Correlations in nuclei

or: on the importance of using $S(k, E)$

Ingo Sick

Historical development of nuclear physics

strongly influenced by mean-field ideas
existence of Quasi-Particle orbits
when use fitted effective interactions
can explain many features of nuclei
but: limited to region Z/A where parameters fitted

More fundamental approach: start from N-N interaction

Faddeev, Variational, MC for A<<
Greens-function MC
Bethe-Bruckner-Goldstone for NM
Correlated Basis Function (CBF) theory for NM
applicable to yet unknown nuclei
decisive at higher densities as *e.g.* in stars

Main difference

account for short-range N-N correlations
scattering of N to orbits $E \gg E_F$, $k \gg k_F$

Ideal approach to expose correlations: CBF theory

appear explicitly as variational functions $f(r_{ij})$ in wave function

$$|N) = \mathcal{G}|N], \quad \mathcal{G} = S\prod_{j>i} F(i,j), \qquad F(i,j) = \sum_n f^n(r_{ij})O^n(i,j)$$

G = many-body correlation operator

$|N]$ = MF state

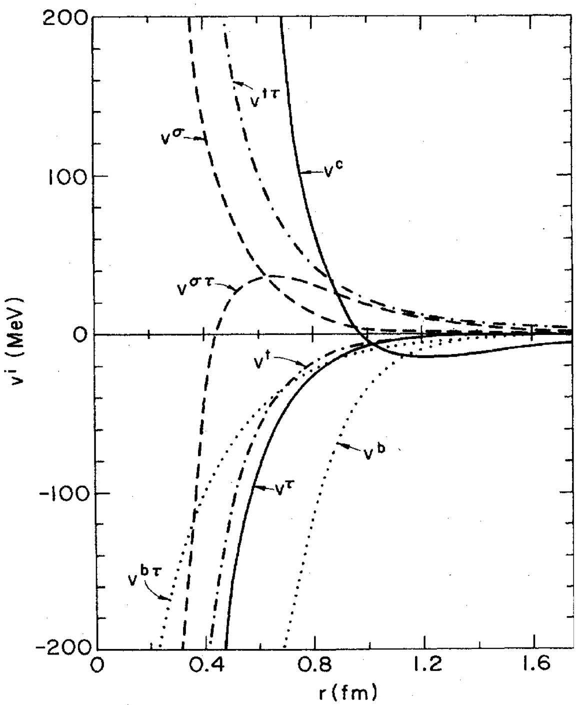
O = operators of V_{NN}

f = correlation functions variationally det.

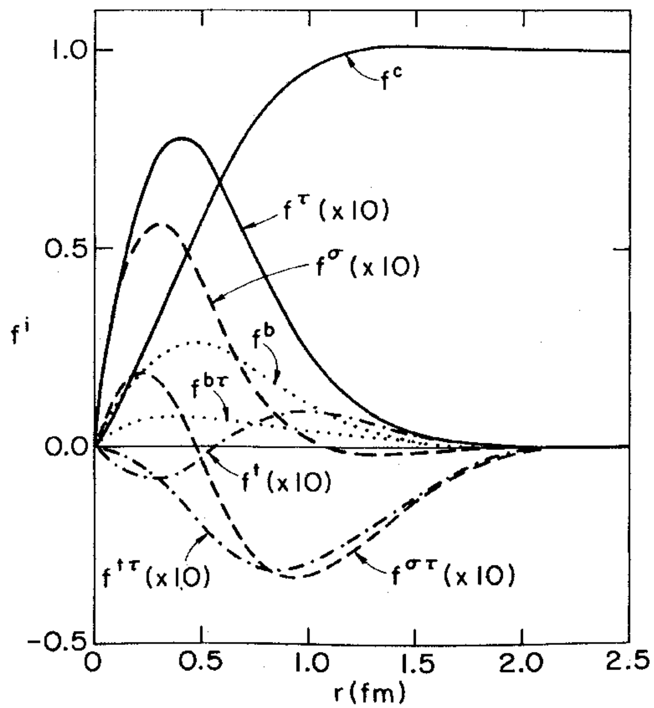
F = two-body correlation operator

Effect of correlations

on components of potential



on f(r_{ij})



correlation hole for some components, short-range enhancements for others

Consequences

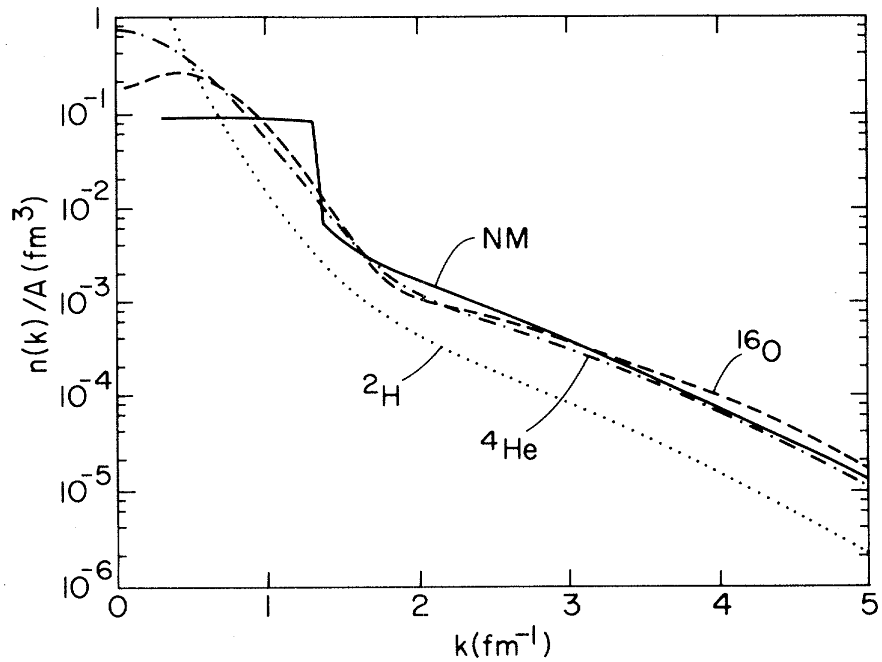
Important high-k components

V_{NN} in some channels strongly repulsive at small r

channel dependence complicates exact solution of Schrödinger equation

core leads to high-k tail of $n(k)$

rather universal for nuclei A=2...∞



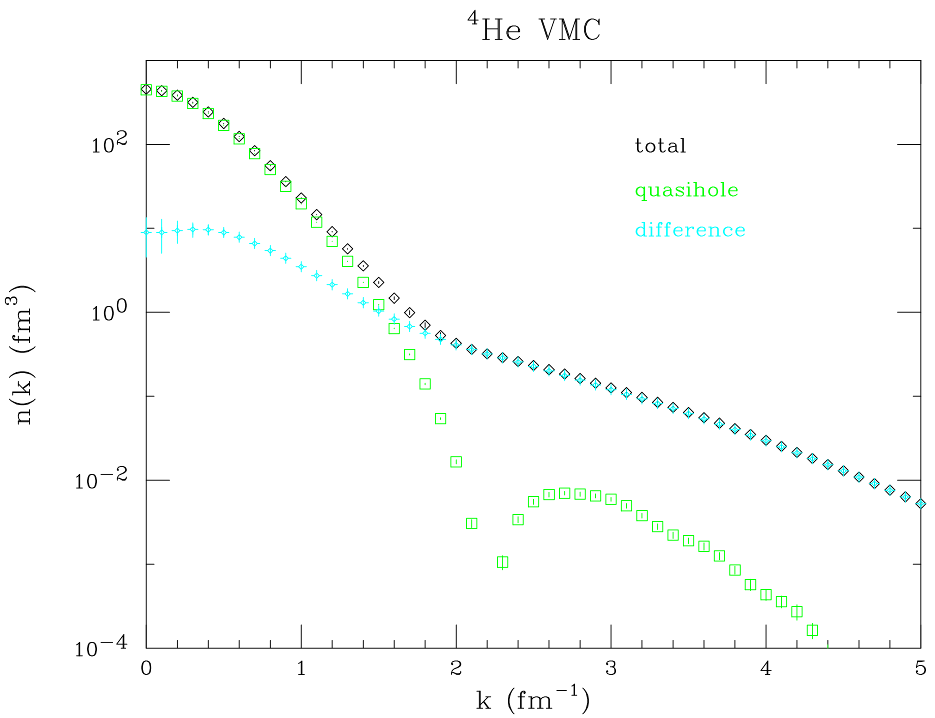
→ search for high-k popular theme... leading mostly to failures!

