

# **Overview of Nuclear Parton Distribution Functions**

**Shunzo Kumano**

**High Energy Accelerator Research Organization (KEK)  
J-PARC Center (J-PARC)**

**Graduate University for Advanced Studies (GUAS)**

**<http://research.kek.jp/people/kumanos/>**

**Neutrino-Nucleus Interactions**

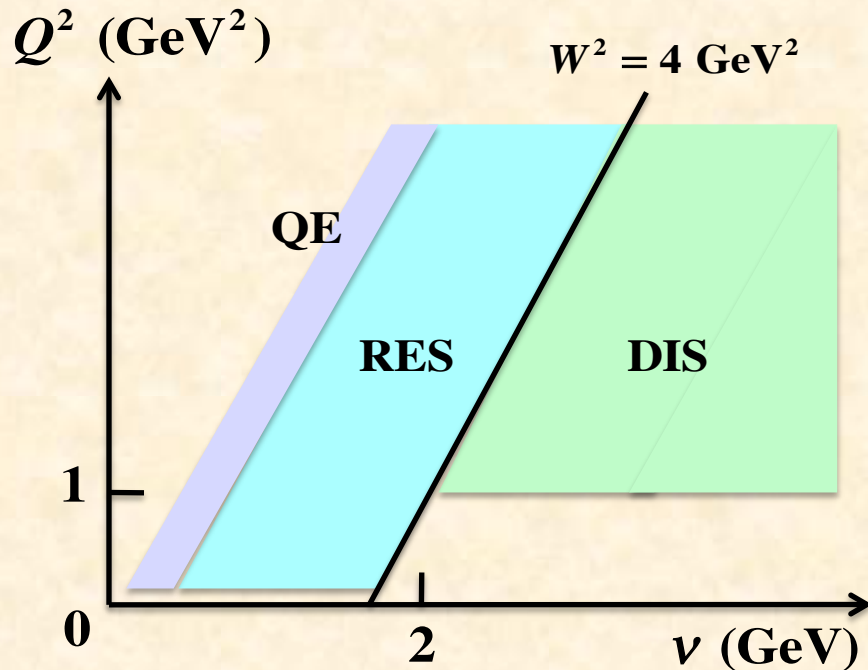
**for Current and Next Generation Neutrino Oscillation Experiments (INT-13-54W)**

**December 3-13, 2013, INT, University of Washington, USA**

**<http://www.int.washington.edu/PROGRAMS/13-54w/>**

**December 9, 2013**

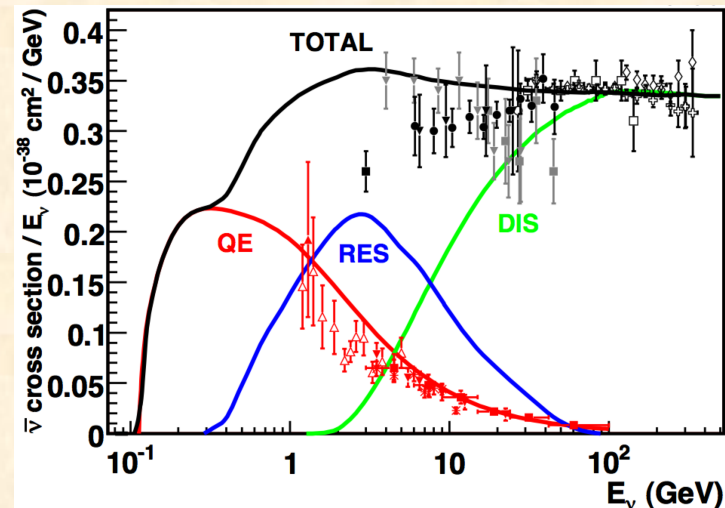
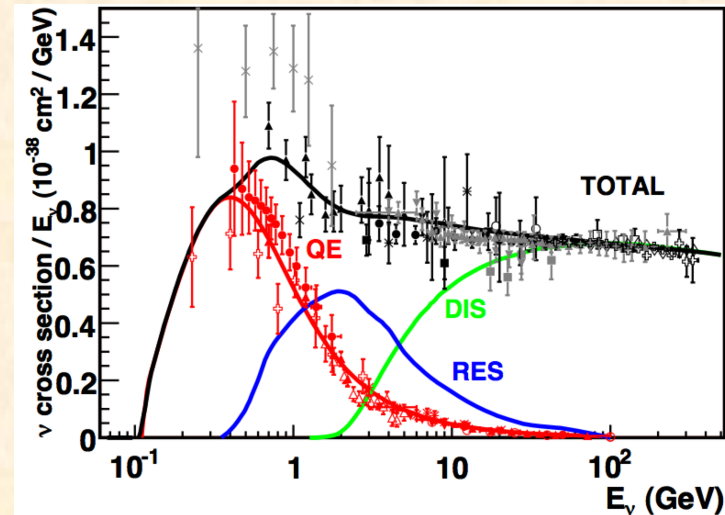
# Kinematical regions of neutrino-nucleus scattering



Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic)

Activities at the J-PARC branch, KEK theory center  
<http://j-parc-th.kek.jp/html/English/e-index.html>



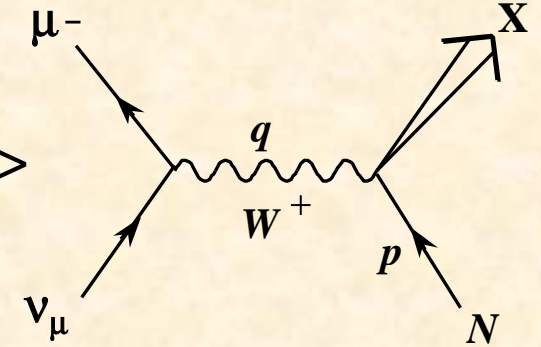
J.L. Hewett *et al.*, arXiv:1205.2671,  
Proceedings of the 2011 workshop  
on Fundamental Physics at the Intensity Frontier

# Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k \cdot p} \frac{1}{2} \sum_{spins} \sum_X (2\pi)^4 \delta^4(k + p - k' - p_X) |M|^2 \frac{d^3k'}{(2\pi)^3 2E'}$$

$$M = \frac{1}{1 + Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1 - \gamma_5) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1 + Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$



$$L^{\mu\nu} = 8 \left[ k^\mu k'^\nu + k'^\mu k^\nu - k \cdot k' g^{\mu\nu} + i \underline{\varepsilon^{\mu\nu\rho\sigma}} k_\rho k'_\sigma \right], \quad \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) + \frac{i}{2M^2} \underline{W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}$$

$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

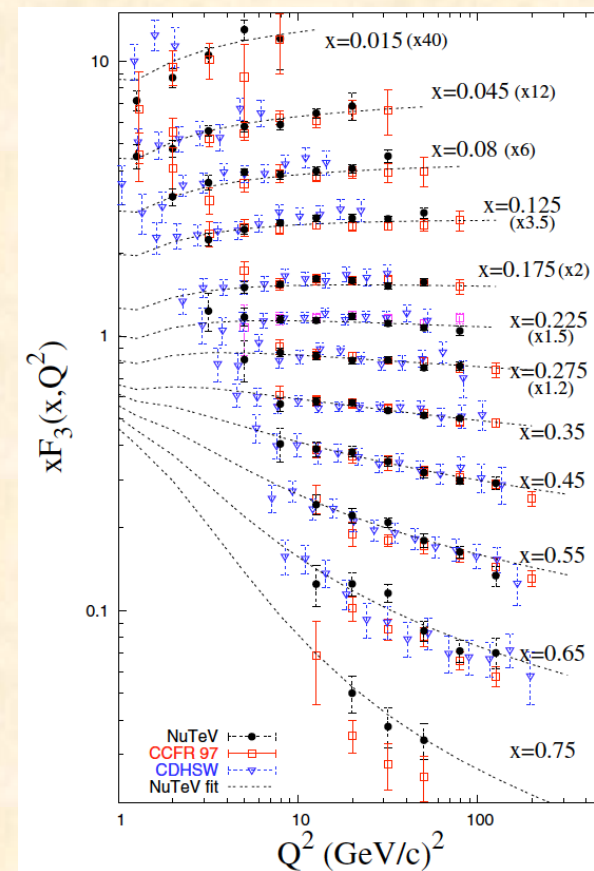
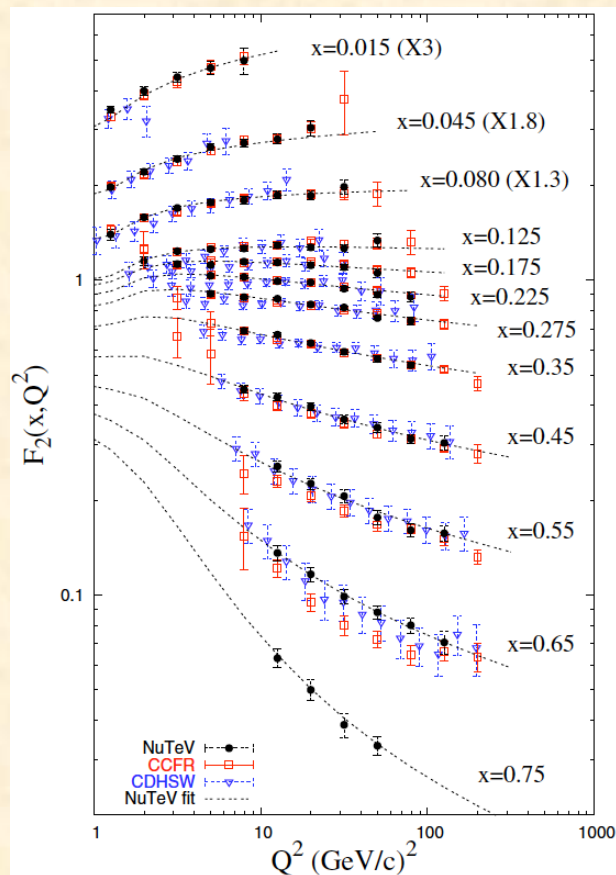
$$\frac{d\sigma_{\nu, \bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1 + Q^2/M_W^2)^2} \left[ x y^2 F_1^{CC} + \left( 1 - y - \frac{M x y}{2E} \right) F_2^{CC} \pm x y \left( 1 - \frac{y}{2} \right) \underline{F_3^{CC}} \right]$$

# Neutrino DIS experiments

Experiment	Target	$\nu$ energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

