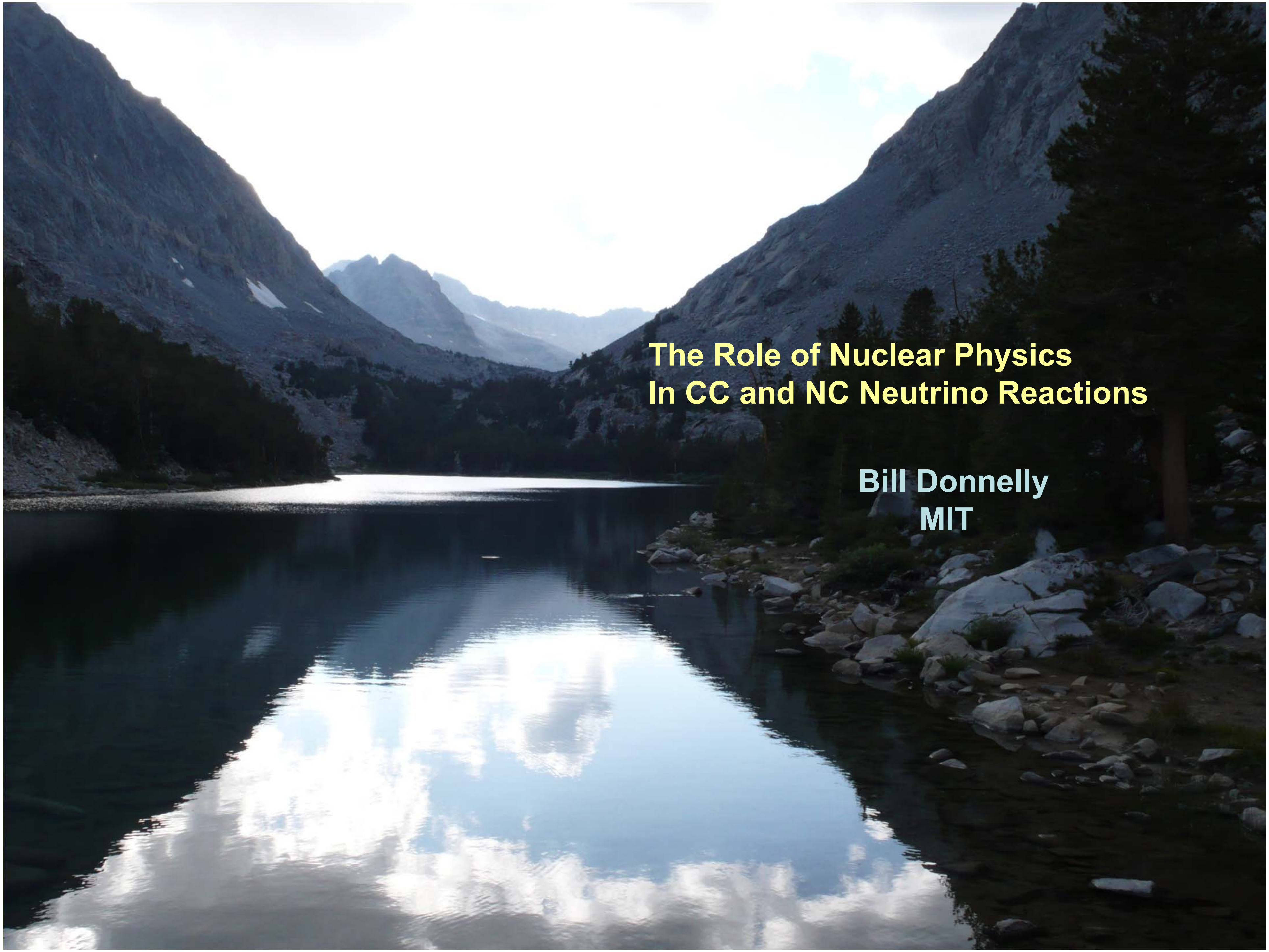

The Role of Nuclear Physics In CC and NC Neutrino Reactions

Bill Donnelly
MIT

Outline:

- Introduction
- Inclusive Scattering
  - Relativistic versus non-relativistic modeling
  - RFG, shell model, rPWIA, RMF approaches
  - Meson-exchange currents
- Semi-inclusive Processes
  - 1-particle spectral function: general form
  - Specific model spectral functions

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1. **Any model that does not succeed for electron scattering is very unlikely to be valid for neutrino reactions.**

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Accordingly,

1. **Any model that does not succeed for electron scattering is very unlikely to be valid for neutrino reactions.**

In this talk I will freely switch between EM responses and CC/NC weak interaction responses.

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3. Dynamical effects in the wave functions themselves

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2. Boost effects on the single-particle current matrix elements
3. Dynamical effects in the wave functions themselves

1. **Kinematic effects:**

   At high energies the final-state ejected nucleon should obey relativistic kinematics, $E = (p^2 + m^2)^{1/2}$ when on-shell. Of course, when interacting the initial- and final-state nucleons in the nucleus are off-shell. A non-relativistic model can be roughly relativized for such effects by replacing the energy transfer ω by $\omega\,(1 + \omega/2m)$, which places the QE peak at essentially the correct position, namely, $|Q^2|/2m$ rather than $q^2/2m$.

Relativistic effects arise from three sources (which are not distinct):

1. Kinematic effects
2. **Boost effects on the single-particle current matrix elements**
3. Dynamical effects in the wave functions themselves

2. Boost effects on the single-particle current matrix elements:

When making a non-relativistic approximation to the (on-shell) single-particle matrix elements of the vector and axial-vector currents there are boost factors that should be included. To leading order these are multiplicative factors typically γ or $1/\gamma$, where $\gamma = |q^2/Q^2|$.

So, for instance the charge response is enhanced by the factor γ (note that this becomes very large as one approaches the lightcone where $\omega = q$ and so Q^2 goes to zero); this is a Lorentz contraction effect on the charge density. The transverse response goes the other way, namely, is decreased by the factor $1/\gamma$.

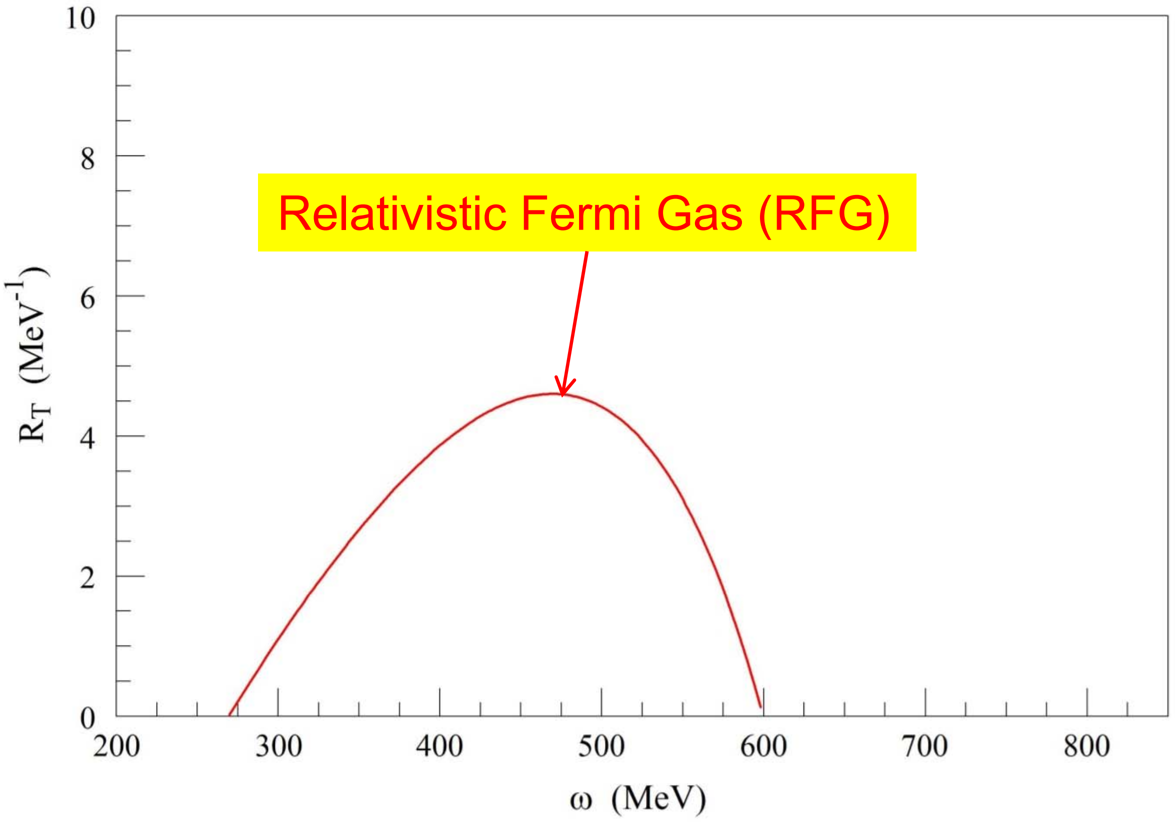
Relativistic effects arise from three sources (which are not distinct):

1. Kinematic effects
2. Boost effects on the single-particle current matrix elements
3. **Dynamical effects in the wave functions themselves**

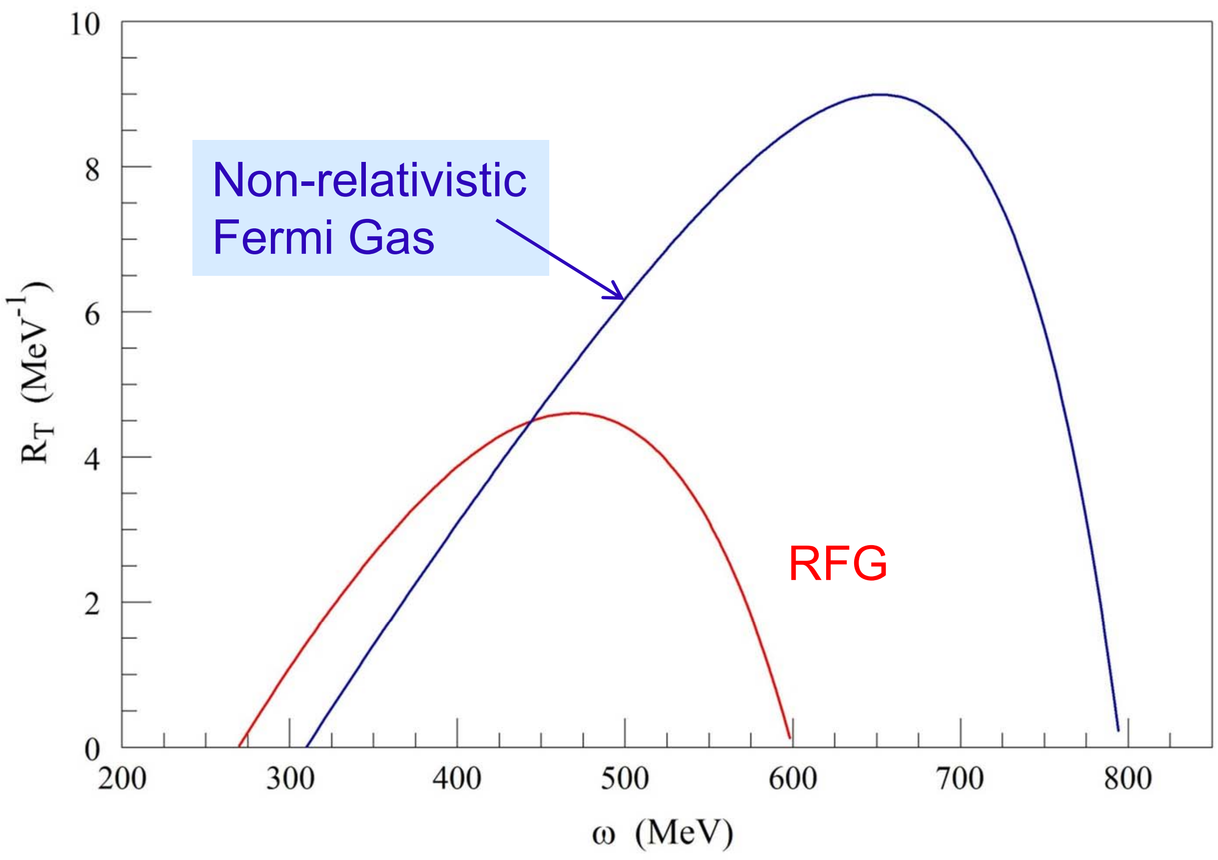
3. Dynamical effects in the wave functions themselves:

The initial-and final-state nucleons in the nucleus are interacting and are therefore off-shell. When relativistic bound and scattering wave functions are employed (for instance in a Dirac Hartree approach) the lower components of the 4-spinors are not related to the upper components by the free-particle relationship and this is manifested in the electroweak responses; typically these amount to 15-20% differences between the various types of response, namely, violations of the so-called scaling of the **zeroth kind** where all of the various responses (longitudinal, vector transverse, axial transverse, VA interference, etc.) scale to a universal function.

Transverse vector response at q = 1 GeV/c



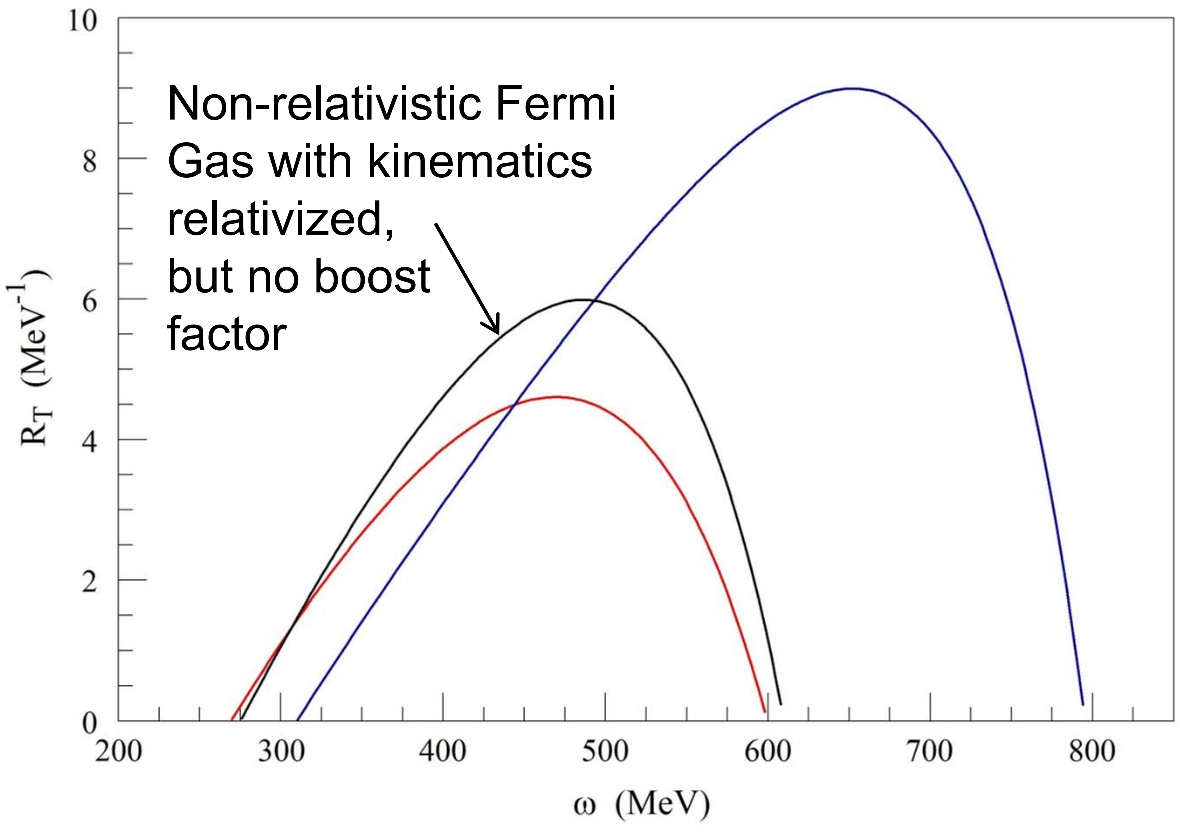
Transverse vector response at q = 1 GeV/c



As an approximation, one can consider "semi-relativistic" modeling where, starting with a non-relativistic model, two steps are made:

1. The kinematic shift introduced above is implemented,
   placing the QE peak in roughly the correct position

Transverse vector response at q = 1 GeV/c

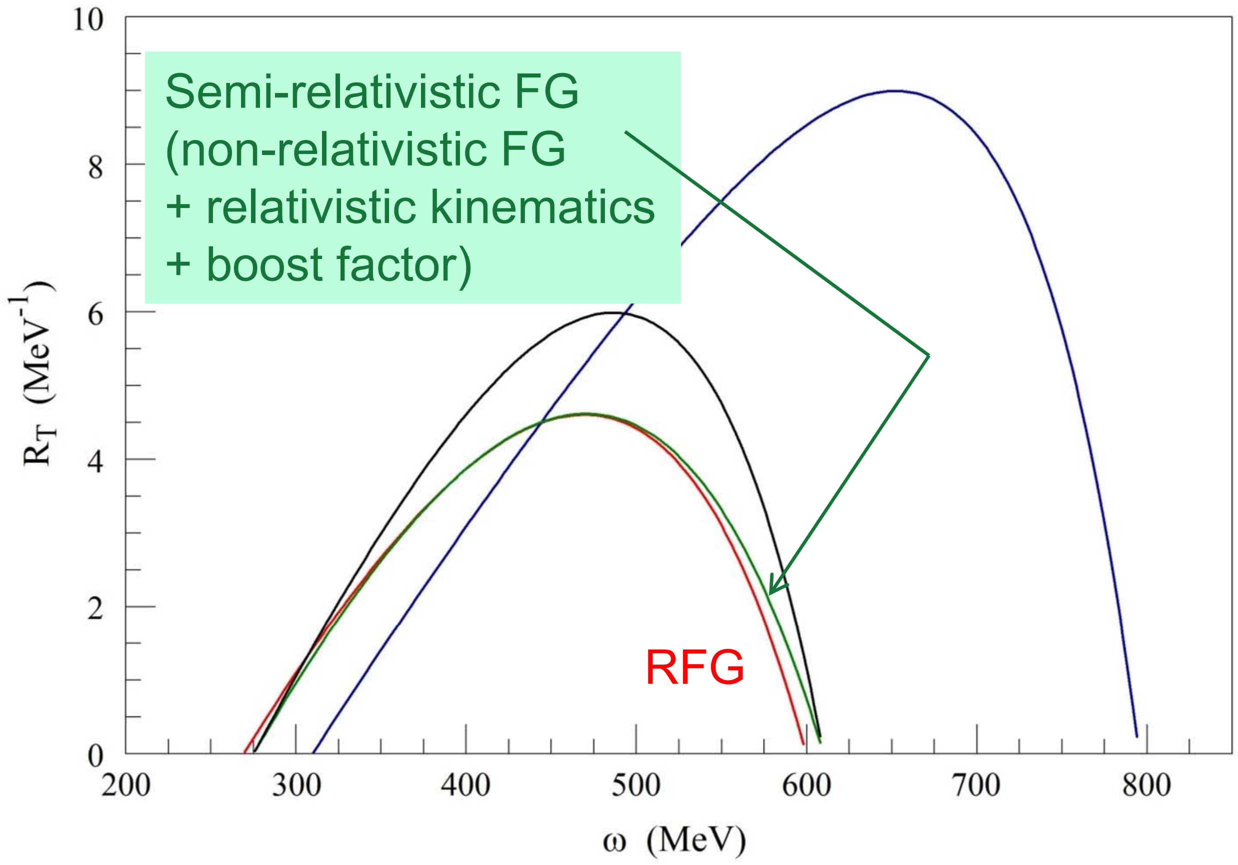


Non-relativistic Fermi Gas with kinematics relativized, but no boost factor

As an approximation, one can consider "semi-relativistic" modeling where, starting with a non-relativistic model, two steps are made:

1. The kinematic shift introduced above is implemented, placing the QE peak in roughly the correct position
2. The boost factors are included in leading order

Transverse vector response at q = 1 GeV/c



Semi-relativistic FG
(non-relativistic FG
+ relativistic kinematics
+ boost factor)

Other models?

… first, consider a non-relativistic shell model,

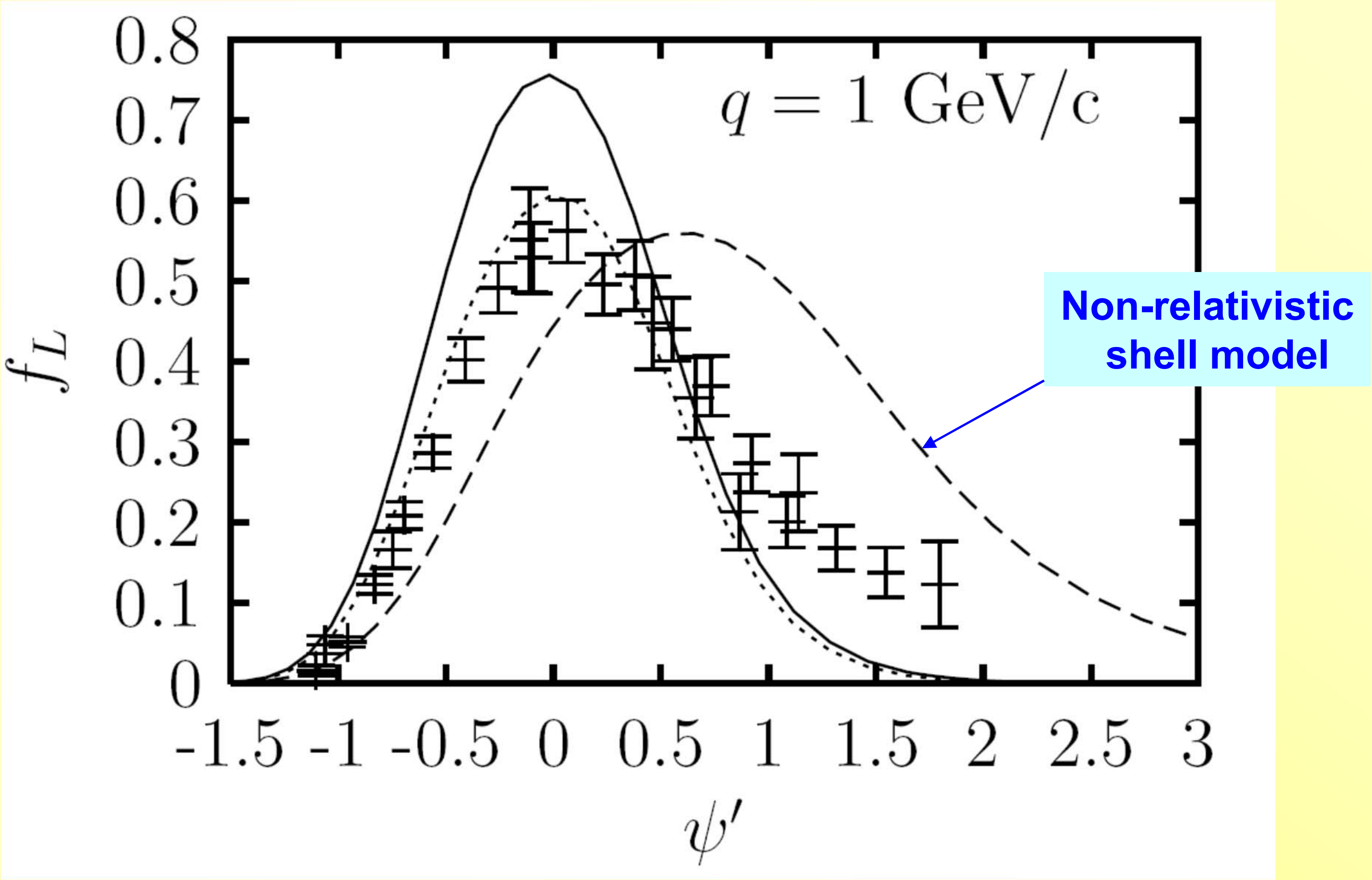
showing, instead of the longitudinal response R_L, the scaled result f_L (where the single-nucleon response has been divided out), and plotting versus ψ' the scaling variable rather than the energy transfer ω

Other models?

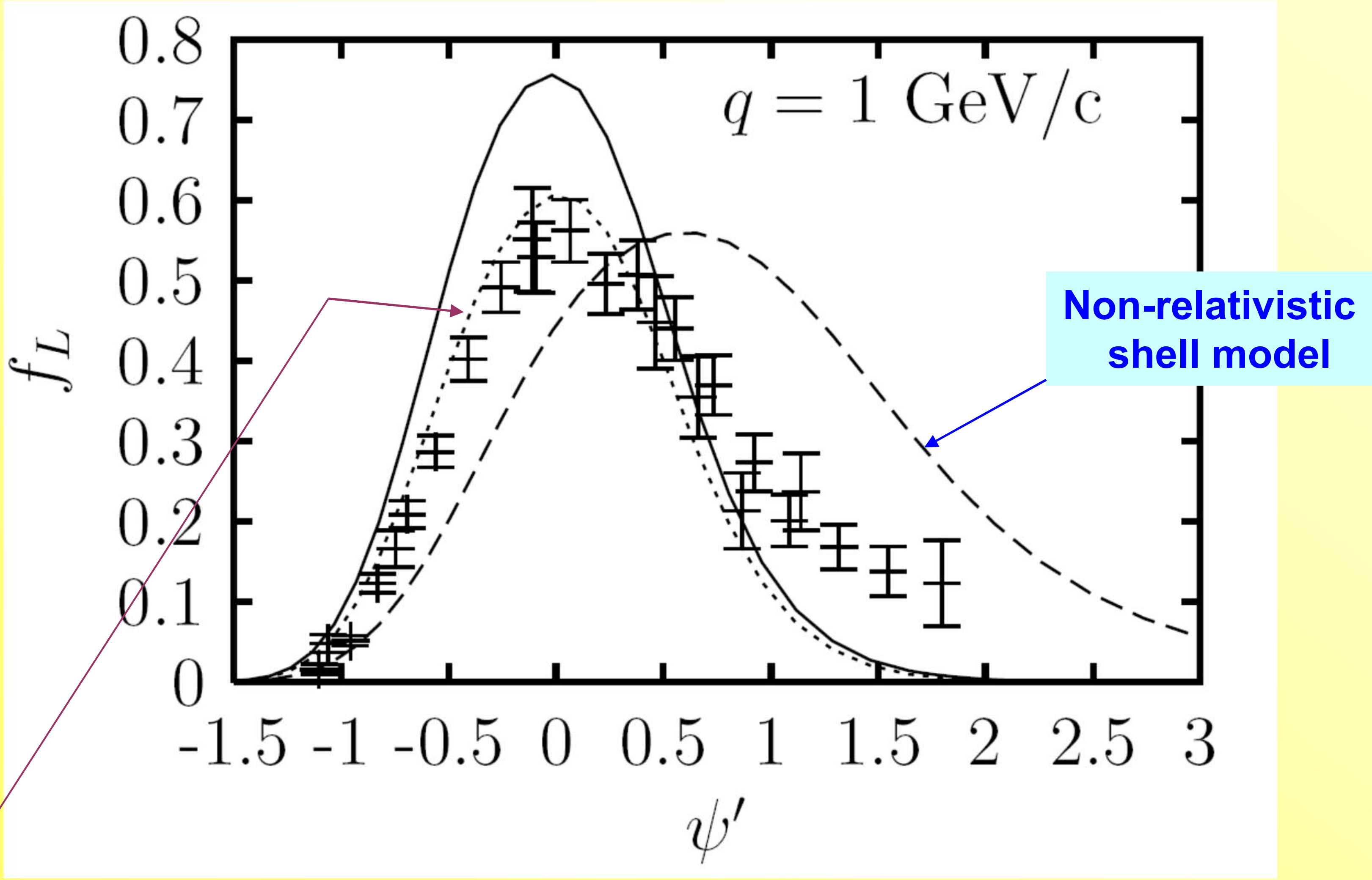
… first, consider a non-relativistic shell model,

showing, instead of the longitudinal response R_L, the scaled result f_L (where the single-nucleon response has been divided out), and plotting versus ψ' the scaling variable rather than the energy transfer ω

Important: the longitudinal response has only very small contributions from meson production and meson-exchange currents, and therefore provides a fair test of the one-body QE cross section. The electron scattering response scales (data shown).

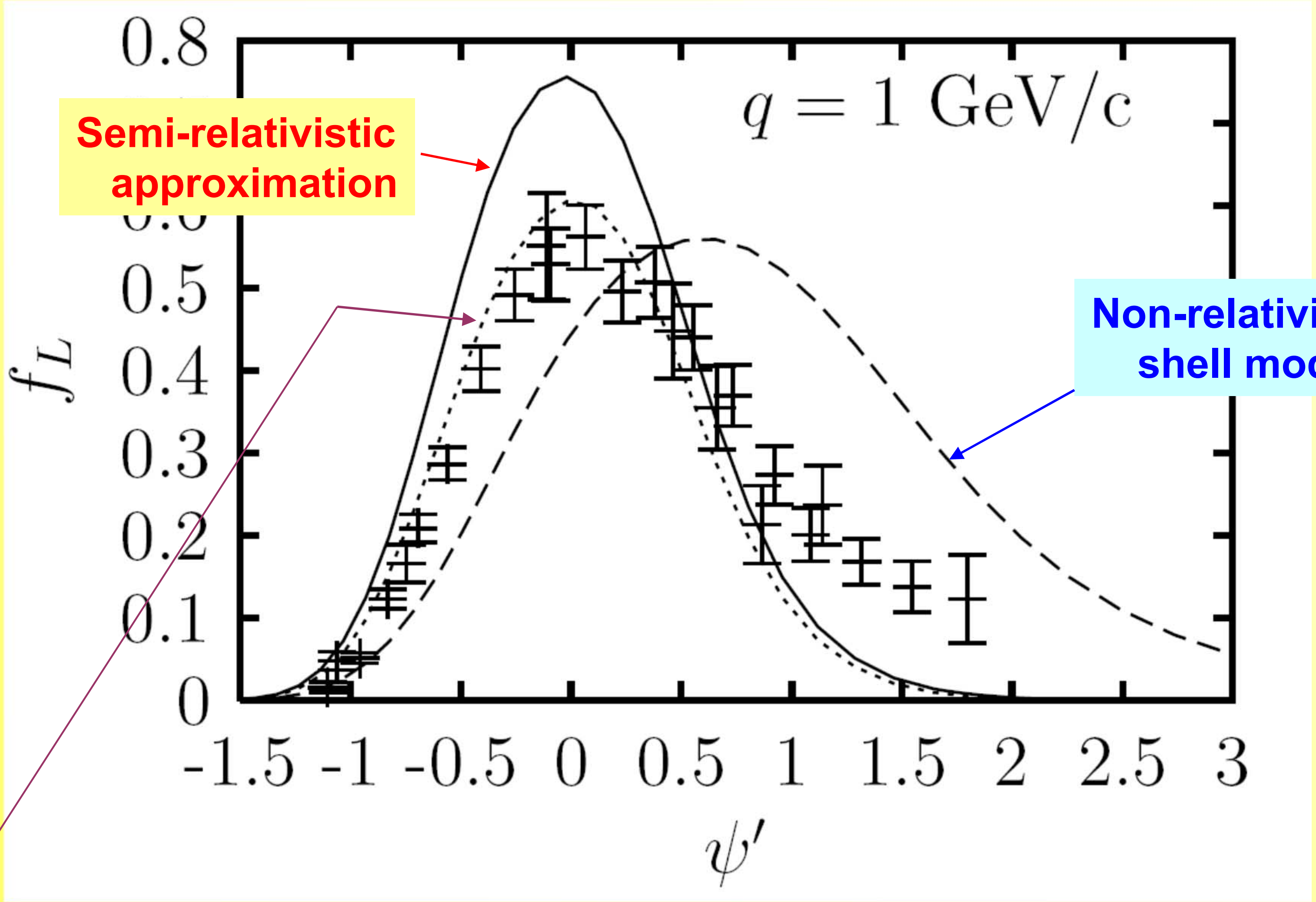


Non-relativistic shell model



Non-relativistic shell model

Non-relativistic current operators (no boost effects), but with relativistic kinematics