Important difference quasi-particle $\leftrightarrow$ correlated strength

at low E observe QP states

behave in most respects like shell-model states

at large E observe correlated states



□□□□ QP orbital, observed *e.g.* in $^4He(e,e'p)^3H$

drops off rapidly at large k

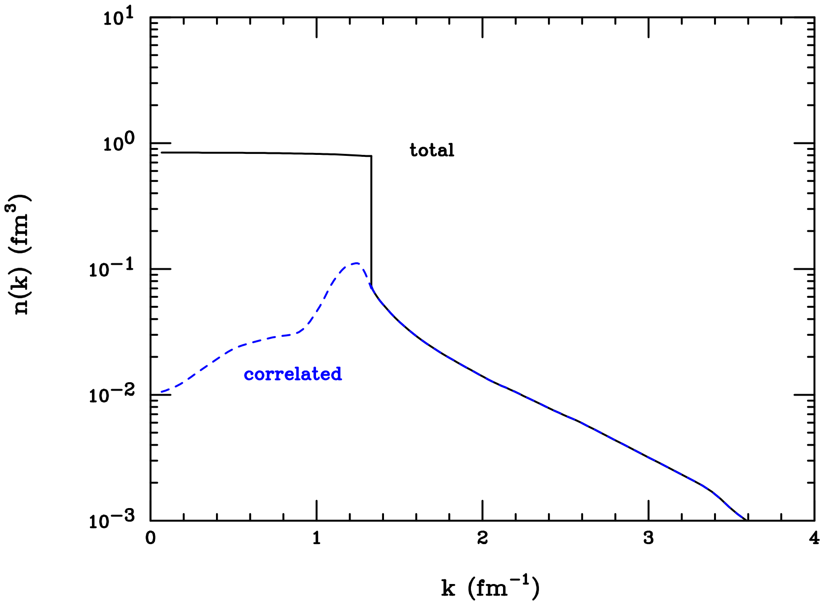
◇◇◇◇ correlated strength in continuum at large E

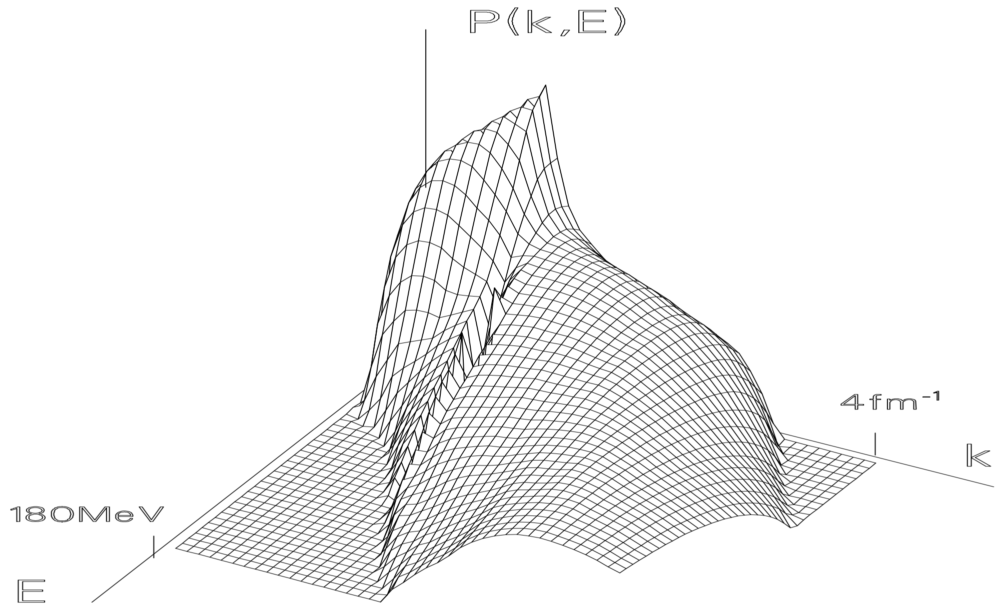
falls off much less quickly, dominates large-k totally

E in continuum $\Rightarrow$ cannot describe properly using $n(k)$

Must describe using spectral function $S(k, E)$

for nuclear matter:





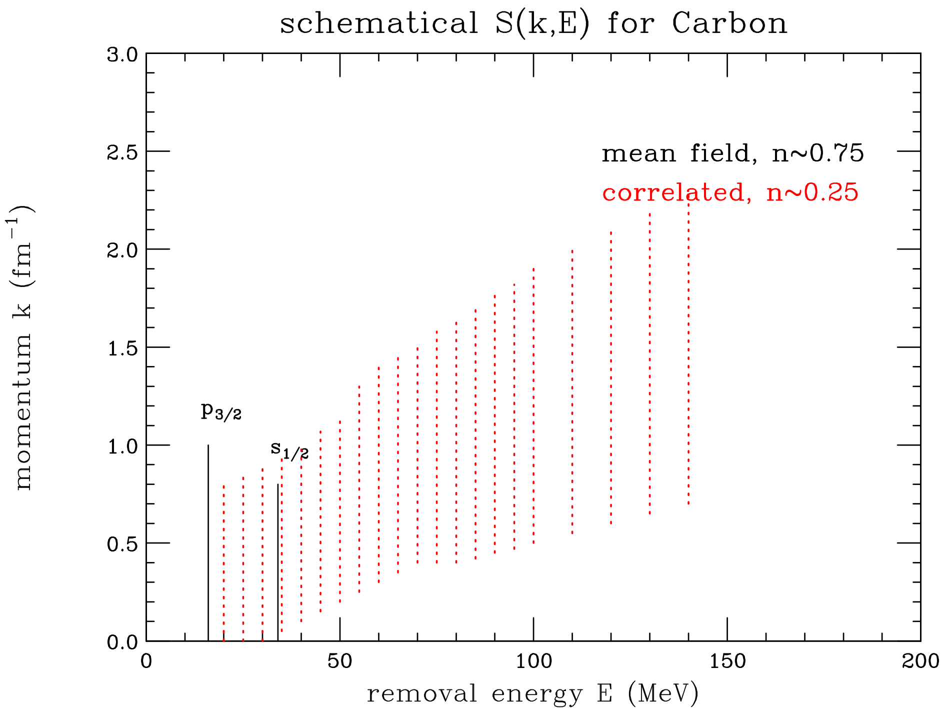
- correlations give strength at *both* large k and E
- strength *very* spread out, hard to identify experimentally
- correlated N have ∼20-25% probability (NM),

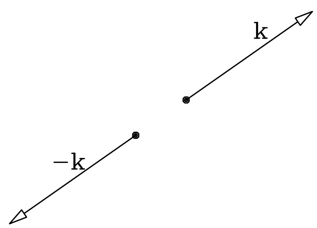
  but give 37% of removal energy

  47% of kinetic energy

- example: for ^{12}C $\bar{E}$= 25MeV from s+p-shells, $\bar{E}$= 52MeV from FHNC

Qualitative structure of $S(k, E)$





Understanding of structure at high k

large k cannot occur in nuclear mean-field

large k occur in 2N-collisions, scattering N to k outside Fermi sphere

if remove one N with large k then second N is set free

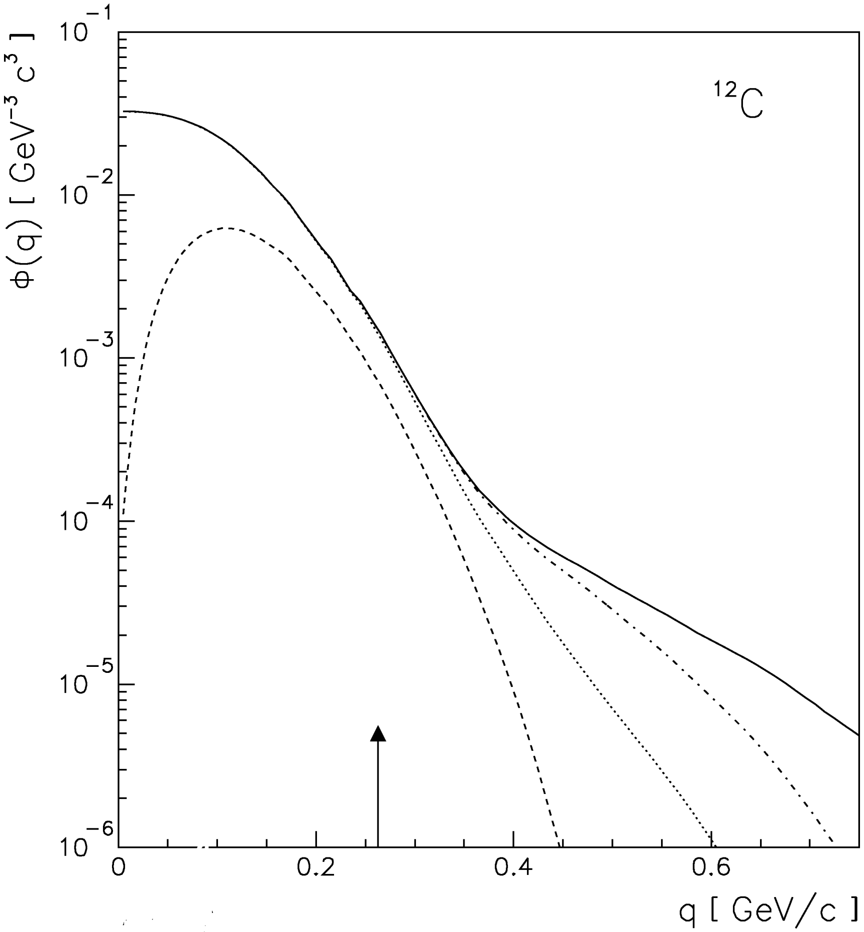
costs energy $E \sim (-k)^2/2M \rightarrow$ large E

verified by (e,e'pp) Shneor et al.