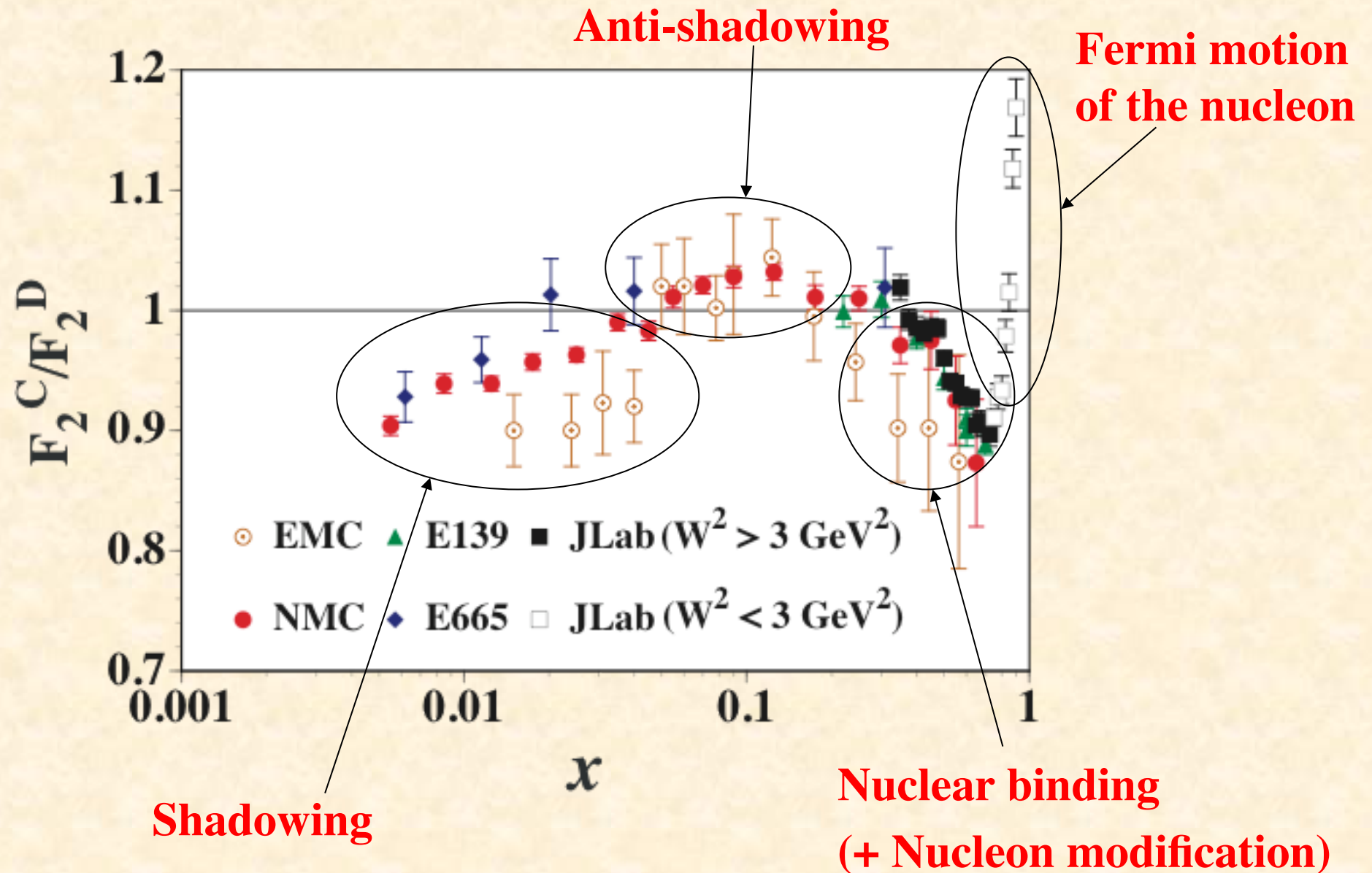
M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.

MINER $\nu$ A (He, C, Fe, Pb), ...



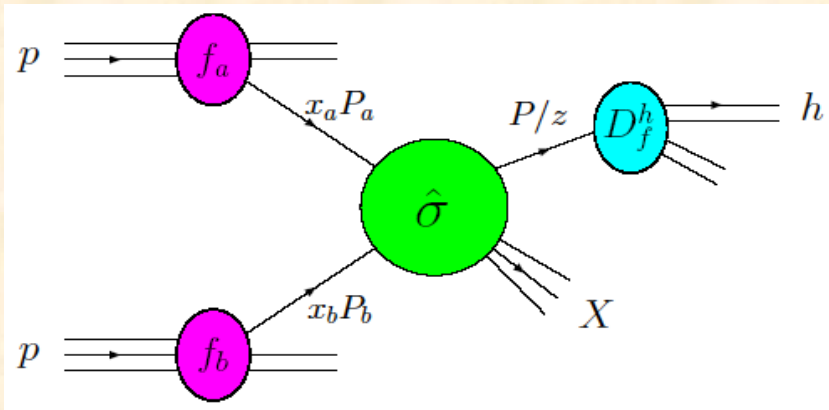


# Nuclear modifications of structure function $F_2$



# High-energy nuclear reactions

Nuclear PDFs are needed for describing high-energy nuclear reactions in order to find any new phenomena.



$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \rightarrow cX) \otimes D_c^h(z, Q^2)$$

$f_a(x_a, Q^2)$ : parton distribution functions

$\hat{\sigma}(ab \rightarrow cX)$ : partonic cross sections

$D_c^h(z, Q^2)$ : fragmentation functions

# Experimental data: total number = 1241

## (1) $F_2^A / F_2^D$

NMC: p, He, Li, C, Ca

SLAC: He, Be, C, Al,  
Ca, Fe, Ag, Au

EMC: C, Ca, Cu, Sn

E665: C, Ca, Xe, Pb

BCDMS: N, Fe

HERMES: N, Kr

+ JLab data

## (2) $F_2^A / F_2^{A'}$

NMC: Be / C, Al / C,

Ca / C, Fe / C,

Sn / C, Pb / C,

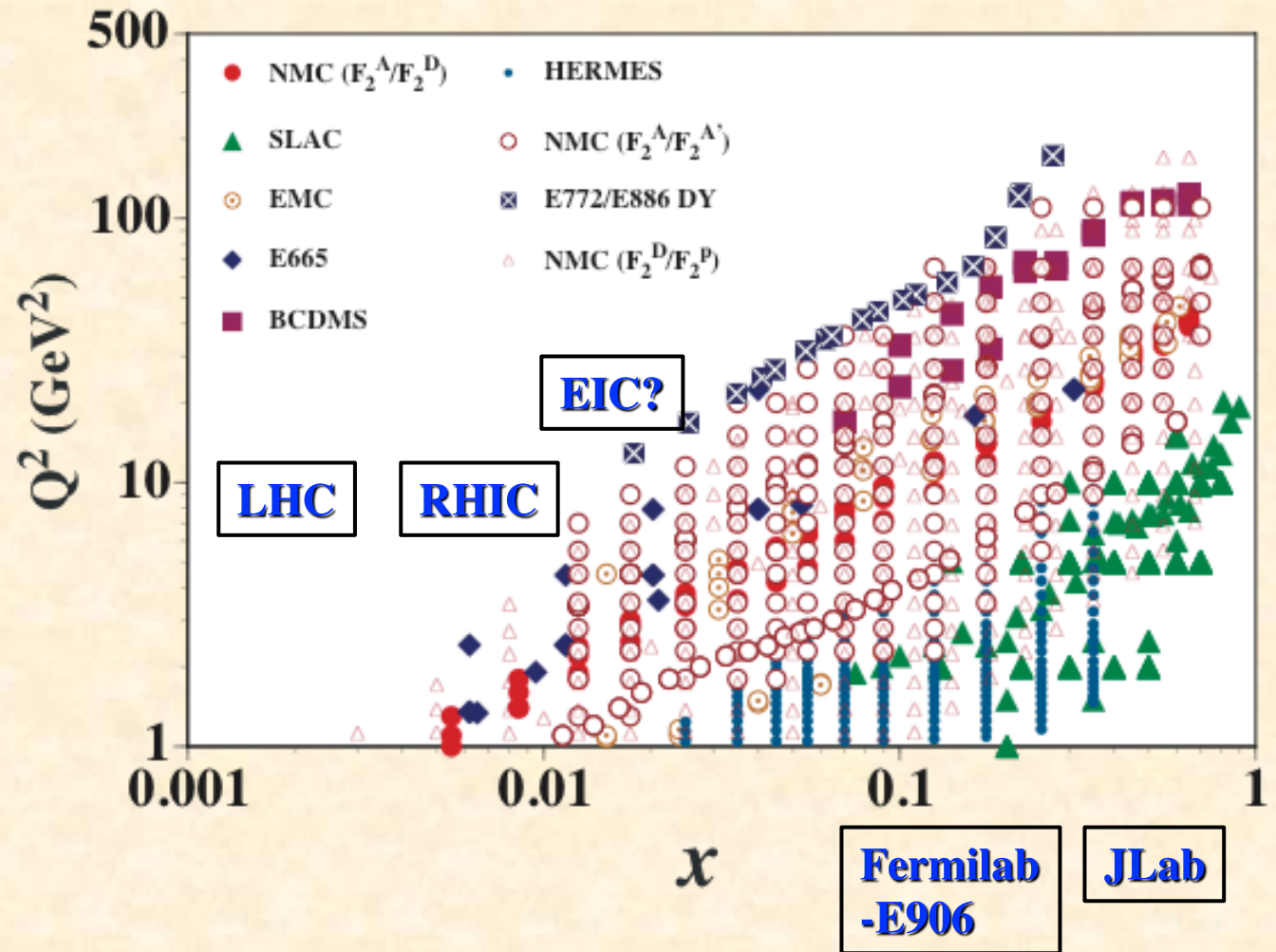
C / Li, Ca / Li

## (3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$

E772: C / D, Ca / D,

Fe / D, W / D

E866: Fe / Be, W / Be



# Situation of data for nuclear PDFs

Available data  
for nuclear PDFs

Jlab at large x

Process/ Experiment	Leading order subprocess	Parton behaviour probed
<b>DIS (<math>\mu N \rightarrow \mu X</math>)</b> $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)*	$\gamma^* q \rightarrow q$	Four structure functions $\rightarrow$ $u + \bar{u}$ $d + \bar{d}$
<b>DIS (<math>\nu N \rightarrow \mu X</math>)</b> $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)*	$W^* q \rightarrow q'$	$\bar{u} + d$ $s$ (assumed = $\bar{s}$ ), but only $\int xg(x, Q_0^2)dx \simeq 0.35$ and $\int (\bar{d} - \bar{u})dx \simeq 0.1$
<b>DIS (small x)</b> $F_2^{ep}$ (H1, ZEUS)*	$\gamma^*(Z^*)q \rightarrow q$	$\lambda$ ( $x\bar{q} \sim x^{-\lambda_s}$ , $xg \sim x^{-\lambda_g}$ )
<b>DIS (<math>F_L</math>)</b> NMC, HERA	$\gamma^* g \rightarrow q\bar{q}$	$g$
<b><math>\ell N \rightarrow c\bar{c}X</math></b> $F_2^c$ (EMC; H1, ZEUS)*	$\gamma^* c \rightarrow c$	$c$ ( $x \gtrsim 0.01$ ; $x \lesssim 0.01$ )
<b><math>\nu N \rightarrow \mu^+ \mu^- X</math></b> (CCFR)*	$W^* s \rightarrow c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{4}(\bar{u} + \bar{d})$
<b><math>pN \rightarrow \gamma X</math></b> (WA70*, UA6, E706, ...)	$qq \rightarrow \gamma q$	$g$ at $x \simeq 2p_T^2/\sqrt{s} \rightarrow$ $x \approx 0.2 - 0.6$
<b><math>pN \rightarrow \mu^+ \mu^- X</math></b> (E605, E772)*	$q\bar{q} \rightarrow \gamma^*$	$\bar{q} = \dots(1-x)^{\eta_s}$
<b><math>pp, pn \rightarrow \mu^+ \mu^- X</math></b> (E866, NA51)*	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$ $u\bar{d}, d\bar{u} \rightarrow \gamma^*$	$\bar{u} - \bar{d}$ ( $0.04 \lesssim x \lesssim 0.3$ )
<b><math>ep, en \rightarrow e\pi X</math></b> (HERMES)	$\gamma^* q \rightarrow q$ with $q = u, d, \bar{u}, \bar{d}$	$\bar{u} - \bar{d}$ ( $0.04 \lesssim x \lesssim 0.2$ )
<b><math>p\bar{p} \rightarrow WX(ZX)</math></b> (UA1, UA2; CDF, D0)	$ud \rightarrow W$	$u, d$ at $x \simeq M_W/\sqrt{s} \rightarrow$ $x \approx 0.13; 0.05$
$\rightarrow \ell^\pm$ asym (CDF)*		slope of $u/d$ at $x \approx 0.05 - 0.1$
<b><math>p\bar{p} \rightarrow t\bar{t}X</math></b> (CDF, D0)	$q\bar{q}, gg \rightarrow t\bar{t}$	$q, g$ at $x \gtrsim 2m_t/\sqrt{s} \simeq 0.2$
<b><math>p\bar{p} \rightarrow \text{jet} + X</math></b> (CDF, D0)	$gg, qg, qq \rightarrow 2j$	$q, g$ at $x \simeq 2E_T/\sqrt{s} \rightarrow$ $x \approx 0.05 - 0.5$

Nuclear target Fe (CCFR, NuTeV)  
Not for p and d.

