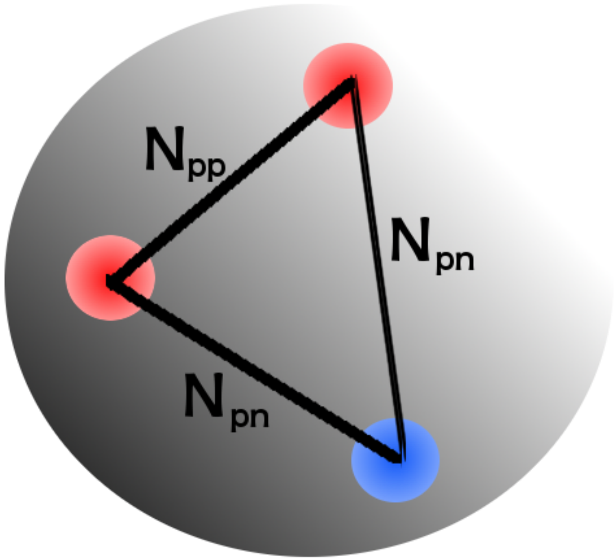
L. Frankfurt, M.Sargsian, M.Strikman *Int. J. Mod. Phys. A**23** (2008) 2991*

- The left figure (TNC model) ***assumes*** deuteron-like high-momentum tail
- Improved calculations actually show a ***rise*** of the ratio $n^A(k)/n^D(k)$
- P.S.: different potentials/methods provide (slightly) different high-momentum components - see Wiringa et al, *arXiv:1309.3794* [nucl-th]; Bogner et al, *PRC**86** (2012)*; Furnstahl, *arXiv:1309:5771* [nucl.th]

3. One-Body Momentum distributions: ^3He

- $n_N(k_1) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}d\boldsymbol{r'}\rho_N(\boldsymbol{r},\boldsymbol{r'})e^{-\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r'})} = \int d\boldsymbol{k}_2\, n^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2)$
- Useful to think in terms of TNC convolution model:

$$n_N(k) \propto \int d\boldsymbol{k}_3\, n_D\left(\left|\boldsymbol{k}-\frac{1}{2}\boldsymbol{k}_3\right|\right) n_{CM}(k_3)$$

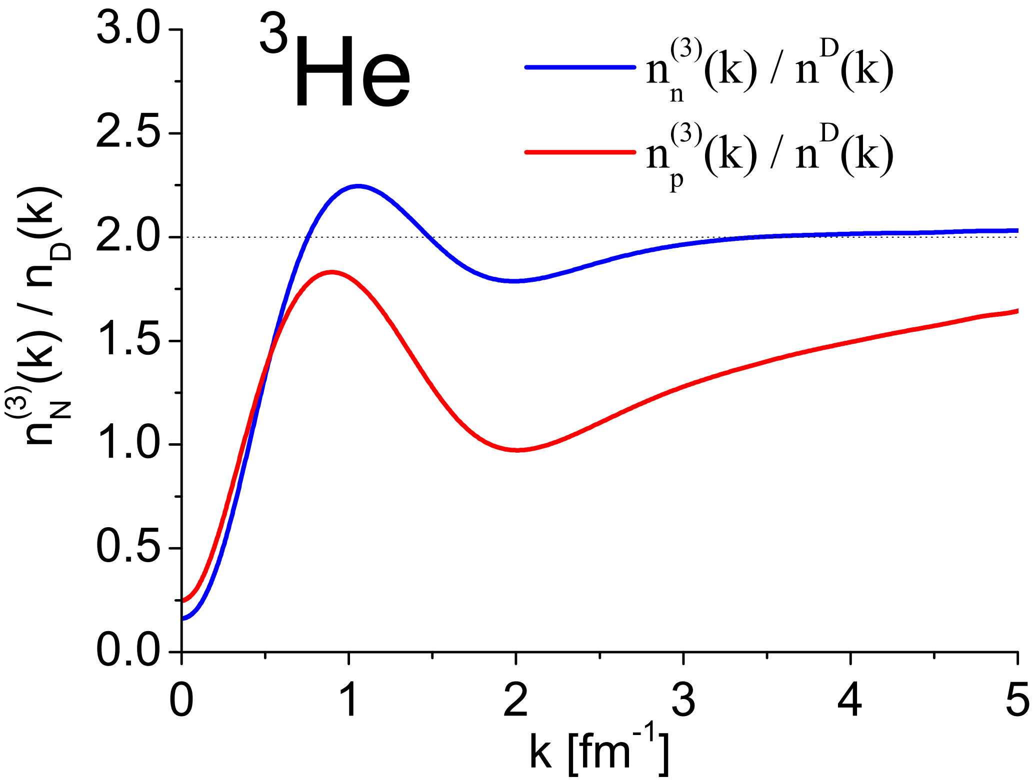


$\longrightarrow n_p(k) \propto N_{pp}(k) + N_{pn}(k)$

$\longrightarrow n_n(k) \propto 2\,N_{pn}(k)$

- in the asymmetric nucleus, proton and neutron have different $n(k)$

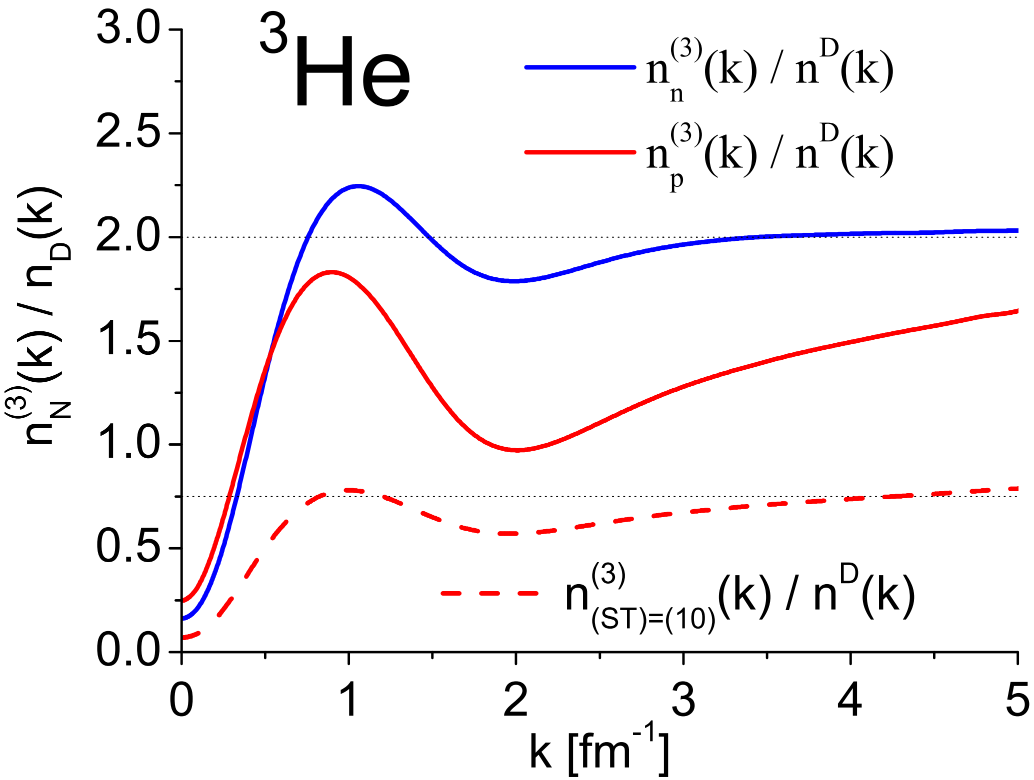
3. One-Body Momentum distributions: ^3He



M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita; ***PRC87*** *(2013),*
and
IntJModPhys E22 *(2013)*

- proton and neutron distributions reflect the different isospin pairs in ^3He;
- the neutron distribution is about twice the deuteron distribution.
- the proton one is larger than the deuteron's.

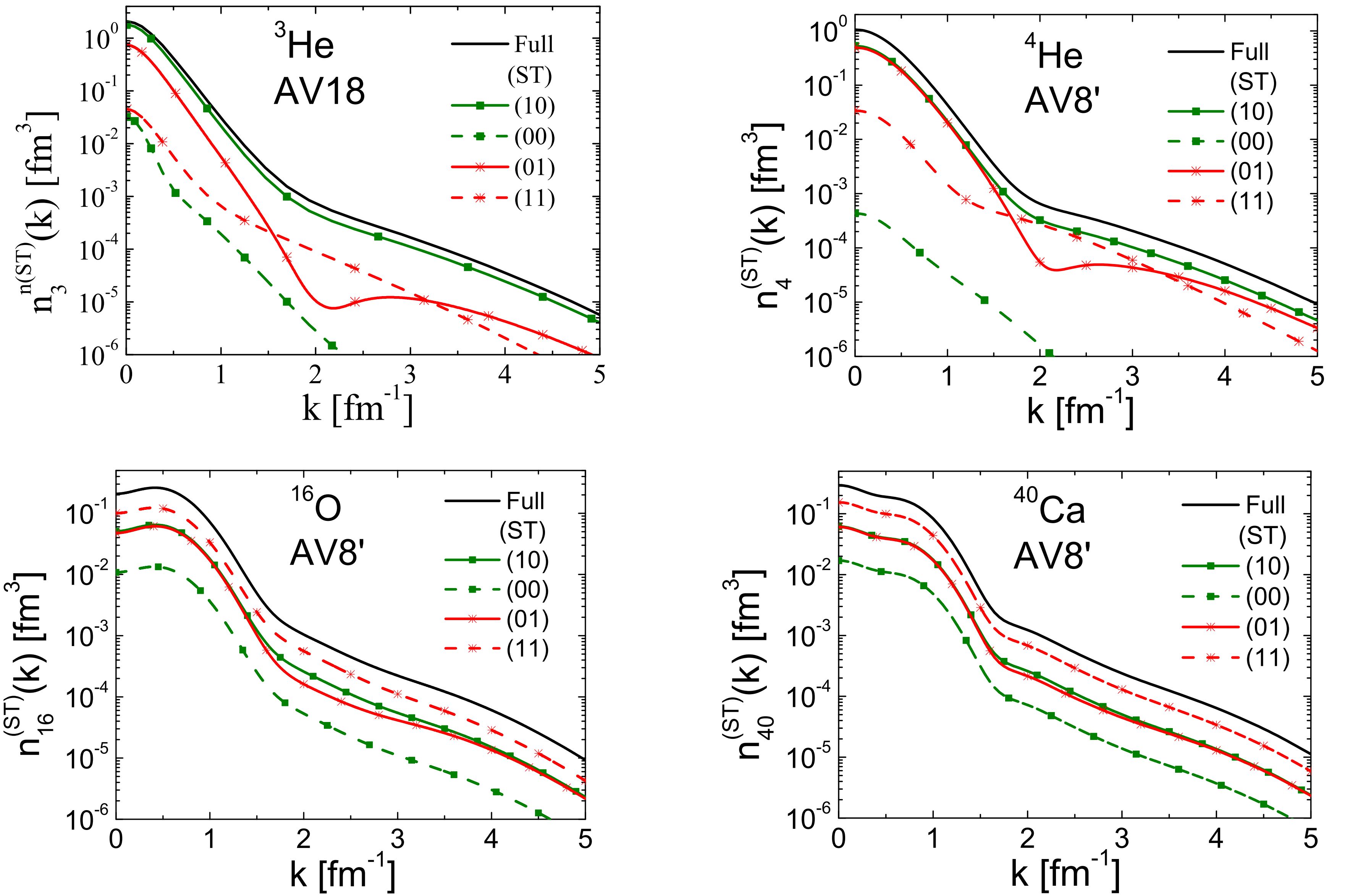
3. One-Body Momentum distributions: ^3He



M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita; ***PRC87*** *(2013),* and ***IntJModPhys E22*** *(2013)*

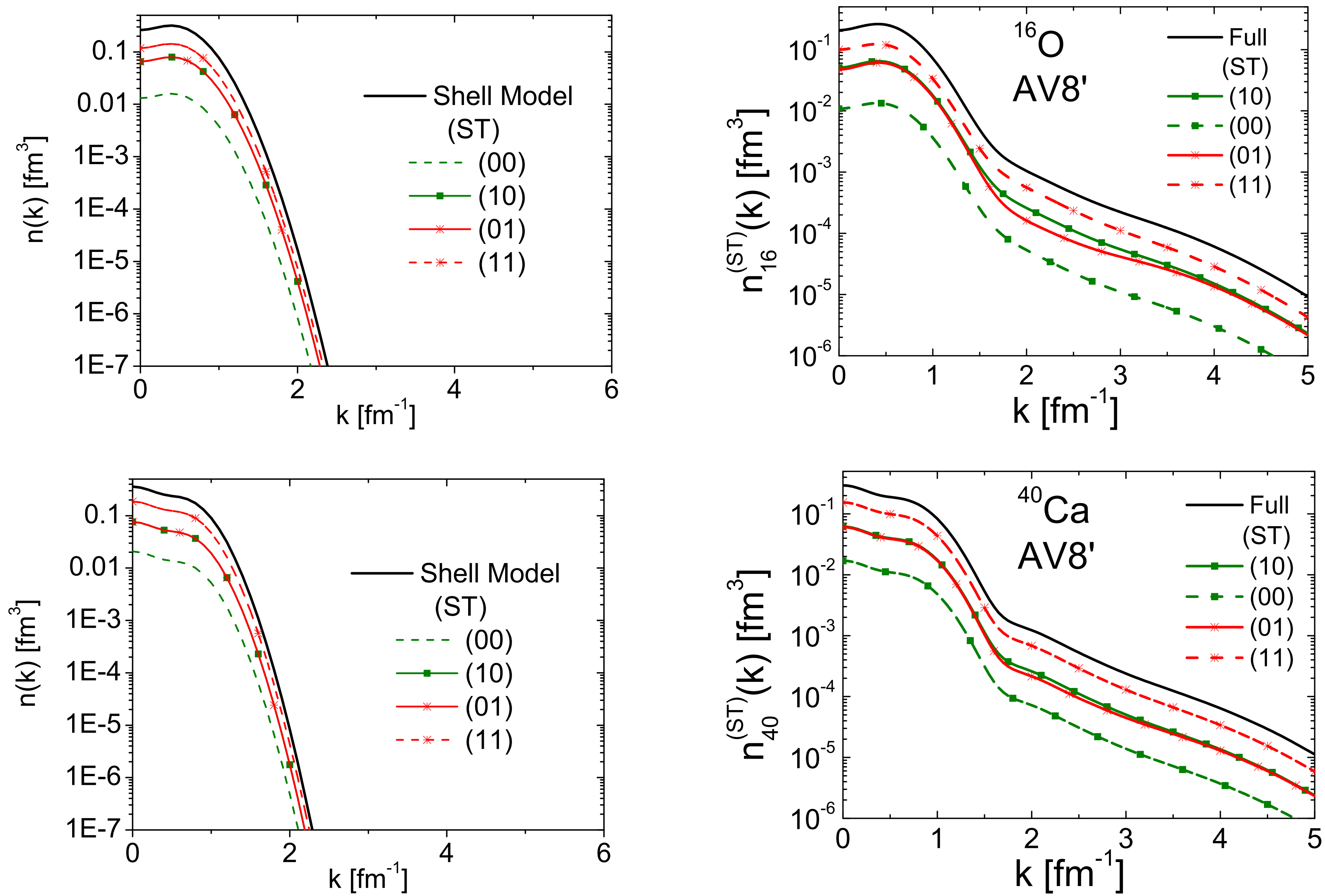
- proton and neutron distributions reflect the different isospin pairs in ^3He;
- the neutron distribution is about twice the deuteron distribution.
- the proton one is larger than the deuteron's.
- Select only (ST)=(10) pairs: *these* are deuteron-like pairs in ^3He.