Large k only appear at large E !!

Drastic consequences for $n(k)$

study $n(k)$ with different cutoffs in E



$E < 30MeV$ - - -
$E < 100MeV$ .....
$E < 200MeV$ -.-.-.
$E < 500MeV$ ——-

At low E find only mean-field strength

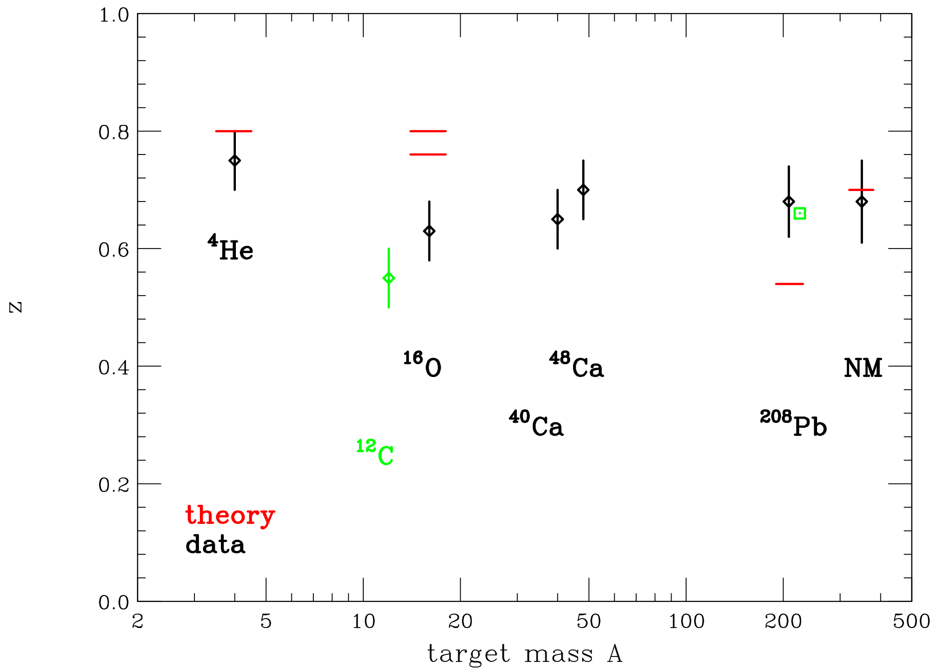
to get at correlations, *i.e.* high-k need *really large* E!

Alternative insight for coupling $\langle T \rangle$, $\langle E \rangle$

Koltun sumrule $\quad \mathrm{BE/A} = 0.5\ (\langle E \rangle - \langle T \rangle)$

large $\langle T \rangle$ implies large $\langle E \rangle$ since BE/A small
average E much larger than usually assumed ($\rightarrow$ position q.e. peak, EMC, ...)

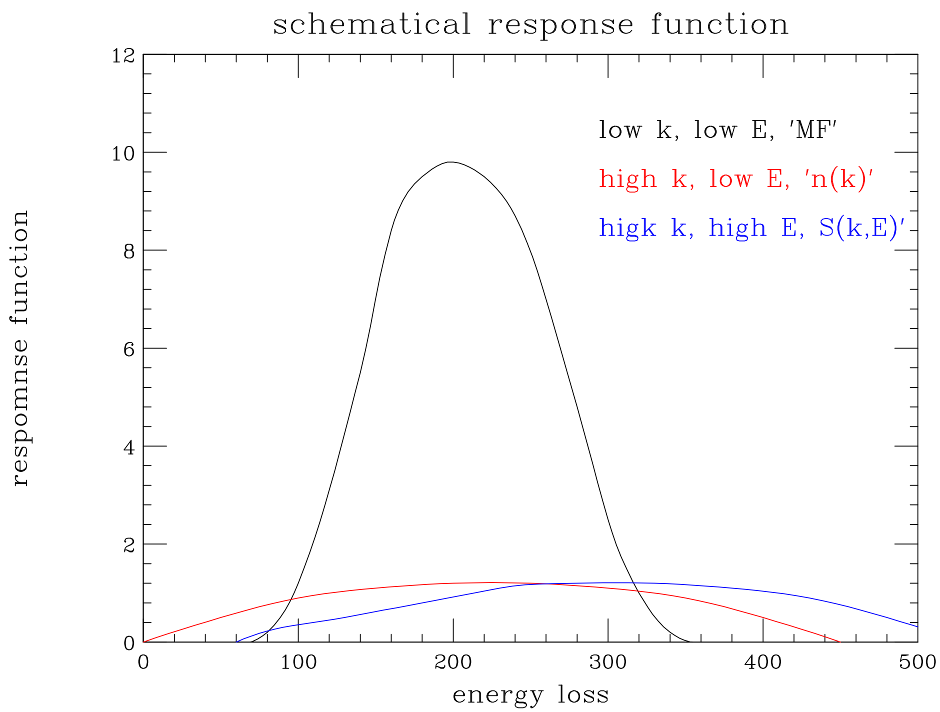
Consequences: partial occupation of MF orbits ∼0.75



Rest of strength

not detectable in transfer reaction experiments (E <10MeV)
can be seen in (e,e'p)

Importance of high-k, E for tails of quasi-elastic peak



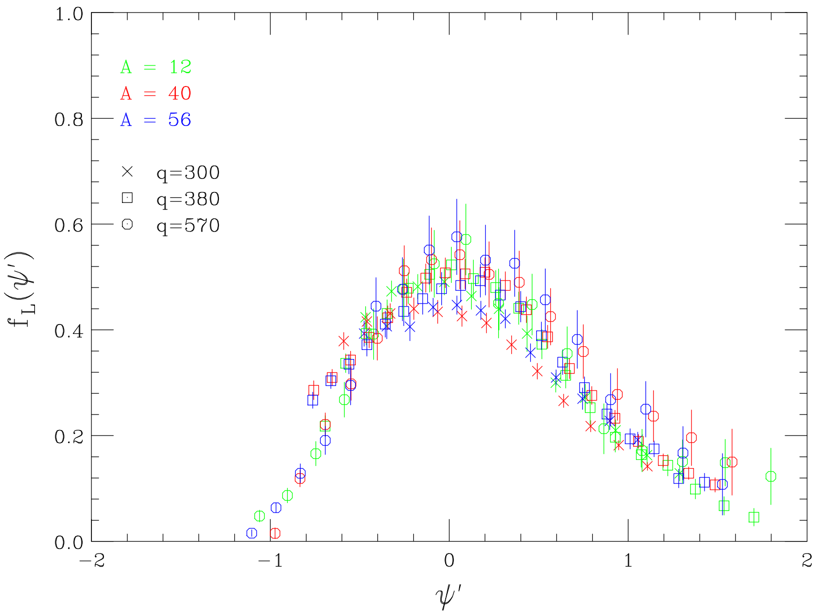
High-k strength is moved to large energy loss ω
disappears under MF piece at low ω
low-ω tail dominated by low-k (+FSI+...)

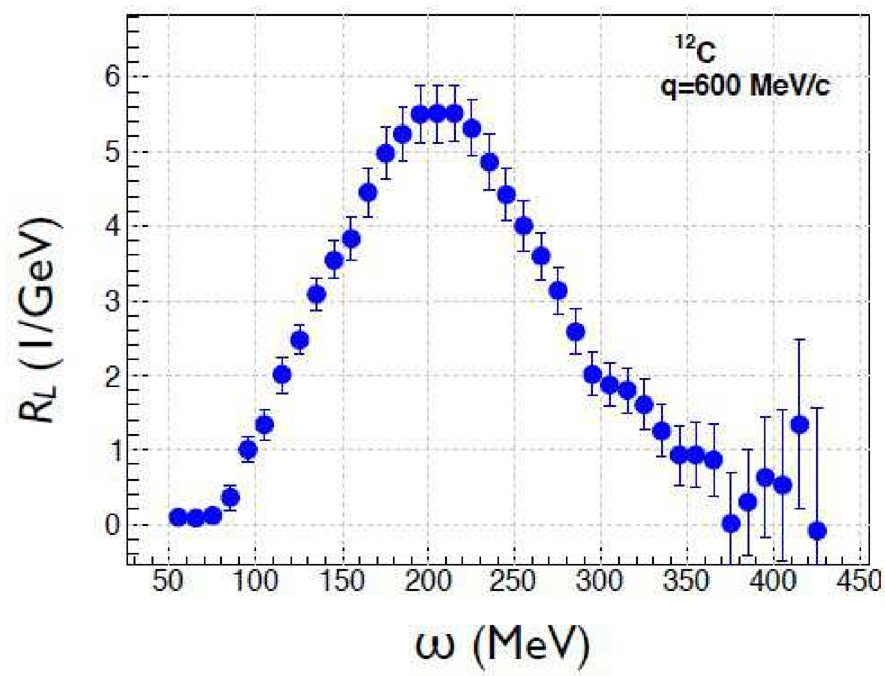
Idea of observing high-k in low-ω tails of q.e. peak naive

Large-ω tail is only place to observe high-k
but is usually obscured by MEC, FSI, ...