Small-x,  
high-energy electron facility  
(EIC, LHeC)?

Fermilab

RHIC, LHC (J-PARC?, GSI?)

RHIC, LHC

hep/ph-9803445

Updated information  
In A. D. Martin *et al.* (MSTW08),  
Eur.Phys.J. C63 (2009) 189.

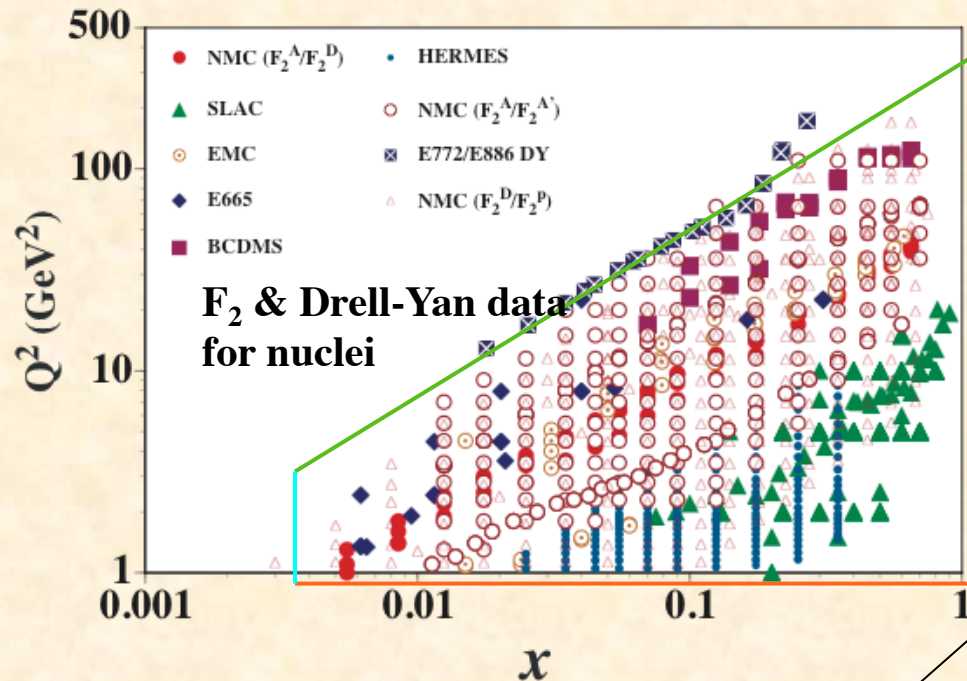


**Current nuclear data are kinematically limited.**

$$x = \frac{Q^2}{2p \cdot q} \approx \frac{Q^2}{ys}$$

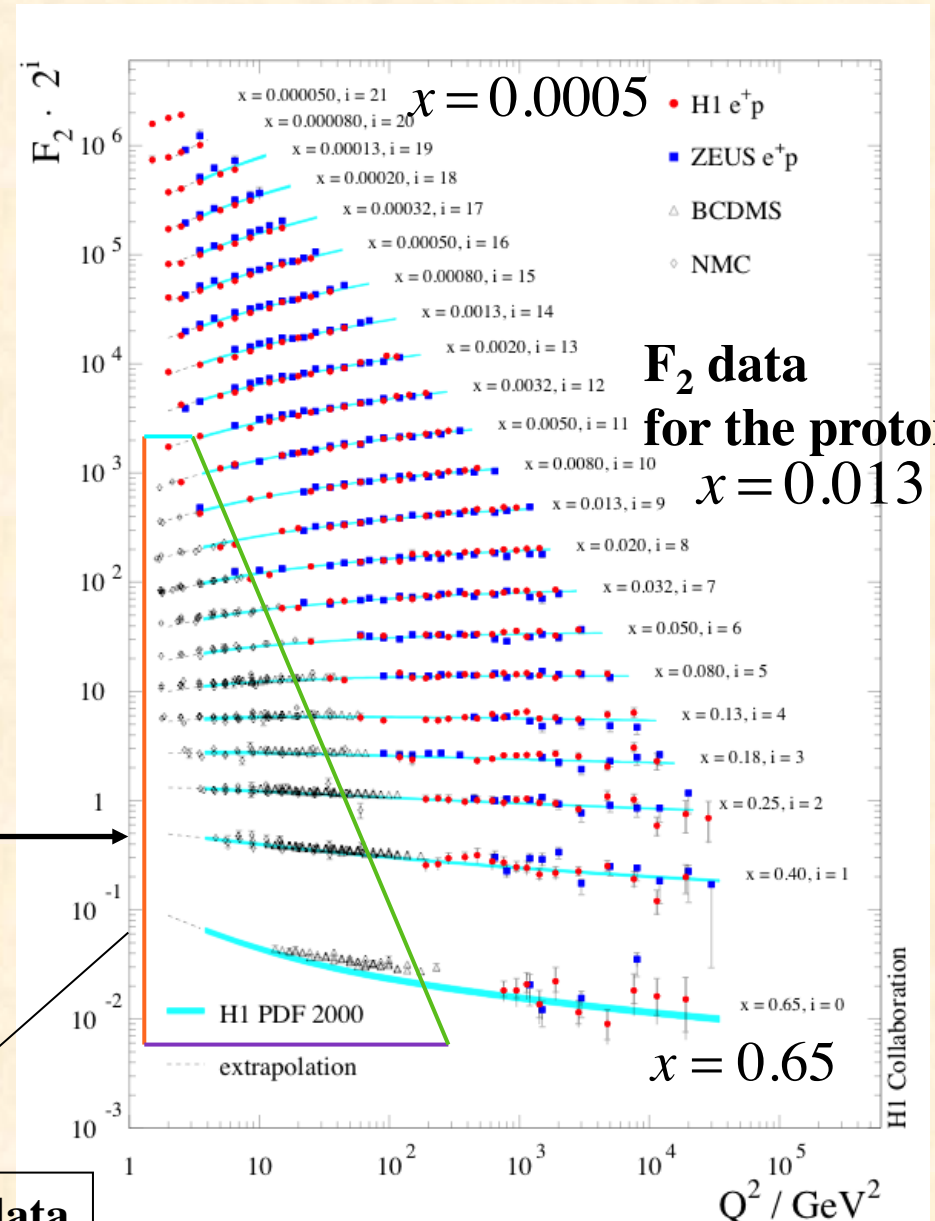
fixed target:  $\min(x) = \frac{Q^2}{2M_N E_{lepton}} \leq \frac{1}{2E_{lepton} \text{ (GeV)}}$   
 if  $Q^2 \geq 1 \text{ GeV}^2$

for  $E_{lepton}$  (NMC) = 200 GeV,  $\min(x) = \frac{1}{2 \cdot 200} = 0.003$



**region of nuclear data**

(from H1 and ZEUS, hep-ex/0502008)



**F<sub>2</sub> data for the proton**  
 $x = 0.013$

$x = 0.65$

H1 Collaboration



# Functional form Nuclear PDFs “per nucleon”

## If there were no nuclear modification

$$Au^A(x) = Zu^p(x) + Nu^n(x), \quad Ad^A(x) = Zd^p(x) + Nd^n(x) \quad p = \text{proton}, \quad n = \text{neutron}$$

## Isospin symmetry:

$$u^n = d^p \equiv d, \quad d^n = u^p \equiv u$$

$$\rightarrow u^A(x) = \frac{Zu(x) + Nd(x)}{A}, \quad d^A(x) = \frac{Zd(x) + Nu(x)}{A}$$

## Take account of nuclear effects by $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x, A) \frac{Zu_v(x) + Nd_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x, A) \frac{Zd_v(x) + Nu_v(x)}{A}$$

$$\bar{u}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{u}(x) + N\bar{d}(x)}{A}, \quad \bar{d}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{d}(x) + N\bar{u}(x)}{A}$$

$$\bar{s}^A(x) = w_{\bar{q}}(x, A) \bar{s}(x)$$

$$g^A(x) = w_g(x, A) g(x)$$

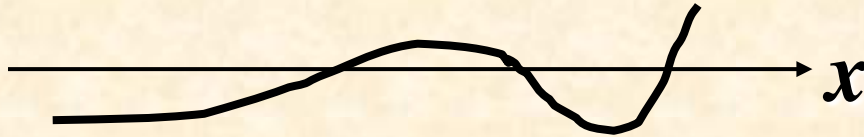
$$\text{at } Q^2 = 1 \text{ GeV}^2 (\equiv Q_0^2)$$

# Functional form of $w_i(x, A)$

$$f_i^A(x, Q_0^2) = w_i(x, A) f_i(x, Q_0^2) \quad i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$$

$$w_i(x, A) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

Note: The region  $x > 1$  cannot be described by this parametrization.



A simple function = cubic polynomial

## Three constraints

**Nuclear charge:**  $Z = A \int dx \left[ \frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right] = A \int dx \left[ \frac{2}{3}u_v^A - \frac{1}{3}d_v^A \right]$

**Baryon number:**  $A = A \int dx \left[ \frac{1}{3}(u^A - \bar{u}^A) + \frac{1}{3}(d^A - \bar{d}^A) + \frac{1}{3}(s^A - \bar{s}^A) \right] = A \int dx \left[ \frac{1}{3}u_v^A + \frac{1}{3}d_v^A \right]$

**Momentum:**  $A = A \int dx \left[ u^A + \bar{u}^A + d^A + \bar{d}^A + s^A + \bar{s}^A + g \right]$   
 $= A \int dx \left[ u_v^A + d_v^A + 2(\bar{u}^A + \bar{d}^A + \bar{s}^A) + g \right]$

**Parton distribution functions are determined by fitting various experimental data.**

- **electron/muon:**  $\mu + p \rightarrow \mu + X$
- **neutrino:**  $\nu_{\mu} + p \rightarrow \mu + X$
- **Drell-Yan:**  $p + p \rightarrow \mu^{+} \mu^{-} + X$
- ...