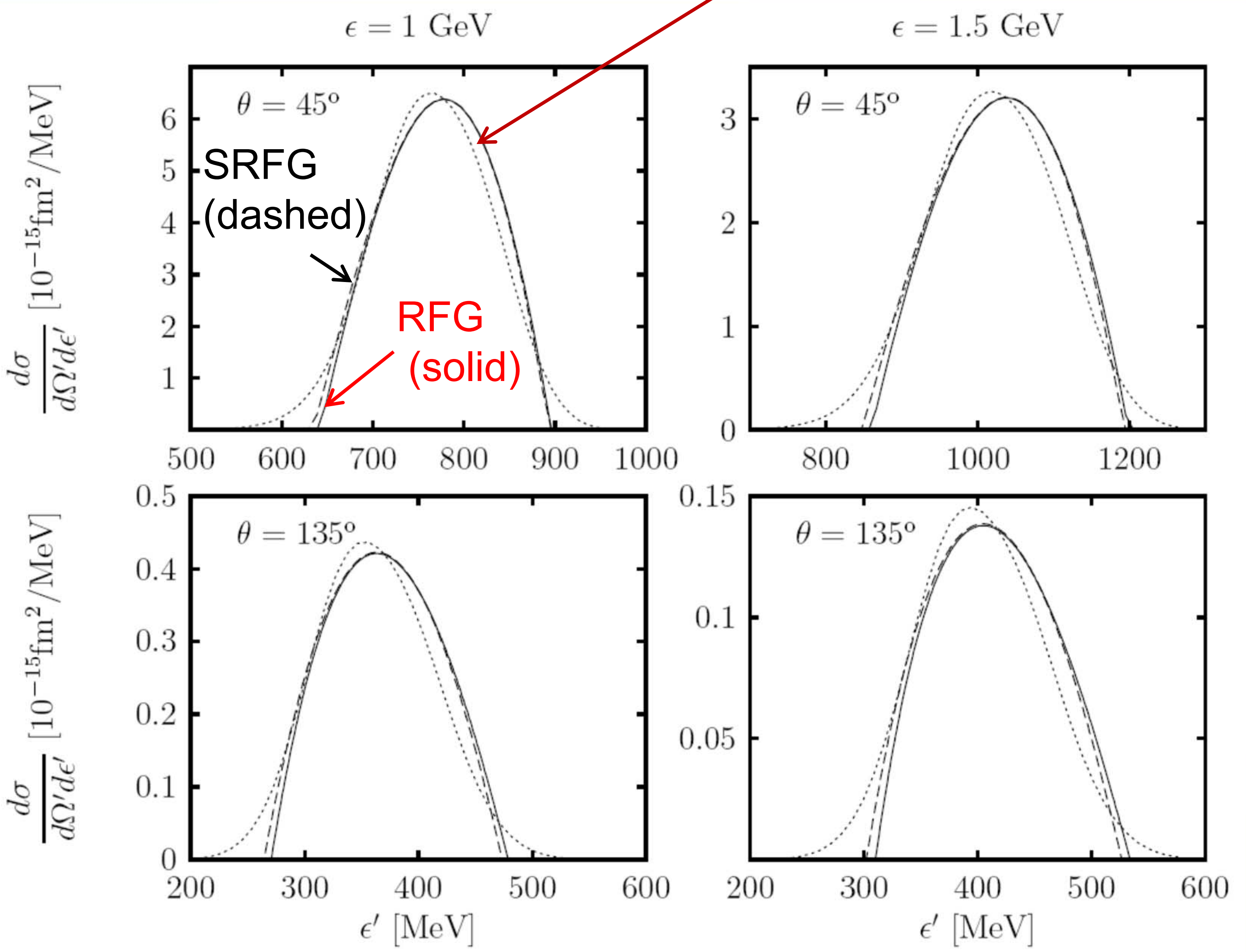
Semi-relativistic approximation

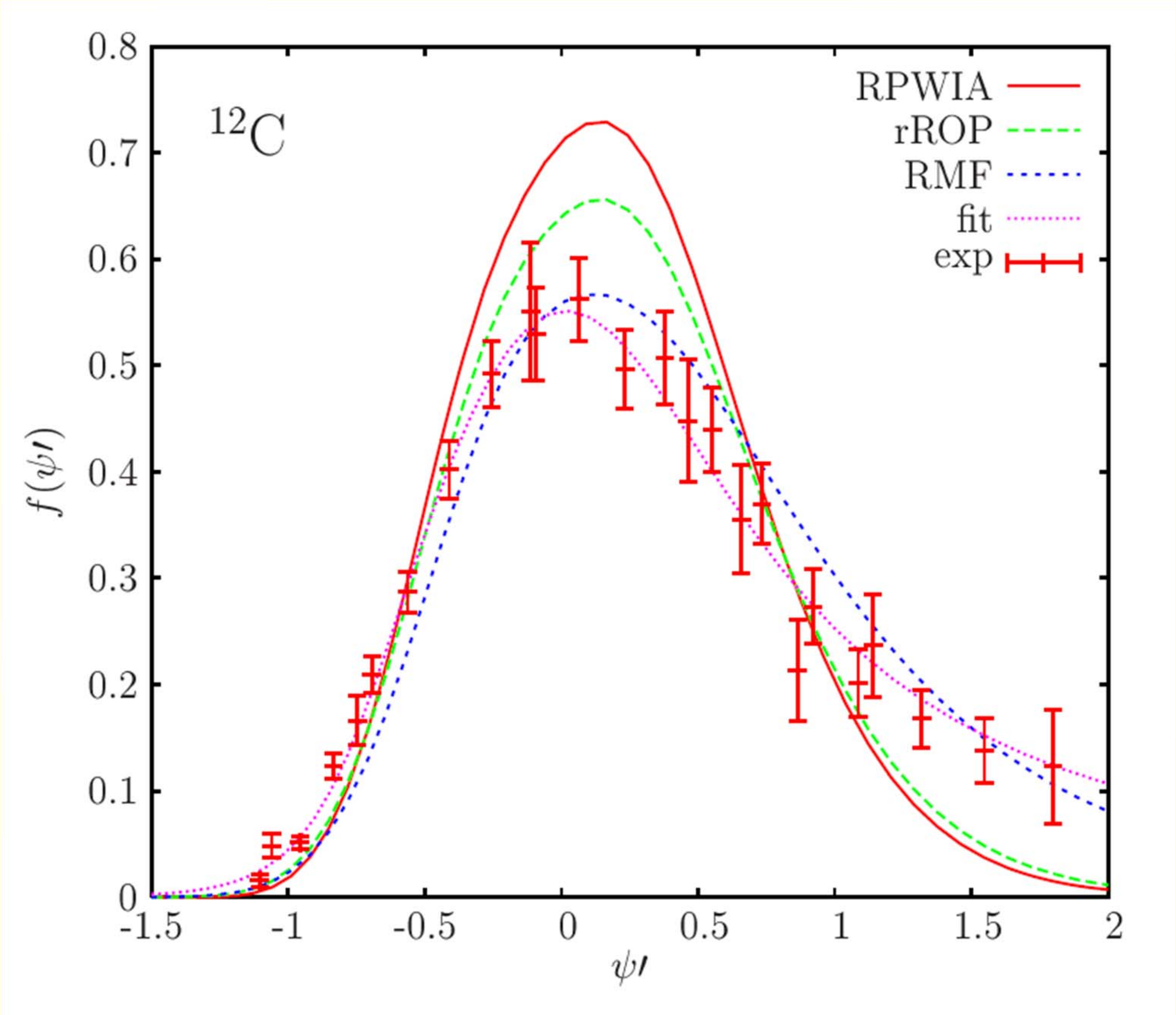
Non-relativistic shell model

Non-relativistic current operators (no boost effects), but with relativistic kinematics

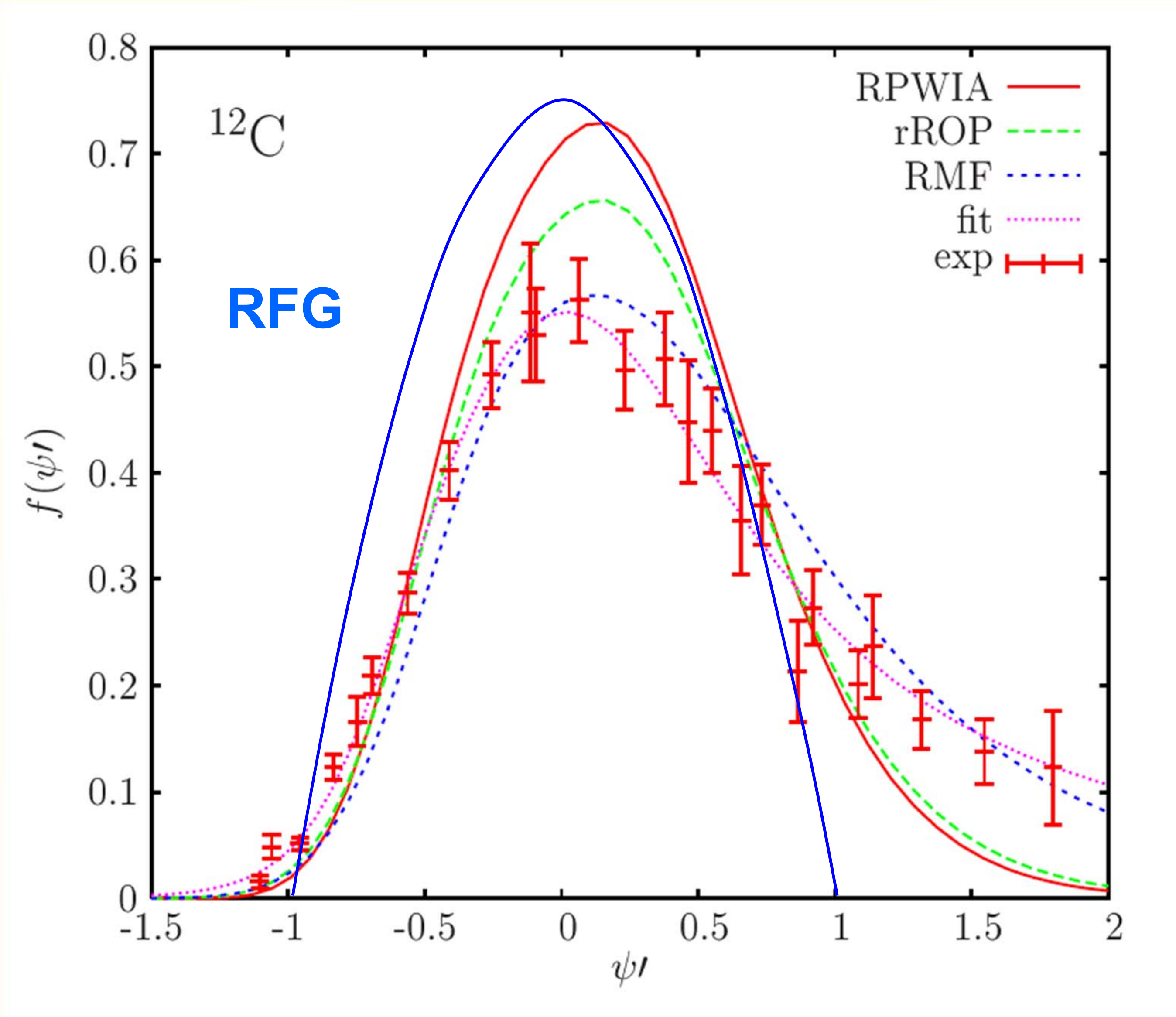
CC Neutrino Reactions (^{12}C)

Continuum Shell Model (dotted)





^{12}C
RPWIA
rROP
RMF
fit
exp
$f(\psi')$
ψ'
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
-1.5
-1
-0.5
0
0.5
1
1.5
2


^{12}C
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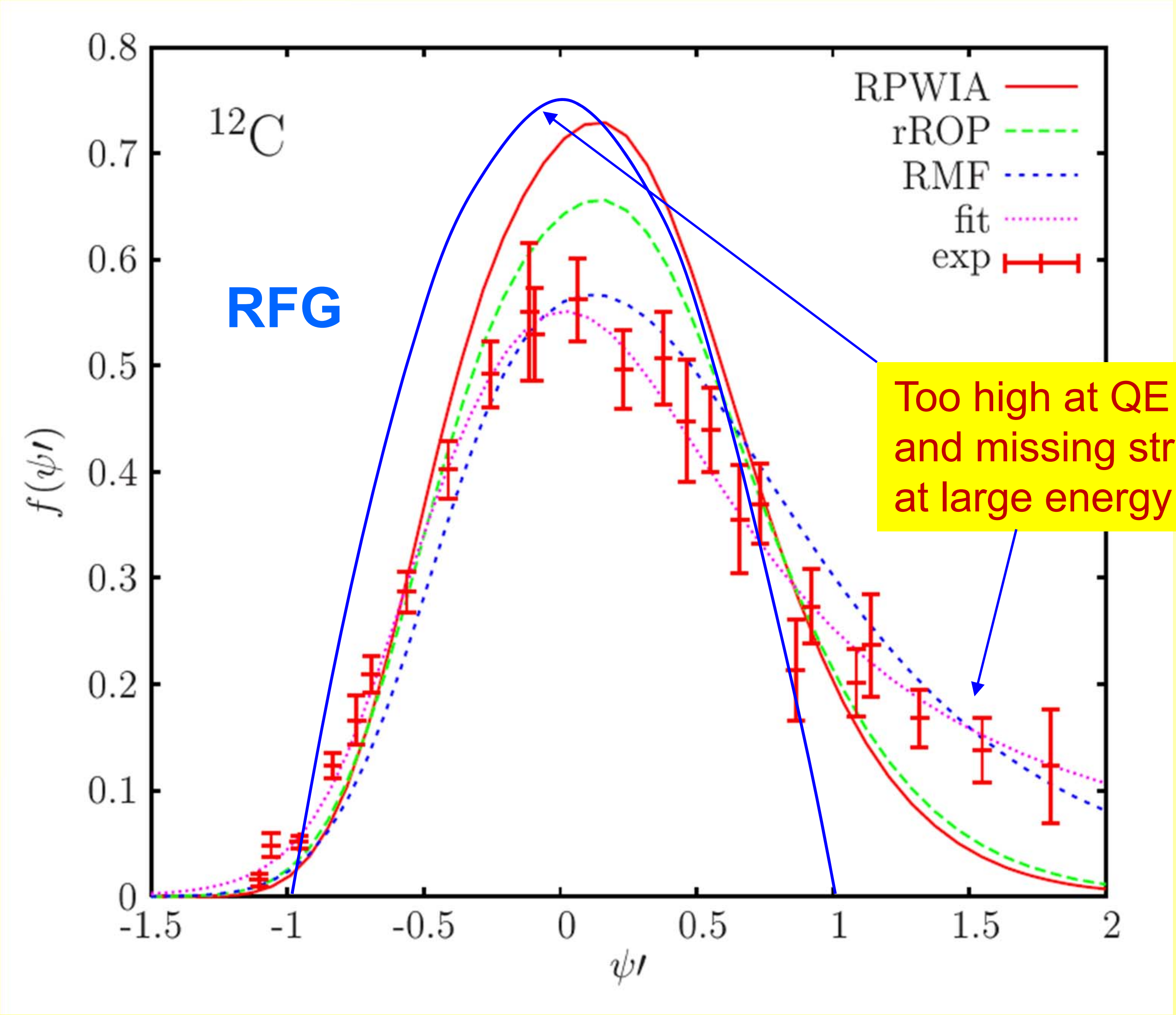
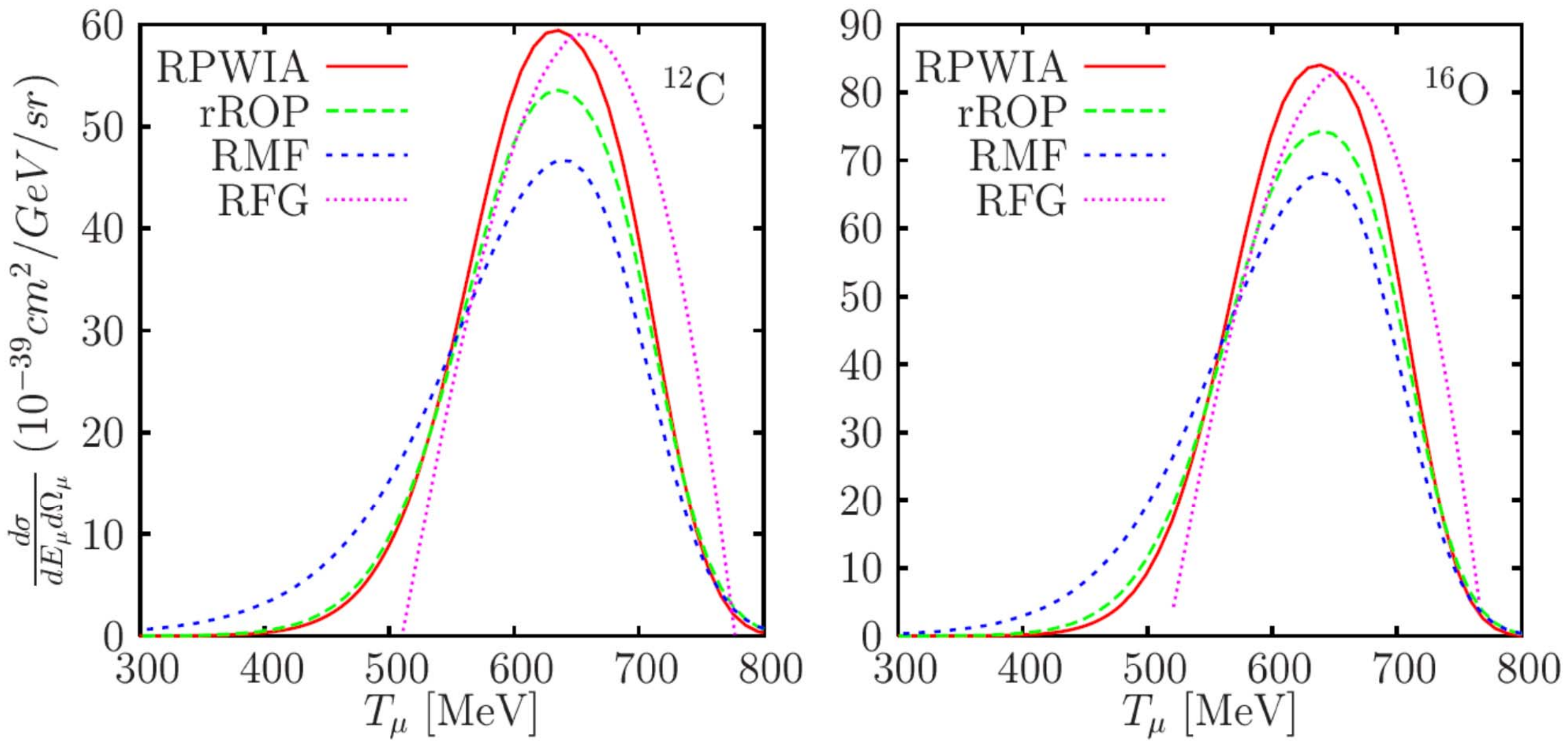


Too high at QE peak
and missing strength
at large energy transfers

CC Neutrino Reactions (MiniBooNE conditions)



Summary so far:

2. Relativistic effects from kinematics and boost factors are essential.
3. Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail to reproduce the data, while for inclusive scattering RMF theory is much better.