Tail visible in longitudinal response

from superscaling





Oh *et al.*

Shape of quasi-elastic peak asymmetrical, far from Fermi-gas!

rarely appreciated
neglect of tail = main reason for troubles with CSR
affects other observables such as in ν-scattering

What do we know even *without* measuring high-k, high-E?

1. $n(k)$ from exact calculations for A=3÷11,16,∞

can today solve Schrödinger equation for best NN-potentials
 Faddeev, CBF, AFMC, GFMC, ..
calculations are phenomenally successful
 explain many observables

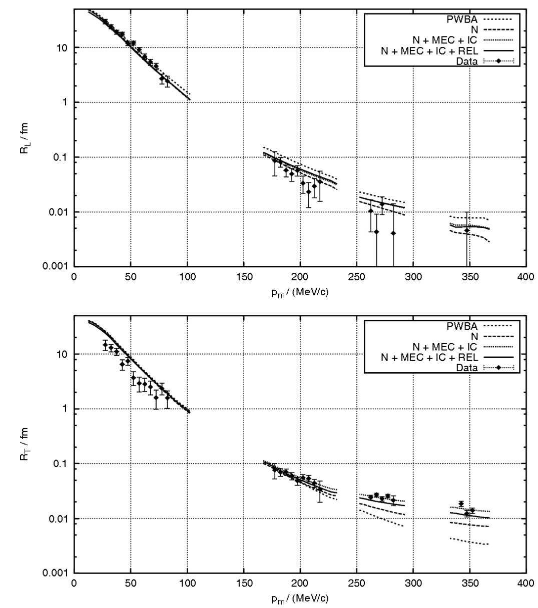
in particular explain binding energy

Koltun sumrule BE/A $= (\langle E \rangle - \langle T \rangle)/2$
 $\pm$ 1MeV $\sim$50MeV

$\langle T \rangle$ quite accurate $\rightarrow$ can trust $\langle E \rangle$ and predictions for large k, E

2. $S(k, E)$ for A=3,4 and ∞

calculated using exact methods
 situation similar to the one for $n(k)$



3. Large-k fall-off same as for deuteron

same short-range V_{NN} $\rightarrow$ same fall-off
know quite well from experiment

4. Integrated correlated (high-k, E) strength known
occupation s_{MF} of mean-field orbits measured
$1-s_{MF}$ yields integrated correlated strength
agrees well with theoretical predictions

We know a lot!
new work *must* start from this knowledge

Minimum requirement when trying to extract large k, E:
calculate observable with $S(k, E)$ in PWIA (easy!)

If σ_{PWIA} deviates by more than 30% from σ_{exp} then non-IA processes dominate
no point in trying to determine $S(k, E)$ or $n(k)$

Sources for $S(k, E)$ for nuclei

Calculations using NMBT

^{3}He: Dieperink *et al.*, Sauer *et al.*, Prosperi *et al.*

^{4}He: ATMS

SCGF theory: finite nuclei such as ^{12}C, Müther et al.

NM: CBF Benhar + Fabrocini

both total $S(k, E)$ and correlated $S(k, E)_{corr}$

Model-$S(k, E)$ for finite nuclei

Ciofi degli Atti + Simula

HF-type calculation for MF piece + convoluted deuteron large-k tail
+ fitted amplitude

Combination MF from data + correlated part from NMBT

get MF $n(k)$ of individual shells from (e,e'p), or WS-fit of (e,e)

alternative: from MF calculations such as DDHF

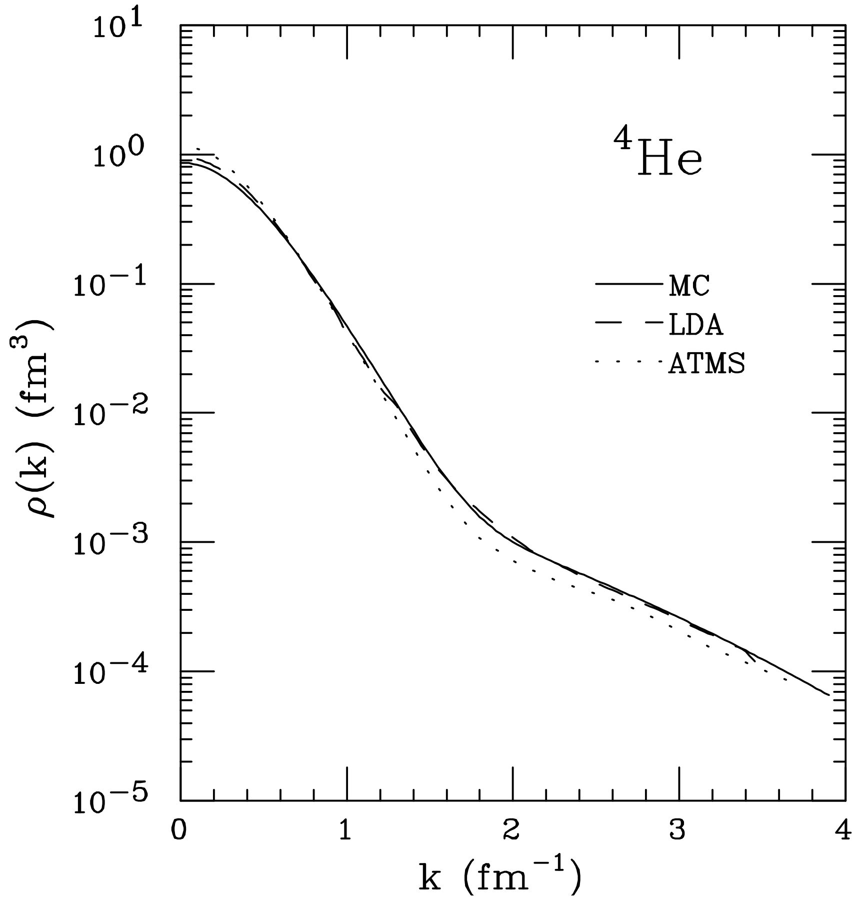
add correlated part, calculated for different NM densities, in LDA

excellent approximation as NN-correlations = short-range properties
where LDA makes sense

Extreme example: $S(k, E$ for ^4He

Calculation of $S(k, E)$ in LDA

integrate to get $n(k)$ in order to compare to MC



excellent agreement MC... LDA although LDA for A=4 really questionable

Experimental measurements of $S(k, E)$: rare

a priori best tool: (e,e'p)
with highest p energies possible to minimize FSI

Difficulties

strength very spread out
cross section small

rescattering of proton
moves strength to larger (apparent) E
can only be minimized by optimal kinematics

perpendicular kinematics worst!
even lowest-E MF states obscured by rescattered p
already for s/d-shell nuclei s-shell obscured

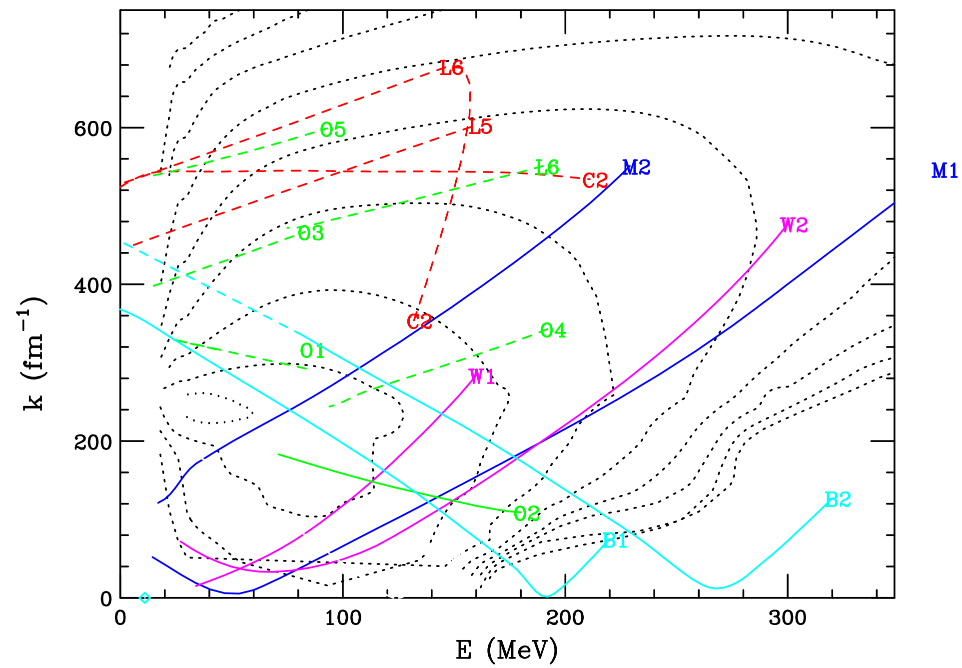
parallel kinematics best
(calculation by C. Barbieri)
note: parallel, *not* anti-parallel