3. One-Body Mom distrs: Few- and Many-Body nuclei



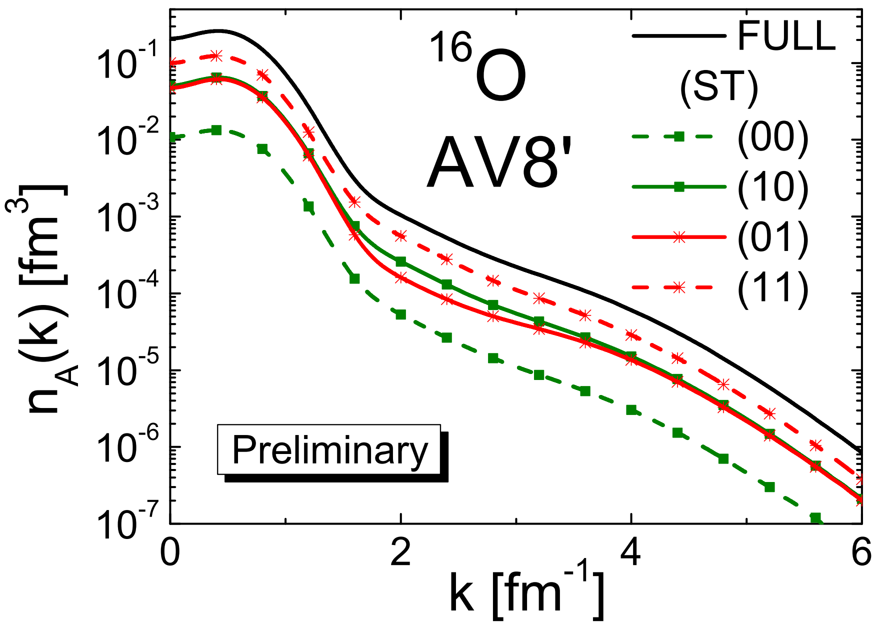
M. Alvioli et al., ***PRC87*** *(2013); IntJModPhys E22 (2013)*

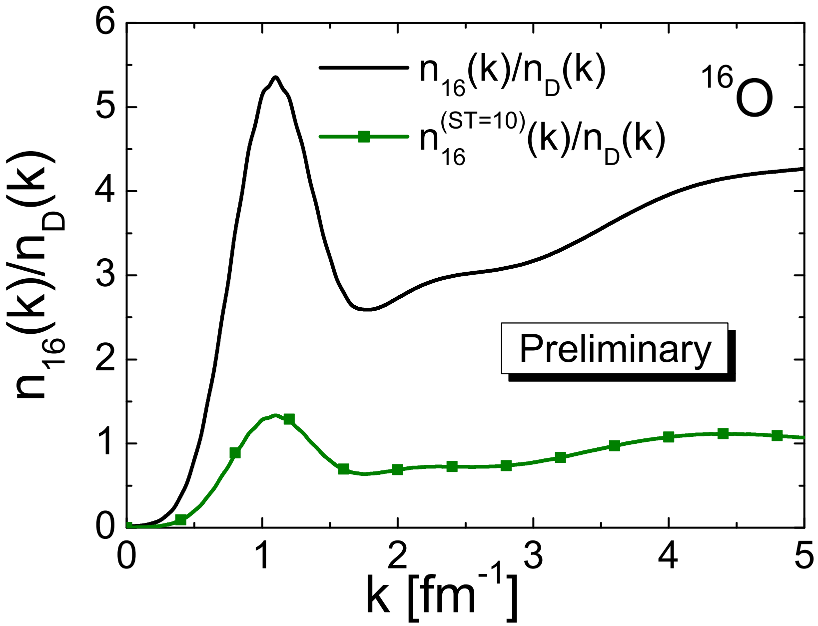
3. One-Body Mom distrs: Many-Body nuclei



*M. Alvioli et al., **PRC87** (2013); IntJModPhys E22 (2013)*

3. One-Body Momentum distributions: ^{16}O

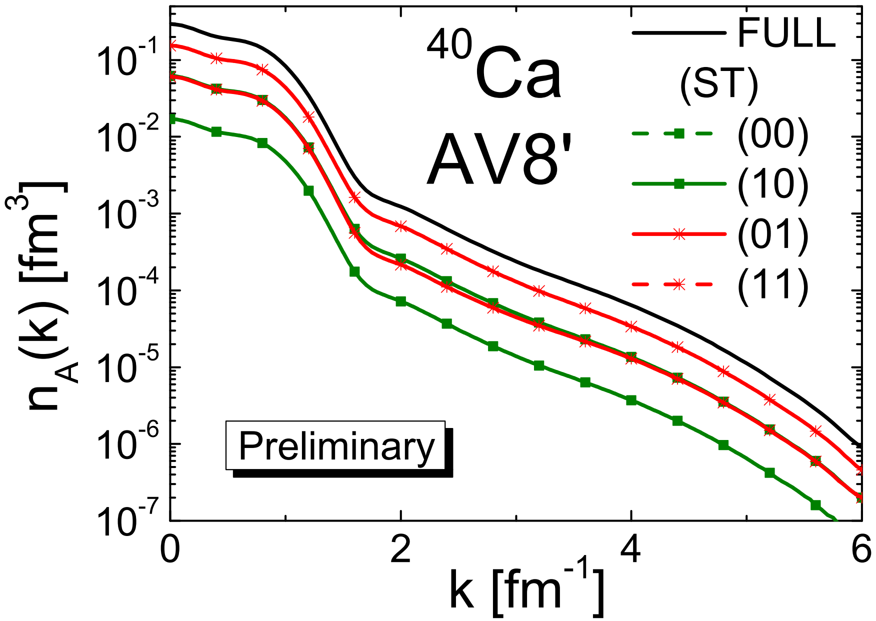


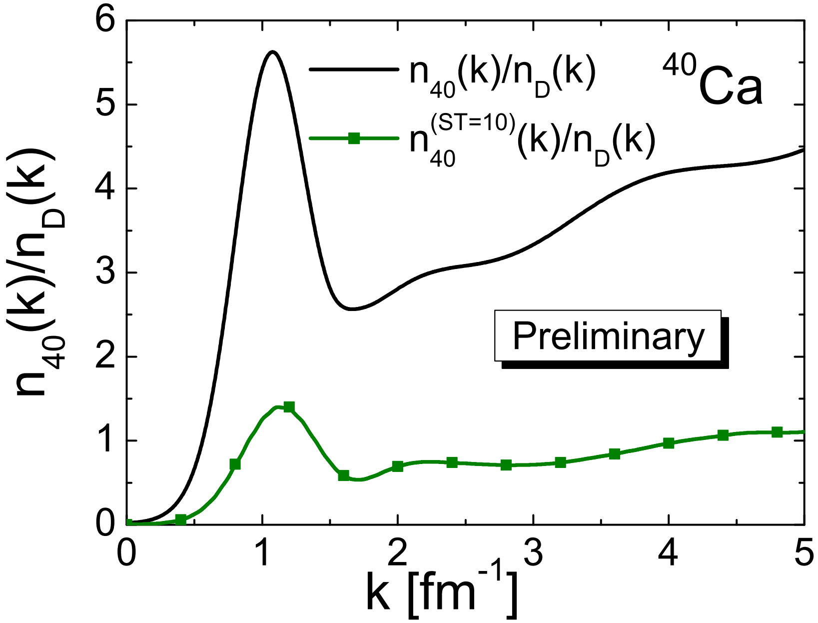


M. Alvioli et al., ***PRC87*** *(2013); IntJModPhys E22* (2013)

- we may compute a complete (ST) separation as in few-body nuclei;
- Select nucleons in (ST)=(10) pairs: the high-momentum tail of $n(k)$ becomes proportional to the deuteron's;
- We use $AV8'$ variational wave function; the $AV14$ one is known to produce a larger ratio, but similar conclusions.

3. One-Body Momentum distributions: ^{40}Ca

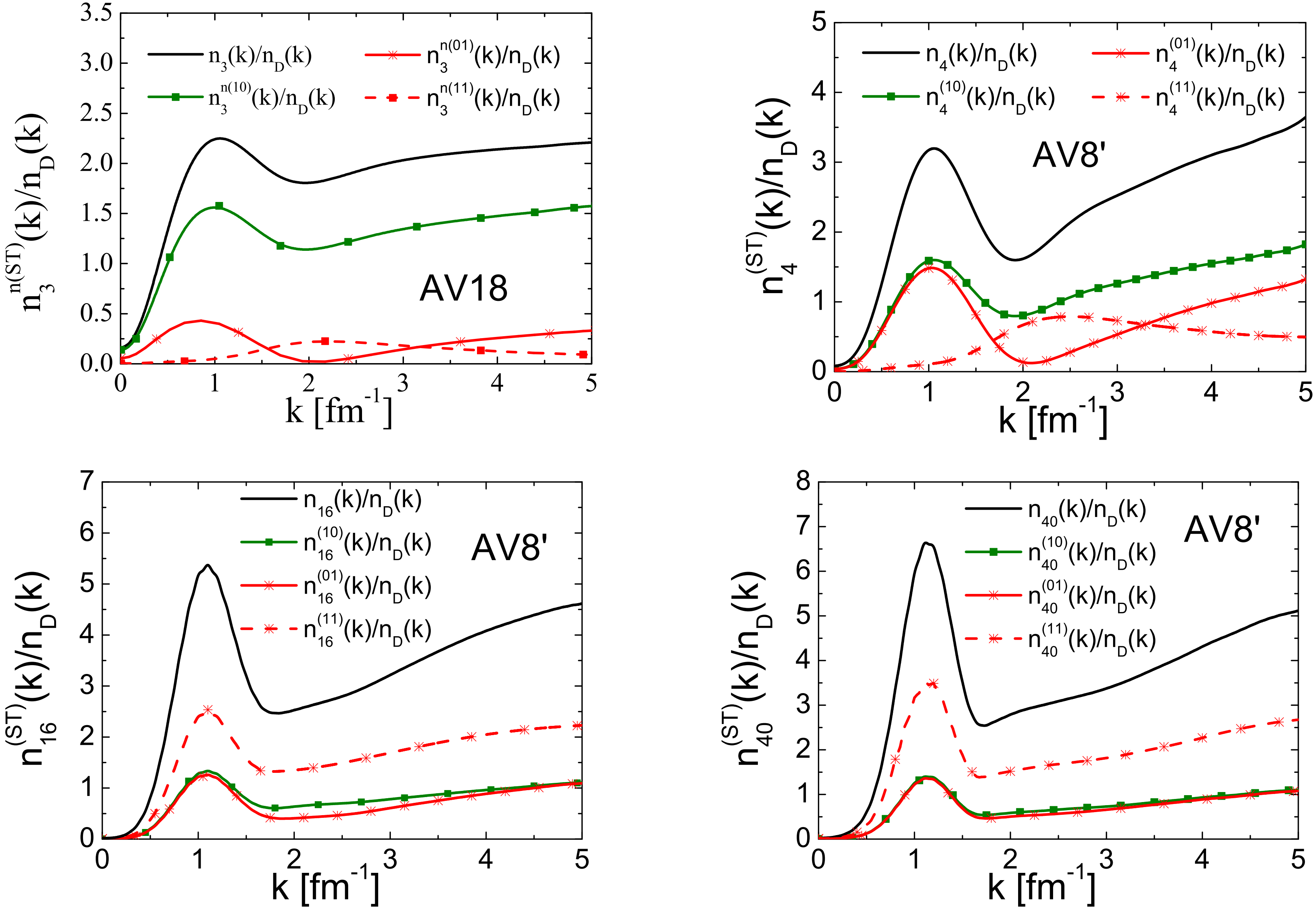




M. Alvioli et al., ***PRC87*** *(2013); IntJModPhys E22* (2013)

- we may compute a complete (ST) separation as in few-body nuclei;
- Select nucleons in (ST)=(10) pairs: the high-momentum tail of $n(k)$ becomes proportional to the deuteron's;
- We use $AV8'$ variational wave function; the $AV14$ one is known to produce a larger ratio, but similar conclusions.

3. One-Body Mom distrs: Few- and Many-Body nuclei



*M. Alvioli et al., **PRC87** (2013); IntJModPhys E22 (2013)*

4. FSI in $A(e,e'p)$: Glauber approach

- *distorted spectral function* & factorization approximation

$$\frac{d\sigma}{dQ^2 d\nu d\boldsymbol{p}} = K\,\sigma_{ep}\,P_D(E_m,\boldsymbol{p}_m)\,; \quad \boldsymbol{p}_m = \boldsymbol{q}-\boldsymbol{p}\,; \quad \theta = \theta_{p\,p_m}$$

- basic quantity: *distorted momentum distribution*

$$n_D(p_m,\theta) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}d\boldsymbol{r}'\; e^{i\boldsymbol{p}_m\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho_D(\boldsymbol{r},\boldsymbol{r}') = \int dE_m\,P(E_m,\boldsymbol{p}_m)$$

- distorted one-body, non diagonal density:

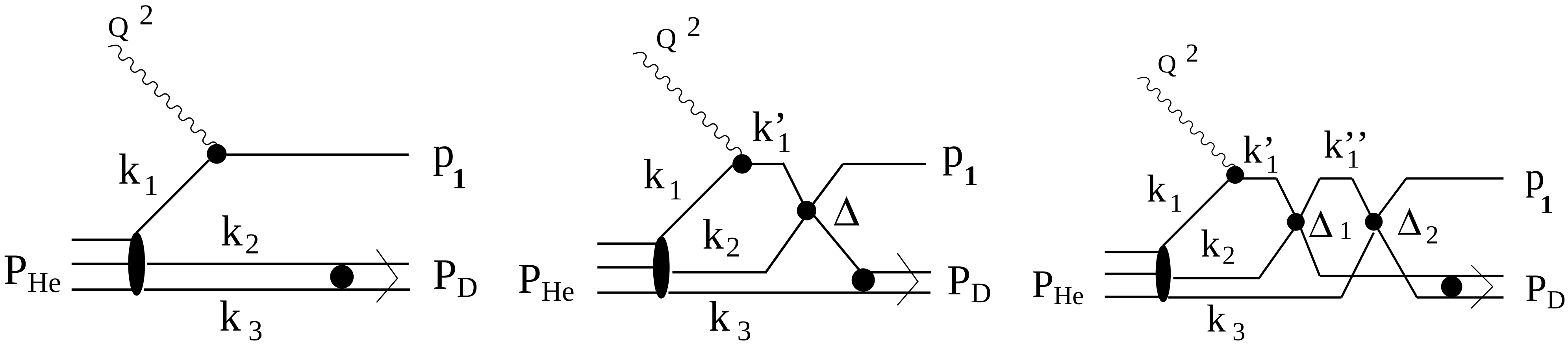
$$\rho_D(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int \prod_{i=2}^{A} d\boldsymbol{r}_i\,\psi^\star(\boldsymbol{r}_1,...,\boldsymbol{r}_A)\,\hat{S}^\dagger\,\hat{S}\,\psi(\boldsymbol{r}'_1,\boldsymbol{r}_2,...,\boldsymbol{r}_A)$$

- energy-dependent Glauber operator:

$$\hat{S} = \prod_{j=2}^{A}\hat{G}(1j) = \prod_{j=2}^{A}\left[1-\theta(z_j-z_1)\,\frac{\sigma_{NN}^{tot}}{4\pi B_0^2}\,exp\left[\left(\boldsymbol{b}-\boldsymbol{b}_j\right)^2/2B_0^2\right]\right]$$

4. diagrammatic GEA approach to FSI: ${}^3He(e,e'p)^2H$

Δ: momentum transfer in the NN rescattering



pwia | single rescattering | double rescattering

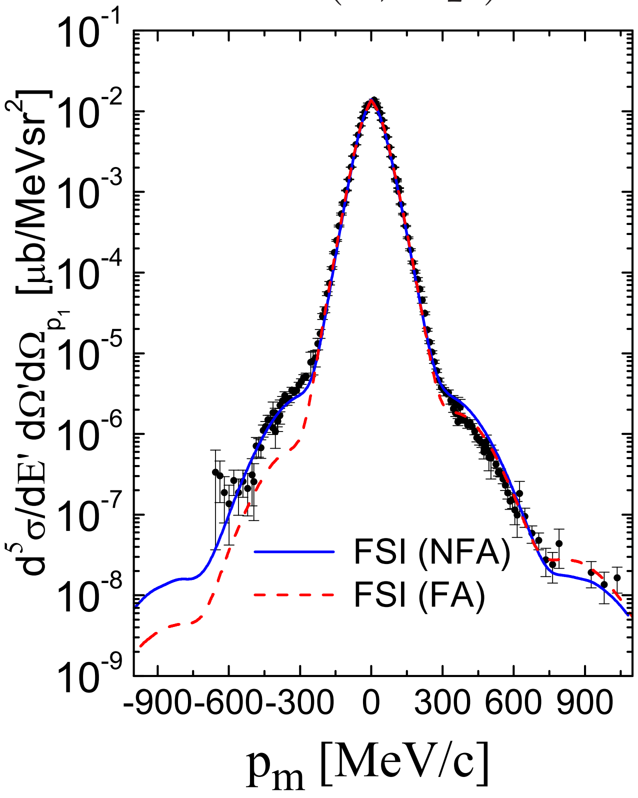
$$S(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3) = \prod_{j=2}^{3}\left[1-\theta\left(z_j - z_1\right)\Gamma\left(\boldsymbol{b}_1 - \boldsymbol{b}_j\right)\right]$$