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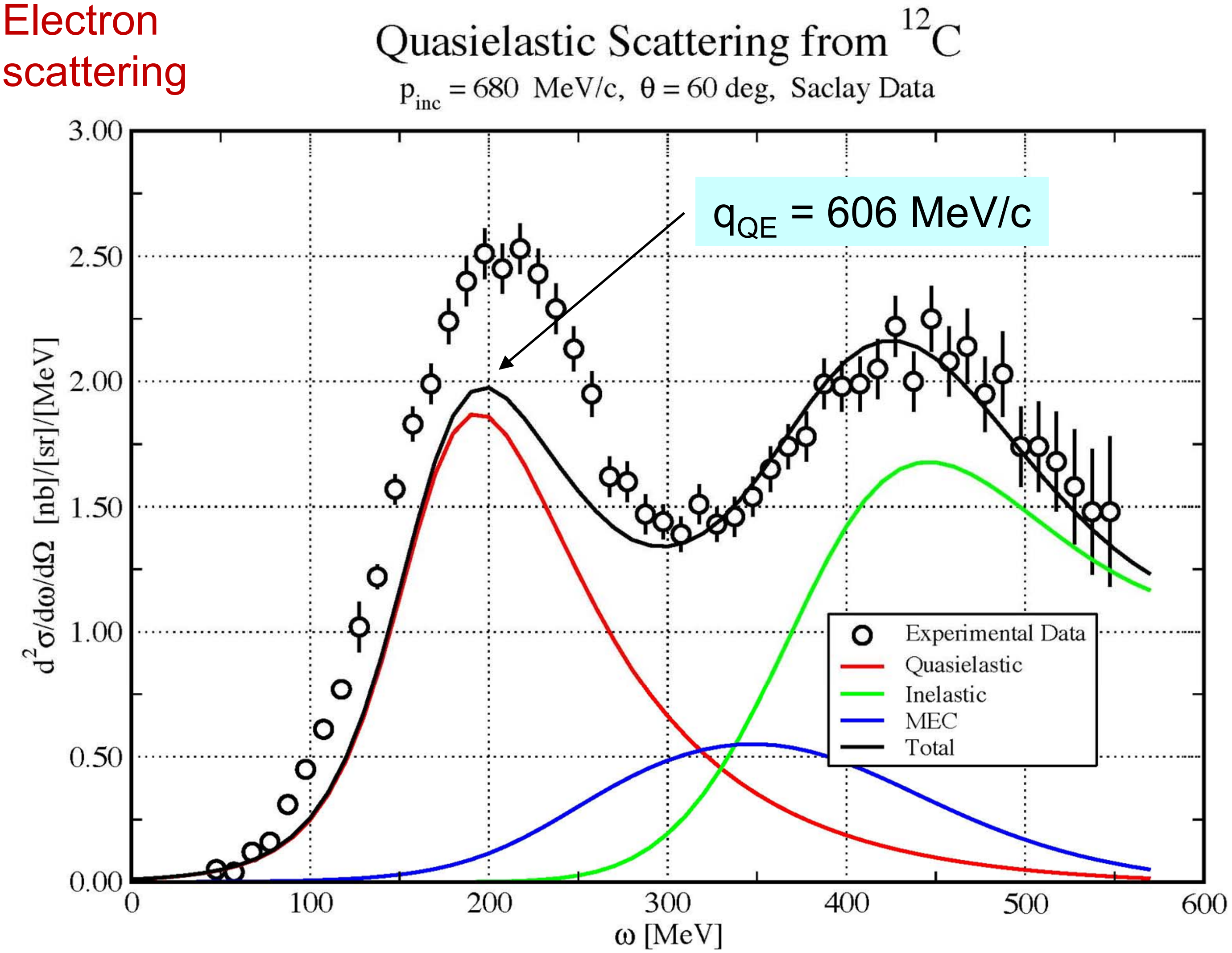
2. **Relativistic effects from kinematics and boost factors are essential.**
3. **Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail to reproduce the data, while for inclusive scattering RMF theory is much better.**

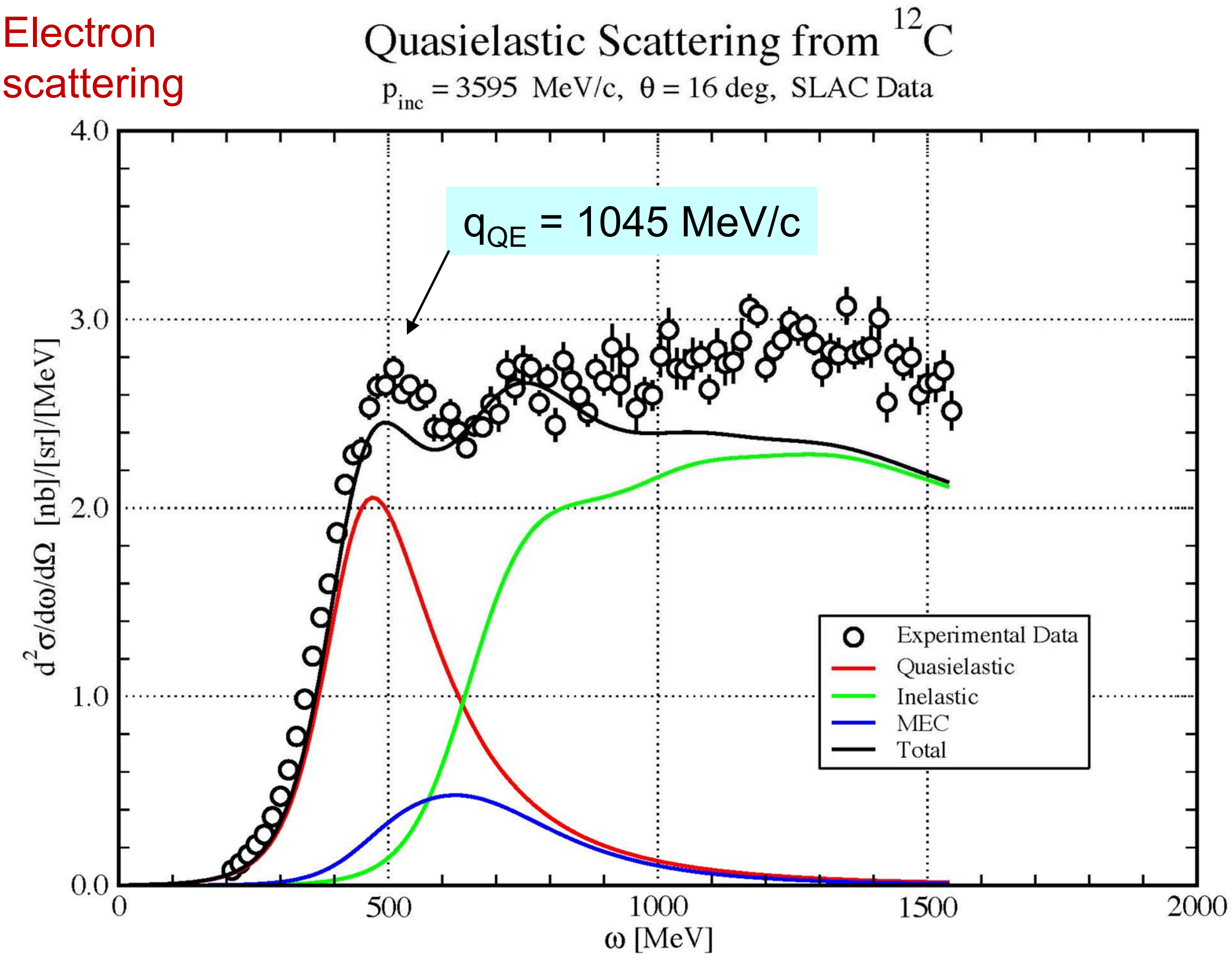
… additionally, so far only one-body currents have been discussed and one should account for the contributions of two-body Meson-Exchange Currents (MEC), especially those that contain an intermediate Δ and π exchange