**(1) assume functional form of PDFs at fixed  $Q^2 (\equiv Q_0^2)$ :**

**e.g.  $f_i(x, Q_0^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1 + \gamma_i x)$ ,**

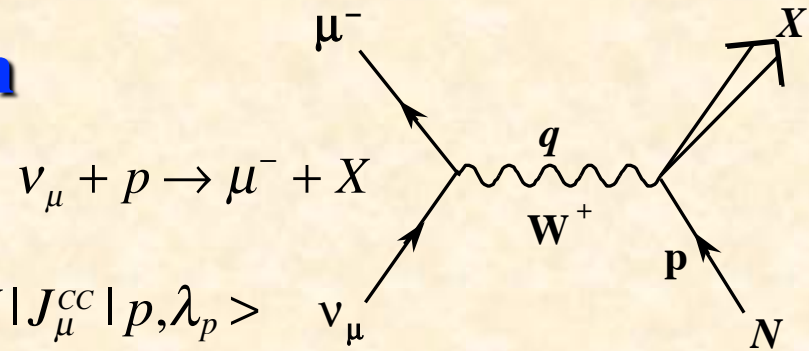
**where  $i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$**

**(2) calculate observables at their experimental  $Q^2$  points.**

**(3) then, the parameters  $A_i, \alpha_i, \beta_i, \gamma_i$  are determined so as to minimize  $\chi^2$  in comparison with data.**

# Determination of each distribution

## Valence quark



$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1 - \gamma_5) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = 8 \left[ k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k' + i \varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right] \quad \text{where } \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left( g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_N^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) + \frac{i}{2M_N^2} W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu, \bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[ xy^2 F_1^{CC} + \left( 1 - y - \frac{Mxy}{2E} \right) F_2^{CC} \pm xy \left( 1 - \frac{y}{2} \right) F_3^{CC} \right]$$

$$\frac{1}{2} [F_3^{\nu p} + F_3^{\bar{\nu} p}]_{CC} = \underline{u_v + d_v} + s - \bar{s} + c - \bar{c}$$

**Note:** Nuclear corrections in CCFR/NuTeV ( $\nu + \text{Fe}$ ).

**Valence:** also  $F_2$  at large  $x$

# Sea quark

## e/μ scattering

$$F_2^N = \frac{F_2^p + F_2^n}{2} = \frac{x}{2} \left[ \frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s}) + \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s}) \right] = \frac{x}{2} \left[ \frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right]$$

$$= \frac{x}{2} \left[ \frac{5}{9}(u_v + d_v) + \frac{10}{9}(\bar{u} + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right] = \frac{5}{18}x(u_v + d_v) + \frac{2}{18}x \left[ \underline{5(\bar{u} + \bar{d}) + (s + \bar{s})} \right]$$

## Drell-Yan (lepton-pair production)

$$p_1 + p_2 \rightarrow \mu^+ \mu^- + X$$

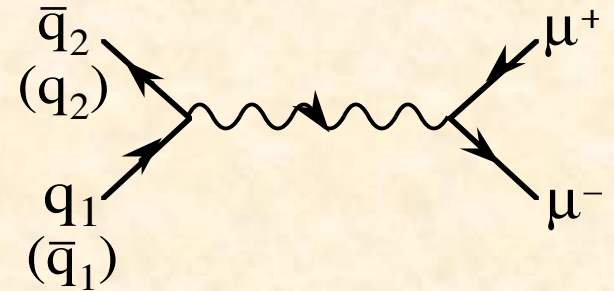
$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$

at large  $x_F = x_1 - x_2$

projectile target

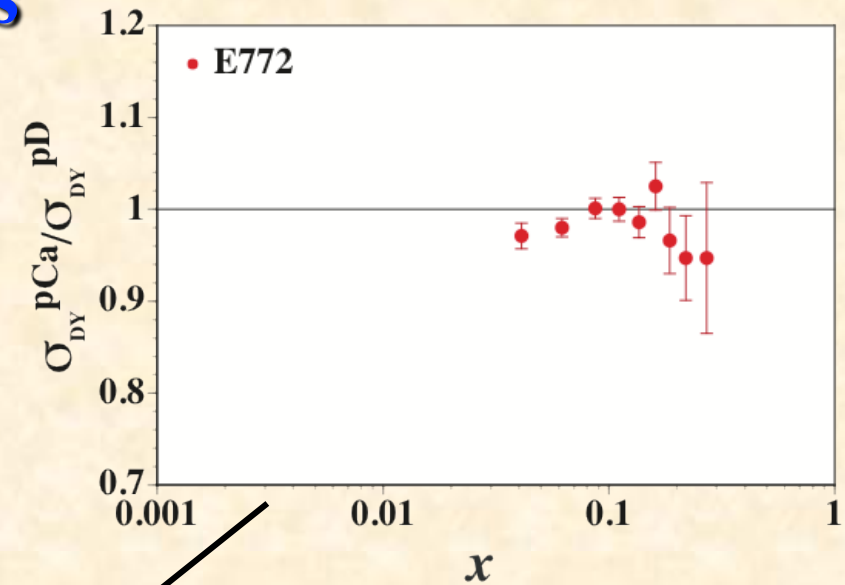
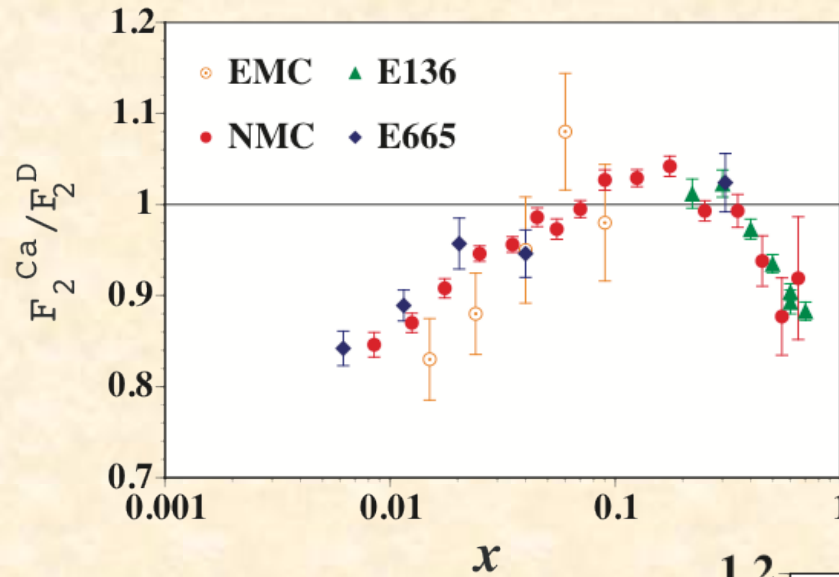
$$d\sigma \propto q_V(x_1)\bar{q}(x_2)$$

$\bar{q}(x_2)$  can be obtained if  $q_V(x_1)$  is known.

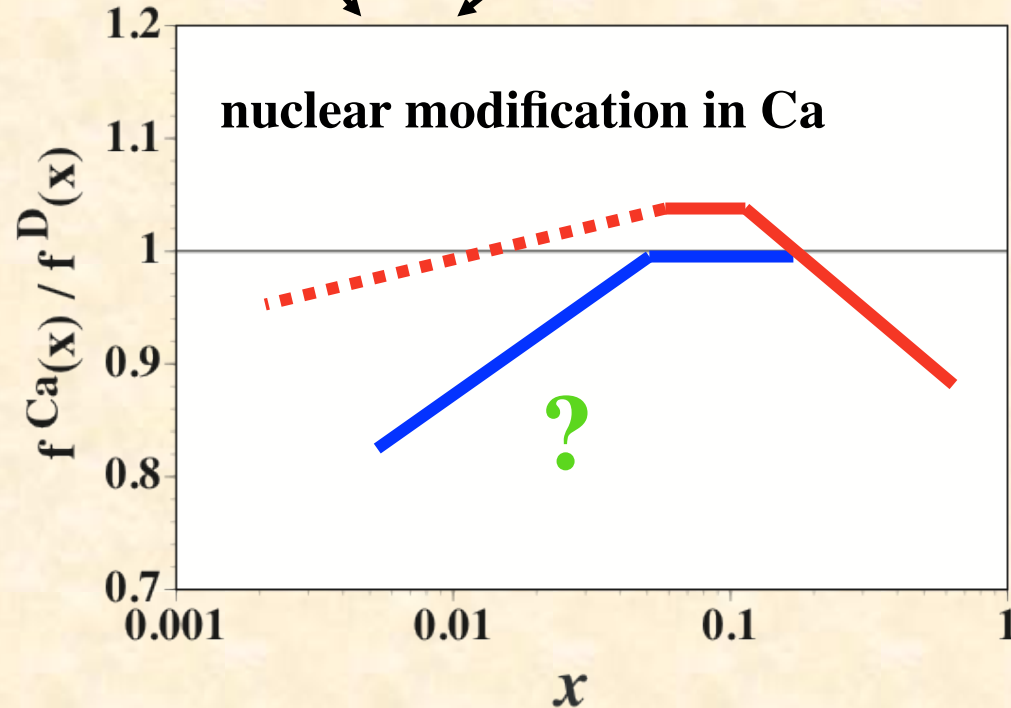




# Nuclear modification of PDFs



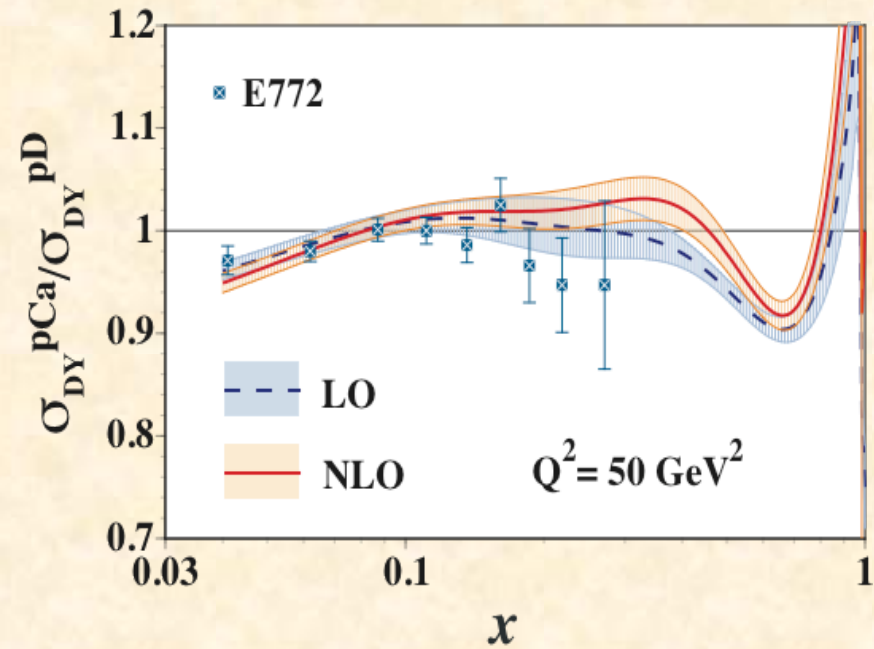
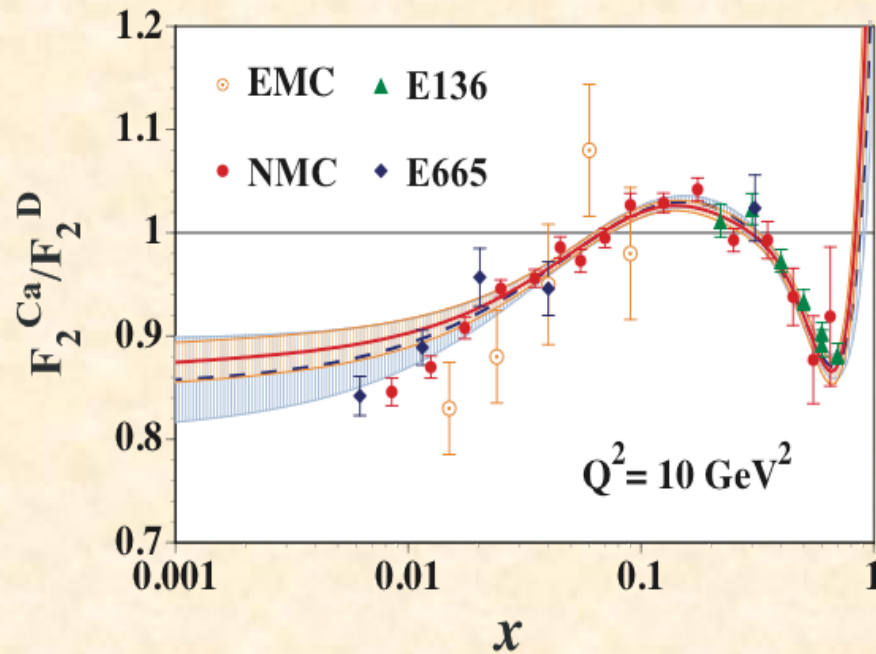
- valence quark
- antiquark
- gluon