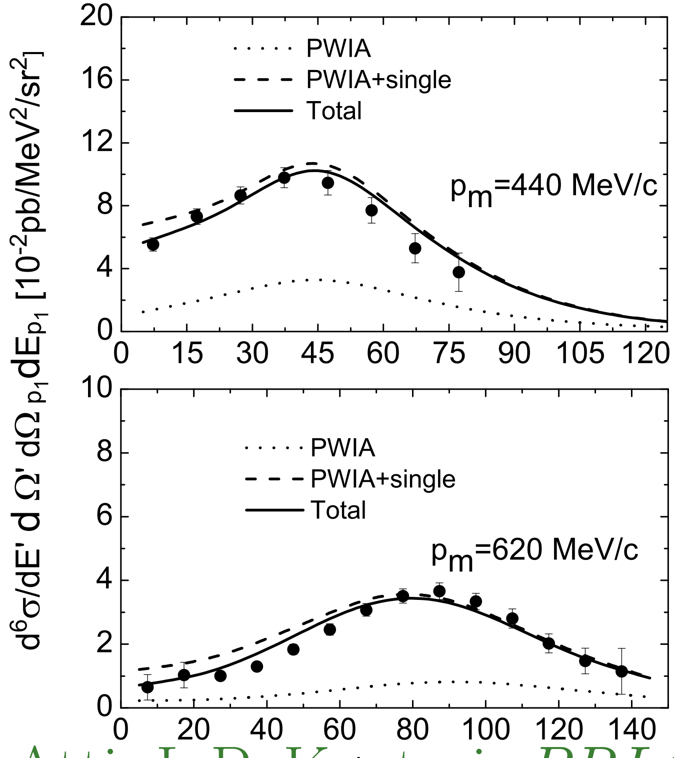
$$\Gamma(\boldsymbol{b}) = \frac{1}{2\pi i K^2}\int d^2\kappa_\perp f_{NN}(\kappa_\perp) e^{-i\kappa_\perp \boldsymbol{b}}$$

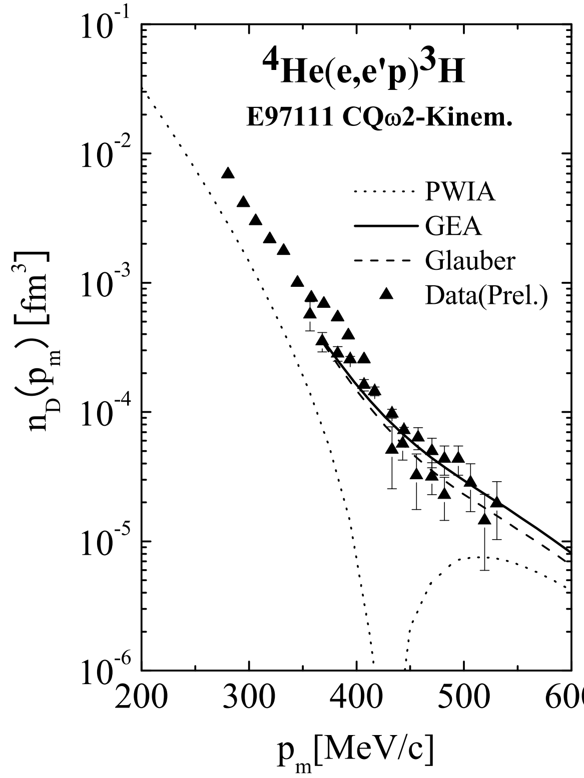
$$f_{NN}(\boldsymbol{\Delta}_\perp) = K\frac{\sigma^{tot}(i+\alpha)}{4\pi}e^{-b^2\boldsymbol{\Delta}_\perp^2/2}$$

4. Exclusive Processes with Few-Body Nuclei

$^3He(e,e'p)^2H$ $\quad$ $^3He(e,e'p)pn$ $\quad$ $^4He(e,e'p)^3He$





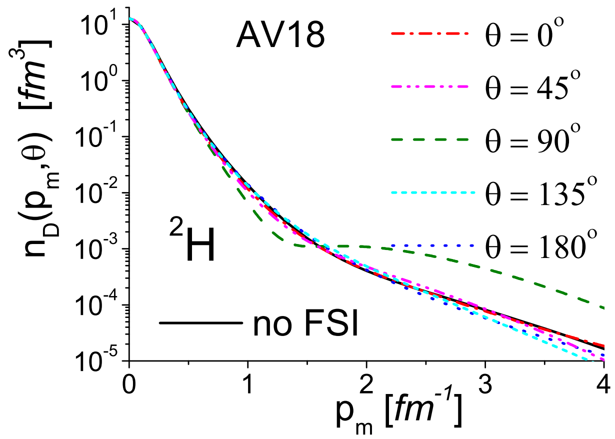


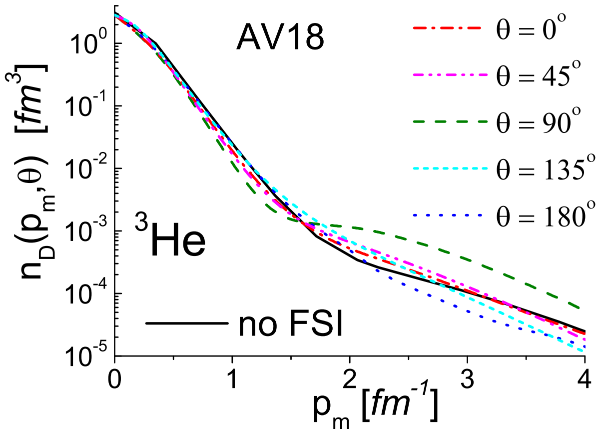
Left: C.Ciofi degli Atti, L.P. Kaptari, *PRL* ***100*** *(2008) 162503*

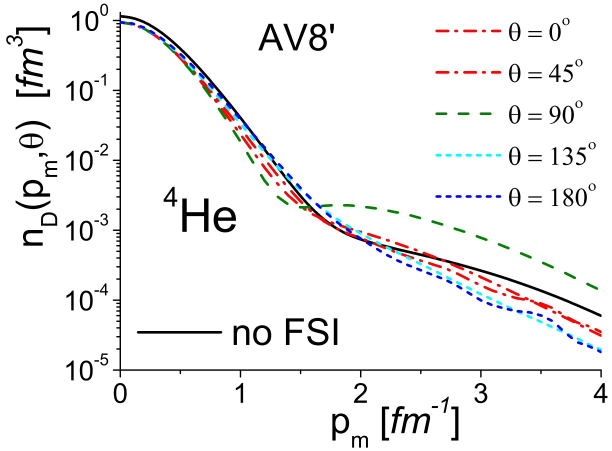
Center: M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, *PRC* ***81*** *(2010) 021001*

Right: C.Ciofi degli Atti, L.P. Kaptari, H. Morita, *FBS* ***43*** *(2008) 39*

Use of ^2H, ^3He and ^4He state-of-the-art wave functions and Glauber multiple-scattering theory for high energies hadron-hadron scattering deuteron $n_D(p_m, \theta) \longrightarrow$

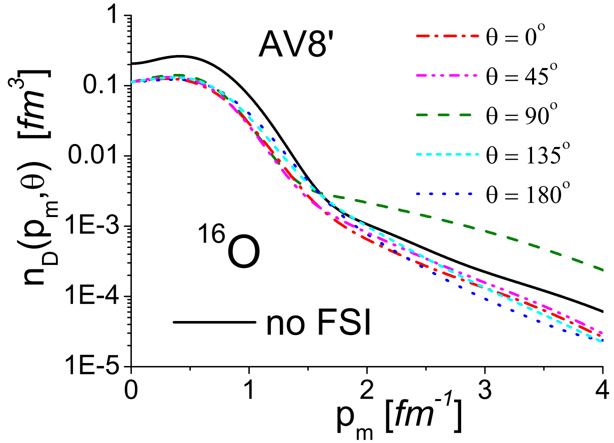


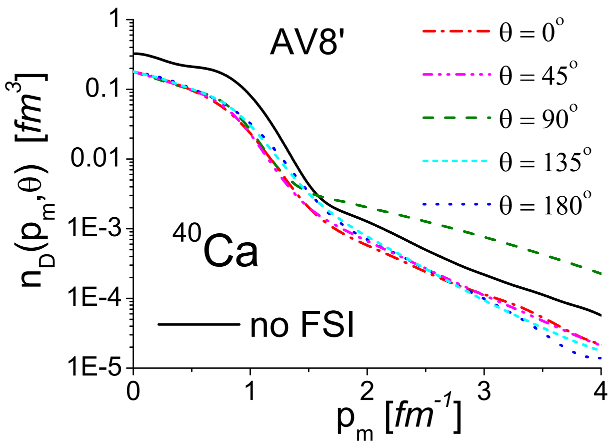




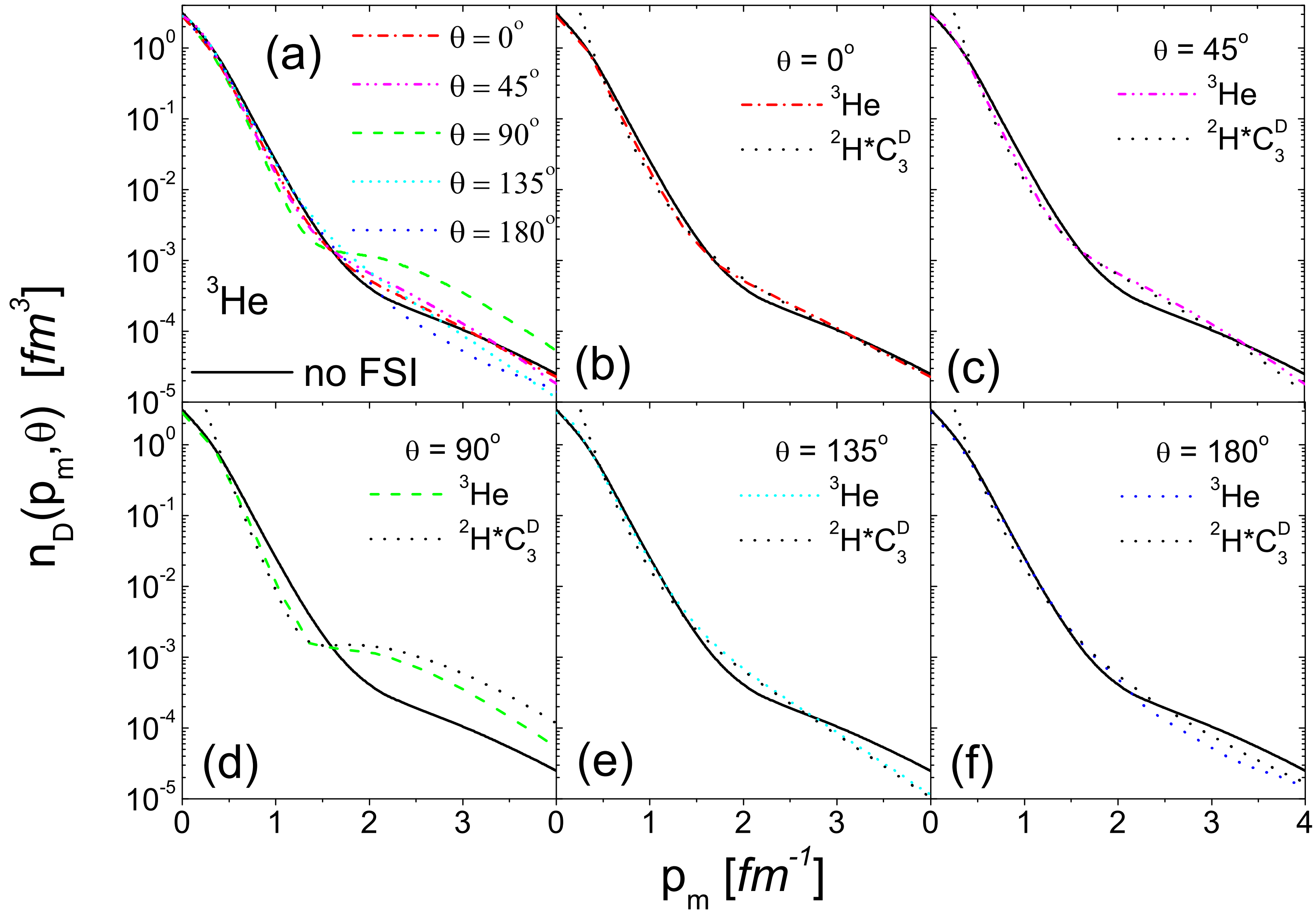
$A > 2$:

same pattern as in deuteron for given θ!