Electron scattering

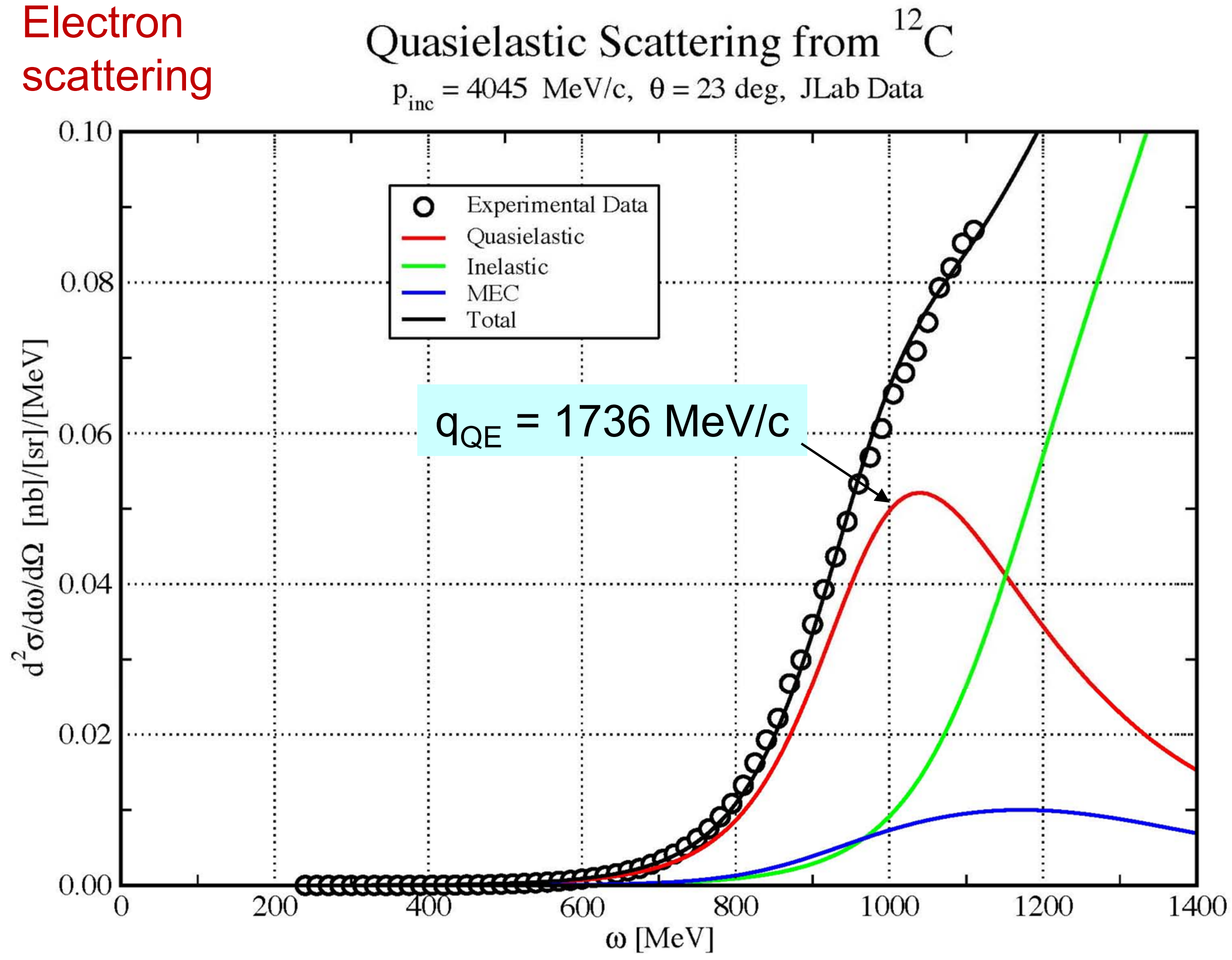


q_{QE} = 606 MeV/c

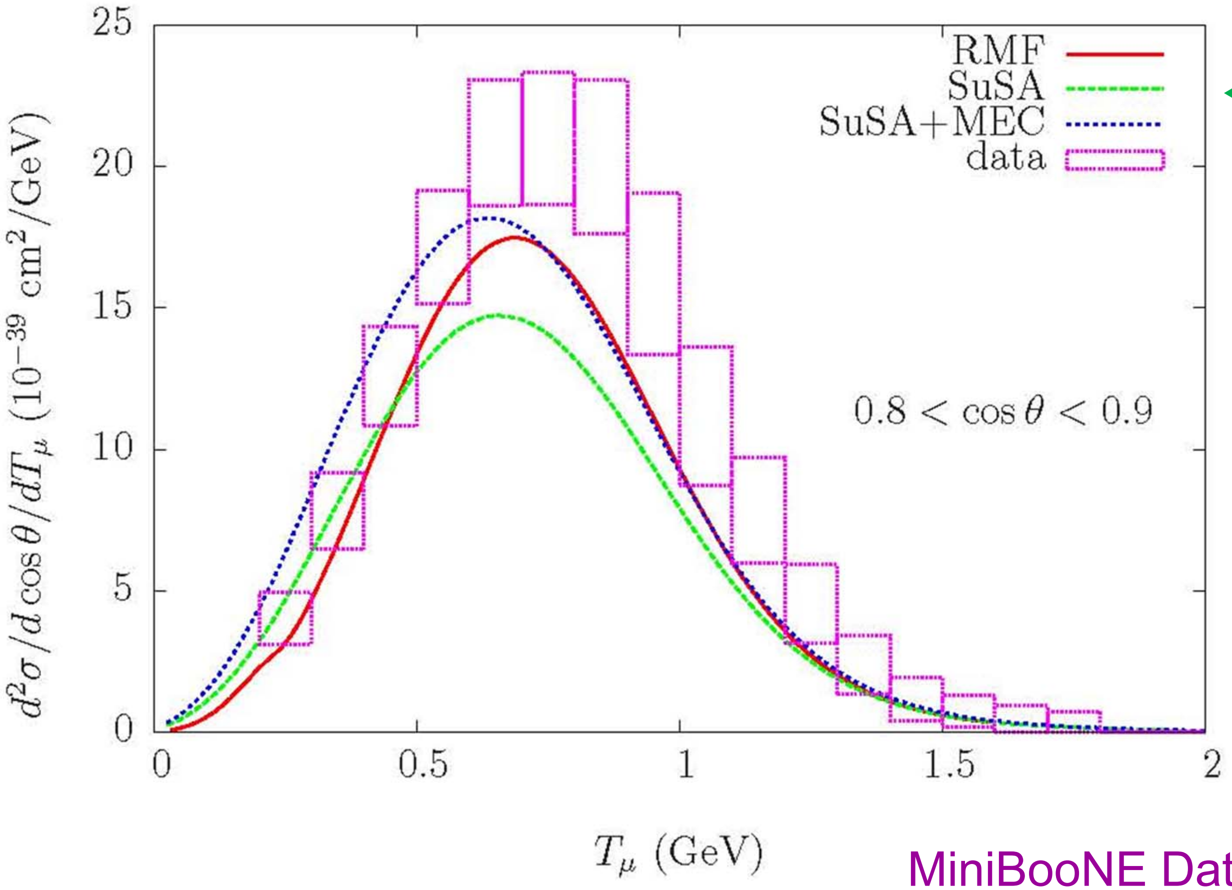
Electron scattering



Electron scattering

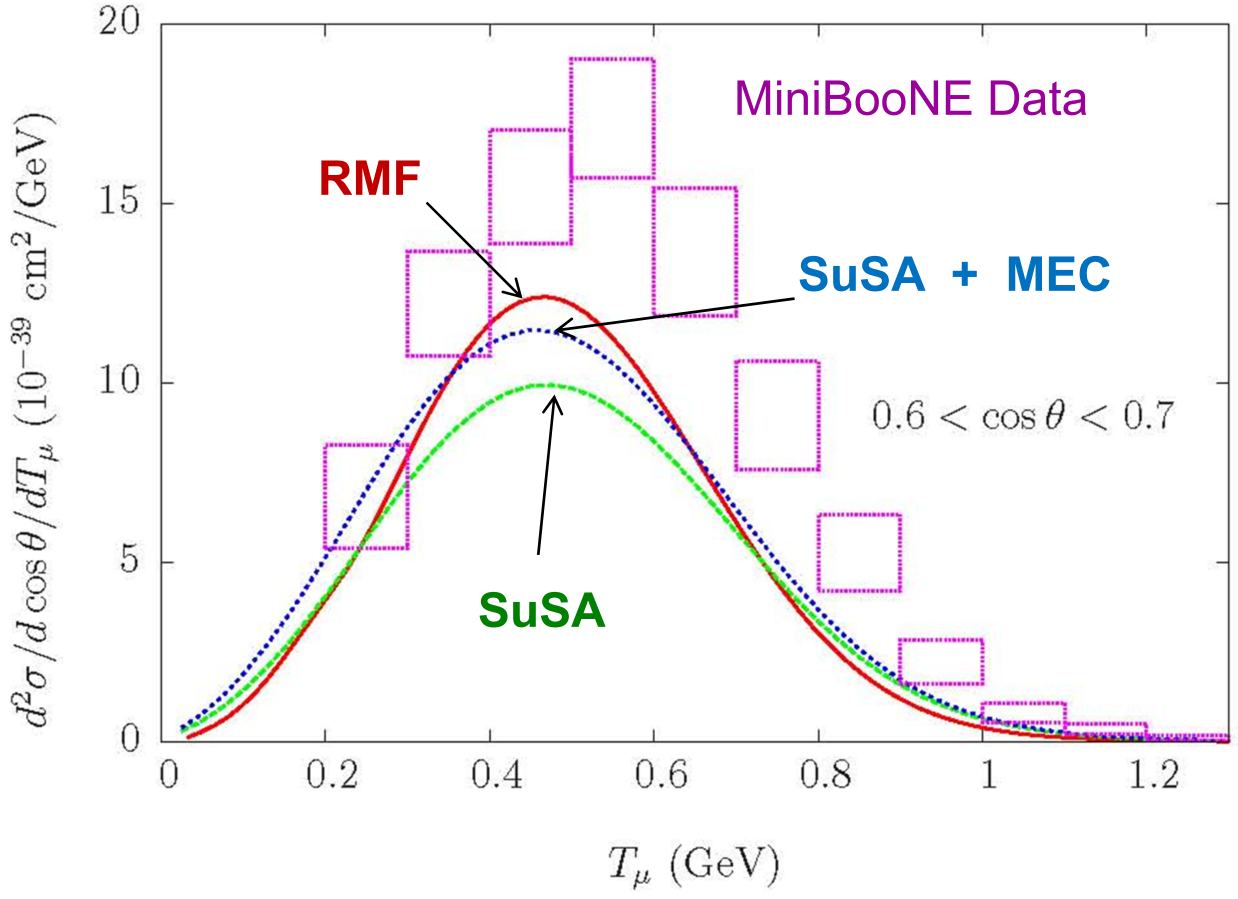


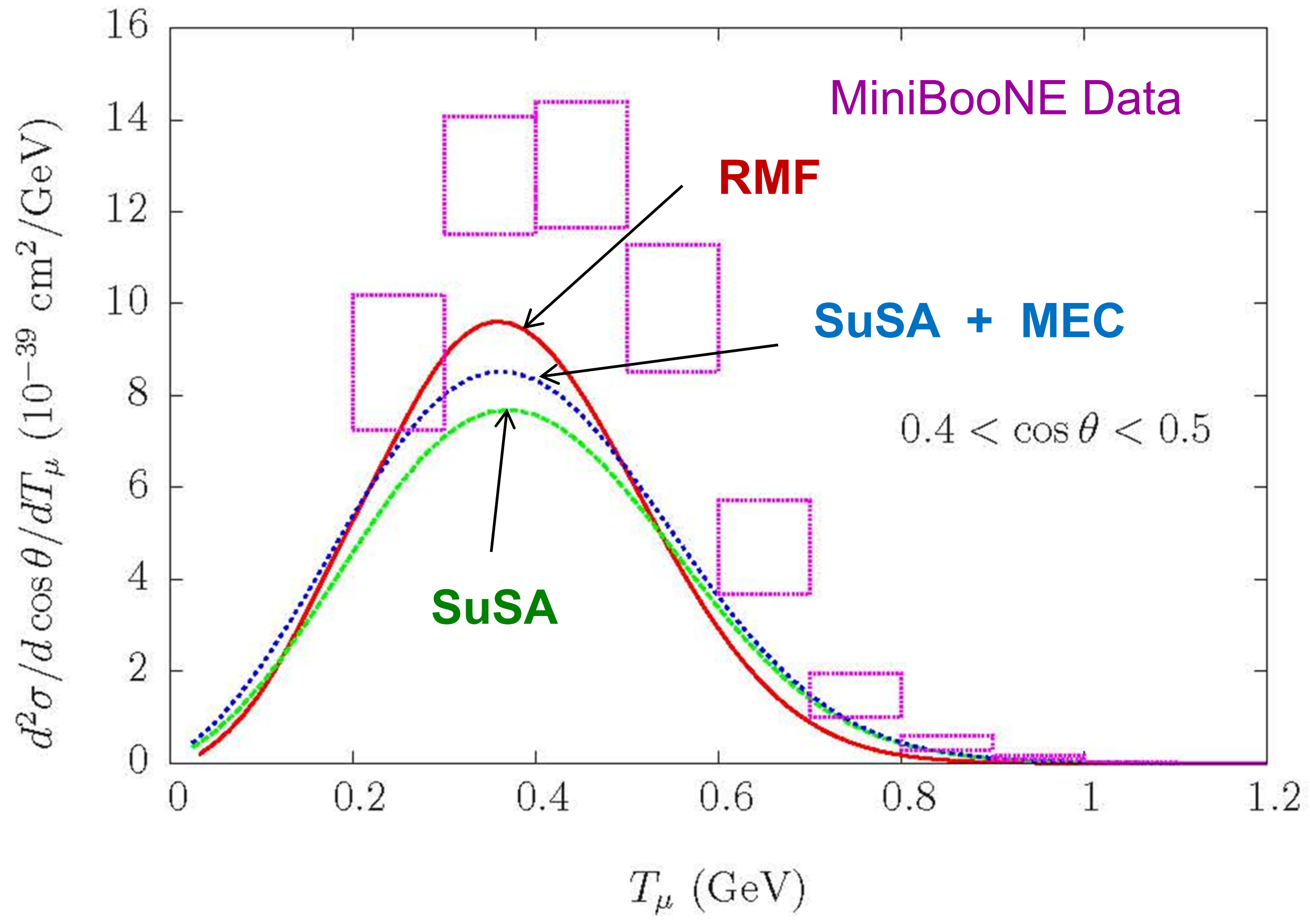
q_{QE} = 1736 MeV/c

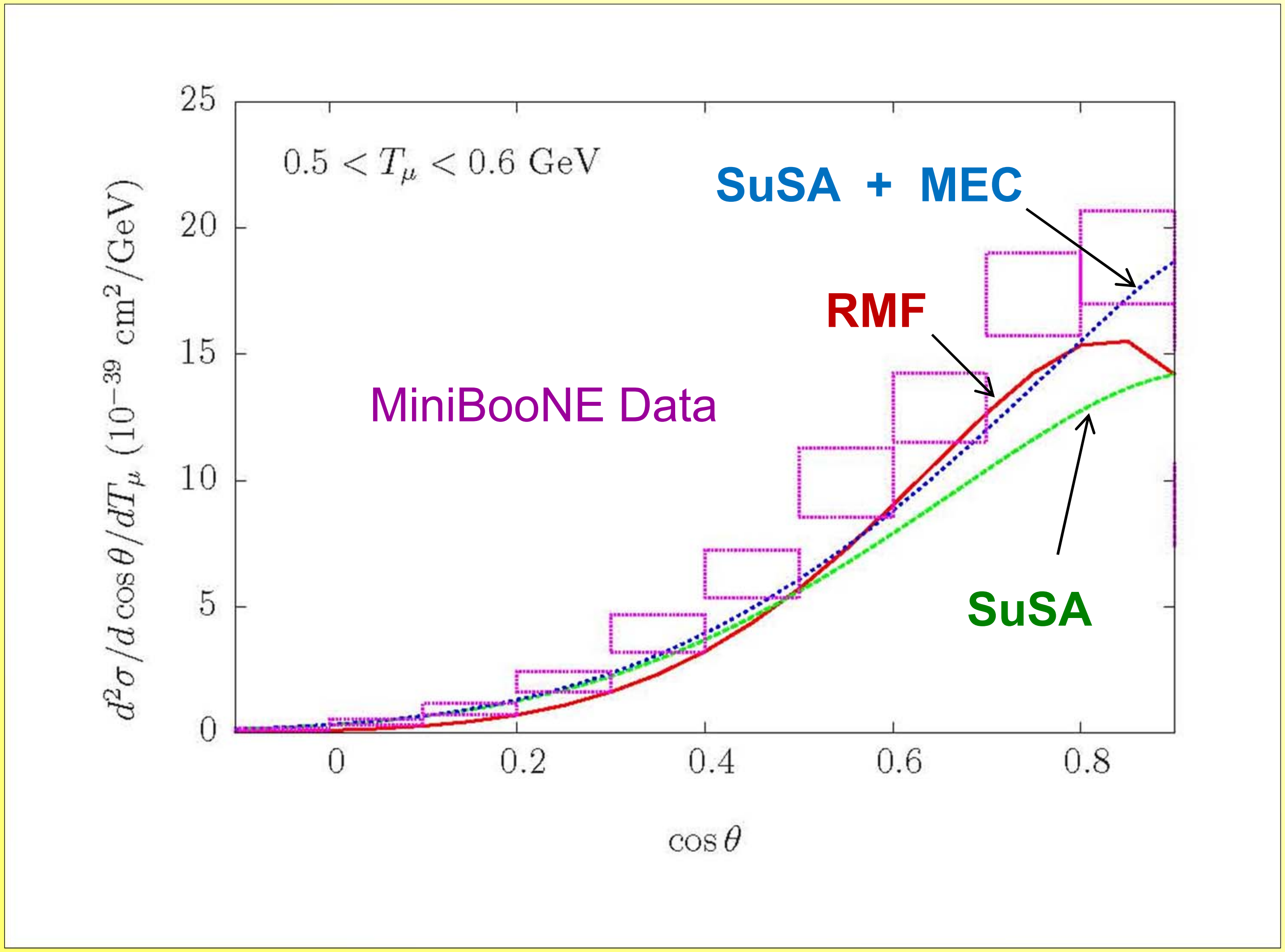


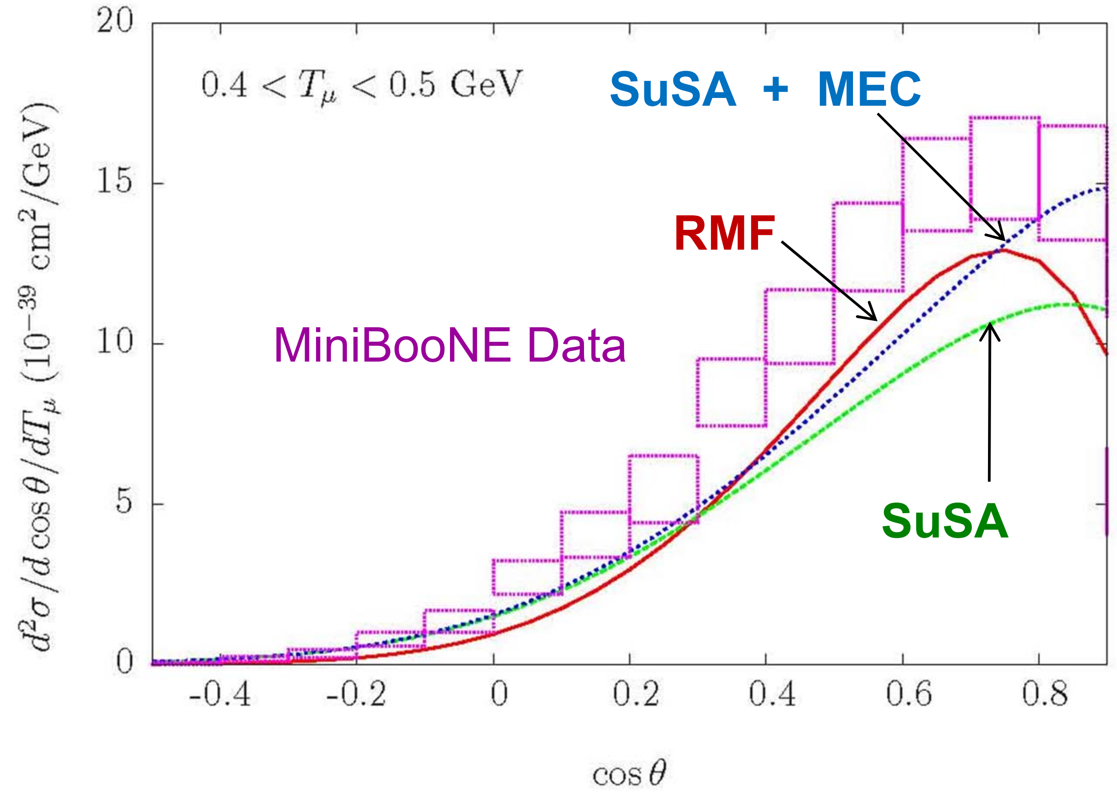
MiniBooNE Data

← **SuperScaling Analysis (SuSA)**


MiniBooNE Data
RMF
SuSA + MEC
SuSA
$0.6 < \cos\theta < 0.7$
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
T_μ (GeV)
0
5
10
15
20
0
0.2
0.4
0.6
0.8
1
1.2


MiniBooNE Data
RMF
SuSA + MEC
SuSA
$0.4 < \cos\theta < 0.5$
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
T_μ (GeV)
0
2
4
6
8
10
12
14
16
0
0.2
0.4
0.6
0.8
1
1.2


$0.5 < T_\mu < 0.6$ GeV
SuSA + MEC
RMF
MiniBooNE Data
SuSA
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
$\cos\theta$
0
5
10
15
20
25
0
0.2
0.4
0.6
0.8