# Comparison with $F_2^{\text{Ca}}/F_2^{\text{D}}$ & $\sigma_{\text{DY}}^{\text{pCa}}/\sigma_{\text{DY}}^{\text{pD}}$ data

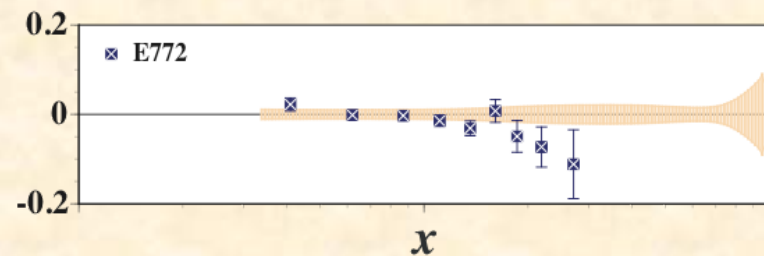
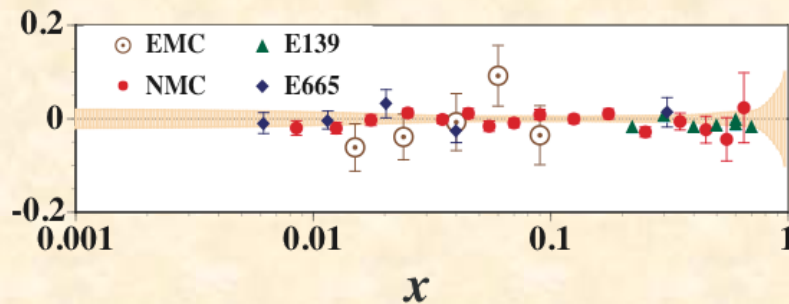
**LO analysis**

**NLO analysis**



**$(R^{\text{exp}} - R^{\text{theo}})/R^{\text{theo}}$  at the same  $Q^2$  points**

**$R = F_2^{\text{Ca}}/F_2^{\text{D}}, \sigma_{\text{DY}}^{\text{pCa}}/\sigma_{\text{DY}}^{\text{pD}}$**



# Gluon

## scaling violation of $F_2$

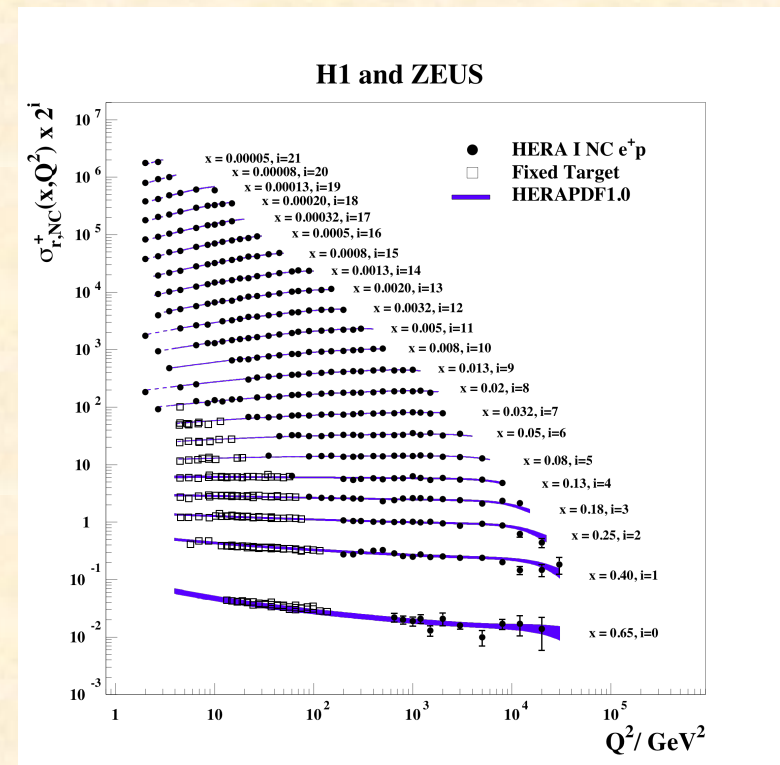
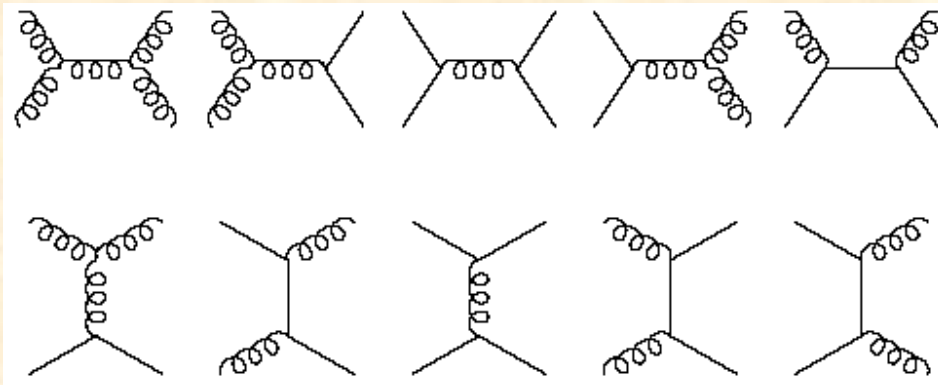
$$\frac{\partial}{\partial(\ln Q^2)} \begin{pmatrix} q_s(x,t) \\ g(x,t) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}(x/y) & P_{qg}(x/y) \\ P_{gq}(x/y) & P_{gg}(x/y) \end{pmatrix} \begin{pmatrix} q_s(y,t) \\ g(y,t) \end{pmatrix}$$

**H1 and ZEUS  
JHEP01(2010)109**

at small  $x$  
$$\frac{\partial F_2}{\partial(\ln Q^2)} \approx \frac{10 \alpha_s}{27\pi} g$$

**K. Prytz, Phys. Lett. B311 (1993) 286.**

## jet production



# Scaling Violation and Gluon Distributions

$$\frac{\partial}{\partial \log Q^2} q_i^+(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ \sum_j P_{qq_j}(x/y) q_j^+(y, Q^2) + \underline{P_{qg}(x/y) g(y, Q^2)} \right]$$

dominant term at small x

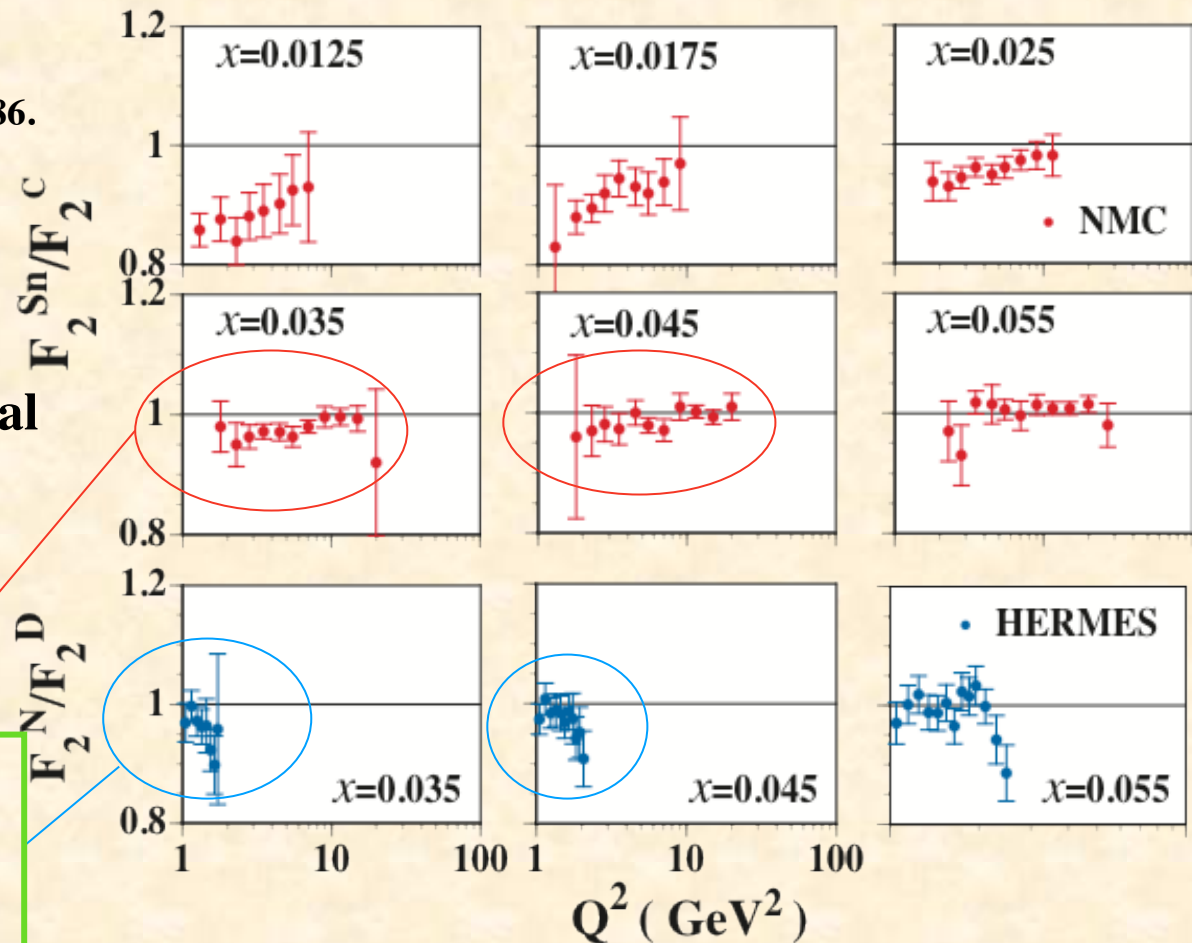
$$q_i^+ = q_i + \bar{q}_i$$

at small x K. Prytz, PLB 311 (1993) 286.