4. Detailed comparison: $^3He/^2H$

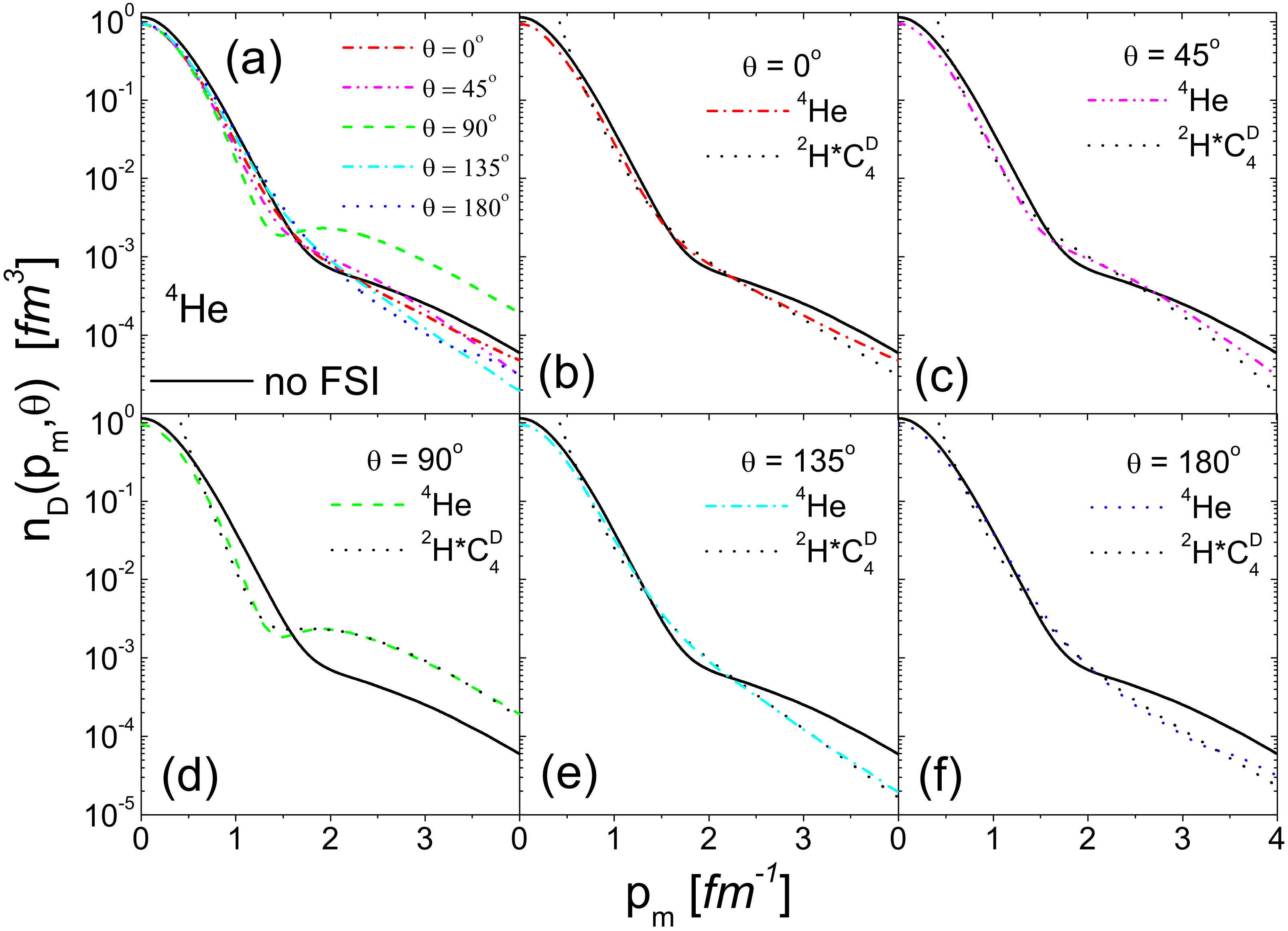


^2H: exact AV18

^3H: Pisa AV18

M. Alvioli *et al.*, ***to appear***

4. Detailed comparison: $^4He/^2H$

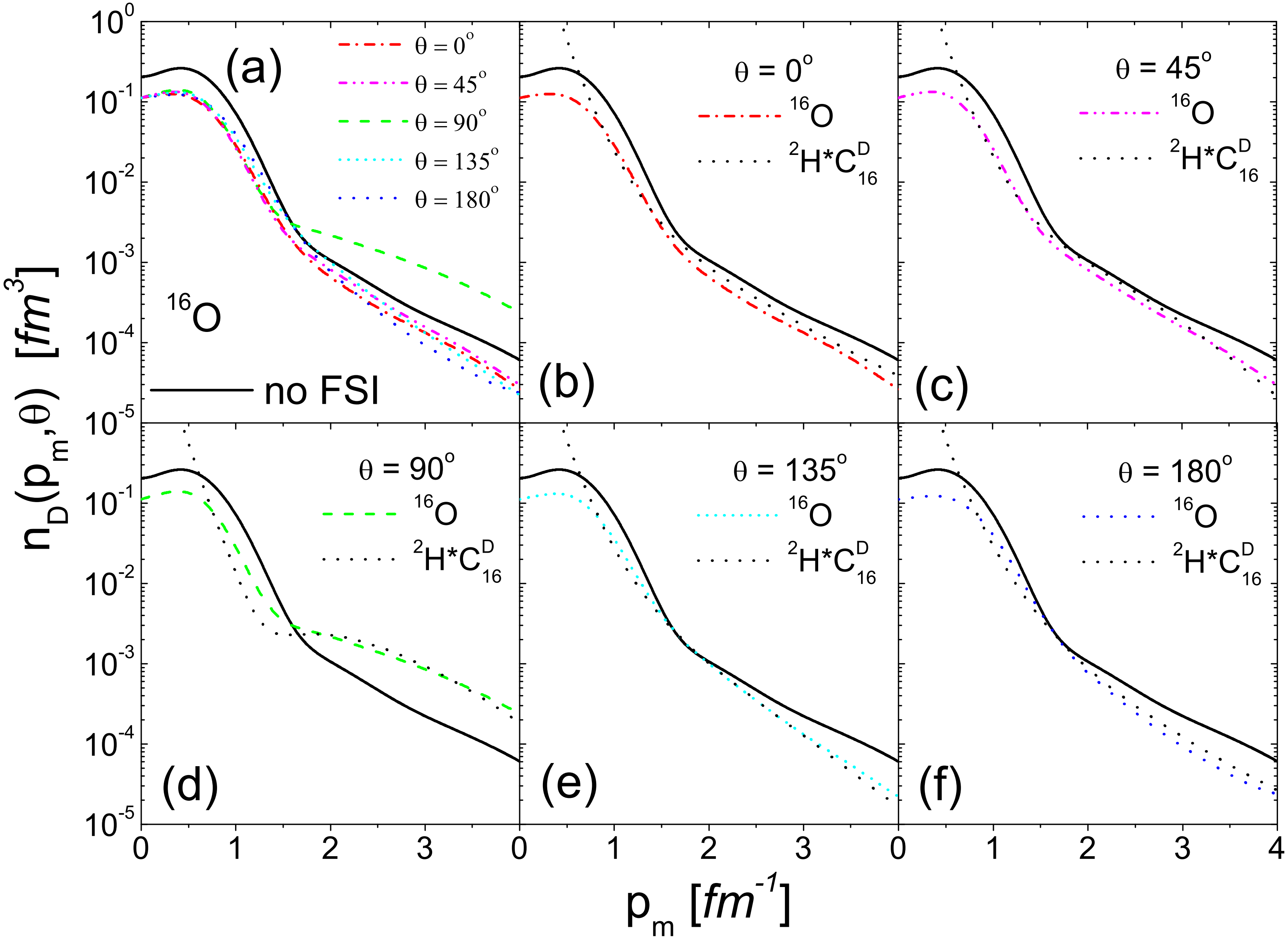


^2H: exact AV18

^4H: ATMS AV8′

M. Alvioli *et al.*, ***to appear***

3. Detailed comparison: $^{16}O/^{2}H$



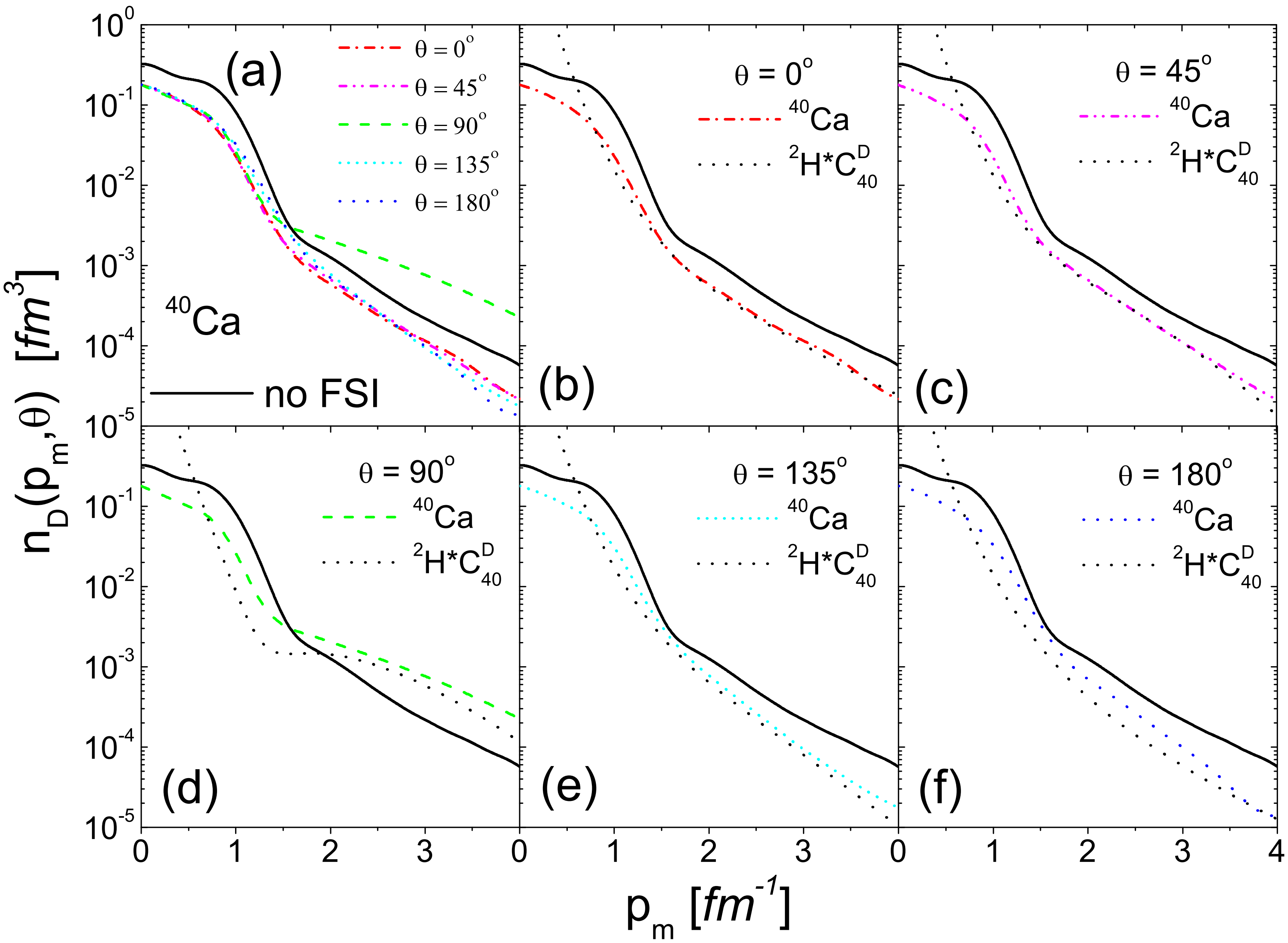
^2H: exact AV8′

^{16}O: variational AV8′ + cluster expansion

M. Alvioli, ***PhD Thesis*** *(2003)*

M. Alvioli *et al.*, ***to appear***

4. Detailed comparison: $^{40}Ca/^{2}H$

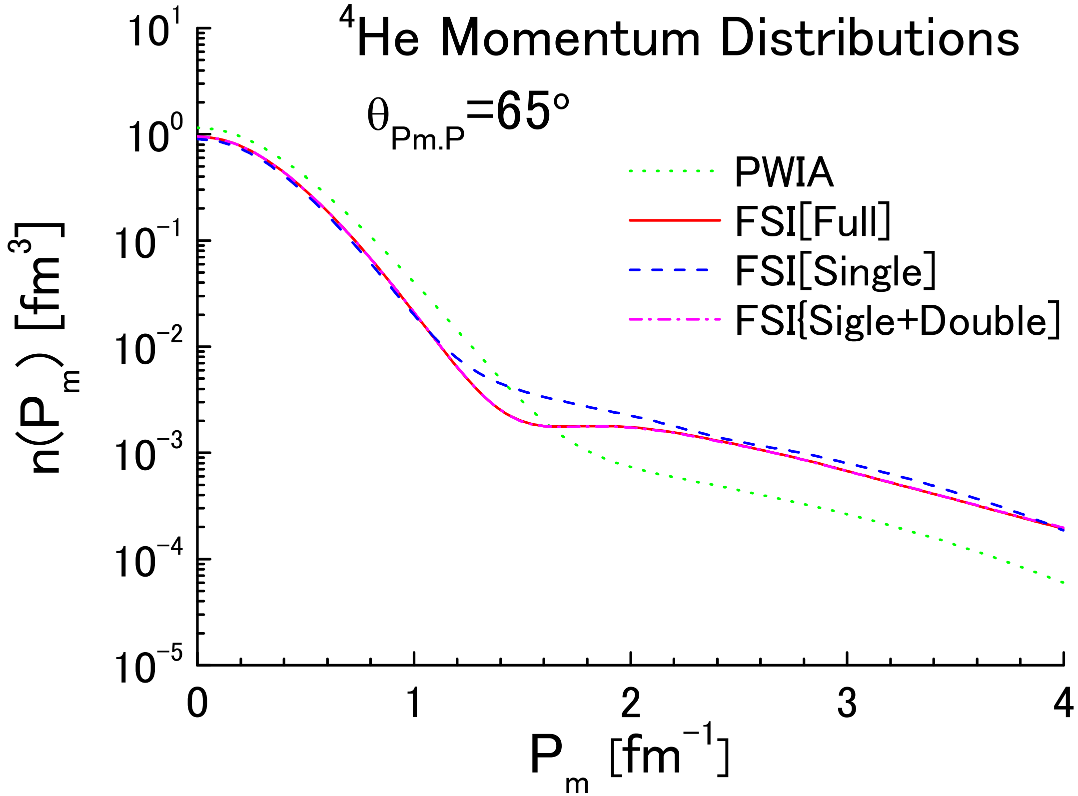


^{2}H: exact AV8′

^{40}Ca: variational AV8′ + cluster expansion *M. Alvioli,* ***PhD Thesis*** *(2003)*

M. Alvioli *et al.*, ***to appear***

4. Single rescattering in FSI



M. Alvioli *et al.*, ***to appear***

5. Perspectives: calculations of cross-sections with our ***realistic*** two-body densities

- Two-body overlap method based on a given two-body density:

$$\Phi_\alpha(\boldsymbol{x}_1,\boldsymbol{x}_2) = \langle\, \Psi^{(C)}\,|\, a(\boldsymbol{x}_1)\, a(\boldsymbol{x}_2)\,|\,\Psi^{(A)}\,\rangle$$

$$\rho^{(2)}(\boldsymbol{x}_1,\boldsymbol{x}_2;\boldsymbol{x}'_1,\boldsymbol{x}'_2) = \langle\, \Psi^{(A)}\,|\, a^\dagger(\boldsymbol{x}_1)\, a^\dagger(\boldsymbol{x}_2)\, a(\boldsymbol{x}'_1)\, a(\boldsymbol{x}'_2)\,|\,\Psi^{(A)}\,\rangle =$$

$$= \sum_\alpha \Phi^\star_\alpha(\boldsymbol{x}_1,\boldsymbol{x}_2)\,\Phi_\alpha(\boldsymbol{x}'_1,\boldsymbol{x}'_2)$$

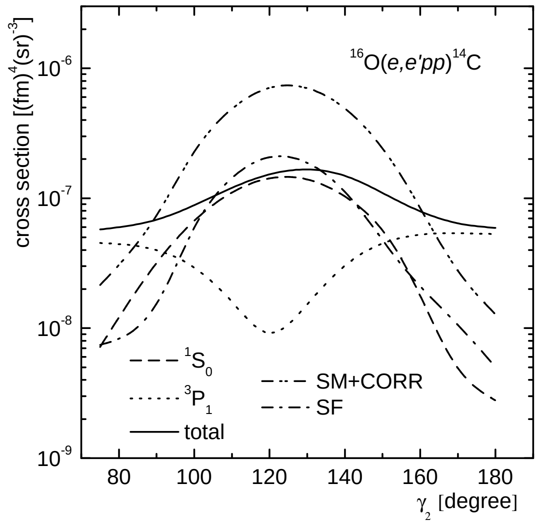
see *e.g.* A.N. Antonov, S.S. Dimitrova, M.V. Stoitsov, D. Van Neck, P. Jeleva, *Phys. Rev.* ***C59*** *(1999) 722*

- used in the calculation of $^{16}O(e,e'pp)^{14}C$ cross-section with ***Jastrow*** correlations at NIKHEF/MAMI kinematics

D.N. Kadrev, M.V. Ivanov, A.N. Antonov, C. Giusti, F.D. Pacati, *Phys. Rrev.* ***C68*** *(2003)*

- inclusion of fully-correlated ^{16}O WF:

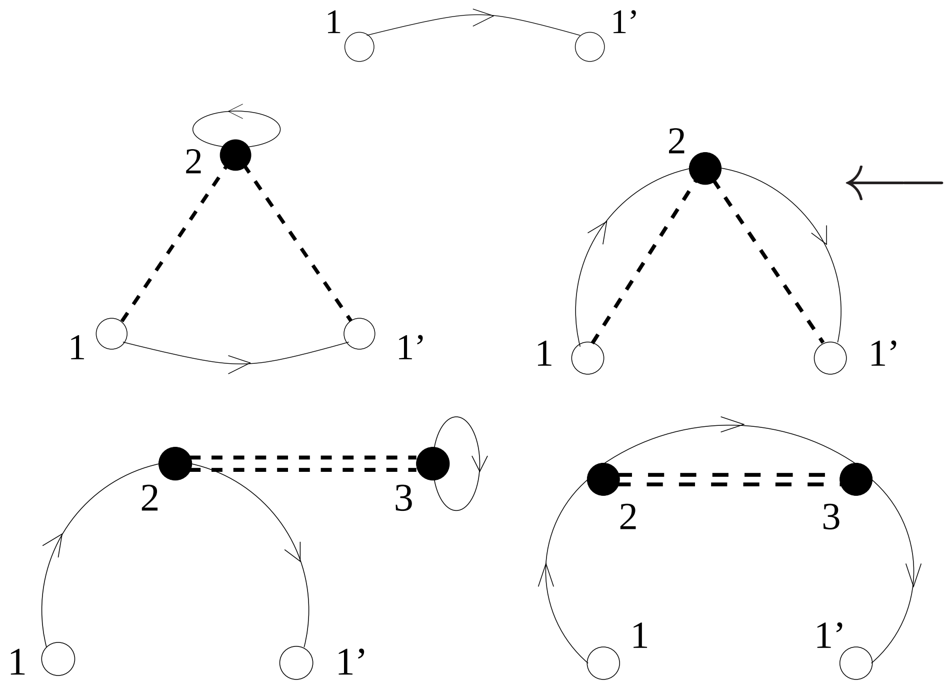
Alvioli, Giusti, Kadrev *to appear*



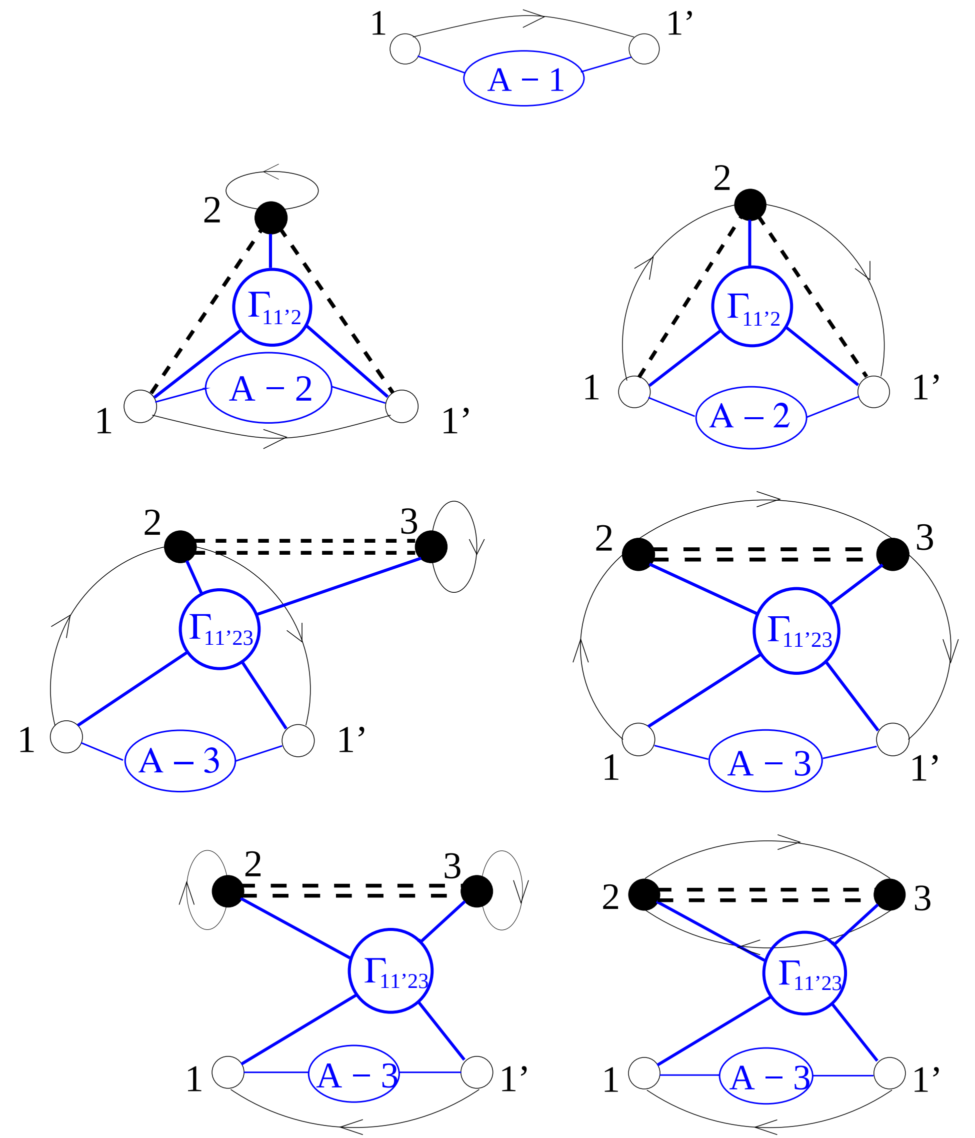
Summary

- Reliable calculations can be performed with realistic wave functions embodying full *short range* structure & *high momentum* components
- Few-body nuclei with exact wave functions; many-body within cluster expansion approximation: any one- and two-body quantity can be calculated
- *Universality of correlations*: I) scaling of two-body momentum distributions to the deuteron one; exact scaling if appropriate (ST)=(10) quantum numbers for the pair are selected
- *Universality of correlations*: II) rise of the nucleus-to-deuteron ratio of one-body $n(k)$ understood by the same argument, deuteron-like quantum numbers
- *Universality of correlations*: III) distorted momentum distributions show similar patterns at large p_{mis} in the case of deuteron and different nuclei; we argue that FSI is mainly due to the (*correlated*) pair

Additional Slides



$\longleftarrow$ *one-body, non-diagonal* $\rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*



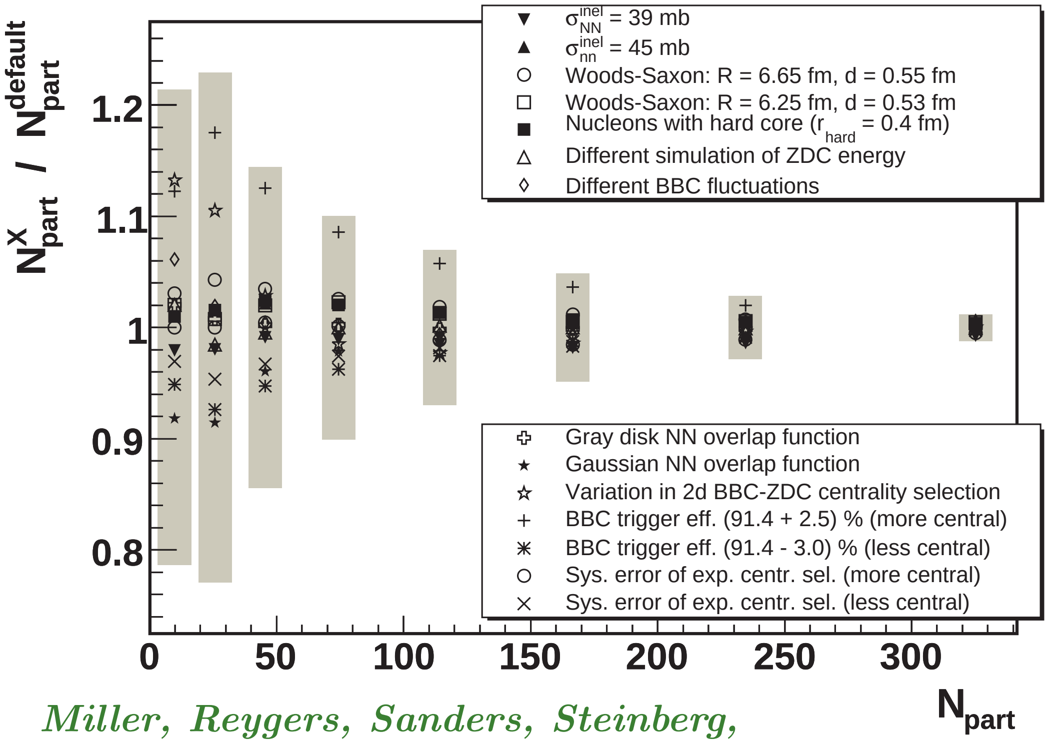
one-body, non-diagonal
distorted $\rho_D(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$

$$\Gamma(11'2) = G^\dagger(12)G(1'2) - 1$$

$$\Gamma(11'23) = G^\dagger(12)G(1'2)G^\dagger(13)G(1'3) - 1$$

M. Alvioli

1. Monte Carlo Glauber (MCG) description: fluctuations



Miller, Reygers, Sanders, Steinberg, Ann. Rev. Nucl. Part. Sci. 57 (2007)

effects of different sources
of fluctuations and
parameter dependencies
within MGC
and detector simulation

We will focus on initial
fluctuations due to:

- inclusion of NN correlations in preparing nuclear configurations
- avoid black-disk approximation for NN scattering $(r_{ij} < \sqrt{\sigma_{NN}^{in}/\pi})$

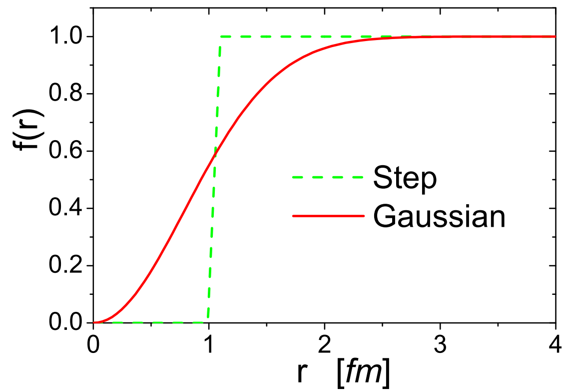
.. and apply these methods in:

- ***spectator nucleons*** excitation and emission for studies of centrality
- fluctuations effects on ecentricity and triangularity of ***participant*** nucleons distribution

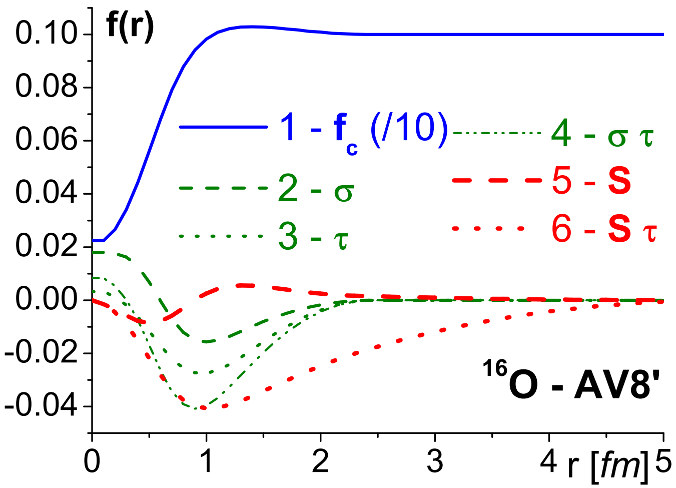
- We used $|\Psi|^2$ as a Metropolis weight function

$$\Psi(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) = \prod_{i<j}^{A} \hat{f}(r_{ij})\, \Phi(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)$$

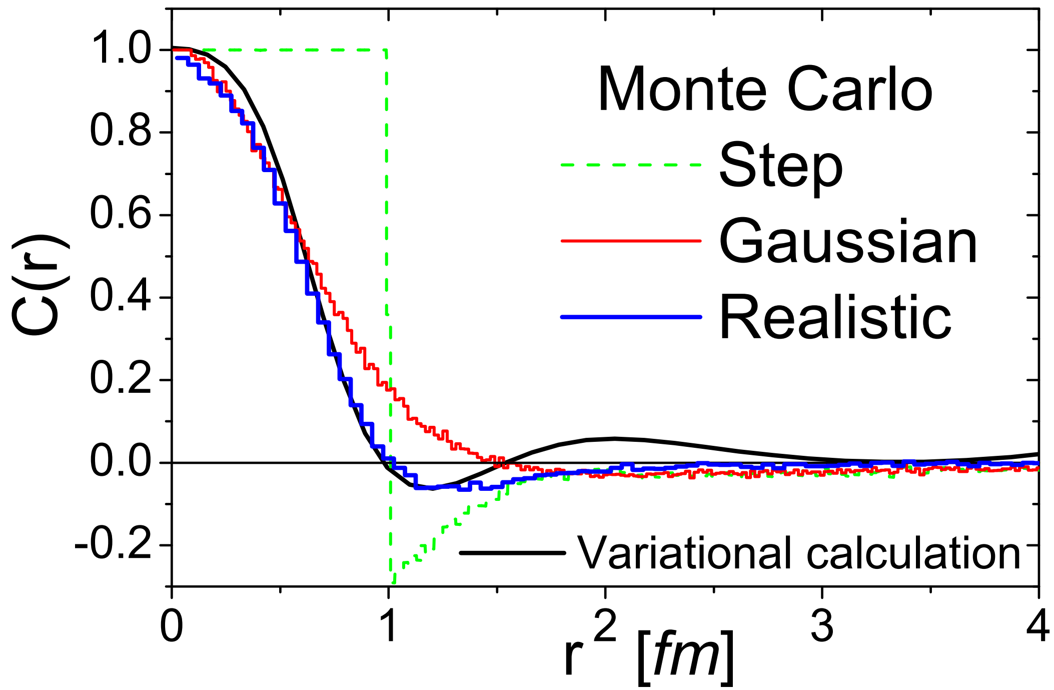
where Φ is given by the independent particle model.



$\Downarrow$



$\Longrightarrow$



$\Uparrow$

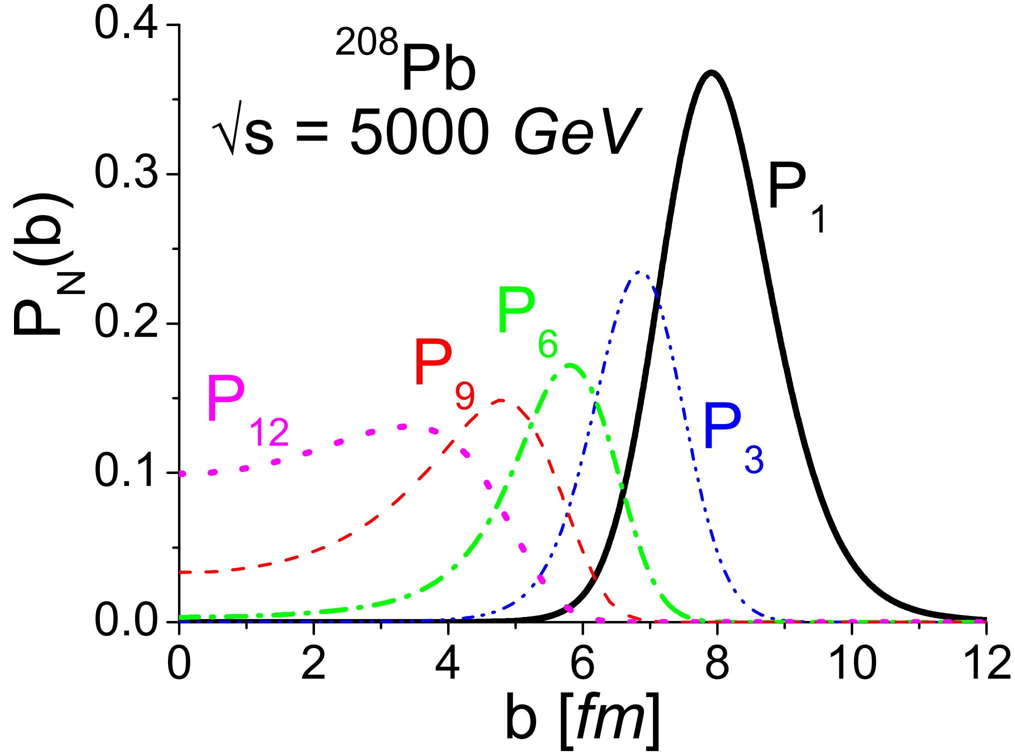
- We use **realistic correlation functions** from variational calculation

probabiltity distribution functions in pA collisions

- probability of interaction with nucleon i: $P(\boldsymbol{b}, \boldsymbol{b}_i) = 1 - [1 - \Gamma(\boldsymbol{b} - \boldsymbol{b}_i)]^2$
- black disk approximation replaced by the Glauber profile $\Gamma(\boldsymbol{b})$:

$$\Gamma(\boldsymbol{b}) = \frac{\sigma_{NN}^{tot}}{4\pi B} e^{-b^2/(2B)}$$

- probability of interaction with N nucleons, *vs* impact parameter $\boldsymbol{b} \rightarrow$



given by:

$$P_N(\boldsymbol{b}) = \sum_{i_1,\ldots,i_N}^{N} P(\boldsymbol{b}, \boldsymbol{b}_{i_1}) \cdot \cdots \cdot P(\boldsymbol{b}, \boldsymbol{b}_{i_N}) \prod_{j \neq i_1,\ldots,i_N}^{A-N} [1 - P(\boldsymbol{b}, \boldsymbol{b}_j)]$$

Effects of NN Correlations in High-Energy Processes

- SRC: are they relevant only in dedicated experiments?
- high-energy scattering processes $\longrightarrow$ Glauber multiple scattering
- exact expansion of the many-body WF (*Glauber, Foldy & Walecka*):

$$|\Psi(\boldsymbol{r}_1,...,\boldsymbol{r}_A)|^2 = \prod_{j=1}^{A} \rho(\boldsymbol{r}_j) + \sum_{i<j=1}^{A} \Delta(\boldsymbol{r}_\mathbf{i},\boldsymbol{r}_\mathbf{j}) \prod_{k\neq(il)}^{A} \rho_1(\boldsymbol{r}_k) + $$

$$+ \sum_{(i<j)\neq(k<l)} \Delta(\boldsymbol{r}_\mathbf{i},\boldsymbol{r}_\mathbf{j})\, \Delta(\boldsymbol{r}_\mathbf{k},\boldsymbol{r}_\mathbf{l}) \prod_{m\neq i,j,k,l} \rho_1(\boldsymbol{r}_m) + \dots$$

$$\simeq \prod_{j=1}^{A} \rho(\boldsymbol{r}_j) \longleftarrow \textit{usual approximation}: \textbf{ is it reliable}?$$

our two-body $\Delta(\boldsymbol{r}_\mathbf{i},\boldsymbol{r}_\mathbf{j}) = \rho_2(\boldsymbol{r}_i,\boldsymbol{r}_j) - \rho_1(\boldsymbol{r}_i)\,\rho_1(\boldsymbol{r}_j)$ provides:

$$\boxed{\int d\boldsymbol{r}_2\, \rho_2(\boldsymbol{r}_1,\boldsymbol{r}_2) = \rho_1(\boldsymbol{r}_1)} \longrightarrow \boxed{\int d\boldsymbol{r}_2 \Delta(\boldsymbol{r}_1,\boldsymbol{r}_2) = 0}$$