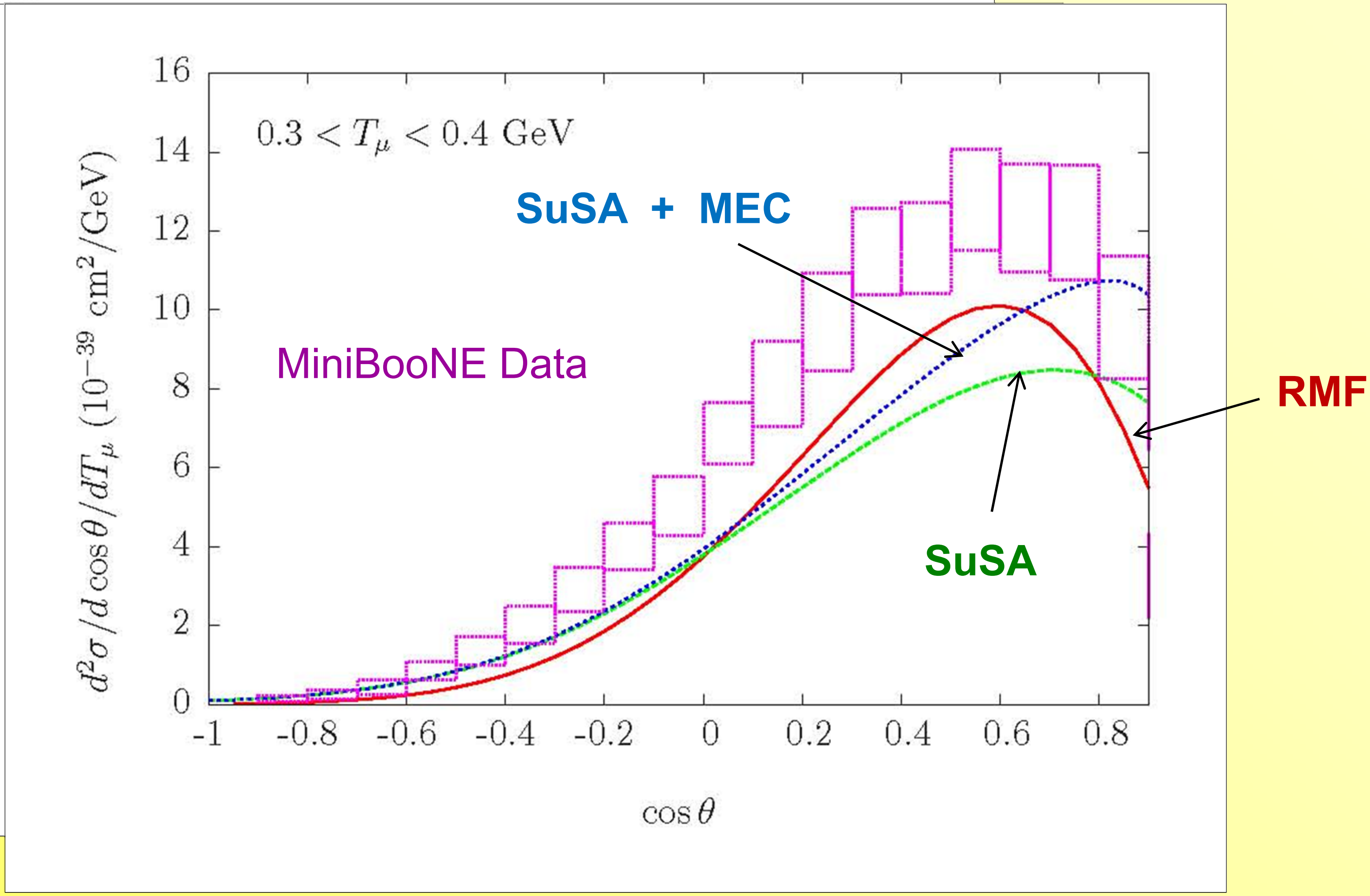


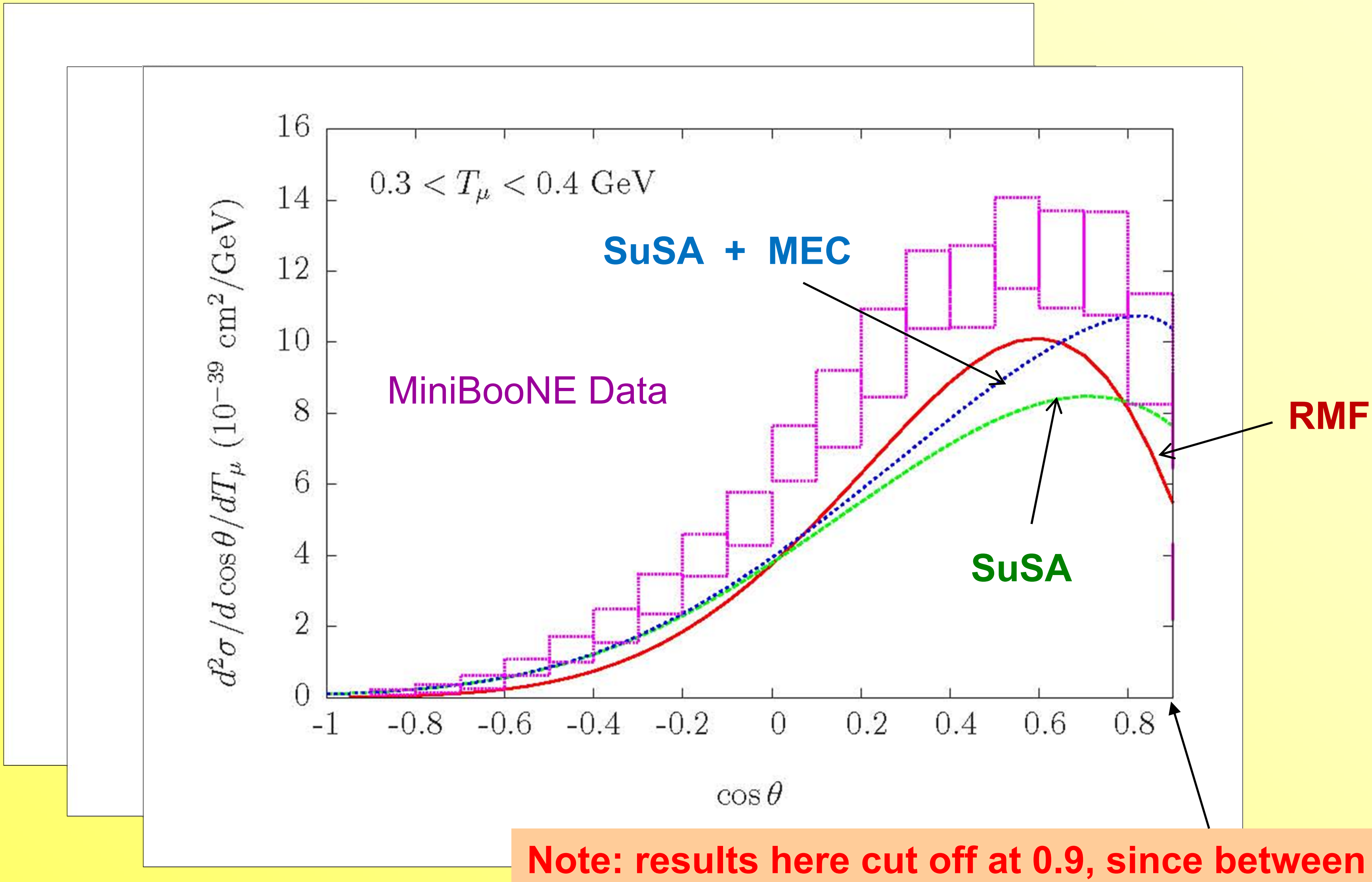
SuSA + MEC

RMF

MiniBooNE Data

SuSA


0.3 < T_μ < 0.4 GeV
SuSA + MEC
MiniBooNE Data
RMF
SuSA
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
16
14
12
10
8
6
4
2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
$\cos\theta$



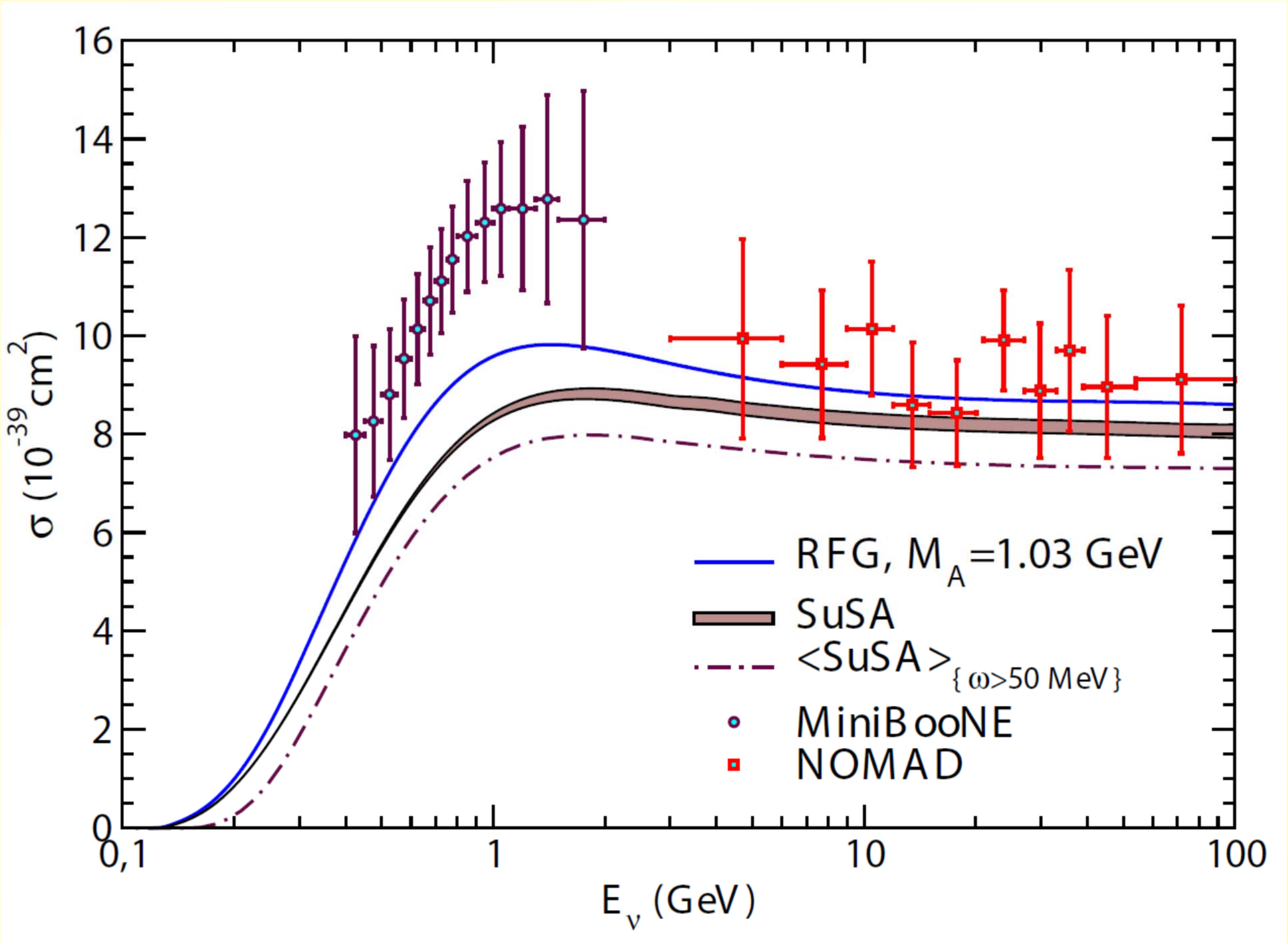
SuSA + MEC

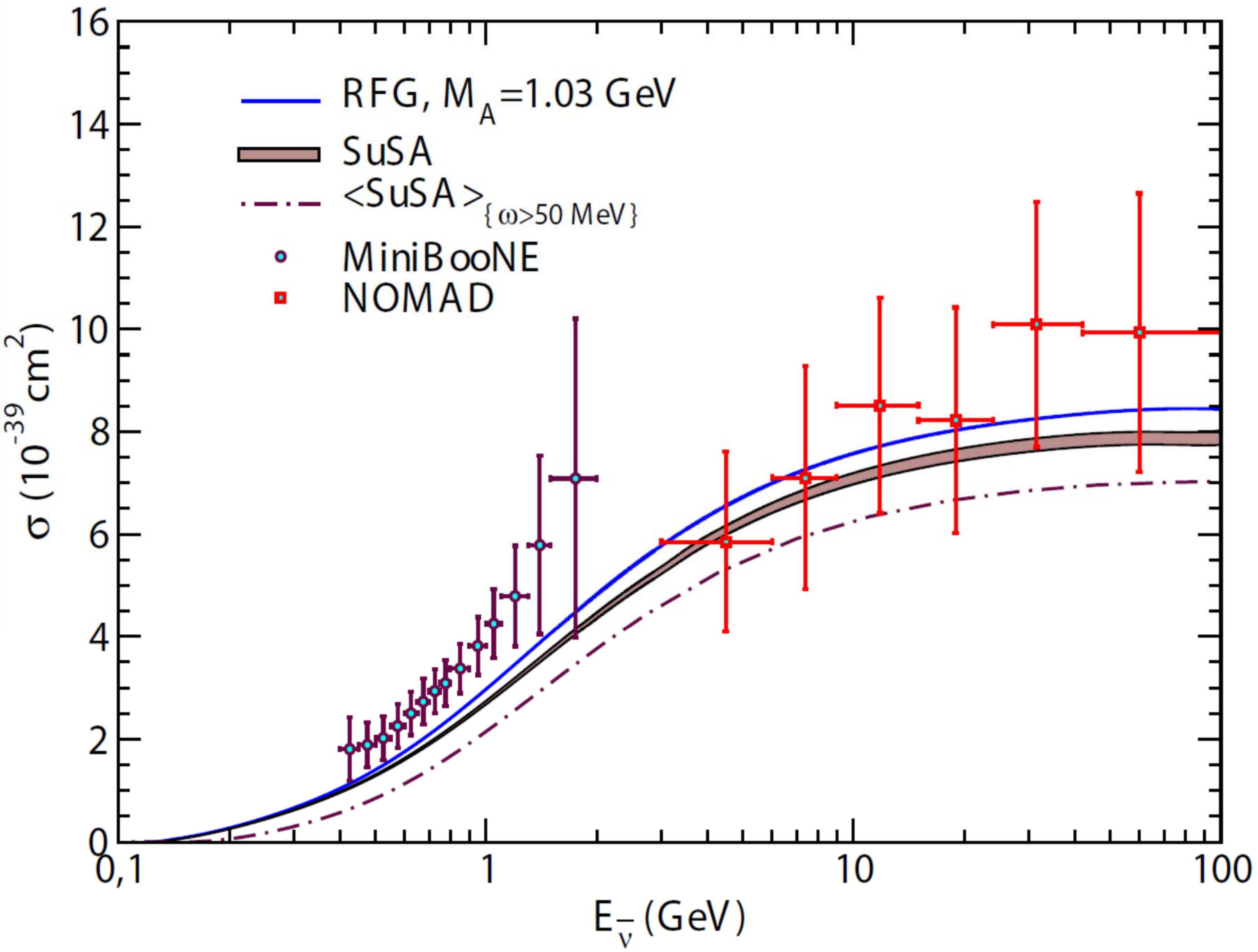
MiniBooNE Data

RMF

SuSA

Note: results here cut off at 0.9, since between 0.9 and 1 roughly ½ of the cross section arises from excitations below 50 MeV


16
14
12
10
8
6
4
2
0
$\sigma\,(10^{-39}\mathrm{cm}^2)$
0,1
1
10
100
E_ν (GeV)
RFG, M_A=1.03 GeV
SuSA
$\langle$SuSA$\rangle_{\{\omega>50\ \mathrm{MeV}\}}$
MiniBooNE
NOMAD


RFG, M_A=1.03 GeV
SuSA
<SuSA>$_{\{\omega>50\ \mathrm{MeV}\}}$
MiniBooNE
NOMAD
σ (10^{-39} cm^2)
$E_{\bar{\nu}}$ (GeV)
0
2
4
6
8
10
12
14
16
0,1
1
10
100

Summary here:

4. MEC effects are significant (and should be modeled relativistically).
5. Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail at the 25% level or so to reproduce the data, while for inclusive scattering RMF theory is much better.