$$\frac{\partial F_2}{\partial (\ln Q^2)} \approx \frac{20 \alpha_s}{27\pi} xg$$

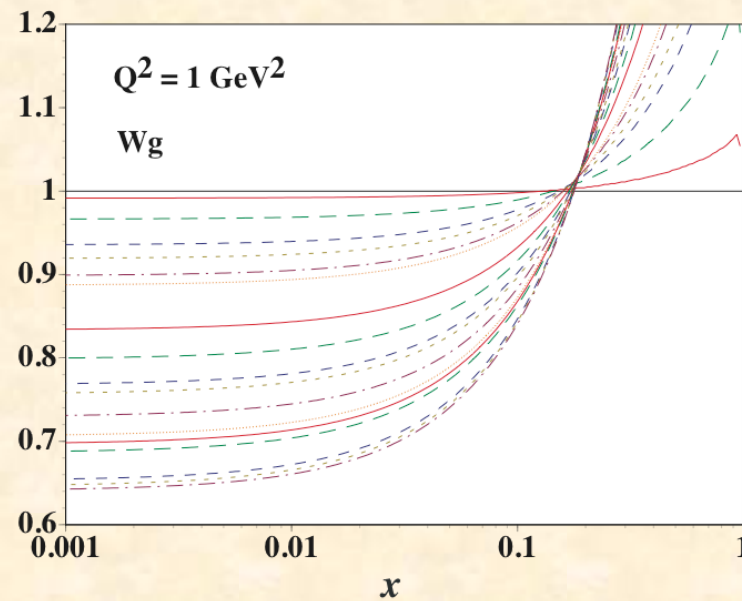
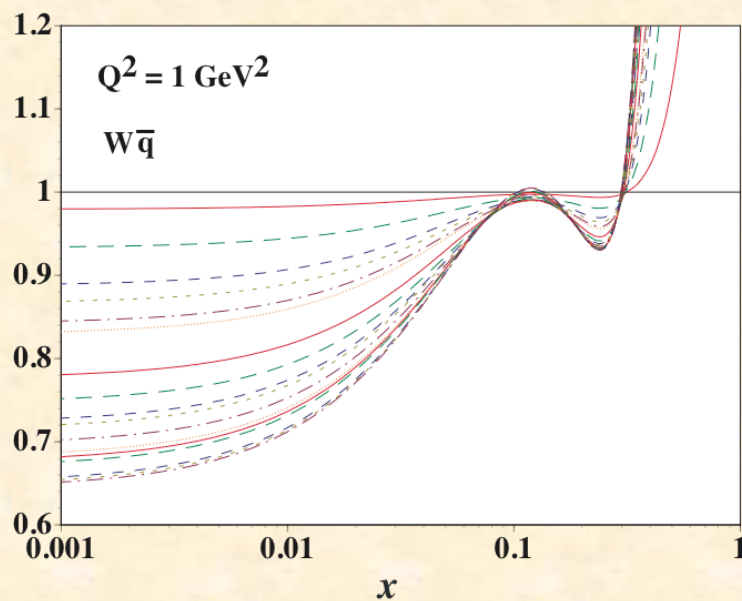
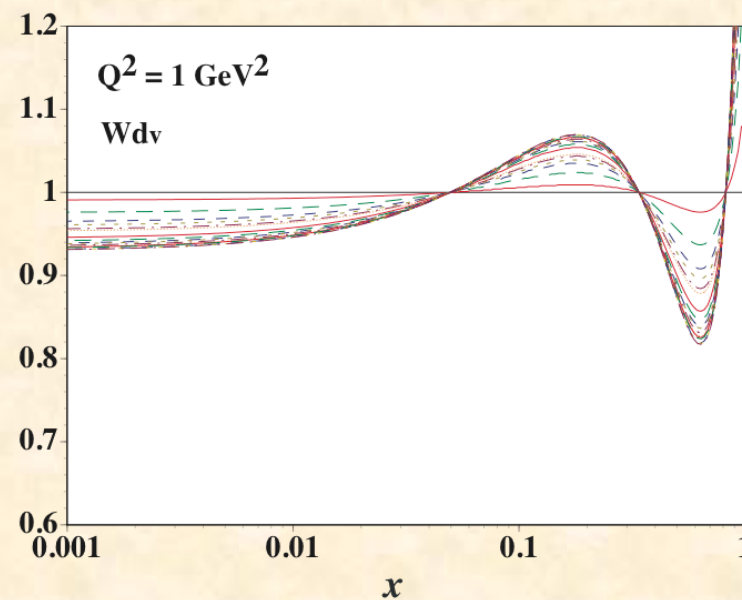
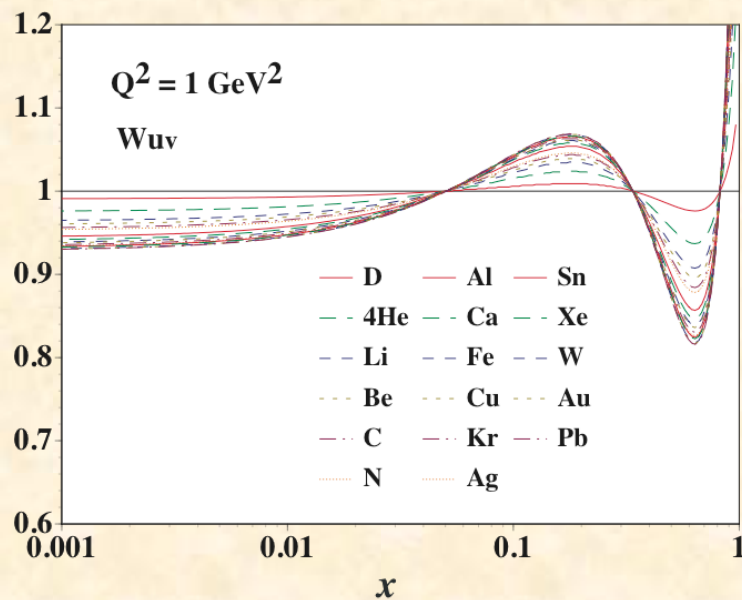
$Q^2$  dependence of  $F_2$  is proportional to the gluon distribution.

No experimental consensus of  $Q^2$  dependence!  
 →  $G^A(x)$  determination is difficult.



# Nuclear PDFs

M. Hirai, S. Kumano, T.-H. Nagai, PRC 76 (2007) 065207.  
<http://research.kek.jp/people/kumanos/nuclp.html>





# Recent global analyses on nuclear PDFs

I may miss some papers.

## – HKN07

- M. Hirai, S. Kumano, and T. -H. Nagai, *Phys. Rev. C* 76 (2007) 065207.
- Charged-lepton DIS, DY.

## – EPS09

- K. J. Eskola, H. Paukkunen, and C. A. Salgado, *JHEP* 04 (2009) 065.
- Charged-lepton DIS, DY,  $\pi^0$  production in  $dAu$ .

## – CTEQ

- I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. I. Olness, J. F. Owens, *Phys. Rev. D* 77 (2008) 054013; D80 (2009) 094004; K. Kovarik *et al.*, *PRL* 106 (2011) 122301; arXiv:1307.3454.
- Neutrino DIS, Charged-lepton DIS, DY.

## – DSZS12

- D. de Florian, R. Sassot, P. Zurita, M. Stratmann, *Phys. Rev. D* 85 (2012) 074028.
- Charged-lepton DIS, DY, RHIC- $\pi$

See also L. Frankfurt, V. Guzey, and M. Strikman, *Phys. Rev. D* 71 (2005) 054001;  
*Phys. Lett. B* 687 (2010) 167; *Phys. Rept.* 512 (2012) 255; arXiv:1310.5879.  
S. A. Kulagin and R. Petti, *Phys. Rev. D* 76 (2007) 094023; *C* 82 (2010) 054614.  
A. Bodek and U.-K. Yang, arXiv:1011.6592.

# Kulagin and Petti's analysis

S. A. Kulagin and R. Petti, Phys. Rev. D76 (2007) 094023.

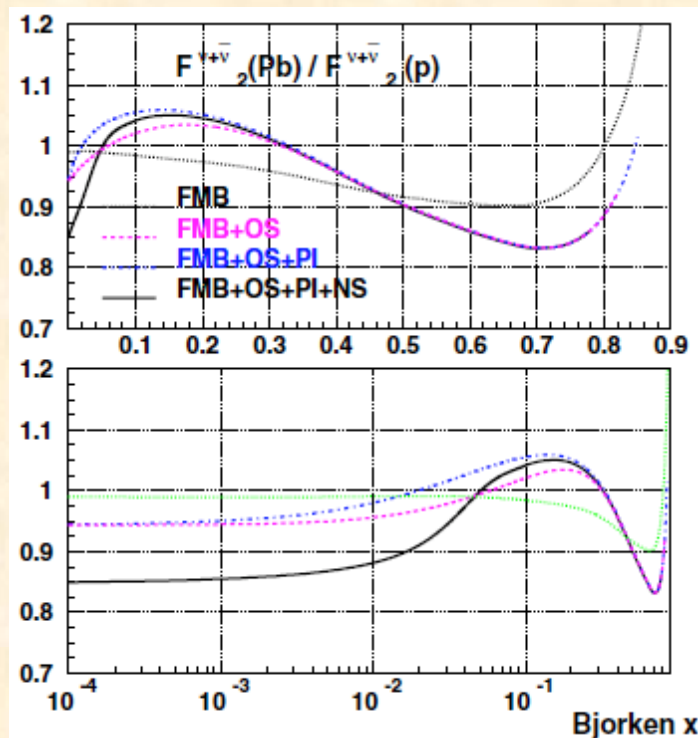
- Although most global analyses assume rather a model-independent functional form with a number of parameters, their approach is different in the sense that **only the “off-shell” effects in the nucleon are parametrized.**
- They tried to obtain **structure functions rather than the PDFs.**

## Nuclear structure functions

= **Conventional nuclear physics + Nucleon modifications in nuclear medium**

= **Binding, Fermi motion + Pion excess + Shadowing (Multiple scattering)**

**+ Off-shell effects (with parameters to be determined from data)**



$$\text{Off-shell effects: } F_2(x, Q^2, p^2) = F_2(x, Q^2) \left[ 1 + \delta f_2(x) \frac{p^2 - M_N^2}{M_N^2} \right]$$

$$\text{Parametrization: } \delta f_2(x) = C_N (x - x_1)(x - x_0)(1 + x_0 - x)$$

**FMB: Fermi Motion + Binding**

**OS: +Off-Shell**

**PI: +Pion**

**NS: +Nuclear Shadowing**

# Functional form of initial distributions at $Q_0^2$

Initial nuclear PDFs at

$$f_i^A(x) = \frac{1}{A} \left[ Z f_i^{p/A}(x) + (A-Z) f_i^{n/A}(x) \right] \quad f_i^{N/A}(x): \text{ PDF of bound nucleon in the nucleus}$$

Isospin symmetry is assumed:  $u \equiv d^n = u^p, d \equiv u^n = d^p$

## Functional forms

- HKN07 ( $Q_0^2 = 1 \text{ GeV}^2$ )

$$f_i^A(x) = w_i(x, A, Z) \frac{1}{A} \left[ Z f_a^p(x) + (A-Z) f_a^n(x) \right], \quad w_i(x, A, Z) = 1 + \left( 1 - \frac{1}{A^{1/3}} \right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{0.1}}$$

- EPS09 ( $Q_0^2 = 1.69 \text{ GeV}^2$ )

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{CTEQ6.1M}}(x, Q_0^2), \quad R_i^A(x) = \begin{cases} a_0 + (a_1 + a_2 x) [\exp(-x) - \exp(-x_a)] & (x \leq x_a : \text{shadowing}) \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & (x_a \leq x \leq x_e : \text{antishadowing}) \\ c_0 + (c_1 - c_2 x) (1-x)^{-\beta} & (x_e \leq x \leq 1 : \text{EMC\&Fermi}) \end{cases}$$

- CTEQ-08 ( $Q_0^2 = 1.69 \text{ GeV}^2$ )

$$x f_i^{N/A}(x) = \begin{cases} A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1 + e^{A_4 x})^{A_5} & : i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s} \\ A_0 x^{A_1} (1-x)^{A_2} + (1 + A_3 x) (1-x)^{A_4} & : i = \bar{d} / \bar{u} \end{cases}$$

- DSZS12 ( $Q_0^2 = 1.0 \text{ GeV}^2$ )

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{MSTW2009}}(x, Q_0^2), \quad R_v^A(x) = \varepsilon_1 x^{\alpha_v} (1-x)^{\beta_1} [1 + \varepsilon_2 (1-x)^{\beta_2}] [1 + a_v (1-x)^{\beta_3}]$$

$$R_s^A(x) = R_v^A(x) \frac{\varepsilon_s}{\varepsilon_1} \frac{1 + a_s x^{\alpha_s}}{1 + a_s}, \quad R_g^A(x) = R_v^A(x) \frac{\varepsilon_g}{\varepsilon_1} \frac{1 + a_g x^{\alpha_g}}{1 + a_g}$$

# Comparison of nuclear PDFs

Different analysis results are consistent with each other because they are roughly within uncertainty bands.