Mean Field *vs.* Correlated Formulæ

$$\sigma_{tot}^{pA} = 2Re\int d\boldsymbol{b}\left\{1 - exp\left[-\frac{\sigma_{tot}^{pN}}{2}T_A^p(\boldsymbol{b})\right]\right\} \longrightarrow 2Re\int d\boldsymbol{b}\left\{1 - exp\left[-\frac{\sigma_{tot}^{pN}}{2}\Big(T_A^p(\boldsymbol{b}) - \Delta T_A^p(\boldsymbol{b})\Big)\right]\right\}$$

where

$$T_A^p(\boldsymbol{b}) = \frac{2}{\sigma_{tot}^{pN}}\int d\boldsymbol{s}\,\Gamma_{pN}(\boldsymbol{s})\,T_A(\boldsymbol{b}-\boldsymbol{s}) = \int d\boldsymbol{s}\,\Gamma_{pN}(\boldsymbol{s})\,A\int dz_s\,\rho(\boldsymbol{b}-\boldsymbol{s}, z_s)\,;$$

$$\Delta T_A^p(\boldsymbol{b}) = \frac{A^2}{\sigma_{tot}^{pN}}\int d\boldsymbol{s}_1 d\boldsymbol{s}_2\,\Gamma_{pN}(\boldsymbol{s}_1)\,\Gamma_{pN}(\boldsymbol{s}_2)\int dz_1 dz_2\,\boldsymbol{\Delta}\Big(\boldsymbol{b}_1 - \boldsymbol{s}_1, z_1;\, \boldsymbol{b}_2 - \boldsymbol{s}_2, z_2\Big)\,.$$

Similarly, for the elastic and quasi-elastic cross sections:

$$\sigma_{el}^{pA} = \int d\boldsymbol{b}\left|1 - exp\left[-\frac{\sigma_{tot}^{pN}}{2}\Big(T_A^p(\boldsymbol{b}) - \Delta T_A^p(\boldsymbol{b})\Big)\right]\right|^2$$

$$\sigma_{qe}^{pA} = 2Re\int d\boldsymbol{b}\left\{exp\left[-\frac{\sigma_{in}^{pN}}{2}\Big(T_A^p(\boldsymbol{b}) - 2\frac{\sigma_{in}^{pN}}{\sigma_{tot}^{pN}}\Delta T_A^p(\boldsymbol{b})\Big)\right] - exp\left[-\frac{\sigma_{tot}^{pN}}{2}\Big(T_A^p(\boldsymbol{b}) - \Delta T_A^p(\boldsymbol{b})\Big)\right]\right\}$$

Example: total *neutron-Nucleus* cross section at high energies

$$\boxed{\sigma_{\mathbf{tot}} = \frac{4\pi}{k} Im\left[F_{00}(0)\right] \qquad F_{00}(\boldsymbol{q}) = \frac{ik}{2\pi}\int d^2 b_n e^{i\mathbf{q}\cdot\mathbf{b_n}}\left[1 - \mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\mathbf{b_n})}\right]}$$

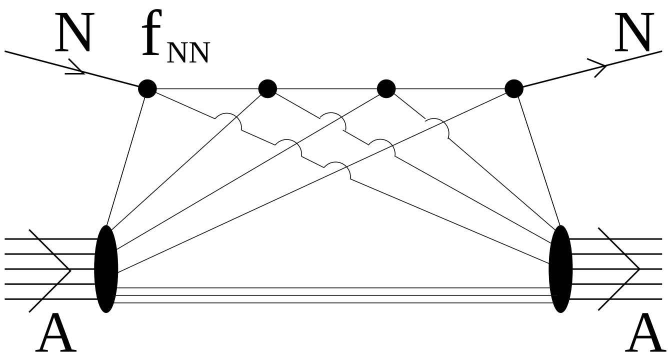
$$\mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\boldsymbol{b}_\mathbf{n})} = \int \prod_{j=1}^{A} d\boldsymbol{r}_j \prod_{j=1}^{A}\left[1 - \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_j)\right] \left|\Psi_0(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_A)\right|^2 \delta\left(\frac{1}{A}\sum \mathbf{r}_j\right)$$

using the $|\Psi_o|^2$ expansion, with:

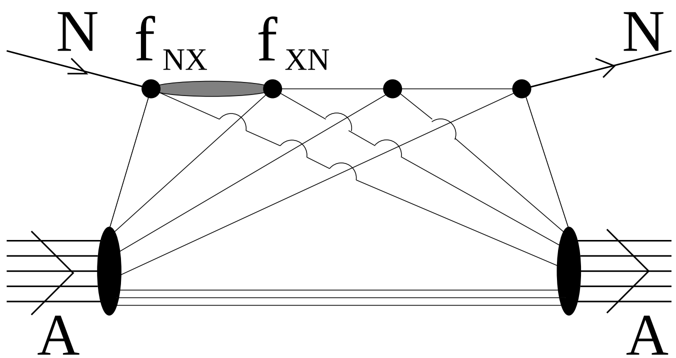
$$\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i}, \boldsymbol{r}_\mathbf{j}) = \rho_2(\boldsymbol{r}_i, \boldsymbol{r}_j) - \rho_1(\boldsymbol{r}_i)\,\rho_1(\boldsymbol{r}_j)\,;$$

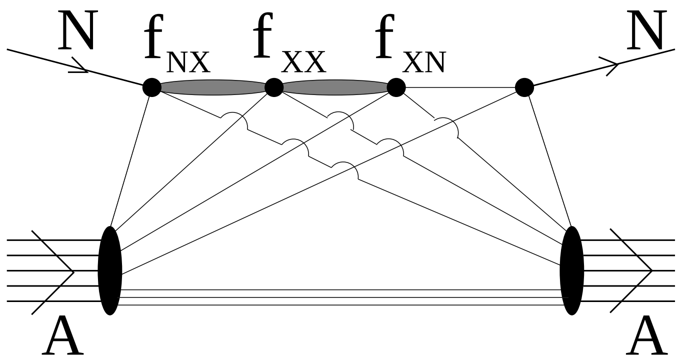
one has:

$$\boxed{\sigma_{\mathbf{tot}} = \sigma_{\mathbf{G}}^{(\mathbf{1})} + \sigma_{\mathbf{G}}^{(\mathbf{2})} + \boldsymbol{\Delta}\sigma_{\mathbf{in}}}$$



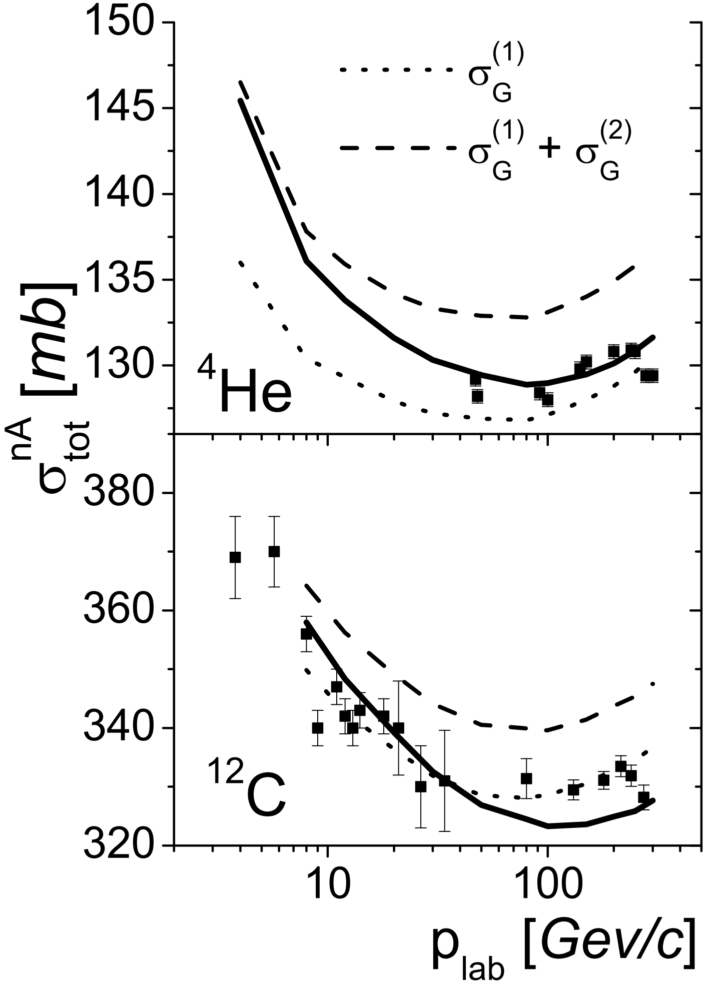
(Glauber)

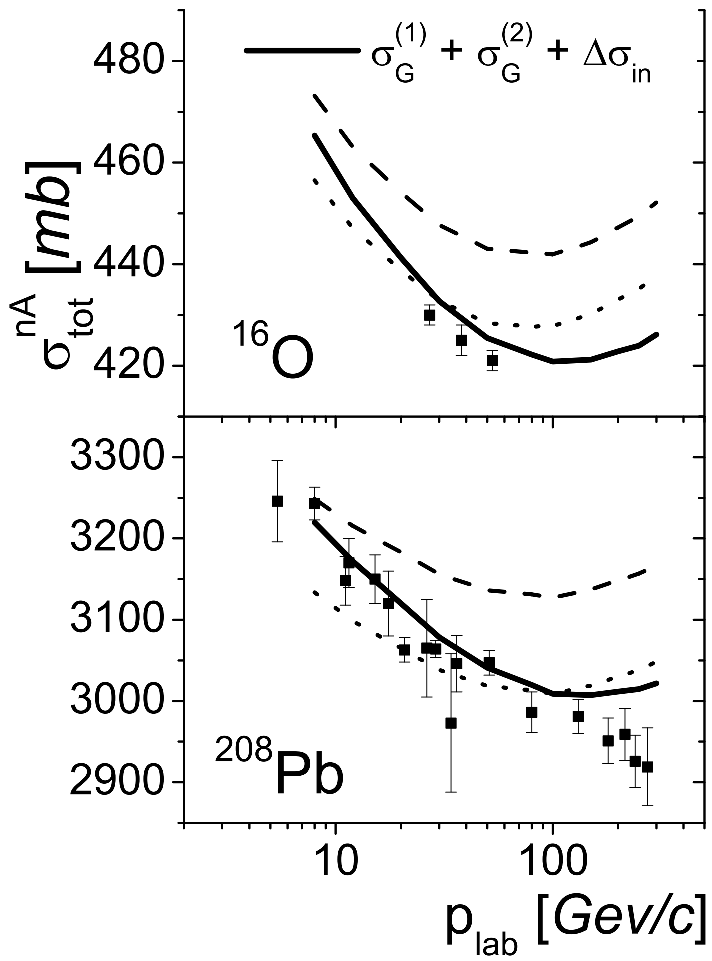




(Inelastic Shadowing)

Effects of NN Correlations in High-Energy Processes



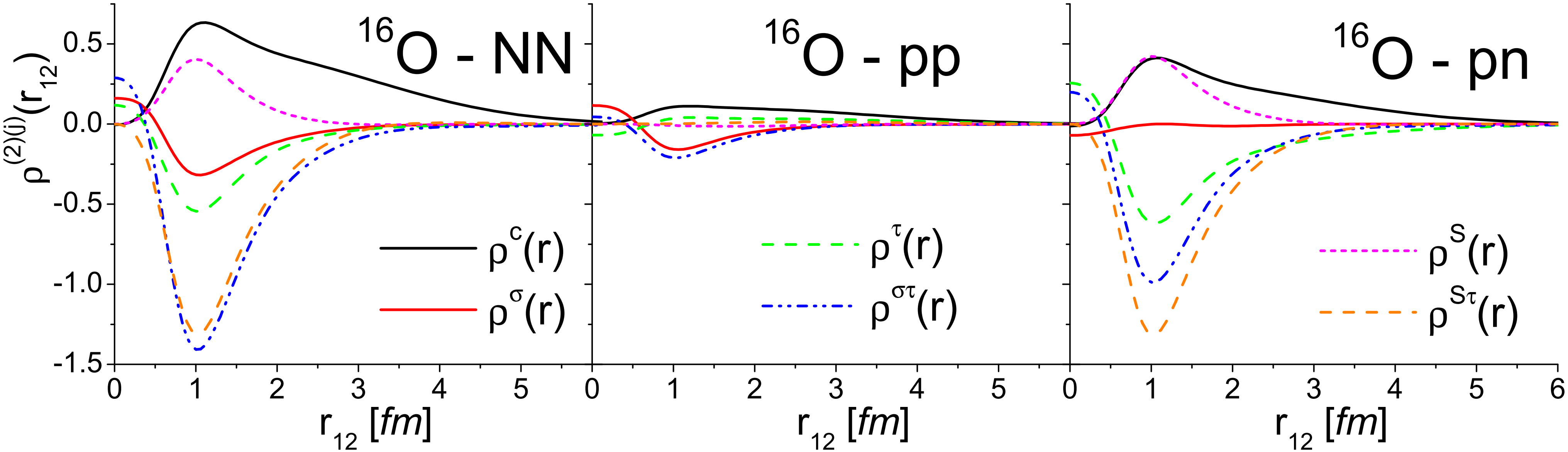


Alvioli, Ciofi degli Atti, Marchino, Morita, Palli, - Phys.Rev.C78 (2008)

Alvioli, Ciofi degli Atti, Kopeliovich, Potashnikova and Schmidt - - Phys.Rev.C81 (2010)

Potential energy: *pn* and *pp* contributions

$$\langle V\rangle_{pN} = \sum_{i<j} V_{ij} = \sum_j \int d\boldsymbol{r}_{12}\, v^{(j)}_{pN}(r_{12})\; \rho^{(2)(j)}_{pN}(r_{12})$$



A	$\langle V\rangle_{pp}(=\langle V\rangle_{nn})$	$\langle V\rangle_{pn}$
16	8%	83%
40	9%	82%

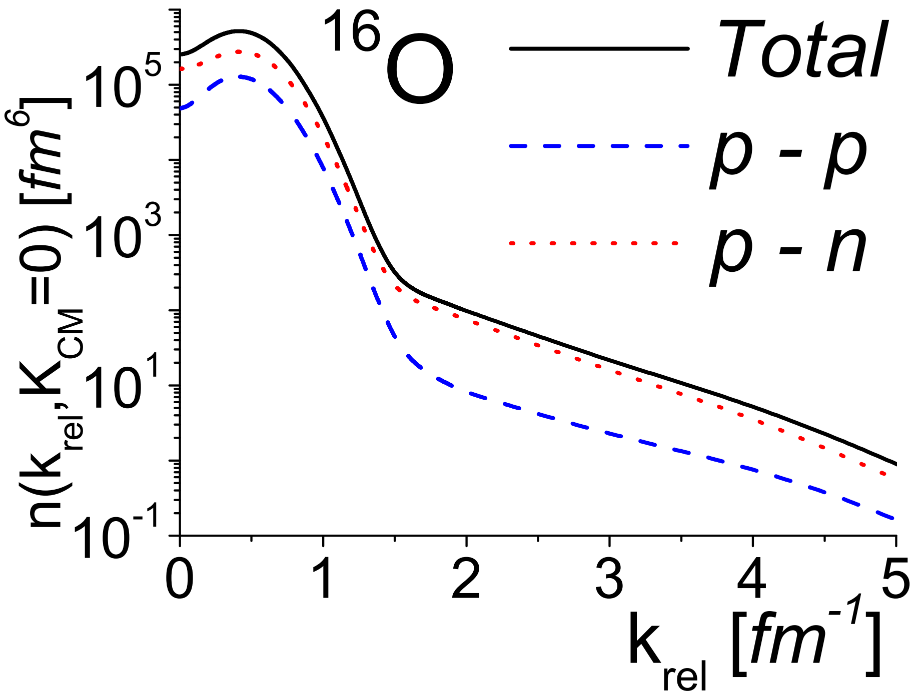
mostly pn pairs

switching off $\Rightarrow$ correlations

A	$\langle V\rangle_{pp}(=\langle V\rangle_{nn})$	$\langle V\rangle_{pn}$
16	23%	53%
40	24%	51%

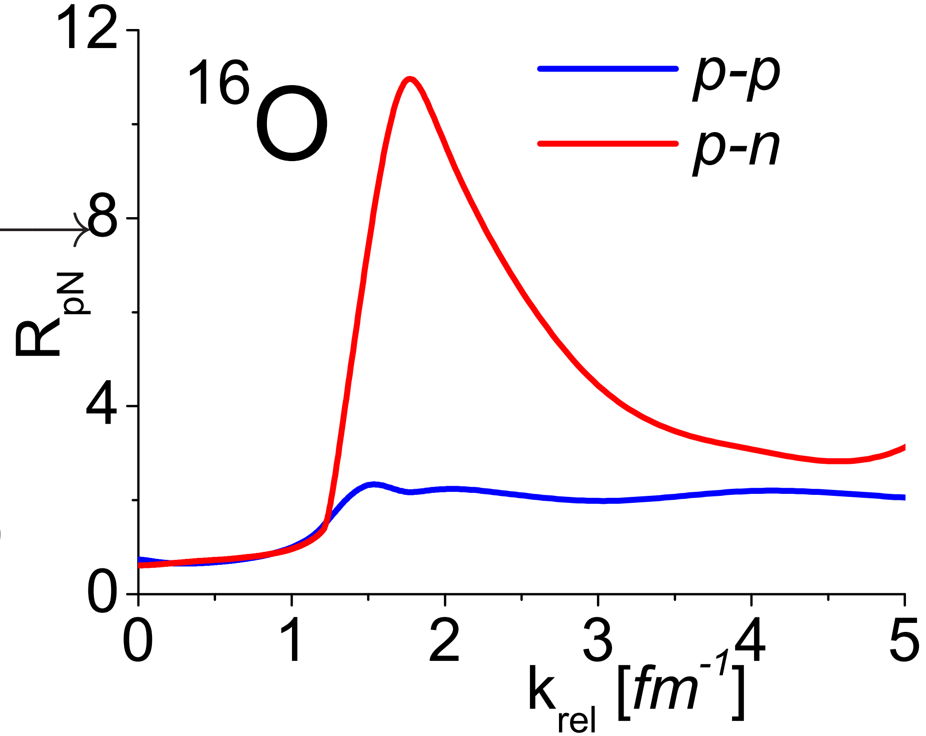
$\Rightarrow$ proportional to # of pairs