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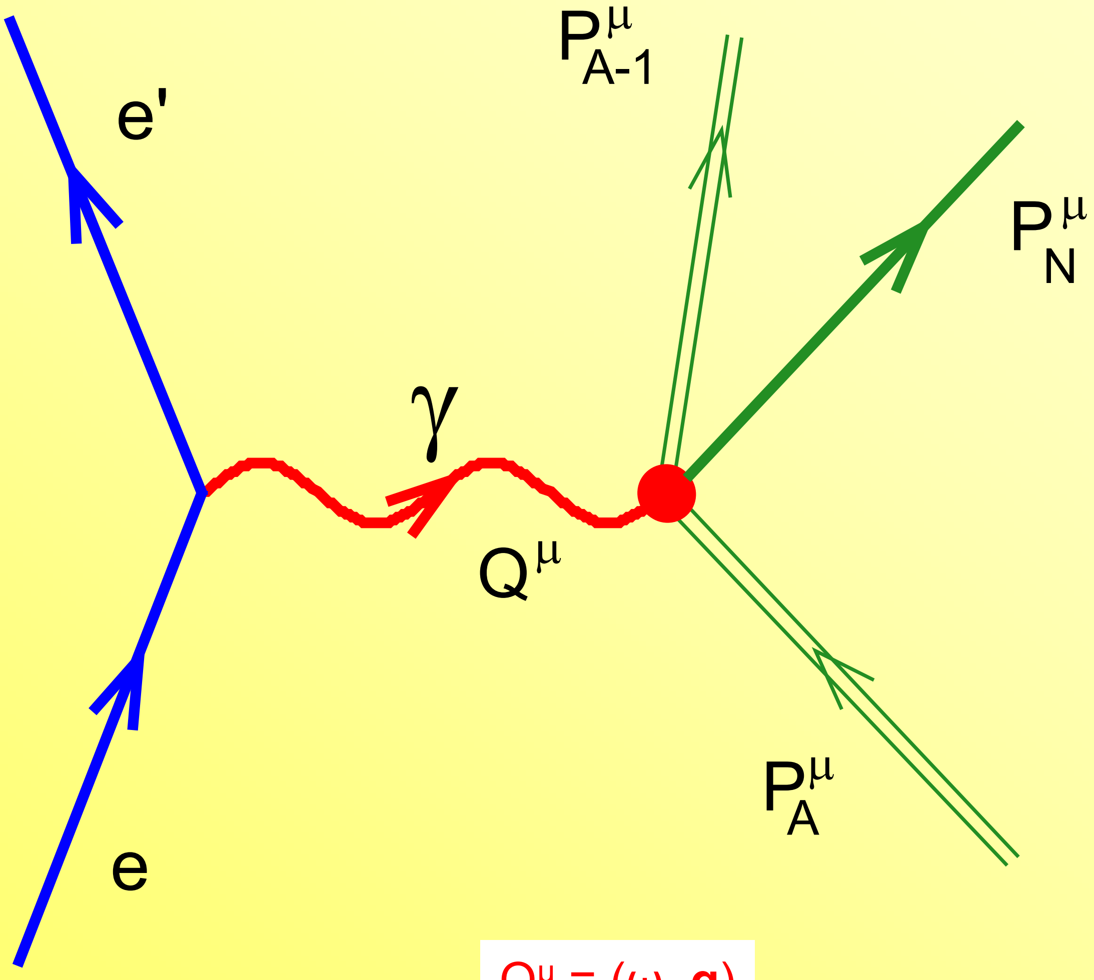
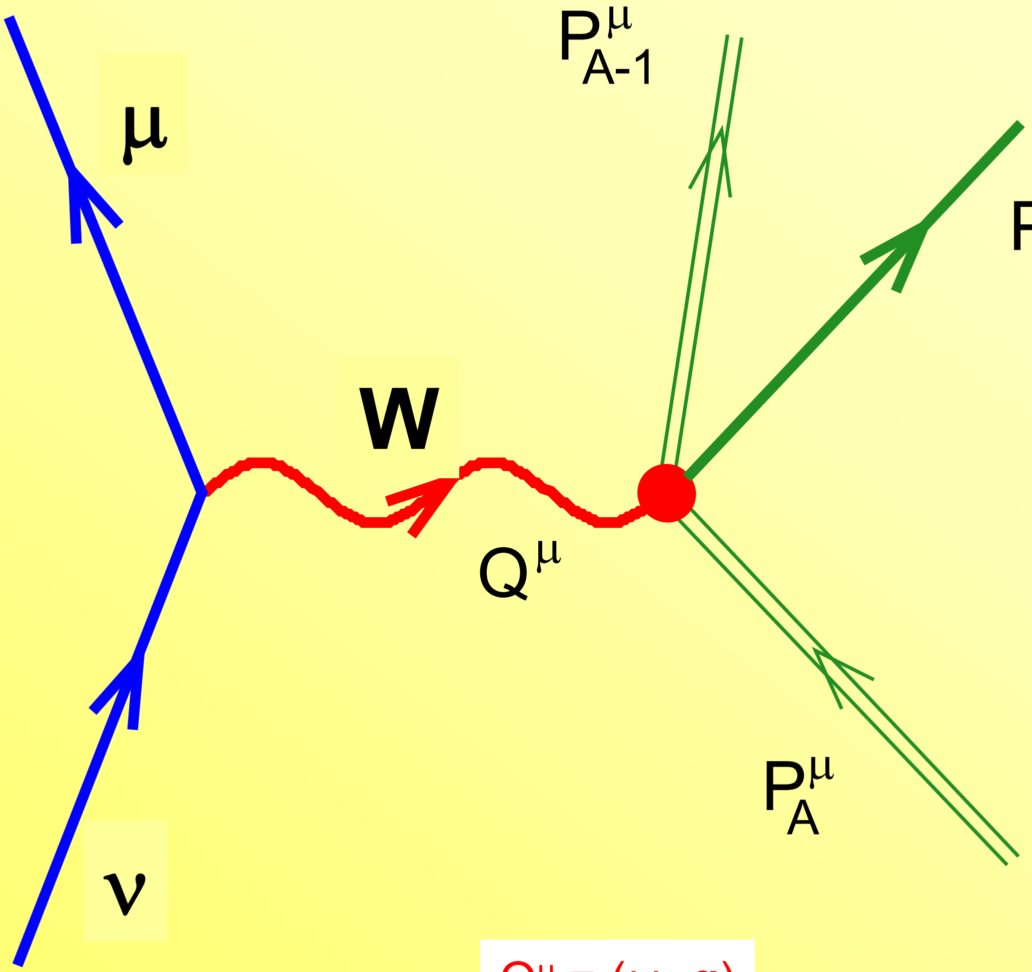
Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^{\mu} = (\omega, \mathbf{q})$

Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^{\mu} = (\omega, \mathbf{q})$

… or by (ν,μN)

The daughter nucleus has 4-momentum

$$P^{\mu}_{A-1} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P^{\mu}_{A} - P^{\mu}_{N}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an “excitation energy” (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M^0_{A-1})^2 + p^2}$$

where

$$M^0_{A-1} = M^0_A - m_N + E_s$$

with E_s the separation energy and M^0_{A-1} the daughter rest mass

Energy conservation gives

$$\begin{aligned} M_A^0 + \omega &= E_N + E_{A-1} \\ &= \sqrt{m_N{}^2 + p_N{}^2} + E_{A-1}^0 + \mathcal{E} \\ &= \sqrt{m_N{}^2 + (\mathbf{q}+\mathbf{p})^2} + \sqrt{(M_{A-1}^0)^2 + p^2} + \mathcal{E} \end{aligned}$$

which can be turned around to yield an expression for the excitation energy:

$$\mathcal{E} = M_A^0 + \omega - \sqrt{(M_{A-1}^0)^2 + p^2} - \sqrt{m_N{}^2 + q^2 + p^2 + 2pq\cos\theta}$$

One can let the angle between *p* and *q* vary over all values and impose the constraints

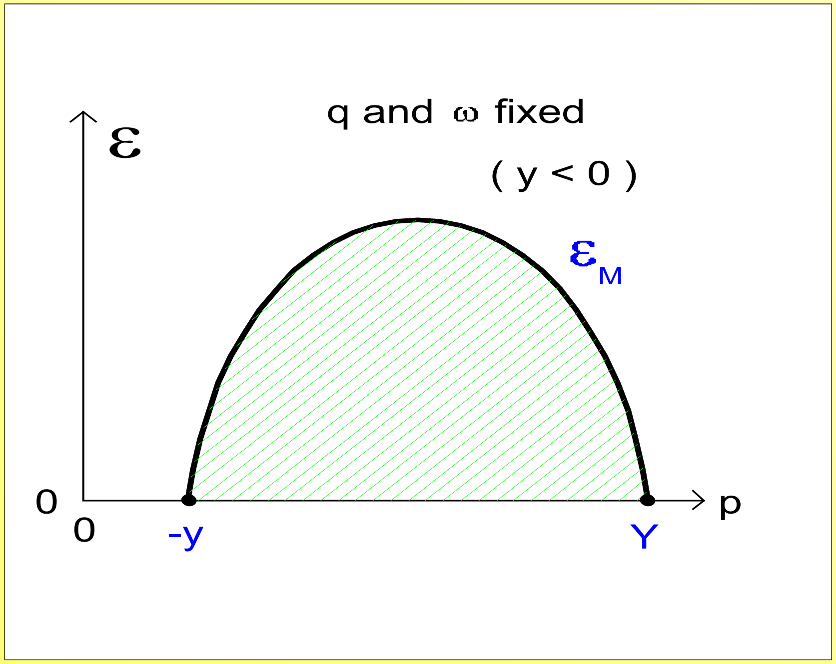
$$p \geq 0$$

$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When

$$\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$$

one finds



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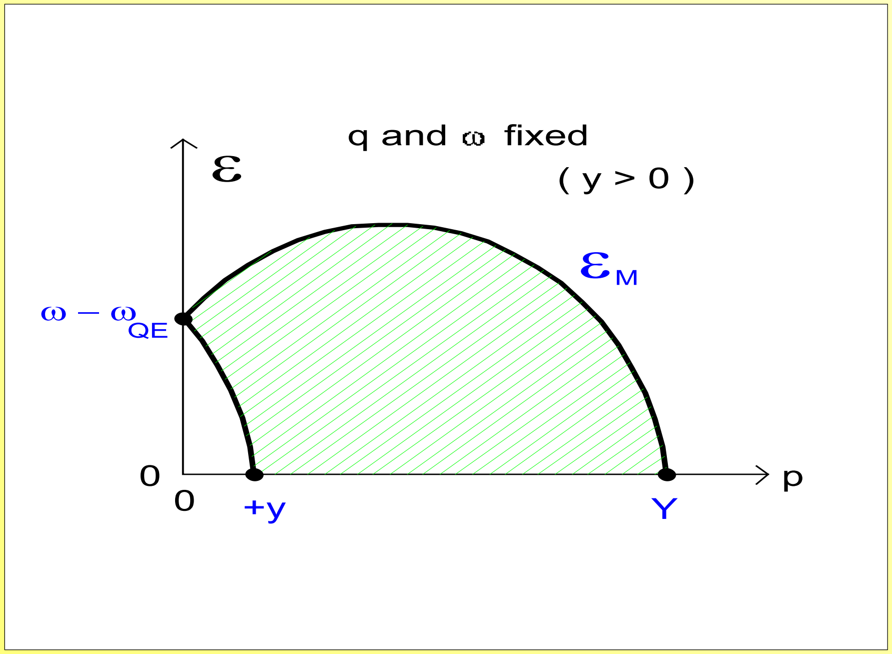
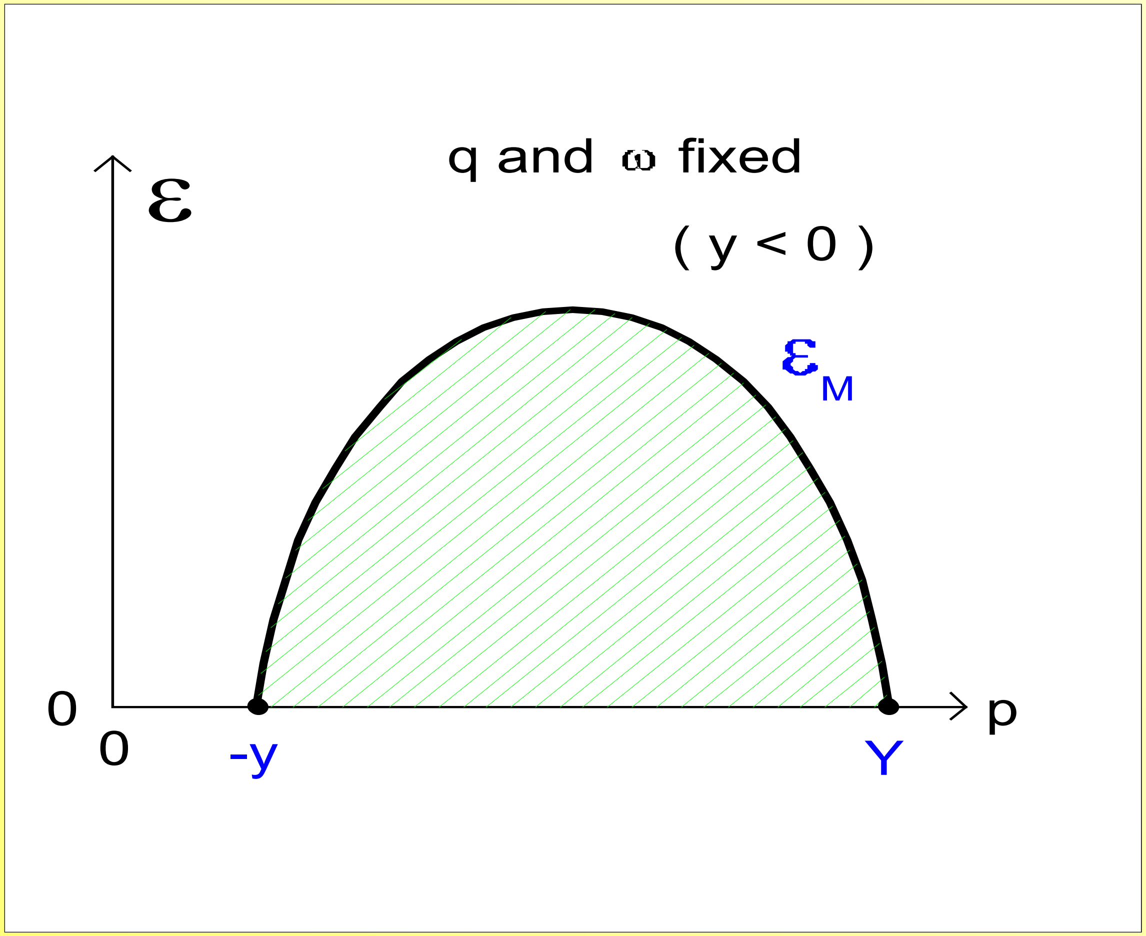
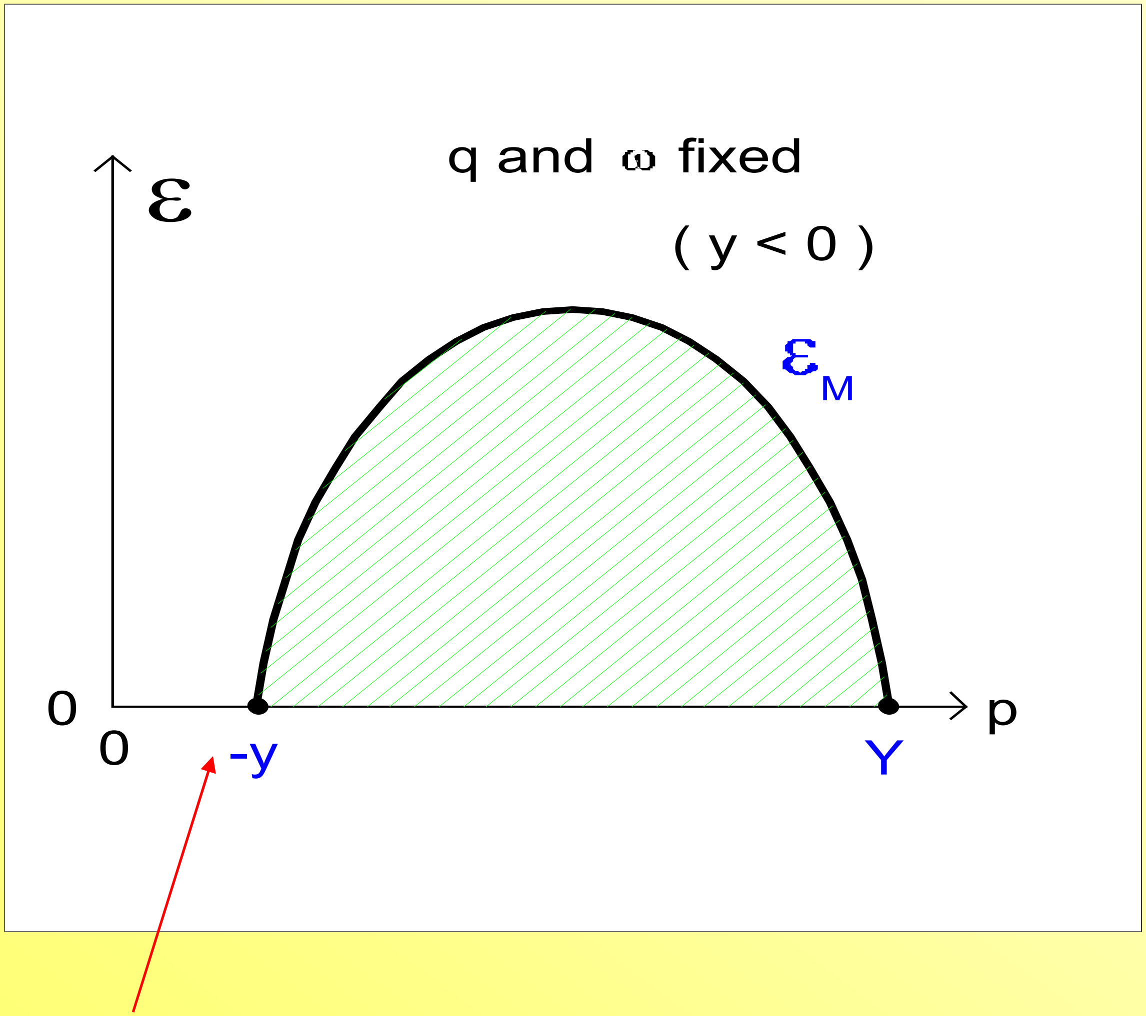
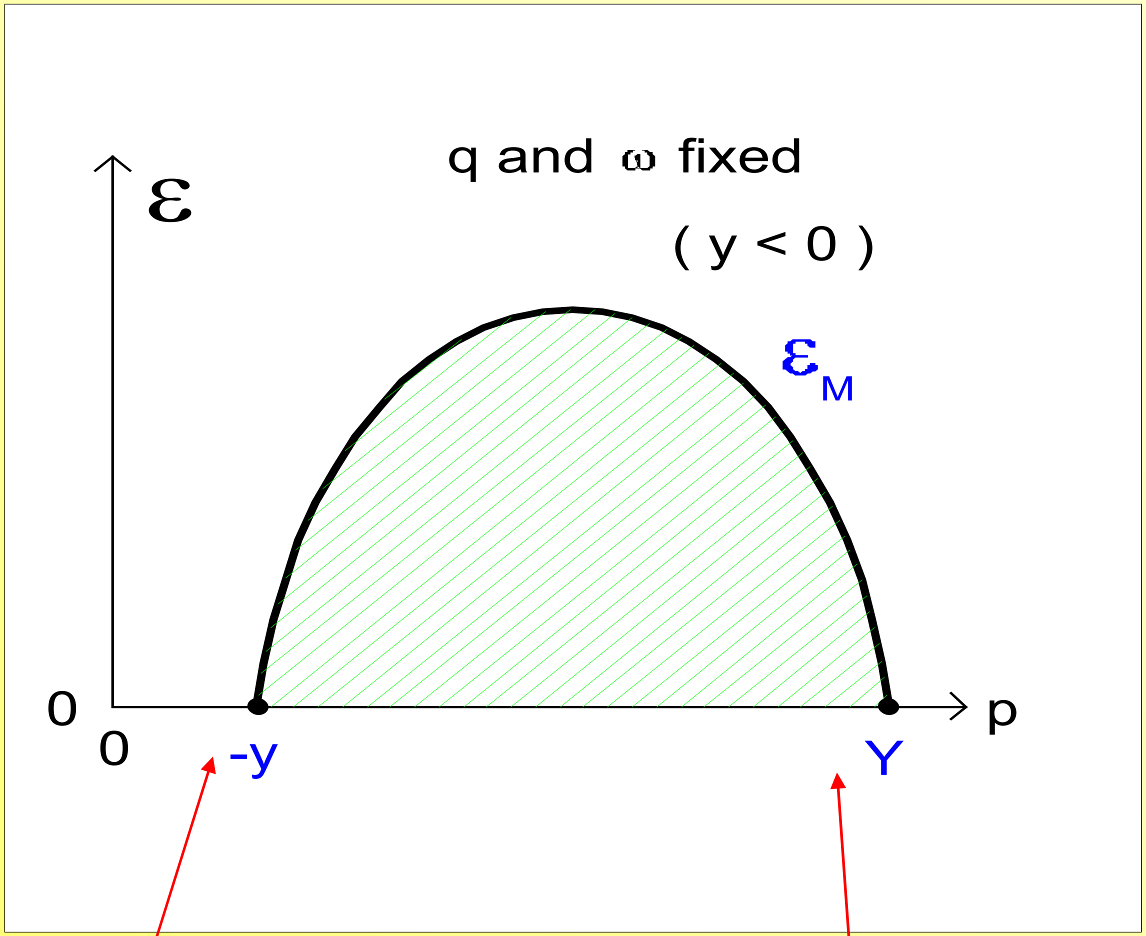