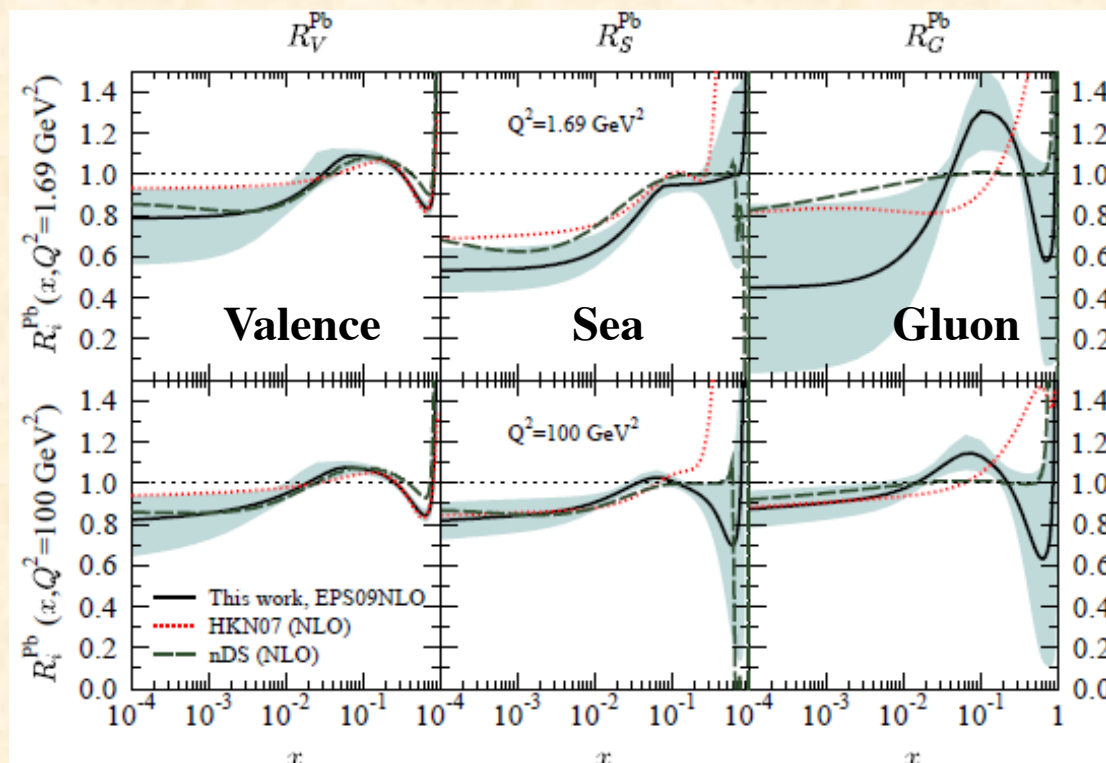
**Valence quark:** Well determined except at small  $x$ .

**Antiquark:** Determined at small  $x$ , Large uncertainties at medium and large  $x$ .

**Gluon:** Large uncertainties in the whole- $x$  region.

$$Q^2 = 1.69 \text{ GeV}^2$$

$$Q^2 = 100 \text{ GeV}^2$$



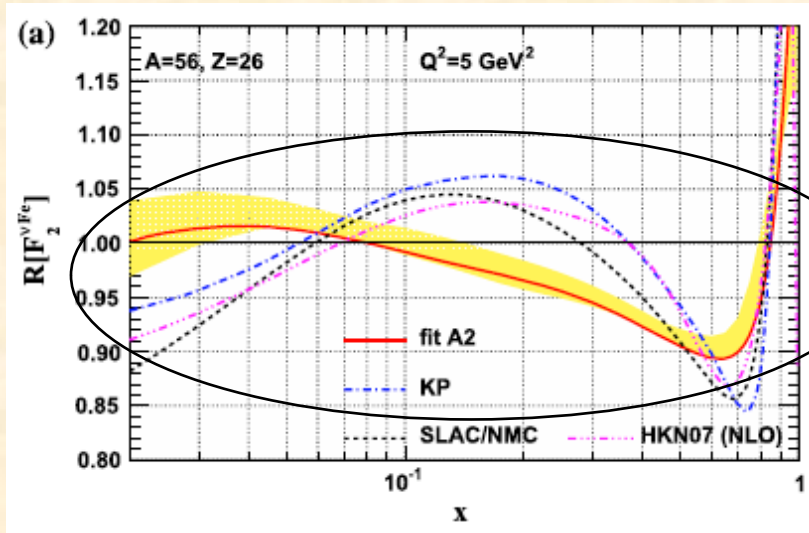
EPS09 (K. J. Eskola *et al.*), JHEP 04 (2009) 065



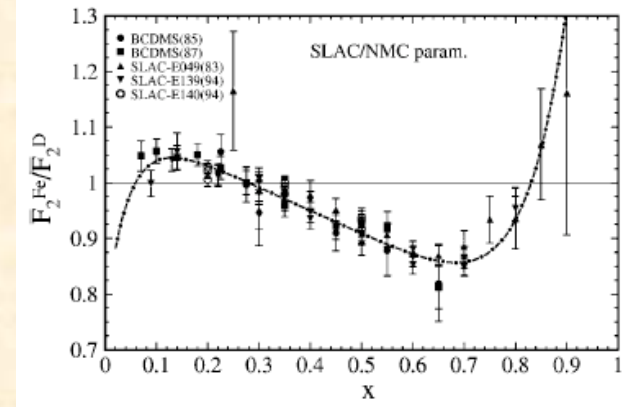
# Analysis of CTEQ-2008 (Schienbein *et al.*)

I. Schienbein *et al.*,  
PRD 77 (2008) 054013

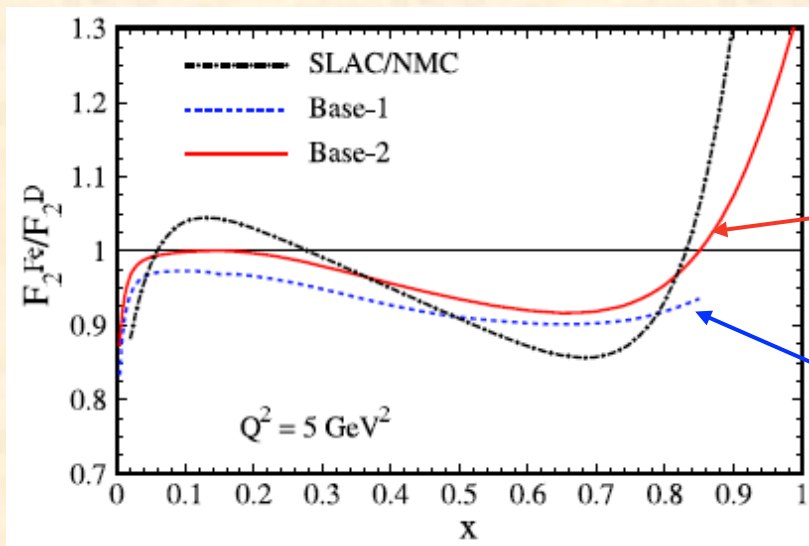
## Charged-lepton scattering



**Differences from typical NPDFs.**



## Neutrino scattering



- Base-1**
  - remove CCFR data
  - incorporate deuteron corrections
- Base-2** corresponds to CTEQ6.1M with  $s \neq \bar{s}$ 
  - include CCFR data

Charged-lepton correction factors are applied.

- $s \neq \bar{s}$

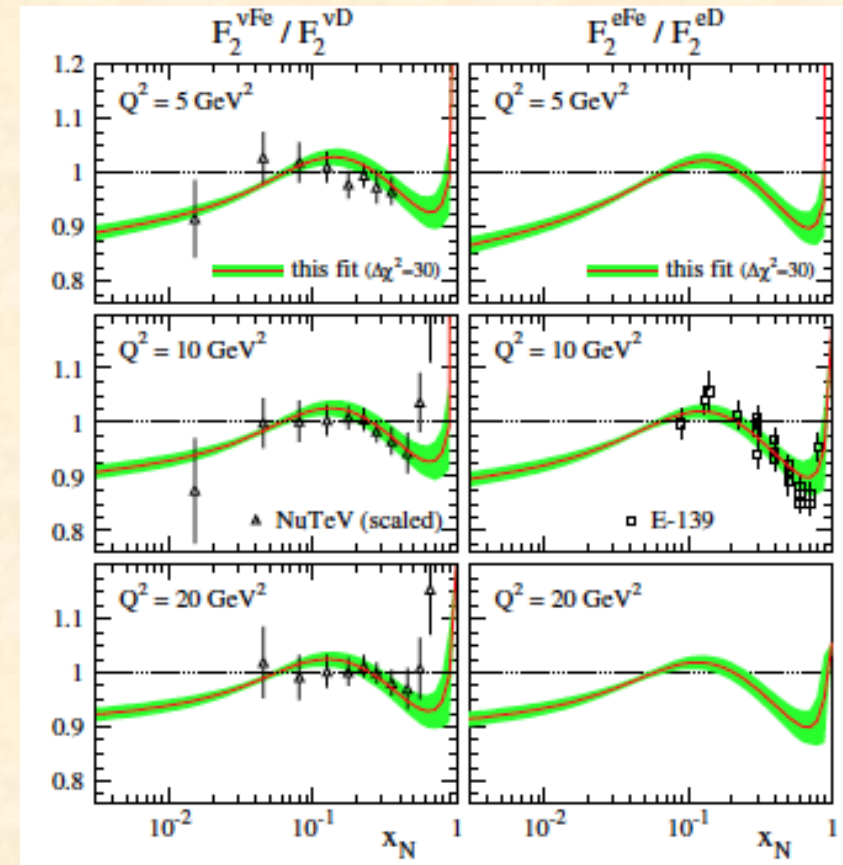
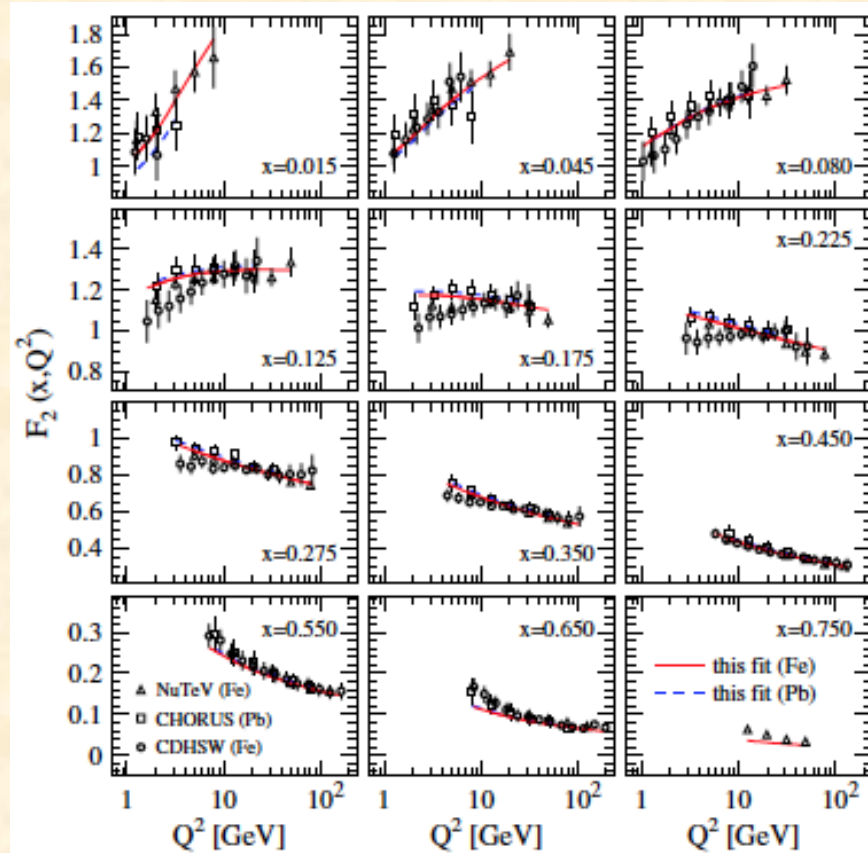
**Base-2:** Using current nucleonic PDFs, they (and MRST) obtained very different corrections from charged-lepton data.