Back-to-Back nucleons: large pn to pp ratio



$\boldsymbol{two\text{-}body}$ $n(\boldsymbol{k}_1, \boldsymbol{k}_2)$:
$\longleftarrow$ ratio pp/pn largely enhanced in the correlation region

$$R_{pN} = n_{pN}^{\mathbf{tensor}}(k_{rel}, 0) \, / \, n_{pN}^{\mathbf{central}}(k_{rel}, 0) \longrightarrow$$



(*M.Alvioli, C.Ciofi degli Atti, H.Morita, PRL* ***100*** *(2008)*)

The Nuclear Many-Body Problem

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \quad with: \quad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

where

$$\hat{v}_{ij} = \sum_n v^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

$$\hat{\mathcal{O}}_{ij}^{(n)} = \Big[1\,,\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\,,\,\hat{S}_{ij}\,,\,(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\,,\,...\Big] \otimes \big[1\,,\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\big]\,.$$

The same operatorial dependence is cast onto Ψ_o:

$$\Psi_o = \hat{\mathbf{F}}\,\phi_o$$

where ϕ_o is the *mean-field* wave function and

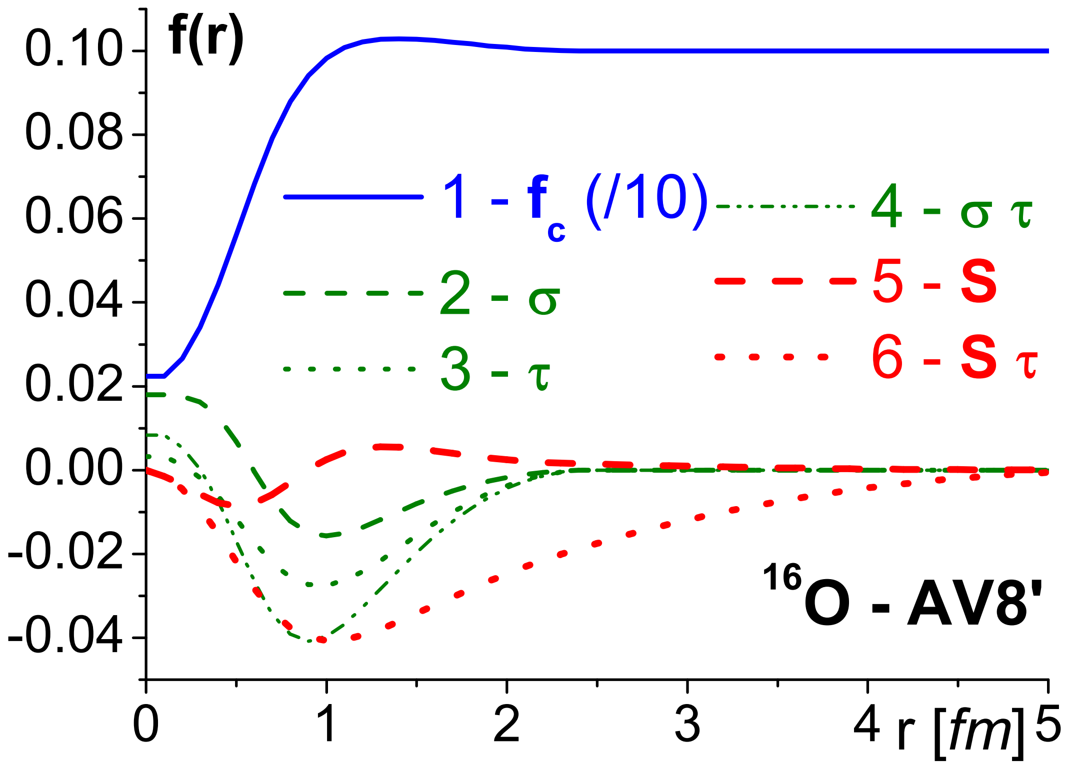
$$\hat{\mathbf{F}} = \hat{S}\prod_{i<j}\hat{f}_{ij} = \hat{S}\prod_{i<j}\sum_n f^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

is a *correlation* operator.

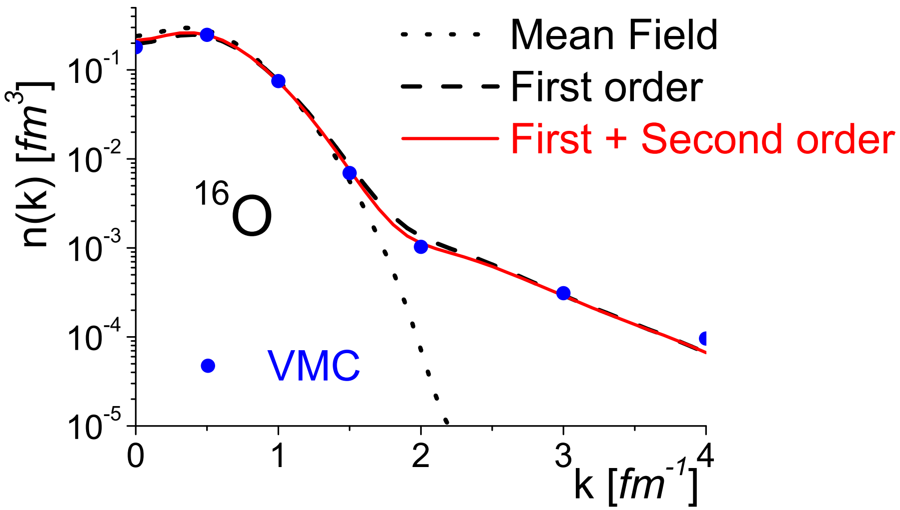
Ground state energy: ^{16}O - Argonne $V8'$

	$<V_c>$	$<V_\sigma>$	$<V_\tau>$	$<V_{\sigma\tau}>$	$<V_S>$	$<V_{S\tau}>$	$<\mathbf{V}>$	$<\mathbf{T}>$	**E**	**E/A** MeV
$\eta-exp$	0.19	-35.88	-9.47	-171.32	-0.003	-172.89	-389.40	323.50	**-65.90**	-4.12
FHNC	0.694	-40.13	-10.61	-180.00	-0.07	-160.32	-390.30	325.18	**-65.12**	-4.07

correlation functions: *Central*, *Spin-Isospin*, *Tensor*



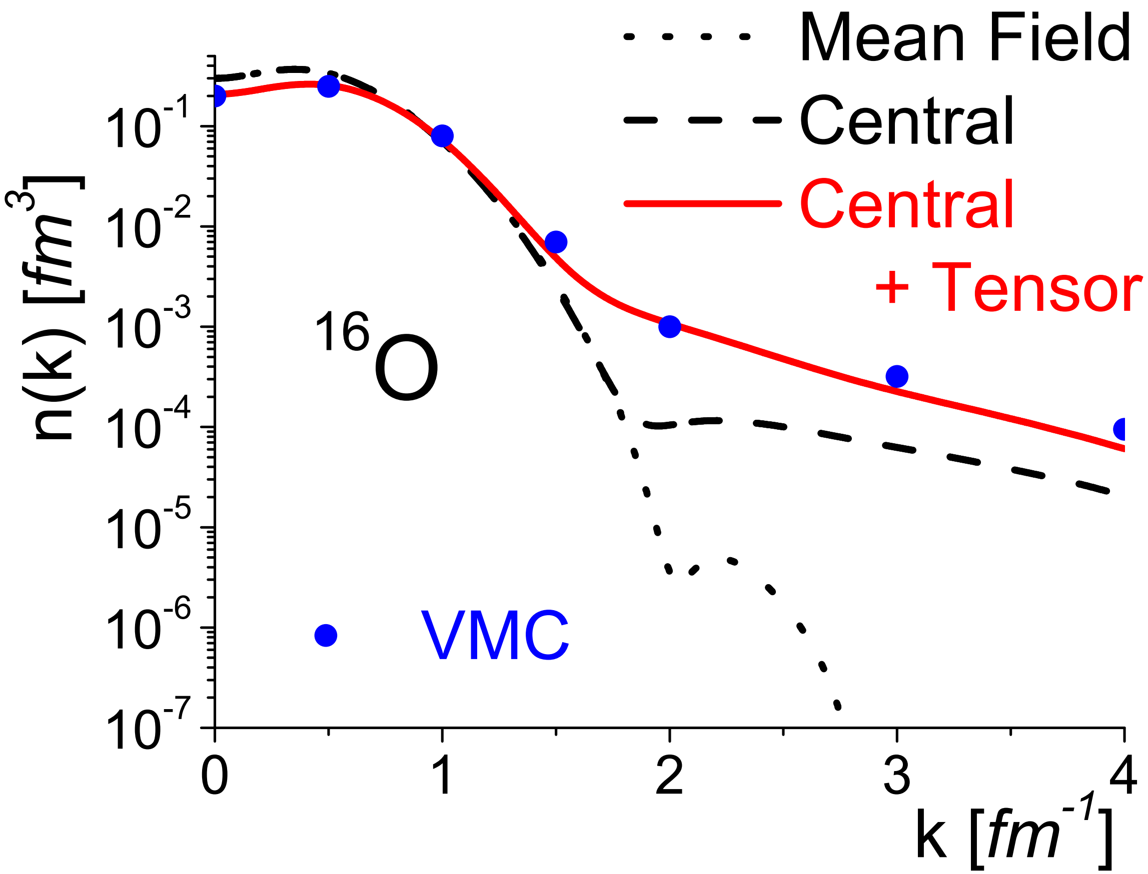
Momentum Distributions and Tensor Forces

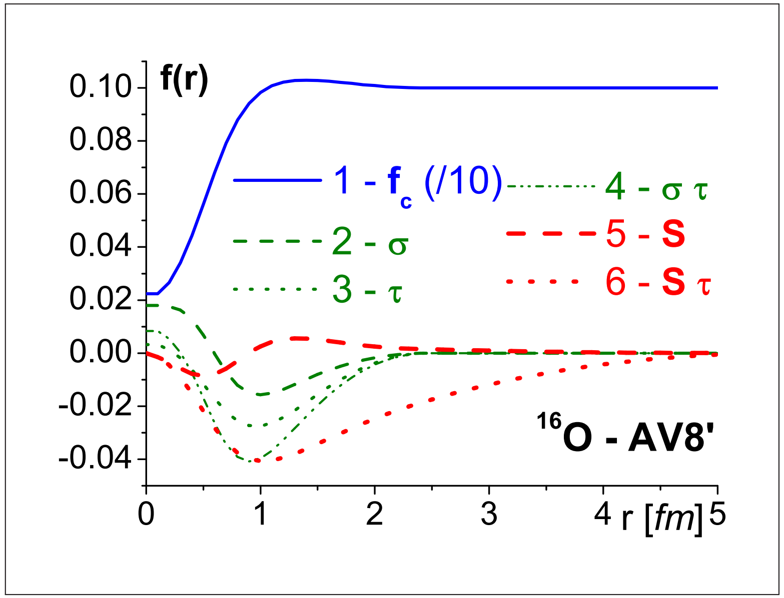


$\longleftarrow$ cluster expansion convergence

Tensor $(S + S \otimes \tau)$ ***correlations!***

$\mathbf{k > 1.5\ fm^{-1}} \longrightarrow$

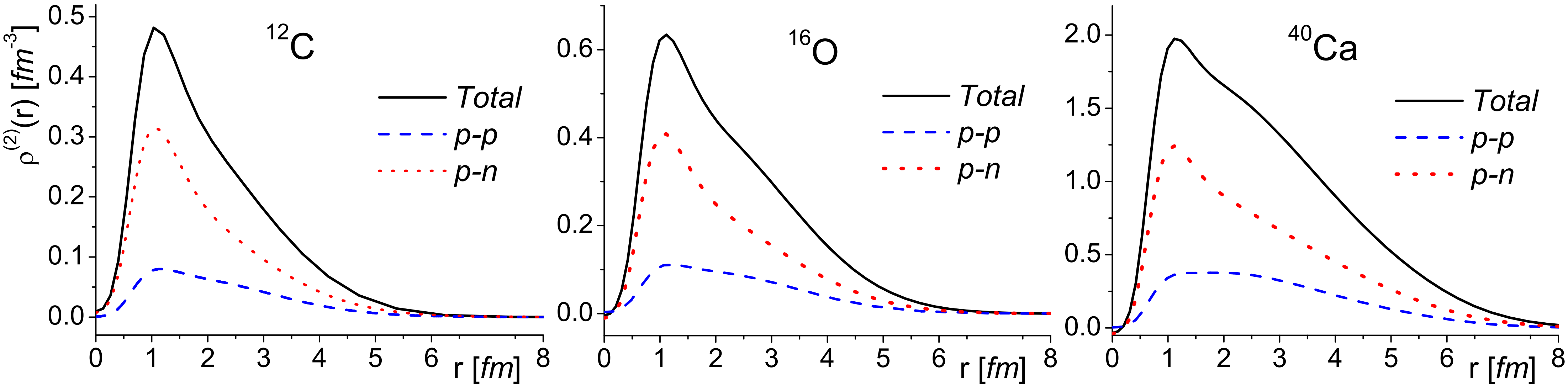




$\longleftarrow$ correlation functions

Two-Body Densities: isospin separation

$$\rho^{(2)}(r) = \int d\boldsymbol{R}\, \rho^{(2)}\left(\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\,\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\,;\,\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\,\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\right)$$

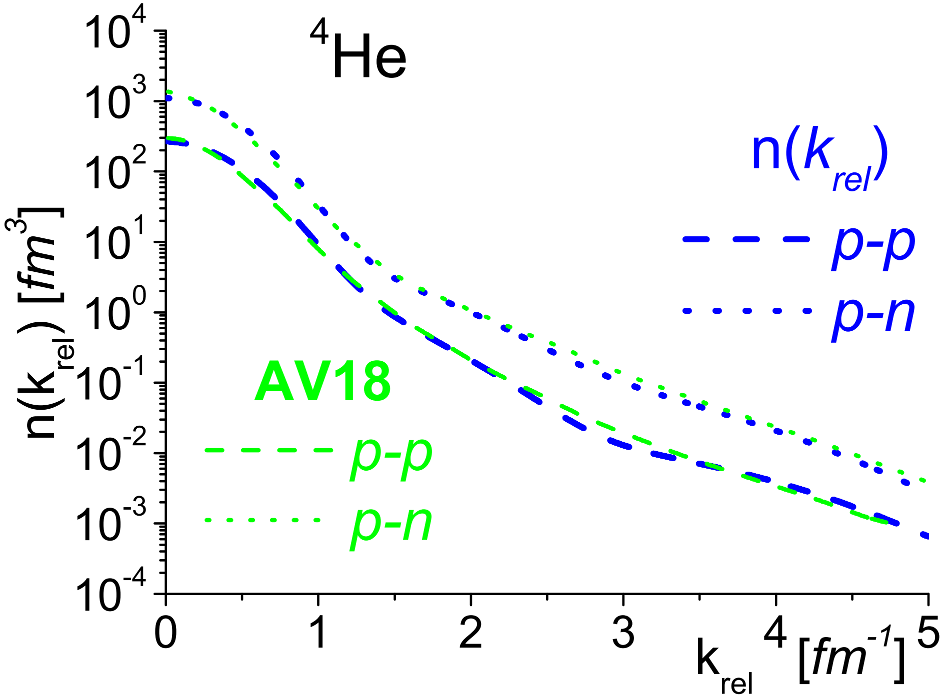


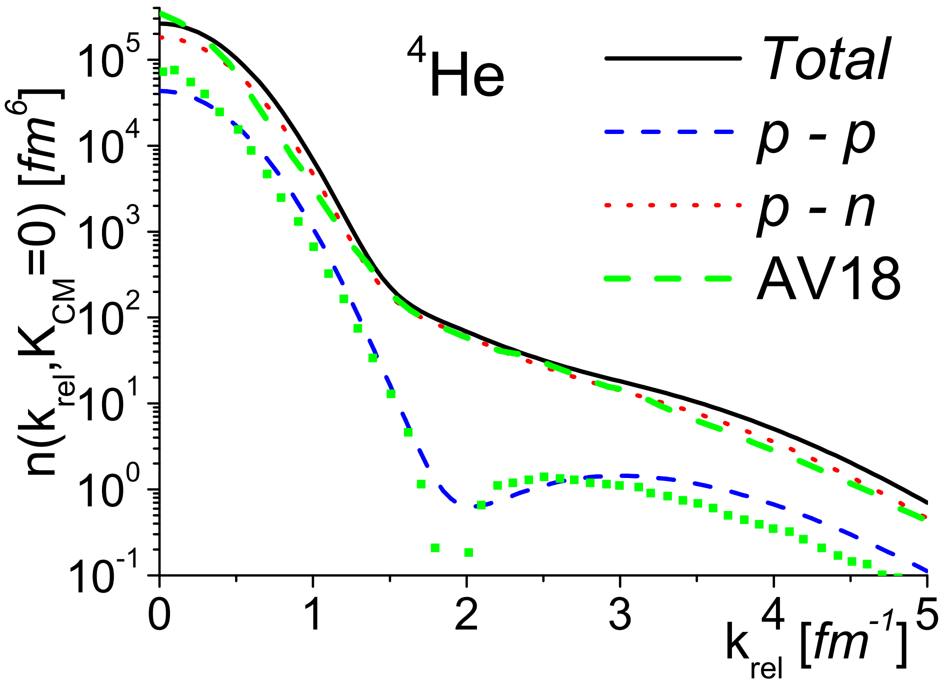
- normalization (`number of pairs`) conserved by the expansion
- `isospin` separation feasible: $\rho^{(2)} = \rho^{pp}_{(2)} + \rho^{pn}_{(2)} + \rho^{nn}_{(2)}$
- We can build **two-body *pp*, *pn* and *nn*** momentum distributions