q and ω fixed
(y < 0)
ε
ε_M
0
0
-y
Y
p



The semi-inclusive cross section is

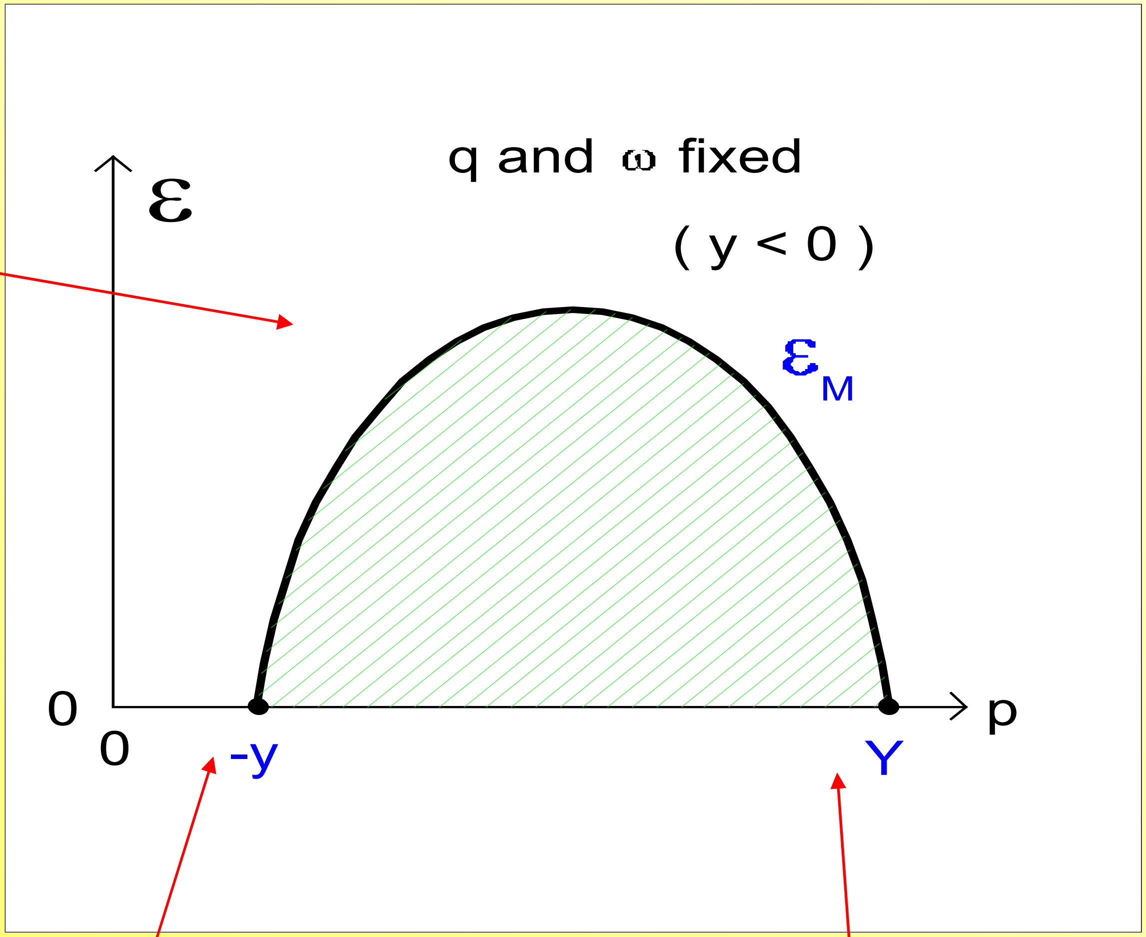
typically largest at small **p** and ε



The semi-inclusive cross section is typically largest at small **p** and $\mathcal{E}$

… and is very small at large **p** and small $\mathcal{E}$

For given **y<0** the region at small **p**, but high $\mathcal{E}$ is inaccessible



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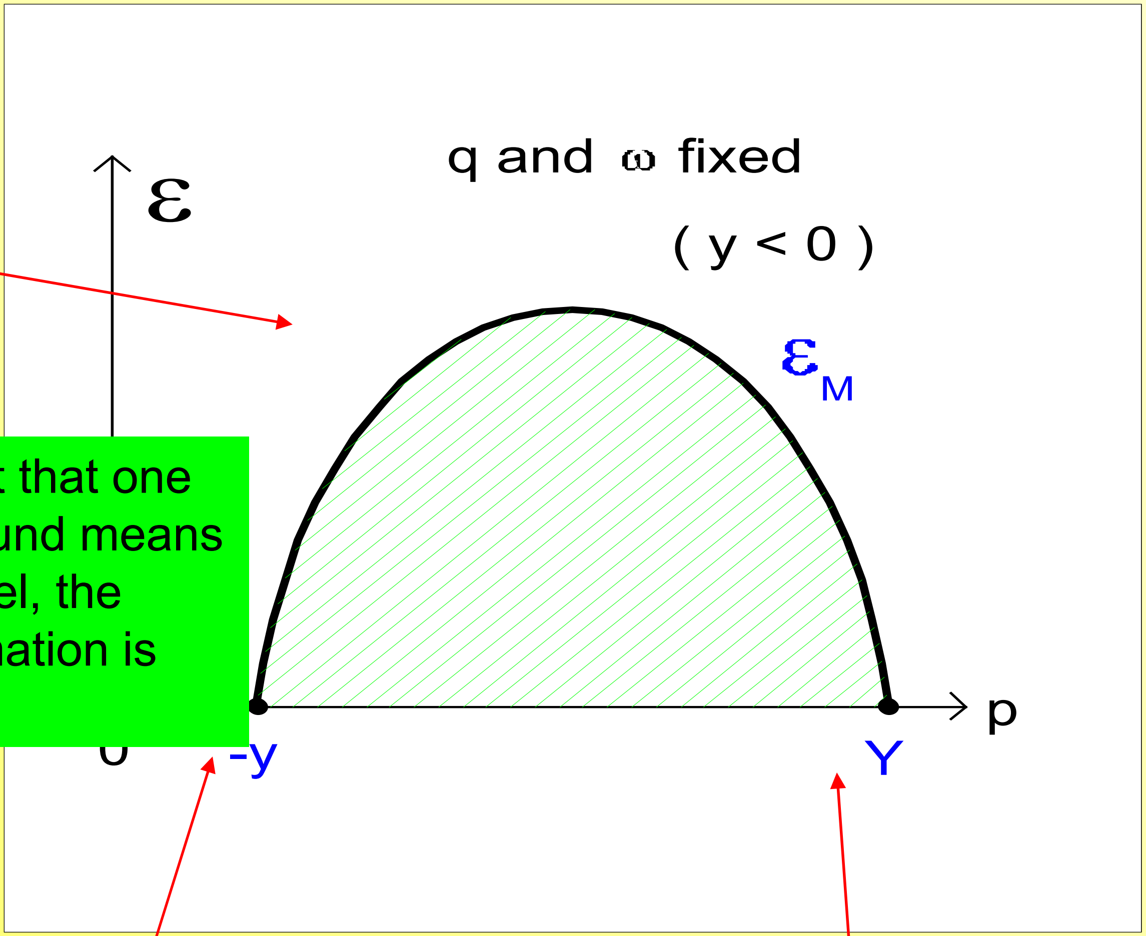

Note that the fact that one has an upper bound means that, at some level, the closure approximation is in error

The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

For given **y<0**
the region at
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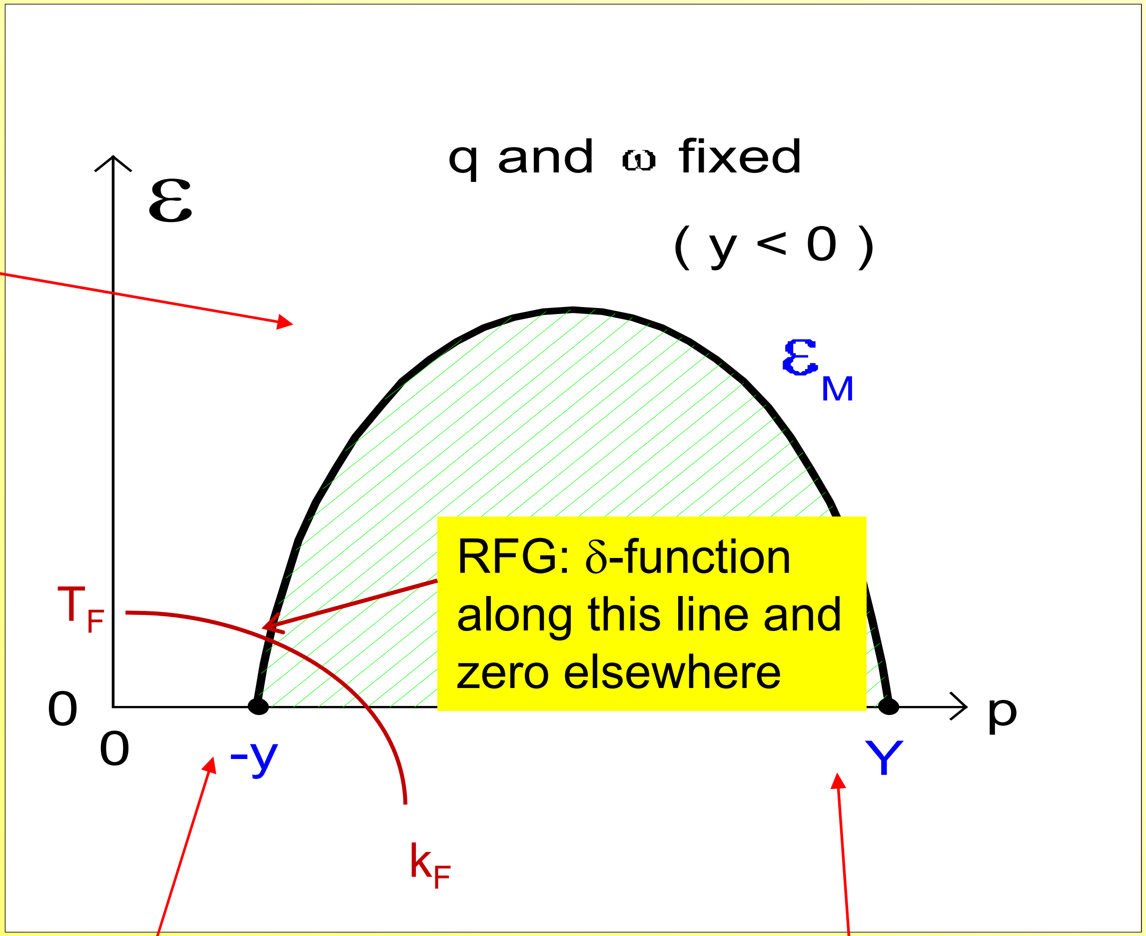

RFG: δ-function
along this line and
zero elsewhere

The semi-inclusive cross section is
typically largest at small **p** and ε

… and is very small at large **p**
and small ε

Given q and ω, and given the missing energy and momentum, one has fixed the 3-momentum p_N and angle θ of the outgoing nucleon.

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And so, just because a specific model does well for **inclusive scattering** (which involves integrals over the regions shown above, summed over appropriate flavors of nucleons, and corrected for double-counting), that model may fail badly for **semi-inclusive scattering**: the strength in the missing energy/momentum plane, and hence the final-state nucleon kinematics, may be wrong. For example, the RFG is infinitely bad almost everywhere.

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This means that adding on final-state interactions to a model that is only suited to inclusive scattering can incur significant errors; a realistic one-particle spectral function should be used for modeling semi-inclusive reactions. For reactions requiring the specification of two or more particles one must go beyond the existing spectral functions.

Summary:

1. Any model that does not succeed for electron scattering is very unlikely to be valid for neutrino reactions.
2. Relativistic effects from kinematics and boost factors are essential.
3. Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail at the 25% level or so to reproduce the data, while for inclusive scattering RMF theory is much better.
4. MEC effects are significant (and should be modeled relativistically).
5. Inclusive "QE" model CC neutrino cross sections fall short of the MiniBooNE data, even when MEC effects are included, whereas for NOMAD kinematics they are much better.
6. While the models discussed here are not too bad for inclusive scattering, they are not suited to semi-inclusive scattering for all choices of missing energy/momentum.
7. For semi-inclusive reactions (detection of one final-state hadron) relativistic one-particle spectral functions are better, although they also involve approximations.
8. For reactions requiring detection of two or more particles one needs relativistic two-particle spectral functions!

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