**Base-1:** However, it depends on the analysis method for determining “nucleonic” PDFs.



# Neutrino DIS $\Leftrightarrow$ Charged DIS issue

D. de Florian, R. Sassot, P. Zurita, and M. Stratmann,  
Phys. Rev. D 85 (2012) 074028.



According to their analysis, the issue does not exist!?

## **Activities in 2013**

- 1. K. Kovarik *et al.* (CTEQ), arXiv:1307.3454 (DIS-2013).**
- 2. H. Honkanen, M. Strikman, and V. Guzey, arXiv:1310.5879.**

# H. Honkanen, M. Strikman, and V. Guzey, arXiv:1310.5879.

Two effects are included:

- Binding effect on the scaling variable:

$$x_p = x(1+r_A), \quad x_p = \frac{Q^2}{2m_p v}$$

$$r_A = \frac{1}{m_p} \left[ (m_n - m_p) \frac{N}{A} - \varepsilon_A \right], \quad \varepsilon_A = \text{binding energy}$$

- QED effect on the nuclear PDFs:

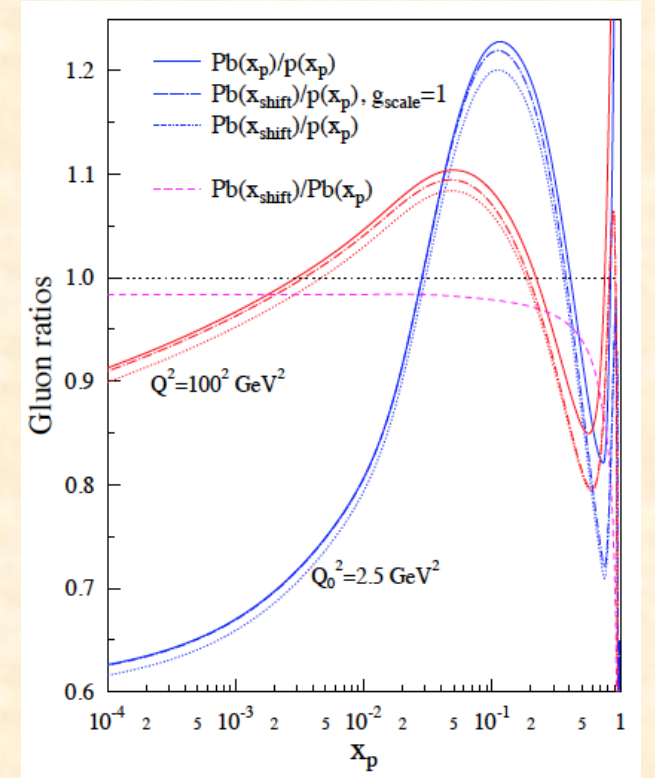
$$\text{Momentum conservation, } \sum_i \int_0^1 dx f_i^A(x, Q^2) = 1 - \eta_\gamma$$

$\eta_\gamma = \text{photon momentum}$

$$\Rightarrow \text{attribute the effect to the gluon modification: } g_{scale} \equiv \frac{\eta_g - \eta_\gamma}{\eta_g}$$

$$xf_i^A(x, Q^2) = \begin{cases} \frac{x_p}{1+r_A} f_i^A\left(\frac{x_p}{1+r_A}, Q^2\right) & \text{for } i = q, \bar{q} \\ g_{scale} \frac{x_p}{1+r_A} f_i^A\left(\frac{x_p}{1+r_A}, Q^2\right) & \text{for } i = g \end{cases}$$

$$\begin{aligned} 0 \leq x_A = \frac{Q^2}{2M_A v} \leq 1, \quad x_A = \frac{Q^2}{2M_A v} = \frac{M_N}{M_A} \frac{Q^2}{2M_N v} \approx \frac{1}{A} x \leq 1 \quad \Rightarrow \quad 0 \leq x \lesssim A \text{ for a nucleus} \\ x_A = \frac{Q^2}{2M_A v} = \frac{Q^2}{2(Zm_p + Nm_n - A\varepsilon_A)v} = \frac{Q^2}{2[(A-N)m_p + Nm_n - A\varepsilon_A]v} = \frac{Q^2}{2[Am_p + N(m_n - m_p) - A\varepsilon_A]v} \\ = \frac{Q^2}{2m_p v A \left[ 1 + \frac{N}{A} \frac{m_n - m_p}{m_p} - \frac{\varepsilon_A}{m_p} \right]} = \frac{1}{A} \frac{x_p}{1+r_A}, \quad x_p = \frac{Q^2}{2m_p v}, \quad r_A = \frac{1}{m_p} \left[ (m_n - m_p) \frac{N}{A} - \varepsilon_A \right] \\ x_{HSG} \equiv \frac{1}{A} x_A = \frac{x_p}{1+r_A} \quad x_p = x_{HSG}(1+r_A) \end{aligned}$$



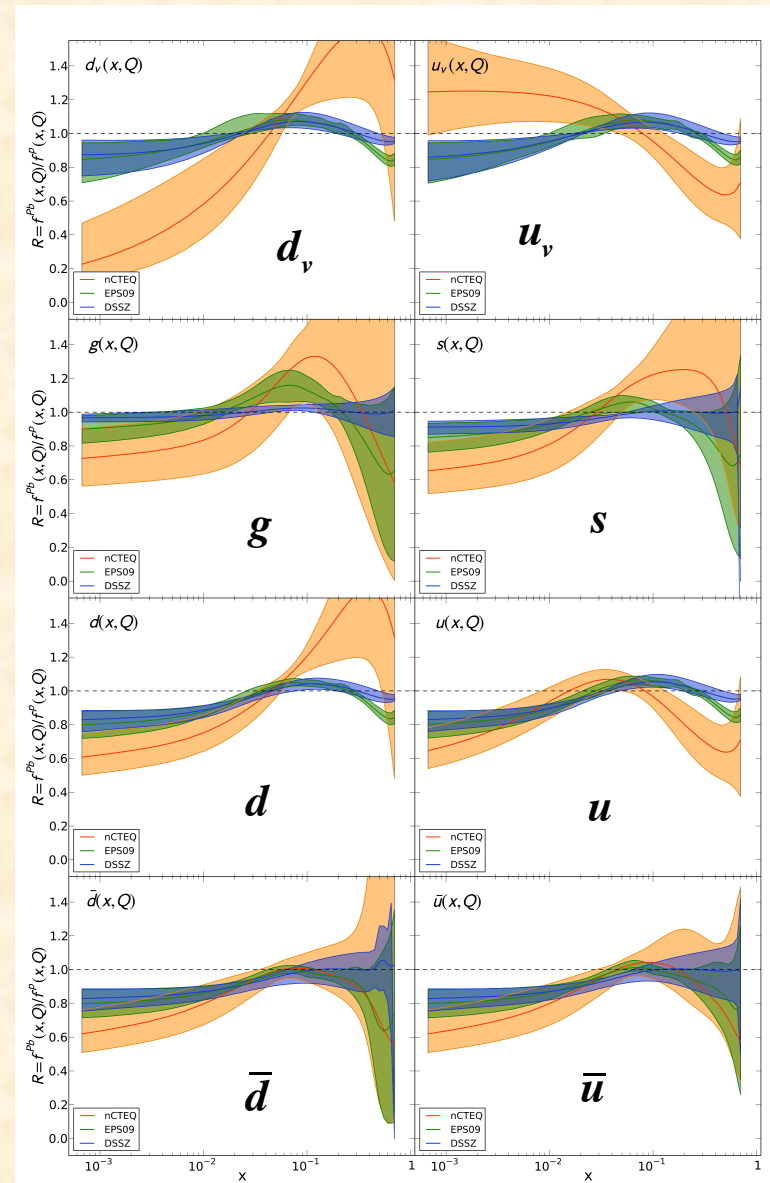
# K. Kovarik *et al.* (CTEQ), arXiv:1307.3454 (DIS-2013)

K. Kovarik, T. Jezo, A. Kusina, F. I. Olness,  
I. Schienbein, T. Stavreva, J. Y. Yu,  
Charged lepton DIS + Drell-Yan

$$x f_k(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5},$$

$$\bar{d}(x, Q_0) / \bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4},$$

$^{208}\text{Pb}$ ,  $Q^2 = 100 \text{ GeV}^2$



# Summary on nuclear-PDF determination

Global analyses for the nuclear PDFs

by using data of charged-lepton, neutrino DIS, pA, AA collisions

**Valence quark:** reasonably good, in progress at JLab for large  $x$

**Antiquark:** good only at  $x=0.1$ , in progress at Fermilab (E906)  $x=0.1 \sim 0.4$ .

**Gluon:** large uncertainties in the whole- $x$  region, LHC

## Issues

- Charged-lepton DIS  $\Leftrightarrow$  Neutrino DIS
- Matching with resonance model
- Gluon distributions

## New data

- JLab, Fermilab-DY, Minerva, LHC, ... (J-PARC high-momentum?)

Large grant was approved for neutrino-research activities in Japan from 2013.

We hope that we answer some of these issues in the near future.



**The End**

**The End**