Insight from data: study of all (e,e'p) experiments

compare experimental and calculated $d\sigma/d\Omega d\omega$
in IA, using realistic $S(k,E)$

use $R(k)^{MF} + S^{corr}_{NM}(\rho)$ in LDA

look if data $\simeq$ or $>>$ theory



find

- most experiments give $\sigma_{exp} \gg \sigma_{IA}$
- standard perpendicular kinematics worst, parallel kinematics best

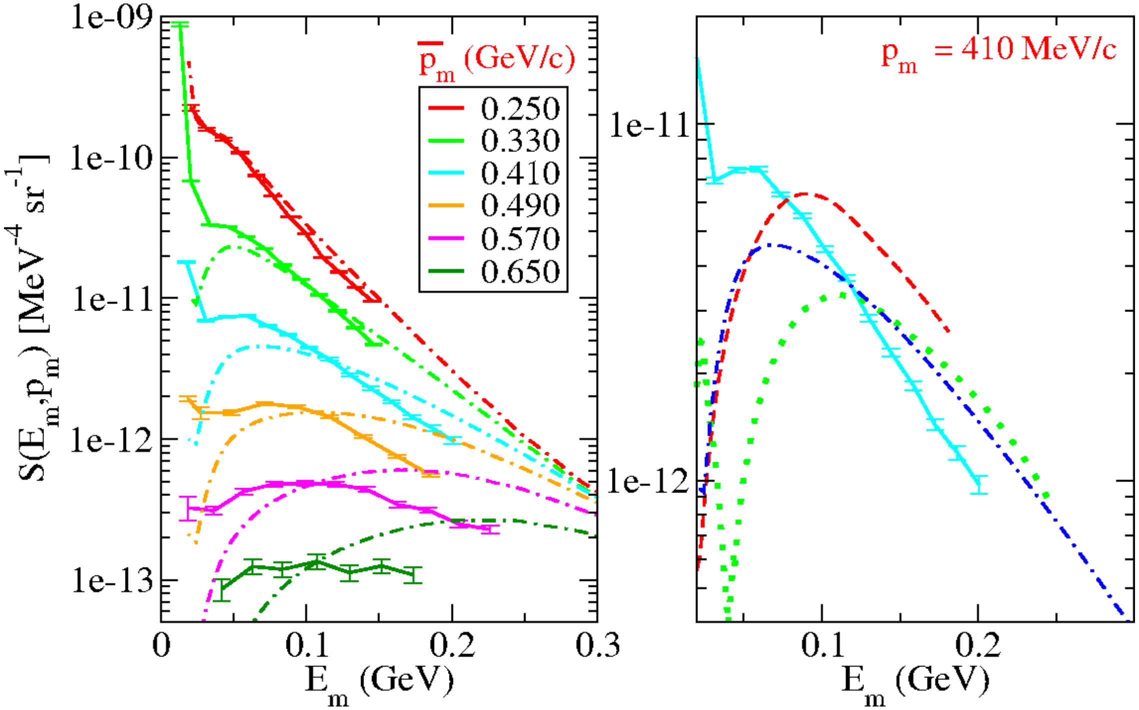
studies of *kinematics* of rescattering processes:

understand how $(p,p'N)$ and $(e,e'p\pi)$ move strength
identify optimal kinematics: parallel (standard: perpendicular!)
same conclusion as from MC calculations of Barbieri

JLab hall-C experiment by Rohe *et al.*, 2004

as close to parallel kinematics as was practical

Results: Spectral function

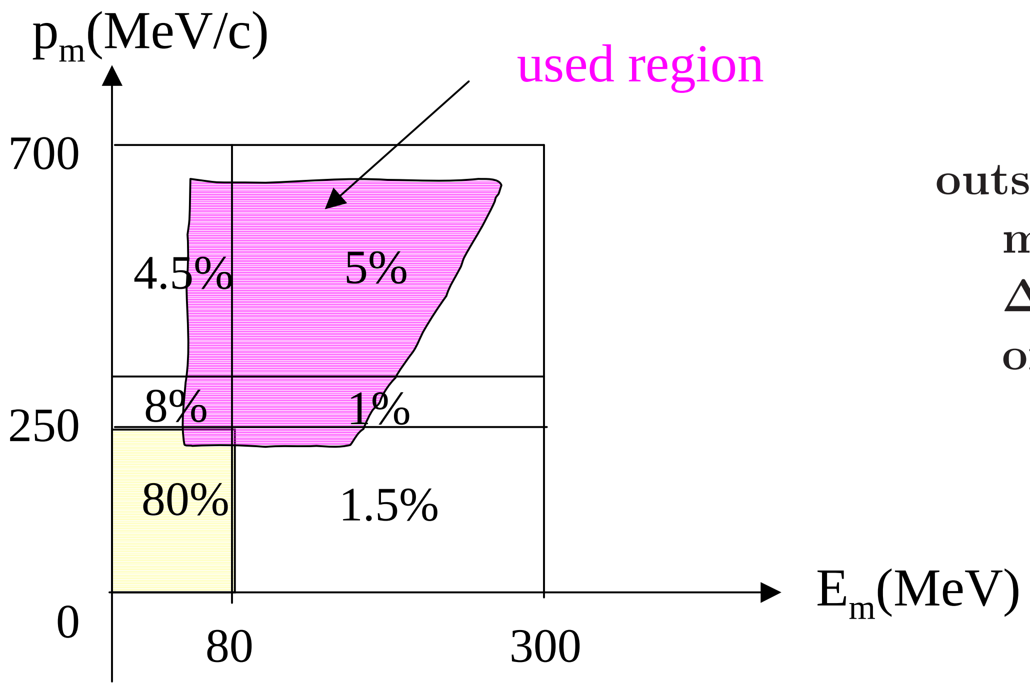


Find ± satisfactory correspondence with theory

in detail: find shift of $S(k, E)$ to smaller E

at present not understood

Comparison of integrated strength: possible for restricted region

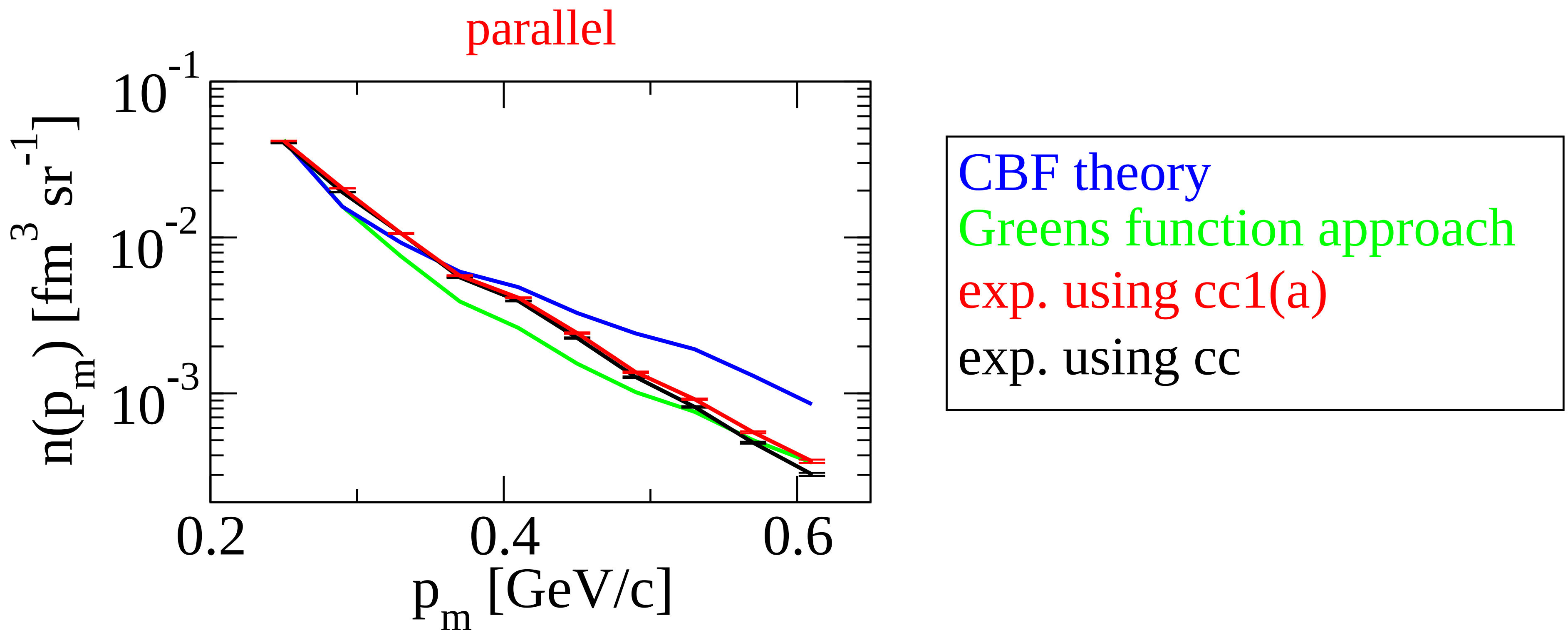


outside used region:
- mean-field dominates
- **Δ** too important
- or no data

# of correlated protons in ^{12}C	used region	total
integral over S from experiment	0.59	
integral over S from CBF	0.64	1.32
integral over S from SCGF	0.61	1.27