  (*M. Alvioli, C.Ciofi degli Atti, H.Morita, PRL* ***100*** *(2008)*)

^4He: comparison with VMC

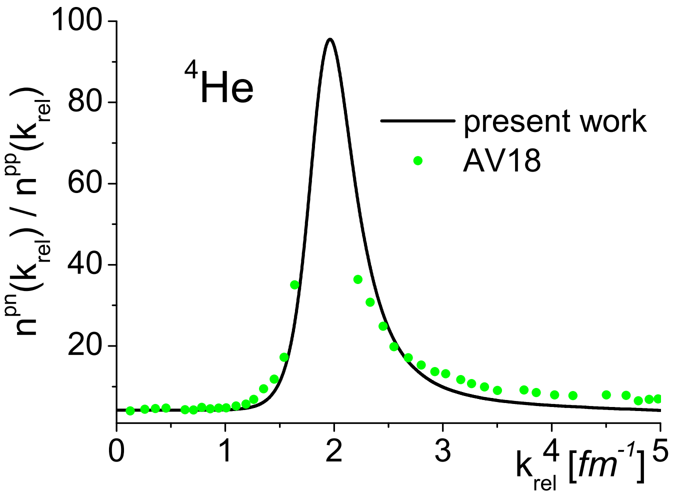
$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) \qquad n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM} = 0)$$





- good agreement with VMC calculations
- $n_{pn}(k_{rel}, 0)/n_{pp}(k_{rel}, 0)$ peak location ok $\longrightarrow$

(**AV18**: *Schiavilla at al.* **PRL98** (2007))



M. Alvioli

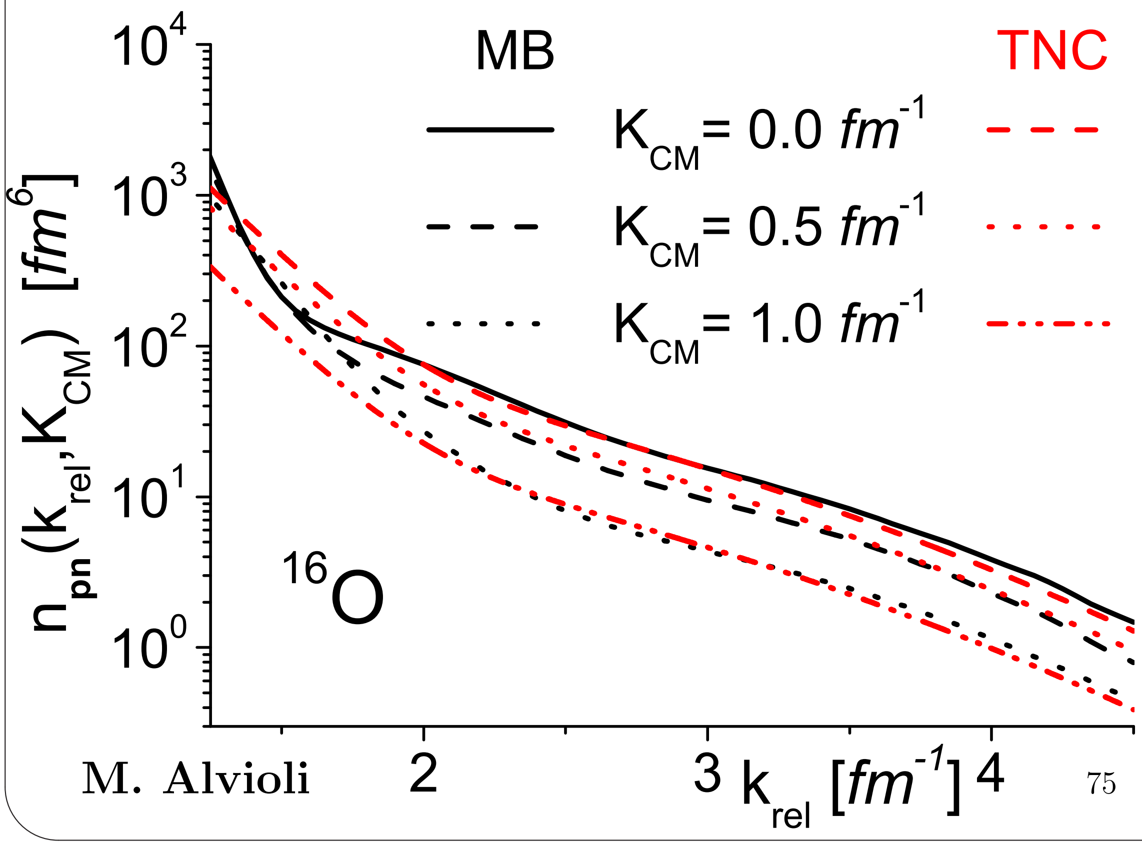
Spectral Function properties at low K_{CM} and high k_{rel}

1-body SF:

$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{K}_{cm}\; n_{rel}^A(|\mathbf{k} - \mathbf{K}_{cm}/2|)\, n_{cm}^A(|\mathbf{K}_{cm}|)\, \delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)}\left(\mathbf{k} - \frac{(A-1)\mathbf{K}_{cm}}{(A-2)}\right)^2\right]$$

2-body SF:

$$P_2^A(\mathbf{k}, \mathbf{K}_{cm}, E) = n_{rel}^A(|\mathbf{k} - \mathbf{K}_{cm}/2|)\, n_{cm}^A(|\mathbf{K}_{cm}|)\, \delta\left(E - E_{thr}^{(2)}\right)$$



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MB: $n_{pn}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{cm})$

TNC: Ciofi, Simula PRC53, (1996)

$C_A\; n_{^2H}(k_{rel})\; n_{cm}(K_{cm})$

factorization of $\mathbf{n_{pn}}$

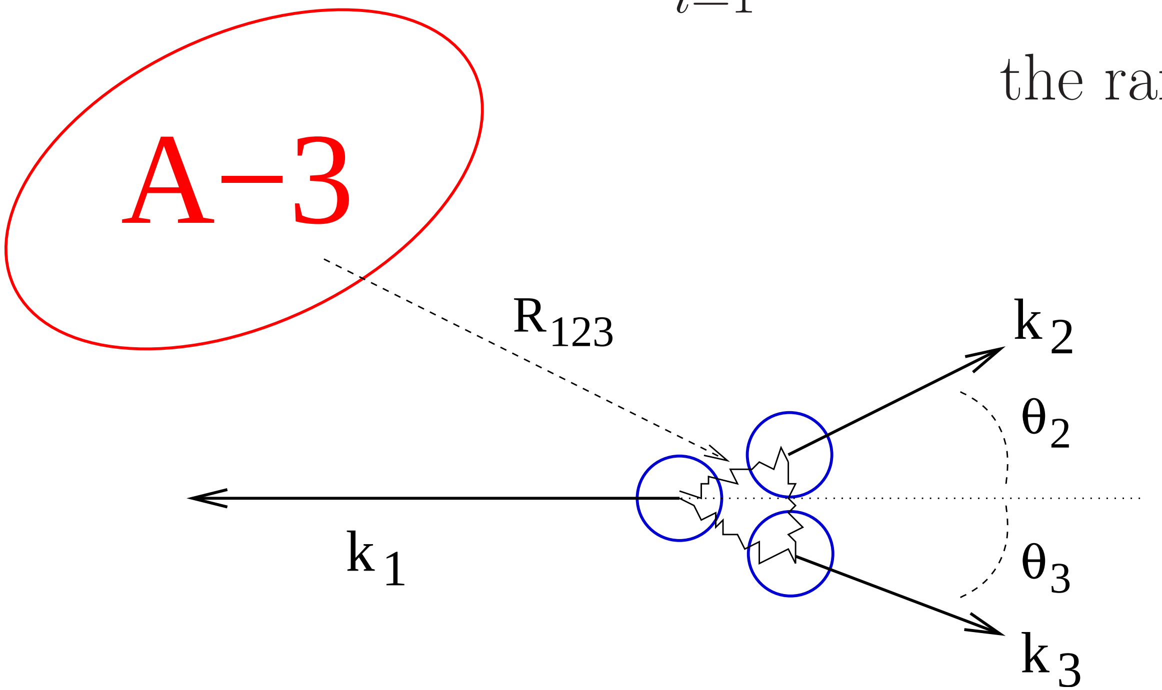
more on three-body correlations?

We can easily ☺ evaluate within the cluster expansion the three-body density

$$\rho^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; \boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}'_3)$$

and calculate, for given values of $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$

$$n(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{1}{(2\pi)^9} \int \prod_{i=1}^{3} d\boldsymbol{r}_i d\boldsymbol{r}'_i \, e^{i \sum_{j=1}^{3} \boldsymbol{k}_j \cdot (\boldsymbol{r}_j - \boldsymbol{r}'_j)} \rho^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; \boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}'_3)$$

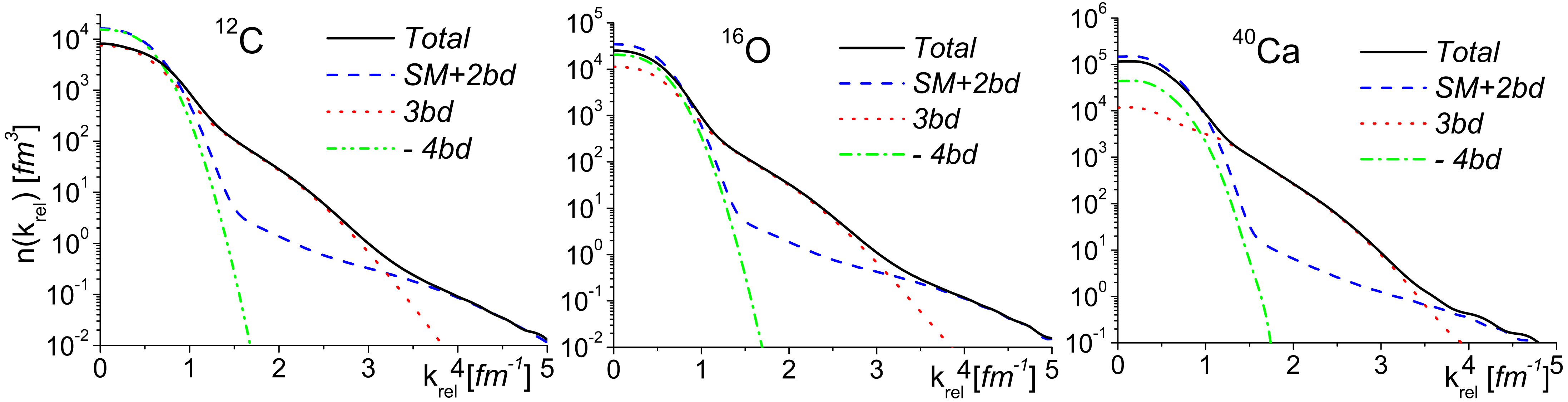


the random "*noise*" to be subtracted:

$$n^{(1)}(\boldsymbol{k}_1) n^{(2)}(\boldsymbol{k}_2, \boldsymbol{k}_3) +$$
$$+ n^{(1)}(\boldsymbol{k}_2) n^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_3) +$$
$$+ n^{(1)}(\boldsymbol{k}_3) n^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) +$$
$$+ n^{(1)}(\boldsymbol{k}_1) n^{(1)}(\boldsymbol{k}_2) n^{(1)}(\boldsymbol{k}_3)$$

$n_{NN}(k_{rel})$ for Complex Nuclei

$$n_{NN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$



- normalization (`number of pairs`) conserved by the expansion
- `isospin` separation feasible
- closed `j-shell` nuclei included in the formalism
- *three* and *four-body* diagrams `essential`

0. Cluster Expansion

- The expansion acts on the numerator and denominator of an operator expectation value ($F^2 = \prod f^2 \simeq \sum (1+\eta)$):

$$\frac{\langle\psi_o|\,\hat{\mathcal{O}}\,|\psi_o\rangle}{\langle\psi_o|\,\psi_o\rangle} = \frac{\langle\phi_o|\,F^\dagger\,\hat{\mathcal{O}}\,F\,|\phi_o\rangle}{\langle\phi_o|\,F^\dagger\,F\,|\phi_o\rangle} \overset{!}{=} \frac{\langle\phi_o|\,F^2\,\hat{\mathcal{O}}\,|\phi_o\rangle}{\langle\phi_o|\,F^2\,|\phi_o\rangle} =$$

$$\simeq \frac{\langle\phi_o|\,(1+\eta)\,\hat{\mathcal{O}}\,|\phi_o\rangle}{1+\langle\phi_o|\,\eta\,|\phi_o\rangle} =$$

$$\simeq \left[\langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle + \langle\phi_o|\sum_{i>j}\eta_{ij}\,\hat{\mathcal{O}}\,|\phi_o\rangle\right]\cdot\left[1 - \langle\phi_o|\sum_{i>j}\eta_{ij}\,|\phi_o\rangle\right] =$$

$$\simeq \langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle + \langle\phi_o|\sum_{i>j}\eta_{ij}\,\hat{\mathcal{O}}\,|\phi_o\rangle - \langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle\,\langle\phi_o|\sum_{i>j}\eta_{ij}\,|\phi_o\rangle\,,$$

in which we dropped terms $\propto\ \mathcal{O}(\eta^2) \equiv \mathcal{O}(f^4)$ ...

... obtaining: $\langle\hat{\mathcal{O}}\rangle = \mathcal{O}_0 + \mathcal{O}_1 + \mathcal{O}_2 + \ldots,$ with:

$$\mathcal{O}_0 = \langle\hat{\mathcal{O}}\rangle \equiv \langle\phi_o|\hat{\mathcal{O}}|\phi_o\rangle\,,$$

$$\mathcal{O}_1 = \langle\sum_{ij}\eta_{ij}\,\hat{\mathcal{O}}\rangle - \mathcal{O}_0\,\langle\sum_{ij}\eta_{ij}\rangle \equiv \langle\sum_{ij}\eta_{ij}\,\hat{\mathcal{O}}\rangle_{Linked}\,;$$

where at each order n of the expansion only powers of

$$\eta^{n/2} \equiv f^n$$

appear, and we use the notation:

$$\eta_{ij}\,\hat{\mathcal{O}} \equiv f_{ij}\,\hat{\mathcal{O}}\,f_{ij} - \hat{\mathcal{O}}\,;$$

$$\eta_{ij}\,\eta_{kl}\,\hat{\mathcal{O}} \equiv f_{ij}\,f_{kl}\,\hat{\mathcal{O}}\,f_{kl}\,f_{ij} - f_{ij}\,\hat{\mathcal{O}}\,f_{ij} - f_{kl}\,\hat{\mathcal{O}}\,f_{kl} + \hat{\mathcal{O}}\,.$$