→ good agreement
→ can believe total from theory
→ 21%, integrated over $\boldsymbol{k}, \boldsymbol{E}$
→ agrees with $1 - s_{MF}$

Momentum distribution in ”used region”



measure believable high-*k*-tail for first time
find rather good agreement with theory

..... but both data and theory could stand some improvement

Question: can experimentally determine $n(k)$ without ”detour” via $S(k, E)$?

Can measure $n(k)$ at large k directly?

Popular topic since 1/2 century! Many simple-minded ideas:

(x,p) with high-k backward going p (x=γ, π, p,...)
(x,p) with energy of x subthreshold for reaction on N
(e,e') at high q, x >1
.....

Common characteristics

1. All processes dont work once consider that large k involve large E
   systematically ignored although known since the 70^{ies}

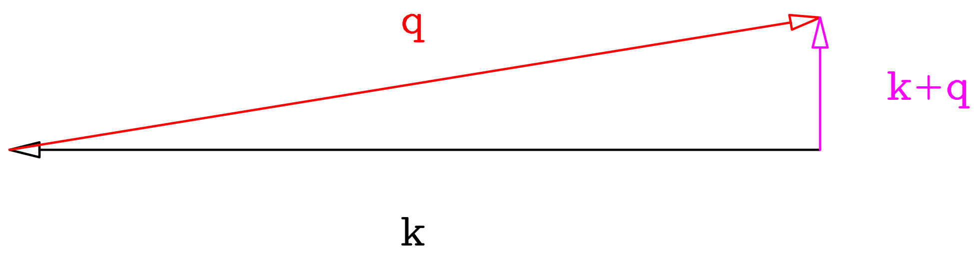
2. PWIA calculation with realistic $S(k, E)$ never done, although easy
   if would do, would find $\sigma_{exp} \gg \sigma_{PWIA}$
   then would know that FSI, MEC, ... dominate

3. Low-q processes suffer from Amado-Woloshyn disease
   in limit q $\sim$ 0 FSI cancels high-k contribution

Example: inclusive electron scattering at large q, low ω, $x > 1$

Naive idea: low $\omega \sim (\vec{k} + \vec{q})^2/2M$ *and* large q

means $\vec{k} \sim -\vec{q}$ *i.e.* large k



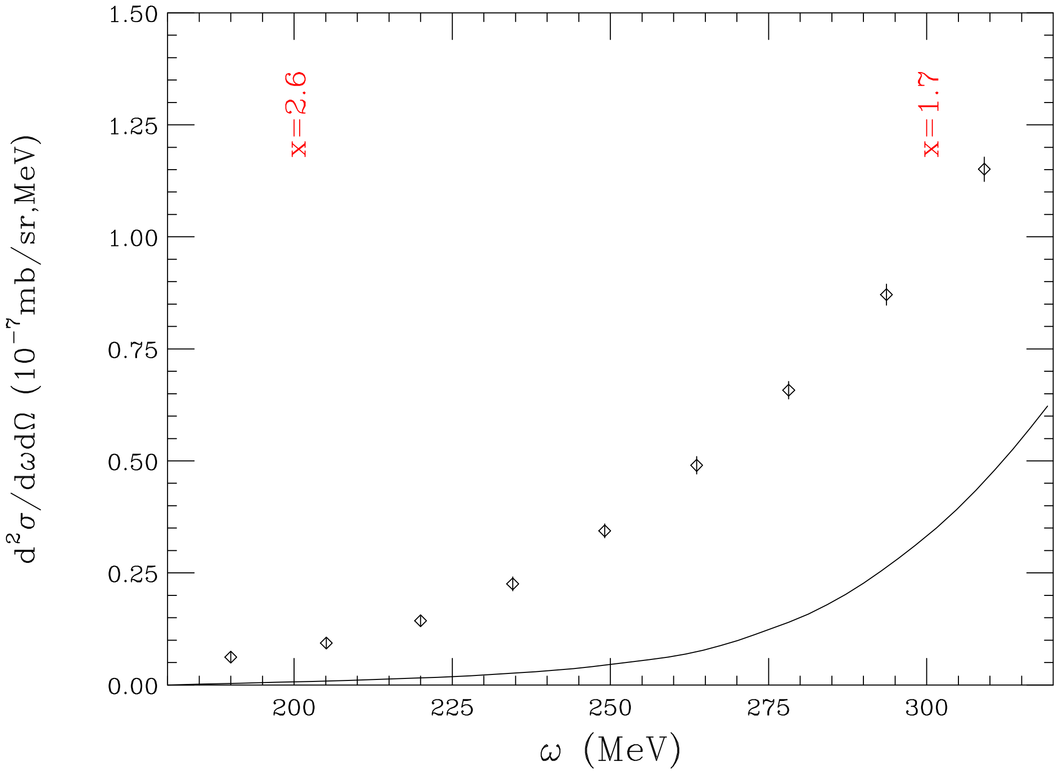
Problem: low $\vec{k} + \vec{q} \rightarrow$ large FSI

is important in tail of quasi-elastic peak
more difficult to calculate than $S(k, E)$
cannot be removed by taking ratios
is additive, not multiplicative! (remember sumrule)
rescattering moves strength from place where large to place where small

Elementary check:
first calculate cross section in PWIA
only if close to σ_{exp} think about correlated nucleons

Specific case: ^{3}He(e,e') in threshold region, $x \sim 1.5 \div 3$

For ^3He have *exact* $S(k, E)$ from Faddeev calculation, as good as deuteron $n(k)$



Find σ_{PWIA} at large x factor 3÷10 too small

need FSI to get close to data

Cross section scales in terms of y

only explainable as consequence of FSI! σ_{PWIA} does *not* scale
experimental $F(y, q)$ converges from *above*, but $F(y, q)$ from $S(k, E)$ from *below*

FSI in inclusive scattering

can be calculated, no need for hand waving arguments

FSI in q.e. scattering of thermal neutrons on L^{4}He

Main interest to condensed matter physics:

% Bose condensate in superfluid L^{4}He $\rightarrow \delta(k=0)$ peak

$\delta(y=0)$ not visible in data. Reason: FSI

Detailed studies of FSI-effects

main effect: folding of IA (n,n') response

width of folding function proportional to σ_{tot} of He-He interaction

smears out δ-function peak

FSI in q.e. electron scattering

see talk of Omar Benhar

derives convolution approach using zero'th order ladder approximation

folding function from *particle* spectral function

calculated in Eikonal approximation

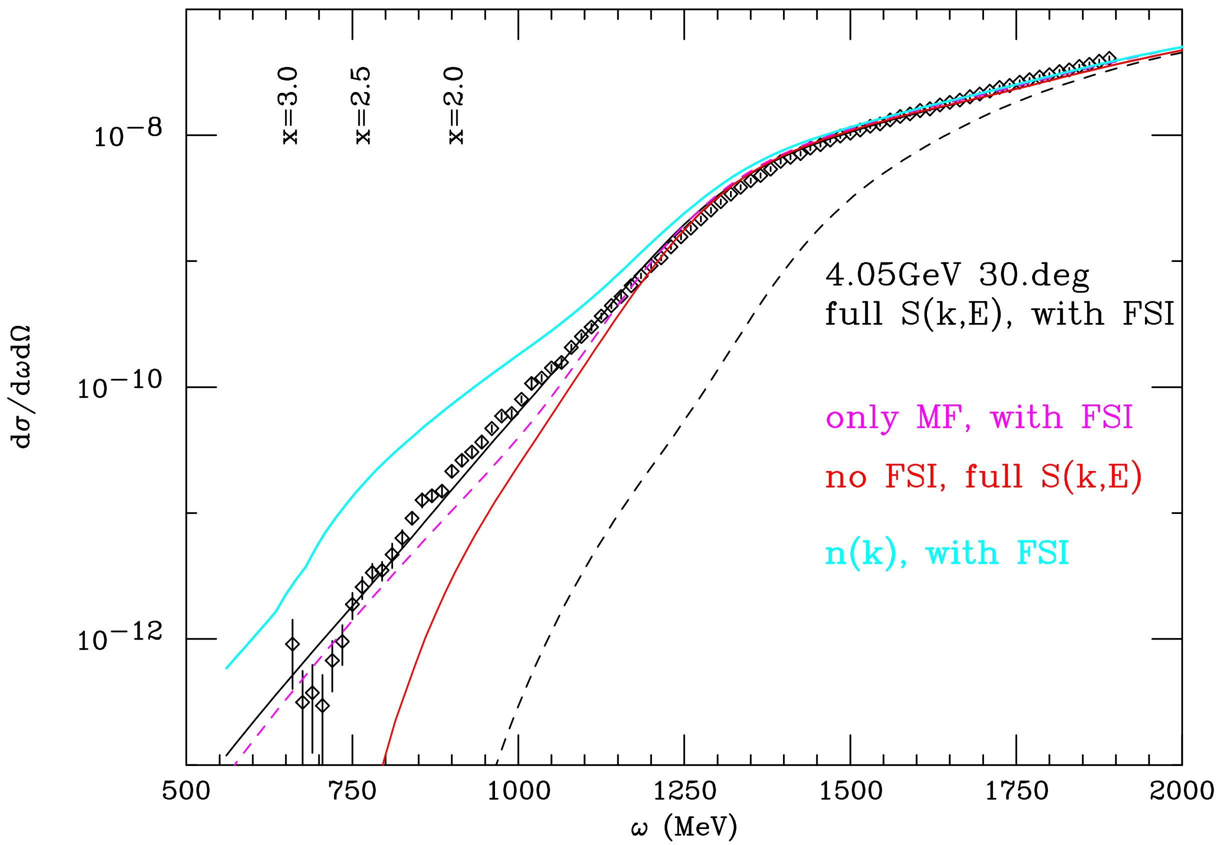
for pedagogical, simplified case (zero-range interaction, no correlations)

folding width proportional to small-angle f_{NN}, density

the only short-range ingredient is $g(r-r')$, *reduces* FSI

Moves strength from top of q.e. peak to tails

Example: recent ^{12}C(e,e') at $x \sim 2 \div 3$: 4GeV, 30°



σ_{PWIA} at large x much too small
Effect of large-k minimal, FSI dominates (Benhar 2013)
Difference between $S(k,E)$ and $n(k)$ huge

Cross section ratios $\frac{\sigma_A}{\sigma_{A'}}$ $\Longrightarrow$ ratios of FSI, not ratios of $n(k)$

Popular a_2 measure FSI, *not* high-k

Deeper origin of problems with large k, E

k, E identified from kinematics via momentum+energy - conservation
valid for *all* processes

Since large k essentially occur at large E

cannot get k or E individually

Consequence for exclusive processes, *e.g.* (e,e'p)

can, in PWIA, determine k and E *together*, measure $S(k, E)$
if kinematics such that corrections to PWIA manageable

Consequences for inclusive processes, *e.g.* (e,e'), (x,p),...

cannot get k or $n(k)$ (or similarly E or $n(E)$)
must input $S(k, E)$ to calculate σ
and then compare to data

Upshot

don't even think about measuring $n(k)$ at large k
it is not possible

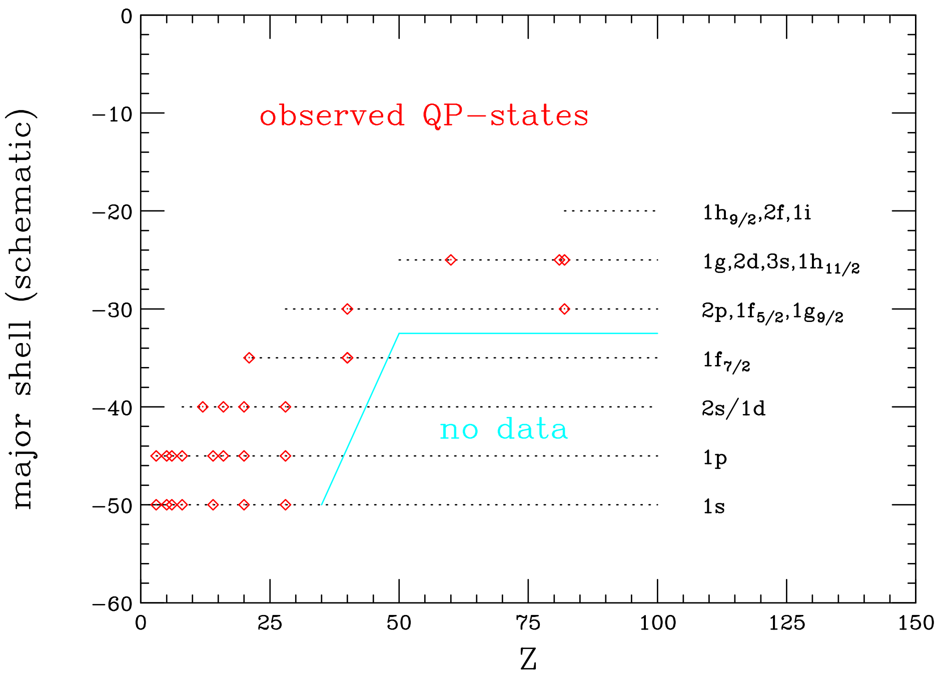
If absolutely want $n(k)$ at large k

measure $S(k, E)$ over largest range in E
then integrate over E

More work needed on $S(k,E)$

several aspects not adequately covered

- $S(k,E)$ for heavier nuclei and lower SM states



have no experimental information on lowest MF states

neither E, nor width, nor $n(k)$

could be obtained via (e,e'p)

certainly better than with (p,2p) (where deep MF orbits seen)

should have been a JLab Hall-A job

- **Better data on large-E/large-k-region, only 1 experiment done**

  want: strictly parallel kinematics

  want different ranges of outgoing-proton energies
  $\rightarrow$ better control of corrections beyond IA

- **Transport code/Glauber calculations for (e,e'p) needed**

  must follow proton and reactions through nucleus
  only then can remove rescattered strength

Orthogonal look:

where correlated strength in r-space?

motivation: difficulties with QP-R(r)

- QP radial wave functions fitted to $\rho(r)$

  poorly explain F(q) of QP-dominated transitions
- QP wave functions poorly explain $\rho(r)$ at small r

reason: $\rho(r)$ contains correlated contribution

presumably radial shape correlated $\rho \neq$ QP shape

$\Rightarrow$ **question: radial distribution of correlated strength = ?**

Two opposing tendencies:

- large E pulls correlated strength to small r
- higher (angular) momenta tend to shift it to larger r

which wins?

2 independent answers:

- study via selfconsistent Green's function theory SGFT

  H. Müther
- determine from (e,e) and (e,e′p)

$S(k,E)$ from Green's function method (Müther, Polls, ..)

split S into QP plus correlated piece

$$\begin{aligned}\rho(r) &= \sum_{lj} S_{lj}^{QP}(r,r) + \sum_{lj}\int_{\varepsilon_{2h1p}}^{\infty} dE\, S_{lj}^{cont}(r,r;E) \\ &= \rho_{QP}(r) + \rho_{corr}(r)\,,\end{aligned}$$

CD-Bonn NN interaction → 1.0 correlated protons (low?)

observations

ρ_{corr} concentrated much more towards small r

does not contribute at large r

there tail of QP dominates completely

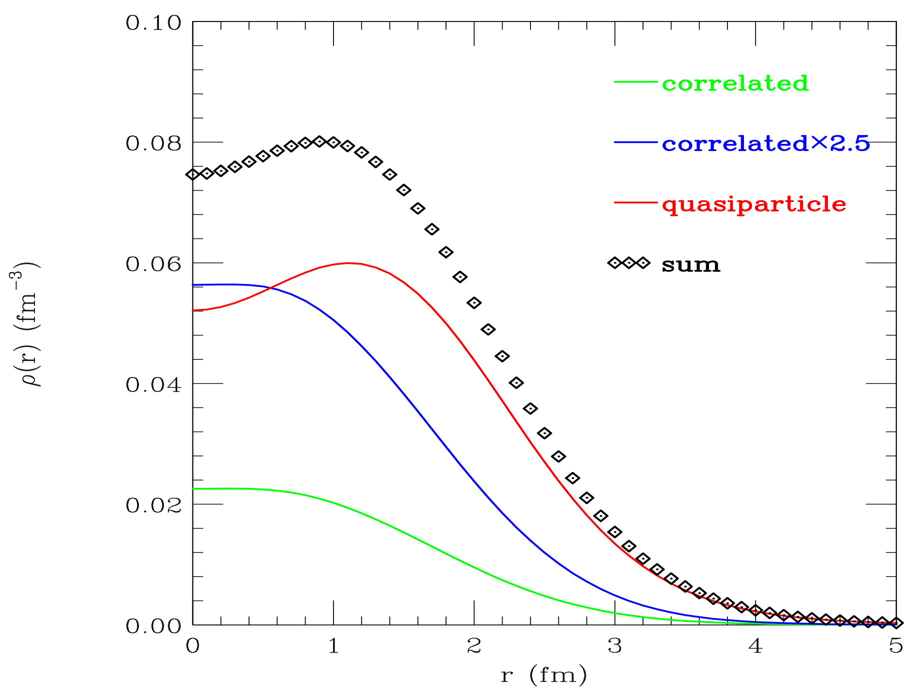
ρ_{corr} at small r *despite* contributions of large l

31% l=0, 37% l=1, rest large l

large E of states pulls R(r) to small r

at small r ρ_{corr} contributes ∼30% of $\rho(r)$

explains failure of QP wave functions



ρ_{corr} from (e,e)+(e,e′p) data

$$\rho_{corr}(r) = \rho(r)_{point} - \sum_{QP-orbits} FBT(R_{QP}(k))^2$$

point density of C

have very precise (e,e) data up to large q
have μ-X-ray data
do modelindependent analysis (SOG)
→ charge density with small $\delta\rho$

unfold nucleon size to get point density

QP wave functions from (e,e′p)

extensive set of (e,e′p) data

- low-q from NIKHEF, Saclay
  analyzed with DWBA
  optical potentials from (p,p)

- high-q data from SLAC, JLAB
  analyzed with theoretical transparencies
  confirmed by data

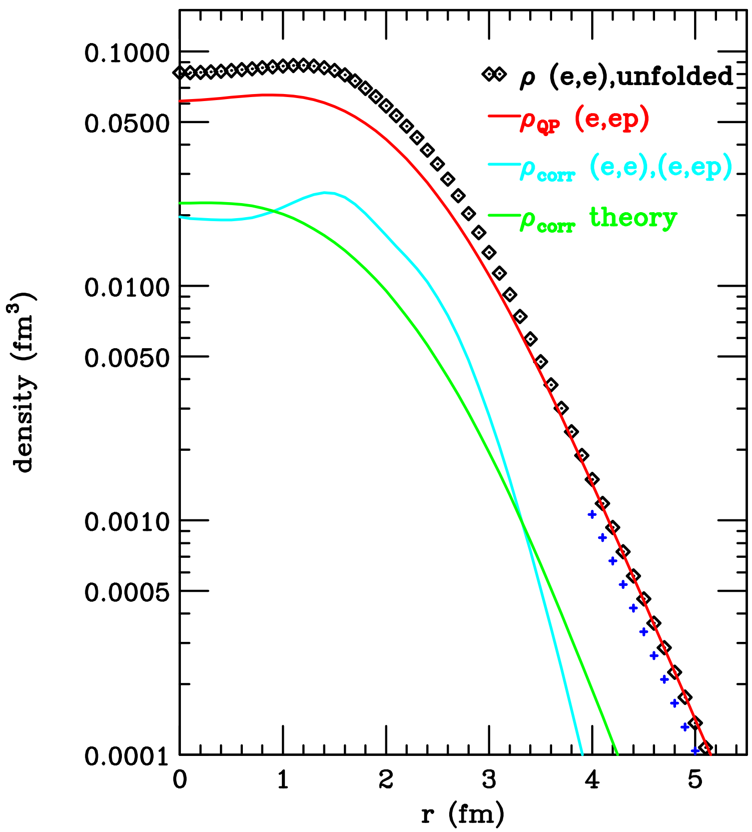
ρ_{corr} from (e,e)+(e,e′p)

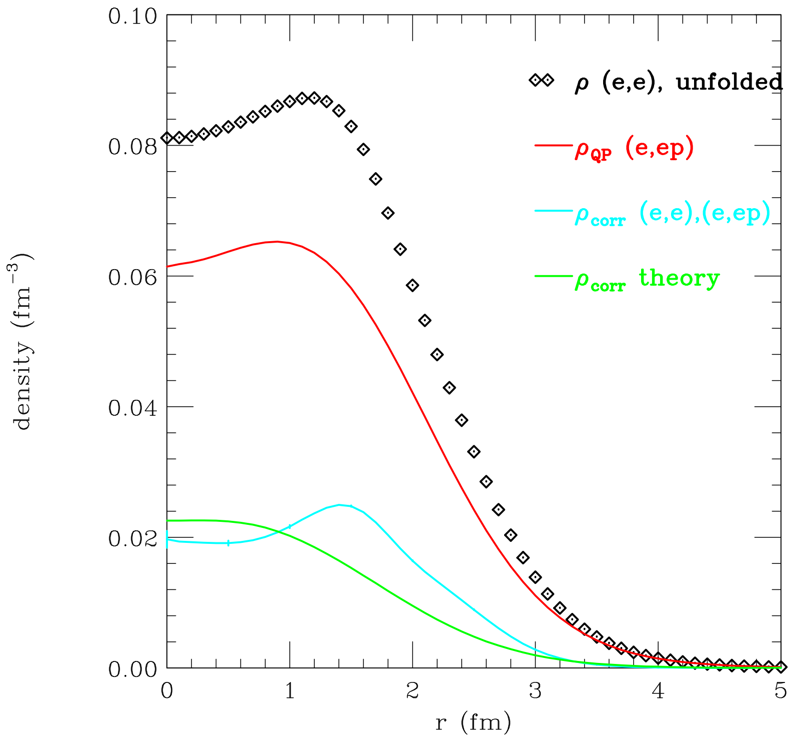
start with point density

subtract QP contribution, Fourier-Bessel-transformed $R(k)$

using high-q (corrected) occupation

result





observations

ρ_{corr} concentrated towards small r
as was seen in theory

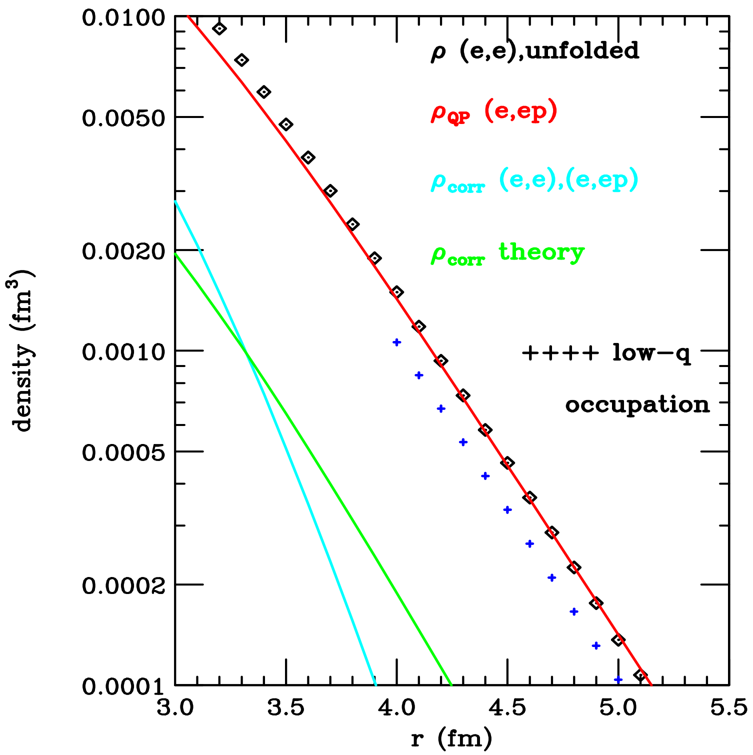
ρ_{corr} gives $\sim$30% contribution at small r
explains failure of QP models

reasonable agreement with theory
(uncertainty of ρ_{corr} $\sim$20%)

in exp. density perhaps more $l > 1$ strength

important consistency check: large r
perfect agreement ρ_{QP} ... ρ_{point}
should occur as ρ_{corr} cannot contribute

large-r = *the* region where MF $\pm$ OK



Conclusions of r-space study

shape of ρ_{corr} differs strongly from shape of ρ_{QP}

ρ_{corr} gives 30% contribution in nuclear interior
explains failure of QP models, cannot be 'compensated' using e_{eff}, etc.

reasonable agreement with Green's function theory

Overall conclusions

for quantitative understanding must go beyond MF

to describe correlated N must use $S(k, E)$
only quantity that accounts for both large k *and* large E

have finally data on correlated strength
... some 15 years after CBF calculation

$\pm$ agrees with modern many-body theories
... which were amazingly good!

for good $S(k, E)$ of *finite* nuclei
look forward to results from FHNC, GFMC calculations

Some references

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